A definition of quantum asymptotic phase function for analyzing quantum synchronization from the Koopman operator viewpoint

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In this poster, we propose a fully quantum-mechanical definition of the asymptotic phase for quantum nonlinear oscillators, a fundamental quantity in the theory of classical nonlinear oscillations.

Synchronization of rhythmic dynamical systems is ubiquitously observed in science and technology, including chemical oscillations, biological rhythms, electrical oscillations, and mechanical vibrations. Recent developments in experimental methodologies have already reached micro- and nano-scales and will soon enter the quantum regime, and the demand for theoretical studies of quantum synchronization is rapidly growing [1]. Several novel features in quantum synchronization have been theoretically analyzed recently, such as multiple-phase locking [2], which are explicit quantum effects arising from the discrete nature of the energy spectrum.

In analyzing synchronization properties of classical nonlinear oscillators, the asymptotic phase of the oscillator is essentially important. It provides the basis for phase reduction, a standard theoretical method for analyzing systems of nonlinear oscillators. It enables us to describe the nonlinear multi-dimensional dynamics of the oscillator by a simple phase equation and has been extensively used to unveil universal synchronization properties of coupled oscillator systems. The collective synchronization transition in a population of coupled oscillators (the Kuramoto model) is the most prominent result predicted by the theory, and the wobbling of the Millennium footbridge in London caused by synchronization of many pedestrians is a well-known real-world example of this universal phenomenon [3].

In our previous study [4], we formulated the phase reduction theory for quantum nonlinear oscillators in the semiclassical regime where the system is represented by a phase-space state fluctuating along a classical trajectory due to small quantum noise. However, this theory is not applicable in the strong quantum regime, because we cannot define the asymptotic phase of the system by using the classical deterministic trajectory. In this study, to overcome this fundamental difficulty, we introduce the asymptotic phase of quantum nonlinear oscillators in a fully quantum-mechanical way, thereby extending its applicability to the strong quantum regime and enabling analysis of nontrivial quantum synchronization phenomena.

We propose a fully quantum-mechanical definition of the asymptotic phase for quantum nonlinear oscillators, a fundamental quantity in the theory of classical nonlinear oscillations. Our definition of the asymptotic phase is based on the eigenoperator of the adjoint Liouville superoperator of the open quantum system. It is inspired by the study on the asymptotic phase of classical stochastic oscillators by Thomas and Lindner [5], which is also natural from the recently developing Koopman-operator viewpoint on dynamical systems. We analyze a quantum van der Pol oscillator with the Kerr effect and show that our quantum asymptotic phase yields appropriate results in both semiclassical and strong quantum regimes [6, 7].

Quantum synchronization, a burgeoning topic at the boundary between quantum physics and nonlinear physics, is attracting much attention not only in pure and applied physics but also in information science, applied mathematics, and various engineering fields. The quantum asymptotic phase proposed in this study is generally applicable in the strong quantum regime and will serve as a fundamental quantity for characterizing quantum nonlinear oscillators and provide new insights into future applications of quantum synchronization in the evolving field of quantum technologies.

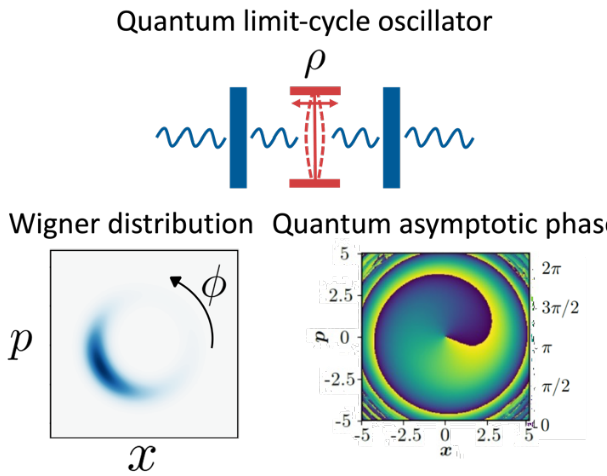

Fig. 1 Quantum limit-cycle oscillators and quantum asymptotic phase

References:

[1] T. E. Lee and H. R. Sadeghpour, "Quantum synchronization of quantum van der Pol oscillators with trapped ions," Phys. Rev. Lett. 111(23), 234101 (2013).

[2] N. Lörch, E. Amitai, A. Nunnenkamp, and C. Bruder, "Genuine quantum signatures in synchronization of anharmonic self-oscillators," Phys. Rev. Lett. 117(7), 073601 (2016).

[3] Y. Kuramoto, Chemical Oscillations, Waves, and Turbulence (Springer, Berlin, 1984).

[4] Y. Kato, N. Yamamoto, and H. Nakao, "Semiclassical phase reduction theory for quantum synchronization," Phys. Rev. Res. 1, 033012 (2019).

[5] P. J. Thomas and B. Lindner, "Asymptotic phase for stochastic oscillators" , Phys. Rev. Lett. 113, 254101 (2014).

[6] Y. Kato and H. Nakao, "A definition of the asymptotic phase for quantum nonlinear oscillators from the Koopman operator viewpoint," Chaos 32, 063133 (2022).

[7] Y. Kato and H. Nakao, "Quantum asymptotic phase reveals signatures of quantum synchronization," New J. Phys. 25 023012 (2023).