CHIMERA STATES FOR COUPLED PENDULA

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Lodz

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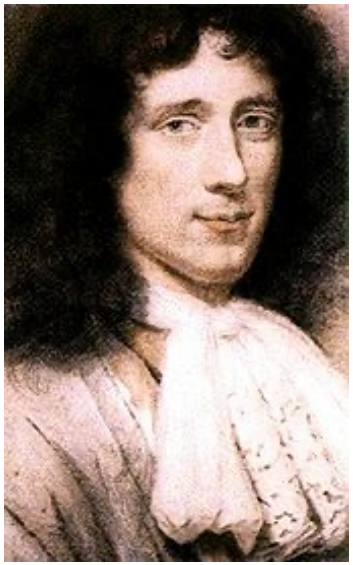
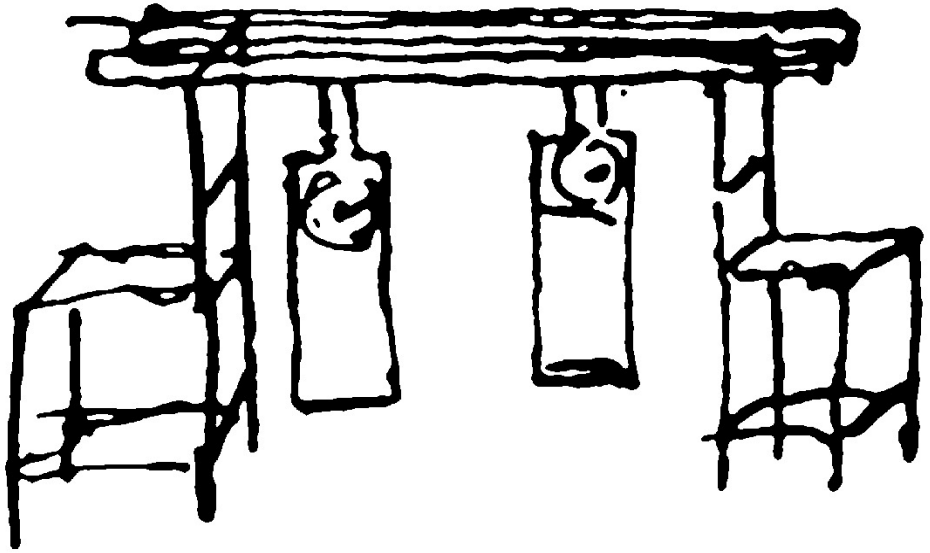
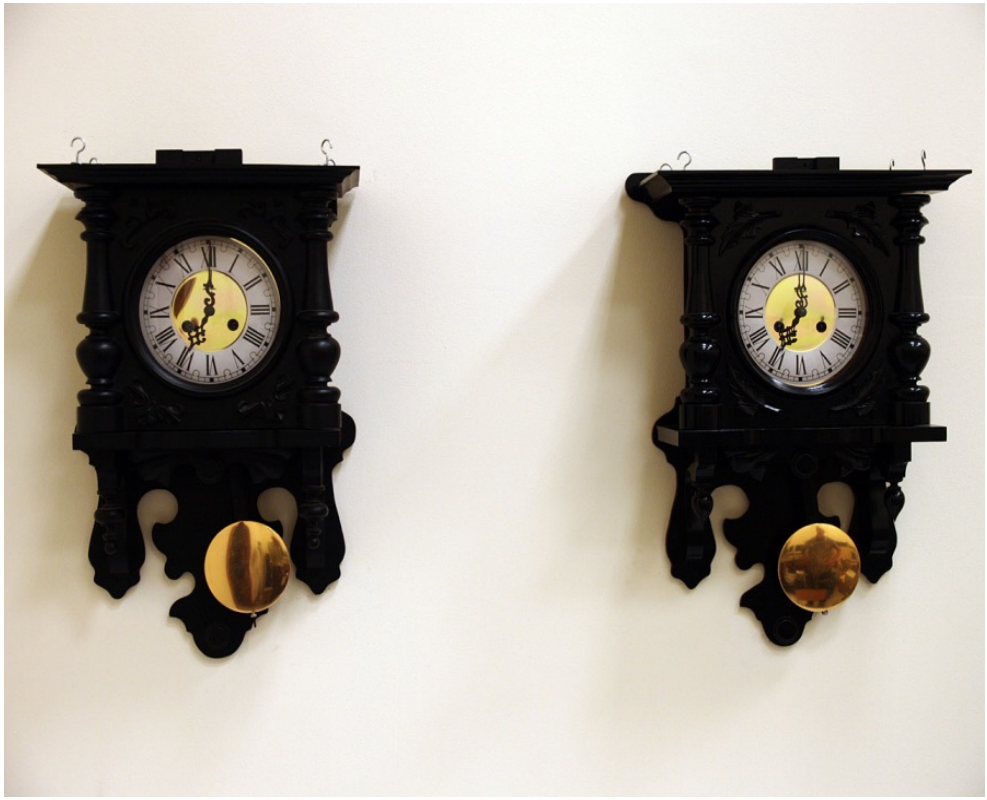
Frank Hellmann,
Paul Schultz,
Juergen Kurths

What is chimera?

Christiaan Huygens

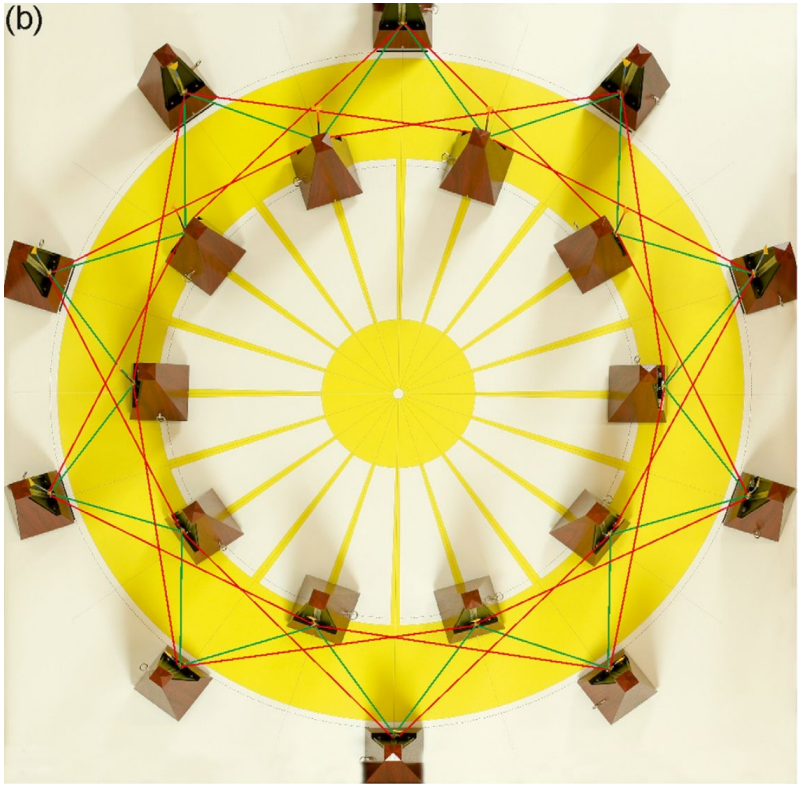




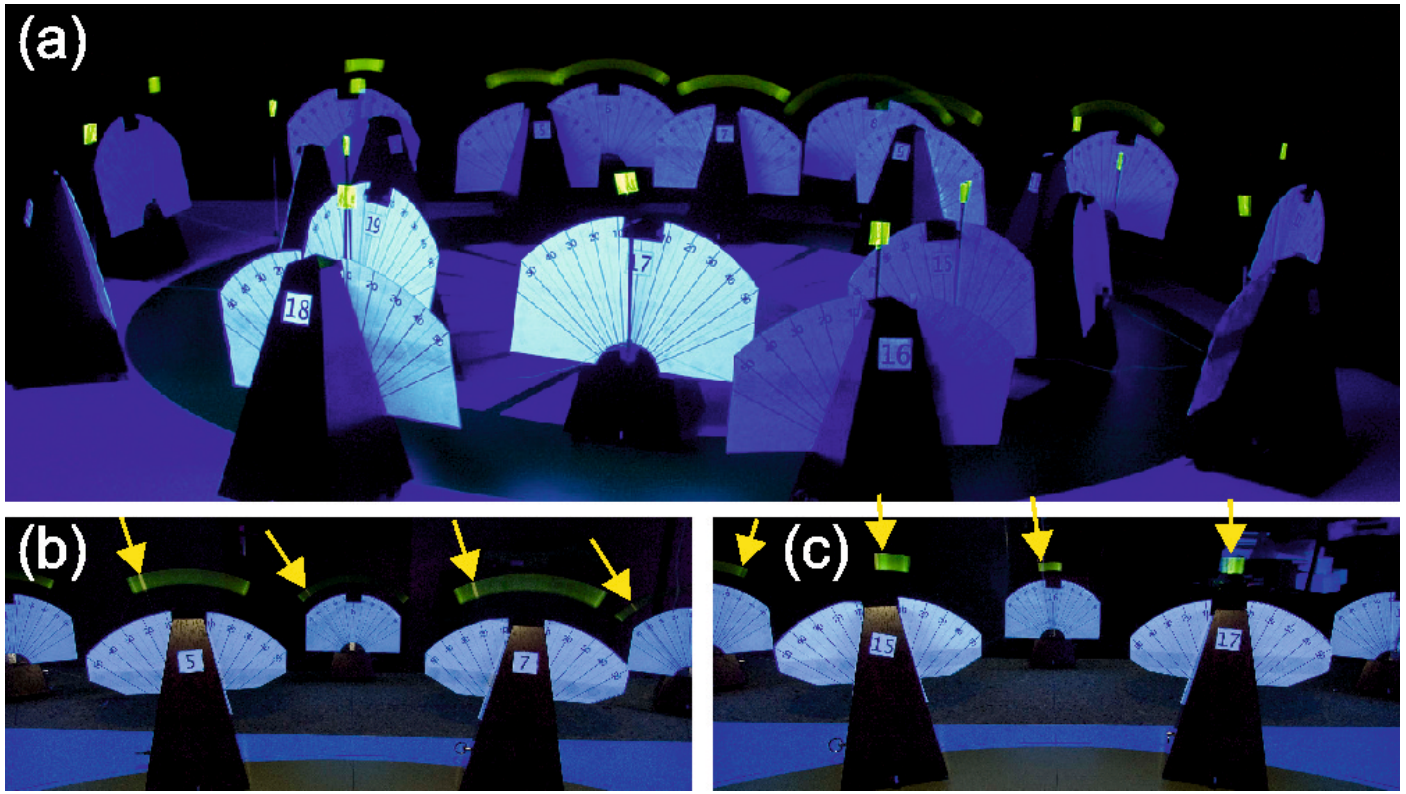


ring of coupled metronomes

- Set of 20 coupled Wittner Maelzel 802K metronomes covering a frequency range of 40 to 208 tics per minute, with a standard deviation of ∼1%. When fully wound up, each metronome ticks for a duration of ∼25 min, corresponding to ∼1,500 oscillation cycles. Frequency of each metronome has been set to 200 tics per minute.



Experimental observation in fluoroscent light



(Sci. Rep. 2014)

equations of motion

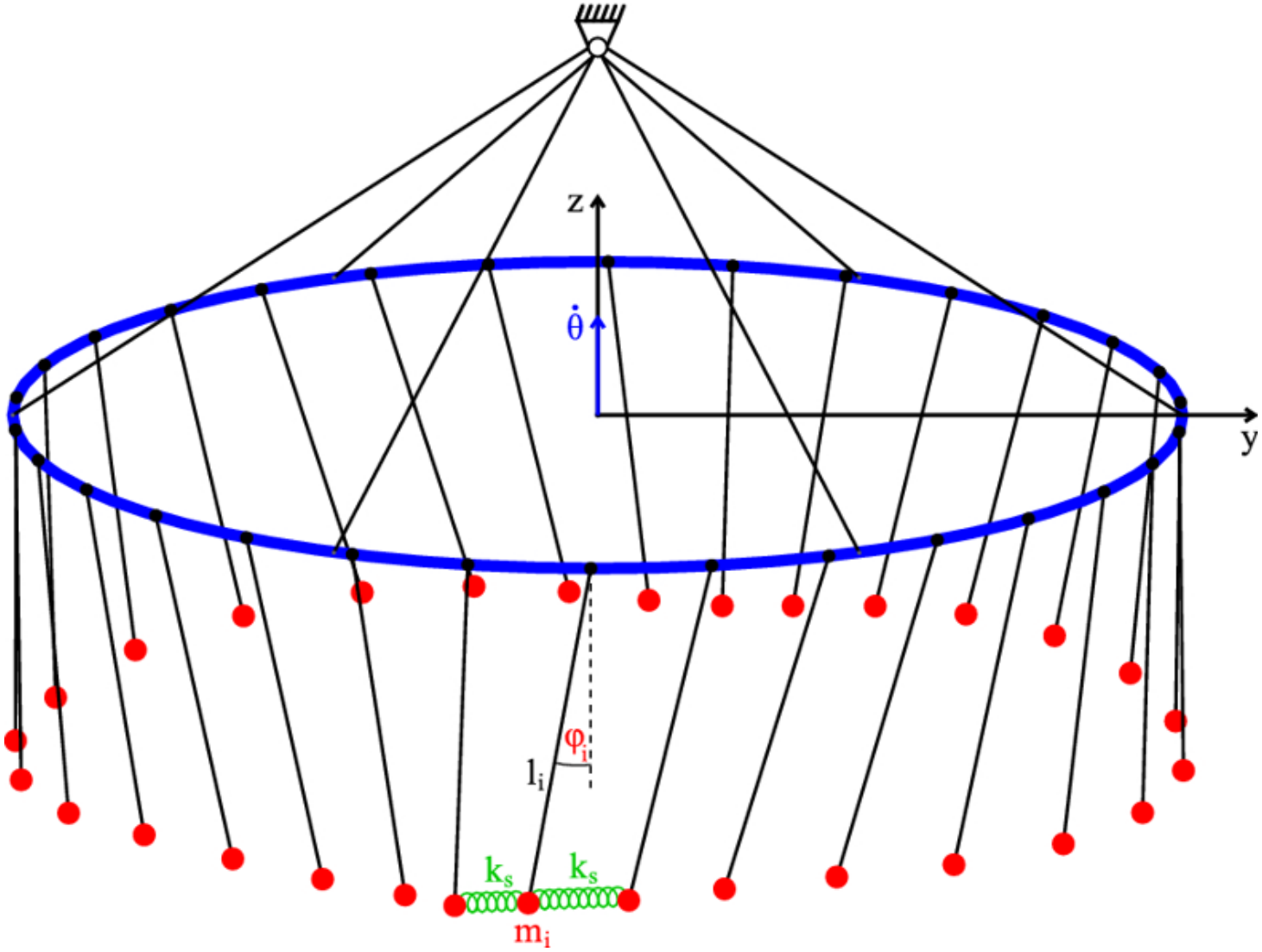
The dynamics of the system of coupled pendula (metronomes) is given by:

$$ml^2\ddot{\varphi}_i + c_\varphi\dot{\varphi}_i + mgl\ \sin\varphi_i +$$
$$+ k_x l_s^2(\varphi_i - \varphi_{i-1}) + k_x l_s^2(\varphi_i - \varphi_{i+1}) + k_x l_s^2(\varphi_i - \varphi_{i-2}) + k_x l_s^2(\varphi_i - \varphi_{i+2}) +$$
$$+ c_x l_s^2(\dot{\varphi}_i - \dot{\varphi}_{i-1}) + c_x l_s^2(\dot{\varphi}_i - \dot{\varphi}_{i+1}) + c_x l_s^2(\dot{\varphi}_i - \dot{\varphi}_{i-2}) + c_x l_s^2(\dot{\varphi}_i - \dot{\varphi}_{i+2}) = M_N\ , \quad (1)$$

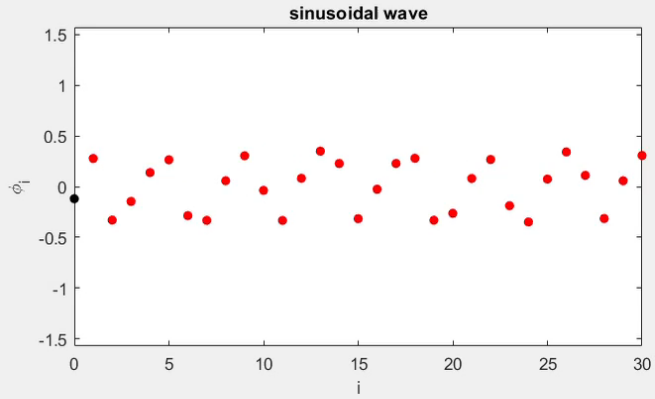
where $i=1,2,\ldots,n$, $\varphi_0=\varphi_n$, $\varphi_{n+1}=\varphi_1$. This system is symmetrical on the ring, i.e., pendulum i is coupled with pendula $i+1$ and $i-1$ (local coupling), and in addition, with $i-2$ and $i+2$ (nonlocal coupling).

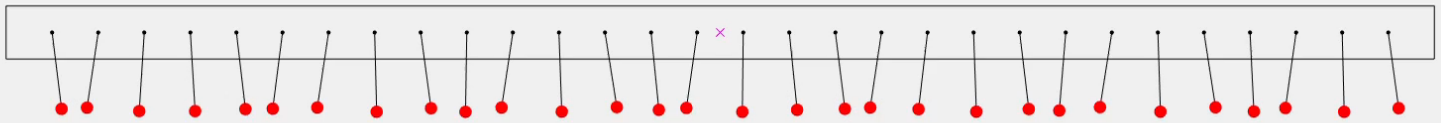
travelling chimeras

(Nonlinear Dynamics, **95**, 1859–1866(2019))

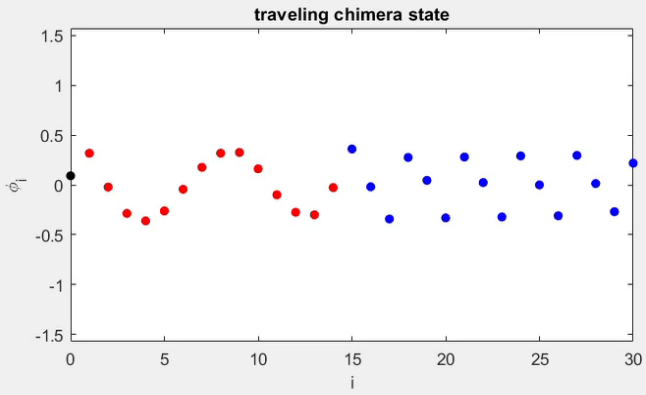


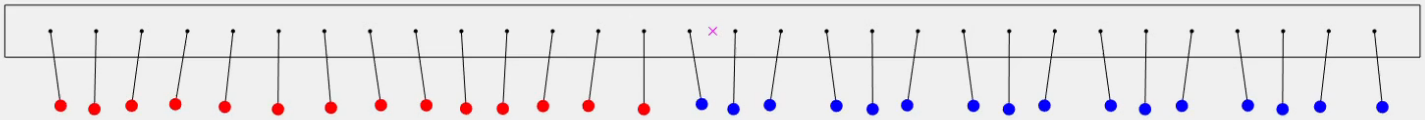
sinusoidal wave



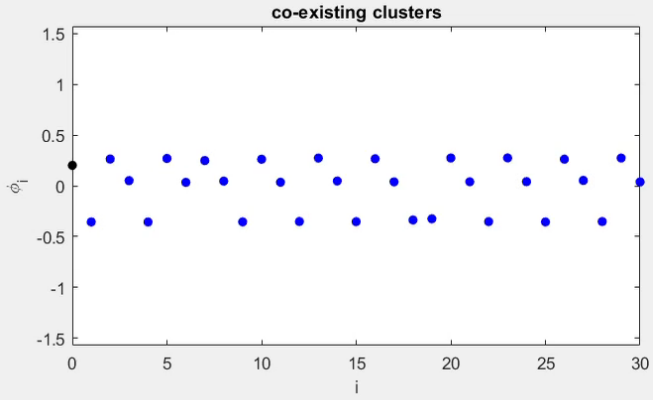


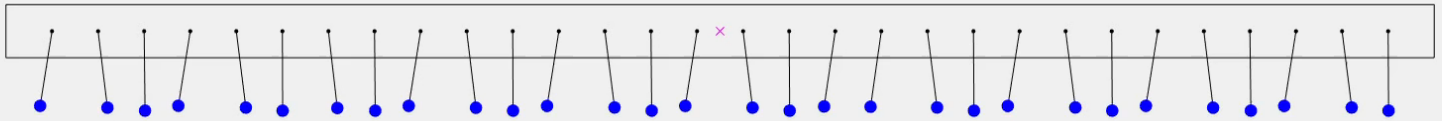
travelling chimera

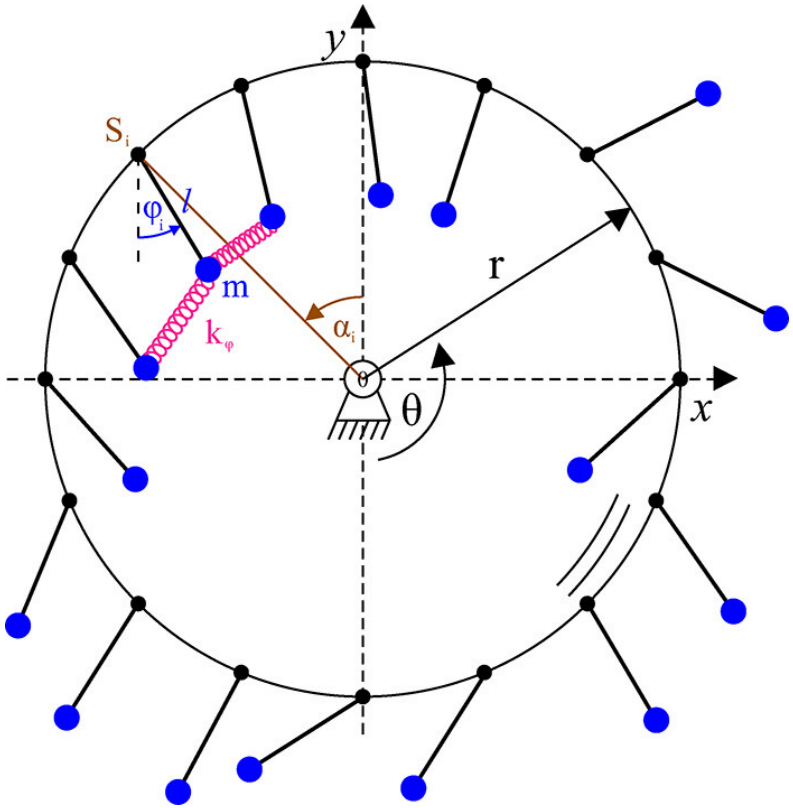




coexisting clusters

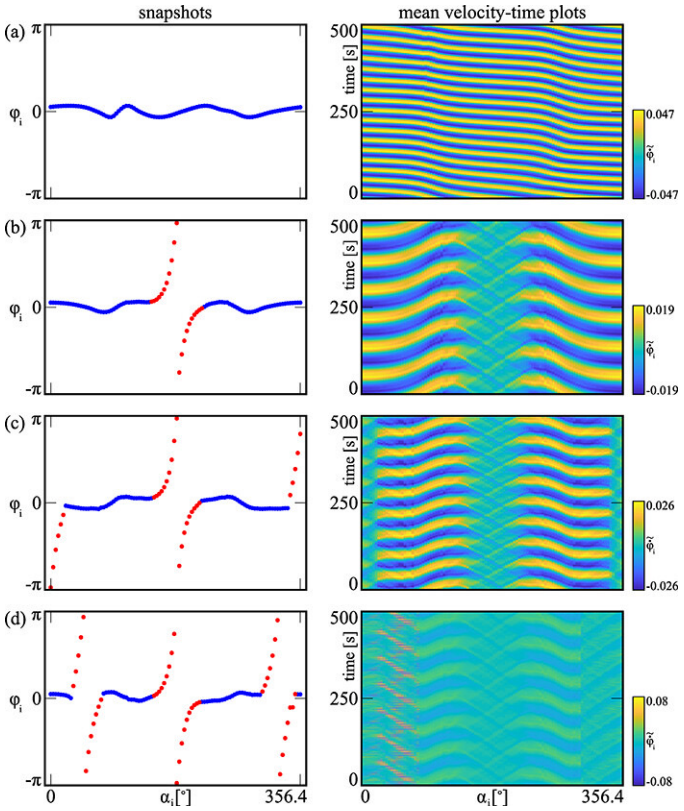






The scheme of the network of locally coupled pendulum clocks hanged on the oscillating wheel (the suspension points marked as the black dots). The number of local connections between the oscillators (marked in pink) is limited for better clarity. Network size: 16 pendula.

Dawid Dudkowski; Krzysztof Czołczyński; TK; *Chaos* **31,** 013135 (2021)
DOI: 10.1063/5.0033519



Dynamical patterns of model (1) for N = 100 pendula. In the left panel, the snapshots of the states are shown, while in the right one, the mean velocity–time plots are presented. From top to bottom: (a) the coherent state (k φ = 1.35 N/m), (b) the one-headed chimera (k φ = 41.85 N/m), (c) the two-headed chimera (k φ = 32.85 N/m), and (d) the three-headed chimera (k φ = 35 N/m). Parameters correspond to the points marked in Fig. 1 by yellow stars (M D = 0.045 N\,m). The visualization of the states can be found in the supplementary material [see Movies M3(a)–M3(d) for details].

Published in: Dawid Dudkowski; Krzysztof Czołczyński; TK; *Chaos* **31,** 013135 (2021)
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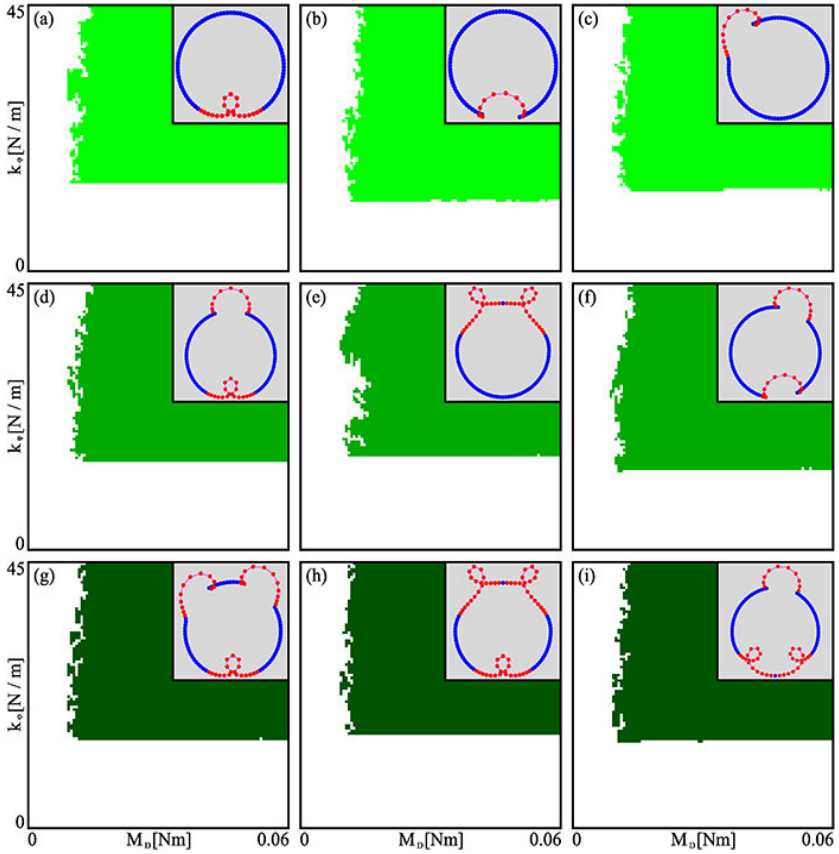

The domains of different chimeric patterns in (M D , k φ) parameters plane (the numerical results). The regions corresponding to the existence of particular states (snapshots included in the inboxes) are shown in green shades. From top to bottom: (a)–(c) one-headed loop chimeras, (d)–(f) two-headed loop chimeras, and (g)–(i) three-headed loop chimeras.

Dawid Dudkowski; Krzysztof Czołczyński; TK; *Chaos* **31,** 013135 (2021)
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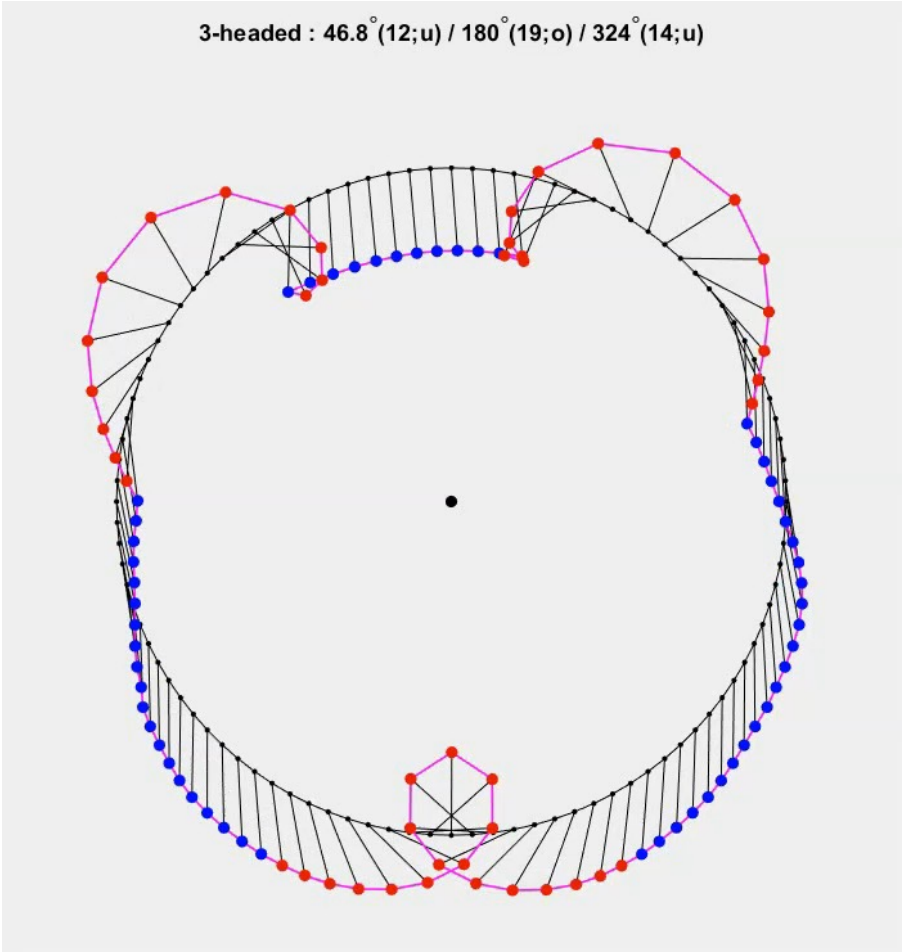
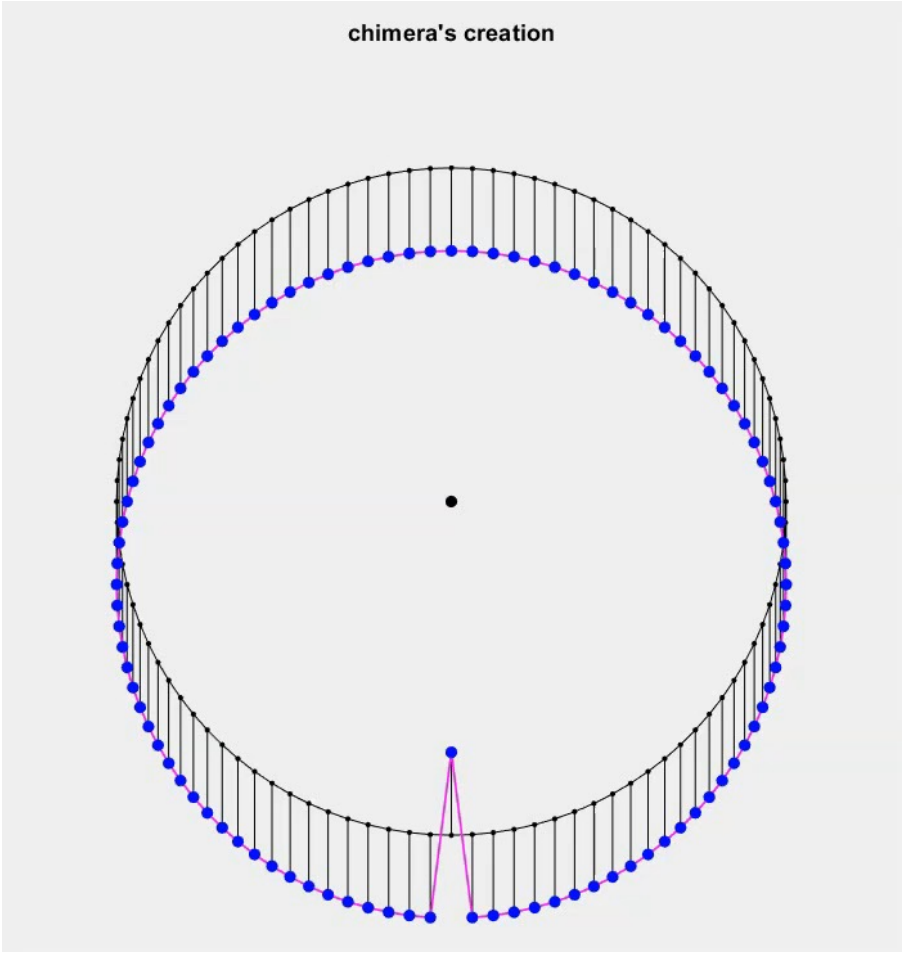
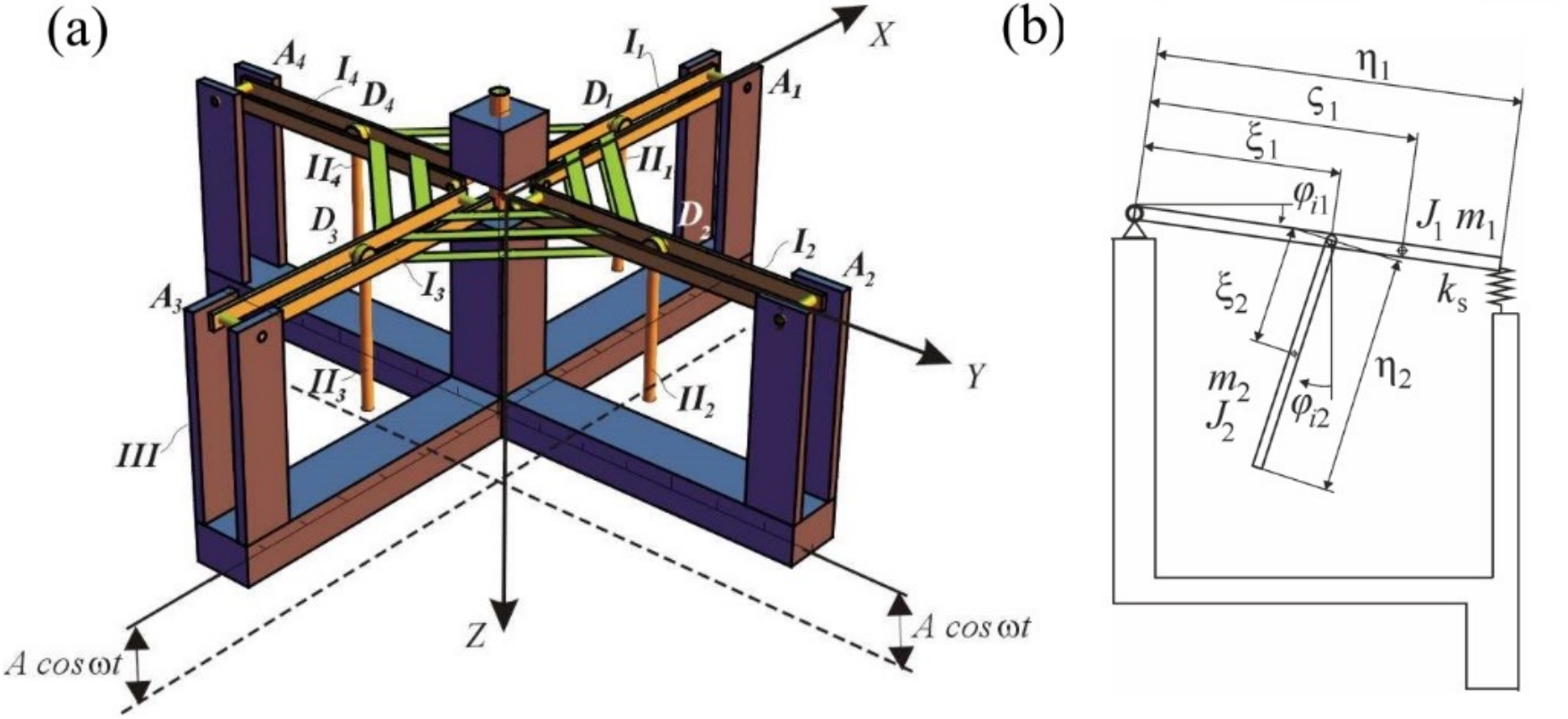

3-headed : 46.8°(12;u) / 180°(19;o) / 324°(14;u)

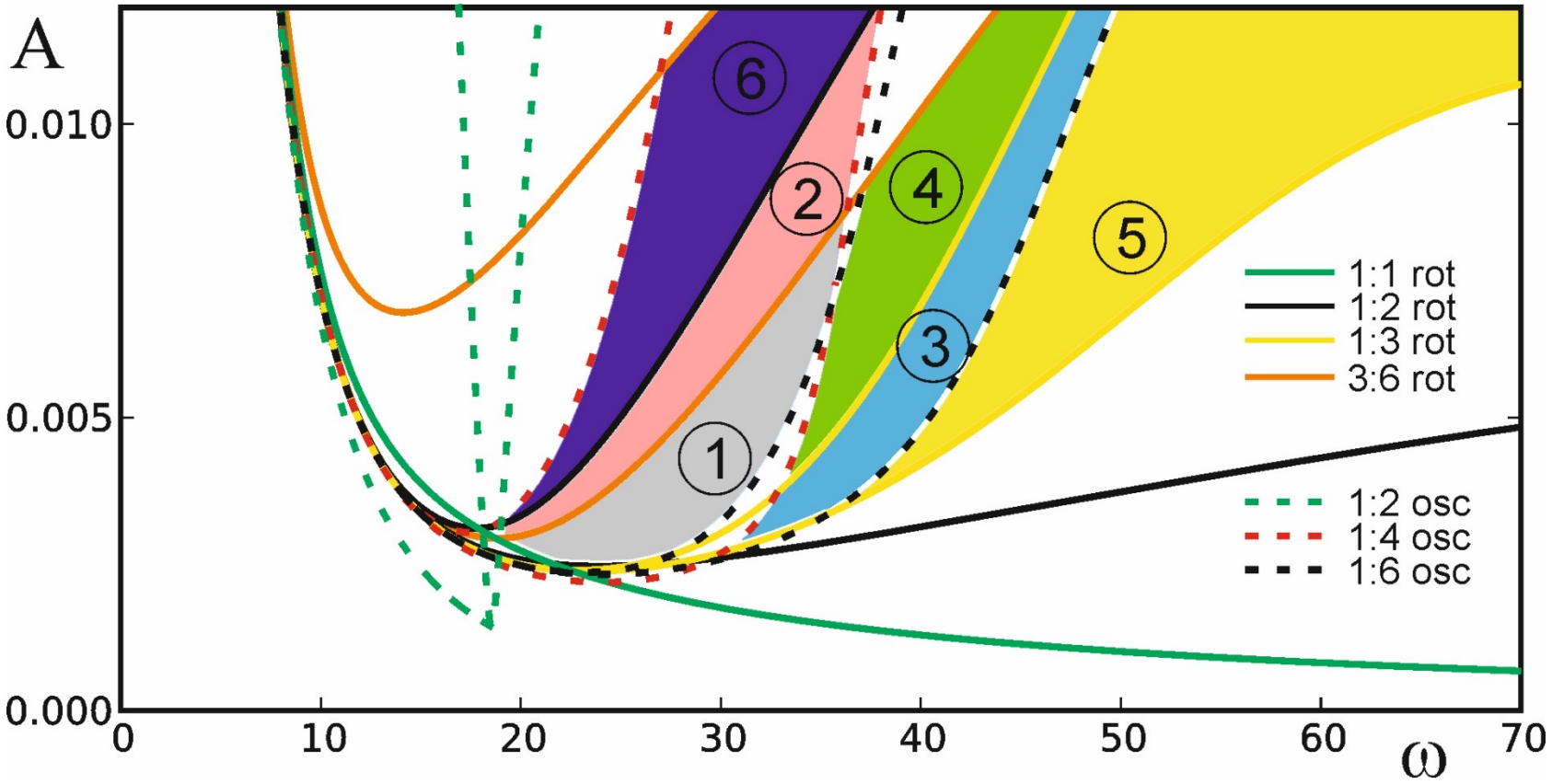

chimera's creation

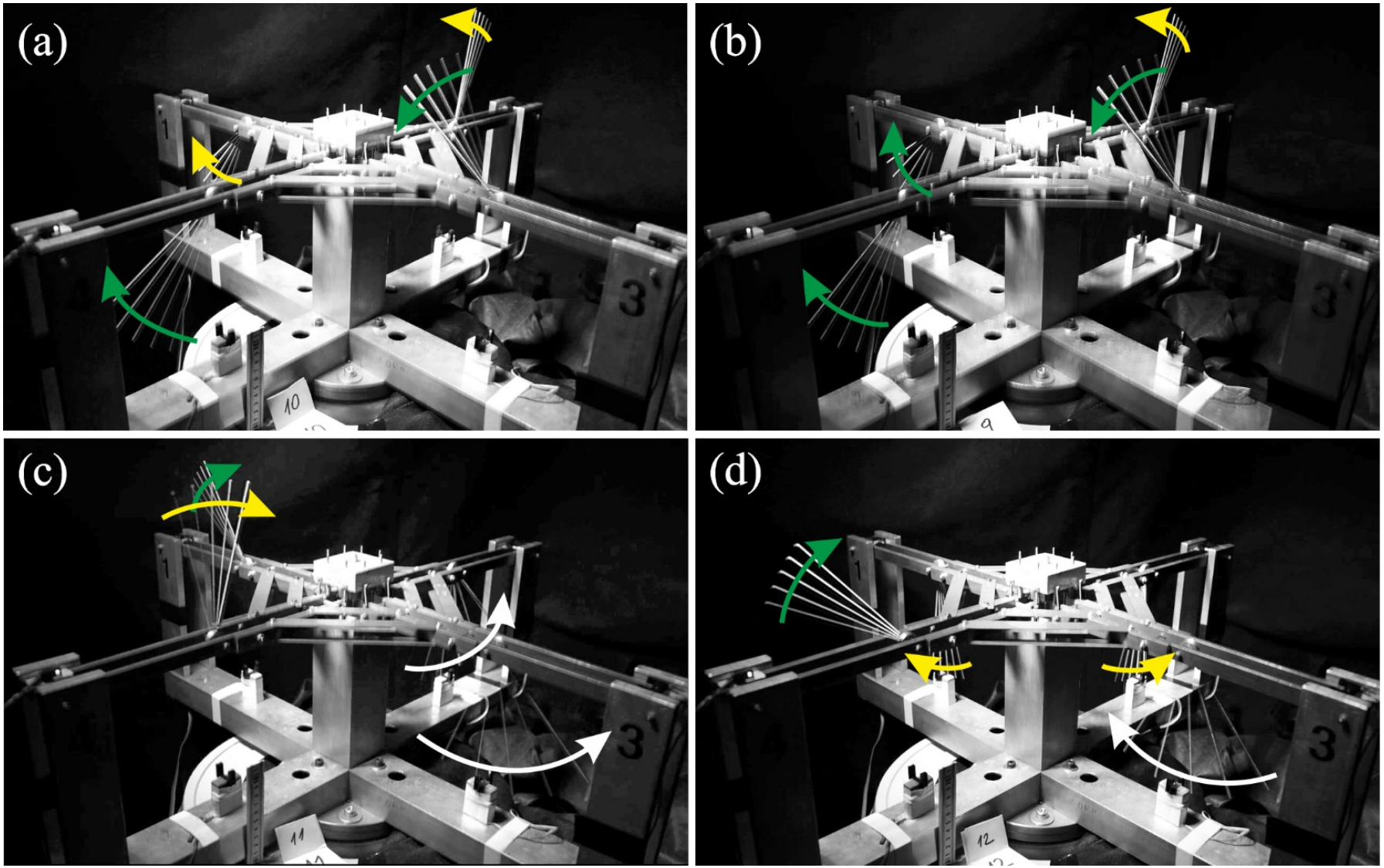
Search for the smallest chimera

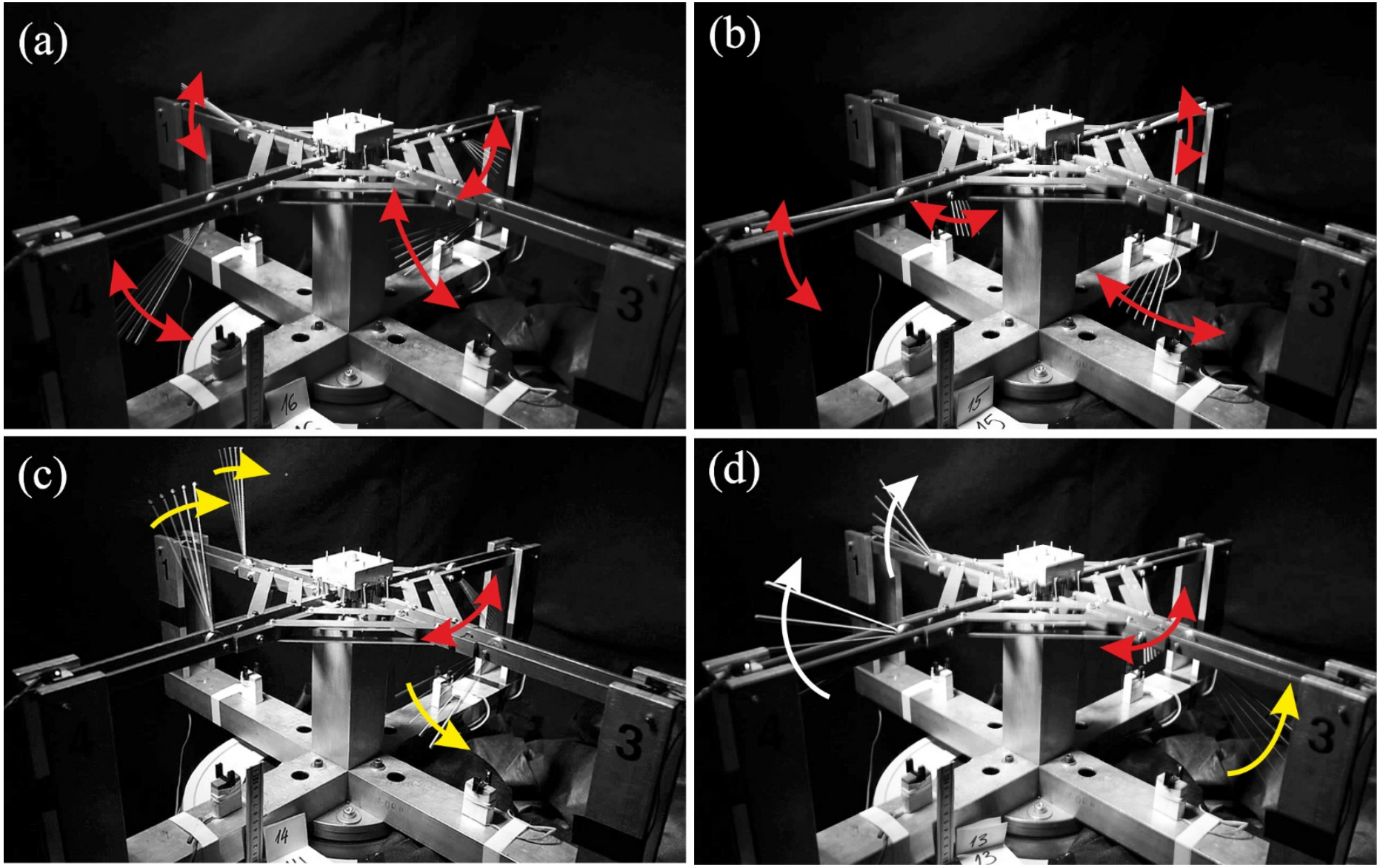
four coupled double pendula

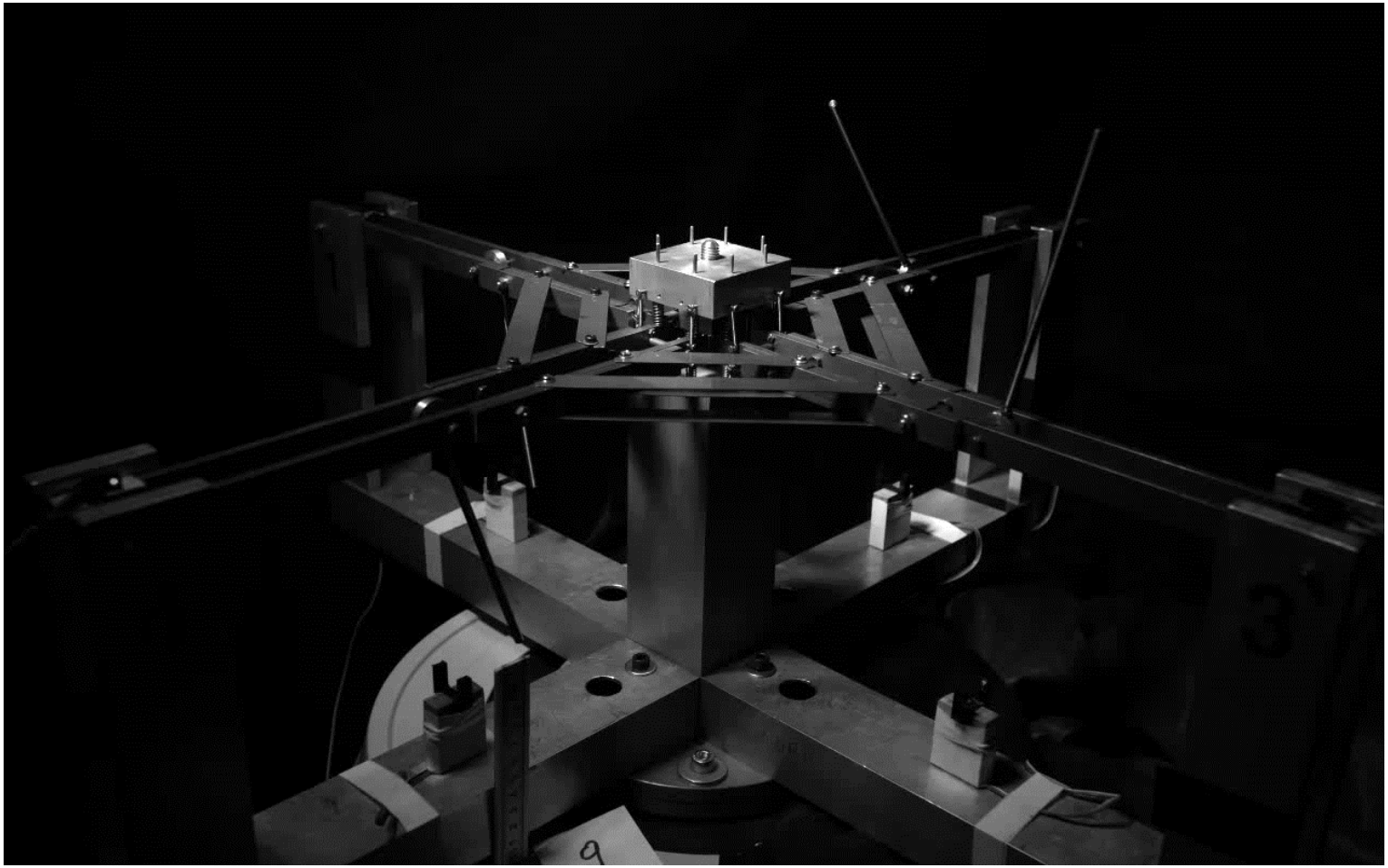


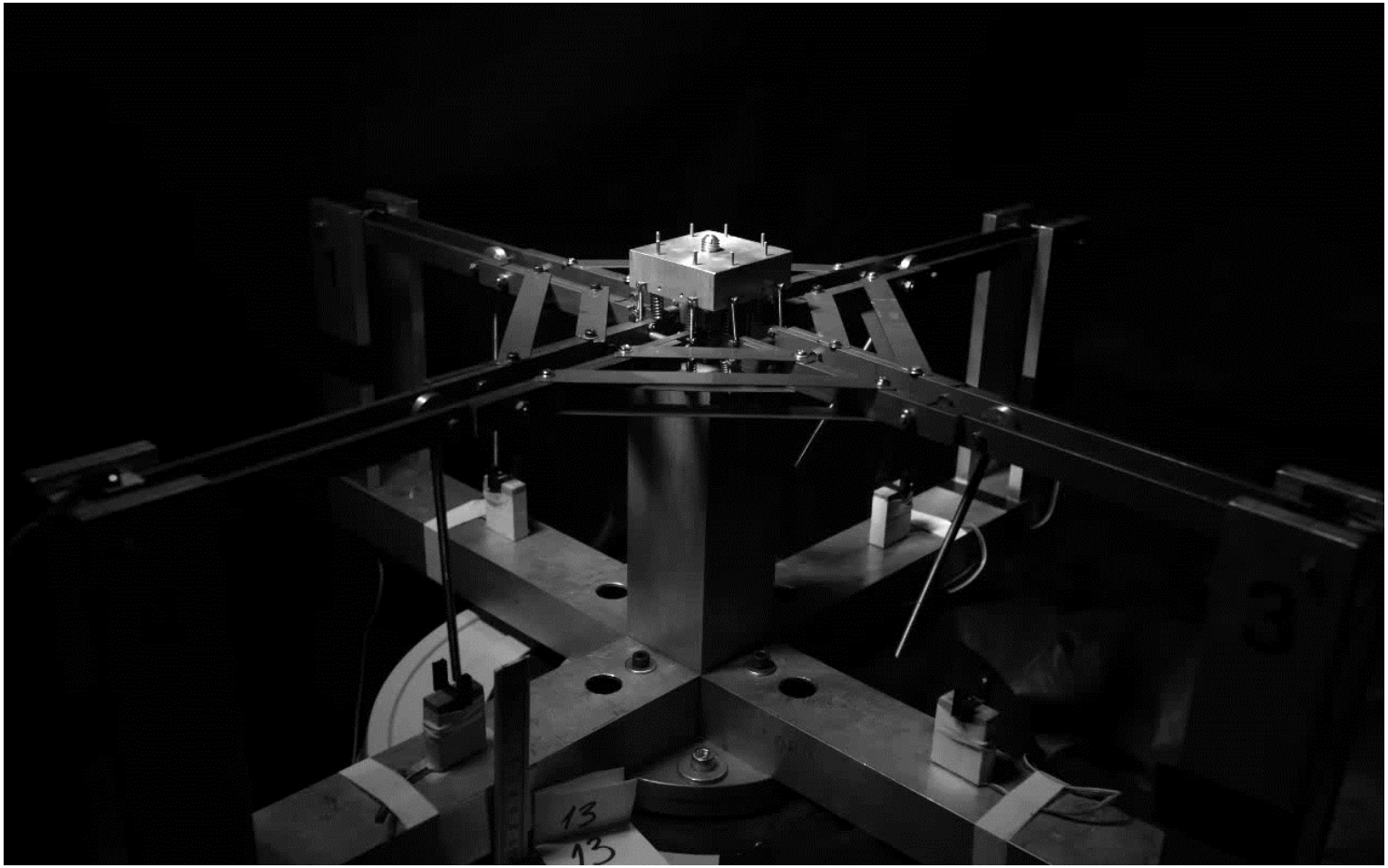
Multistability of the single pendulum



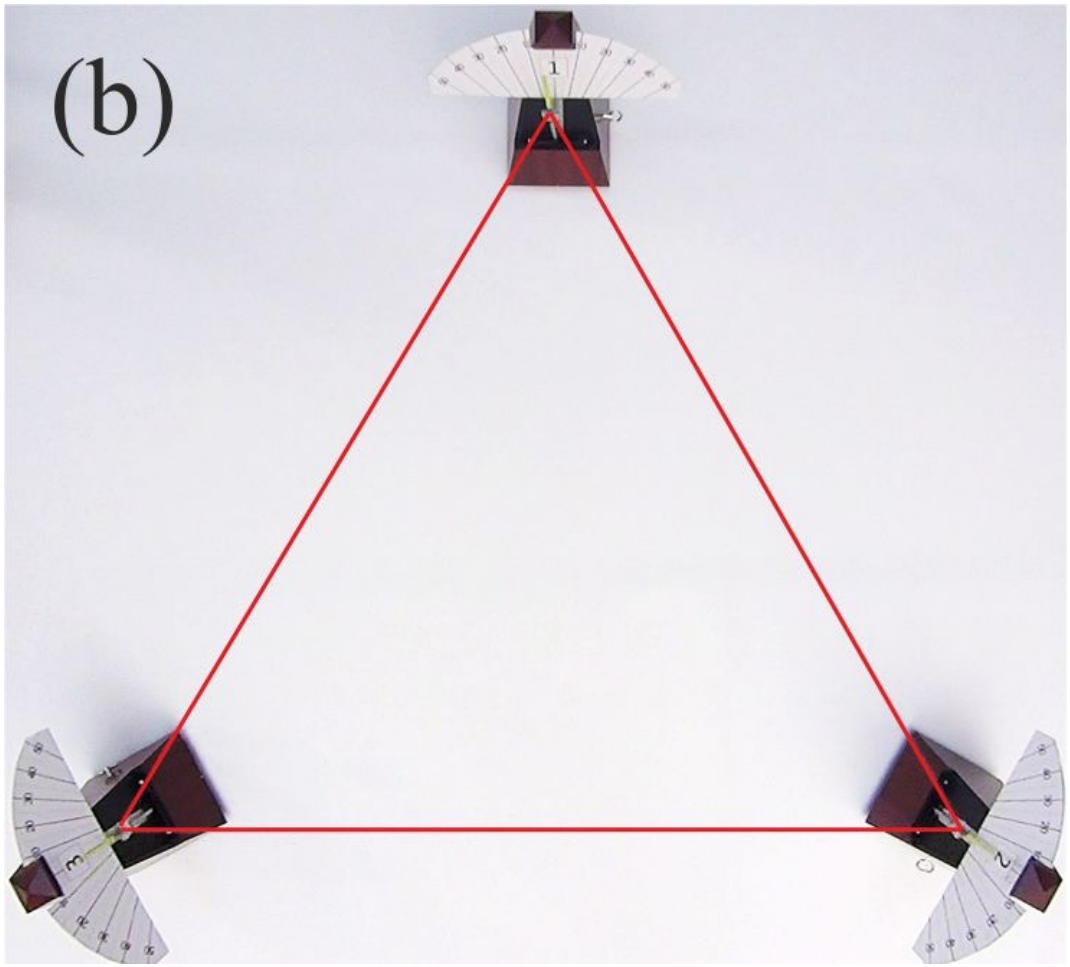

(a)
(b)
(c)
(d)

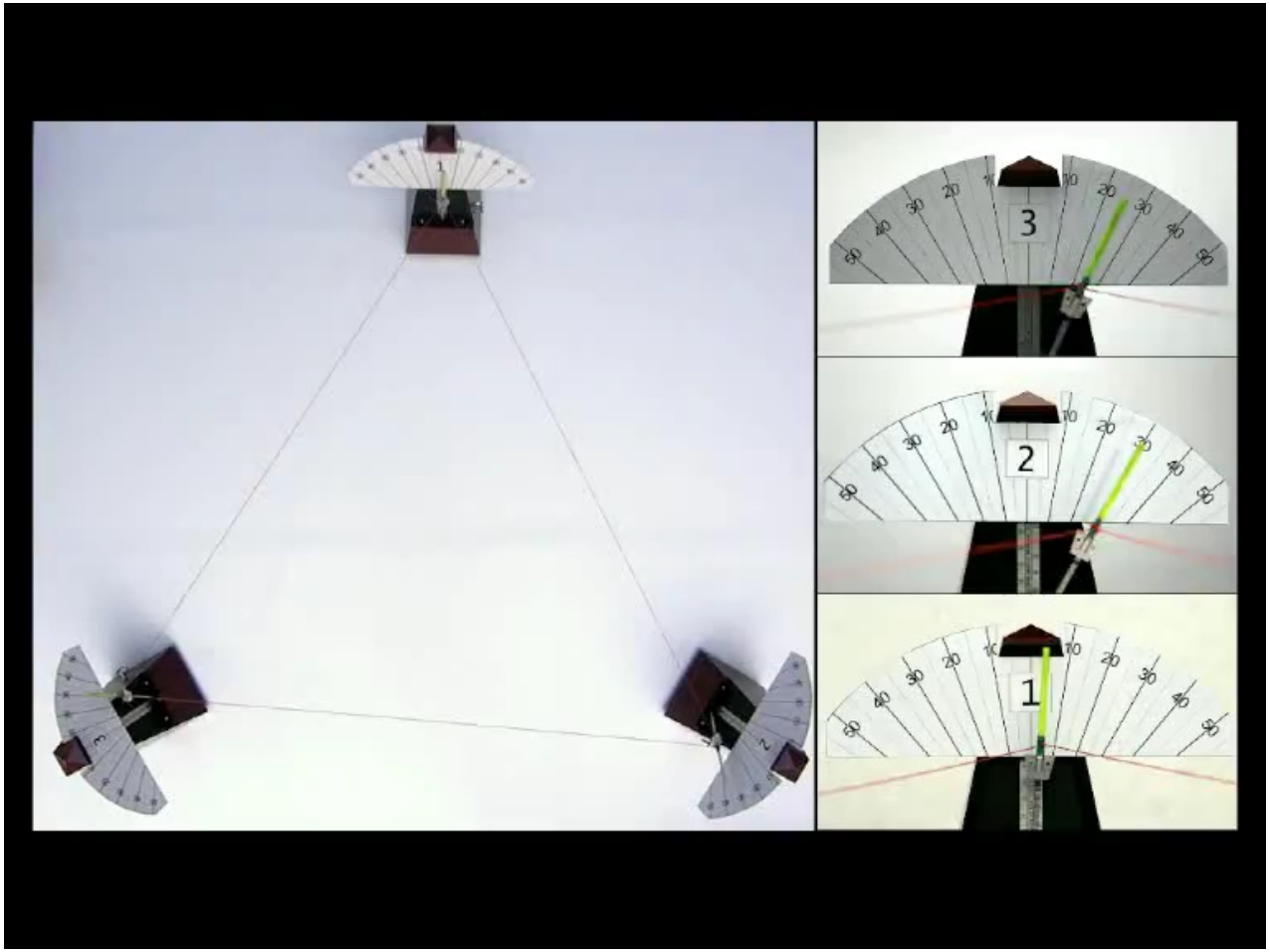

(a)
(b)
(c)
(d)

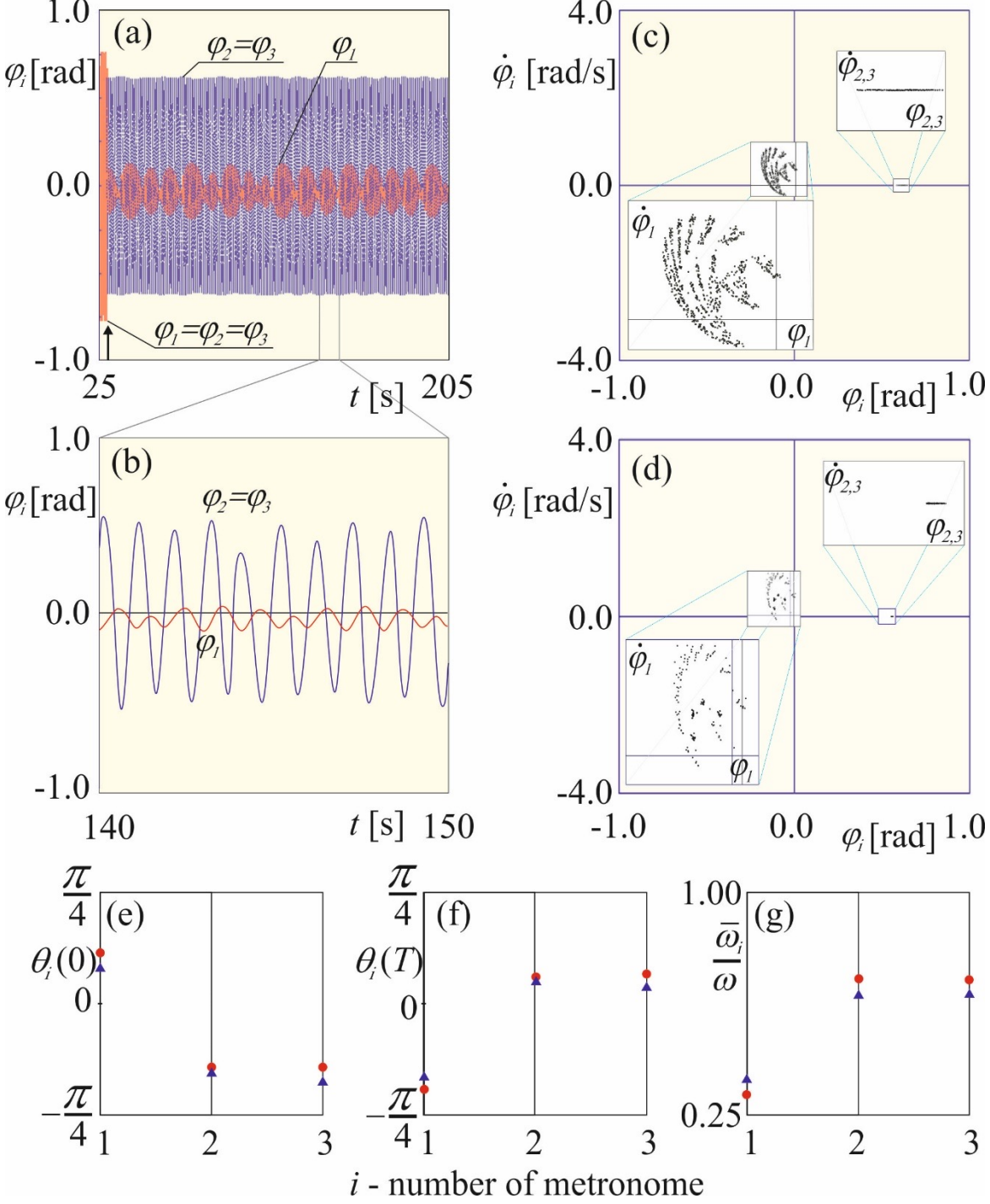


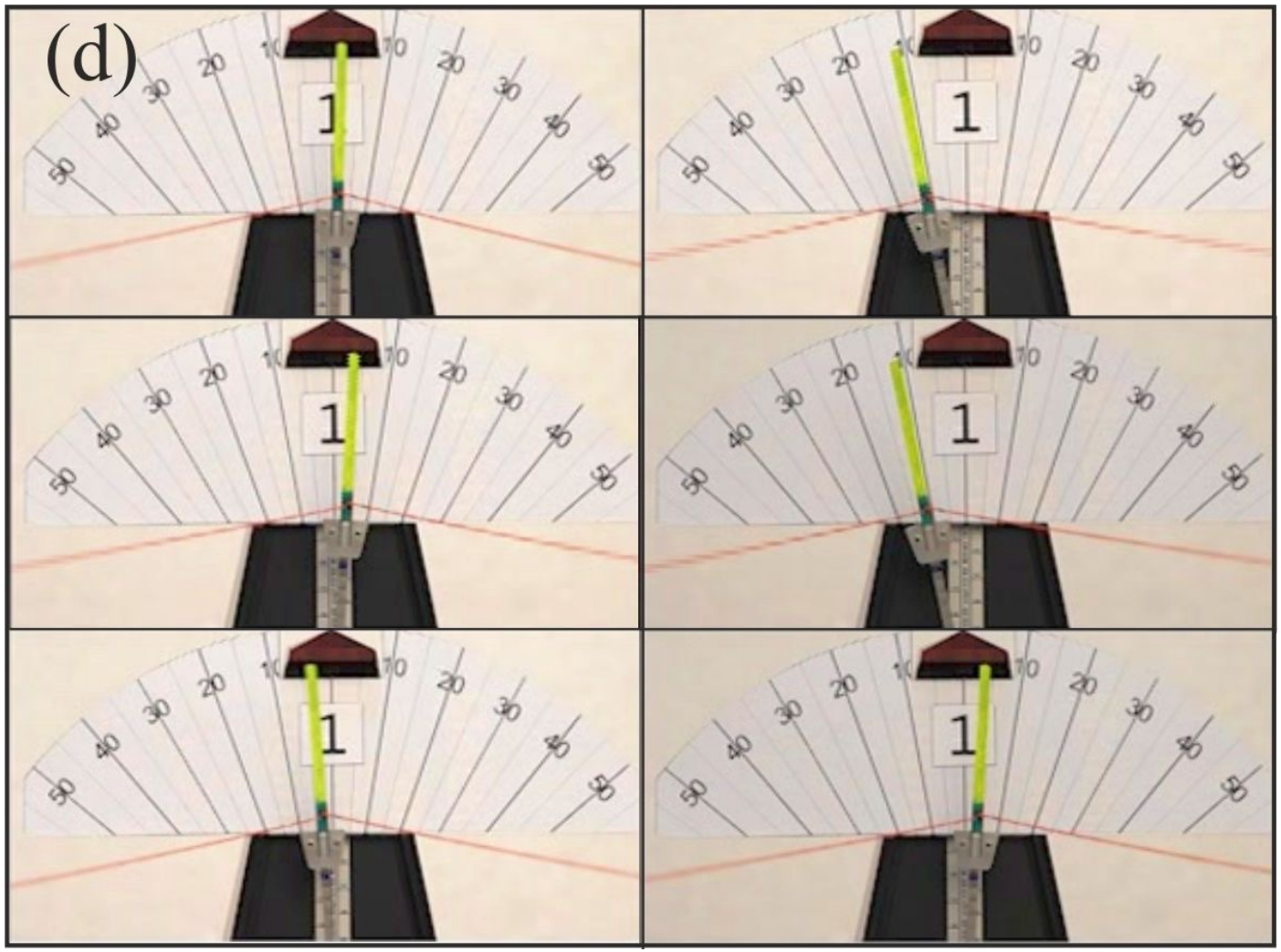

13
13

the smallest chimera




3
2
1

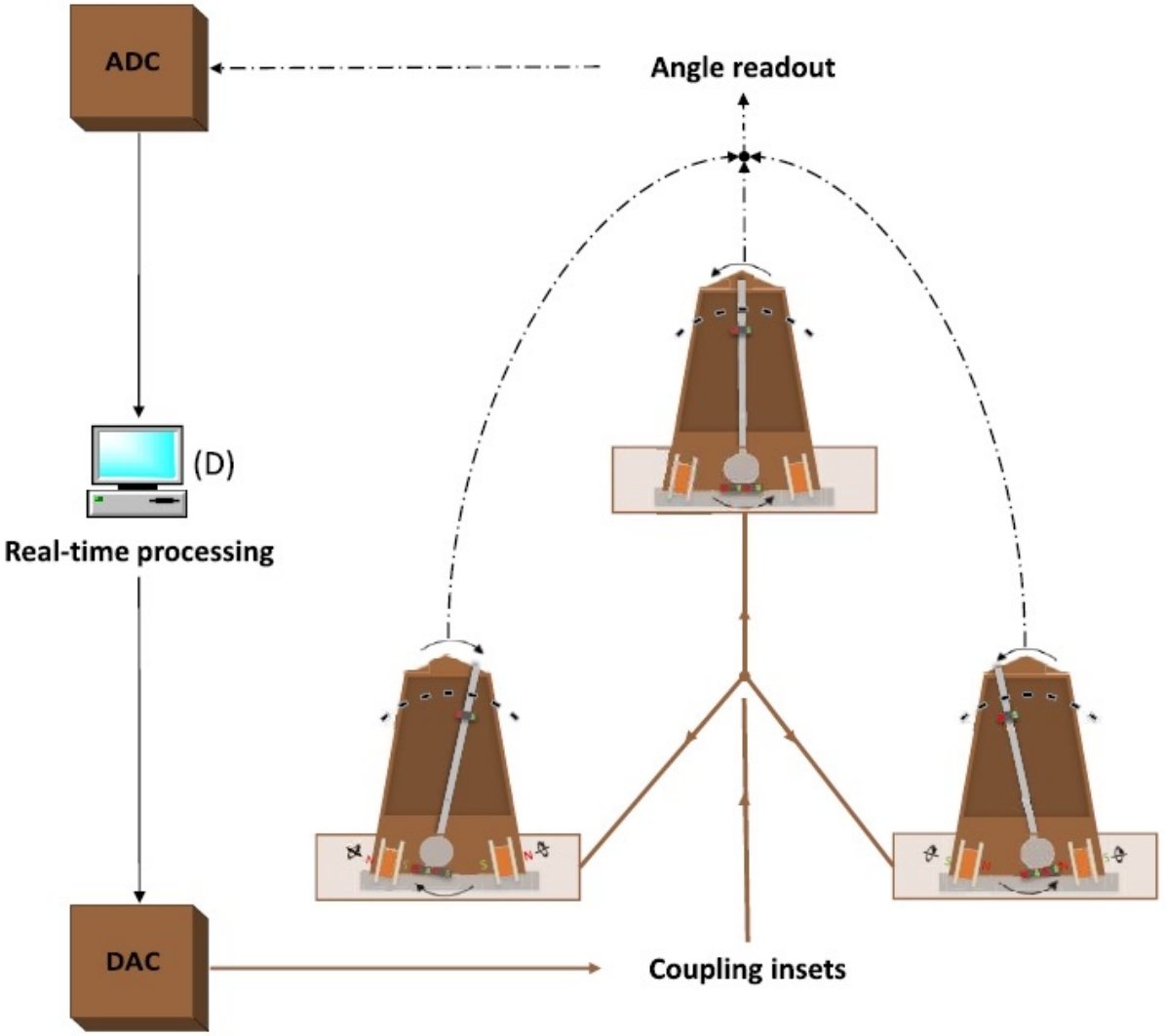

(a)
(b)
(c)
(d)
(e)
(f)
(g)
φ_i[rad]
$\dot{\varphi}_i$ [rad/s]
t [s]
$\varphi_2=\varphi_3$
φ_1
$\varphi_1=\varphi_2=\varphi_3$
$\dot{\varphi}_{2,3}$
$\theta_i(0)$
$\theta_i(T)$
$\frac{\bar{\omega}_i}{\omega}$
i - number of metronome

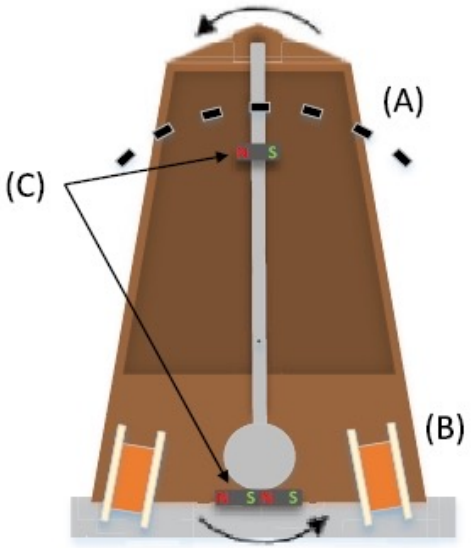

(d)

phase-lag all-to-all coupling

$$B\ddot{\theta}_i + \varepsilon\dot{\theta}_i + mglsin\theta_i + M_D + H(\theta_i - \theta^*)\frac{\mu}{3}\sum_{i=1}^{3}\sin(\theta_j - \theta_i - \alpha)$$

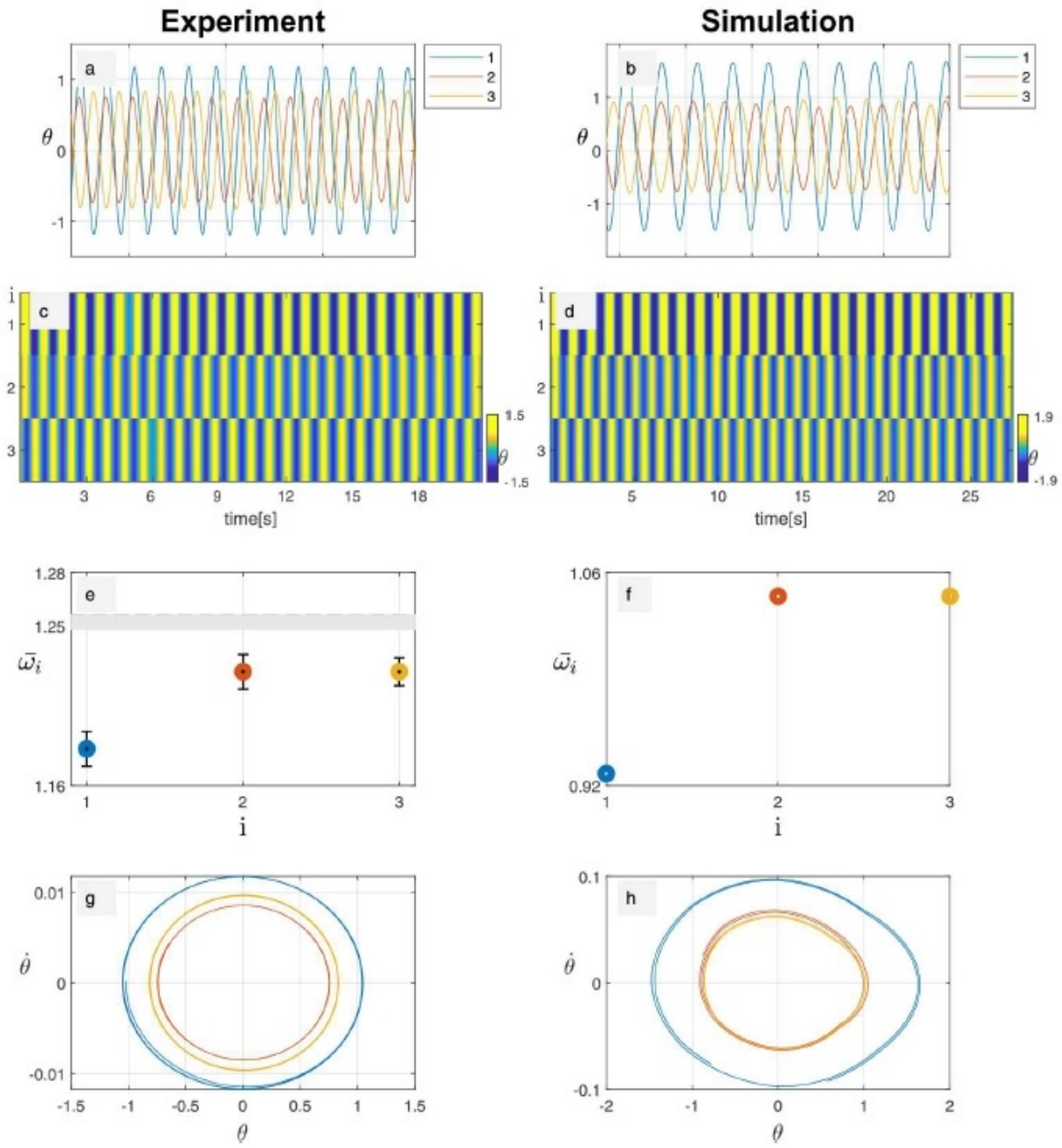
Sinusoidal coupling term is controlled by the Heaviside-like step function $H(\theta - \theta^*)$ where $\theta^* \approx 30^\circ$

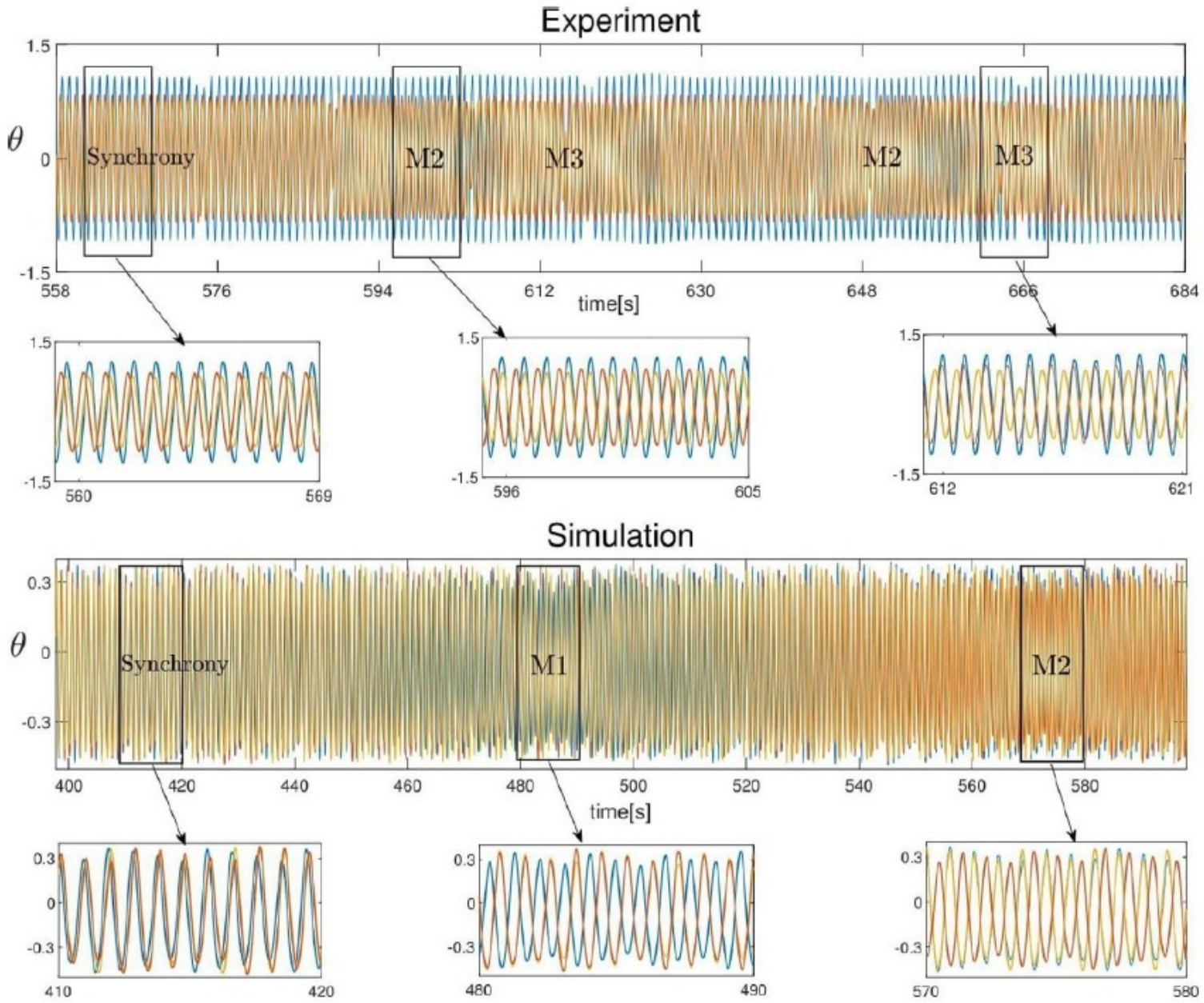




Eur. Phys. J. Special Topics **229**, 2205–2214 (2020)

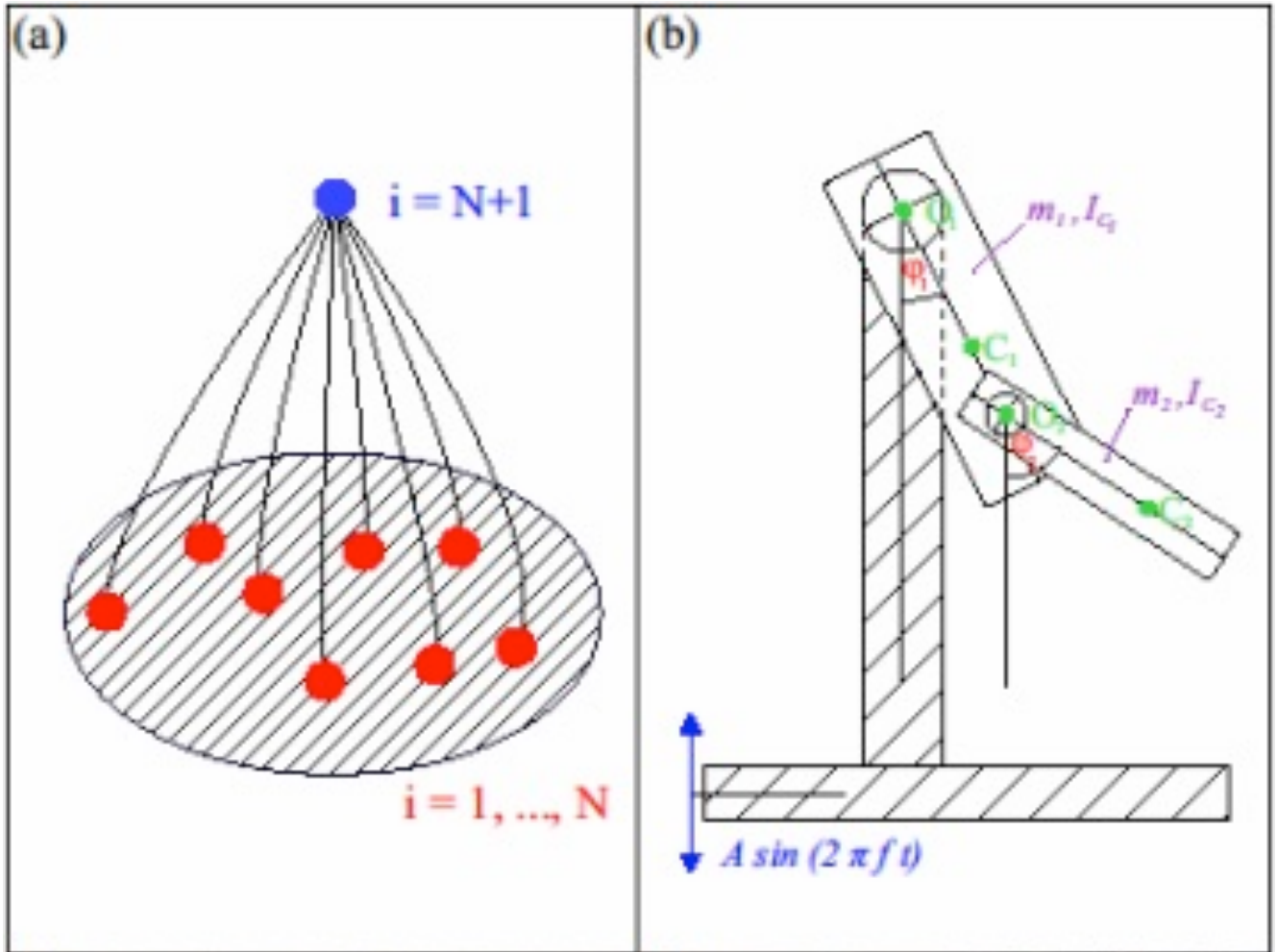
The three metronomes are equipped with an array of magnetic sensors (Hall sensors, A) for contactless measuring of the rod's position and two coils (B) for applying magnetic force onto the counter weight. To increase magnetic signal and force Neodymium magnets are attached to rod and counterweight (C). Hall sensor signals (dashed line) are digitized in the ADC unit. In real-time processing unit (D), the angles of all metronomes are reconstructed based on the ADC data and the coupling terms are calculated according to equation (1). In DAC unit the coupling terms are converted to electrical currents (solid brown lines) passing through the coils and generating magnetic forces

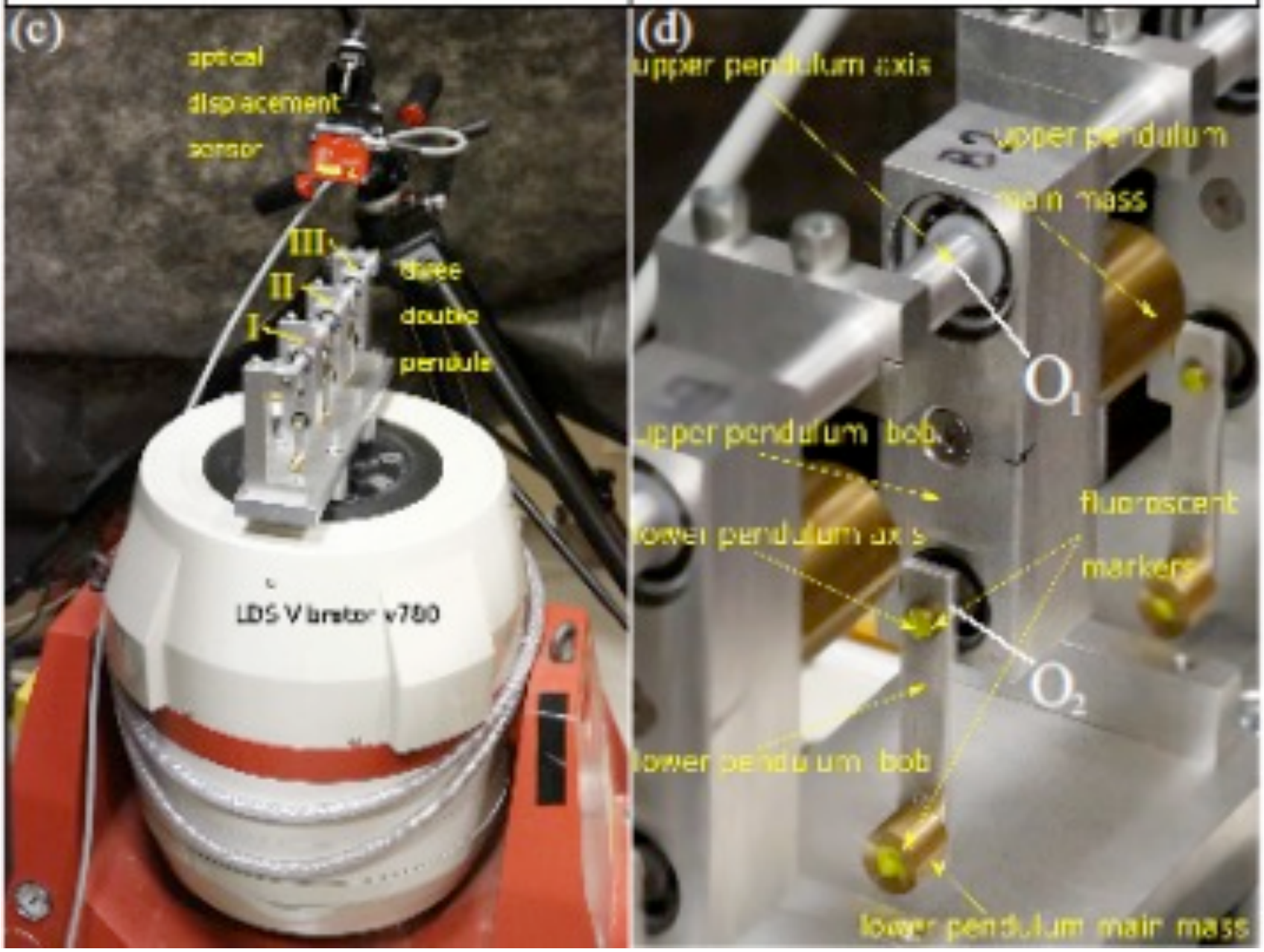

Experiment
Simulation
a
b
c
d
e
f
g
h
1
2
3
θ
time[s]
i
$\bar{\omega}_i$
$\dot{\theta}$

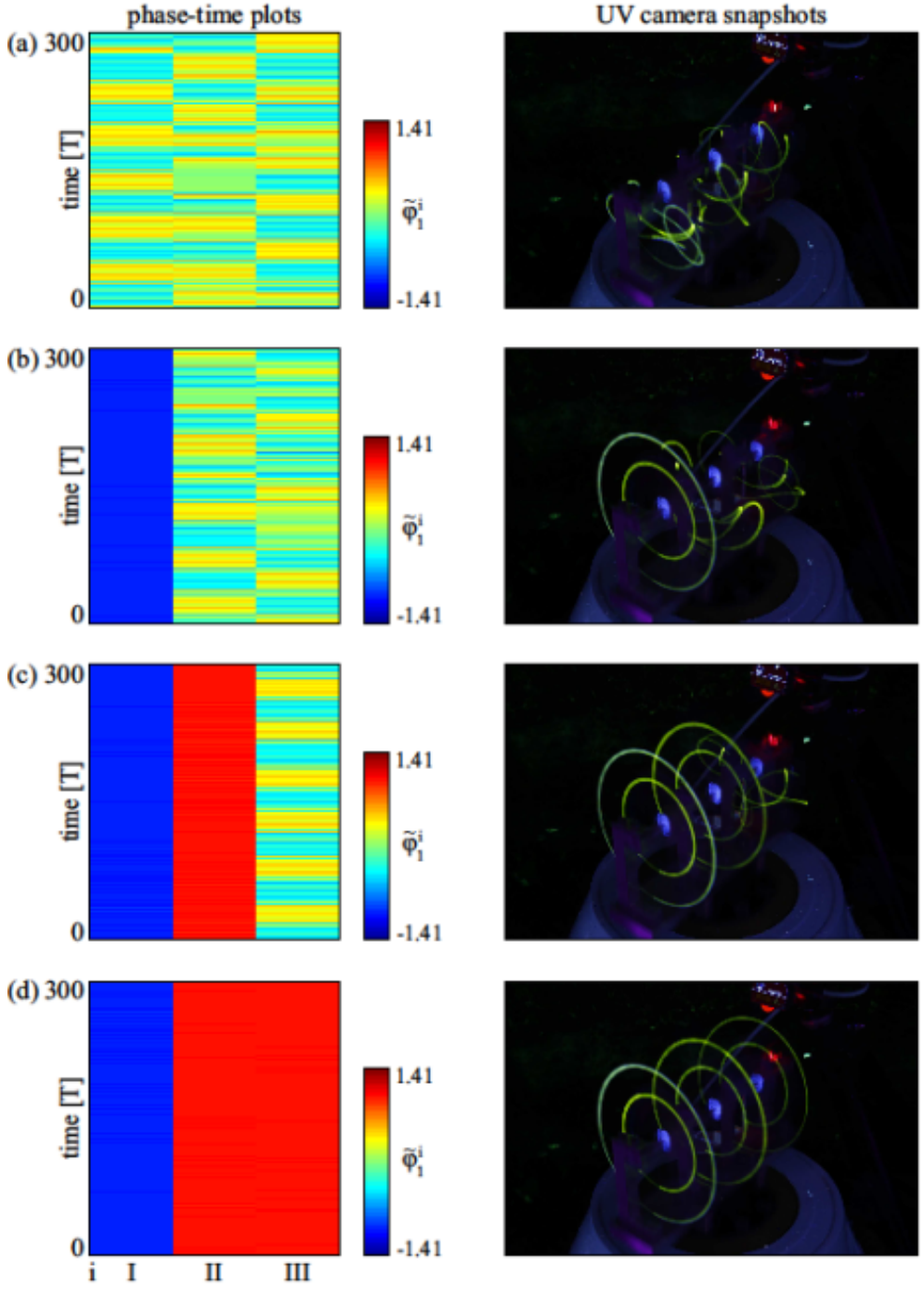

Experiment
θ
Synchrony
M2
M3
M2
M3
time[s]
Simulation
θ
Synchrony
M1
M2
time[s]

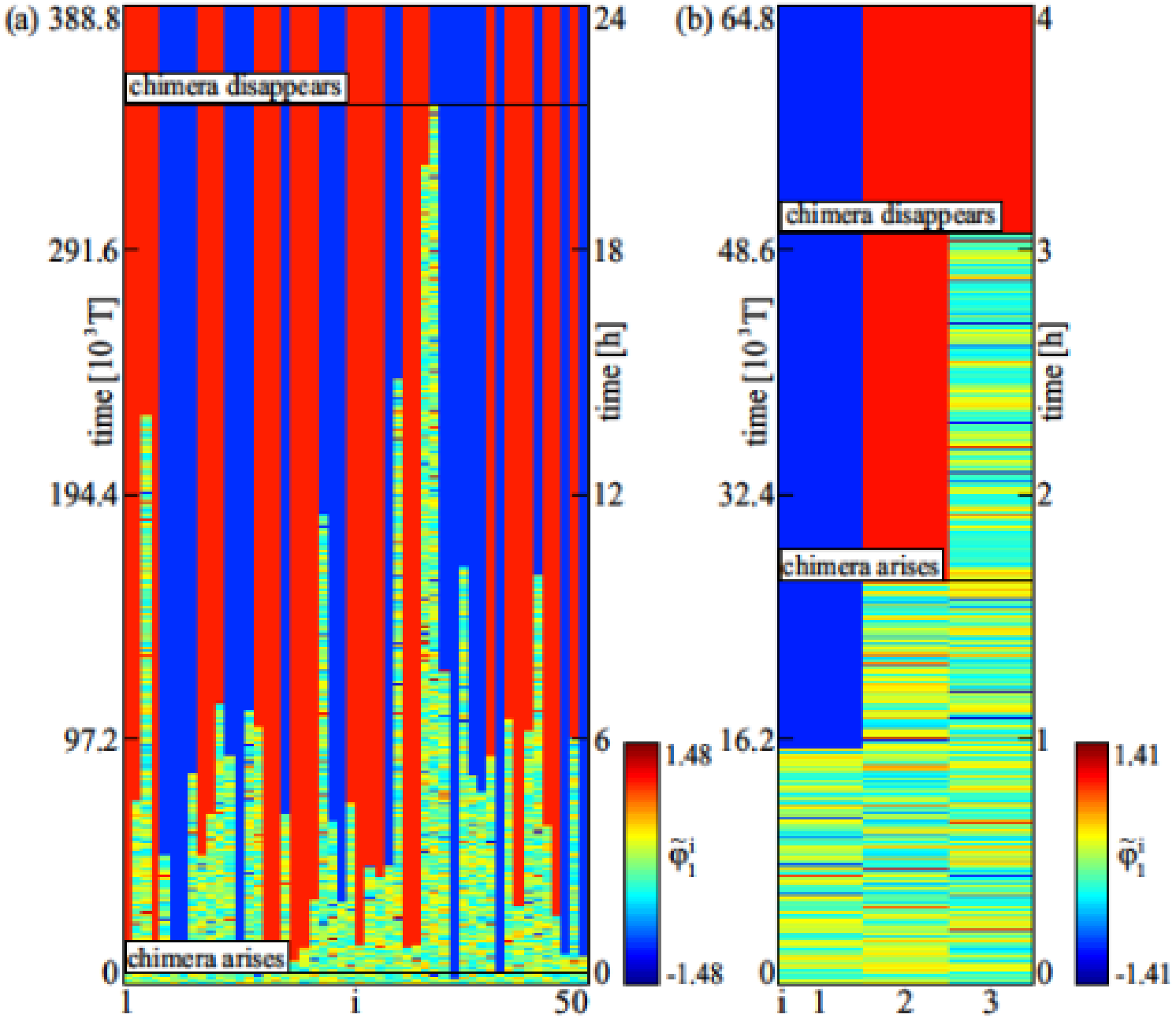
Transient chimera states

(Chaos **30**, 011102 (2020))



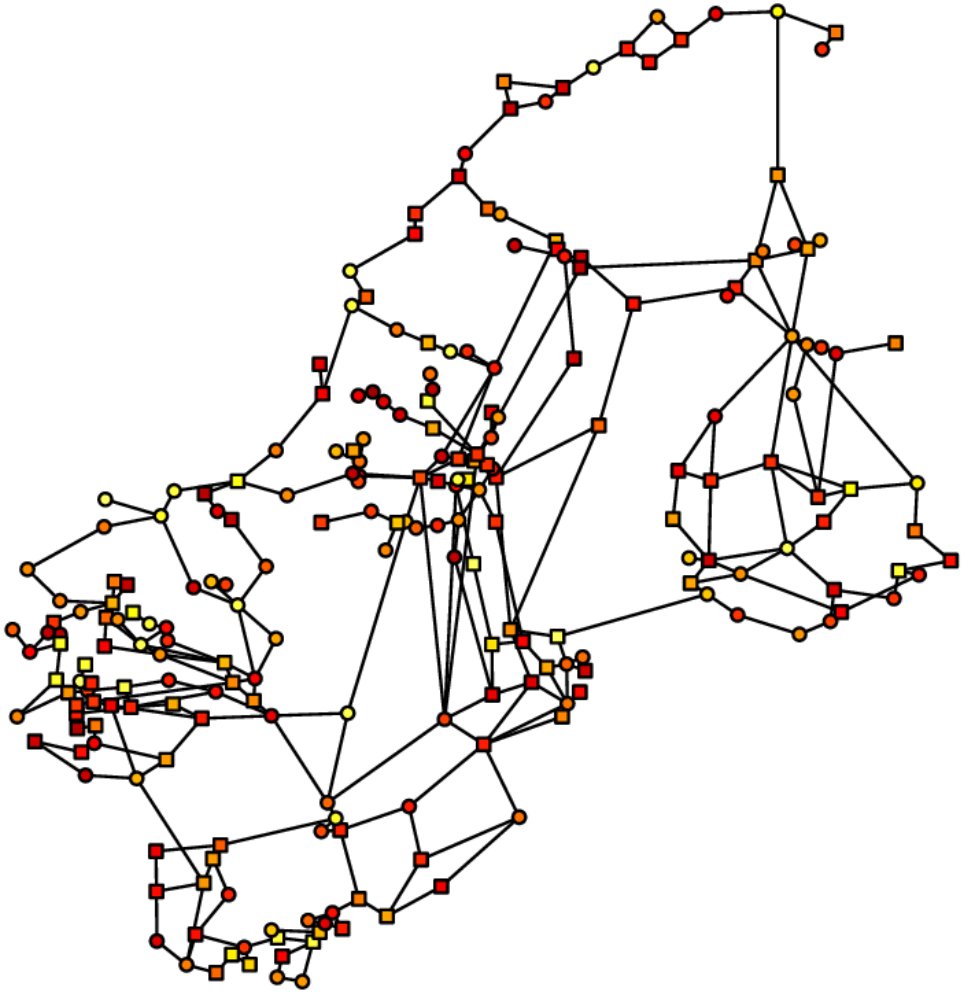

(c)
optical
displacement
sensor
III
II
I
three
double
pendula
LDS Vibrator v780
(d)
upper pendulum axis
upper pendulum
main mass
O1
upper pendulum bob
fluorescent
markers
lower pendulum axis
O2
lower pendulum bob
lower pendulum main mass

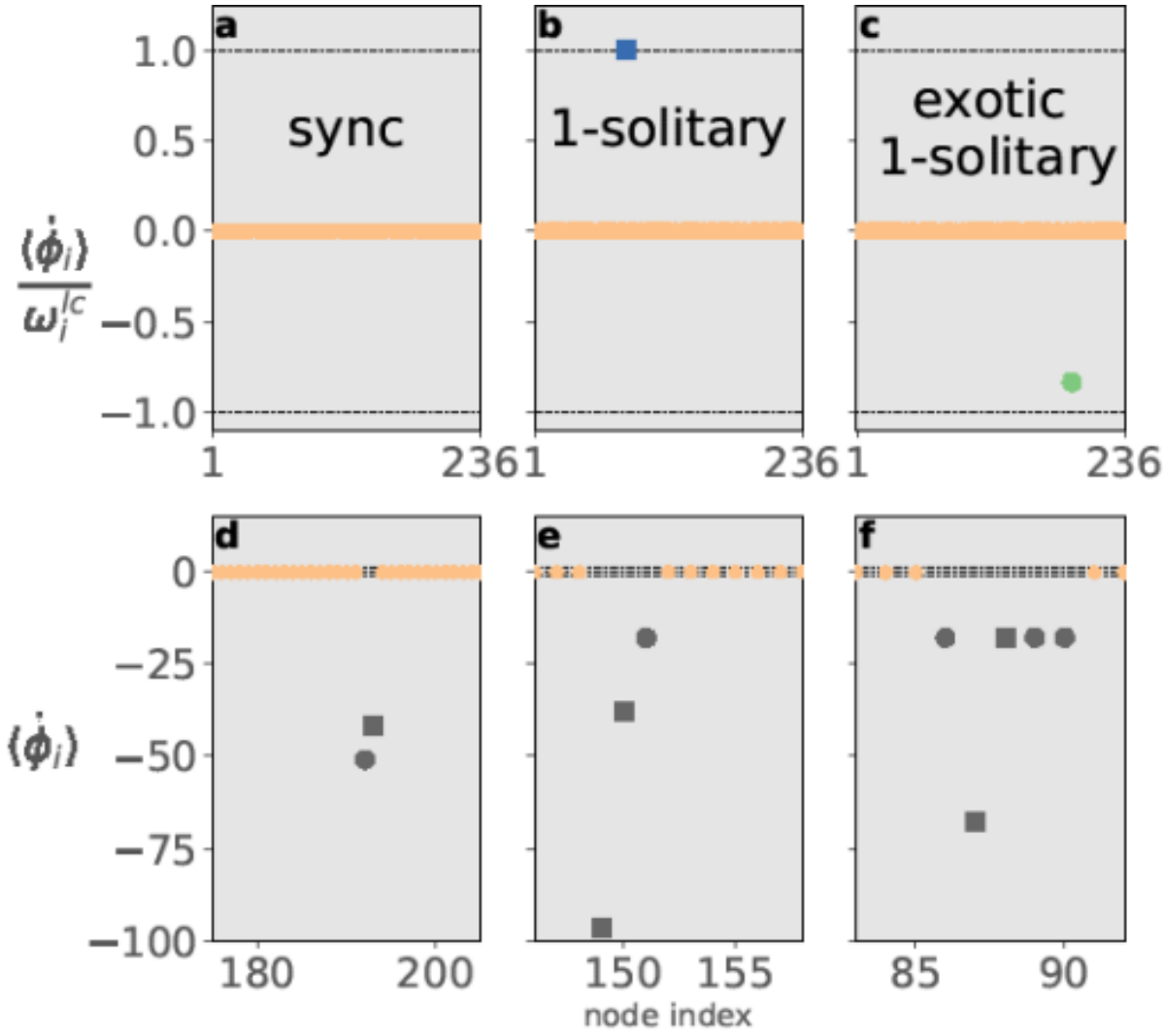

phase-time plots
UV camera snapshots
(a) 300
(b) 300
(c) 300
(d) 300
time [T]
0
1.41
-1.41
$\tilde{\varphi}_1^i$
i
I
II
III


(a)
388.8
chimera disappears
291.6
194.4
97.2
0
chimera arises
time [10^3 T]
24
18
12
6
0
time [h]
1.48
-1.48
φ_1^i
1
i
50
(b)
64.8
chimera disappears
48.6
32.4
chimera arises
16.2
0
time [10^3 T]
4
3
2
1
0
time [h]
1.41
-1.41
φ_1^i
i
1
2
3

Blackout

Frank Hellmann, Paul Schultz, Patrycja Jaros, Roman Levchenko, TK, Juergen Kurths, Y. Maistrenko, Nature Communications (2020)




a
b
c
sync
1-solitary
exotic
1-solitary
1.0
0.5
0.0
−0.5
−1.0
$\frac{\langle\dot{\phi}_i\rangle}{\omega_i^{lc}}$
1
2361
1
2361
1
236
d
e
f
0
−25
−50
−75
−100
$\langle\dot{\phi}_i\rangle$
180
200
150
155
85
90
node index

The six panels depict the time-average $\langle \dot{\varphi}_i \rangle$ of the angular velocity vs. the node index i. The values in the are normalised to the respective natural $\frac{P_i}{D_i}$. **a** phase locking, **b** 1-solitary, **c** exotic 1-solitary, **d** 2-solitary, **e** 3-solitary and **f** a cluster that is the network but synchronised internally, with a solitary in it. The plots in **e**, **f** are restricted to a relevant subset nodes for better visualisation. The asymptotic regimes correspond to different initial conditions of the Scandinaviangrid with a 1:1 prosumer scenario, strongly coupled (K+) with standard damping (D) and $\alpha = 0.24$.

Thank you !