Sync and desync in higher-order networks and several other topics

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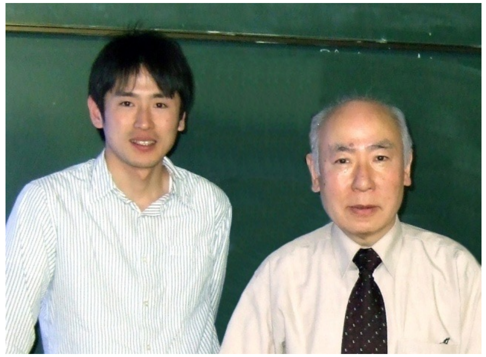
(Talk in DDAP13 on July 3, 2025)

Self introduction

- ～2003 : Kyoto Univ. (master and PhD courses)
  - Supervised by Yoshiki Kuramoto
  - DDAP2 in Hangzhou, my first international conference
- ～2006 : Fritz Haber institute in Berlin (postdoc)
  - In the group of Alexander Mikhailov
  - Observing BPO rhythm every week
  - Playing fussball every week
  - Start collaboration with chemists
- ～2008 : Hokkaido univ. (postdoc)
  - In the group of Yasumasa Nishiura
  - Playing with snow
  - Start collaboration with biologists
- ～2018: Ochanomizu univ (assistant, associate professor)
- ～Now: Univ Tokyo



ECC11

Venue

Takeda Hall

The University of Tokyo

Tokyo, Japan

Two Train Station Routes

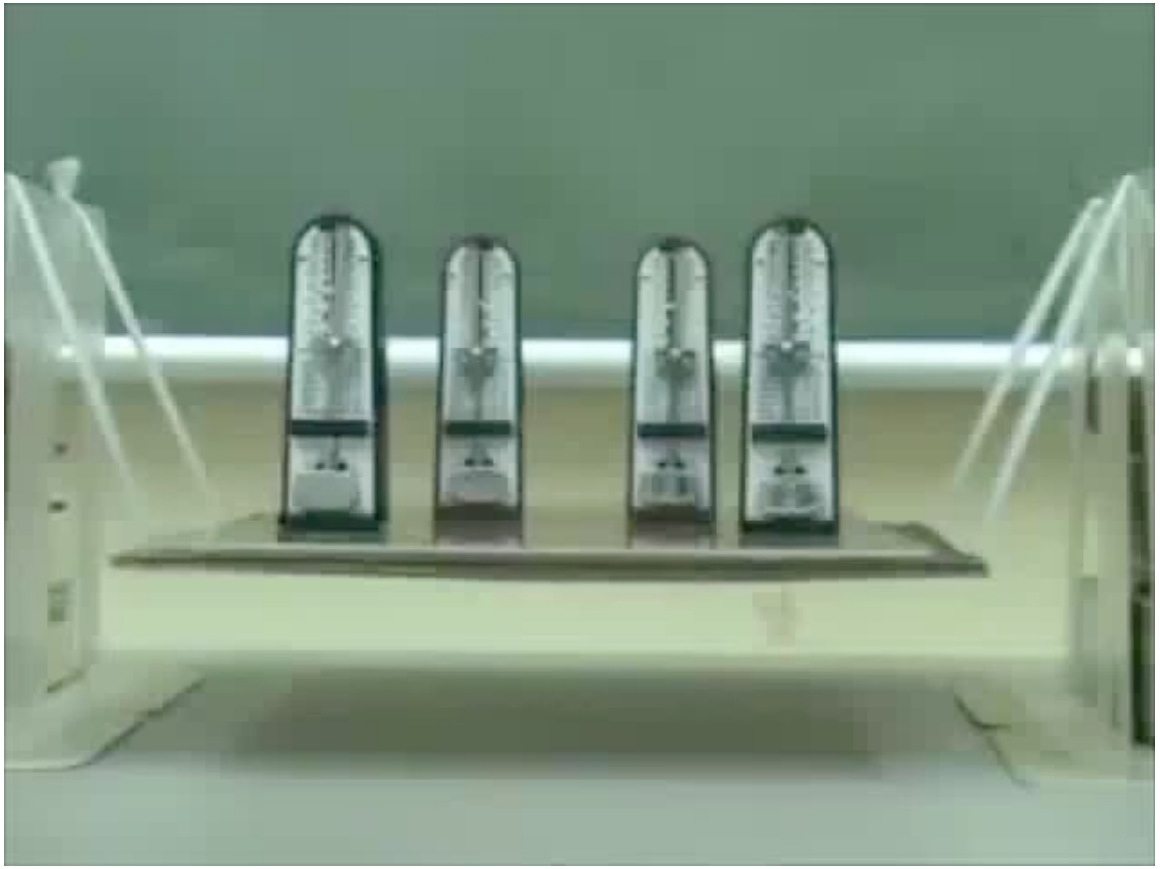
(Google Map)

Yoshiki Kuramoto will give a talk (in person!)

https://sites.google.com/edu.k.u-tokyo.ac.jp/ecc11/home

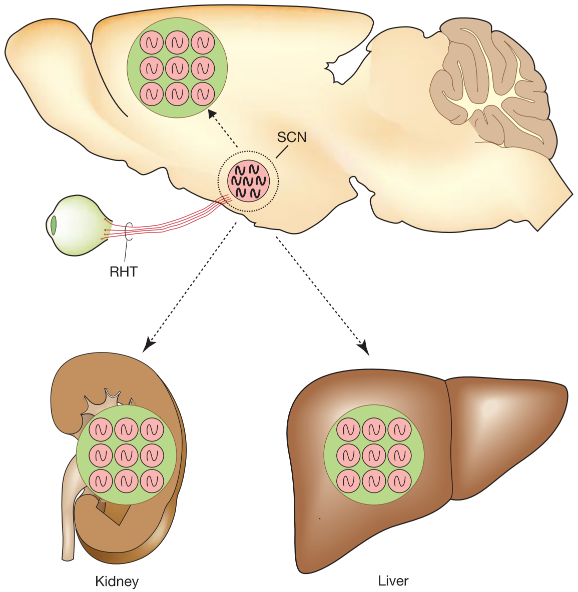
ECC11 twitter

Synchronization of metronomes:
pay attention to interesting transient behavior



Youtube: Synchronization of four metronomes on a suspension bridge

Circadian clock

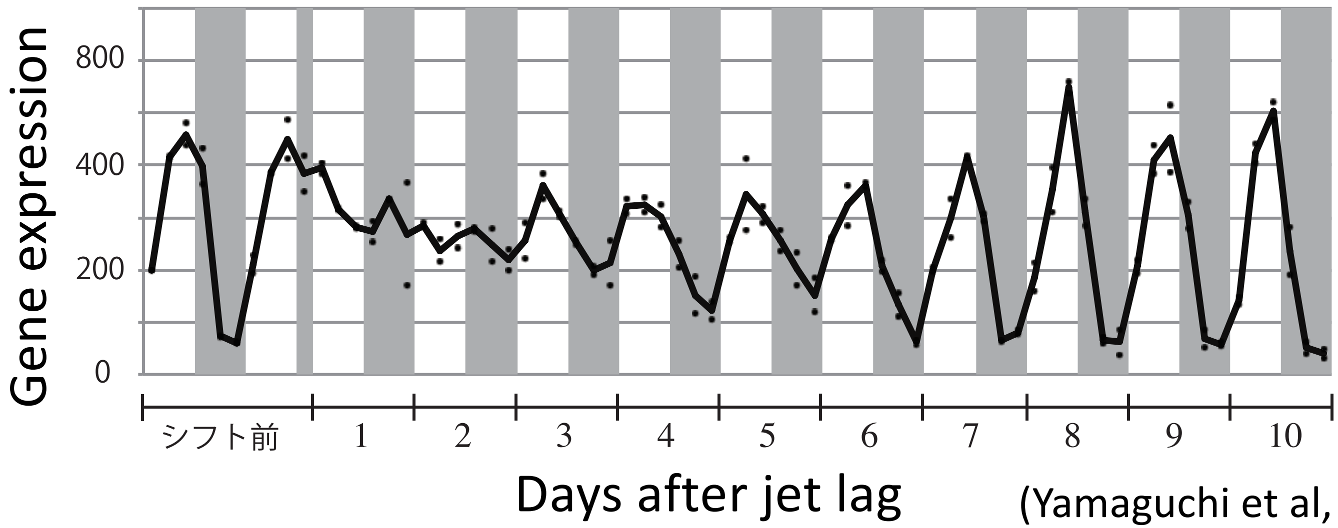


(Reppert & Weaver, 2002 Nature)

Synchronization of clock gene expression among SCN cells (Yamaguchi et al, Science 2003)

Circadian clock in jet lag

When a mouse is subjected to advancing phase shift of light-dark cycles (similar to a trip from Europe to Japan), oscillations of gene expression disappear for a while:



(Yamaguchi et al, Science 2013)

This “oscillation quenching” is thought to be a primal cause of heavy jet-lag symptoms.

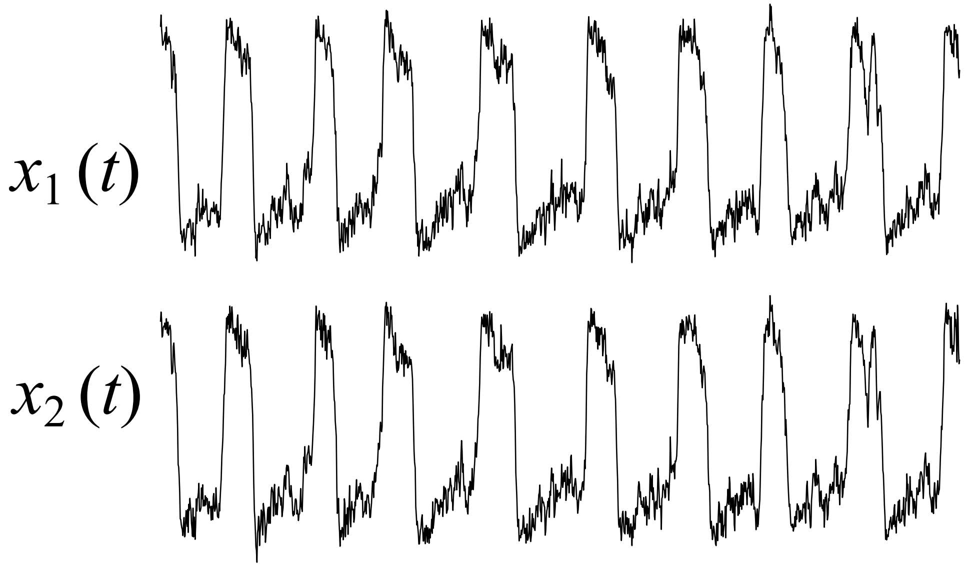
My wish is to conduct theoretical research that will be useful in the real world

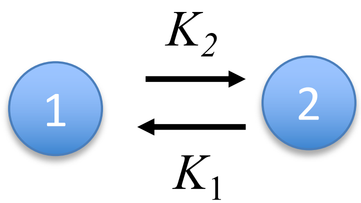
- Developing theoretical frameworks for data-driven approaches
  - Network inference from rhythmic signals
    [Matsuki, HK, Kobayashi, to be submitted] Poster
  - Network inference from spike data
    [Mori & HK, PNAS (2022)] (talk on Thursday)
  - Forecasting dynamics using reservoir computing
    [Kuno & HK, arxiv]
- Oscillation quenching
  - Metronomes [Kato & HK, Sci. Rep. (2023)]
  - Kuramoto model with stochastic turnover [Ozawa & HK, to appear in PRL]
- Energetics of synchronization
  - Coupled Heat engines [Yin, Izumida, HK, PRR (2023)]
- Higher order networks
  - Slow desynchronization process in noisy oscillators [Marui & HK, arxiv]

Developing theoretical frameworks for data-driven approaches

network inference

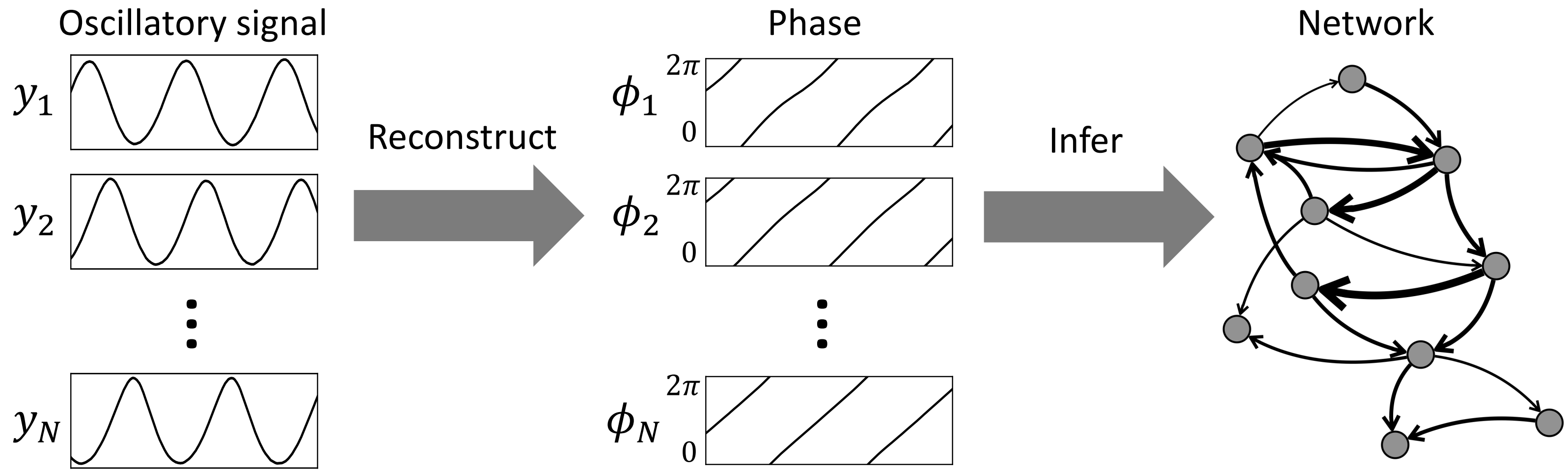
Suppose that we may observe oscillatory signals $x_i(t)$ from a network of noisy oscillators and want to infer coupling network between oscillators.





Inference using phase models

One idea:



$$\dot{\phi}_1 = \omega_1 + K_1 \sin\left(\phi_2 - \phi_1\right) + \sqrt{D_1}\xi_1(t),$$

$$\dot{\phi}_2 = \omega_2 + K_2 \sin\left(\phi_1 - \phi_2\right) + \sqrt{D_2}\xi_2(t),$$

by which coupling strength K_i and noise strength D_i may be inferred.

Many studies have been conducted along this line

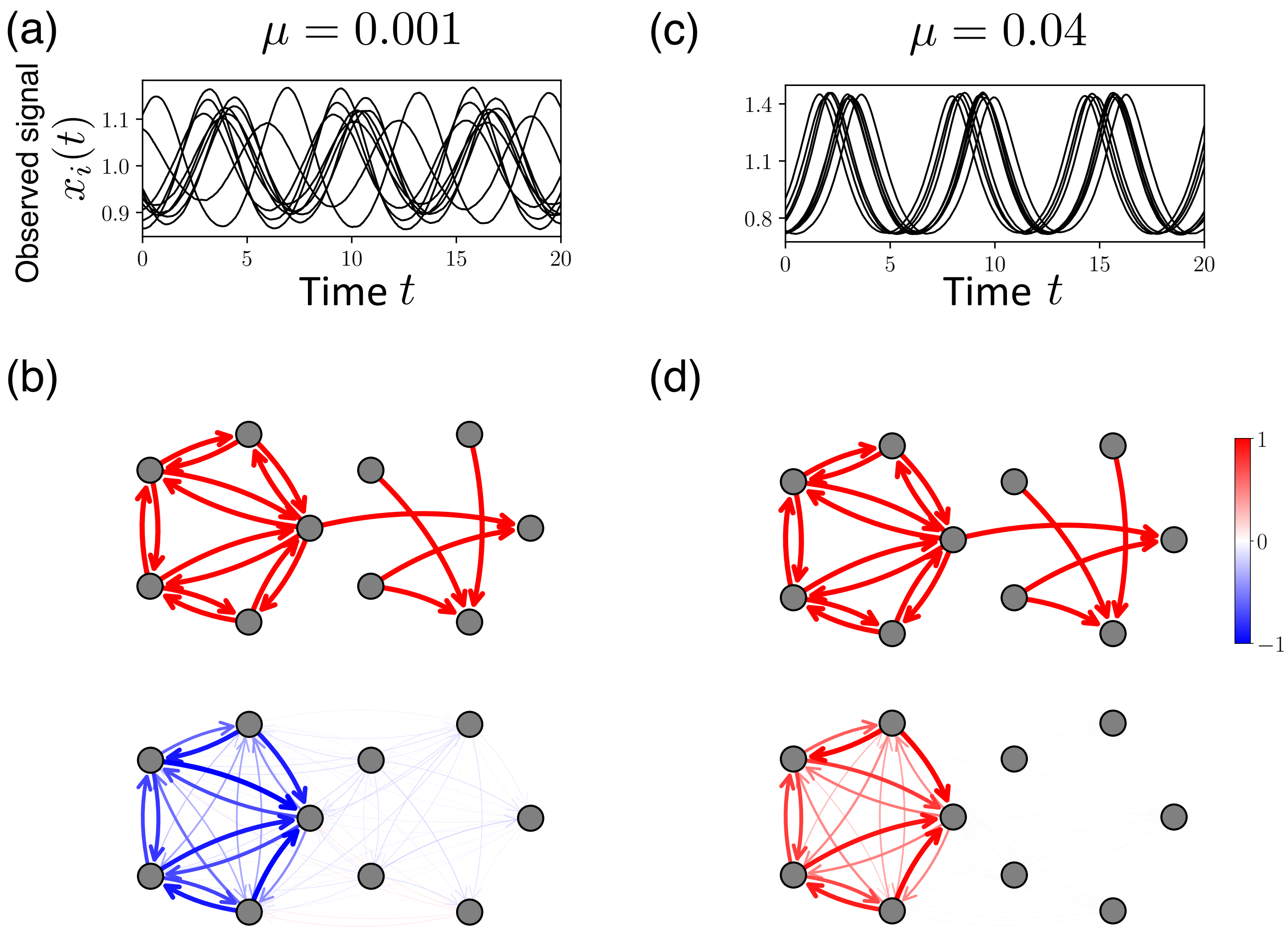
[Tokuda et al (2007); Kralemann et al (2012); Stankovsky et al. (2012); Ota, Aoyagi (2018); …]

Inference does NOT work for well-synchronized networks

Artificial data
(Coupled Brusselators,
μ : Hopf bif parameter)

True network

Inferred network



BUT, it works if we use the circle map

[Matsuki, Kobayshi, HK, in preparation]

Akari Matsuki's Poster

$$\phi_i(t+T) - \phi_i(t) = T\omega_i + T\sum_{j=1}^{N} c_{ij}\sin\left(\phi_j(t) - \phi_i(t) + \alpha\right) + \sqrt{T}\sigma_i\eta_{i,t}$$

Artificial data

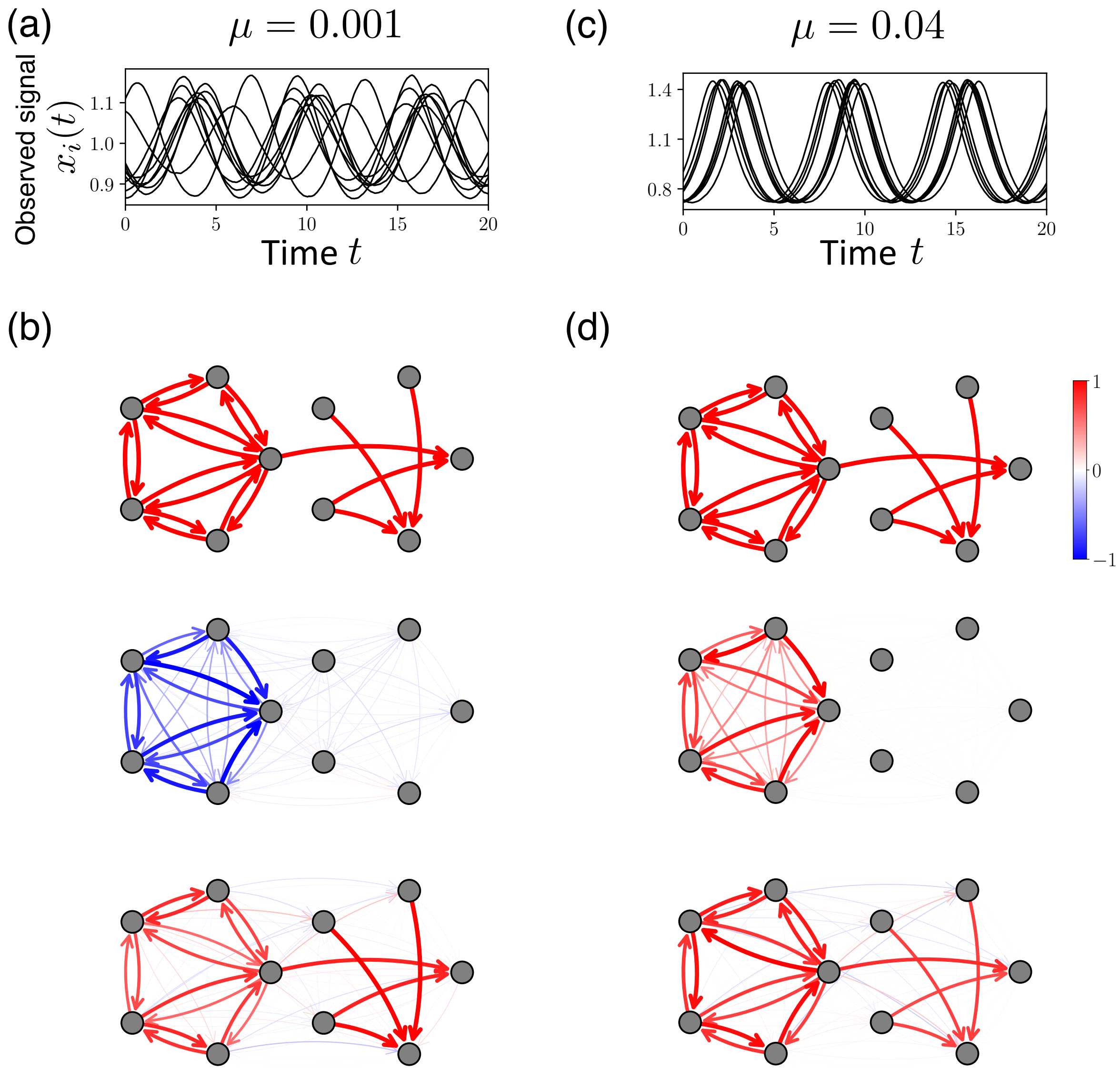
(Coupled Brusselators,

μ : Hopf bif parameter)

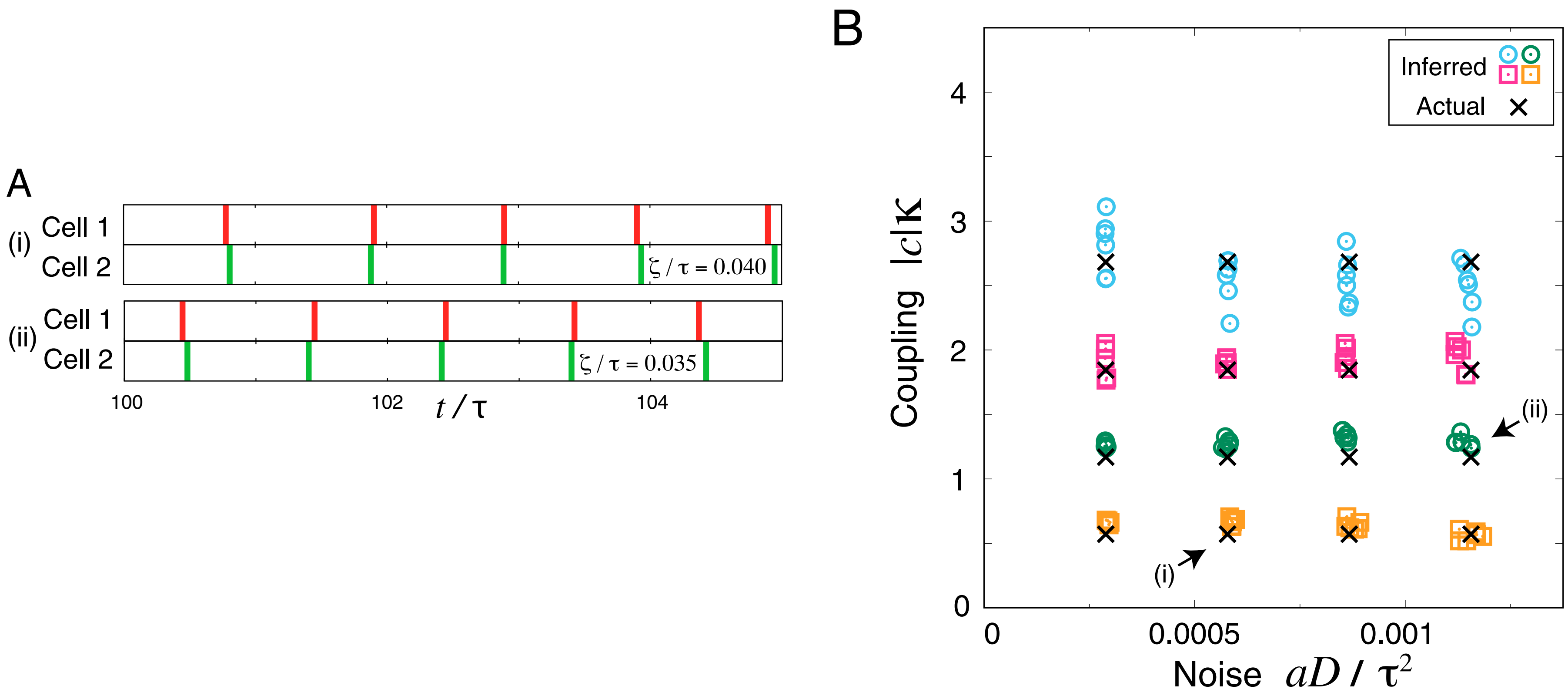
True network

Inferred network (via phase model)

Inferred network (via circle map)



Inference of coupling and noise strength using only spike data [Mori&HK, PNAS (2022)]



This method works for well-synchronized oscillators

Fumito Mori will give a talk on Friday

Forecasting a better shiftwork scheduling using Reservoir computing

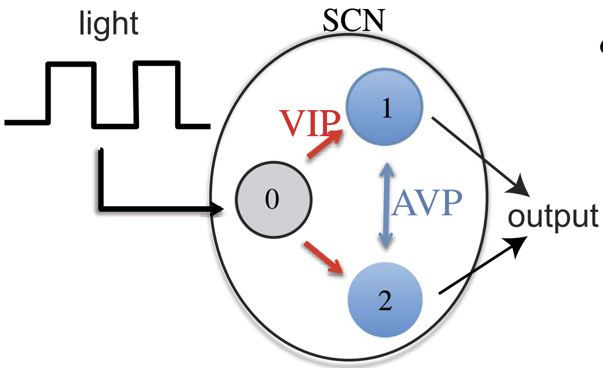
[Kuno & HK, arxiv 2024]

Problems in shift working

- Shift workers are known to be at an increased risk of certain diseases
- They are supposed to be in “chronic jet lag”, which is thought to have a significant impact on their well-being and health

Mathematical models help qualitative understandings and predictions about jet lag

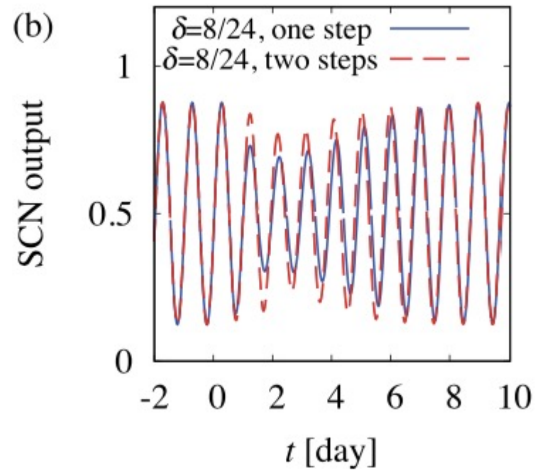
[HK, Yamaguchi, Okamura Sci. Rep. (2017)]

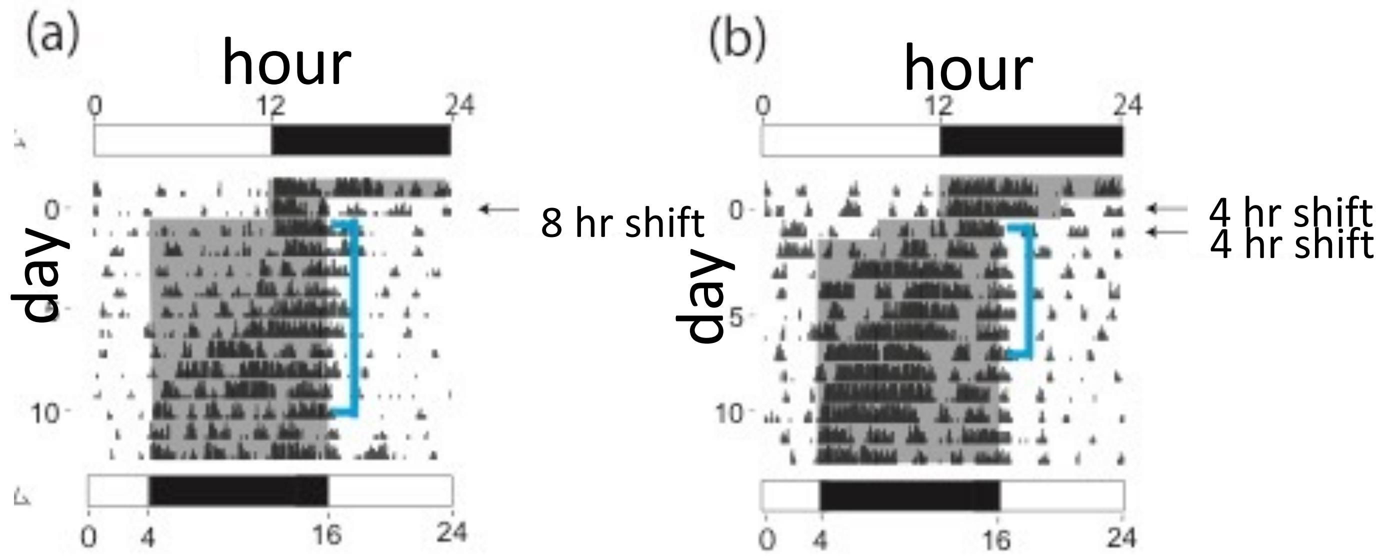


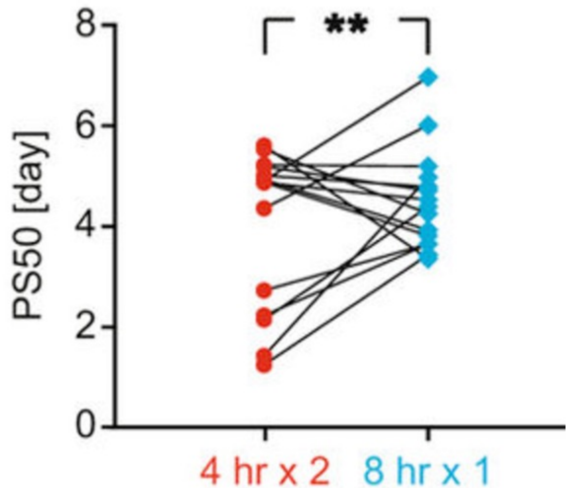
$$\phi_0 = \Omega(t + \Delta t).$$

$$\frac{d\phi_1}{dt} = \omega + K_1 \sin(\phi_0 - \phi_1) + K_2 \sin(\phi_2 + \alpha - \phi_1),$$

$$\frac{d\phi_2}{dt} = \omega + K_1 \sin(\phi_0 - \phi_2) + K_2 \sin(\phi_1 - (\phi_2 + \alpha)),$$

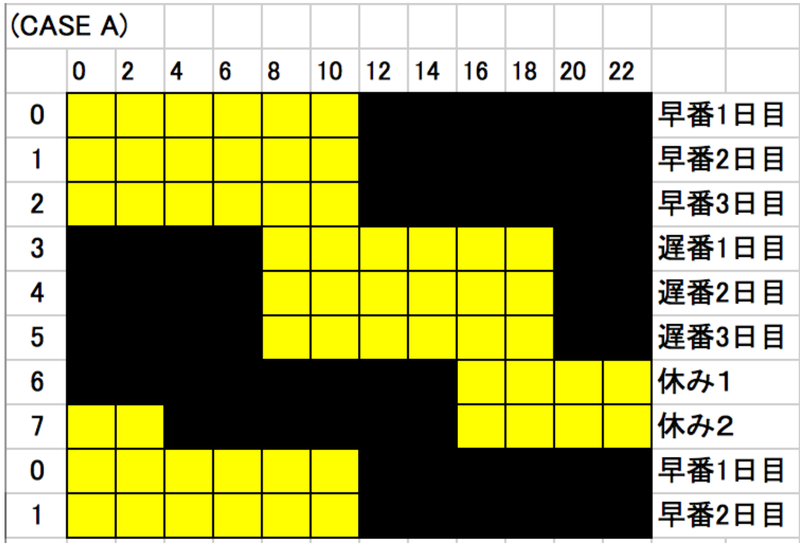


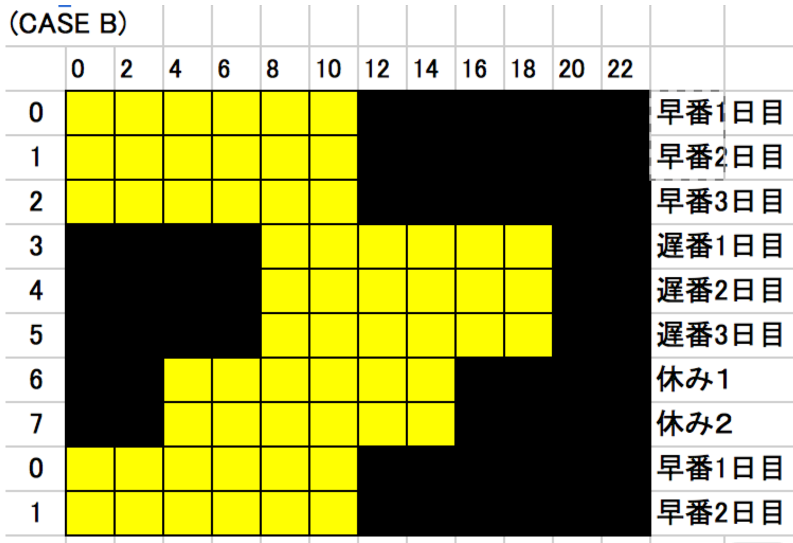


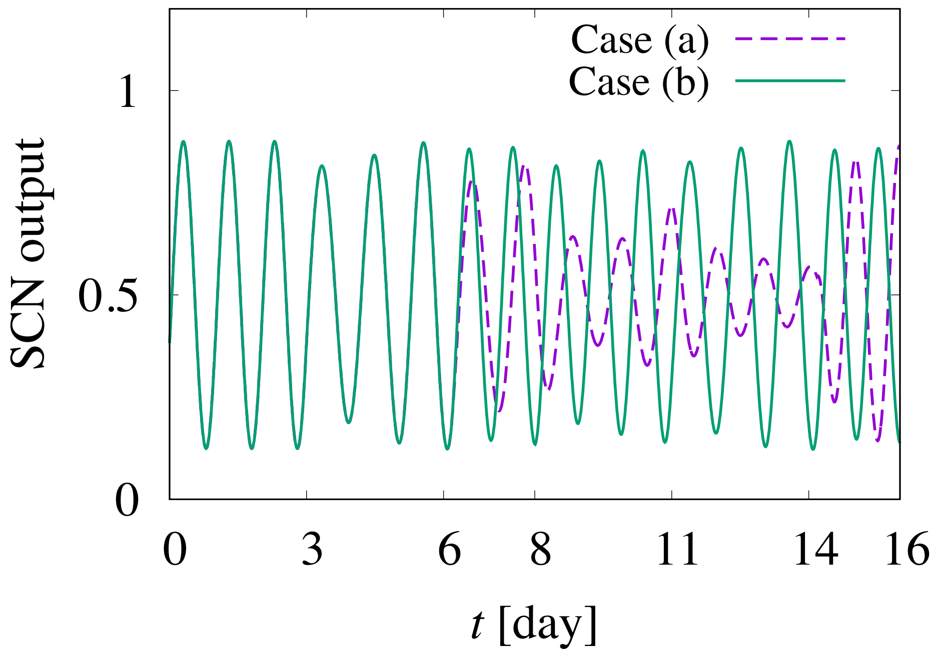


Wake up 4 hours earlier than usual on the day of the eastbound flight!

Mathematical models help qualitative understandings and predictions also for shiftwork scheduling (unpublished)







Our model predicts
a better way to spend holidays

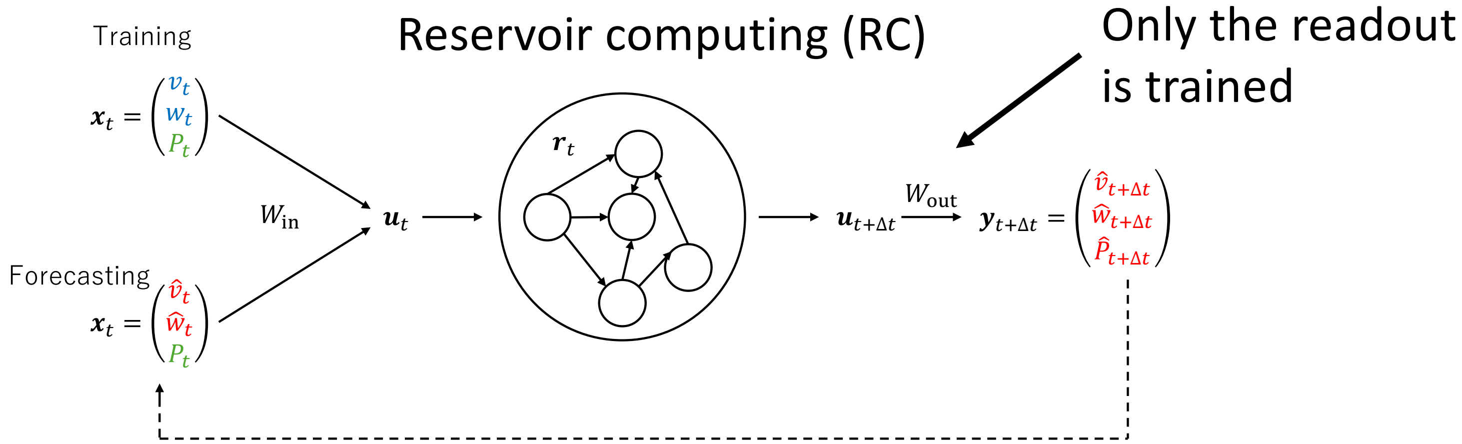
But I consider that this does not work for everyone.
A data-driven approach is needed to provide a reliable prediction to each individual.

Can we assist in schedule decision-making using machine learning?

[Kuno&HK, arxiv]

Suppose that a person who has experienced a certain schedule of shift working will at some point change to a new, different schedule. Our aim is to forecast the dynamics of the circadian clock for the new schedule on the basis of past data and the new schedule of sleep-wake cycles.

Specifically, in this study, we asked whether Reservoir Computing (RC) can predict the dynamics of limit-cycle oscillators subjected to a periodic drive with frequent and abrupt phase shifts.



RC can make quantitative forecast

[Kuno&HK, arxiv 2024]

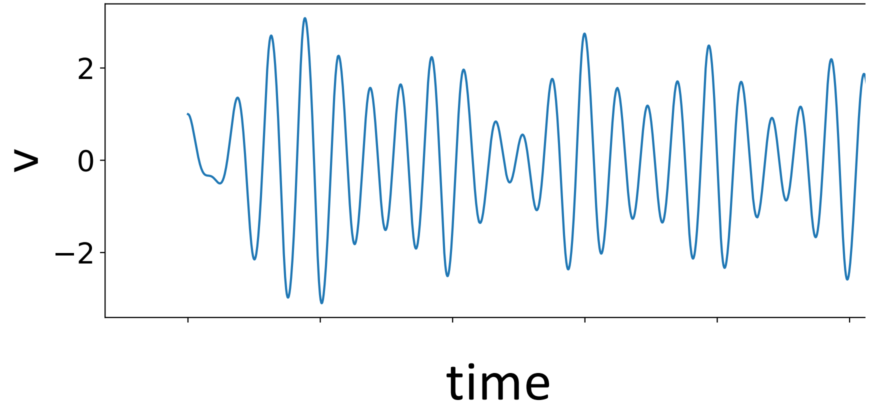
(artificial data)

$$\frac{dv}{dt} = w,$$

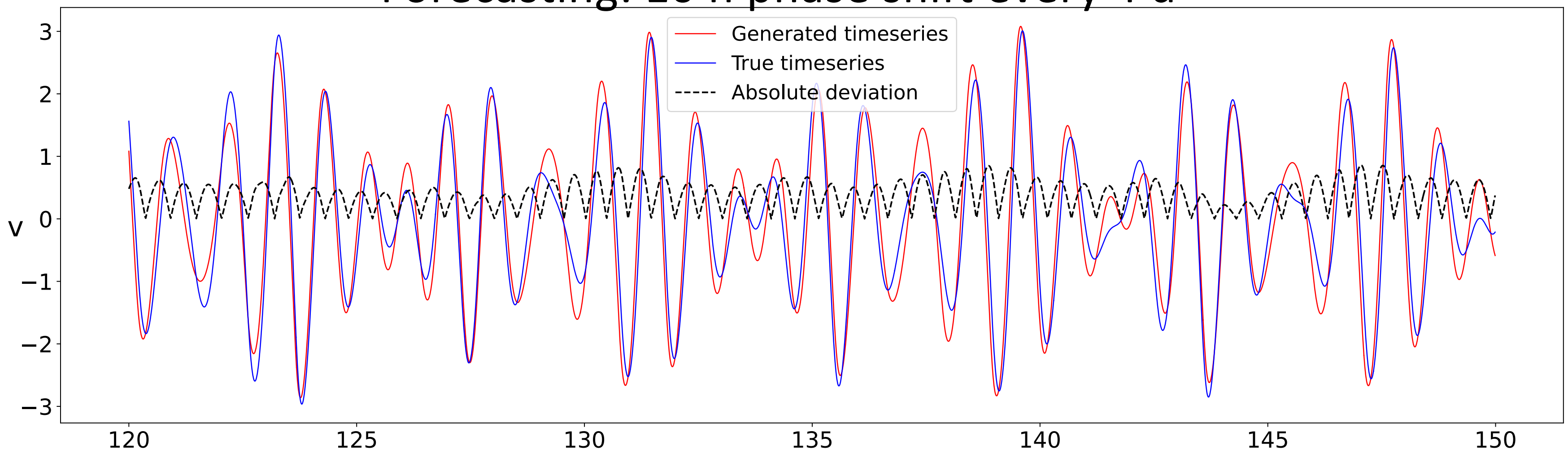
$$\frac{dw}{dt} = \mu(1 - v^2)w - v + \underline{P_n(t)}.$$

describes sleep-wake cycles

Training data: 7 h phase-shift every 4 d



Forecasting: 10 h phase shift every 4 d



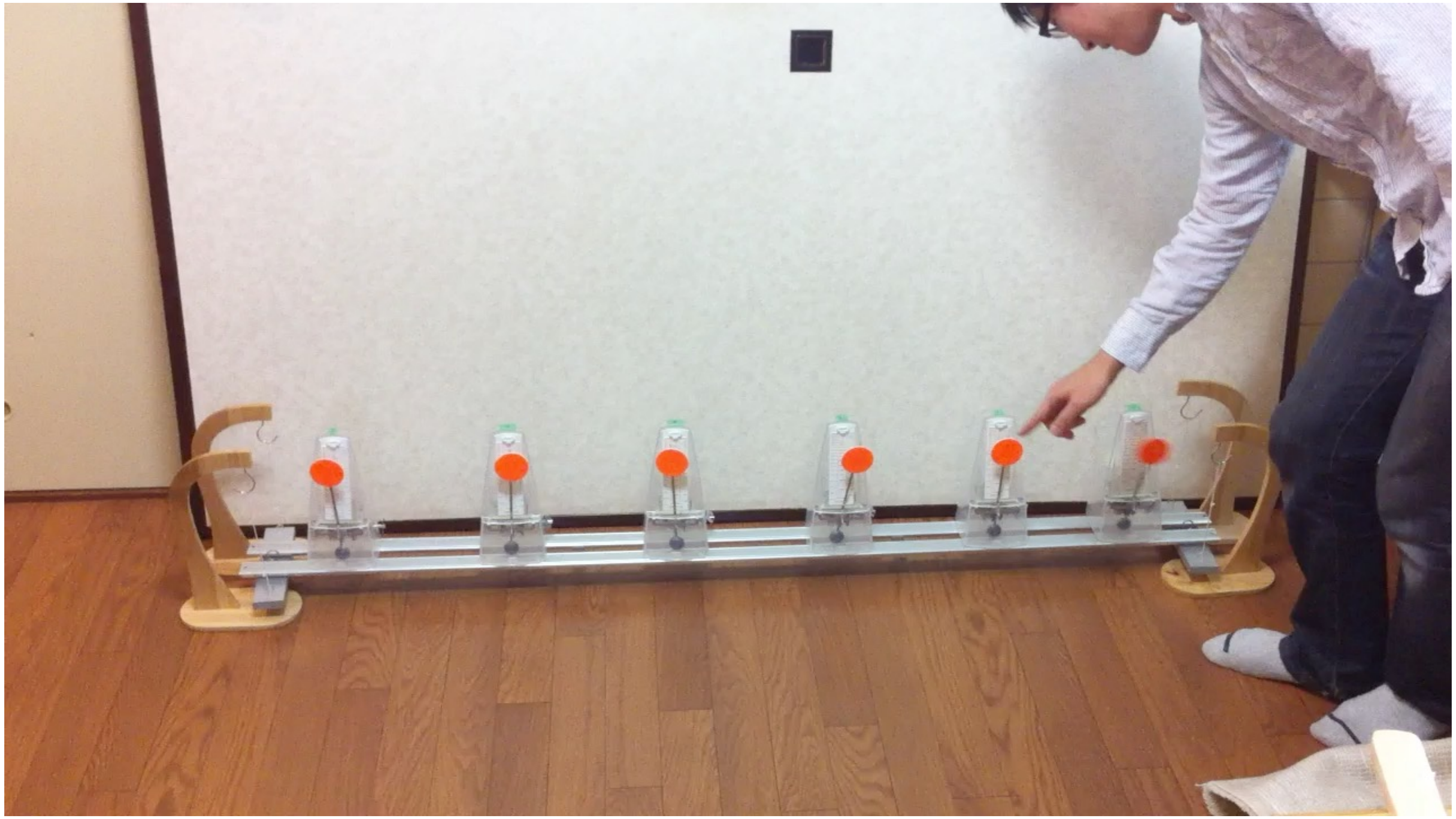
Quantitative forecast is possible for this simple model.

Oscillation quenching

Sometimes, oscillation is desired.
Sometimes, oscillation is harmful, e.g.,
trembles in Parkinson 's disease.

We would like to know the design and control principle
of a system producing (or suppressing) oscillations

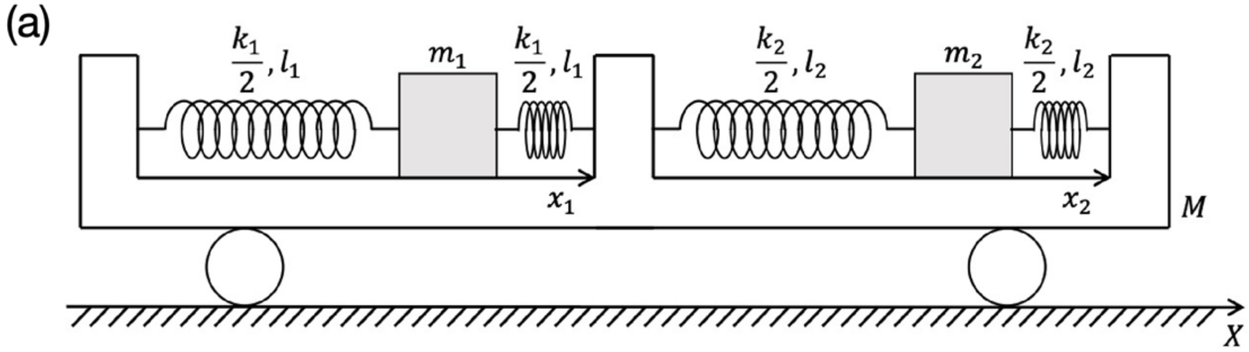
Coupled metronomes



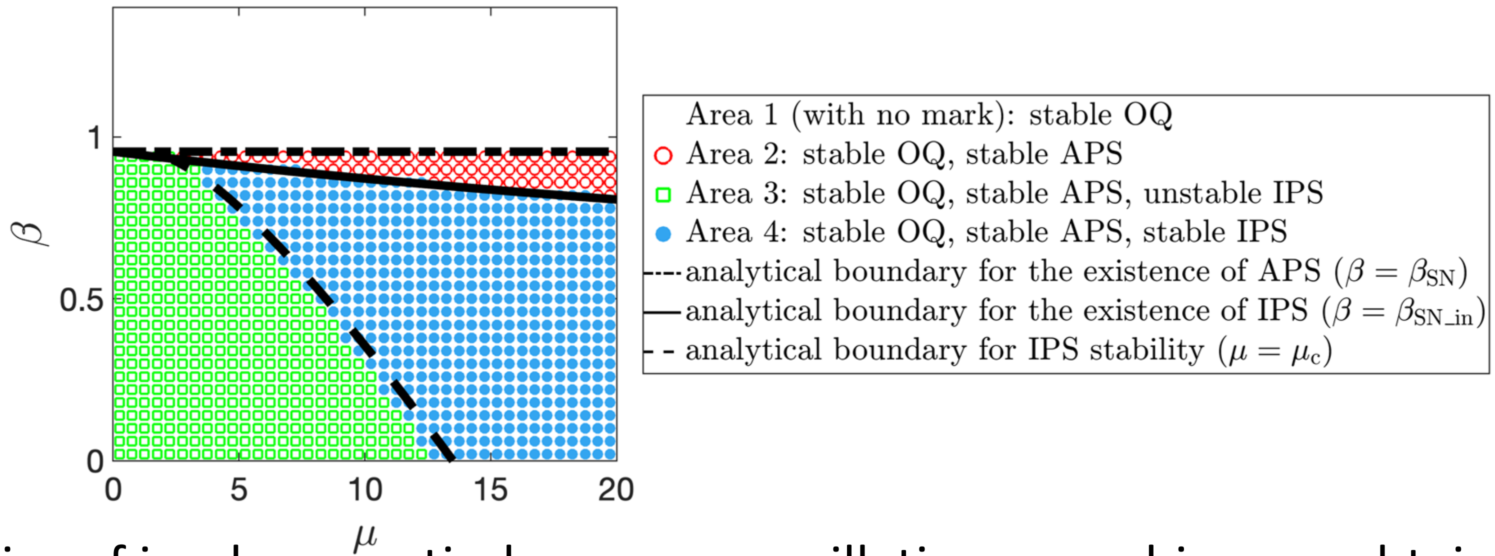
Movie taken by a friend of mine

Weakly nonlinear analysis using a simple model

[MD. Kato & HK, Sci Rep. (2024)]



$$\ddot{x}_i + x_i = -\varepsilon\left[\mu(x_1 + x_2) + \beta\dot{x}_i - g(x_i, \dot{x}_i)\right]$$

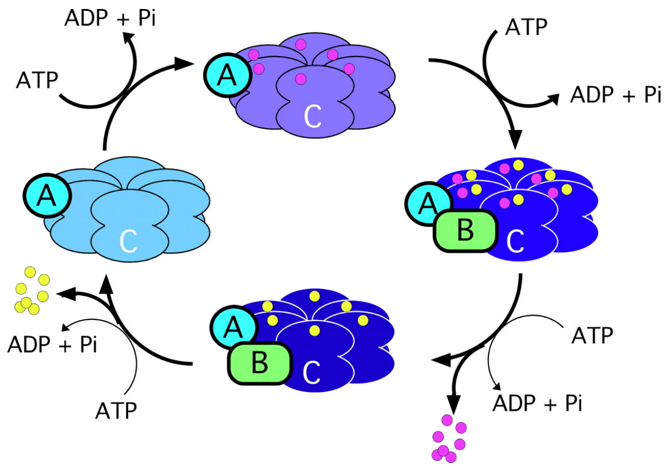


Boundaries of in-phase, anti-phase sync, oscillation quenching are obtained.
Novel interesting behavior, out-of-phase sync and beating phenomenon are found.

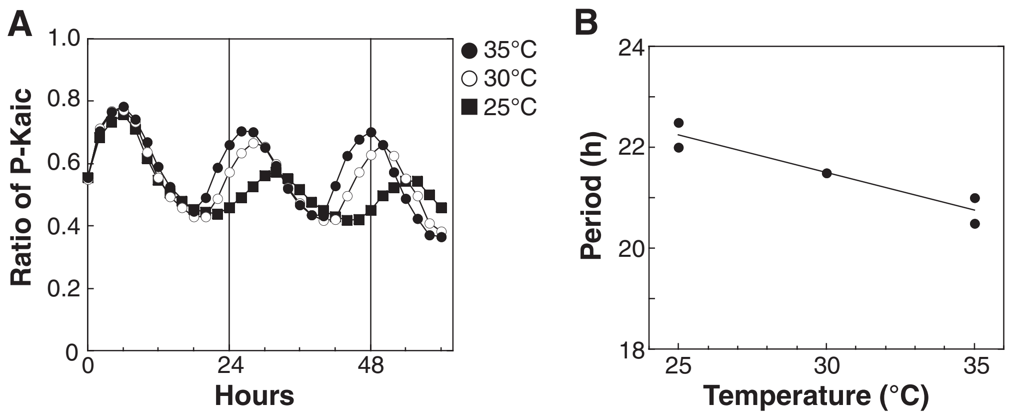
Coupled oscillators with metabolism

[Ozawa & HK, to appear in PRL]

Synchronization in phosphorylation rhythm of Kai proteins



[C. Robertson McClung, PNAS (2007)]



[Nakajima et al, 2005]

Period: about 24 hours
Half-life: about 10 hours

In vivo, there should be considerable effect of turnovers of Kai proteins on synchronization

Coupled oscillators with metabolism

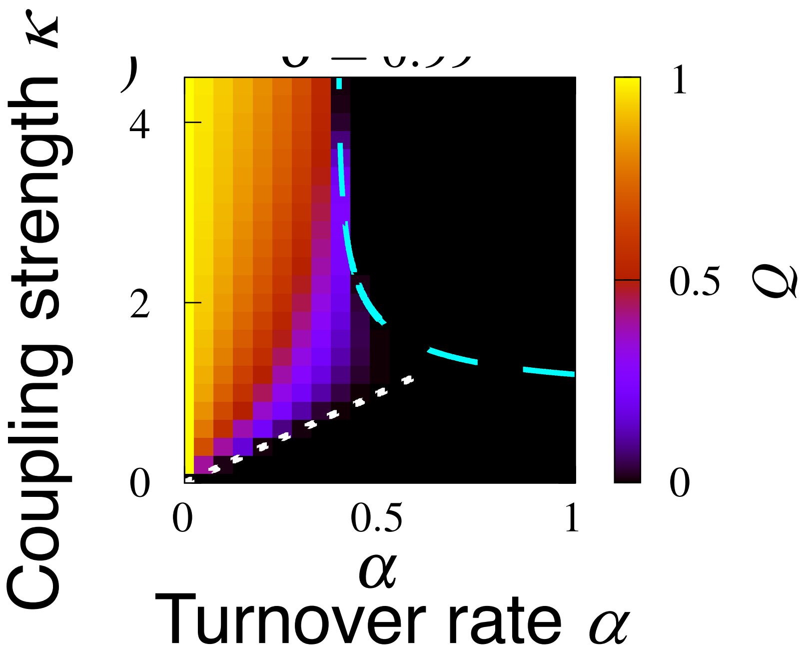
[Ozawa & HK, to appear in PRL]

$$d\theta_i = \left[\omega + \frac{\kappa}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)\right] dt \quad + \quad [-\theta_i + \phi_i]\, dP_i(\phi_i;\alpha)$$

Kuramoto type interaction

Random phase resetting

At each time, oscillators are randomly picked up and reseted (turnonver)



- For small coupling strengths, the turnover induces **desynchronization**
- For stronger coupling strengths, it may induce **stochastic oscillation quenching**.

Energetics of Synchronization

What is the merit of synchronization?

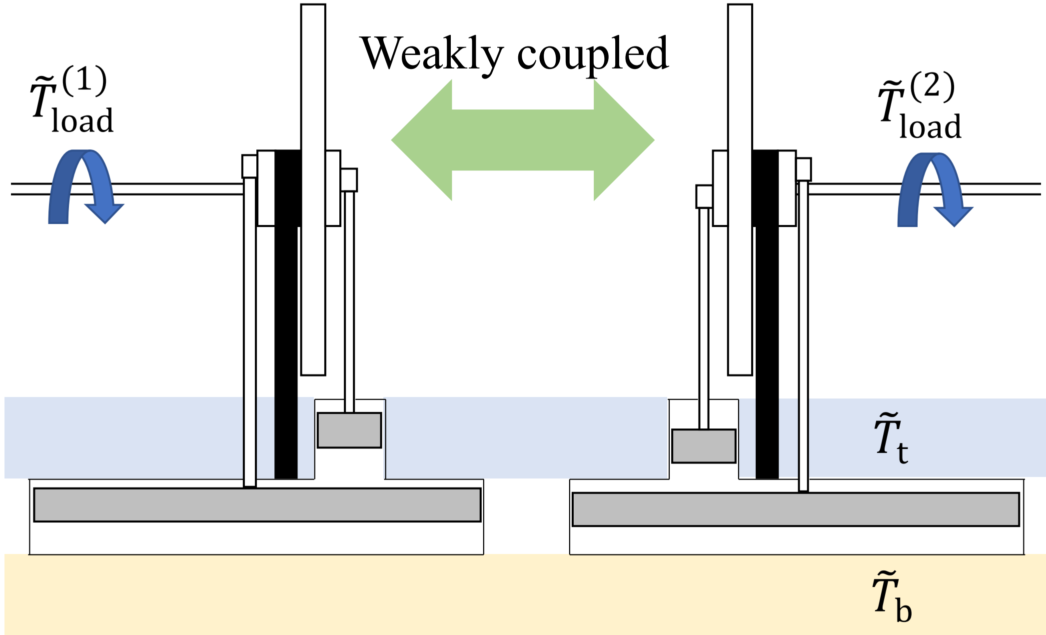
Energy should be a key aspect

Image: American Institute of Physics, June 9 (2017)

Coupled heat engines

[Yin, Izumida, Kori, PRR 2023]

https://www.youtube.com/shorts/YECEFJzvyQY



$$\frac{d\theta_i}{dt} = \omega_i,$$

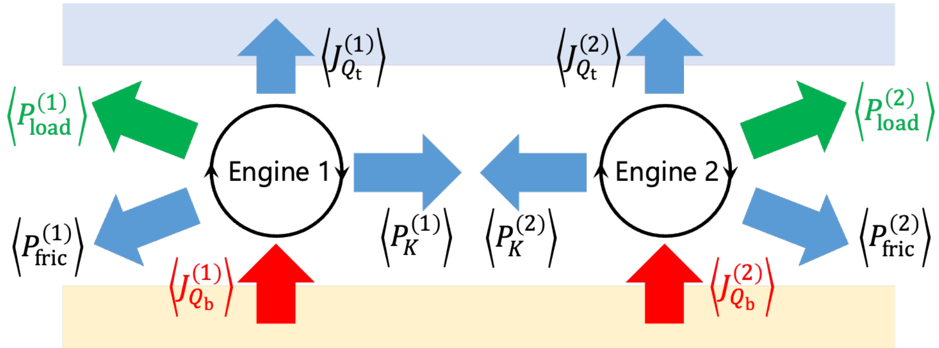
$$\frac{d\omega_i}{dt} = \sigma\left(\frac{T(\theta_i,\omega_i)}{V(\theta_i)} - P_{\mathrm{air}}\right)\sin\theta_i - \Gamma\omega_i - T^{(i)}_{\mathrm{load}}$$
$$-K\sin(\theta_i - \theta_j).$$

Maximum power and thermal efficiency are achieved when engines are synchronized

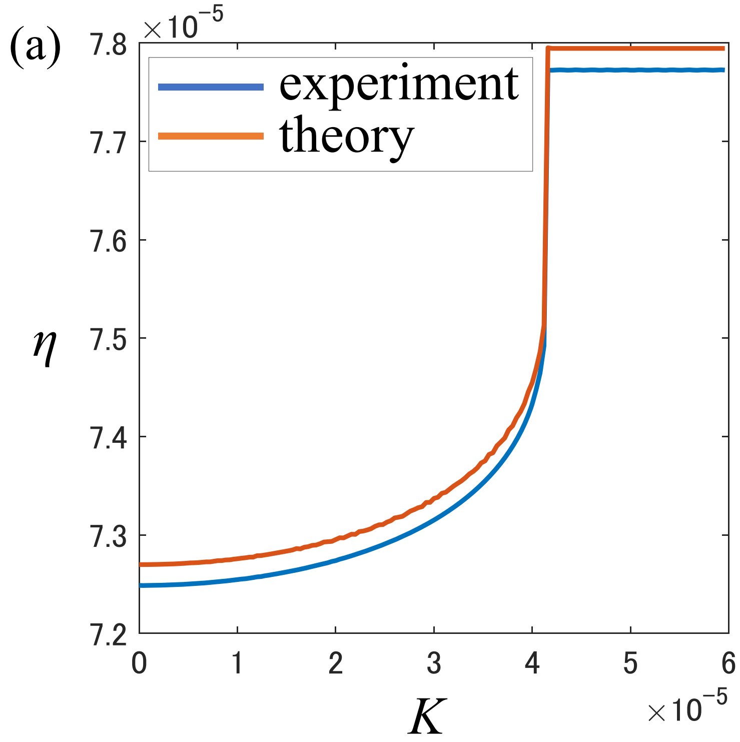
[Yin, Izumida, Kori, PRR 2023]

$$\frac{d\theta_i}{dt} = \omega_i,$$

$$\frac{d\omega_i}{dt} = \sigma\left(\frac{T(\theta_i,\omega_i)}{V(\theta_i)} - P_{\mathrm{air}}\right)\sin\theta_i - \Gamma\omega_i - T_{\mathrm{load}}^{(i)}$$
$$-K\sin(\theta_i-\theta_j).$$



Thermal efficiency η : $\dfrac{\langle P_{\mathrm{load}}^{(1)}\rangle + \langle P_{\mathrm{load}}^{(2)}\rangle}{\langle J_{Q_{\mathrm{b}}}^{(1)}\rangle + \langle J_{Q_{\mathrm{b}}}^{(2)}\rangle}$



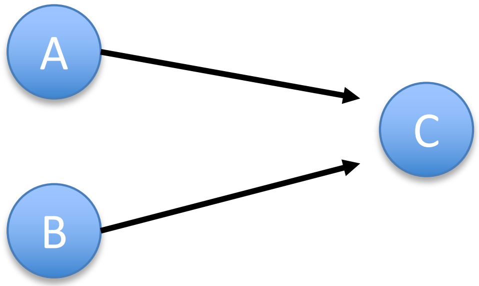
Mechanism: Loads to the engines are evenly distributed by synchronization

Higher order network of noisy oscillators

[Marui & HK, arXiv]

Higher-order networks are ubiquitous

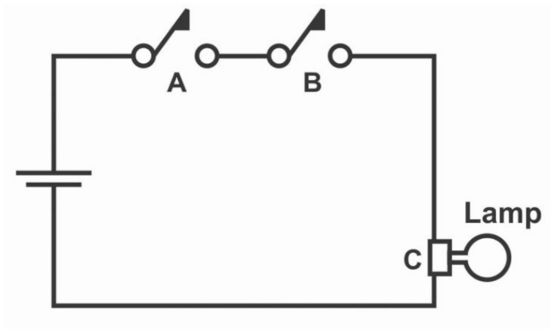
Neural network



$$\frac{d}{dt}v_C = f(v_C) + g(v_A) + g(v_B)$$

Pair-wise network

If neuron C fires only when neuron A and B fire almost simultaneously
this is effectively “and circuit”:



Non-pairwise network

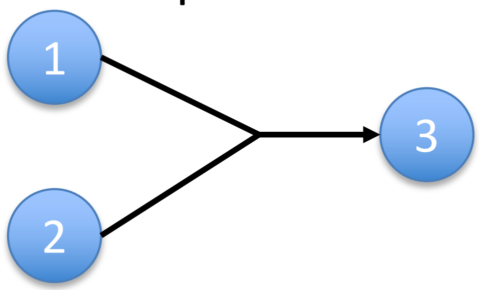


Figure from https://www.electroniclinic.com/logic-and-gate-working-principle-circuit-diagram/

If we take temporal information into account,
some systems can better be modeled as non-pairwise networks

Oscillators in higher-order network

- Oscillators in higher-order networks have extensively been studied, e.g.,
  Tanaka&Aoyagi (2011), Skardal&Arenas (2019), Millan, Torres, Bianconi (2020); Chutani, Tadic, Gupte(2021); Kuehn, Bick (2021); Rajwani, Suman, Jalan(2023); Carletti, Giambagli, Bianconi (2023) …
  - Emergence of multiple attractors (two cluster states)
  - Abrupt desynchronization

- However, noise effects on synchronization in higher-order networks are largely overlooked.

Noisy oscillators in a higher-order network

[Marui & Kori, arXiv (2023)]

Globally coupled phase-oscillators with
two- and three-body interactions and independent white noises

$$\dot{\theta}_m = \omega_m + \frac{K_1}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_m) + \frac{K_2}{N^2}\sum_{j,k=1}^{N}\sin(\theta_j + \theta_k - 2\theta_m) + \xi_m(t),$$

$$\langle \xi_m(t) \rangle = 0,\ \langle \xi_m(t)\xi_n(\tau) \rangle = 2D\delta_{mn}\delta(t-\tau),$$

Using the order parameter $Z = Re^{i\Theta} = \frac{1}{N}\sum_{j=1}^{N} e^{il\theta}$

the model reduces to

$$\dot{\theta}_m = \omega_m + K_1 R \sin(\Theta - \theta_m) + K_2 R^2 \sin 2(\Theta - \theta_m) + \xi_m(t).$$

where R is Kuramoto order parameter, Θ is macroscopic phase

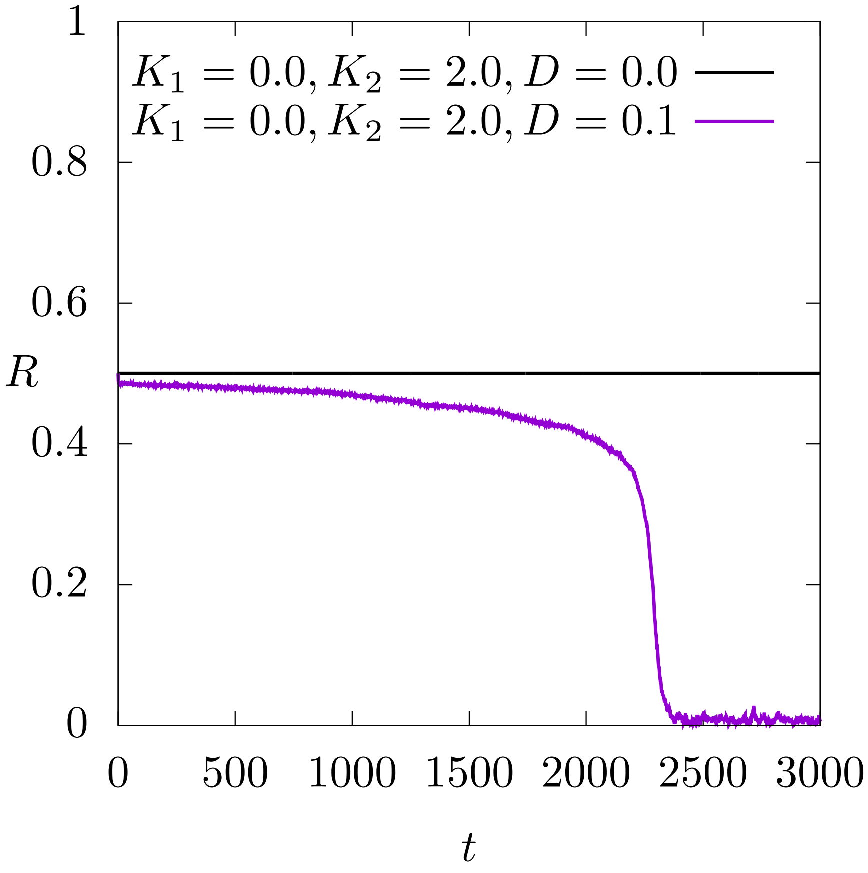
What is the effect of three-body interaction?

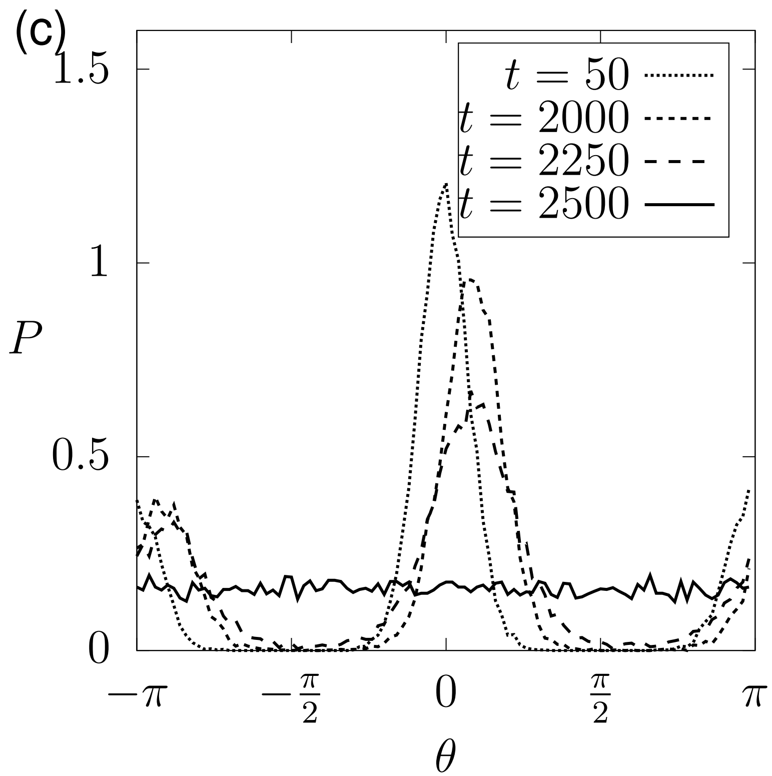
$$\dot{\theta}_m = \omega_m + K_1 R \sin(\Theta - \theta_m) + K_2 R^2 \sin 2(\Theta - \theta_m) + \xi_m(t).$$

By two-simplex interaction (i.e., three-body interaction),
Individual phase θ seems to be locked to Θ or $\Theta + \pi$
(where Θ is mean phase).

Therefore, one can expect that three-body interaction
promotes the formation of two-cluster states,
which is actually the case in noise-free oscillators

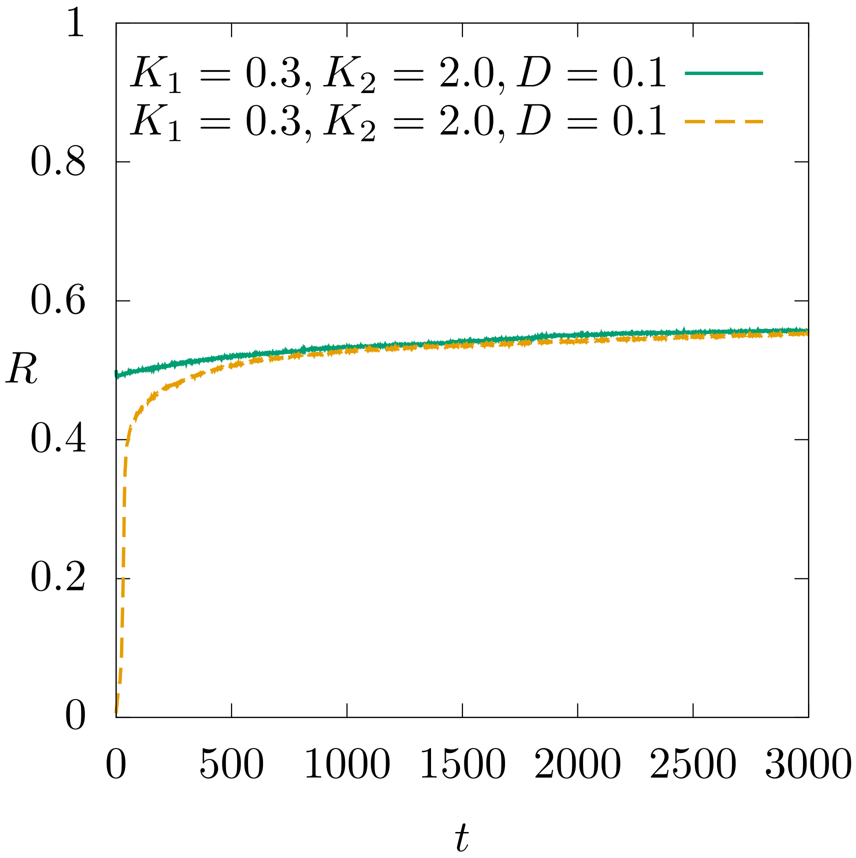
Two cluster states slowly decays and eventually disappear in noisy oscillators with two-simplex interaction alone

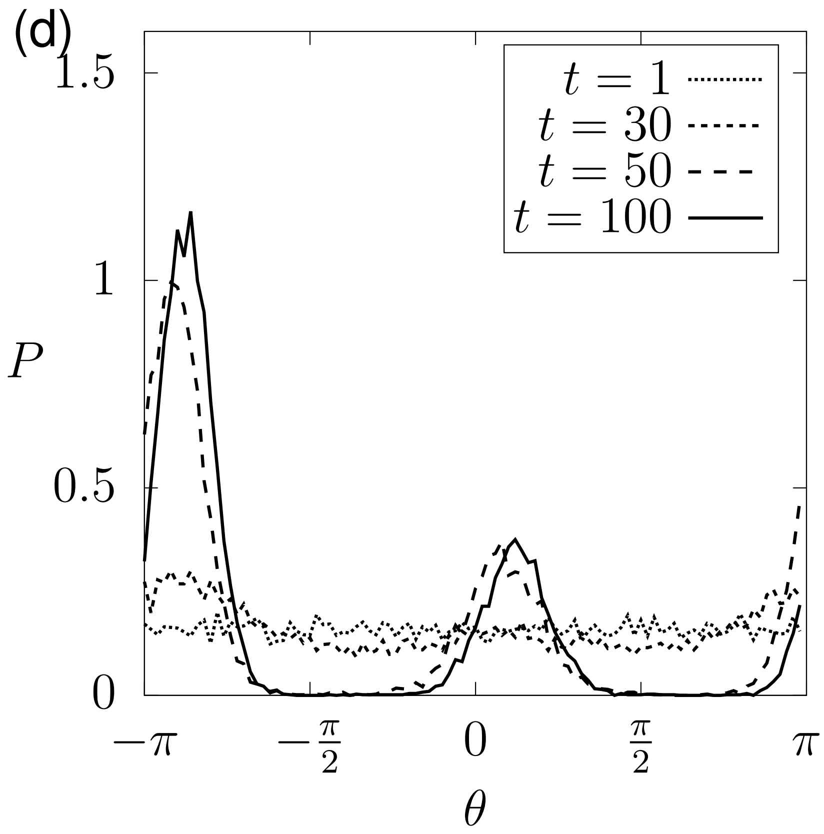




This decay occurs even when the noise is infinitesimally weak

Two clusters becomes persistent when two-body interaction is additionally introduced



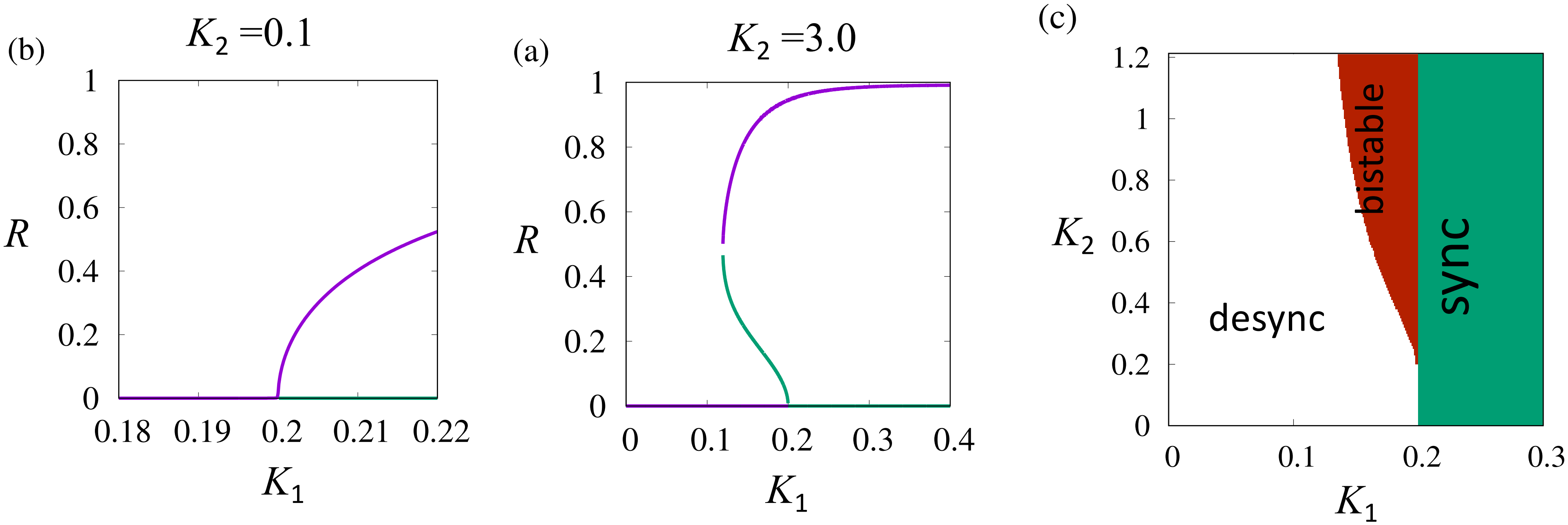


Phase diagram

Self-consistency equation:

$$R = \int_0^{2\pi} P_{\rm st}(\psi, R)\cos\psi \mathrm{d}\psi := g(R)$$

Numerically finding R satisfying this equation, we obtain



$R = 0$ seems to bifurcate at $K_1 = K_c$, where $K_c = 0.2 = 2D$.
This bifurcation seems to be super- and sub-critical for $K_2 < K_c$ and $K_2 > K_c$, respectively.

Super- or subcritical? Weakly nonlinear analysis

Weakly nonlinear analysis for the bifurcation of $R=0$ at $K_1=K_\mathrm{c}$ is as follows.

We set $K_1 = K_\mathrm{c}(1+\varepsilon^2)$ and introduce $\dfrac{\mathrm{d}}{\mathrm{d}t} \to \dfrac{\partial}{\partial t} + \varepsilon^2 \dfrac{\partial}{\partial \tau}$.
The complex order parameter is expanded as

$$Z = \varepsilon Z_1 + \varepsilon^2 Z_2 + \cdots ,$$

Note $R=|Z|$. Using a standar method [Kuramoto, 1984], we derive

$$\frac{\partial}{\partial \tau} Z_1(\tau) = \frac{K_1 - K_\mathrm{c}}{2} Z_1(\tau) - g|Z_1(\tau)|^2 Z_1(\tau),$$

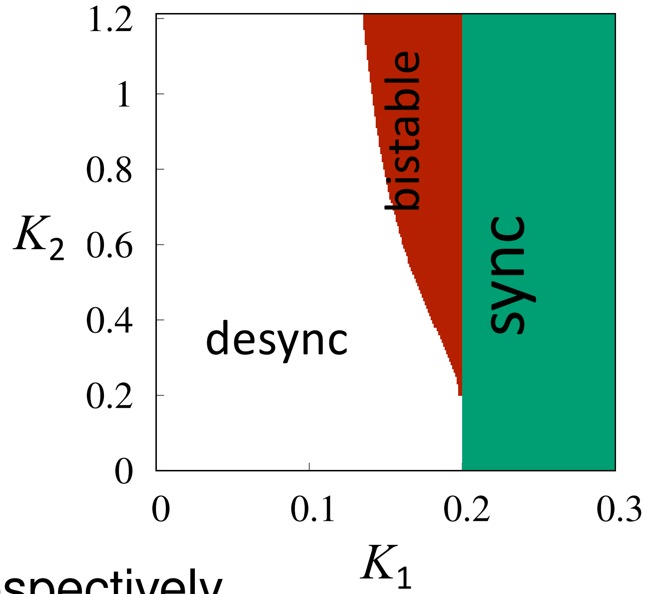
where

$$K_\mathrm{c} = 2D,$$
$$g = \frac{K_\mathrm{c}^2 + K_\mathrm{c} K_2}{8D} - \frac{K_2}{2}.$$

The sign of g changes at $K_2 = K_\mathrm{c}$.
We thus conclude that

- The bifurcation occurs at $K_1 = 2D$
- It is super- and sub-critical for $K_2 < 2D$ and $K_2 > 2D$, respectively.

(c)



Lifetime of synchronized states

We have obtained

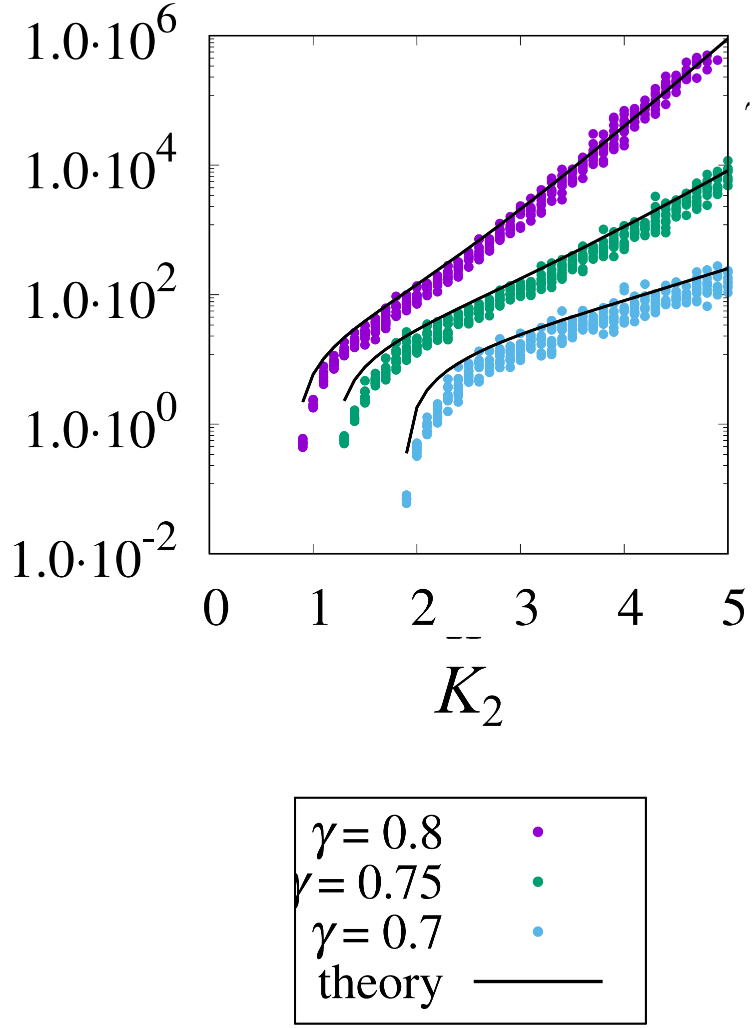
$$\dot{R} = -\frac{4K_2R^3}{\pi}\exp\left(-\frac{K_2R^2}{D}\right).$$

We define the lifetime of the synchronized state by the time within which R varies from R_0 to $R_{\rm thre}$:

$$\tau = \int_{R_0}^{R_{\rm thre}} \frac{{\rm d}t}{{\rm d}R}\,{\rm d}R = \int_{R_{\rm thre}}^{R_0} \frac{\pi\exp\left(\frac{K_2R^2}{D}\right)}{4K_2R^3}\,{\rm d}R,$$

where $R_0 = R(0) = 2\eta(0) - 1$.
Very roughly, we can estimate

$$\tau \propto \exp\left(\frac{K_2R_0^2}{D}\right)$$



Synchronized states persist for long time,
increasing exponentially with three-body coupling strength K2.

Noisy oscillators in a higher-order network: Summary

When noise is absent,
Oscillators are synchronized into various two-cluster states.

When noise is present,
all those synchronized states disappear.

However, the desynchronization process from synchronized state
is extremely slow.

One-simplex interaction (two-body coupling) may
stabilize the synchrony.

My wish is to conduct theoretical research that will be useful in the real world

- Developing theoretical frameworks for data-driven approaches
  - Network inference from rhythmic signals [Matsuki, HK, Kobayashi, to be submitted] Poster
  - Network inference from spike data [Mori & HK, PNAS (2022)] (talk on Thursday)
  - Forecasting dynamics using reservoir computing [Kuno & HK, arxiv]
- Oscillation quenching
  - Metronomes [Kato & HK, Sci. Rep. (2023)]
  - Kuramoto model with stochastic turnover [Ozawa & HK, to appear in PRL]
- Energetics of synchronization
  - Coupled Heat engines [Yin, Izumida, HK, PRR (2023)]
- Higher order networks
  - Slow desynchronization process in noisy oscillators [Marui & HK, arxiv]

Many thanks for your attention and Cheers!