Tuning transduction from hidden observables to optimize information harvesting

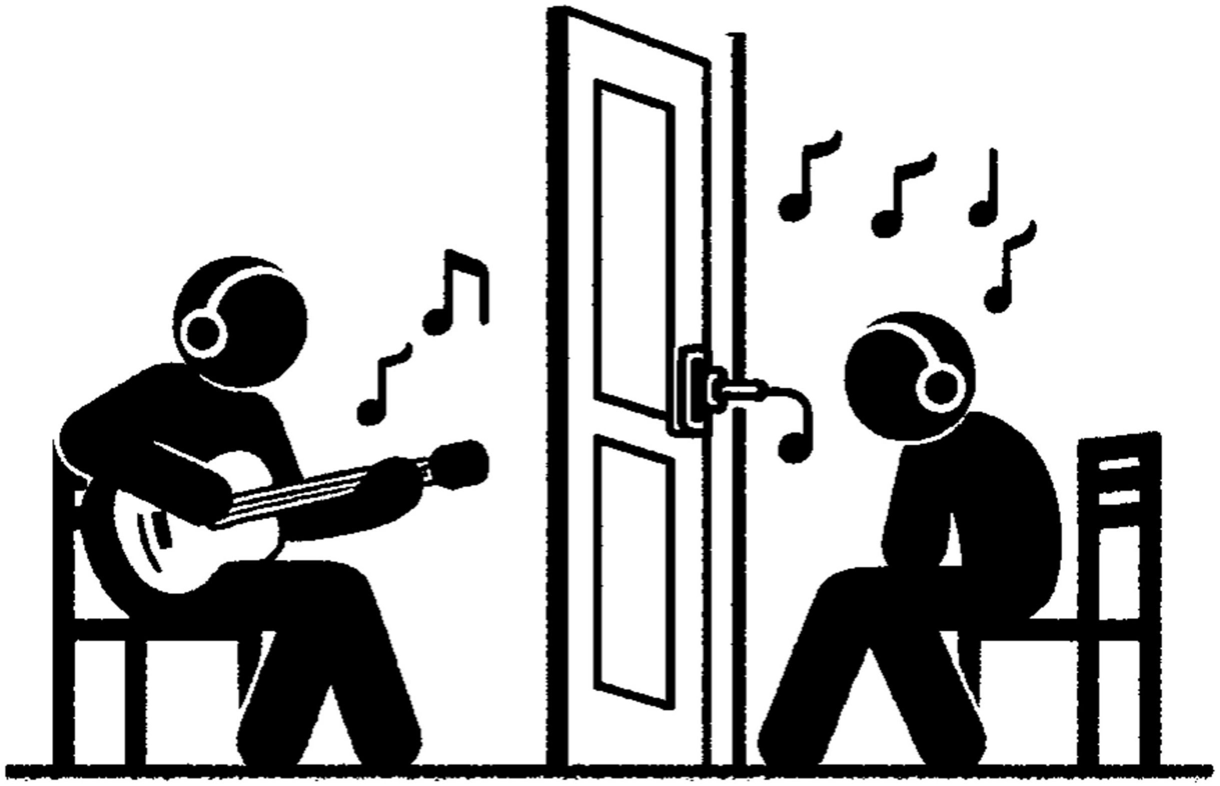
D. M. Busiello

in collaboration with *G. Nicoletti (EPFL)*

Workshop, July 8–19, 2024

Frontiers in Non-equilibrium Physics, YITP, Kyoto

Information from *inaccessible degrees of freedom*

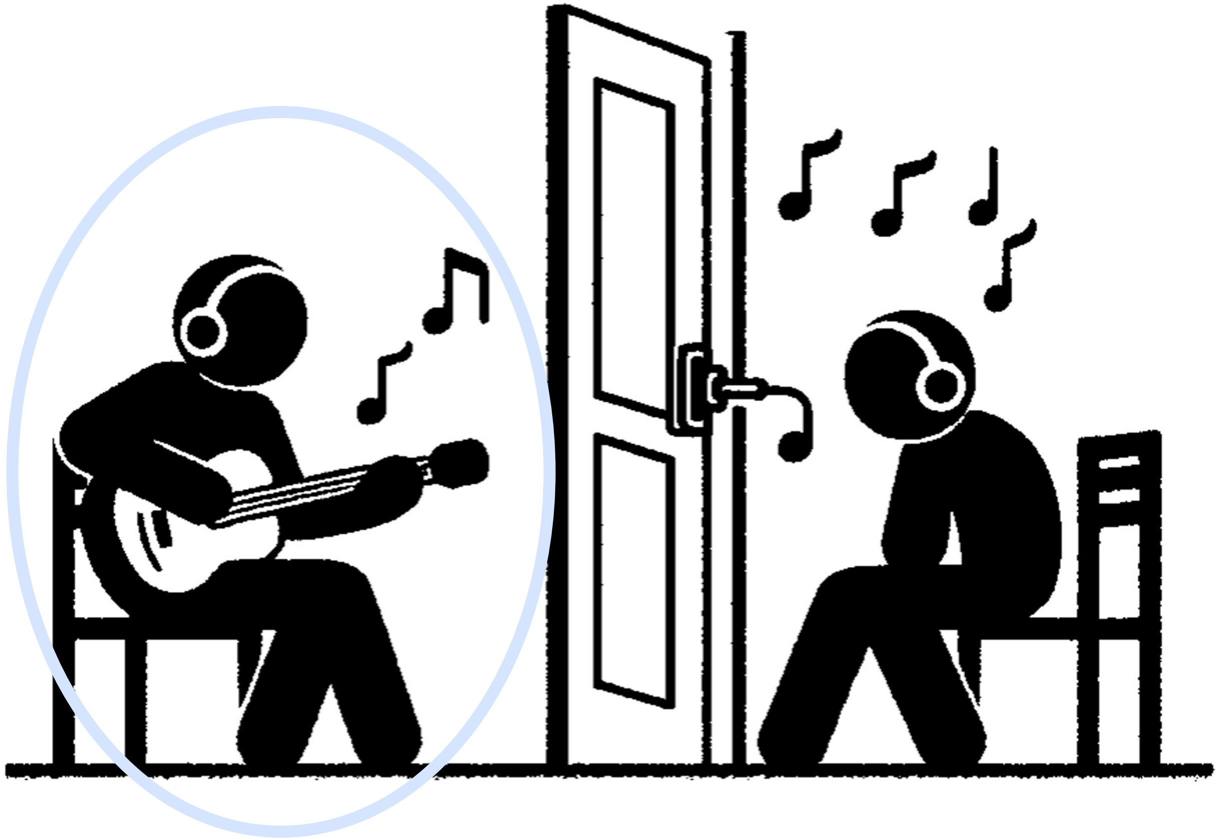


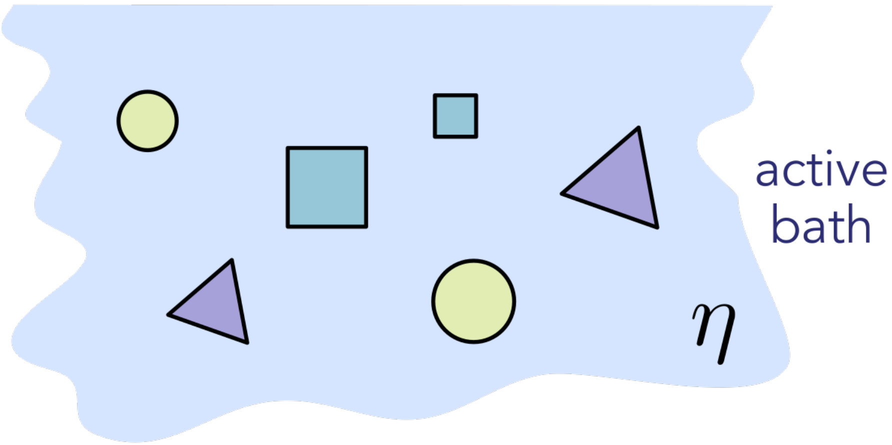
Who is playing the guitar?

- Age
- Nationality
- …

Too many hidden variables
The lack of an underlying model

Information from *inaccessible* *degrees of freedom*





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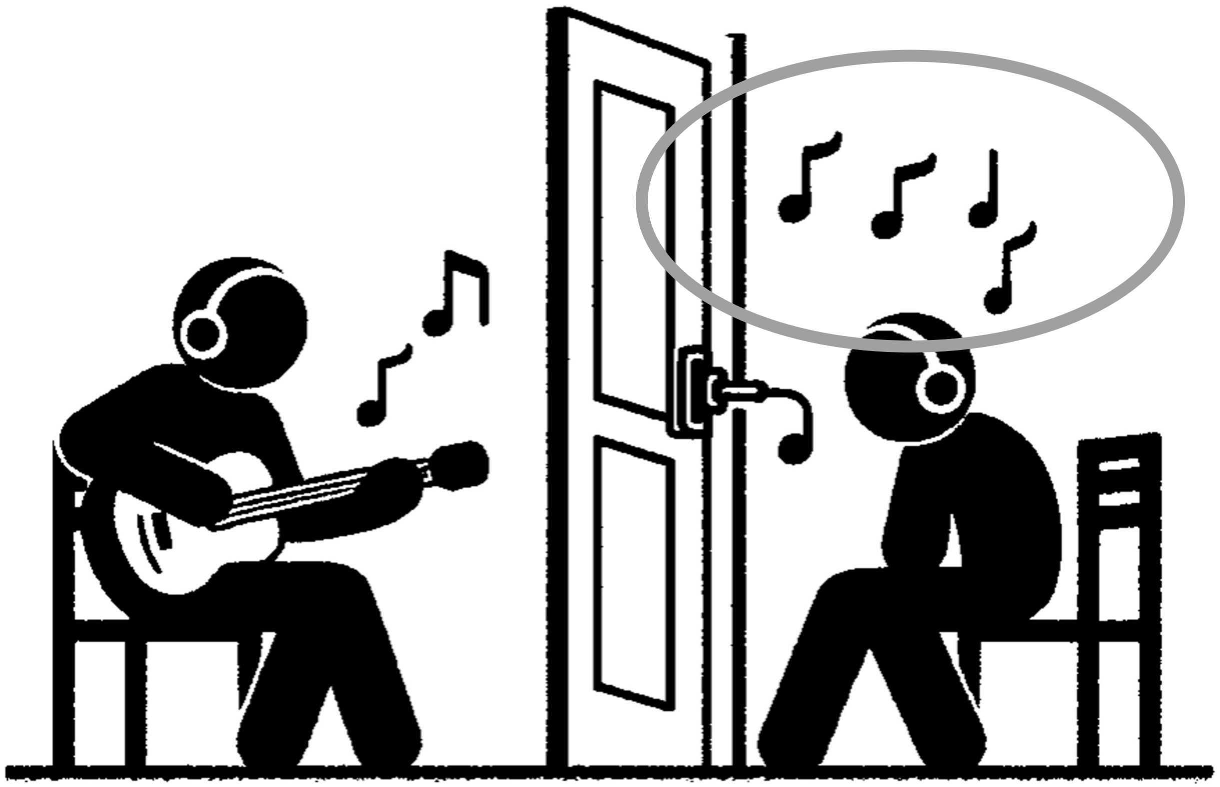

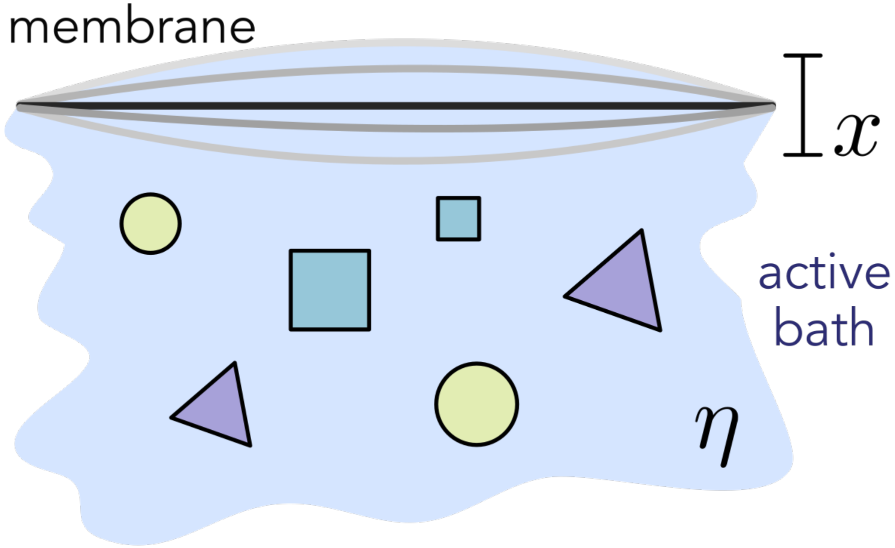

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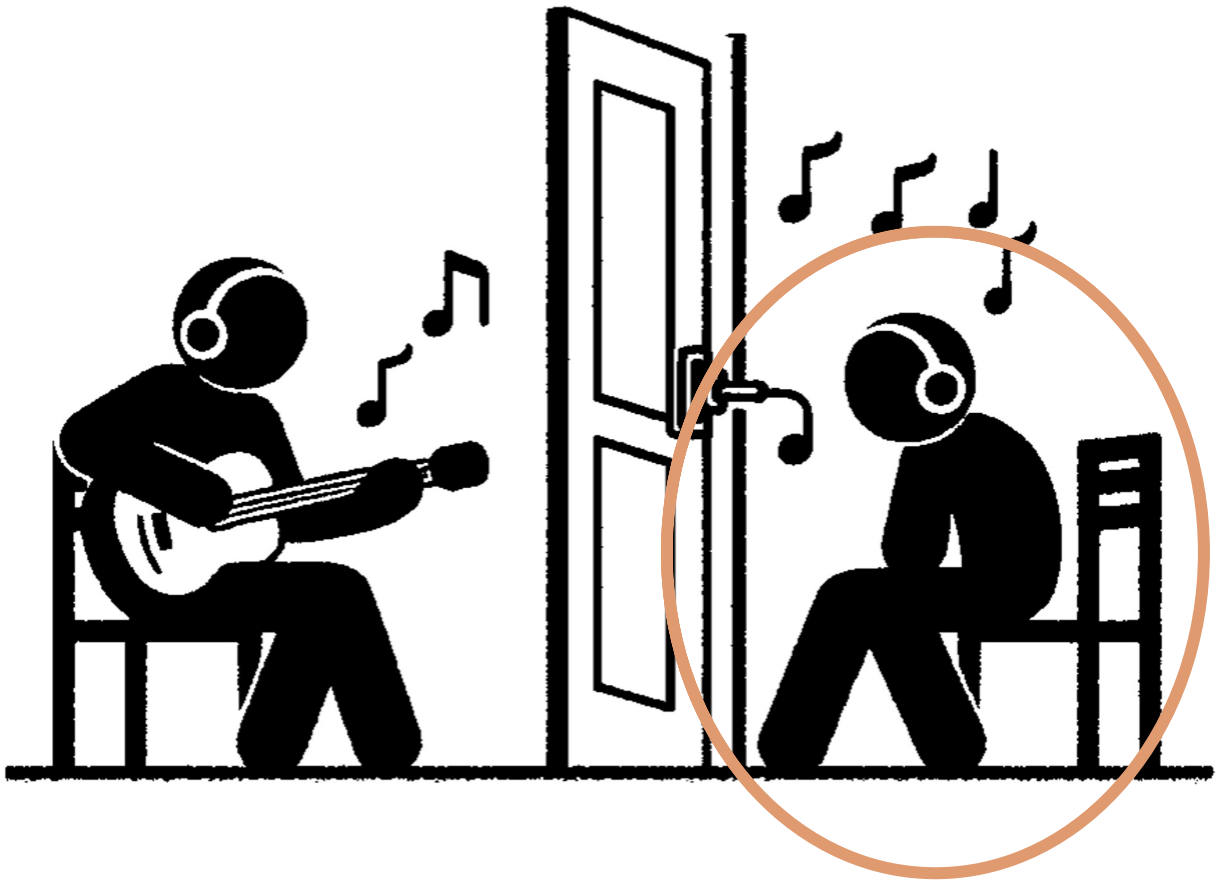

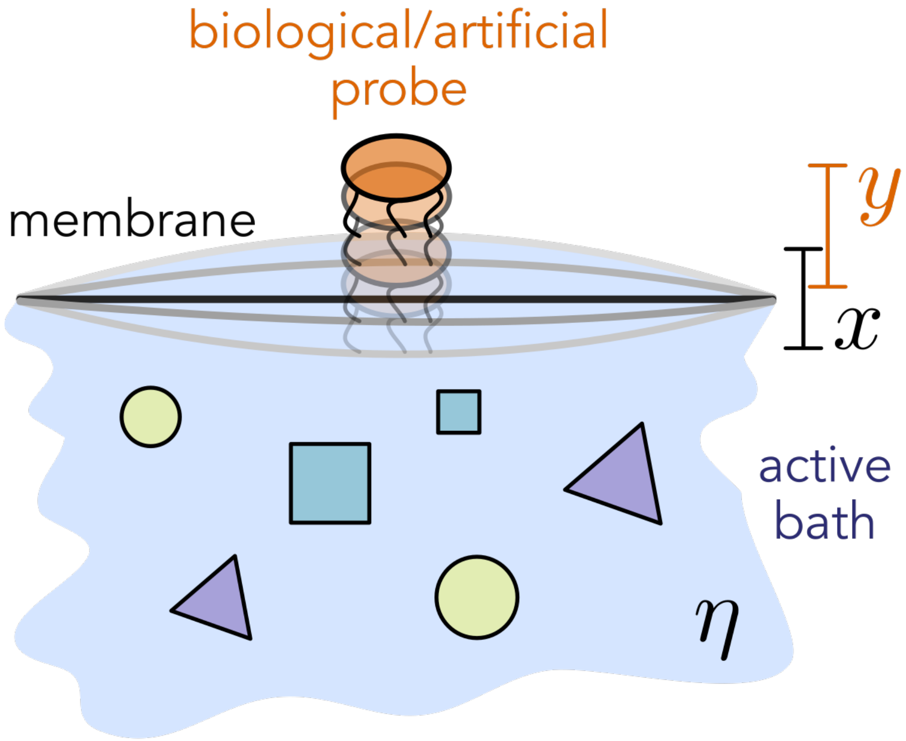

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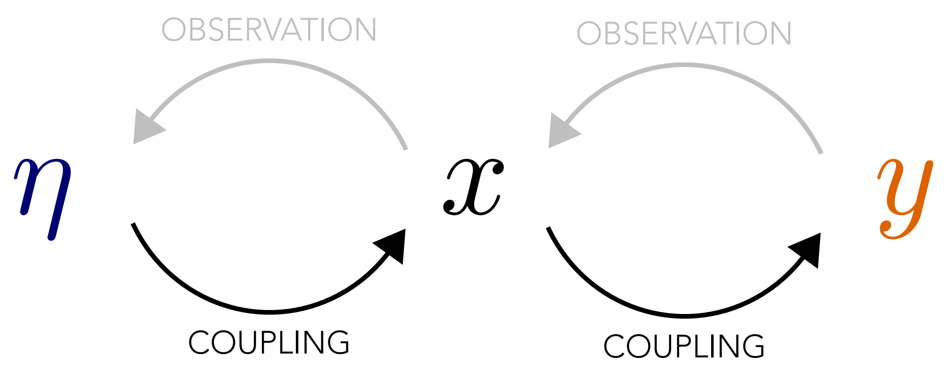
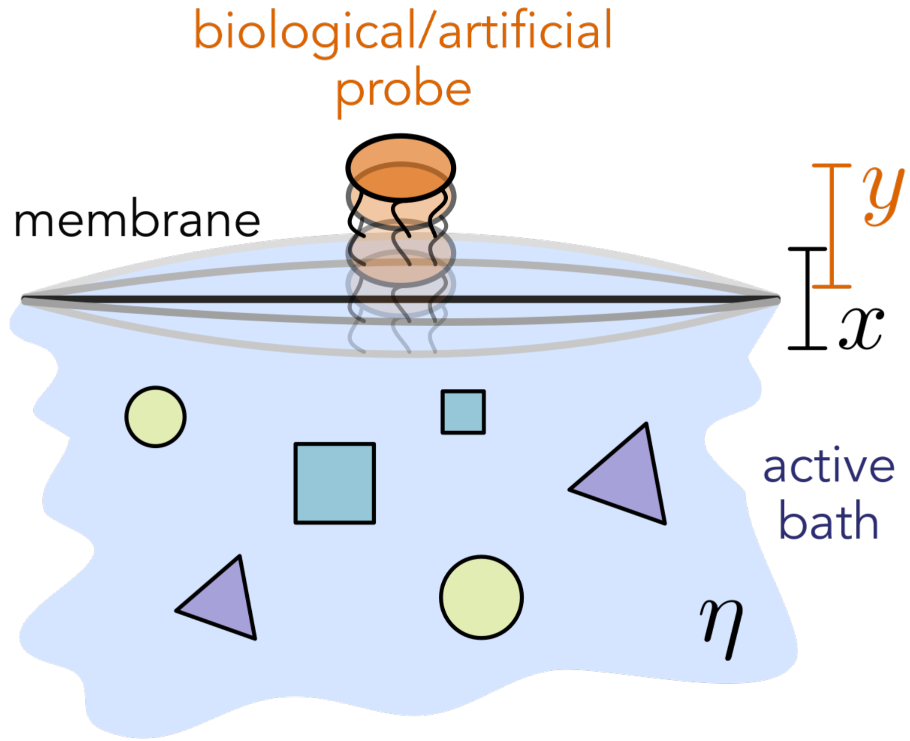


$$
\begin{aligned}
\tau_y \dot{y} &= -y + a x + \sqrt{2\tau D_y}\xi_y(t) \\
\tau_x \dot{x} &= -x + \sigma\eta + \sqrt{2\tau D_x}\xi_x(t) \\
\tau_\eta \dot{\eta} &= -\eta + \sqrt{2\tau_\eta D_\eta}\xi_\eta(t)
\end{aligned}
$$



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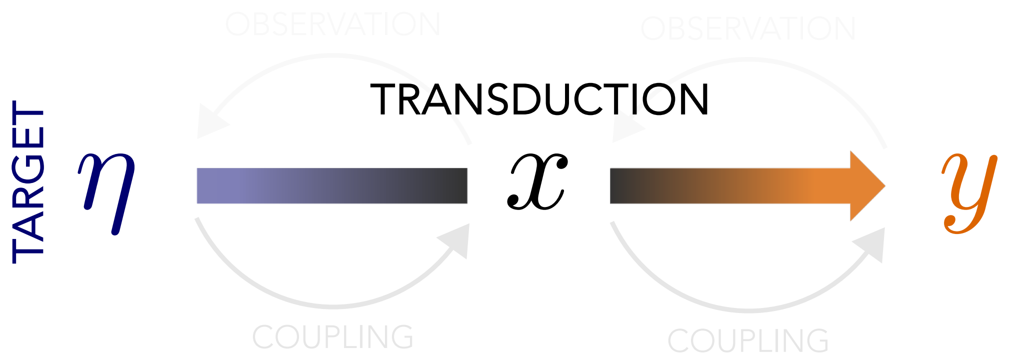

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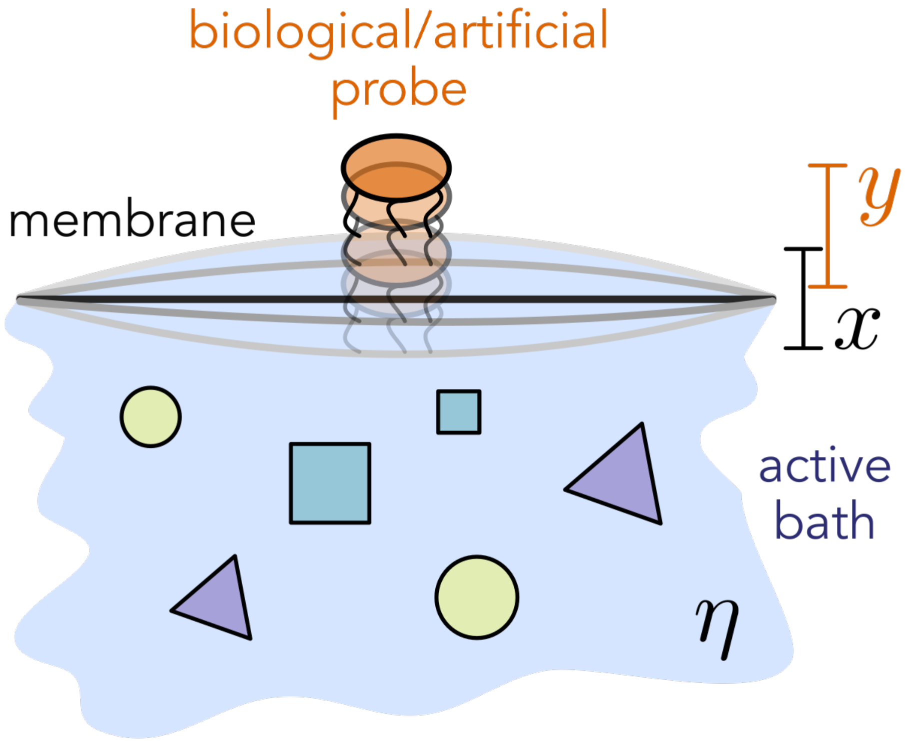
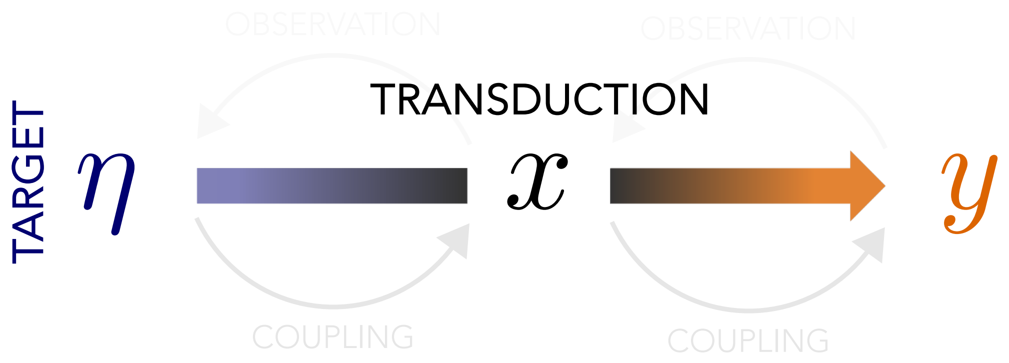


Ideal but unrealizable scenario: *all-knowing probe*



Probe-membrane coupling to be tuned

$$\begin{aligned} \tau_y \dot{y} &= -y + a x + \sqrt{2\tau D_y}\xi_y(t) \\ \tau_x \dot{x} &= -x + \sigma\eta + \sqrt{2\tau D_x}\xi_x(t) \\ \tau_\eta \dot{\eta} &= -\eta + \sqrt{2\tau_\eta D_\eta}\xi_\eta(t) \end{aligned}$$

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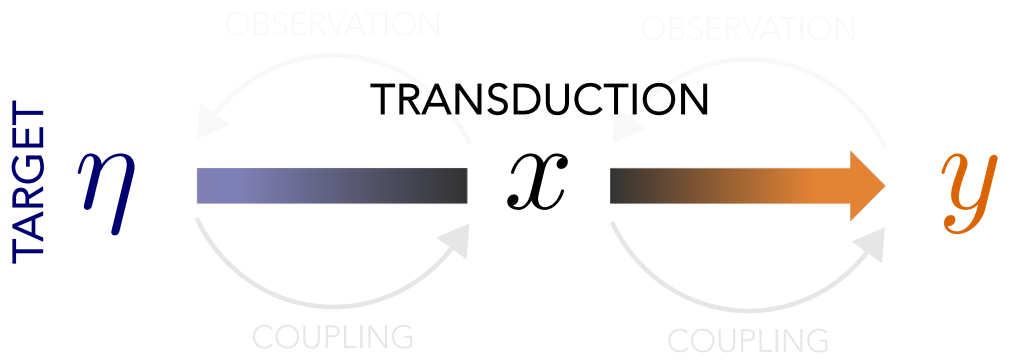

$$\underbrace{\max_a \left(\lambda I_{y\eta} - (1-\lambda) T \dot{S}_a\right)}_{\mathcal{L}(a)}$$

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$$\begin{aligned}
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\end{aligned}$$

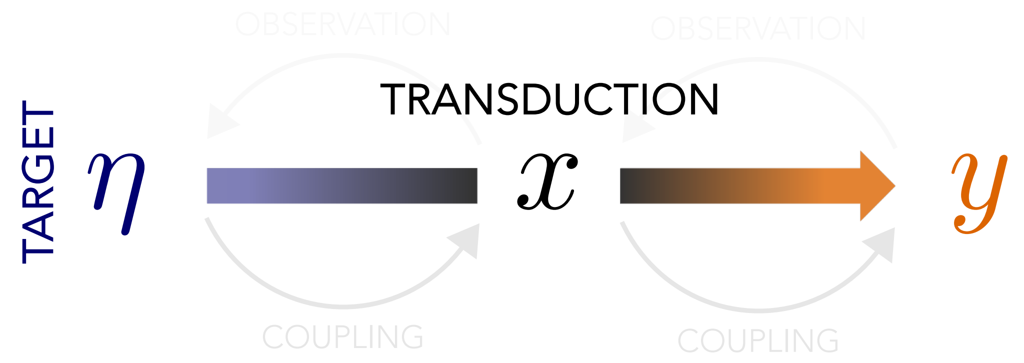
Probe-bath mutual information

$$I_{y\eta} = \int p_{y\eta} \log(p_{y|\eta}/p_y)\, \mathrm{d}y\mathrm{d}\eta$$

Entropy production due to the probe

$$\dot{S}_{\mathrm{tot}} - \dot{S}_{\mathrm{tot}}(a=0)$$

Ideal but unrealizable scenario: *all-knowing probe*



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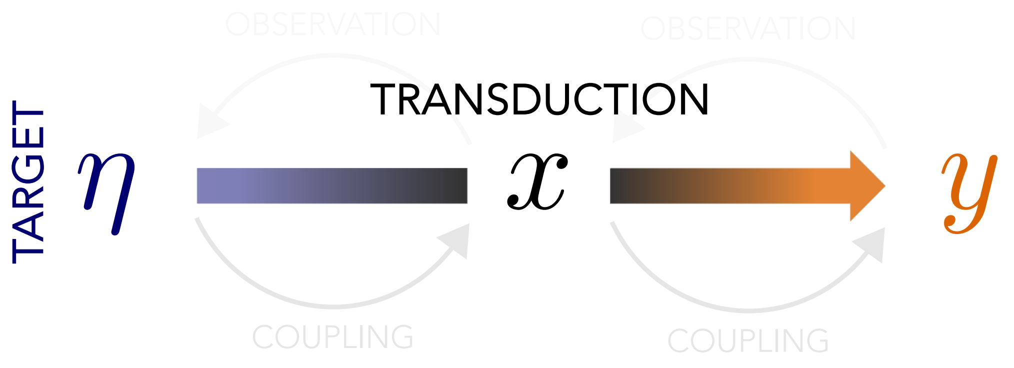
λ represents the "strategy" of the probe
and sets the *energy budget* of the system
Invested into information transduction

Ideal but unrealizable scenario: *all-knowing probe*



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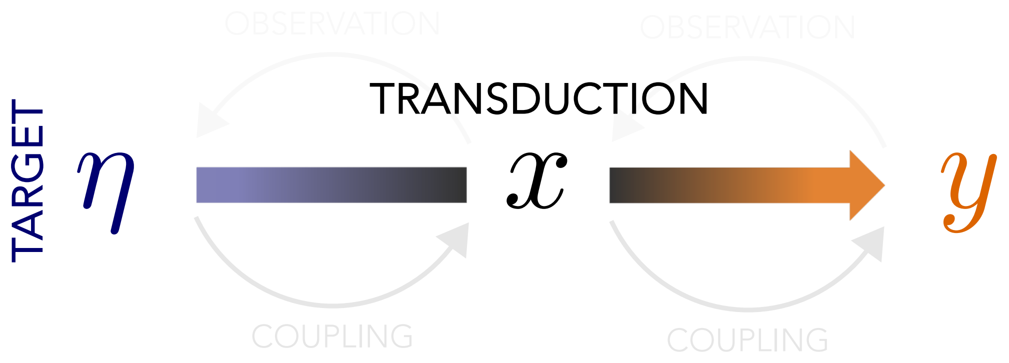
Timescales: $\theta_\eta = \frac{\tau_\eta}{\tau} \gg \frac{\tau_x}{\tau} = \frac{\tau_y}{\tau} = 1$

λ represents the "strategy" of the probe and sets the *energy budget* of the system Invested into information transduction

Ideal but unrealizable scenario: *all-knowing probe*



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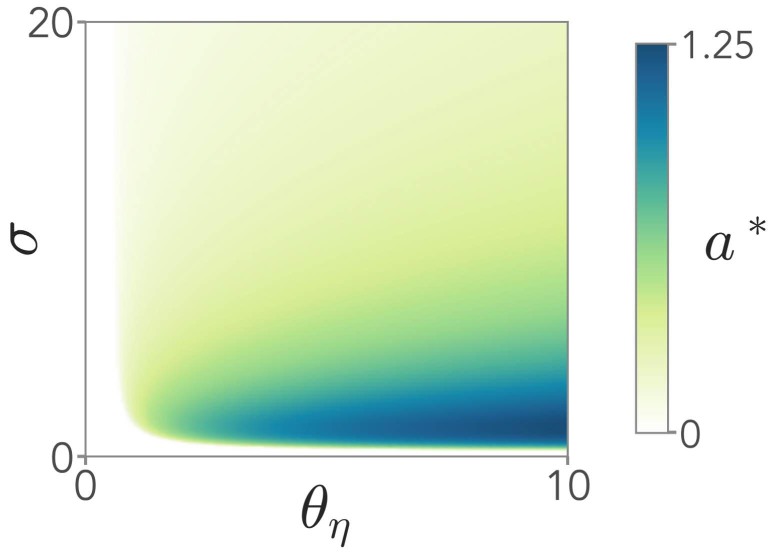

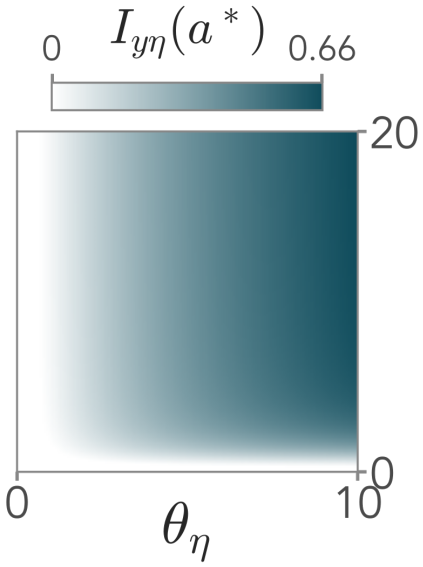

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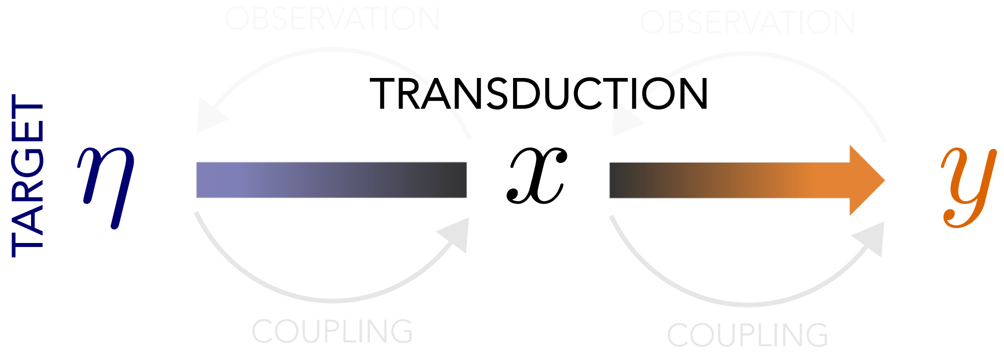

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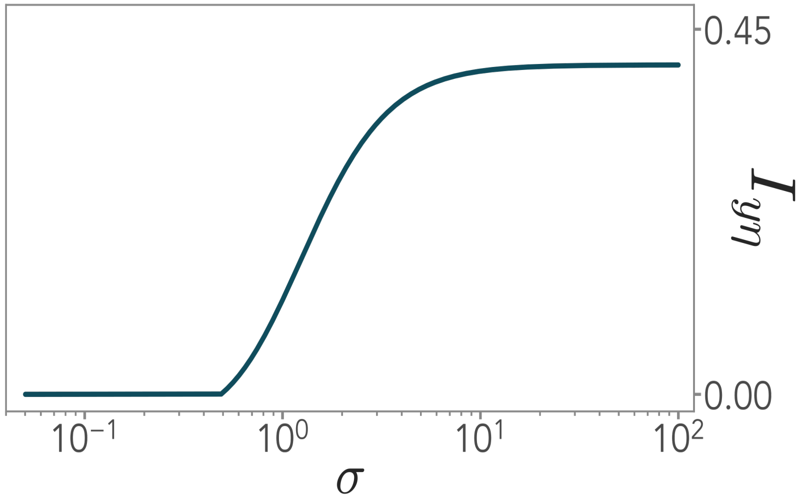
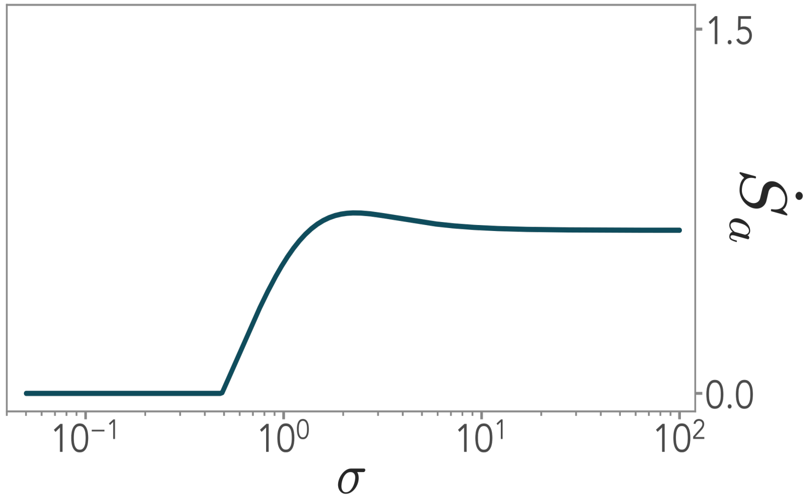
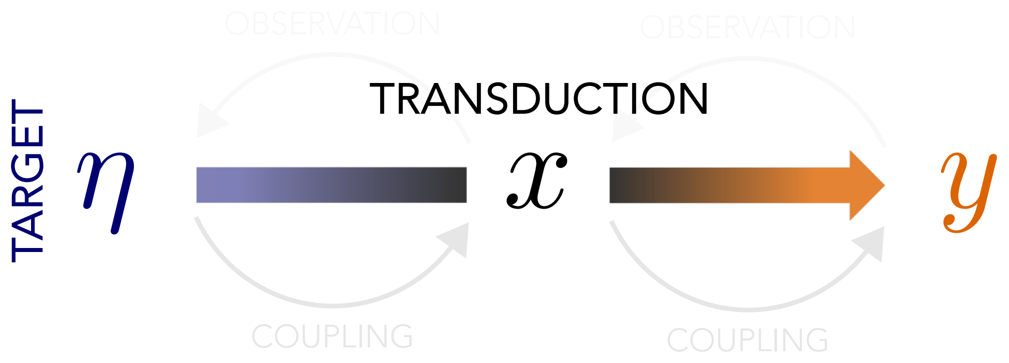


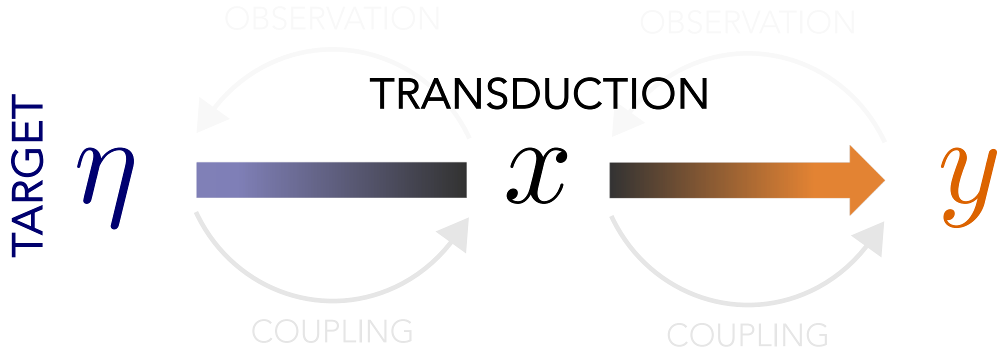


Realistic scenario: *partial information available*



$$\max_a \underbrace{\left(\lambda I_{xy} - (1-\lambda) T \dot{S}_{xy}\right)}_{\mathcal{L}_{\text{eff}}(a)}$$

Realistic scenario: *partial information available*



$$\max_a \underbrace{\left(\lambda I_{xy} - (1-\lambda) T \dot{S}_{xy}\right)}_{\mathcal{L}_{\text{eff}}(a)}$$

Probe-membrane mutual information

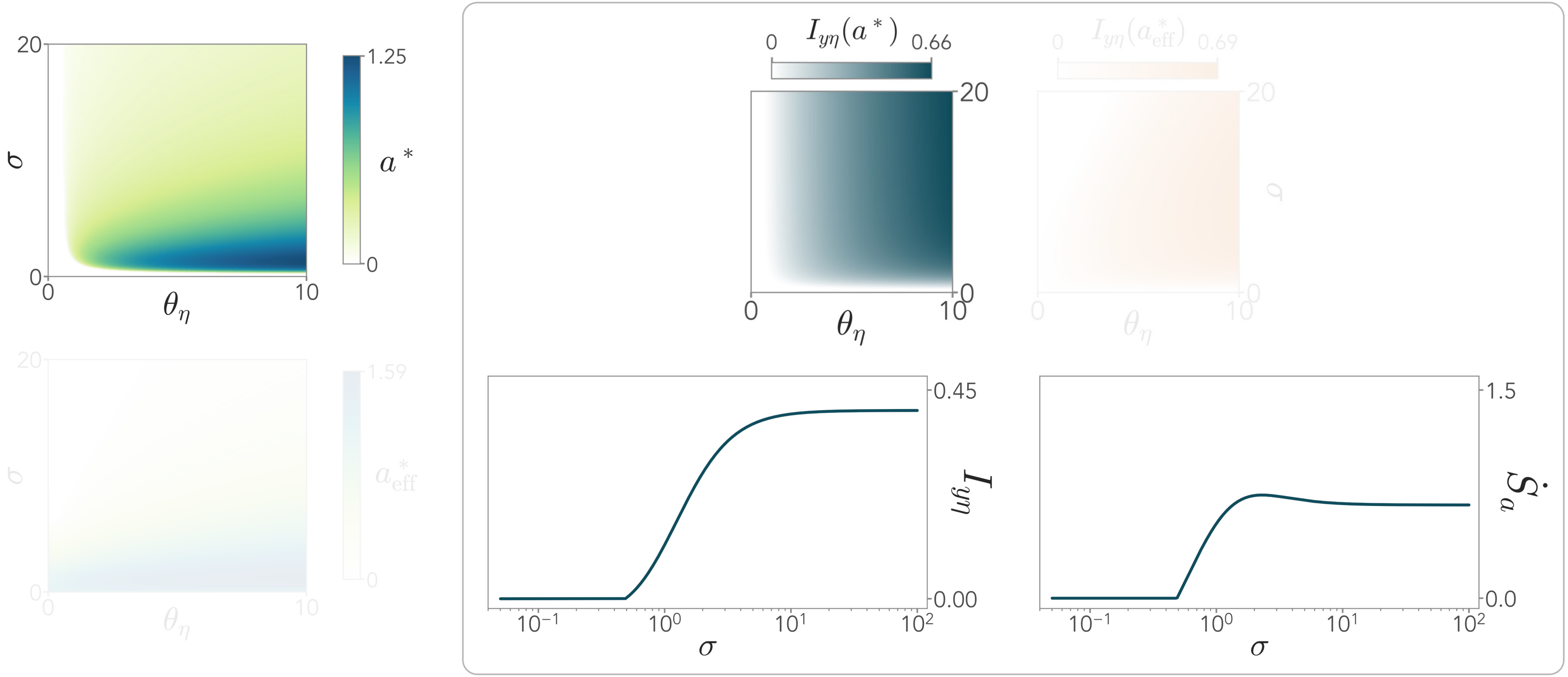
$$I_{xy} = \frac{1}{2} \log \frac{\Sigma_{xx}\Sigma_{yy}}{\det(\mathbf{\Sigma}_{xy})}$$

Entropy production due to the probe

$$\dot{S}_{xy} = \mathrm{Tr}\left[\mathbf{D}_{xy}^{-1} \mathbf{A}_{xy}^{\text{red}} \mathbf{\Sigma}_{xy} \left(\mathbf{A}_{xy}^{\text{red}}\right)^T\right] - \mathrm{Tr}\left(\mathbf{A}_{xy}^{\text{red}}\right)$$

Reduced interaction matrix marginalizing over non-accessible DOFs

Information transduction can be efficient



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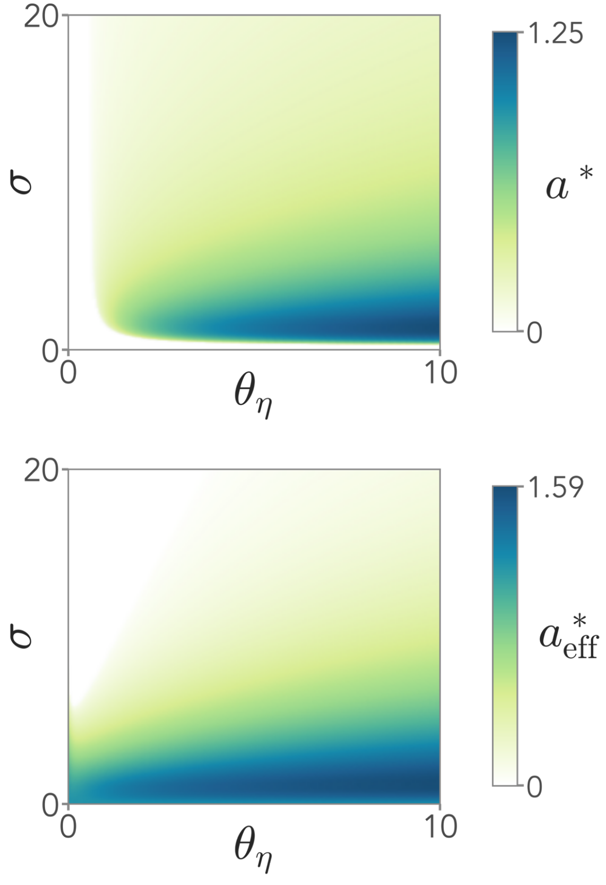

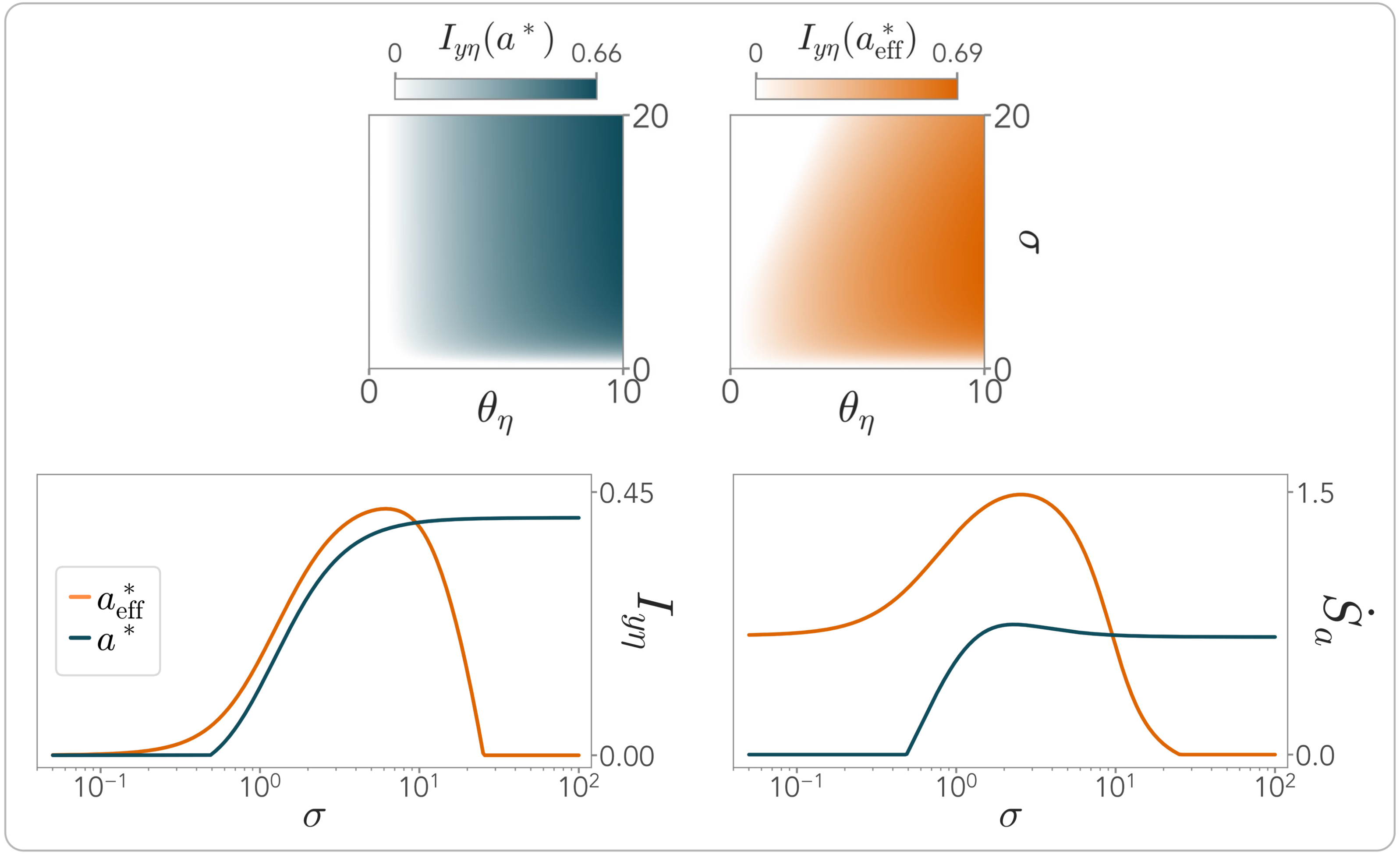

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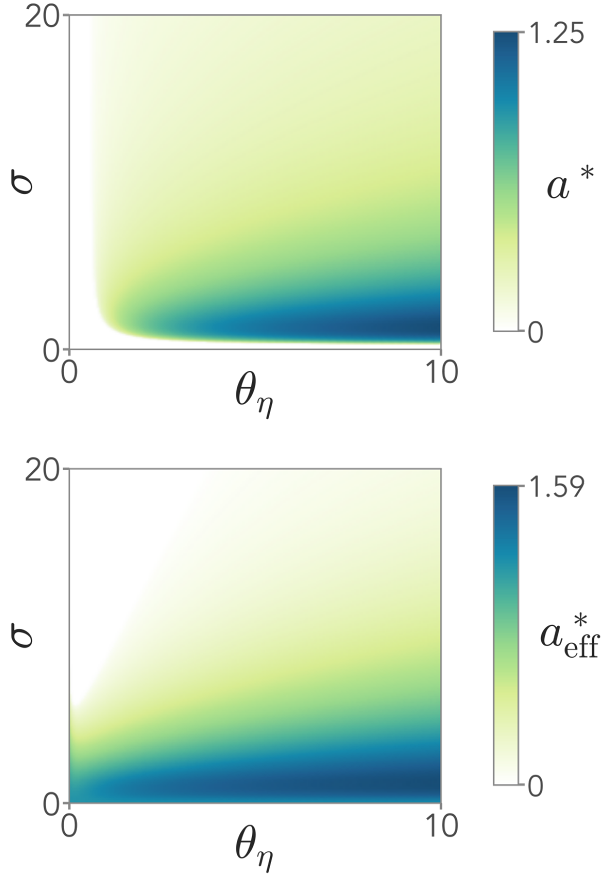

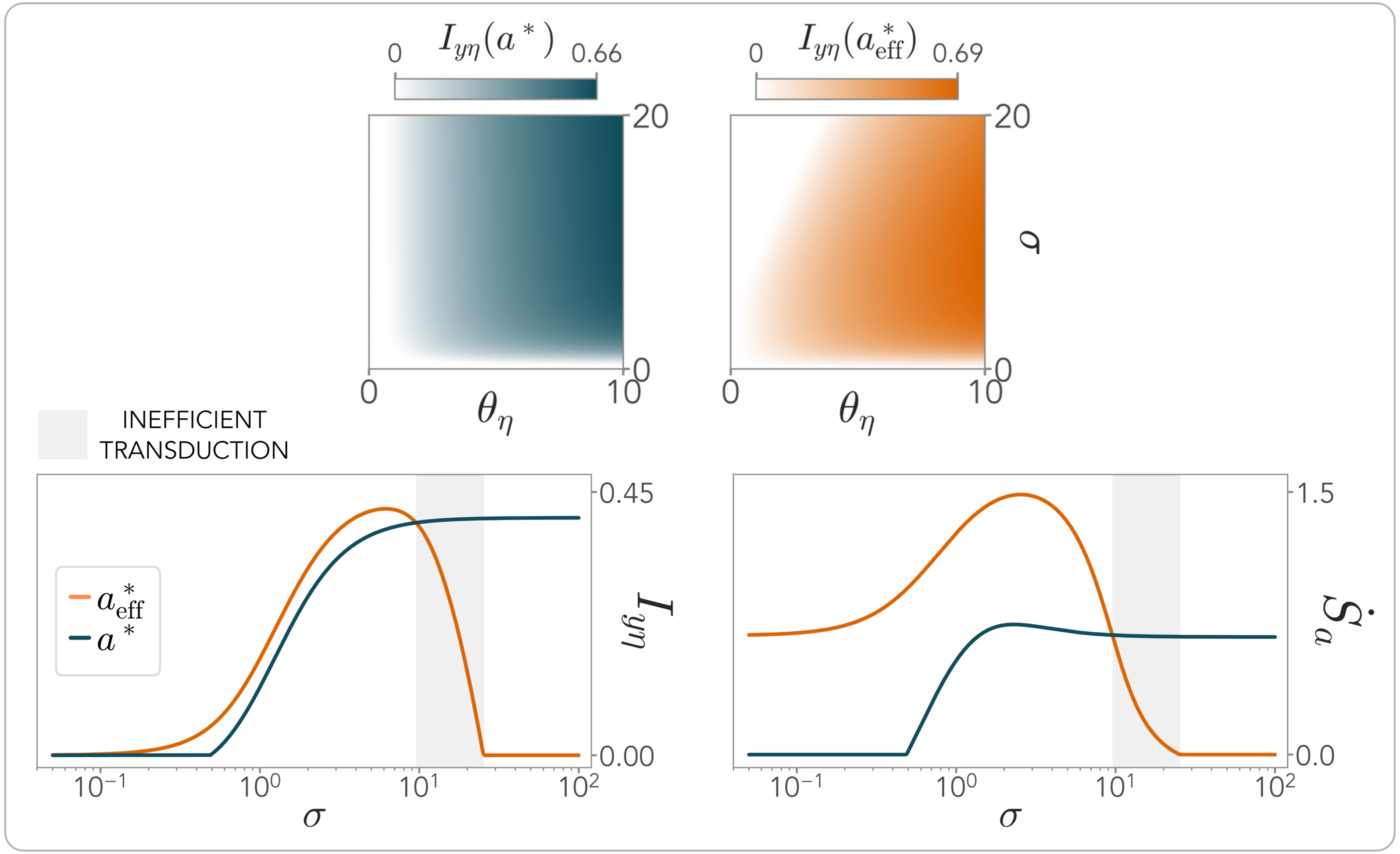

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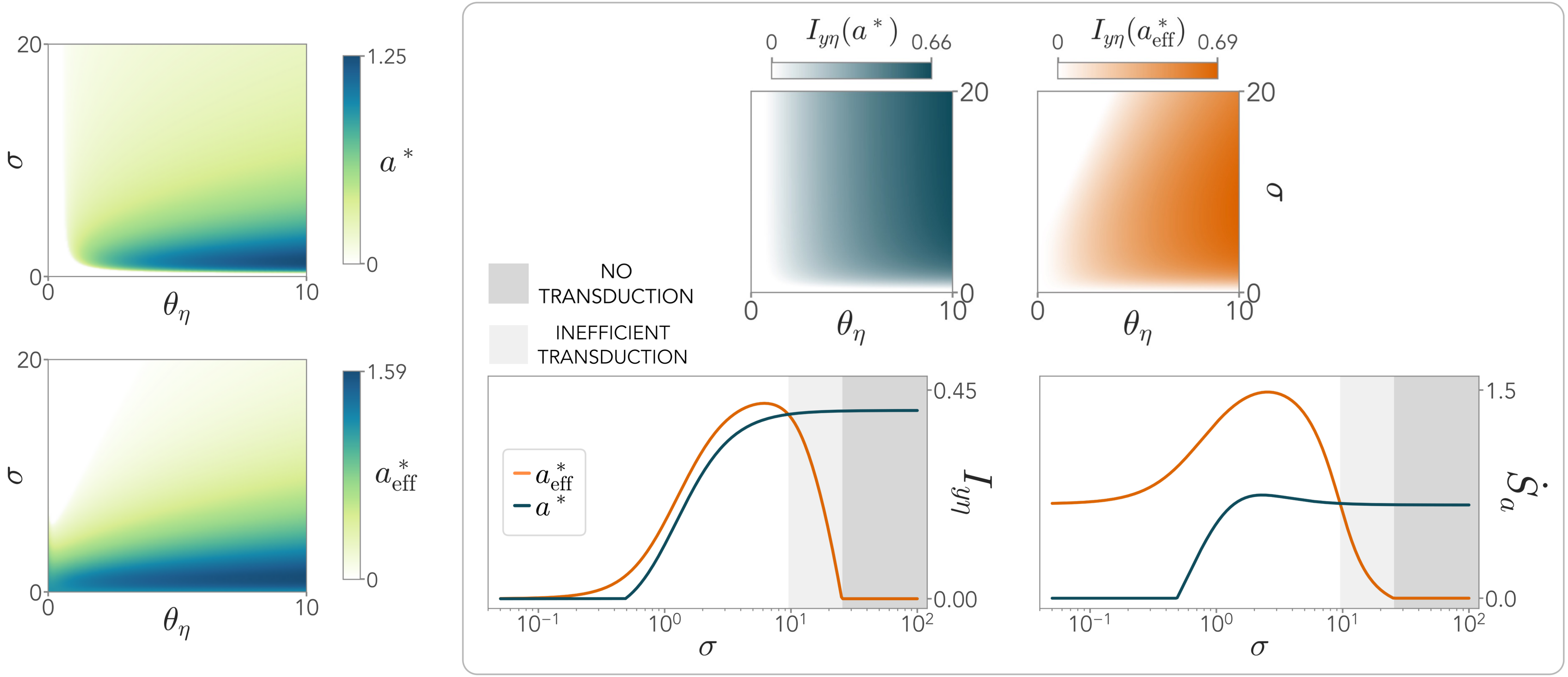
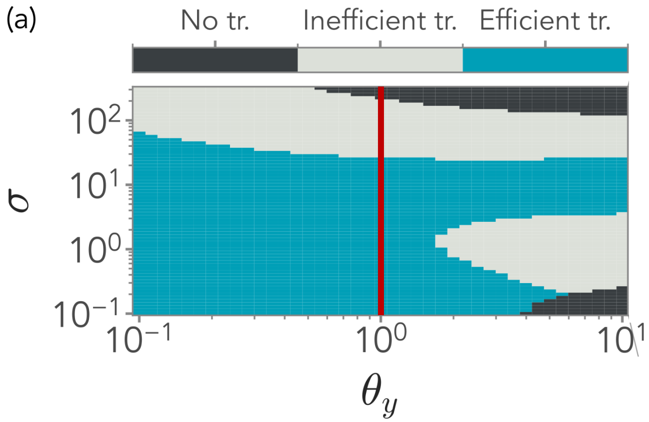
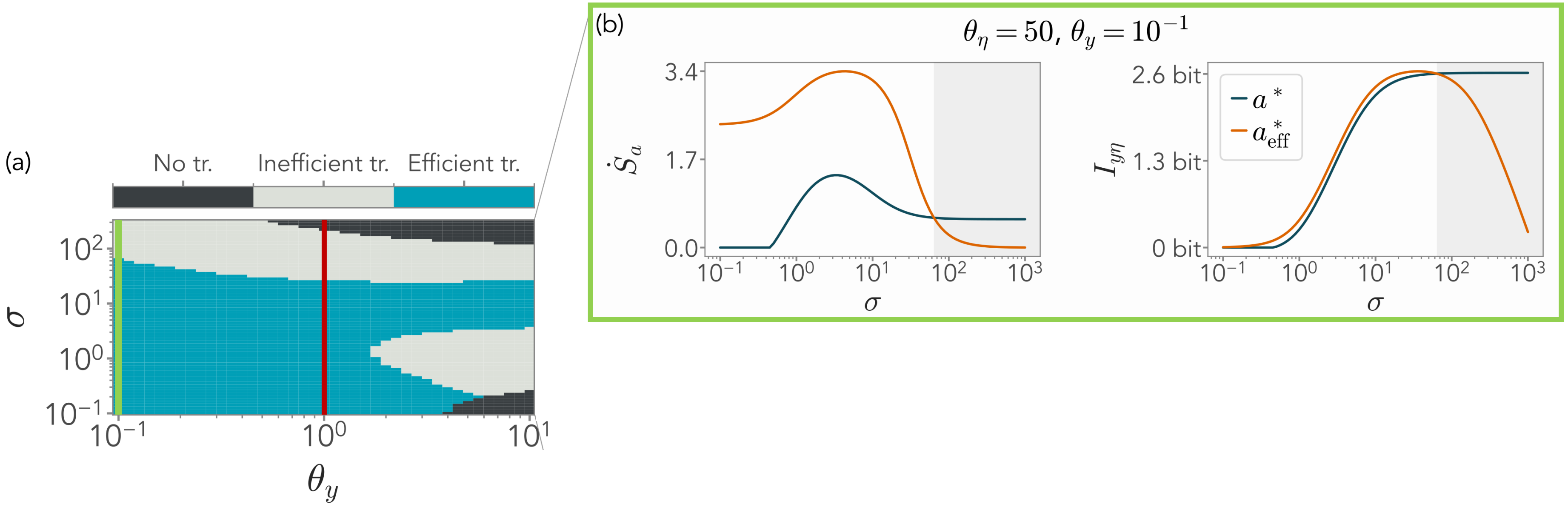


Timescales and *transduction efficiency*

(a)



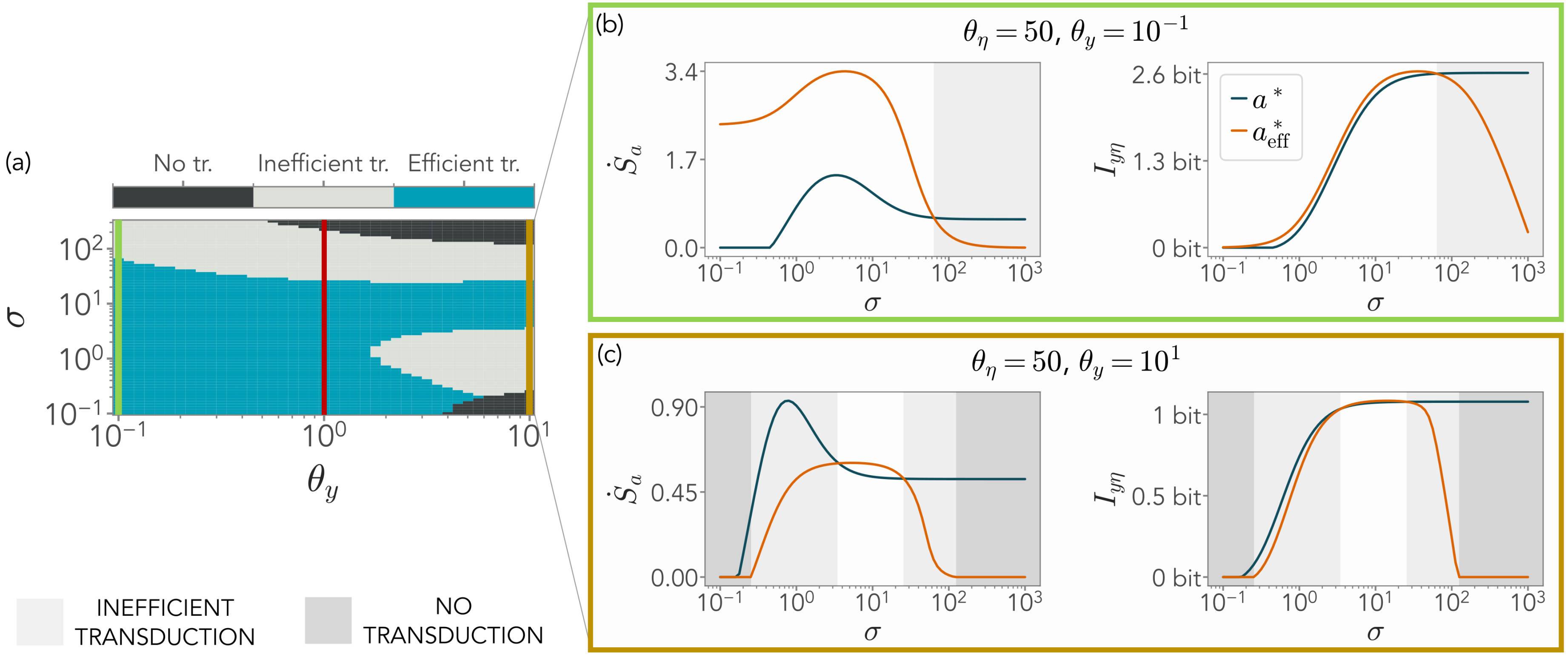
Timescales and *transduction efficiency*



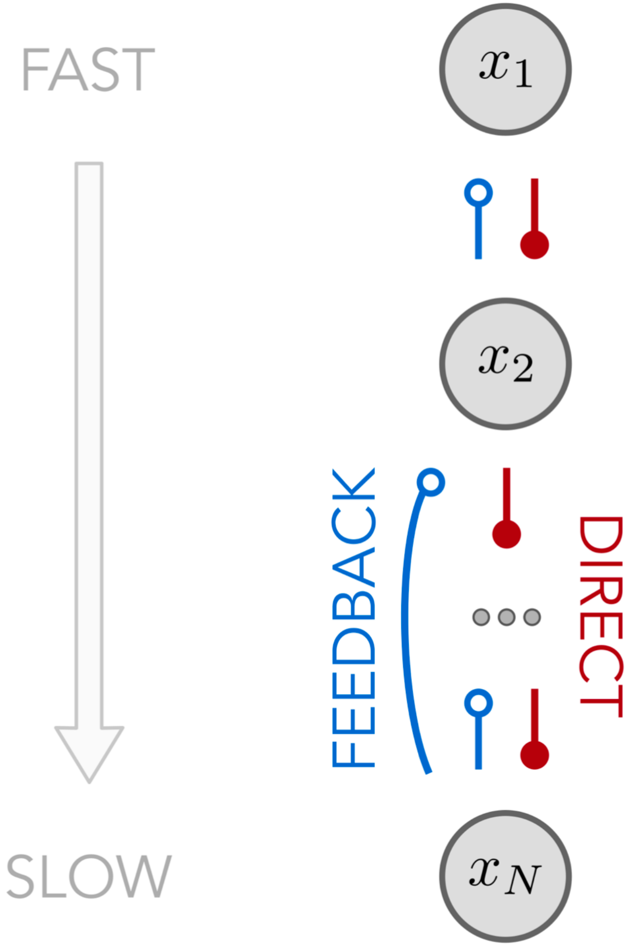
INEFFICIENT TRANSDUCTION

NO TRANSDUCTION

Timescales and *transduction efficiency*

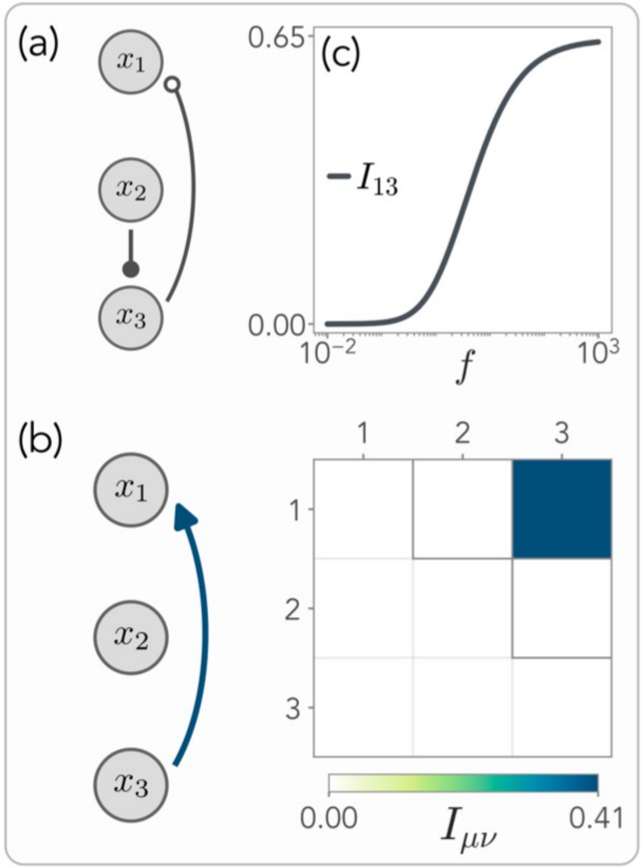


Information propagation across scales



G Nicoletti, DMB, Physical Review X 14, 021007 (2024)

Information propagation across scales



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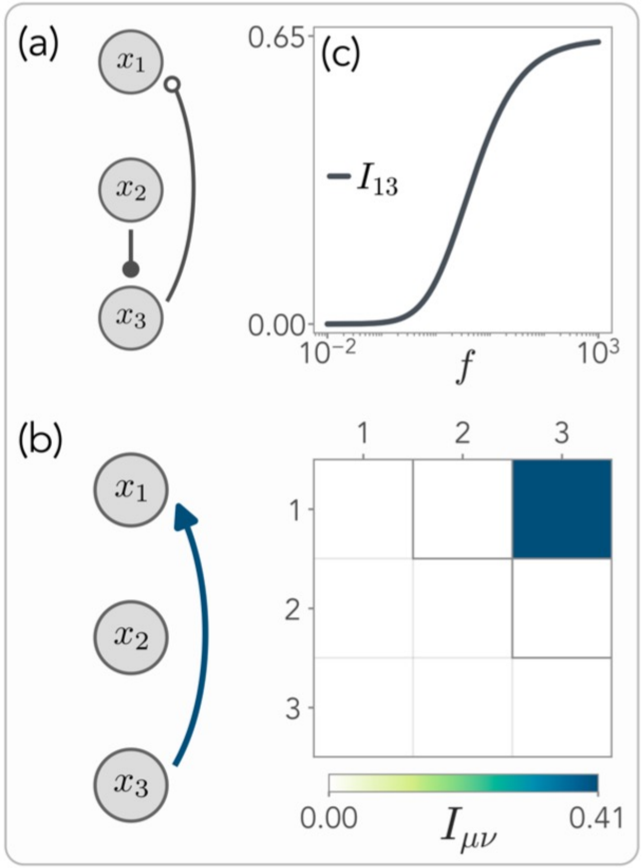

The only contribution to the Mutual Information Matrix for Multiscale Observables (MIMMO) is from 3 to 1

(i) Feedback interactions generate information from slow to fast layers

(ii) Direct interaction alone do not generate information

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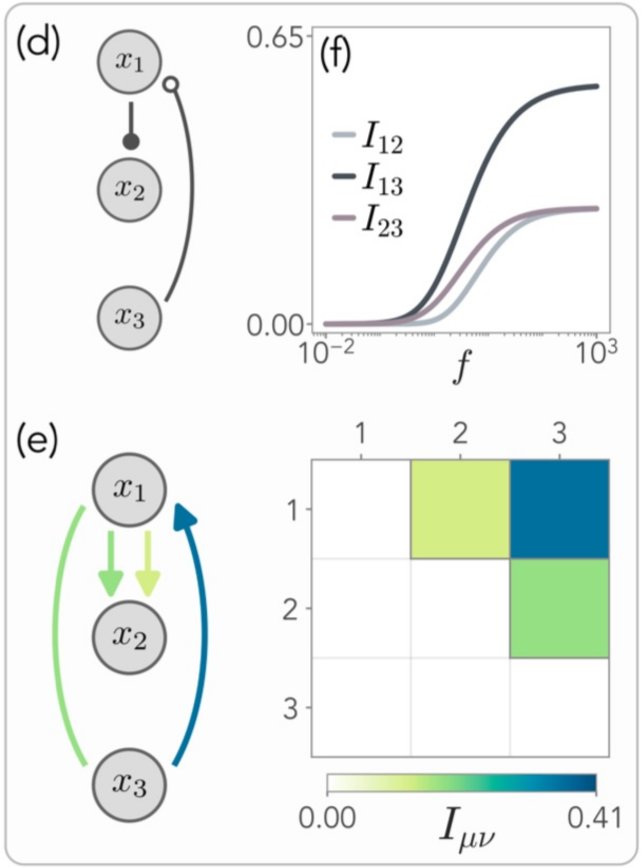
Information propagation across scales



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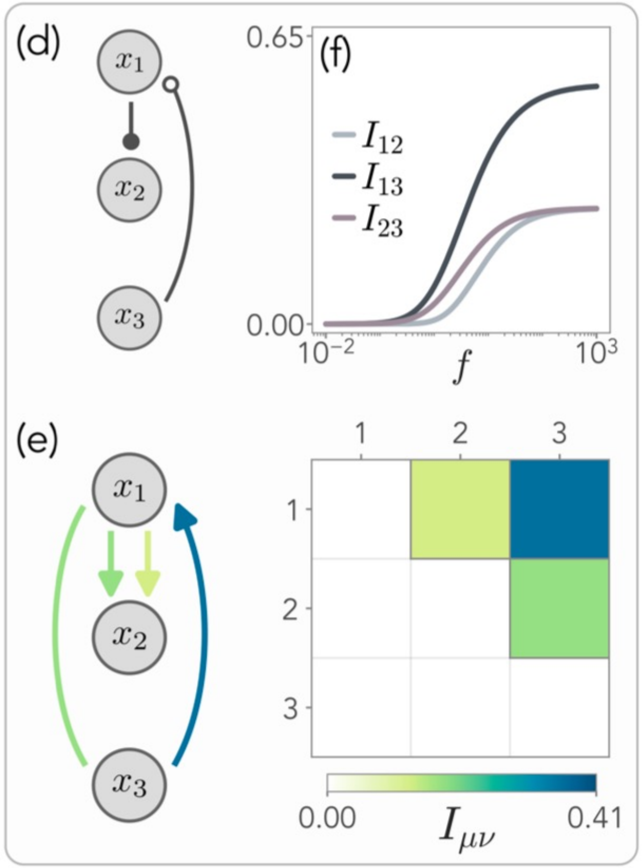
Information propagation across scales

Direct contribution to the MIMMO from 3 to 1

Indirect contribution due to propagation through 1



(iii) Information created by a slow layer can be propagated to faster layers via direct links

G Nicoletti, DMB, Physical Review X 14, 021007 (2024)

Information propagation across scales

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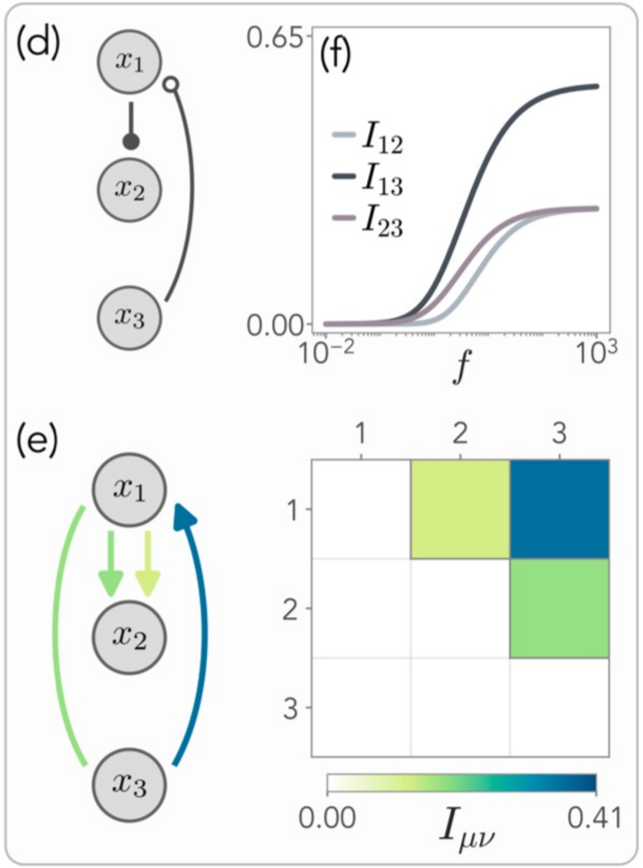
Indirect contribution due to propagation through 1

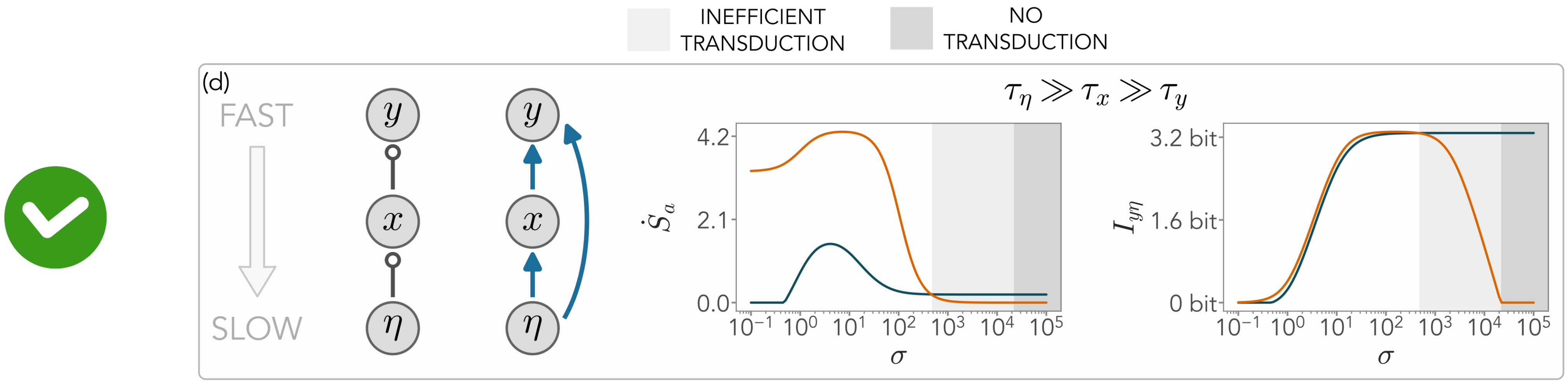


(iii) Information created by a slow layer can be propagated to faster layers via direct links

To propagate information to the other variables, the active bath has to be the slowest DOF

G Nicoletti, DMB, Physical Review X 14, 021007 (2024)

Information propagation across scales



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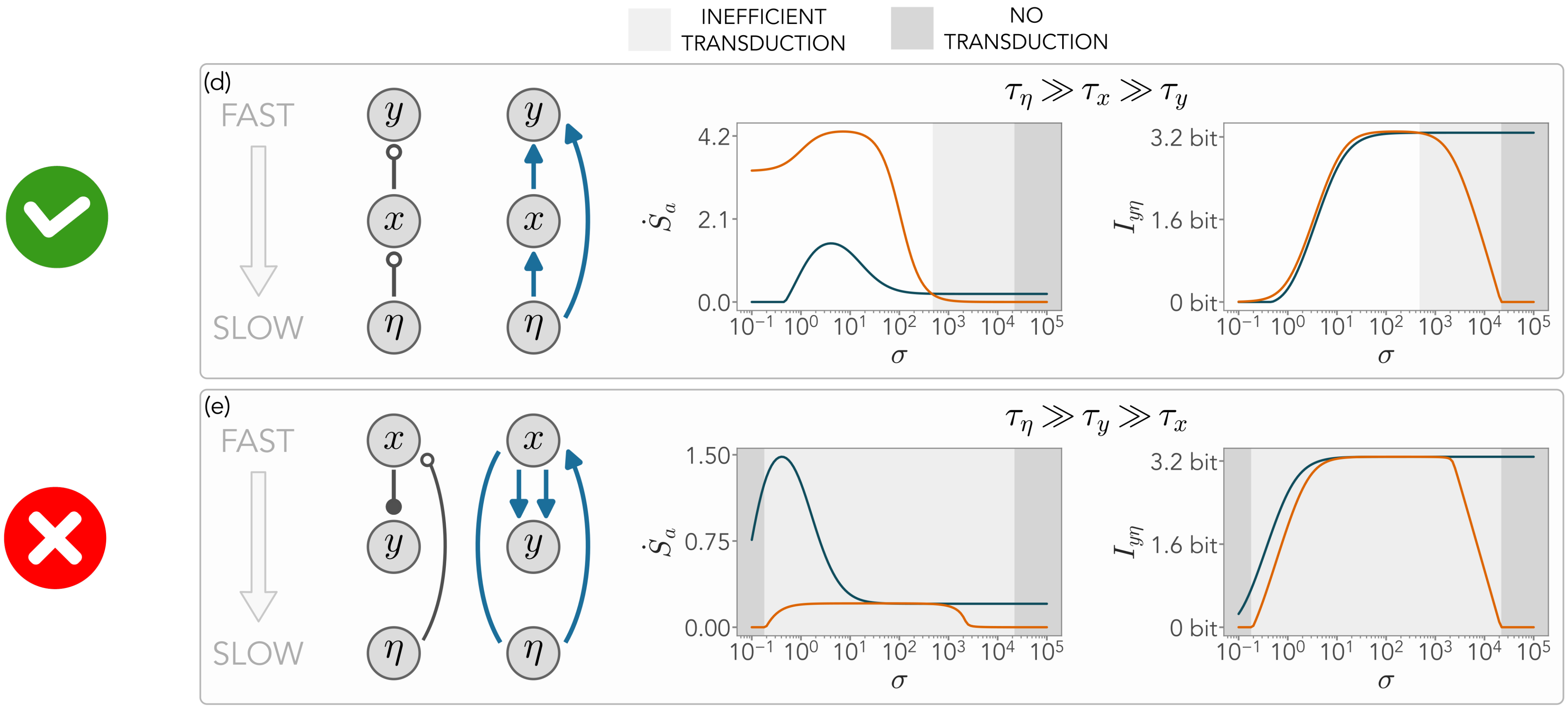
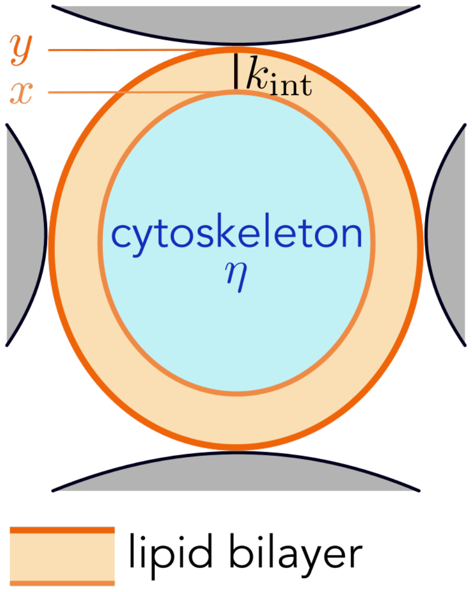


G Nicoletti, DMB, Physical Review X 14, 021007 (2024)

Information transduction in *red blood cells*



Membrane flickering is measured and it contains information on hidden dissipative degree of freedom

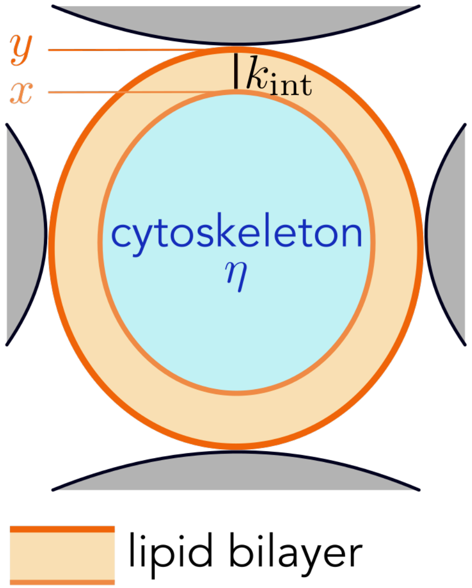
$$\dot{y} = -\mu_y \left(k_y y - k_{\mathrm{int}} x\right) + \sqrt{2 k_B T \mu_y} \xi_y(t)$$

$$\dot{x} = -\mu_x \left(k_x x - k_{\mathrm{int}} y + \eta\right) + \sqrt{2 k_B T \mu_x} \xi_x(t)$$

$$\dot{\eta} = -1/\tau_\eta \eta + \sqrt{2\epsilon^2/\tau_\eta} \xi_\eta(t)$$

[1] I Di Terlizzi et al, Science 383, 971-976 (2024)

Information transduction in *red blood cells*



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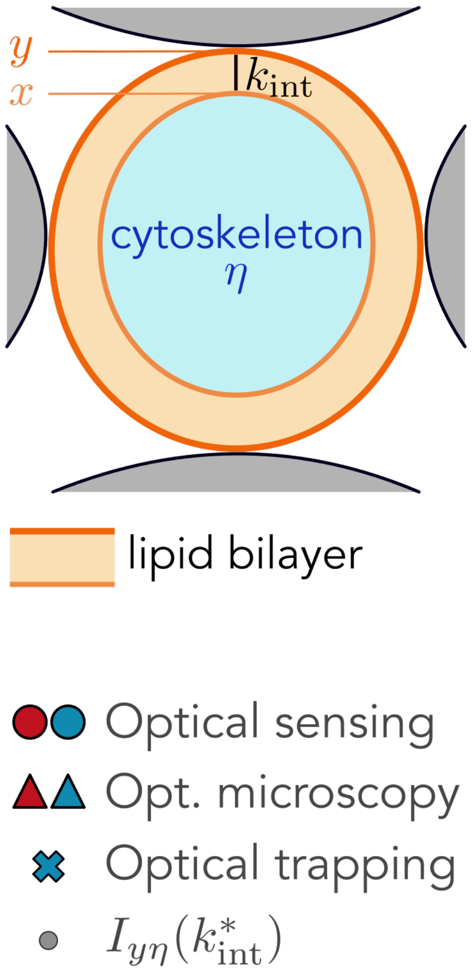
INVERSE PROBLEM:

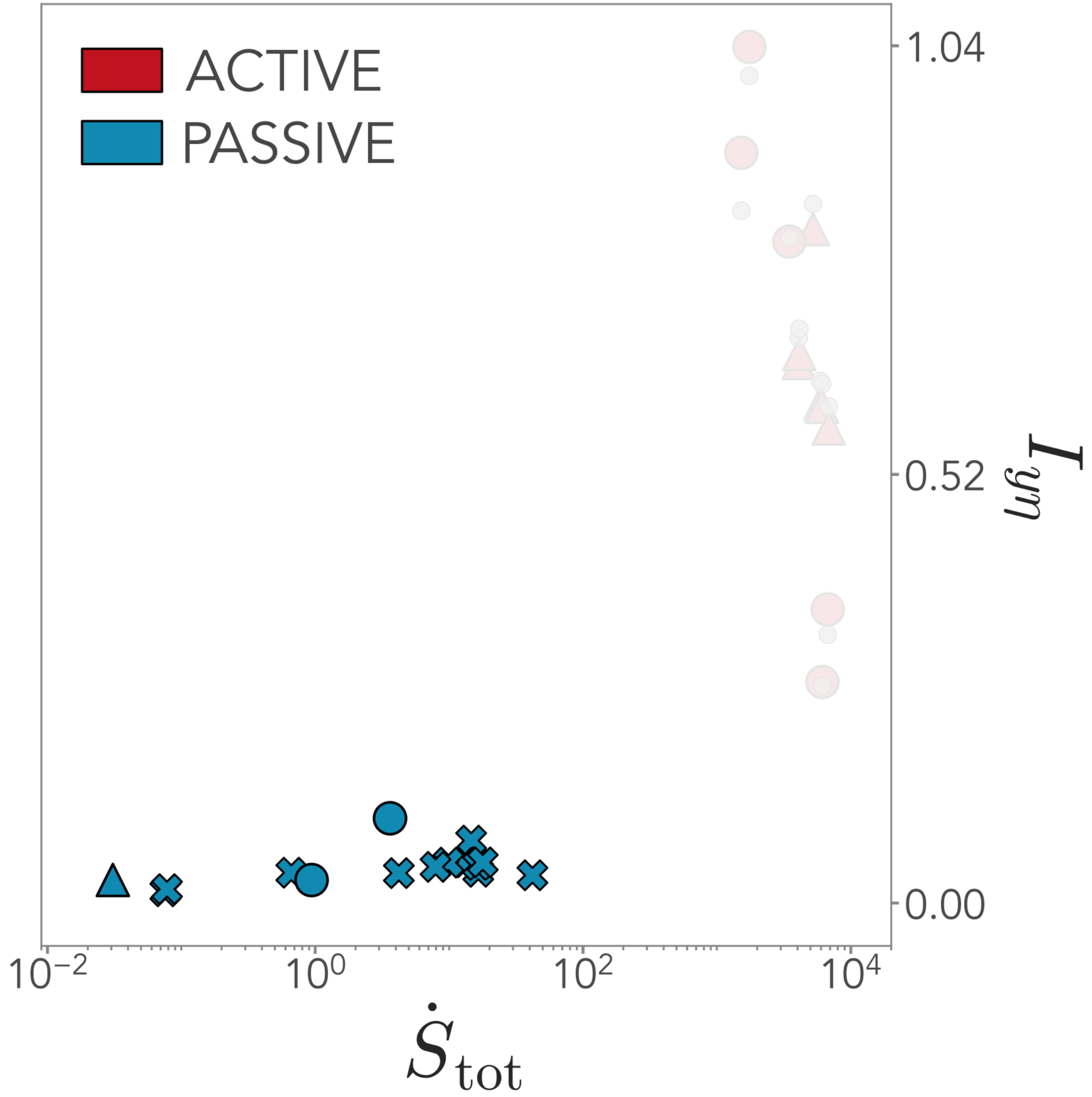
$$k_{\text{int}}^{\text{exp}} = \arg\max_{k_{\text{int}}} \mathcal{L}_{\text{eff}}(\lambda, k_{\text{int}})$$
$$\lambda^{\text{exp}} \;:\; \max_{k_{\text{int}}} \mathcal{L}_{\text{eff}}(\lambda^{\text{exp}}, k_{\text{int}}) = \mathcal{L}_{\text{eff}}(\lambda^{\text{exp}}, k_{\text{int}} = k_{\text{int}}^{\text{exp}})$$

Given the estimated dissipation, how efficiently is the information on cytoskeleton transduced?

[1] I Di Terlizzi et al, Science 383, 971-976 (2024)

Information transduction in *red blood cells*





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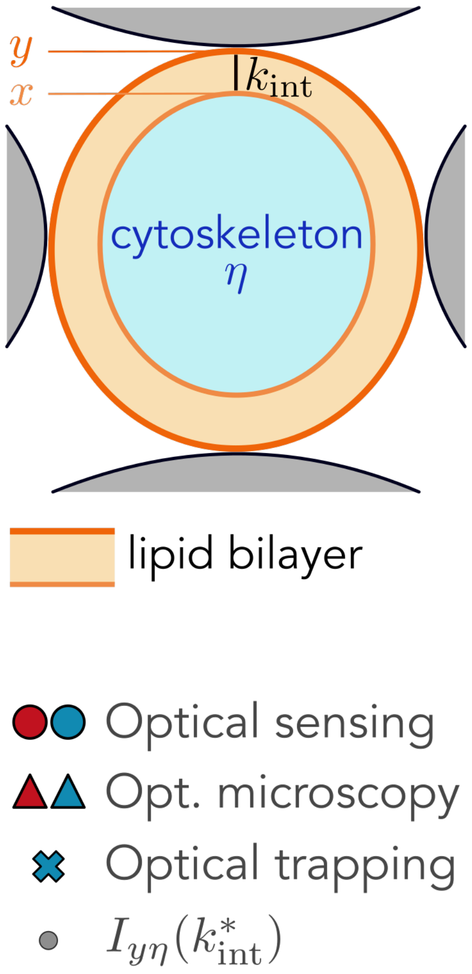
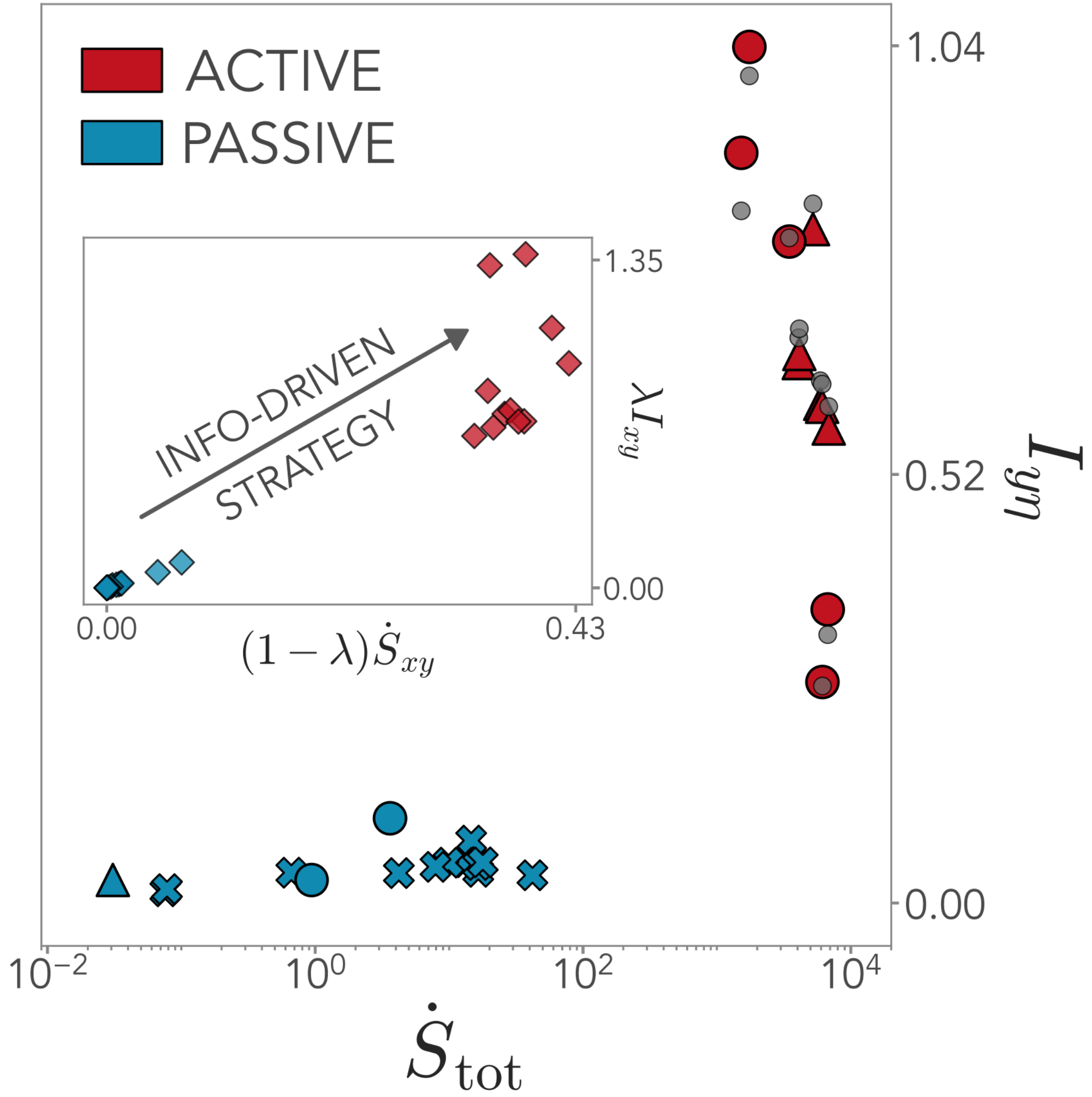




[1] I Di Terlizzi et al, Science 383, 971-976 (2024)

Take-home message

- Information harvesting on hidden (dissipative) degrees of freedom is possible by tuning the coupling to accessible variables
- Transduced information can overcome the ideal case, even starting from finite-time stochastic trajectories (and empirical distributions)
- Interesting connection between transduction efficiency and mechanical conditions can be highlighted from membrane flickering in red blood cells

What's next? Estimation of forces, Multi-variables data, Pareto fronts in parameter space

[1] G Nicoletti, DMB, Physical Review Letters 127, 228301 (2021)
[2] G Nicoletti, DMB, Physical Review X 14, 021007 (2024)
[3] G Nicoletti, DMB, arXiv:2403.04709 (2024)

ARXIV:2403.04709
(2024)

Thanks for your attention!

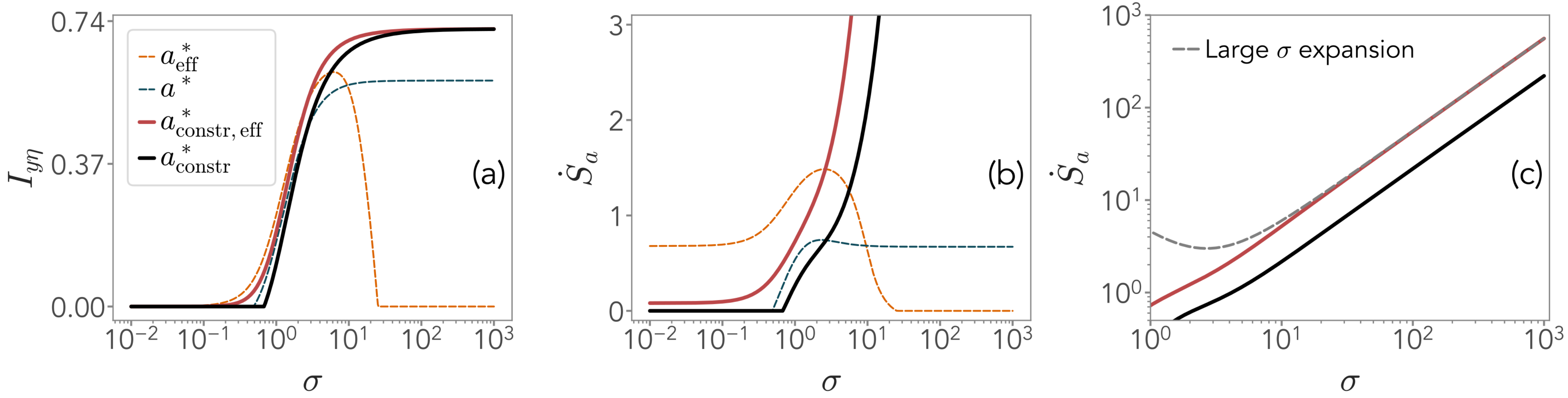
BONUS SLIDES

An *energetic constraint* is necessary

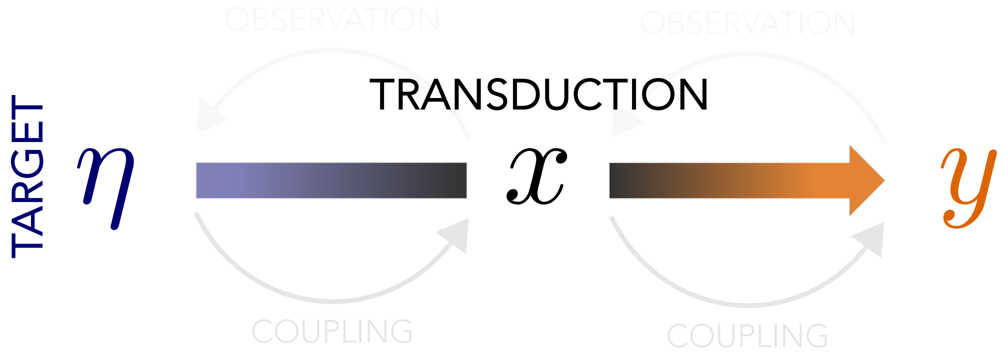
$$\max_a \left(\lambda I_{y\eta} - (1-\lambda)a^2\right)$$

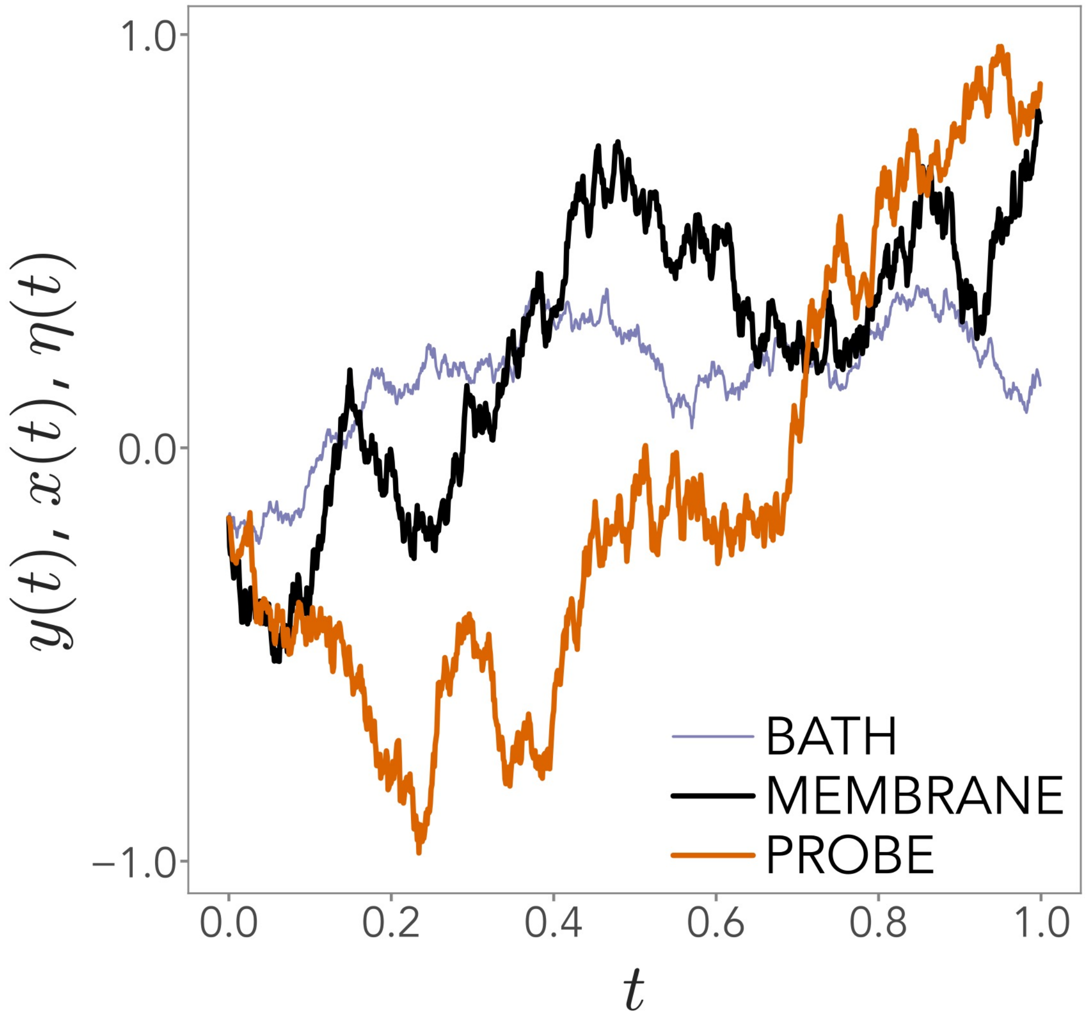
The cost of maintaining an efficient transduction at increasing σ diverges without an energetic constraint

$$\dot{S}_a \approx \frac{\lambda\left(2+\theta_\eta+\sigma^2\right)}{2\sqrt{\theta_\eta(1-\lambda)}\,\sigma}$$



A more realistic scenario: *finite-time trajectories*





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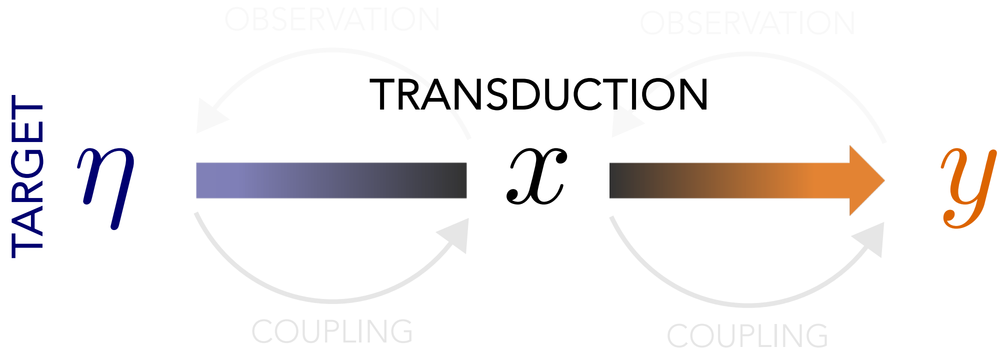
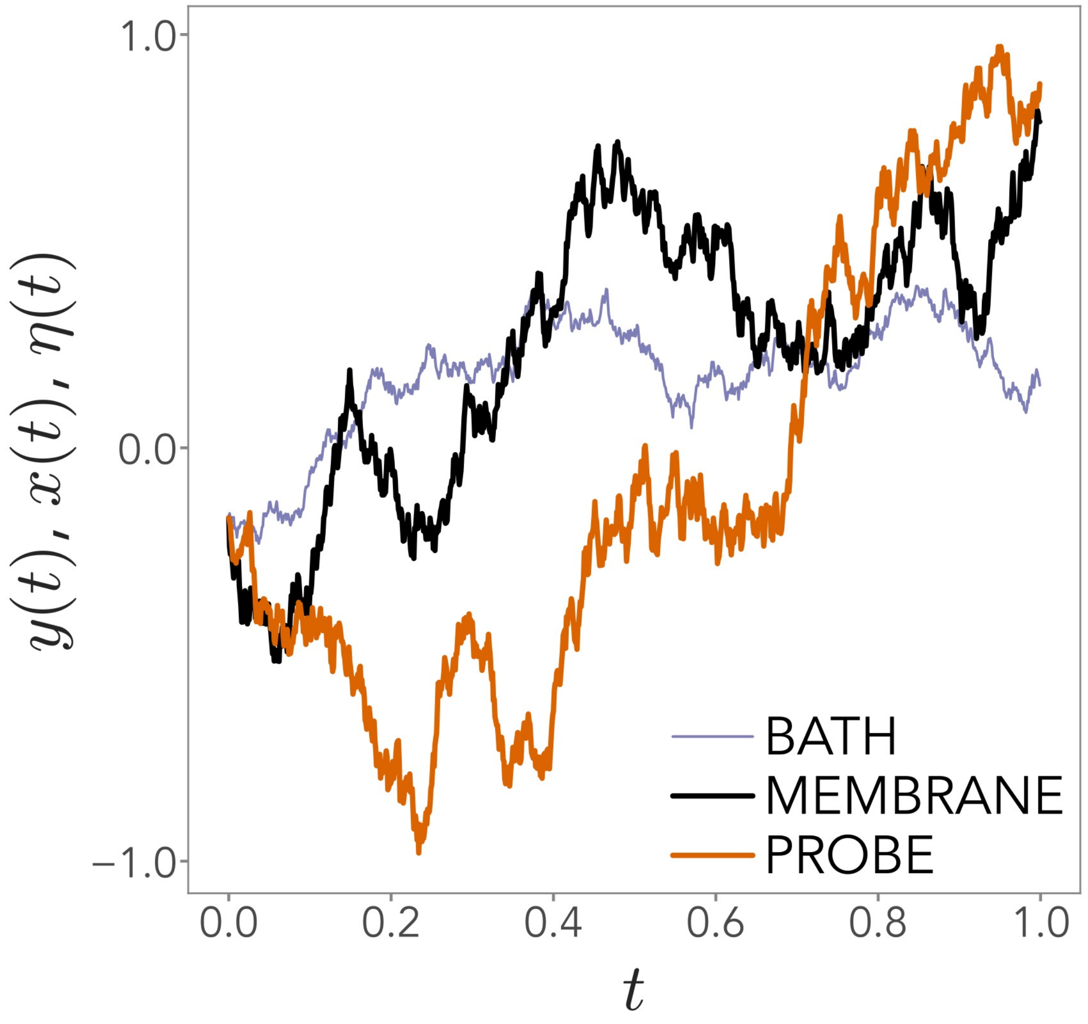


$$I_{xy} \quad \to \quad i_{xy}(t) = \log \frac{p(x(t), y(t))}{p(x(t))p(y(t))}$$

$$\dot{S}_{xy} \quad \to \quad \dot{s}_{xy}(t) = \mathbf{A}_{xy}^{\text{red}} \circ (\dot{x}(t), \dot{y}(t))^T$$



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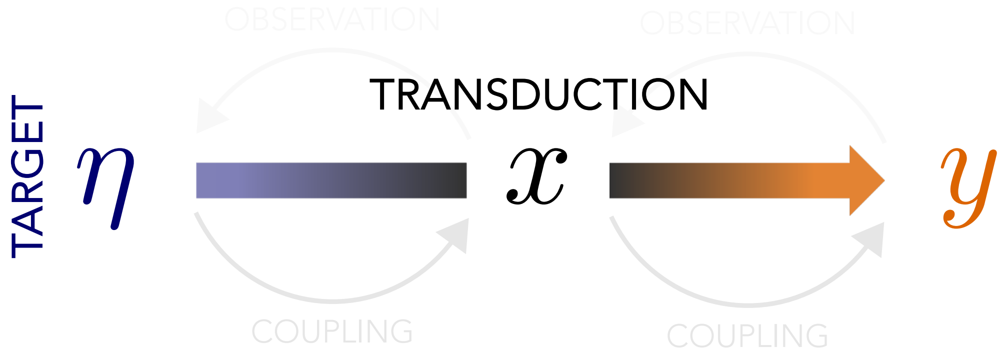


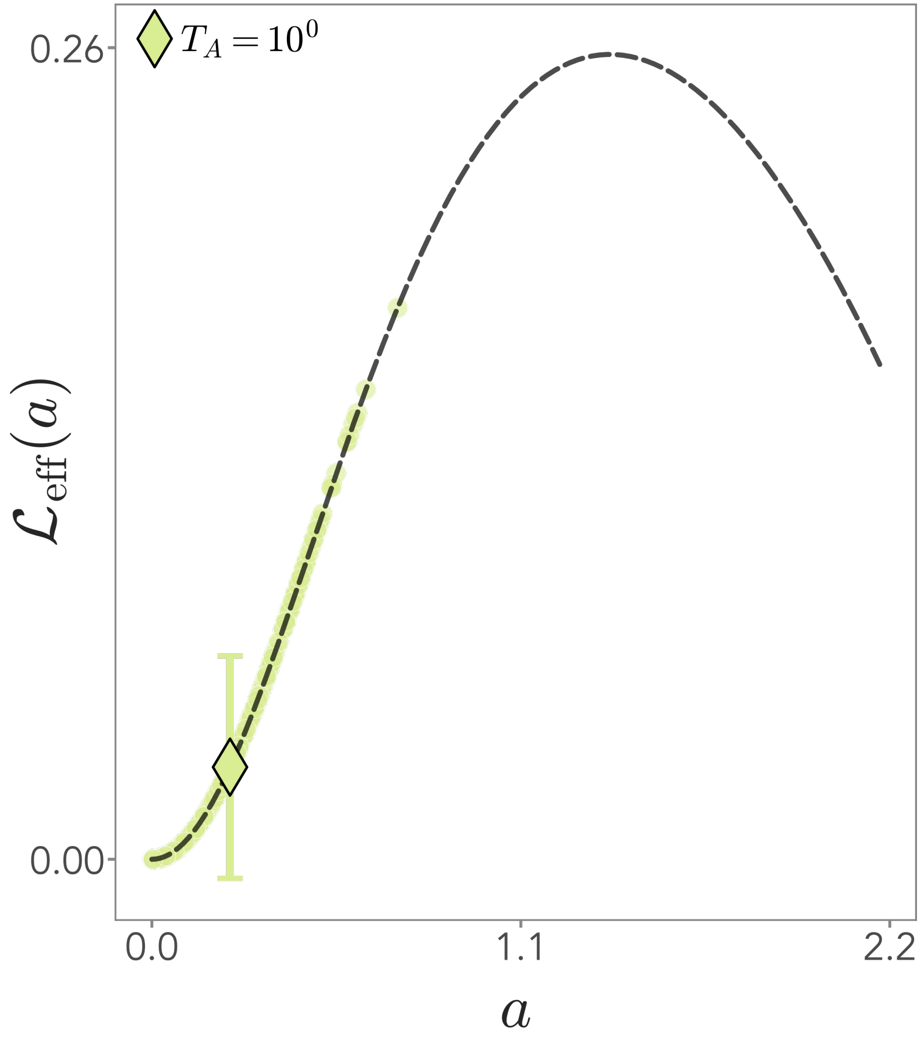
PROPOSE A NEW COUPLING

⇓

MEASURE

⇓

ACCEPT/REJECT THE NEW COUPLING



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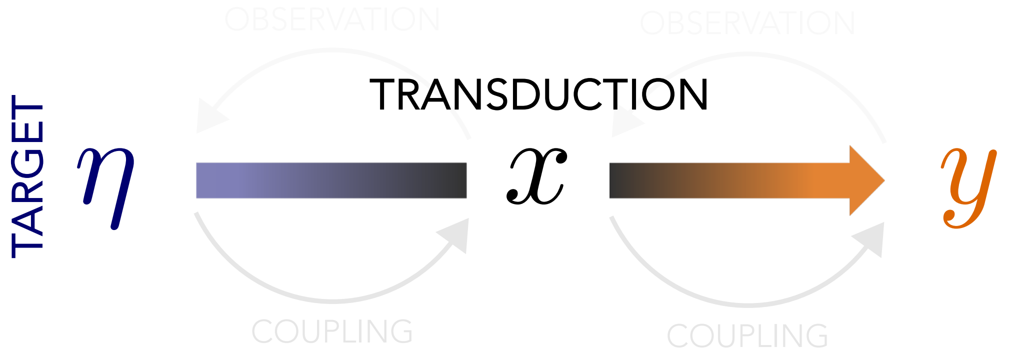


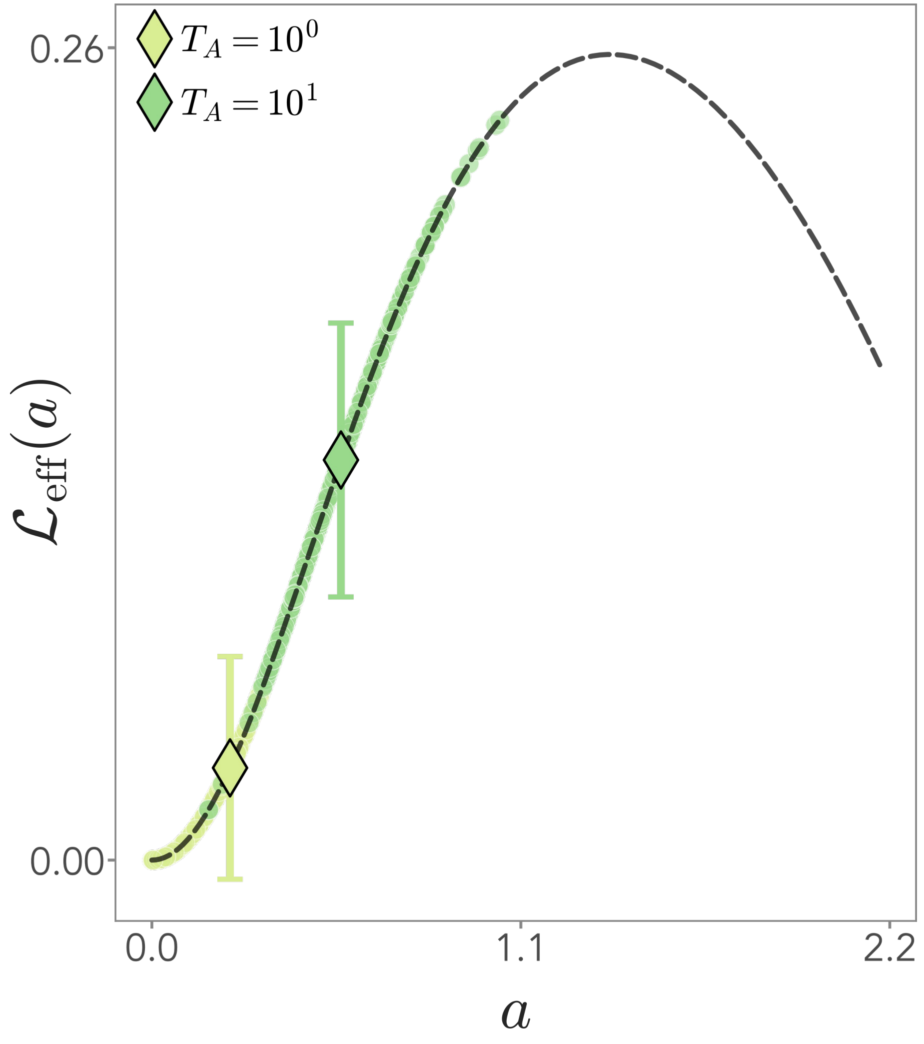
PROPOSE A NEW COUPLING

⇓

MEASURE

⇓

ACCEPT/REJECT THE NEW COUPLING



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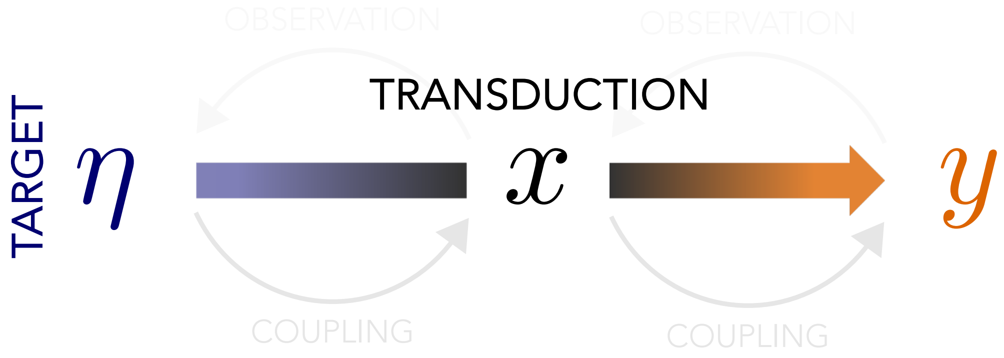

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