Dynamical ergodicity breaking and scaling relations for finite-time first-order phase transition

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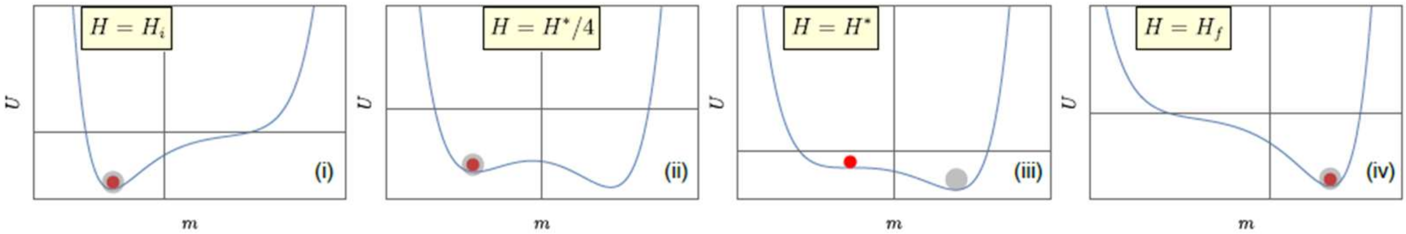
Abstract

Hysteresis and metastable states are typical features associated with ergodicity breaking in the first-order phase transition which occurs in the thermodynamics limit. When the system is quenched across a first-order phase transition, the excess work (enclosed area between the dynamic and static hysteresis) even exhibits universal scaling behavior. Nevertheless, for a system of finite size, how will the features of the first-order phase transitions persist remains unexplored. We study the scaling behavior of the excess work as a function of the quench rate in the Curie-Weiss model. We find the shrinking of the hysteresis when downsizing the system, and the crossover of the scaling of the excess work from $v^{2/3}$ to v. Our study elucidates the interplay between the quench rate and the relaxation rate (system size), which leads to the dynamical ergodicity breaking and different scaling behavior of the excess work.

1D mean-field Curie-Weiss model



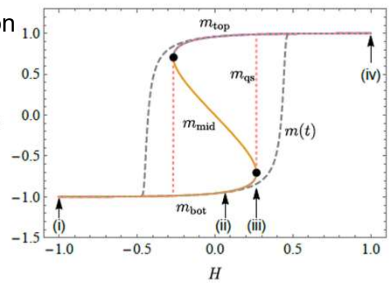
1. Infinitely large system

- Deterministic equation of motion

$$\frac{dm}{dt}=\frac{2}{\tau_0}\Big[\sinh(\beta Jm+\beta H)-m\cosh(\beta Jm+\beta H)\Big]$$

- We consider a linear protocol

$$H(t)=H_i+(H_f-H_i)\frac{t}{t_f},$$



- Expand the EMO around the turning point for small quench rate

$$\frac{d\hat{m}}{d\hat{t}}\approx\frac{2}{\tau_0}\Big[\beta J\sqrt{\beta J-1}\hat{m}^2+\sqrt{\frac{\beta}{J}}v\hat{t}\Big].$$

- $v^{-1/3}$ scaling relation of the delay time and transition time

$$\hat{t}_{\rm del}=-A_1'\Big[4\sqrt{\beta J(\beta J-1)}\frac{\beta}{\tau_0^2}v\Big]^{-1/3},$$

$$\hat{t}_{\rm trans}=-A_1\Big[4\sqrt{\beta J(\beta J-1)}\frac{\beta}{\tau_0^2}v\Big]^{-1/3},$$

where $A_1'\approx-1.019$ and $A_1\approx-2.338$ are zeros of Airy functions.

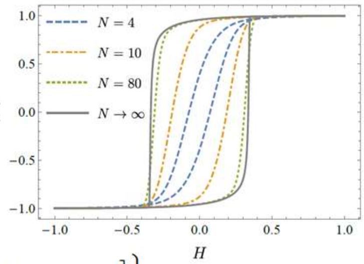
- $v^{2/3}$ scaling relation of the excess work

$$w_{\rm ex}\approx\frac{-A_1(1+\sqrt{1-\frac{1}{\beta J}})}{\Big[4\sqrt{\beta J(\beta J-1)}\frac{\beta}{\tau_0^2}\Big]^{1/3}}v^{2/3}.$$

2. Finite-size system

- Stochastic equation of motion

$$\frac{\partial P(M,t)}{\partial t}=\sum_{\eta=\pm}\Big[W_\eta(M_{-\eta},H(t))P(M_{-\eta},t)-W_\eta(M,H(t))P(M,t)\Big].$$



The transition rates under the external field H are given by

$$W_\pm(M,H(t))=\frac{N\mp M}{2\tau_0}\exp\Big\{\pm\beta\Big[\frac{J}{N}(M\pm1)+H(t)\Big]\Big\},$$

obeying the detailed balance condition.

- The average work can be computed from

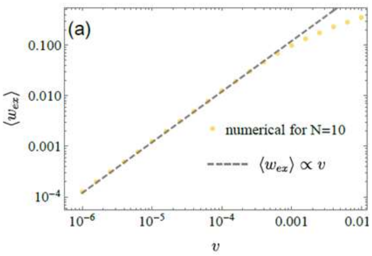
$$\langle w\rangle=-v\int_0^{t_f}dt\langle m(t)\rangle.$$

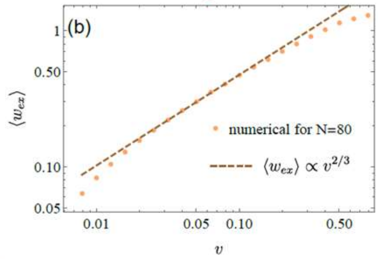
- The average excess work is defined through $\langle w_{ex}\rangle=\langle w\rangle-w_{qs}$.

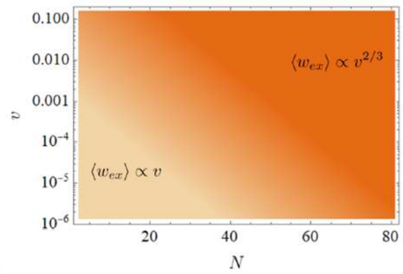
Table I. The scaling relation of the excess work $w_{\rm ex}$ with the quench rate v for different situations.

Finite-time isothermal process [34–39]	$w_{\rm ex}\propto v$
Finite-time adiabatic process [40–45]	$w_{\rm ex}\propto v^2$
Finite-time first-order phase transition[63–69]	$w_{\rm ex}\propto v^{2/3}$
Finite-time second-order phase transition [22, 46]	$w_{\rm ex}\propto v^{\delta_1}$

◆ **Crossover in the scaling relation from $v^{2/3}$ to v in finite N system**







Reference Y. X. Wu, J. F. Chen and H. T. Quan, arXiv:2401.15592 (2024).