Optimal figure of merit of low-dissipation quantum refrigerators

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Abstract

We establish a finite-time external field-driven quantum tricycle model. Within the framework of slow driving perturbation, the perturbation expansion of heat in powers of time can be derived during the heat exchange processes. Employing the method of Lagrange multiplier, we optimize the cooling performance of the tricycle by considering the cooling rate and the figure of merit, which is the product of the coefficient of performance and cooling rate, as objective functions. Our findings reveal the optimal operating region of the tricycle, shedding light on its efficient performance.

1. The control protocol of a finite-time quantum tricycle

The working substance is a two level system (TLS) with time-dependent Hamiltonian $H(t)=\hbar\omega_v(t)\sigma_z/2$, where $\omega_v(t)$ is the energy splitting at time t. A weak coupling between the system and reservoir is considered. The density operator $\rho(t)$ of the TLS evolves according to the Markovian master equation, i,e.,

$$d\rho(t)/dt=\mathcal{L}_v(t)\left[\rho(t)\right],$$

where the generator $\mathcal{L}_v(t)$ represents the quantum Liouvillian superoperator. By introducing the dimensionless time-rescaled parameter $s=t/\tau_v$, the equation can be rewritten as[1]

$$\tilde{\rho}(s)=\tilde{\rho}_{\text{eq},v}(s)+\frac{1}{\tau_v}\tilde{\mathcal{L}}_v^{-1}(s)\frac{d}{ds}\left[\tilde{\rho}_{\text{eq},v}(s)\right],$$

where $\tilde{\mathcal{L}}_v^{-1}(s)$ is the Drazin inverse of $\tilde{\mathcal{L}}_v(s)$. By applying Alicki's definition of heat and the first order perturbation, the amount of heat entering the system from bath during the interval would be

$$Q_v=Q_v^0+Q_v^1=\beta_v^{-1}\left(\Delta S_{\text{eq},v}+\Sigma_v/\tau_v\right),$$

where the first order irreversible corrections of heat can be written by

$$\Sigma_v=\beta_v\int_0^1 ds\text{Tr}\left[\tilde{H}(s)\frac{d}{ds}\left\{\tilde{\mathcal{L}}_v^{-1}(s)\frac{d}{ds}\left[\tilde{\rho}_{\text{eq},v}(s)\right]\right\}\right].$$

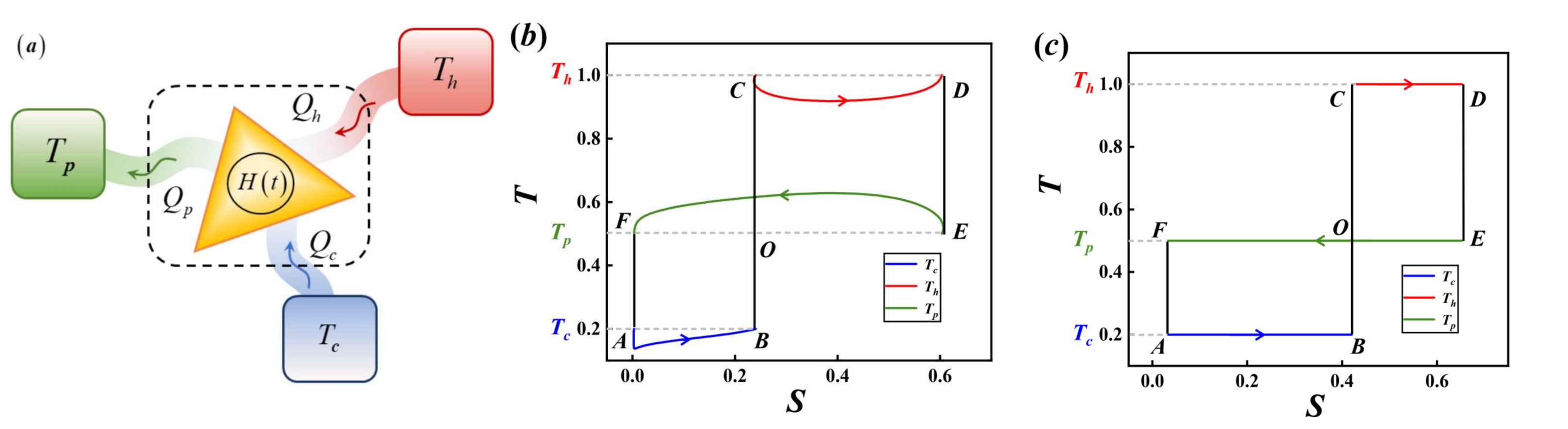

Fig.1. (a) Schematic representation of a quantum tricycle. (b) The temperature-entropy diagram of a FTQTC. (c) The temperature-entropy diagram of a reversible quantum tricycle.

The detail of the control protocols of the quantum tricycle are designed as follows:

A→B: Heat exchange with reservoir c

$$\omega_c(t)=\delta_c\left[\cos\pi(t/\tau_c)+\zeta_c\right]$$

B→C: Diabatic expansion

$$\omega_c(\tau_c)\to\omega_h(0)=(T_h/T_c)\omega_c(\tau_c)$$

C→D: Heat exchange with reservoir h

$$\omega_h(t)=\delta_h[\cos(\pi t/\tau_h)+\zeta_h]$$

D→E: First diabatic expansion p

$$\omega_h(\tau_h)\to\omega_p(0)=(T_p/T_h)\omega_h(\tau_h)$$

E→F: Heat exchange with reservoir

$$\omega_p(t)=\delta_p[\cos(\pi(1-t)/\tau_p)+\zeta_p]$$

F →A: Second diabatic expansion

$$\omega_p(\tau_p)\to\omega_c(0)=(T_c/T_p)\omega_p(\tau_p)$$

2. The relationships between the amplitude and the displacement

$$\omega_c(\tau_c)=(T_c/T_h)\omega_h(0)$$
$$\omega_h(\tau_h)=(T_h/T_p)\omega_p(0)$$
$$\omega_p(\tau_p)=(T_p/T_c)\omega_c(0)$$

$$\frac{\delta_c(\zeta_c-1)}{\delta_h(\zeta_h+1)}=\frac{T_c}{T_h},\quad \zeta_p=\frac{1+\zeta_c\zeta_h}{\zeta_c+\zeta_h},$$

$$\frac{\delta_h(\zeta_h-1)}{\delta_p(\zeta_p-1)}=\frac{T_h}{T_p},\quad \delta_h=\frac{T_h(\zeta_c-1)}{T_c(1+\zeta_h)}\delta_c,$$

$$\frac{\delta_p(\zeta_p+1)}{\delta_c(\zeta_c+1)}=\frac{T_p}{T_c}.\quad \delta_p=\frac{T_p(\zeta_c+\zeta_h)}{T_c(1+\zeta_h)}\delta_c.$$

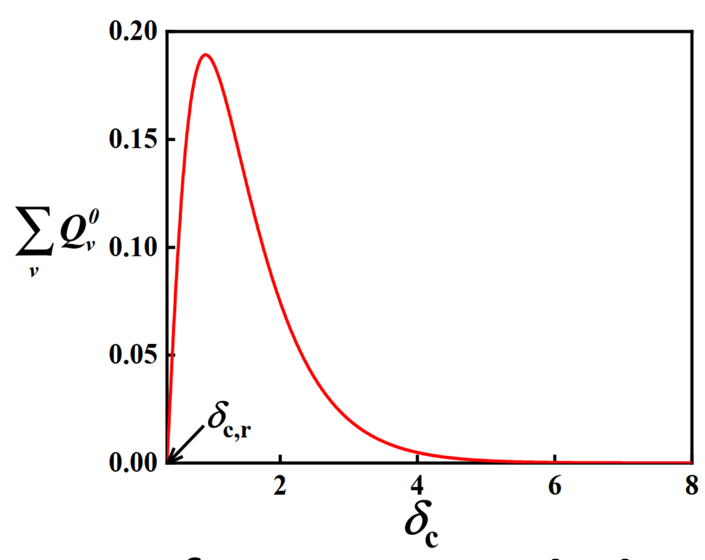

Fig.2. The curves of the sum of the zeroth order approximation of heat varying with the amplitude frequency.

3. Performance optimization in the slow-driving regime

We can generate the optimal curve of the cooling rate varying with the COP for given frequency exponent α. The frequency α exponent determines the spectral density

$$J(\omega_v(t))\propto[\omega_v(t)]^\alpha$$

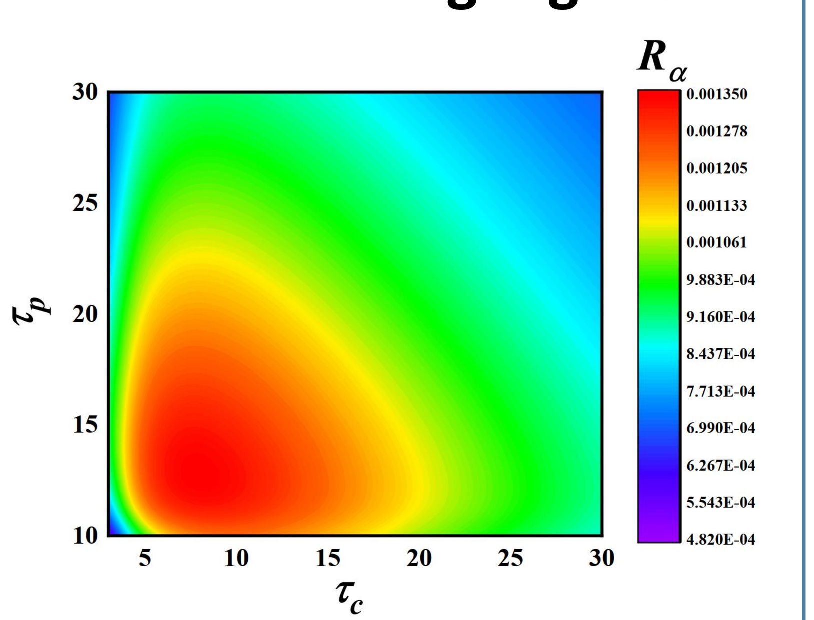

Fig.3. Plot of the cooling rate R with respect to time τ_c and τ_p.

4. Result

$$R=\frac{Q_c}{\tau_c+\tau_h+\tau_p}=\frac{T_c\left[\Delta S_{eq,c}+\Sigma_c/\tau_c\right]}{\tau_c+\tau_h+\tau_p}.$$

$$\psi=\frac{Q_c}{Q_h}=\frac{T_c\left[\Delta S_{eq,c}+\Sigma_c/\tau_c\right]}{T_h\left[\Delta S_{eq,h}+\Sigma_h/\tau_h\right]},$$

$$L(\tau_c,\tau_h,\tau_p)=R+\lambda_1\psi+\lambda_2\left(Q_c+Q_h+Q_p\right),$$

$$\Delta S_{eq,h}\frac{\tau_h^2}{\Sigma_h}+\Delta S_{eq,p}\frac{\tau_p^2}{\Sigma_p}+\Delta S_{eq,c}\frac{\tau_c^2}{\Sigma_c}+2\left(\tau_c+\tau_h+\tau_p\right)=0.$$

$$\tau_h=-\frac{T_h\Sigma_h}{T_p\left(\Delta S_{eq,p}+\Sigma_p/\tau_p\right)+T_c\left(\Delta S_{eq,c}+\Sigma_c/\tau_c\right)+T_h\Delta S_{eq,h}}.$$

The figure of merit $\chi=\psi R$ can be commonly employed as a target function for optimizing the performance of refrigerators

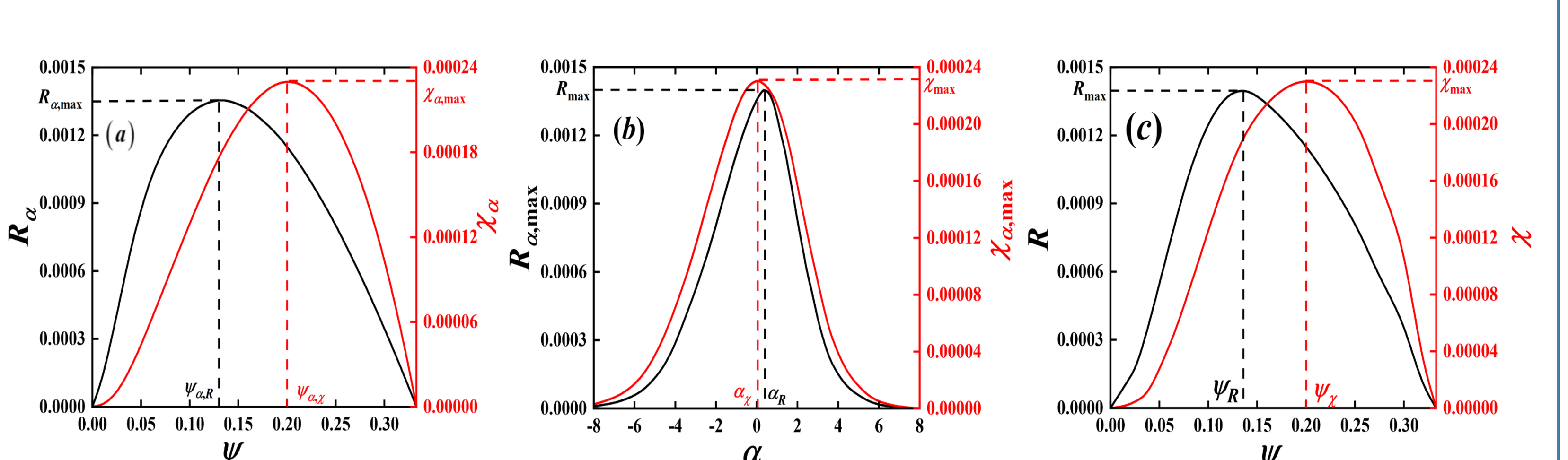

Fig.4. (a) Plot of the cooling rate and figure of merit varying with the COP. (b) Plot of the cooling rate and figure of merit varying with the α. (c) The optimum characteristic curves of the cooling rate and the figure of merit varying with COP .

References: [1] V. Cavina, A. Mari, and V. Giovannetti, Phys. Rev. Lett. 119, 050601 (2017).
[2] J. Y. Chen, S. H. Xia, J. C. Chen, and S. H. Su, Performance optimization of the finite-time quantum tricycle[J]. (Under review)

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