Surprising aspects of Lagrangian dispersion in supersonic turbulence

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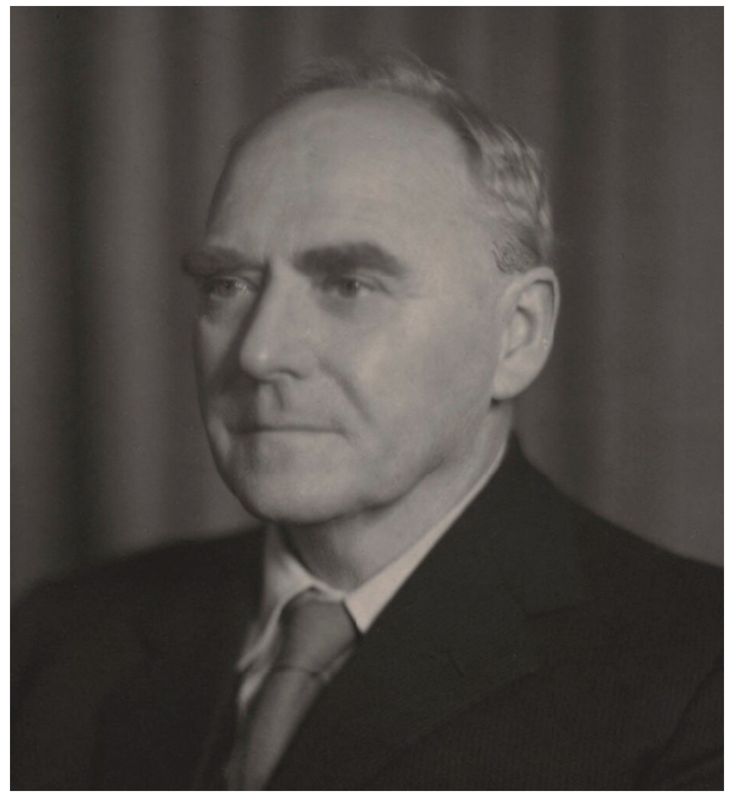
भारतीय विज्ञान संस्थान

Two (of many) pillars of turbulence

Taylor's Diffusion



G. I. Taylor

$$MSD(t) \sim \begin{cases} t^2 & \text{for} \quad t \ll T_L\,, \\ t & \text{for} \quad t > T_L\,. \end{cases}$$

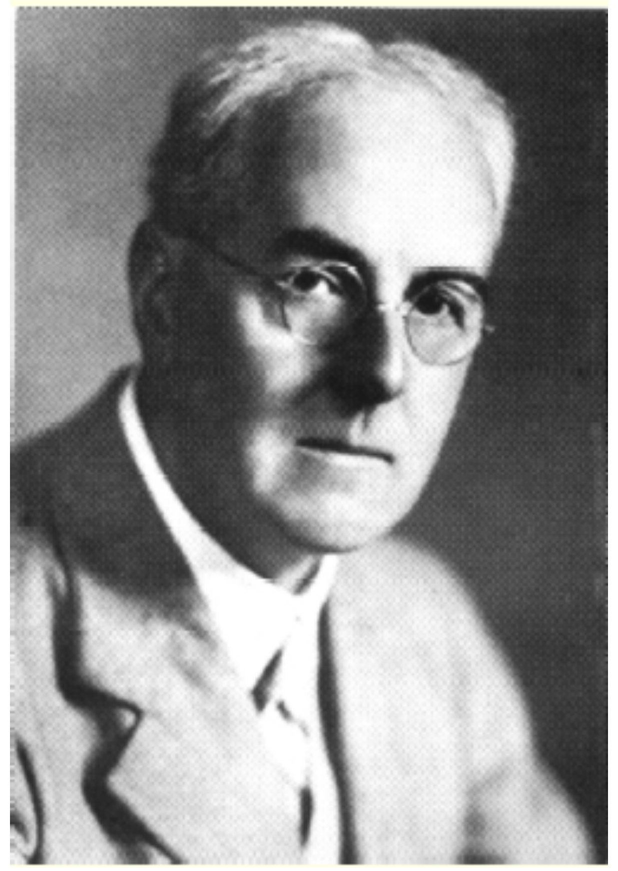
- Good evidence from experiments and DNS's in *incompressible turbulence.*

Richardson's Law

$$\langle R^2(t) \rangle \sim t^3$$

- Derivable from Kolmogorov's simple scaling arguments.

What about $\langle R^p(t) \rangle$ for any p?

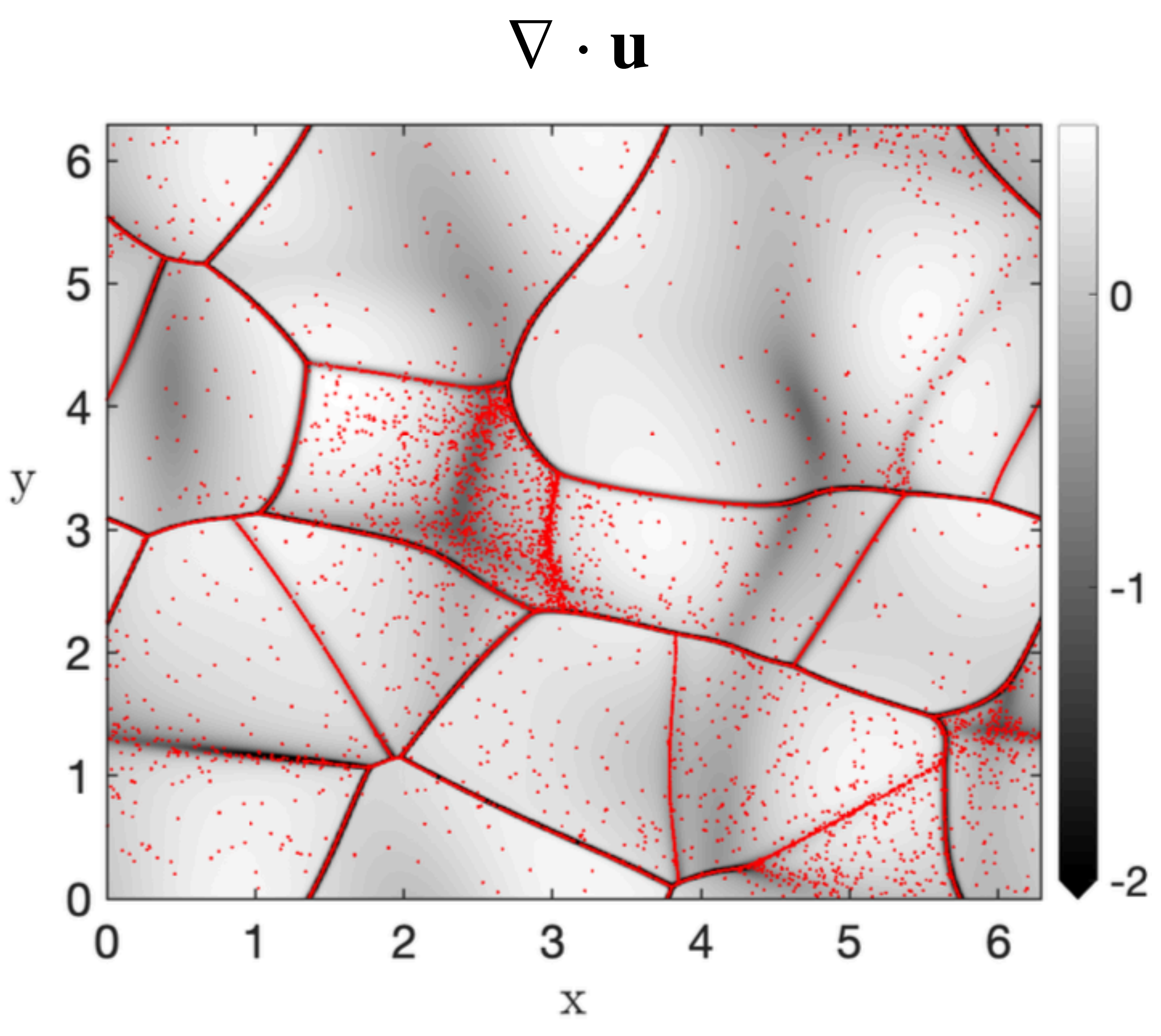


L. F. Richardson

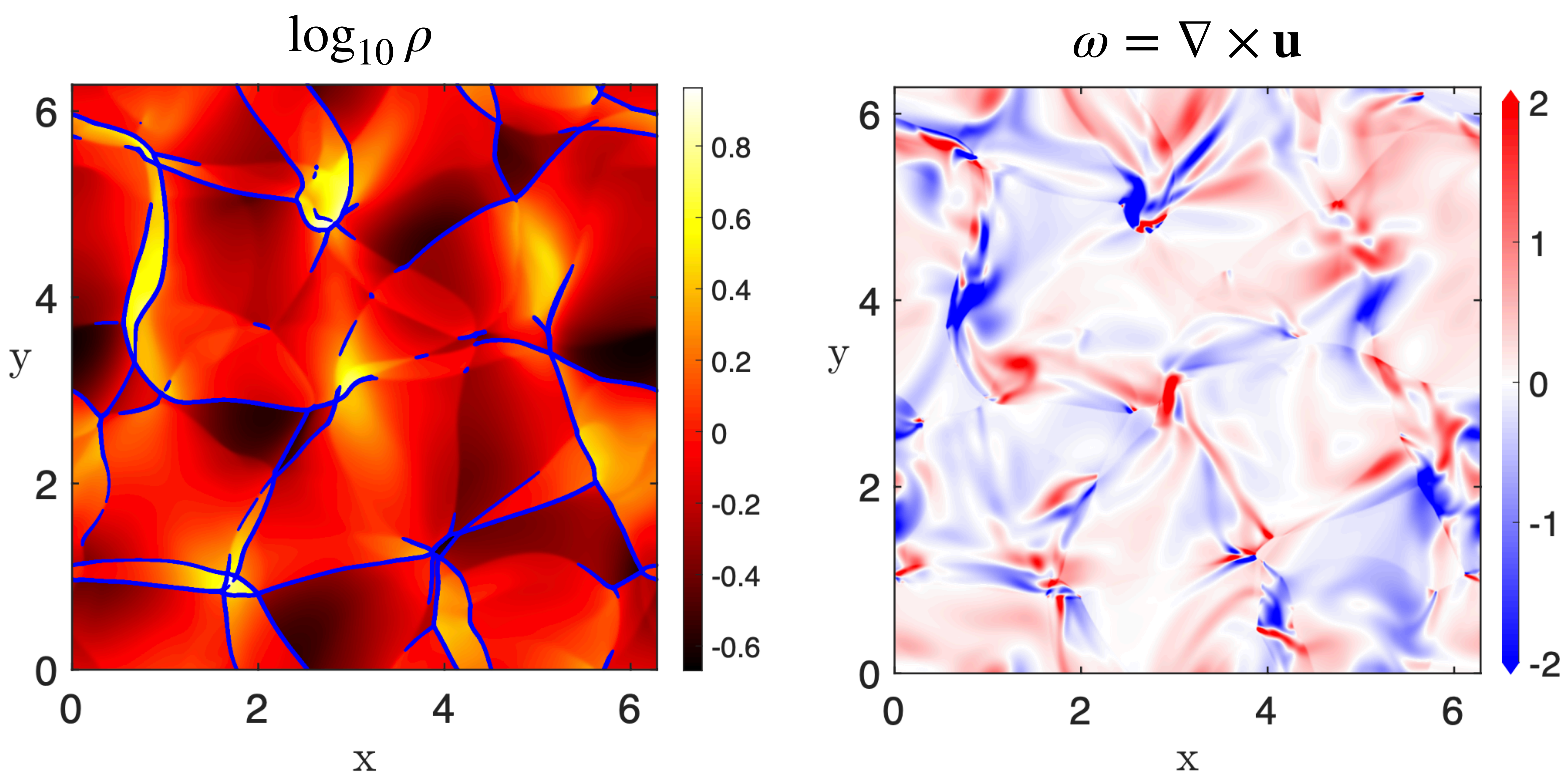
Types of supersonic turbulence

Shock-Dominated — Net kinetic energy in the irrotational modes ≫ solenoidal modes

Vorticity-Dominated — Net kinetic energy in the irrotational modes ≪ solenoidal modes



Burgers turbulence



Shock-dominated Navier-Stokes turbulence

1. Validity of Taylor's conjecture in shock-dominated turbulence

2. Multifractality of pair-dispersion in shock-dominated turbulence

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Beyond Taylor's Diffusion

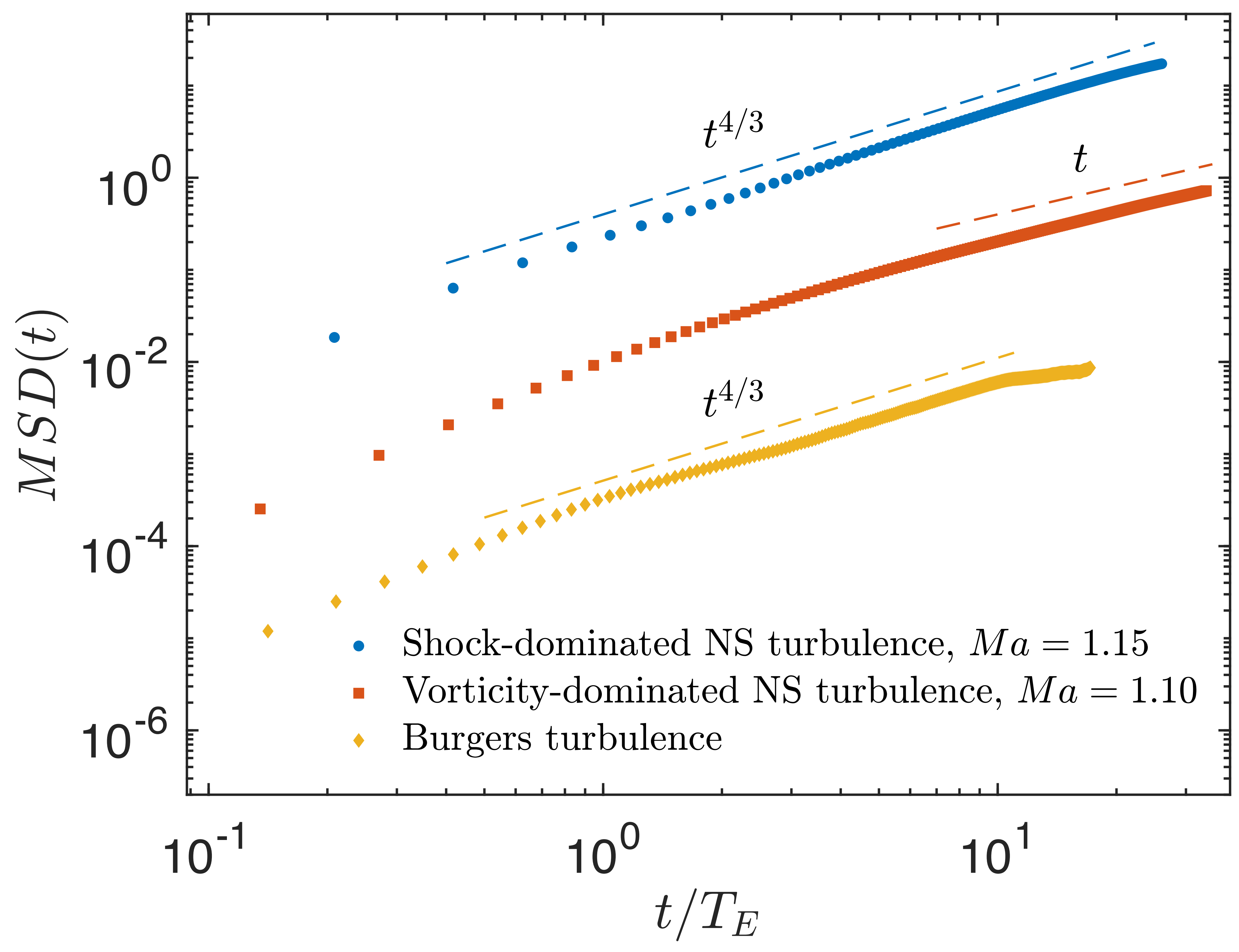
- **Shock-dominated turbulence — Transport is clearly superdiffusive at late times**

— persists upto $t \gg T_E$

— $MSD(t) \sim t^{\gamma}$; $\gamma \approx 4/3$ in both Burgers and Navier-Stokes

- **Vorticity-dominated supersonic turbulence —** No superdiffusion; qualitatively agrees with Taylor.

$T_E \rightarrow$ Large-eddy turnover time



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  — *mainly determined by* χ -> ratio of solenoidal to net KE

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— Small $\chi \rightarrow$ *shock dynamics largely irrotational.*

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— *Navier-Stokes -> Shock dynamics similar to Burgers;* Fastest tracers preferentially sample shocks.

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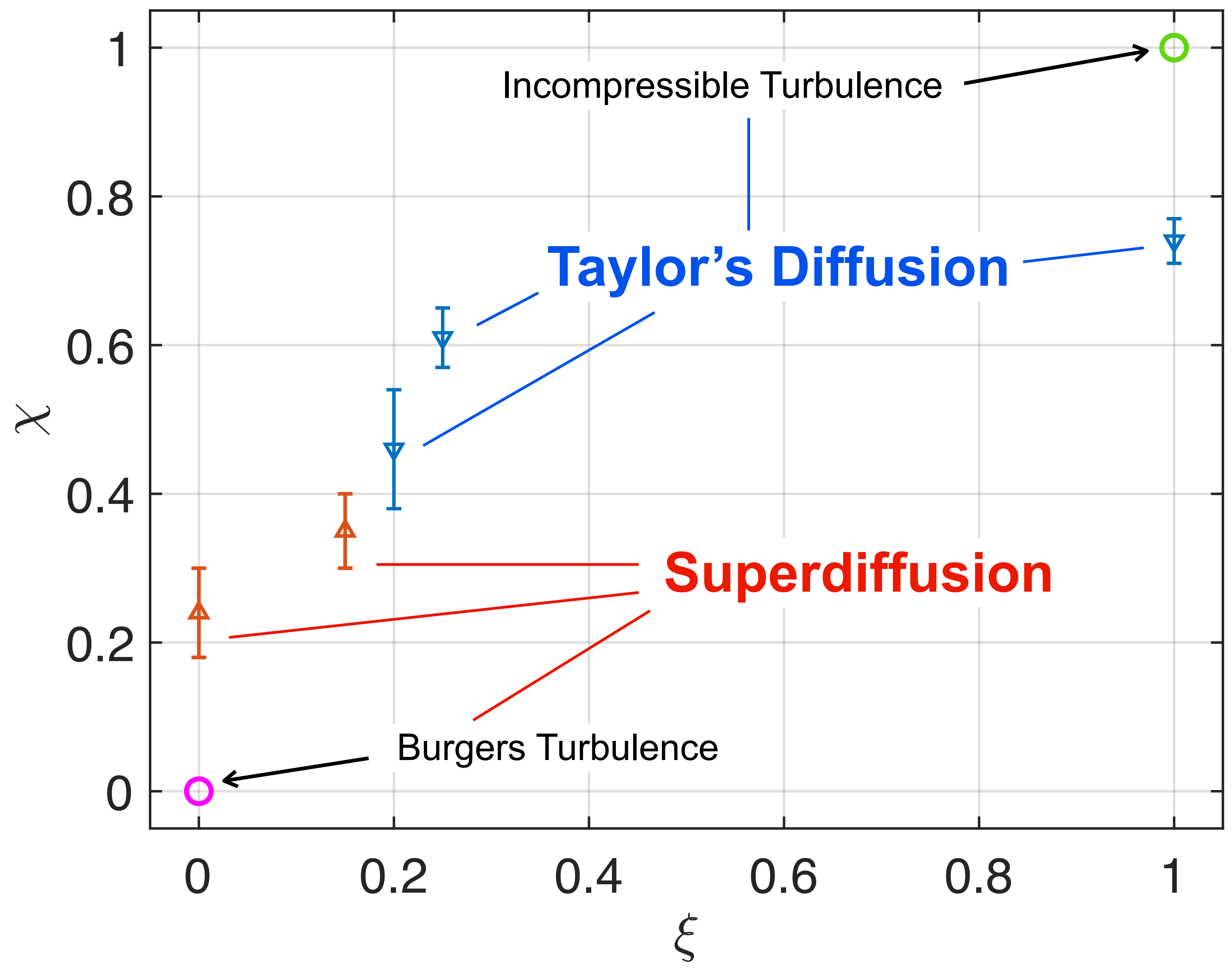
$\xi \rightarrow$ fraction of solenoidal power in the external force



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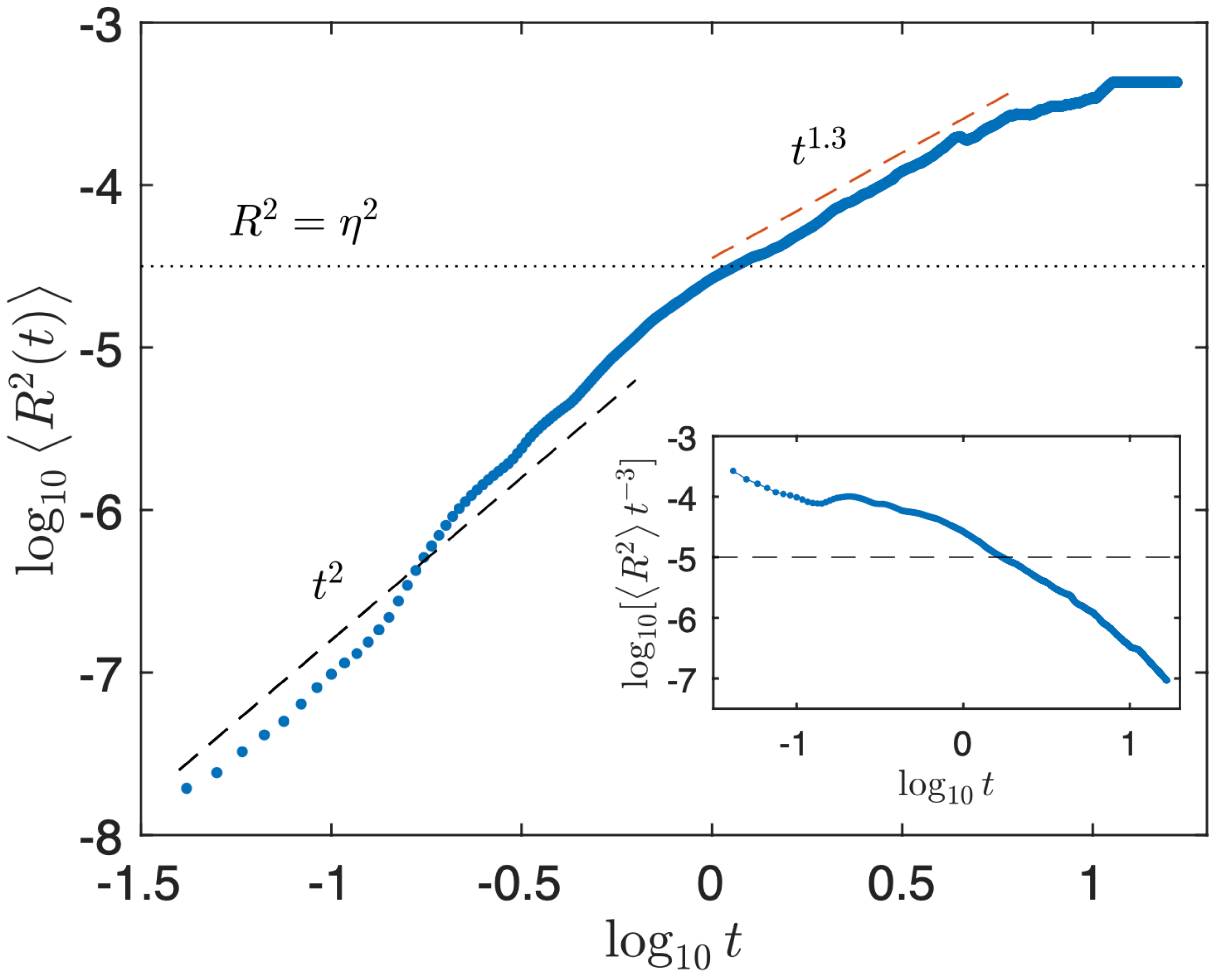
Richardson’s Law?



Clear suppression of Richardson’s scaling due to clustering of tracers on shocks.

Beyond Richardson's Law

[De et al, Phys. Rev. Research 6 (2024)]

Doubling and Halving times in Burgers turbulence

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Doubling and Halving times in Burgers turbulence

$$\left\langle T_{\mathrm{H}}^{-p} \right\rangle \sim R_0^{-\chi_p^{\mathrm{H}}} \qquad \left\langle T_{\mathrm{D}}^{-p} \right\rangle \sim R_0^{-\chi_p^{\mathrm{D}}}$$

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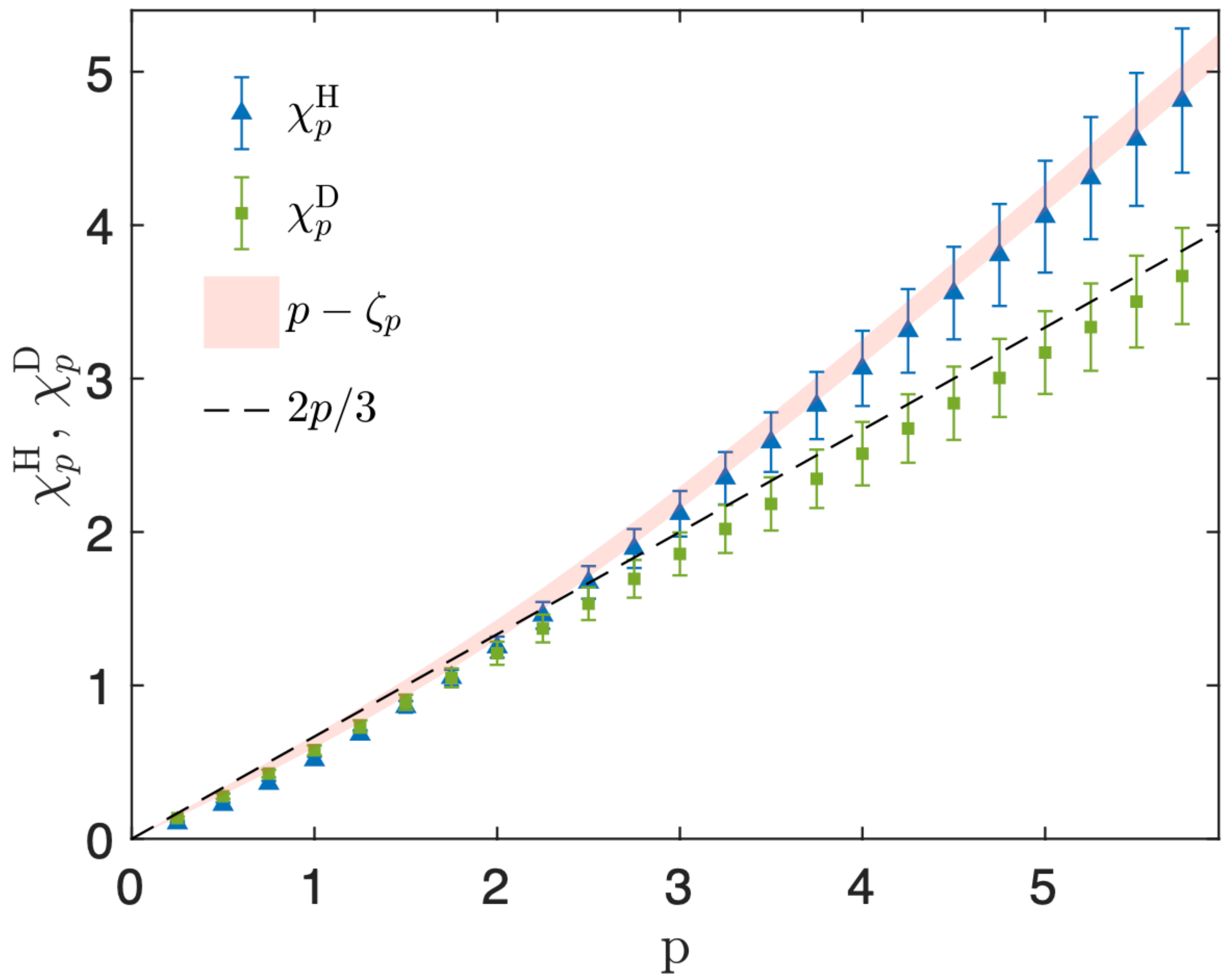

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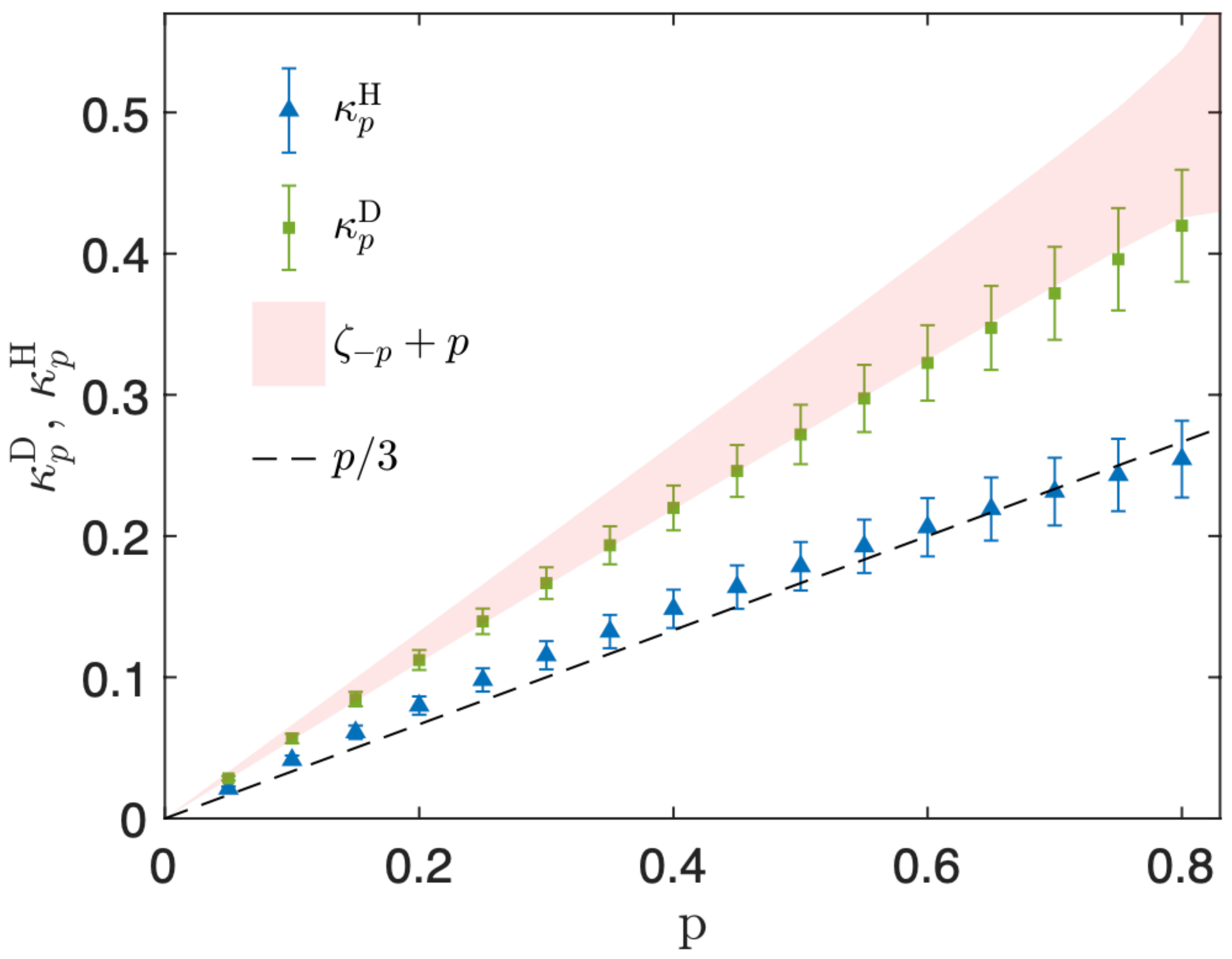

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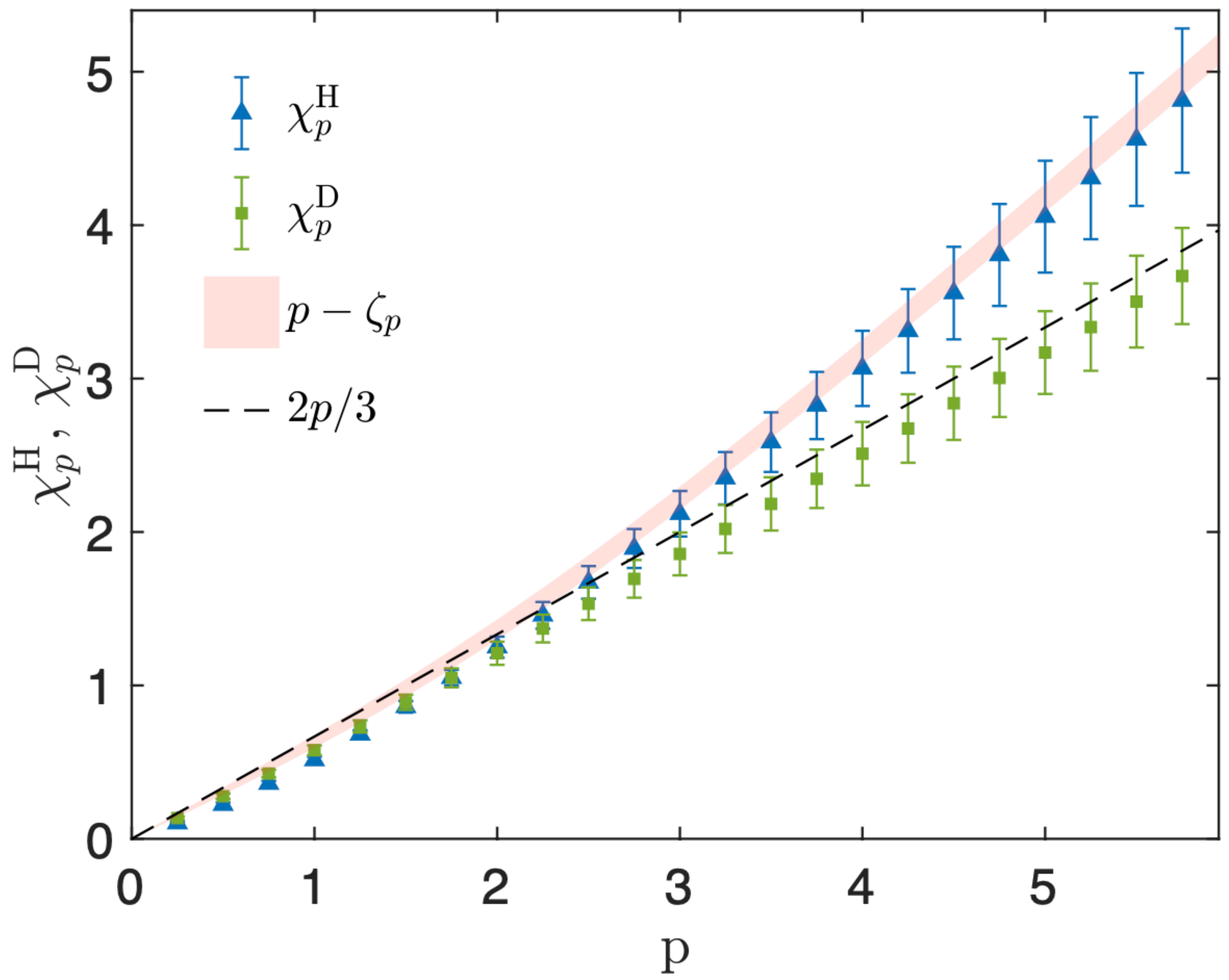

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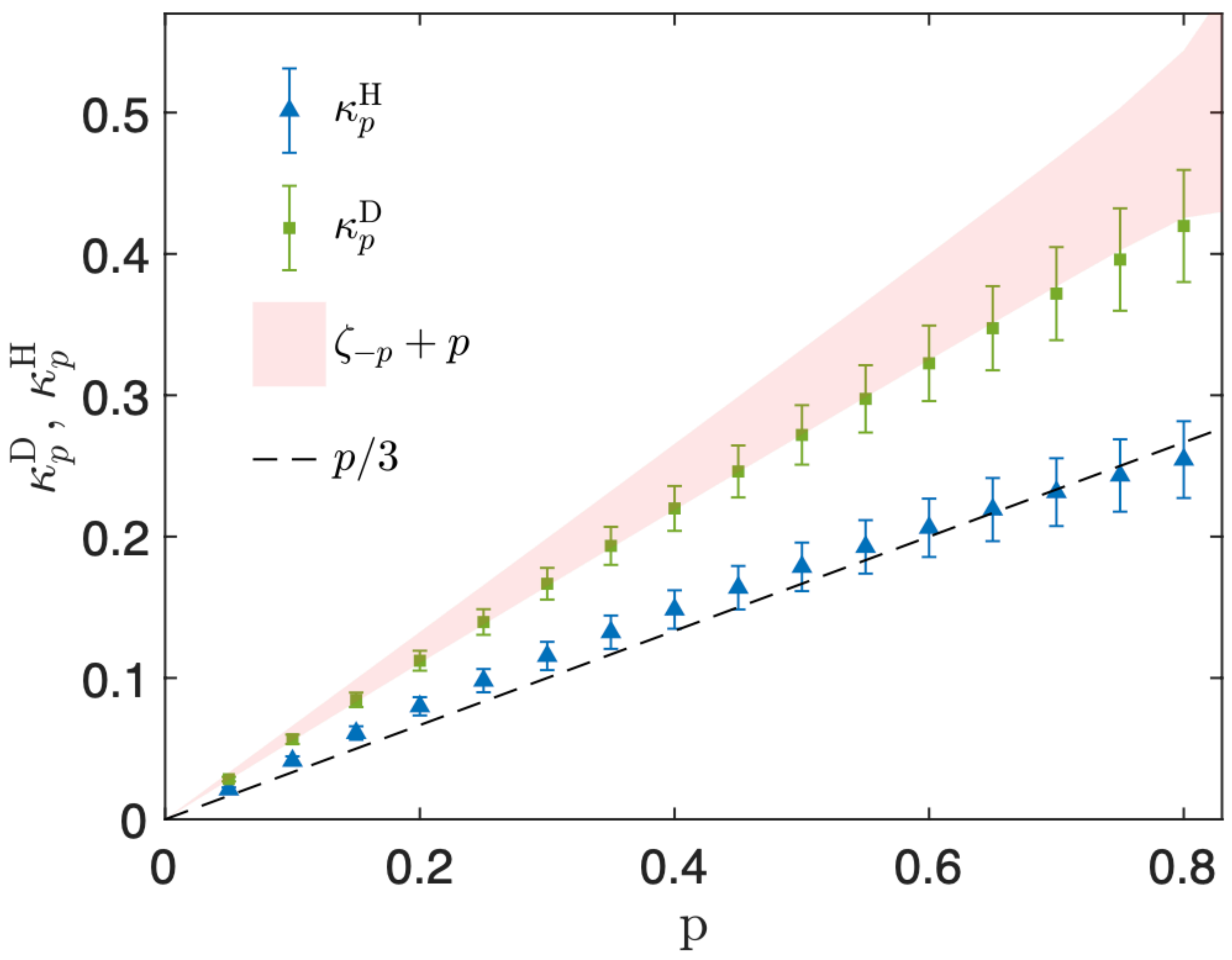

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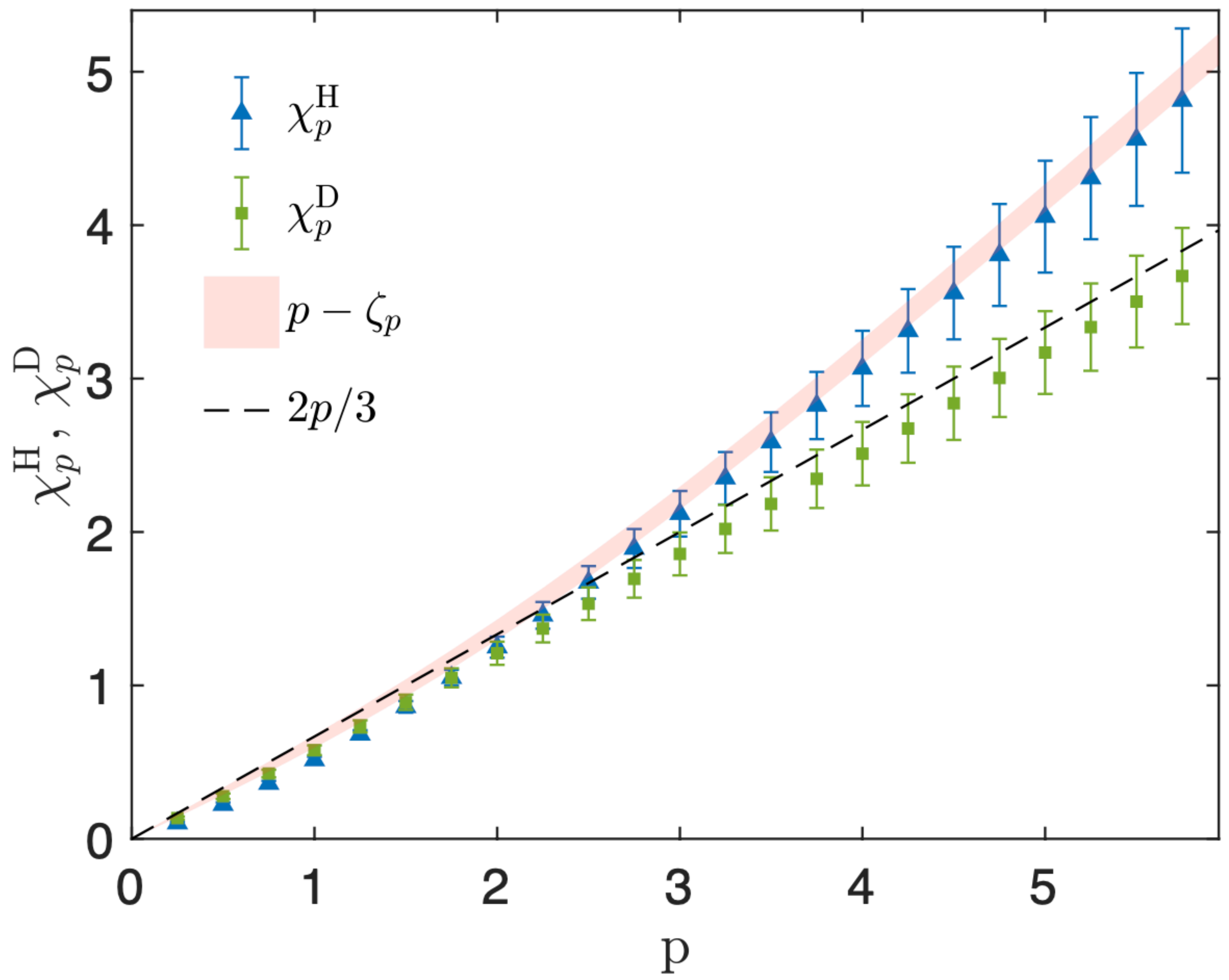

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Our theory -> $\chi_p^{\mathrm{D}} = 2p/3$

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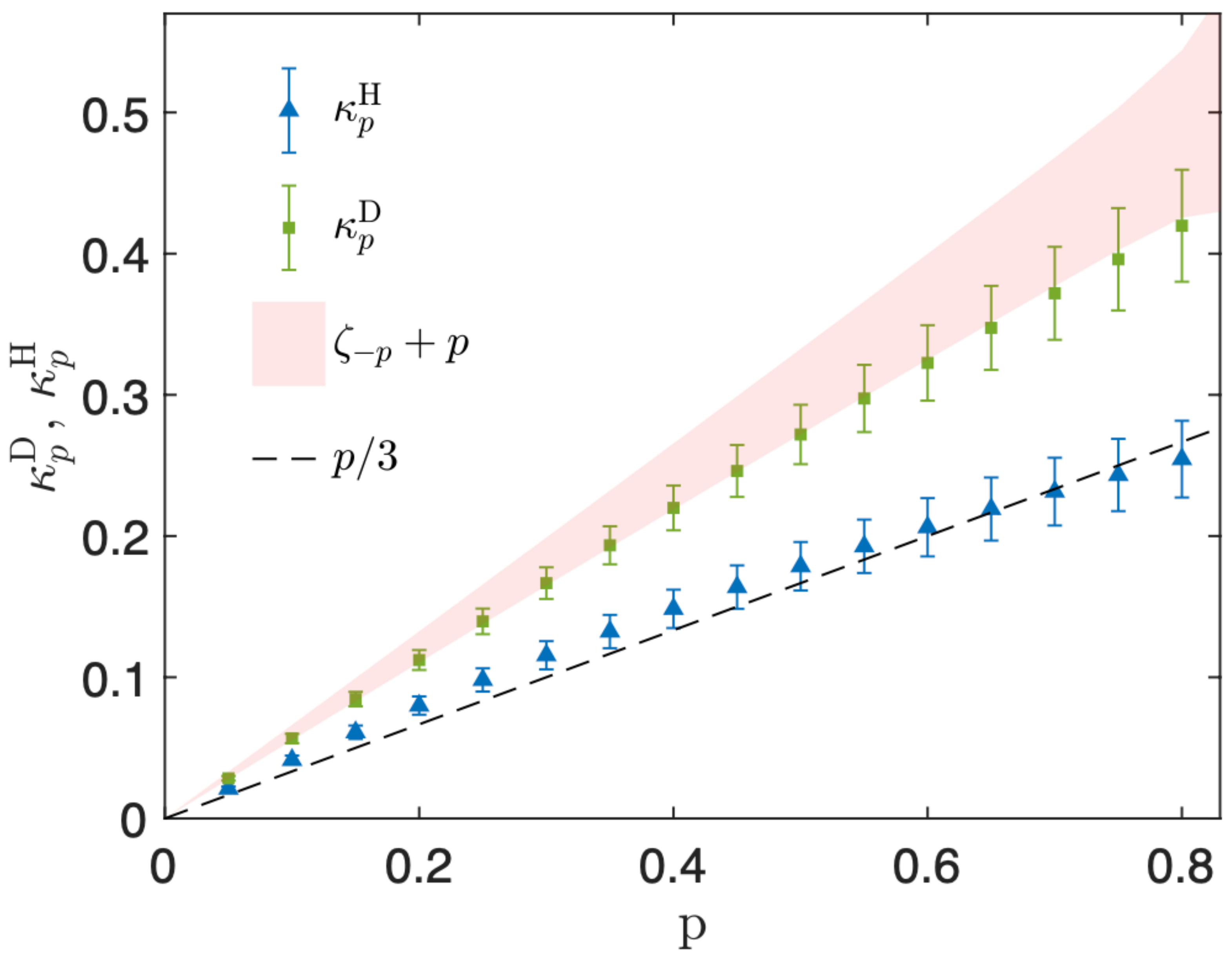

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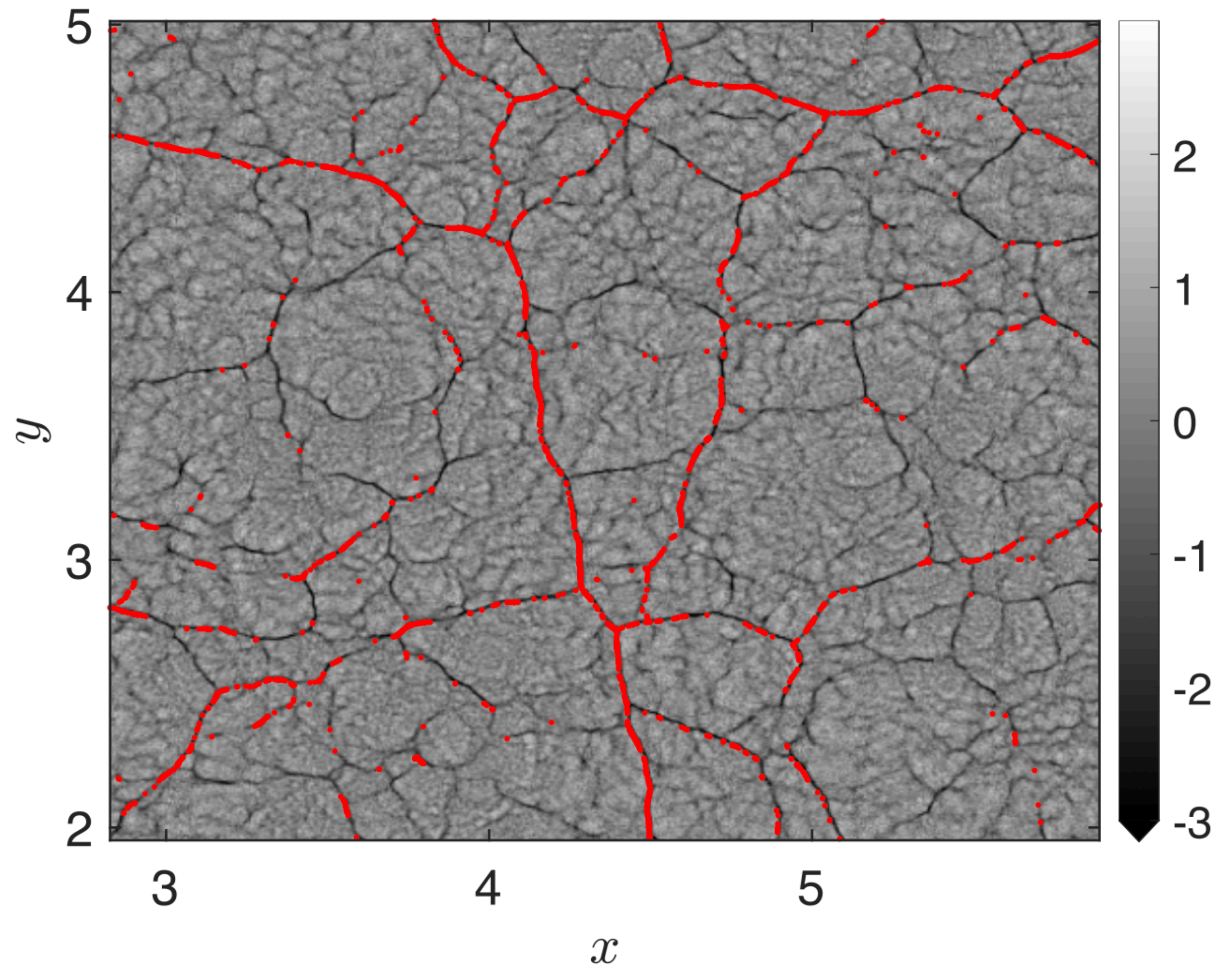

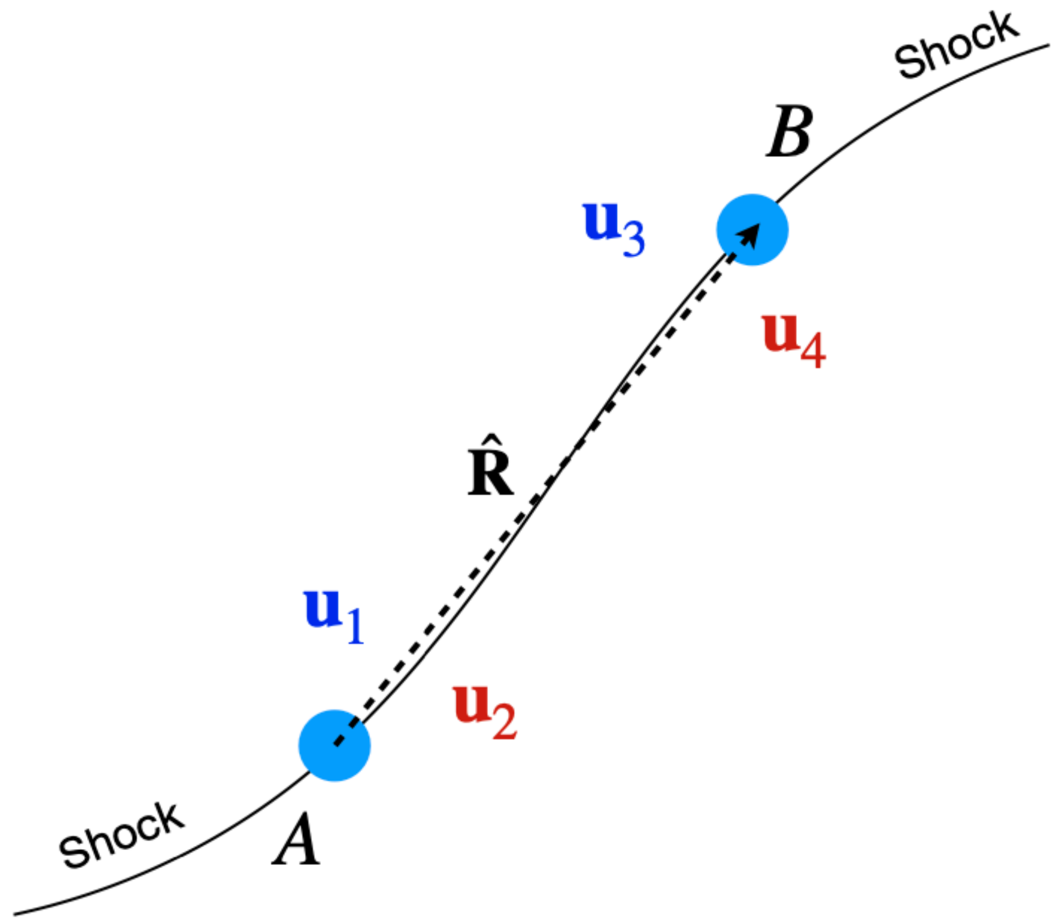

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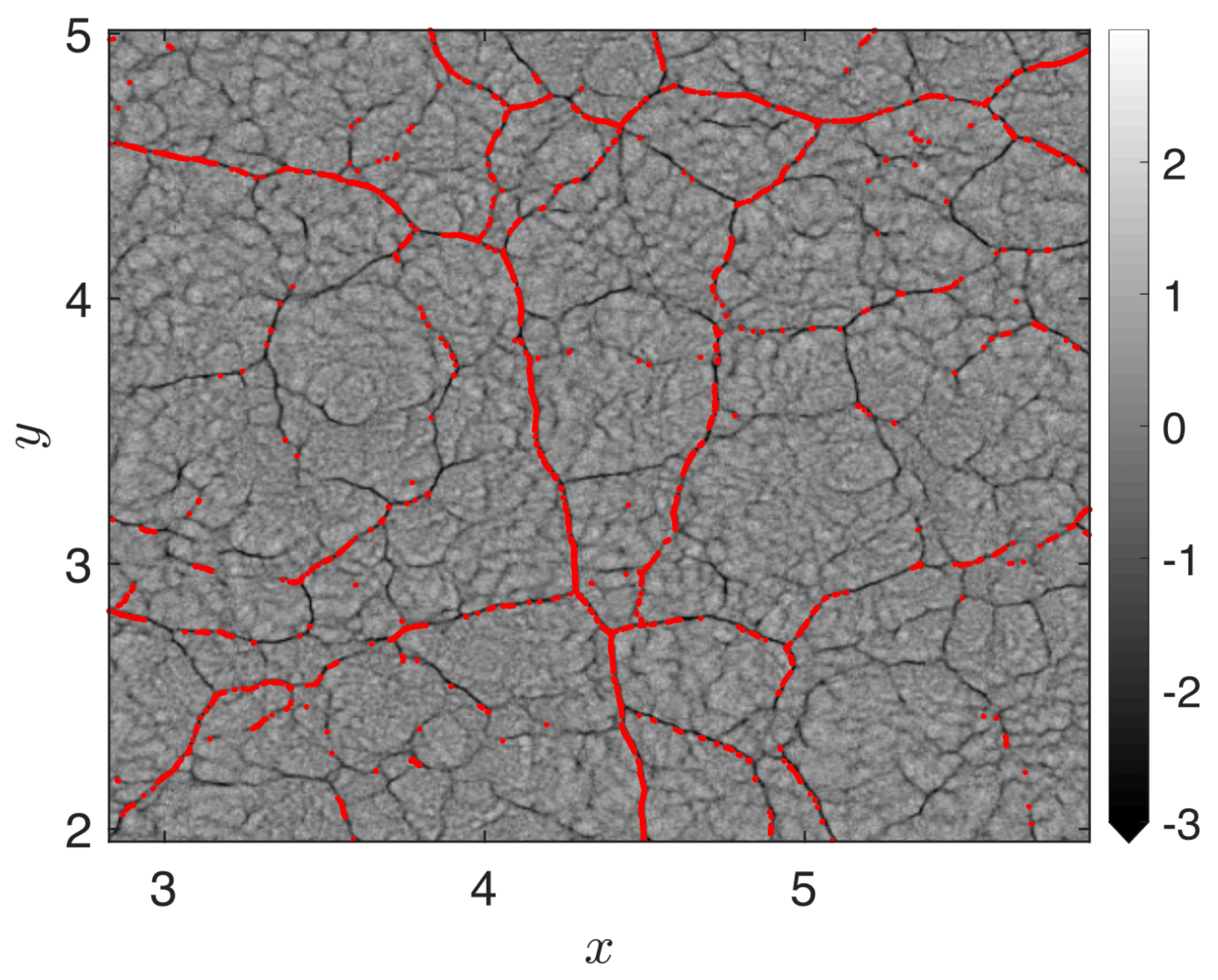

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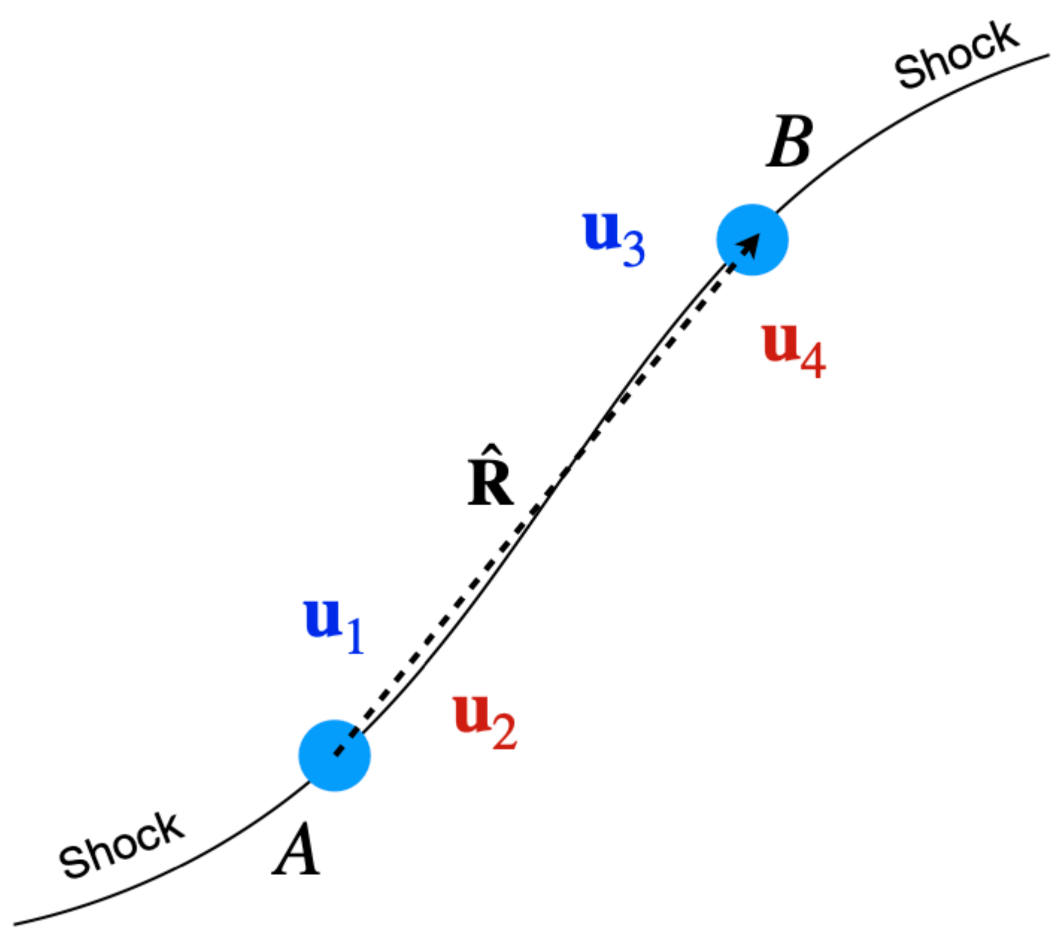
- Solve the corresponding first-passage time problem

$$\partial_t W = \frac{1}{R} \partial_R [R K(R) \partial_R W]$$

→ Survival probability from $W(R, t)$

↓ Exit-time distribution

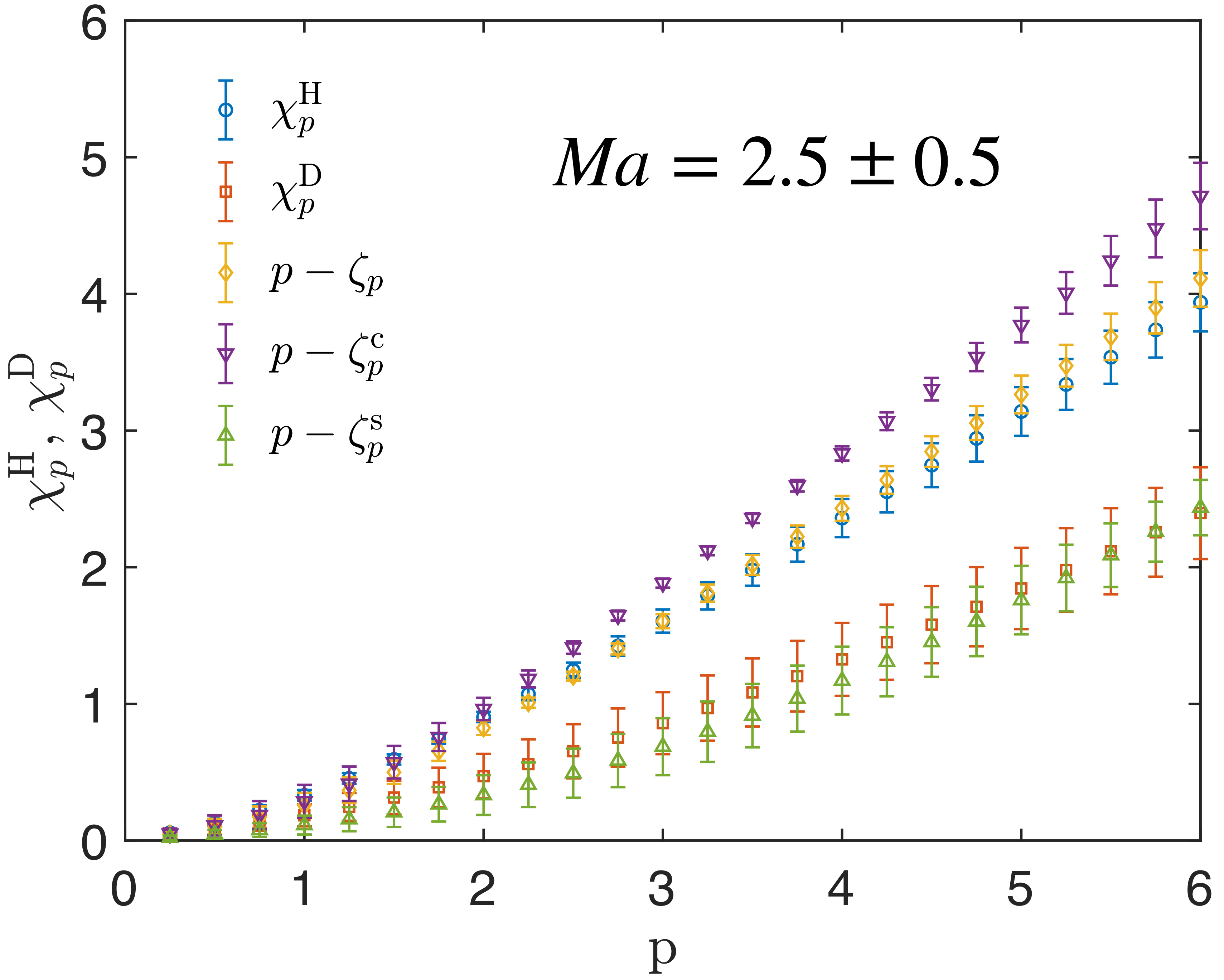
← Moments of exit times



Beyond Richardson's Law

Compressible Navier-Stokes turbulence

Purely solenoidal forcing



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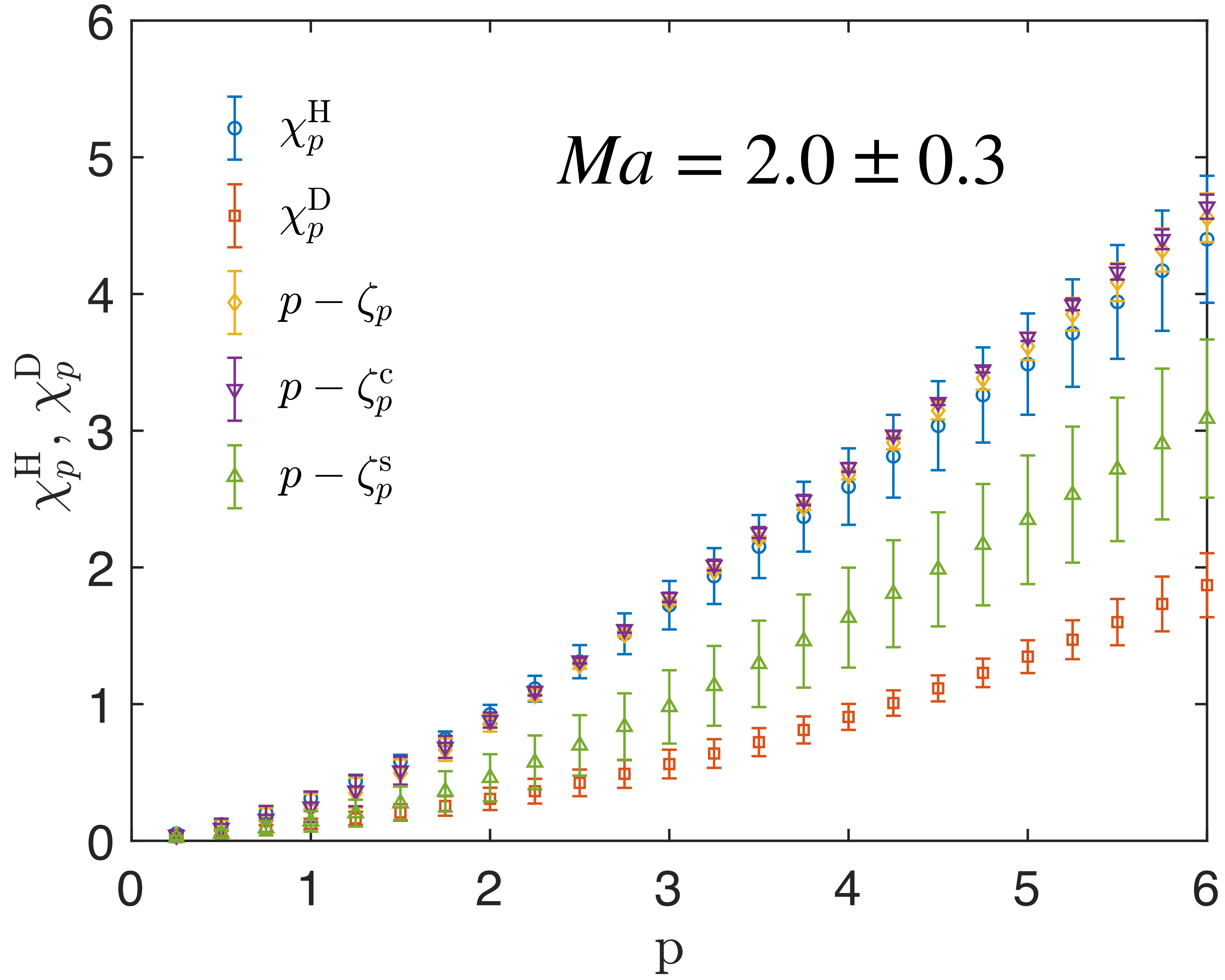

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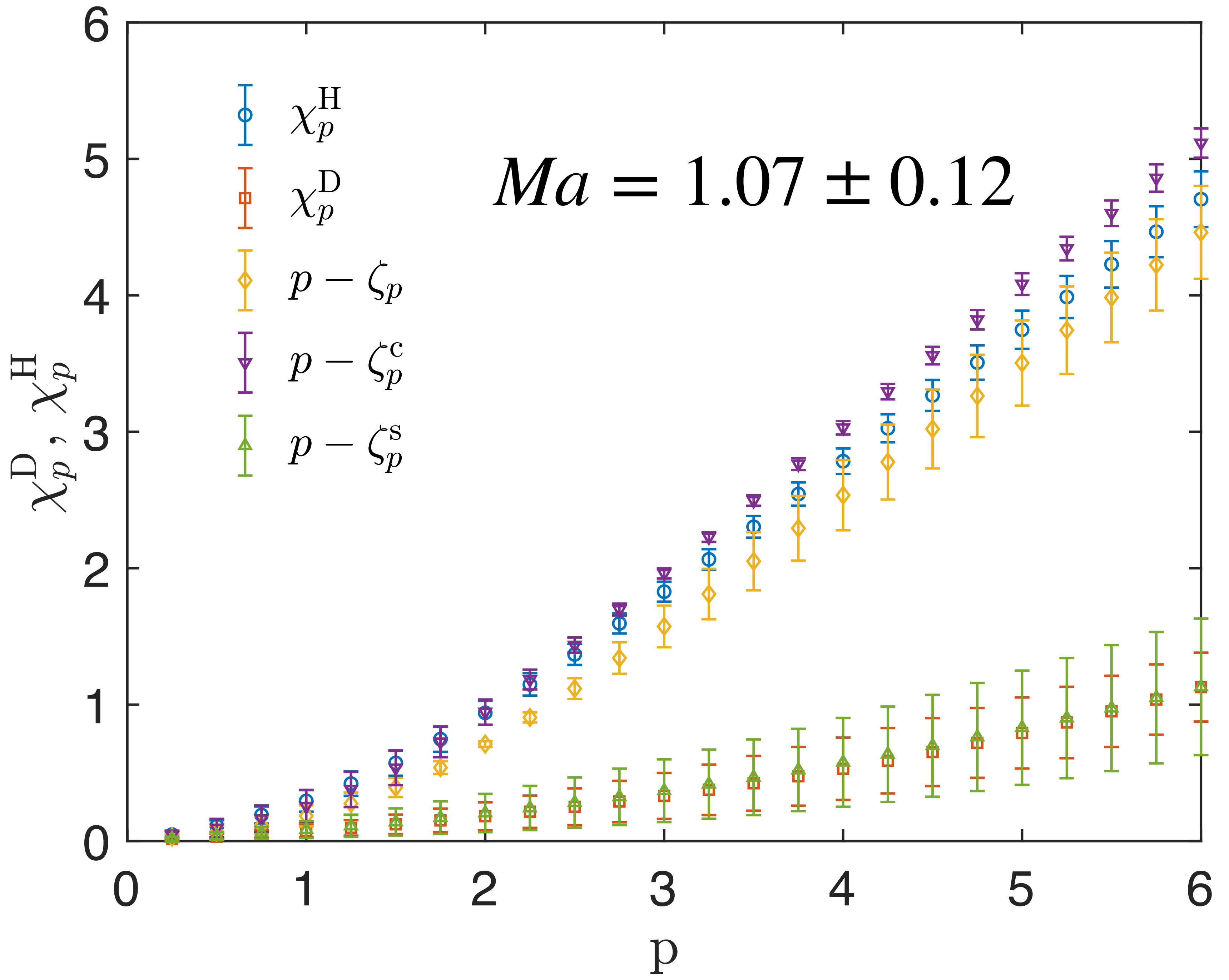

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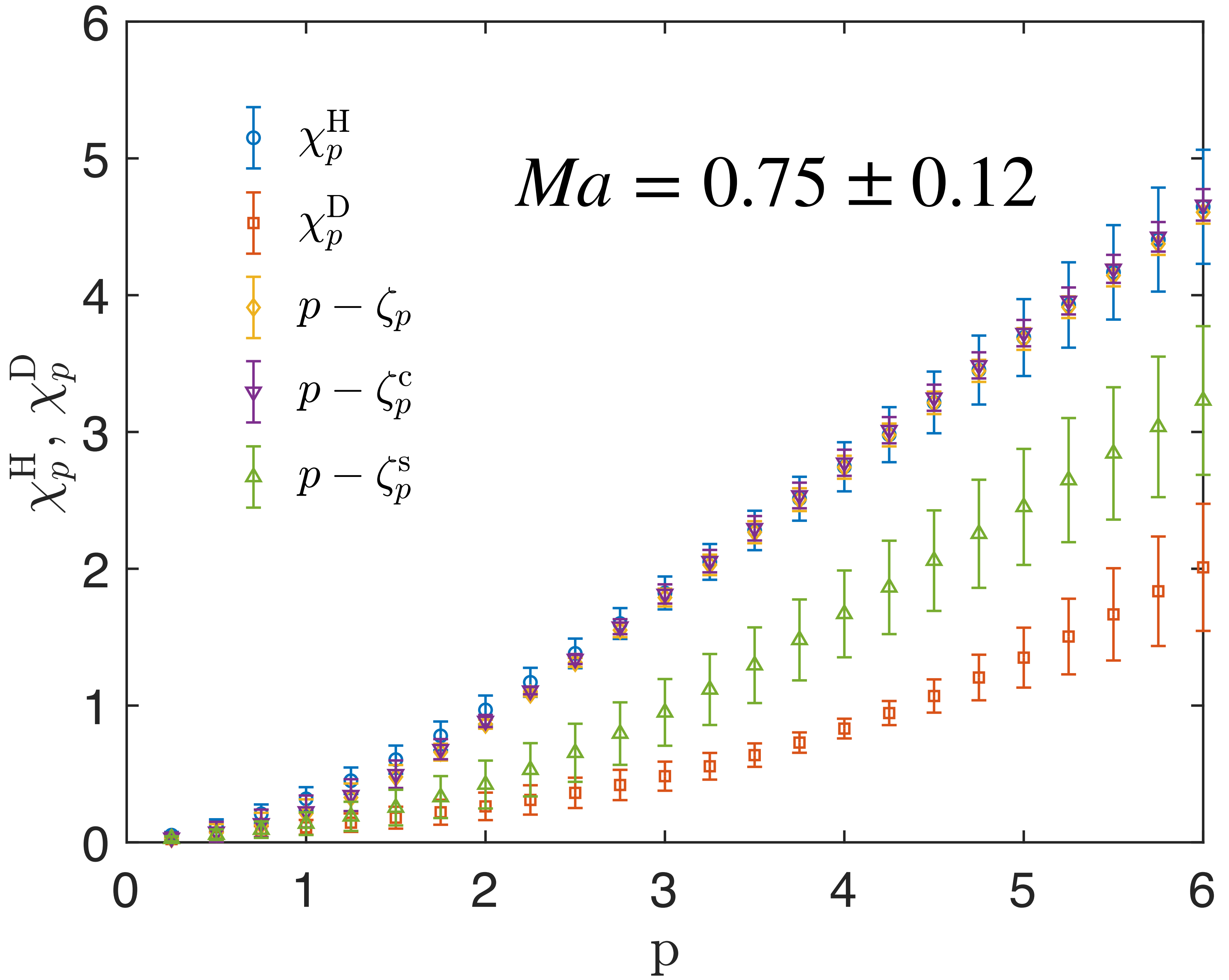

Summary

- Large-scale transport is superdiffusive over a significant range of time-scales in shock-dominated turbulence; not so in vorticity-dominated supersonic turbulence.

- Attributable to the differences in the shock dynamics in these two types of flows.

- Different pair exit times in shock-dominated flows exhibit different scaling properties — *multifractality of pair dispersion.*

- Our theoretical insights capture these differences in Burgers turbulence.

  [S. De, D. Mitra, and R. Pandit, Phys. Rev. Research 6 (2024)]

- Supersonic Navier-Stokes turbulence — Pair dispersion and exit time statistics depend on both Mach number and the nature of external force —> *determined by the small-scale structures of the underlying vorticity and velocity fields.*