Extreme Events Scaling in Finite-Size Abelian Sandpile Model

Abelian BTW Sandpile Model

- The emergent scale-invariant feature remains one of the most remarkable observation occurring in system.
- Such features can arise near the critical point of a continuous transition between order and disorder phases.
- The concept of Self-organized criticality introduced by Bak et al. (1987), explains the underlying origin of scaling in natural systems, which remain far away from equilibrium.
- Observable x can describe the events like size (total toppled sites) s and area (spatial extent of size) a, duration T.

0	3	0
3	4	2
1	1	2

T=1

0	4	0
4	0	3
1	2	2

T=2

2	0	1
0	2	3
2	2	2

T=3

- The probability distribution of the event x obeys a decaying power-law behavior

$$P(x, x_c) = \begin{cases} A x_c^{-\theta} x^{-\tau_x}, & \text{for} \quad x \ll x_c \\ \text{rapid change}, & \text{for} \quad x \approx x_c \end{cases}$$

Generalized Extreme Value Distribution

$$\mathscr{F}(x; \mu, \beta, \xi) = \exp\left\{ -\left[1 + \xi\left(\frac{x - x_c}{\beta}\right)\right]^{-1/\xi} \right\}$$

where x_c, β and ξ are location (or mode), scale, and shape parameters, respectively, having bounds x_c, $\xi \in \mathbb{R}$ and $\beta \in \mathbb{R} \mid \beta > 0$. Depending upon the valued of shape parameter can be categorised into three universality classes

i. $\xi > 0$ implies Fr\'echet class where the parent distribution decaying as a power law.

ii. $\xi < 0$, describes Fisher - Tippette - Gumbel (FTG) class for which the parent distribution decays faster than power law.

iii. $\xi = 0$ represents Gumbel class

Methodology

1. Peak over threshold
2. Block Maxima

Objectives:

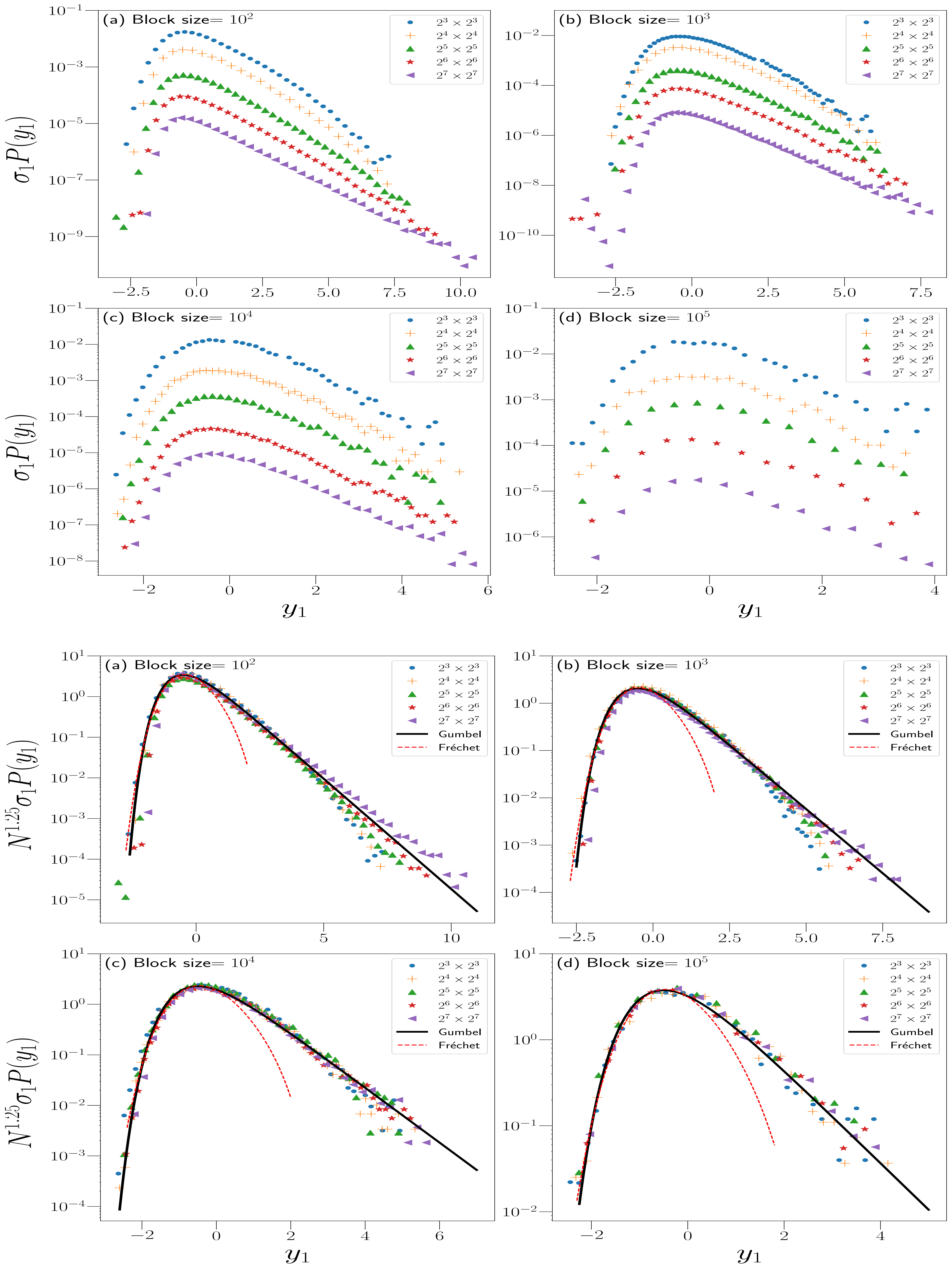
➡ Extreme activity distribution may also be an explicit function of the system size.

➡ The probability, along with the parameters, may vary with the system size and belong to the same class of extreme value distributions.

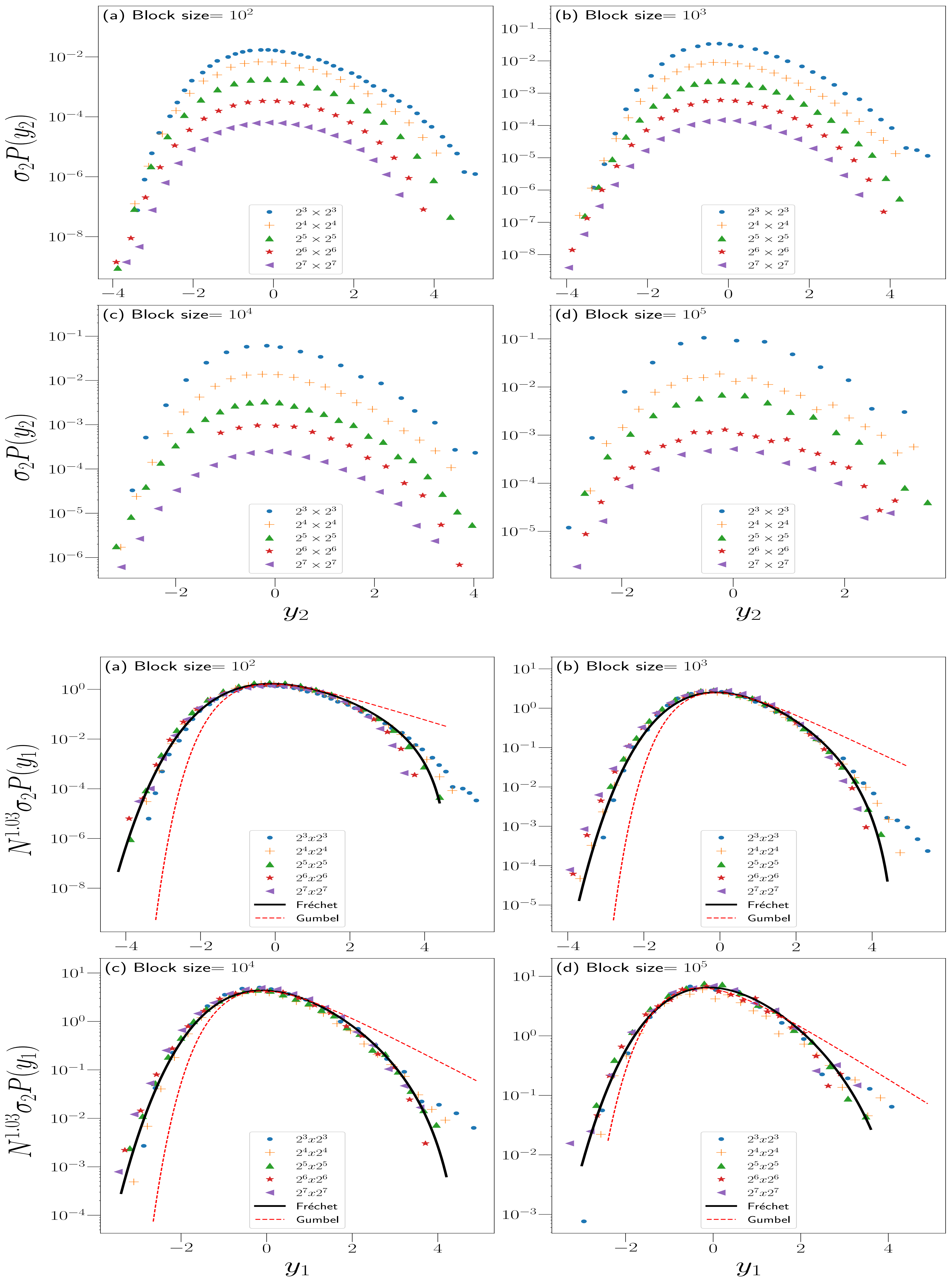
➡ We demonstrate a simple scaling analysis that can capture this characteristic.

Abdul Quadir* and Haider H Jafri, Department of Physics, Aligarh Muslim University, Aligarh INDIA

Results and Discussions



Extreme value distributions and data collapse for maxima of avalanche sizes with different block size



Extreme value distributions and data collapse for maxima of avalanche area with different block size

We found that:

$$\langle x_i \rangle = \langle x_i(N) \rangle \sim N^{\alpha}$$

Rescaling: $y_i = \dfrac{x_i - \langle x_i \rangle}{\sigma_i}$

We observe that:

$$\begin{cases} P(y_1 = y_c) \sim N^{-\gamma_1}, & \gamma_1 = 1.25 \\ P(y_1 = y_c) \sim N^{-\gamma_2}, & \gamma_1 = 1.05 \end{cases}$$

Scaling Function:

$$\psi(x_j; N) = N^{-\gamma_j} \frac{1}{\sigma_j} \mathscr{F}\left(\frac{x_j - \langle x_j \rangle}{\sigma_j} \right)$$

Numerically we found that:

✓ Avalanche size maxima lies into Gumbel family of GEVD with $\xi = 0.0$

✓ Avalanche size maxima lies into Frechet family of GEVD with $\xi = 0.19$