Global bifurcation diagrams of a prescribed curvature problem arising in a generalized MEMS model

Kuo-Chih Hung

Abstract

We study global bifurcation diagrams and exact multiplicity of positive solutions for the one-dimensional prescribed mean curvature problem arising in MEMS

$$\begin{cases} -\left(\dfrac{u'(x)}{\sqrt{1+(u'(x))^2}}\right)' = \dfrac{\lambda}{(1-u)^p}, \quad u<1, \quad -L<x<L, \\ u(-L)=u(L)=0, \end{cases} \tag{1}$$

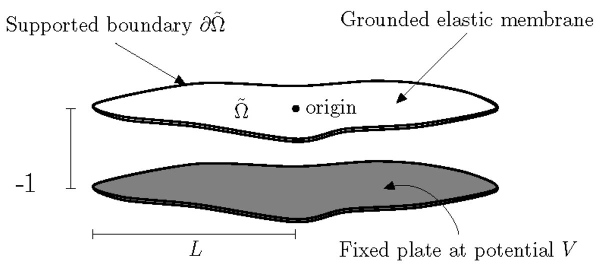
where $\lambda > 0$ is a bifurcation parameter, and $p, L > 0$ are two evolution parameters.

Notice (1) can be written in the equivalent dynamical system

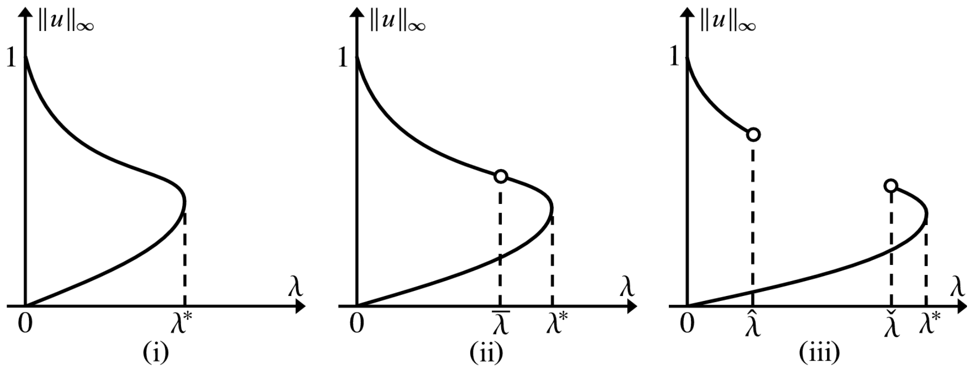
$$\begin{cases} \dot{u} = v, \\ \dot{v} = -\lambda \dfrac{(1+v^2)^{3/2}}{(1-u)^p}, \end{cases} \quad u(-L)=u(L)=0. \tag{1*}$$

The problem is a derived variant of a *canonical* model used in the modeling of electrostatic MEMS device obeying the electrostatic Coulomb law with the Coulomb force satisfying the *inverse p-th power law* with respect to the distance of the two charged objects, which is a function of the deformation variable.

When a voltage λ is applied, the membrane deflects towards the ceiling plate and a snap-through may occur when it exceeds a certain critical value λ^*, referred to as the "pull-in voltage". This creates a so-called "pull-in instability" which greatly affects the design of many devices. Also, in the actual design of a MEMS device, typically, one of the primary device design goals is to achieve the maximum possible stable steady-state deflection (that is, $\|u_{\lambda^*}\|_\infty$ (<1), referred to as the "pull-in distance", with a relatively small applied voltage.

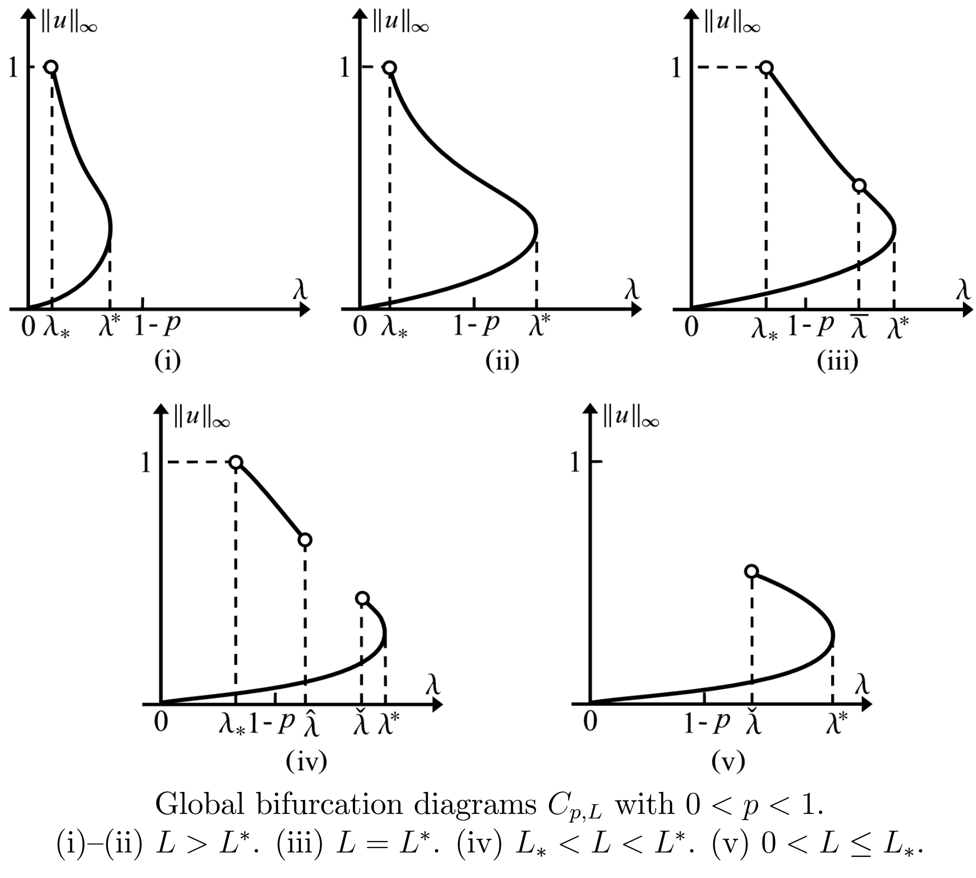


- For $p \geq 1$, the bifurcation diagram (1) undergoes *two* bifurcations. The first is a standard *fold* (or called *saddle-node*) bifurcation, which happens for all positive L at some positive λ^*. The second is a *splitting* bifurcation.



Global bifurcation diagrams $C_{p,L}$ with $p \geq 1$.
(i) $L > L^*$. (ii) $L = L^*$. (iii) $0 < L < L^*$.

- For $0 < p < 1$, the bifurcation diagram (1) undergoes *three* bifurcations. The first is a standard *fold* bifurcation. The second is a *splitting* bifurcation. The third is a *segment-shrinking* bifurcation.



Global bifurcation diagrams $C_{p,L}$ with $0 < p < 1$.
(i)–(ii) $L > L^*$. (iii) $L = L^*$. (iv) $L_* < L < L^*$. (v) $0 < L \leq L_*$.