Inference theory for coupling direction between synchronized oscillators and its experimental verification

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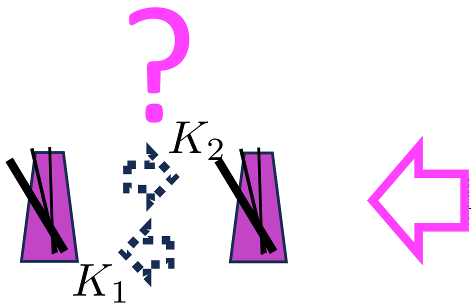
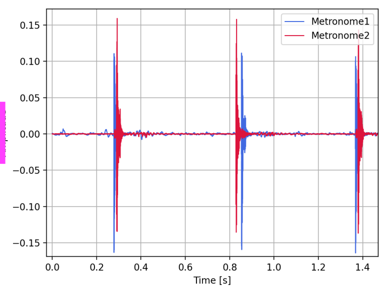
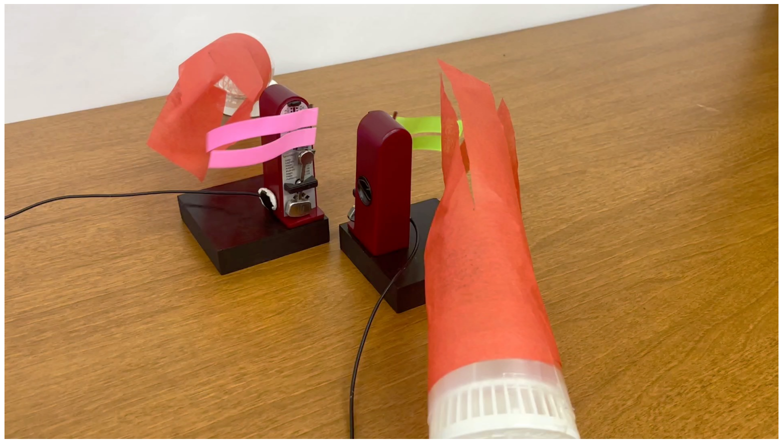
[1]Kyushu Univ., [2]Univ. of Tokyo. Japan

Inference theory



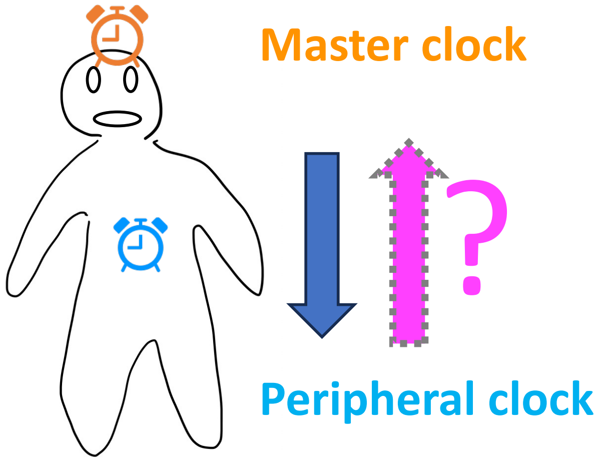


Validation experiments



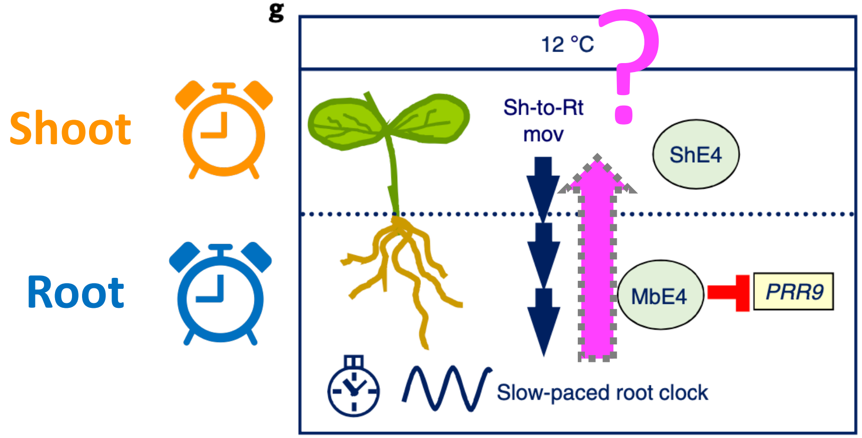
Coupling direction in synchronized oscillators

Circadian clocks (Mammal)

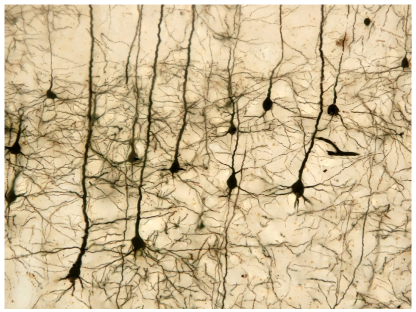


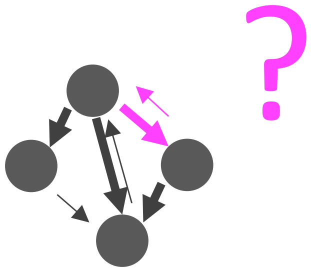
(Plant)

W. W. Chen et al.,
Nat Plants 6, 416-426 (2020).



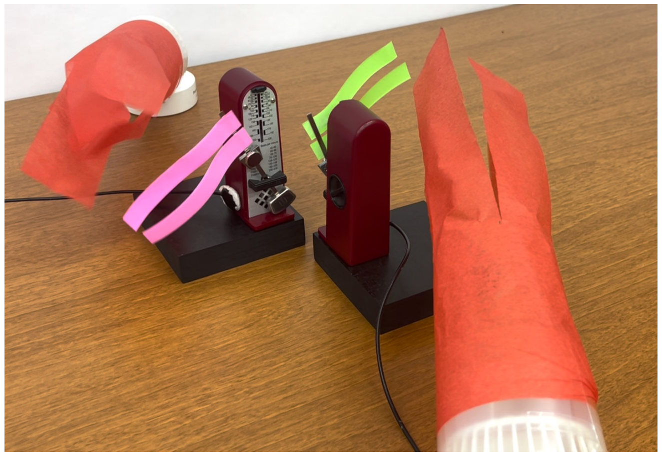
Neurons



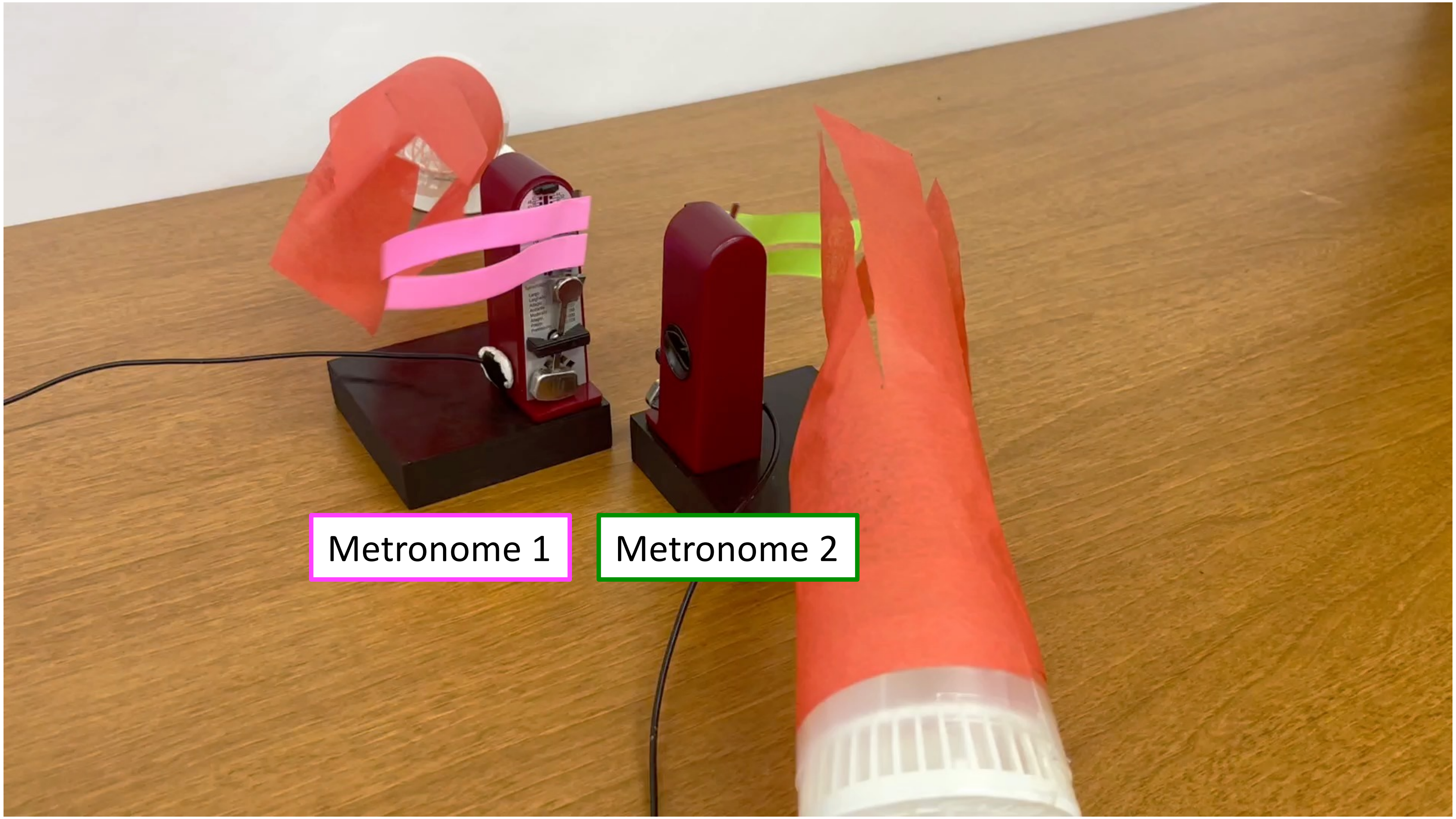


PNAS "Predictions could help neurons, and the brain, learn"

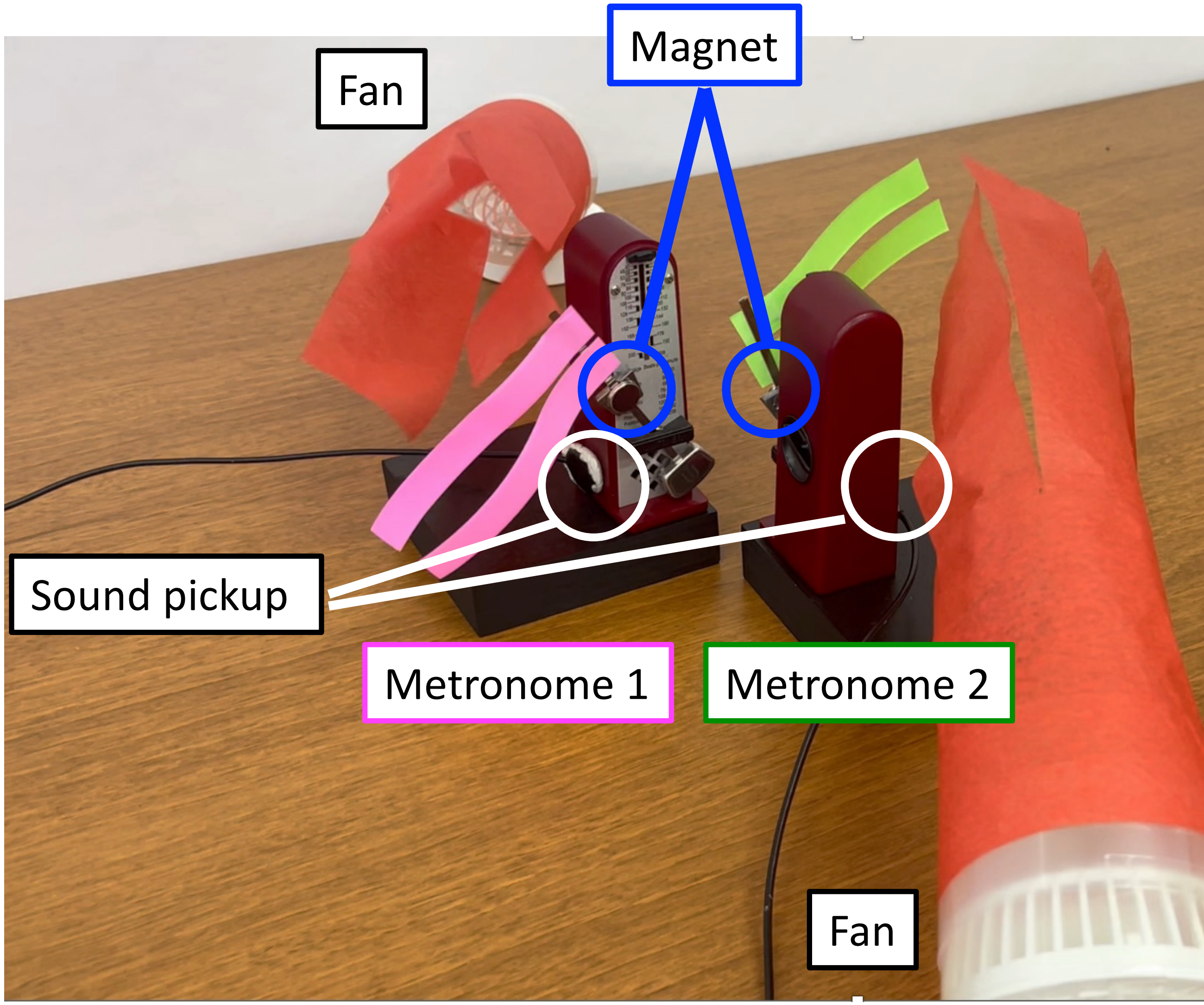
Artificial systems



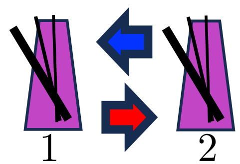
Synchronized metronomes



Synchronized metronomes

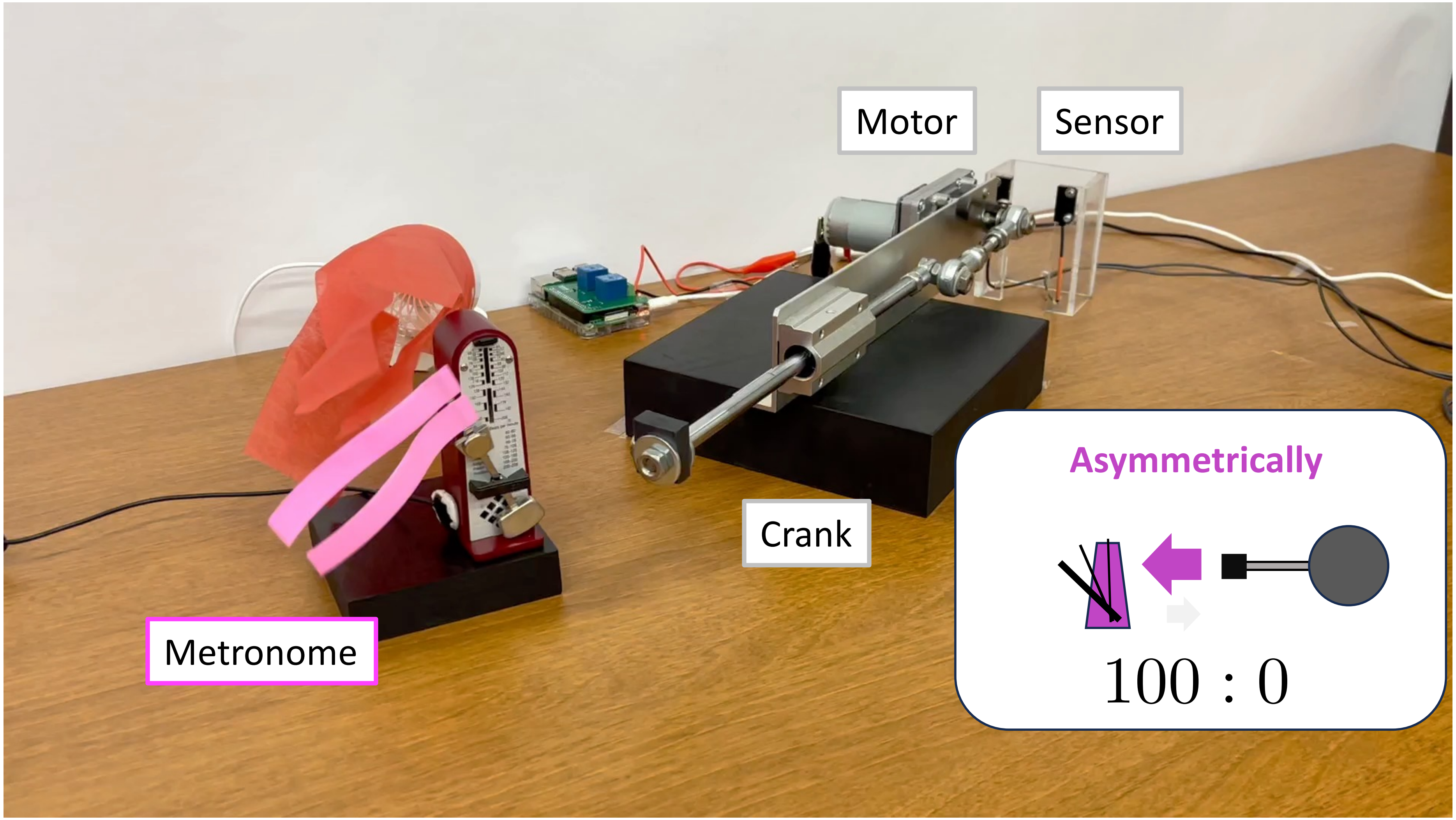


Symmetric

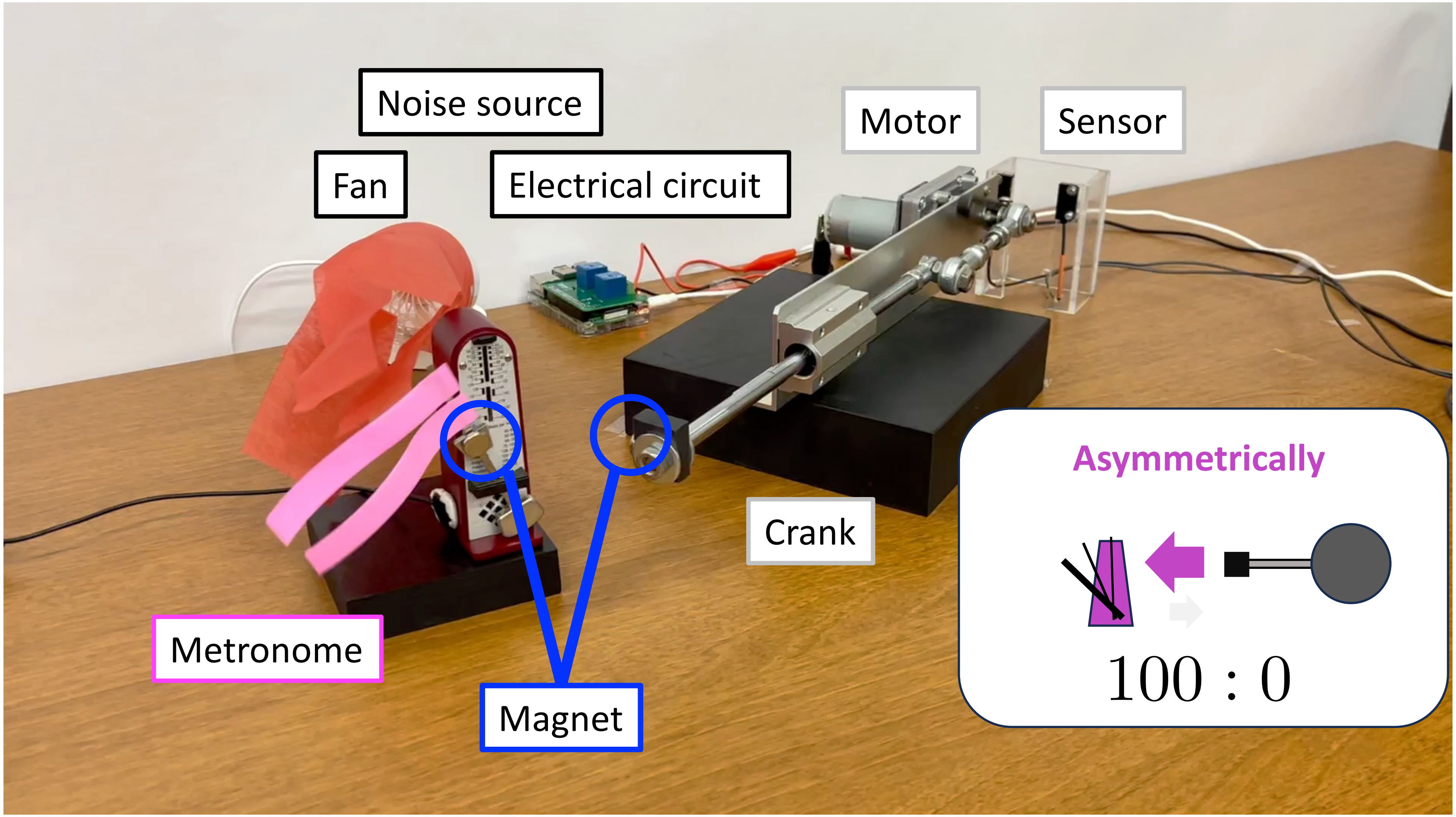


50 : 50 ?

Metronome + Crank

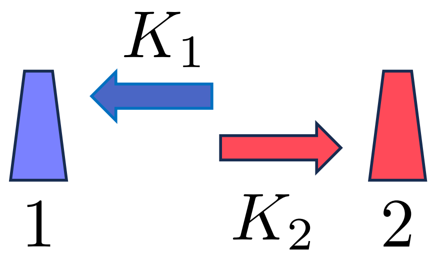


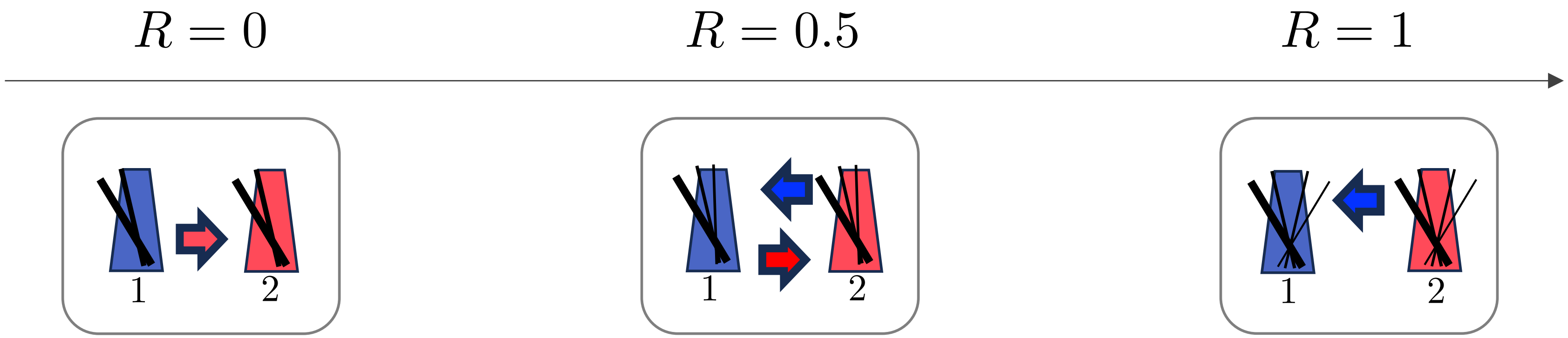
Metronome + Crank



Coupling direction ratio

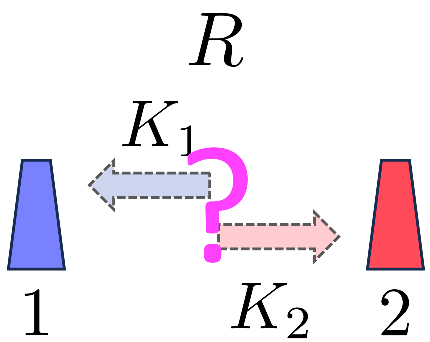
$$R \equiv \frac{K_1}{K_1 + K_2}$$





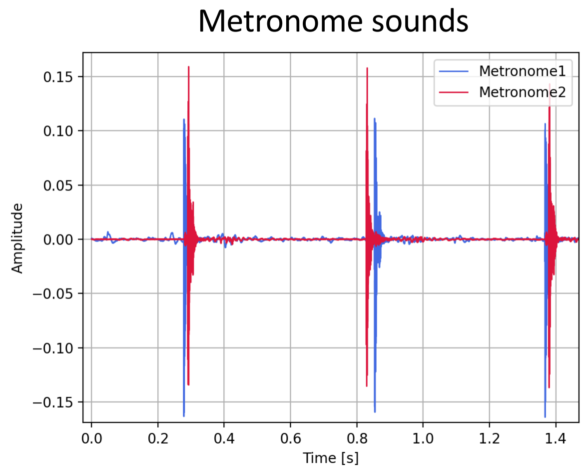
Question

Can R be inferred ?



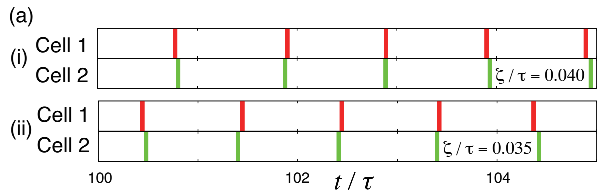
Synchronized oscillators

Spike-timing fluctuations

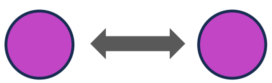


Our previous study

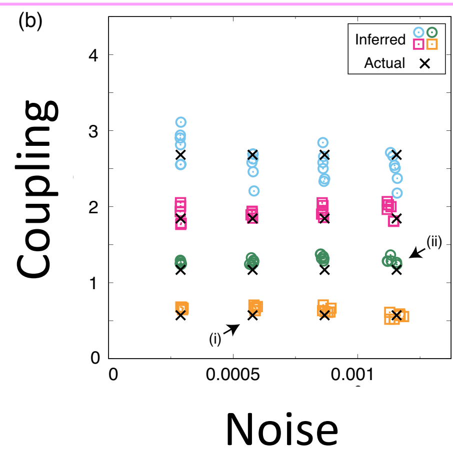
Observed spike-time series (Numerical simulation)



Identical



Inferred!



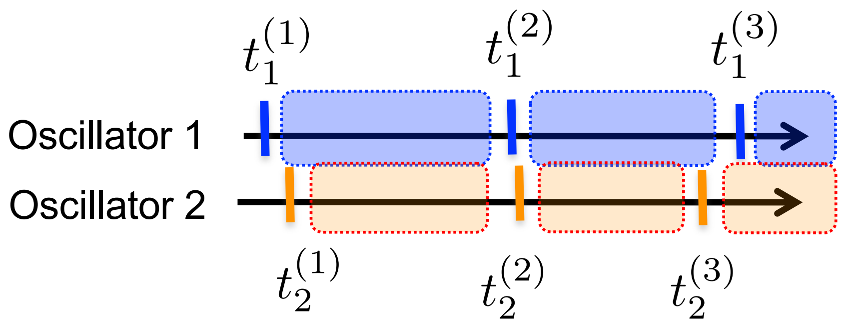
FM and H. Kori, PNAS
Vol. 119 No. 6 e2113620119 (2022)

Inference formula

Extension of the theory
in FM and H. Kori, PNAS 2022 ※Different notations

$$R = \frac{V_2 - V_1 + 2W_2}{2(W_1 + W_2)}$$

Spike-time series



Statistics

V_1 Variance in periods of oscillator 1

V_2 Variance in periods of oscillator 2

W_1 Correlation between periods of oscillator 1 and spike-time lags

W_2 Correlation between periods of oscillator 2 and spike-time lags

Inference formula

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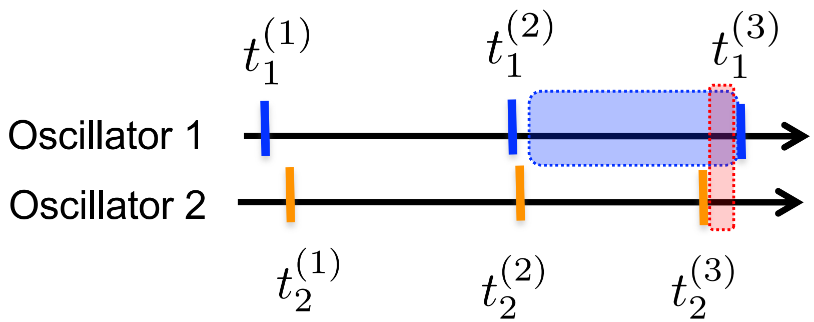
Spike-time series



Statistics

V_1 Variance in periods of oscillator 1

V_2 Variance in periods of oscillator 2

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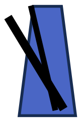
W_2 Correlation between periods of oscillator 2 and spike-time lags

Numerical demonstration

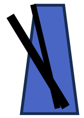
FitzHugh-Nagumo oscillator 1



1

$$\begin{cases} \dfrac{dv_1}{dt} = v_1(v_1 - \alpha)(1 - v_1) - w_1 + \kappa_1(v_2 - v_1) + \sqrt{D}\xi_1(t) \\ \dfrac{dw_1}{dt} = a(v_1 - \beta w_1) \end{cases}$$

FitzHugh-Nagumo oscillator 2



2

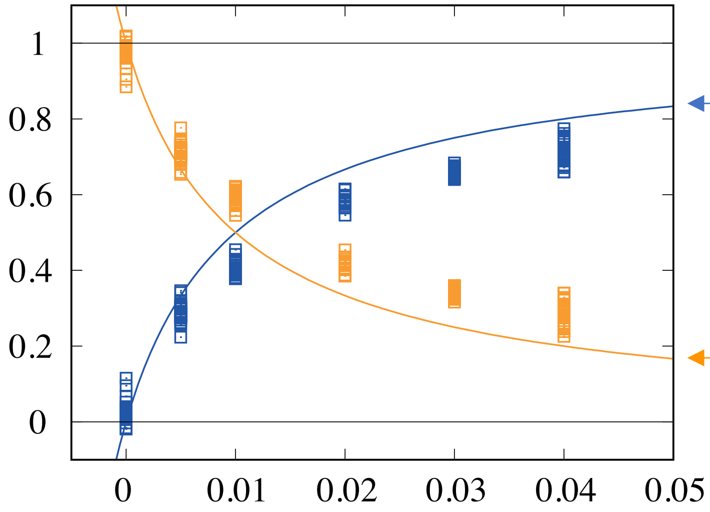
$$\begin{cases} \dfrac{dv_2}{dt} = \cdots + \kappa_2(v_1 - v_2)\cdots \\ \dfrac{dw_2}{dt} = \cdots \end{cases}$$

Stochastic simulation → Statistics → Inference formula

R ■

$1 - R$ ■

It can be inferred.



$$\frac{\kappa_1}{\kappa_1 + \kappa_2}$$

Actual parameter ratio

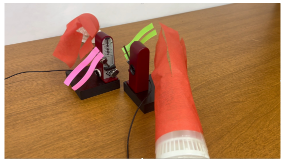
$$\frac{\kappa_2}{\kappa_1 + \kappa_2}$$

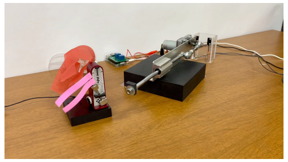
Coupling parameter κ_1

$(\kappa_2 = 0.01)$

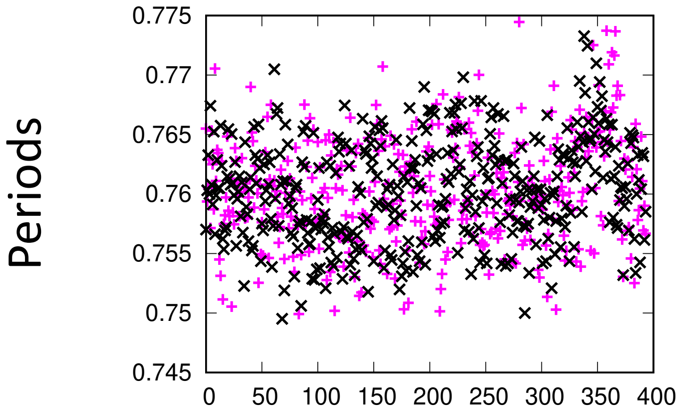
Information of FitzHugh-Nagumo equation were not used.

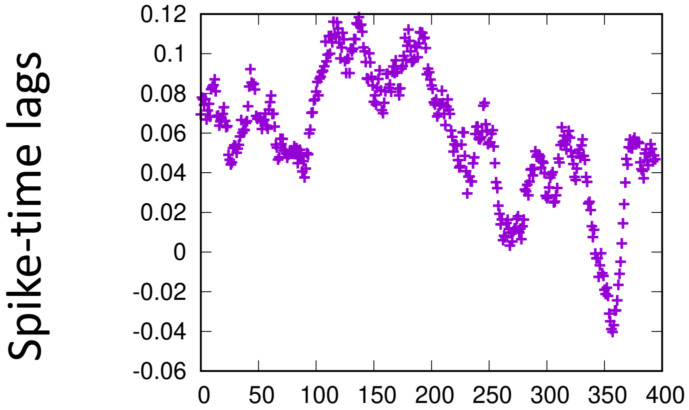
Metronome experiments



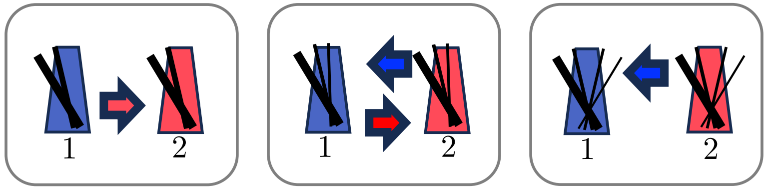


Observed data (3 min)





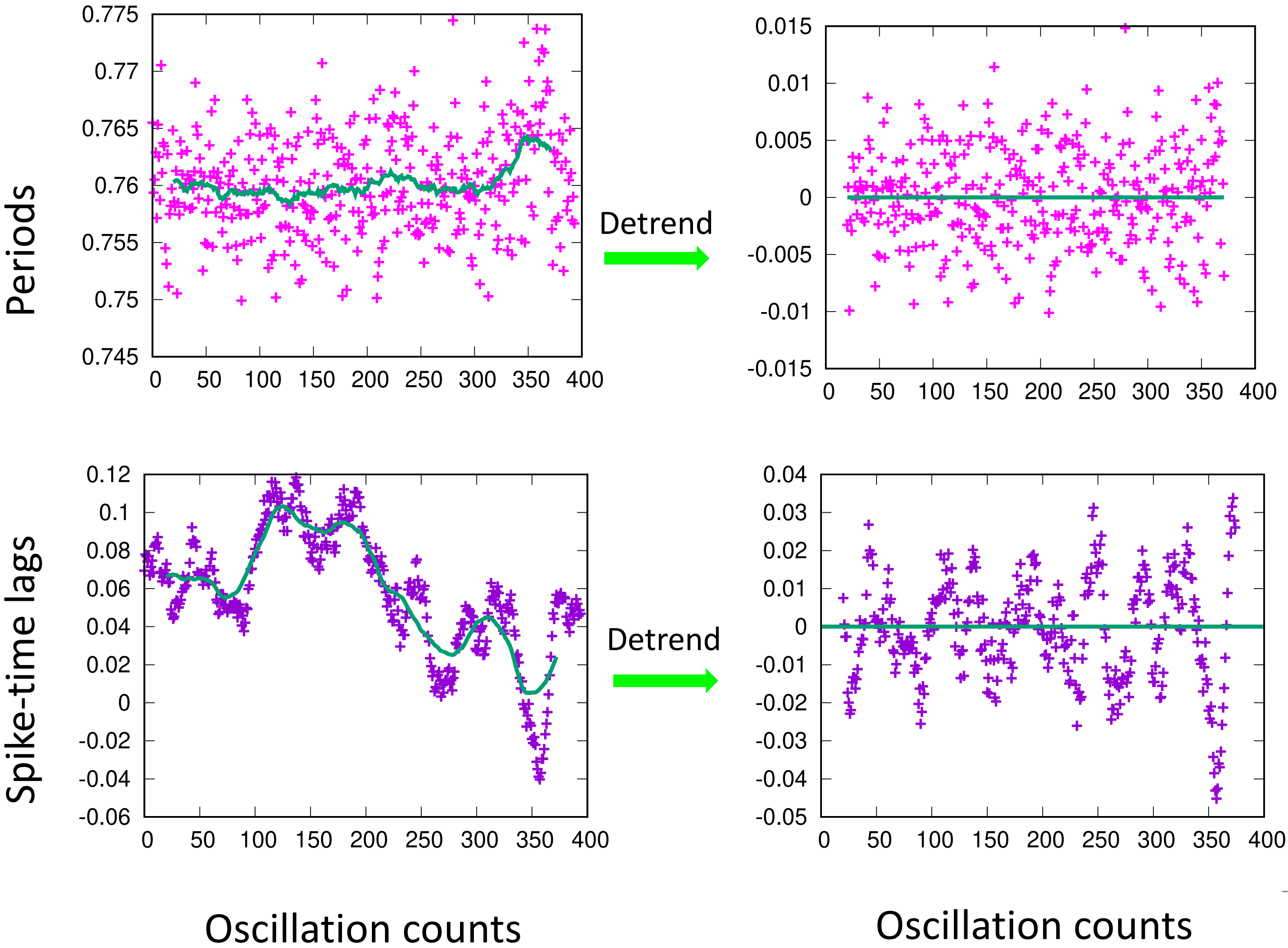
Which case?



Inference formula!

Detrend

The moving average period is L=41.

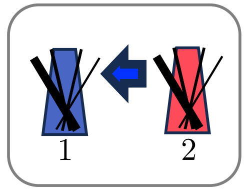


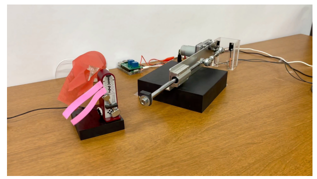
$V_1 \quad V_2$

$W_1 \, W_2$

R=1.16

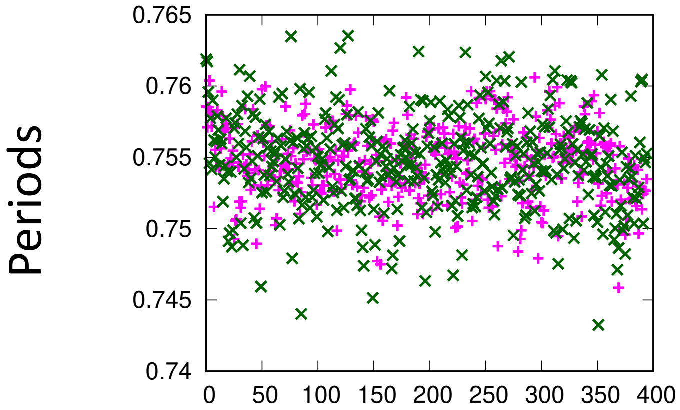
≈1.0

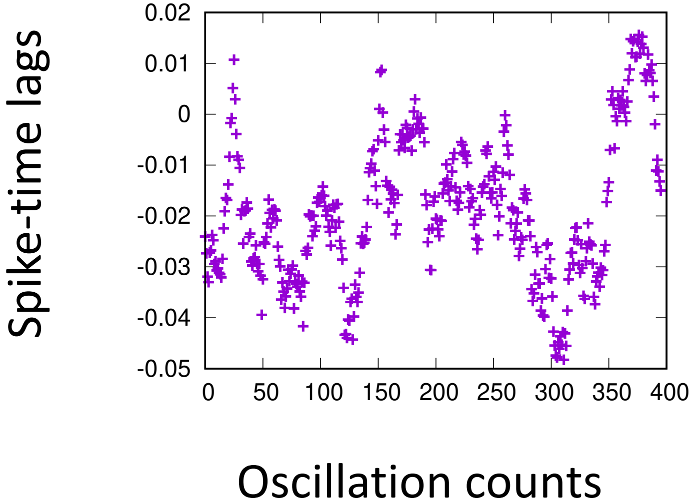




Metronome experiments

Observed data



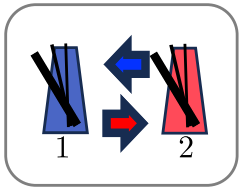


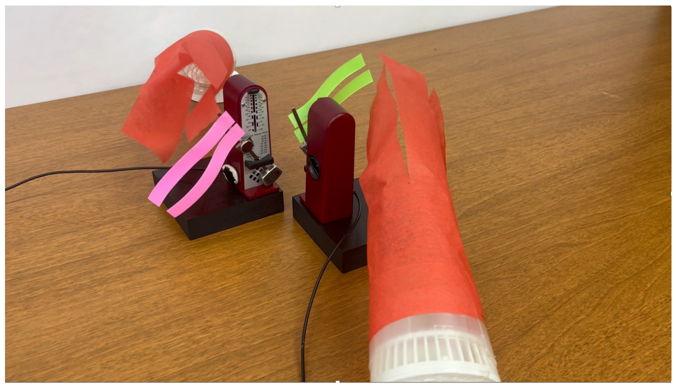
→ Detrend

→ Statistics

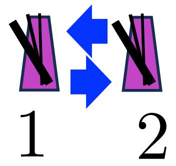
→ Formula

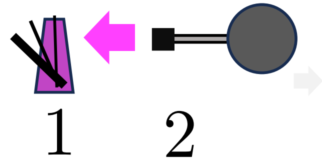
R=0.53



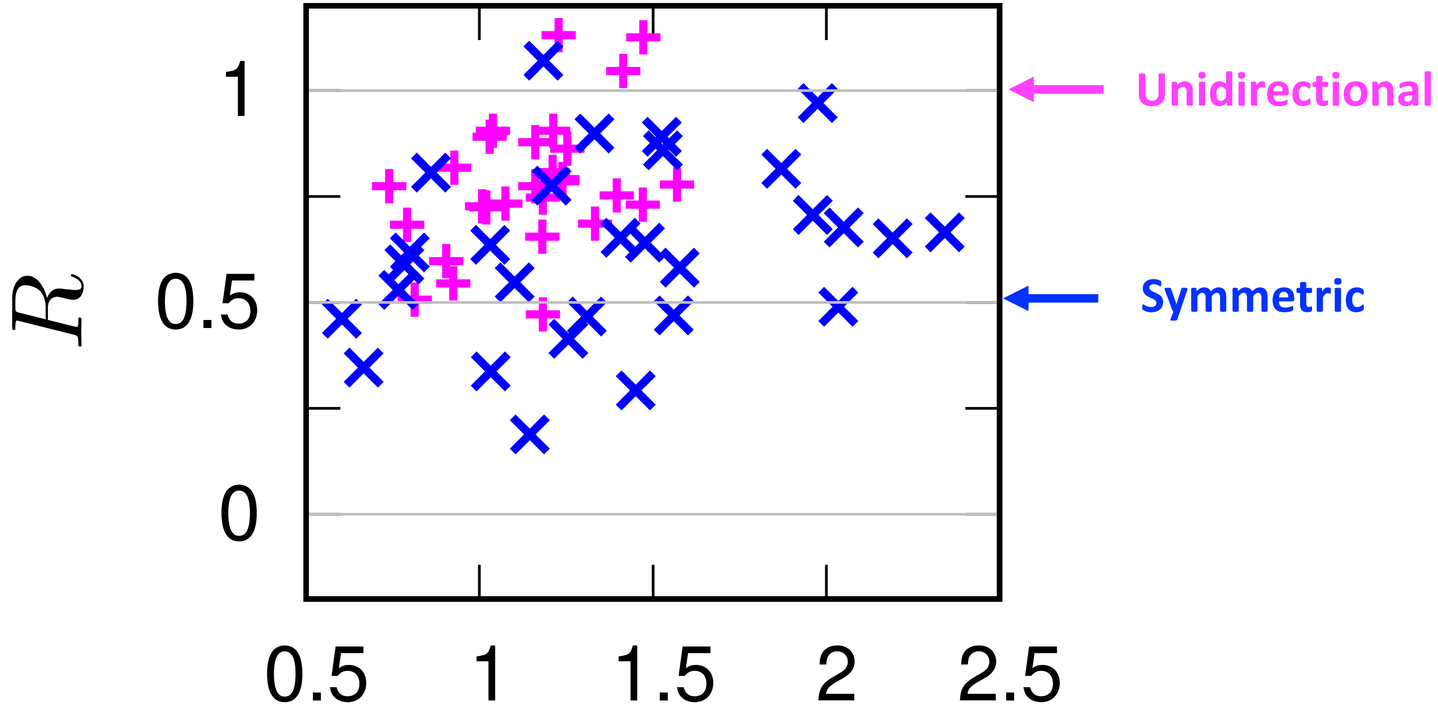


30 times	30 times
Bidirectional ×	**Unidirectional** +





L=41 (Moving average period)



$$\sqrt{\frac{V_1}{V_2}}$$

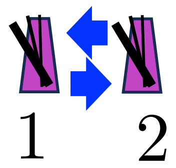
Variance in periods of oscillator 1

Variance in periods of oscillator 2

30 times

Bidirectional

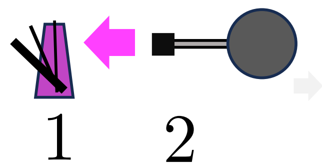
×



30 times

Unidirectional

+



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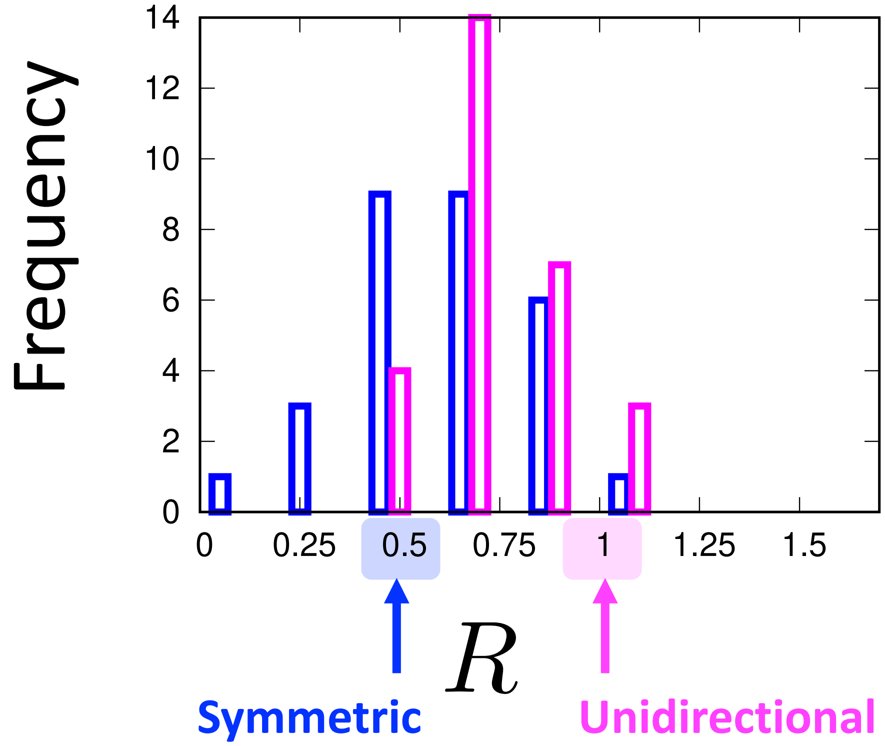
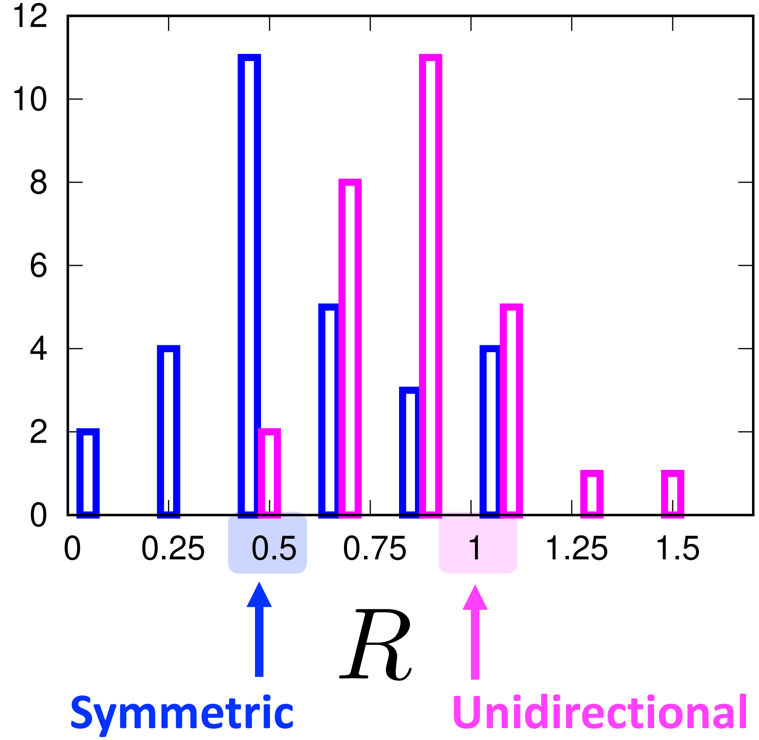


L=61 (Moving average period)



Coupled phase oscillators

No-trending case

Describing weakly coupled limit-cycle oscillators

$$\begin{cases} \dfrac{d\theta_1}{dt} = \omega_1 + \kappa_1 \Gamma_1(\theta_1 - \theta_2) + \epsilon Z_1(\theta_1)\sqrt{D_1}\xi_1(t) \\ \dfrac{d\theta_2}{dt} = \omega_2 + \kappa_2 \Gamma_2(\theta_2 - \theta_1) + \epsilon Z_2(\theta_2)\sqrt{D_2}\xi_2(t) \end{cases}$$

Frequency | Coupling | Phase response | i.i.d noise

[Assumptions]

- Weak noise $\epsilon \ll 1$
- Close frequencies $\omega_1 \simeq \omega_2$
- In-phase synchronization

Modified model

Coupled phase oscillators

Trending case

$$\begin{cases} \dfrac{d\theta_1}{dt} = [\omega_1 + \underline{\tilde{\epsilon}\tilde{\omega}_1(t)}] + [\kappa_1 + \underline{\tilde{\epsilon}\tilde{\kappa}_1(t)}]\Gamma_1(\theta_1 - \theta_2) + \epsilon Z_1(\theta_1)\sqrt{D_1}\xi_1(t) \\ \dfrac{d\theta_2}{dt} = [\omega_2 + \underline{\tilde{\epsilon}\tilde{\omega}_2(t)}] + [\kappa_2 + \underline{\tilde{\epsilon}\tilde{\kappa}_2(t)}]\Gamma_2(\theta_2 - \theta_1) + \epsilon Z_2(\theta_2)\sqrt{D_2}\xi_2(t) \end{cases}$$

Slow modification

[Assumptions]

Weak noise $\epsilon, \tilde{\epsilon} \ll 1$

Well synchronized

- Phase difference → Attractor
- Spike-time lags are small.

Derivation of inference formula

V_1 V_2 W_1 W_2 were obtained by the lowest order approximation.

R

$$V_1 = \text{(Phase diffusion)} - 2\frac{K_1}{K_1 + K_2} W_1$$

$$V_2 = \text{(Phase diffusion)} - 2\frac{K_2}{K_1 + K_2} W_2$$

Variances

Correlations

(1-R)

Not necessary to know
the details of the function forms

Conclusions

[Theory] We derived inference formula that infers coupling direction in synchronized oscillators.

Extension of the theory in FM and H. Kori, PNAS 2022

[Simulations] We numerically confirmed the validity.

[Experiments] Our theory could distinguish between unidirectional and bidirectional coupling systems.