Reparametrization invariance:

from glasses to toy black holes

1. GLASSES

The simplest glass model

$$E_p = -\frac{1}{p!} \sum_{i_1,\dots,i_p} J_{i_1,\dots,i_p} q_{i_1} \dots q_{i_p}$$

Langevin dynamics (also the simplest)

$$\dot{q}_i(t) = -\frac{\partial V}{\partial q_i} + \eta_i(t) \ ,$$

$$\lambda(t)[\textstyle\sum_i q_i^2 - N]$$

constraint

Many other situations, e.g.

Graph colouring:

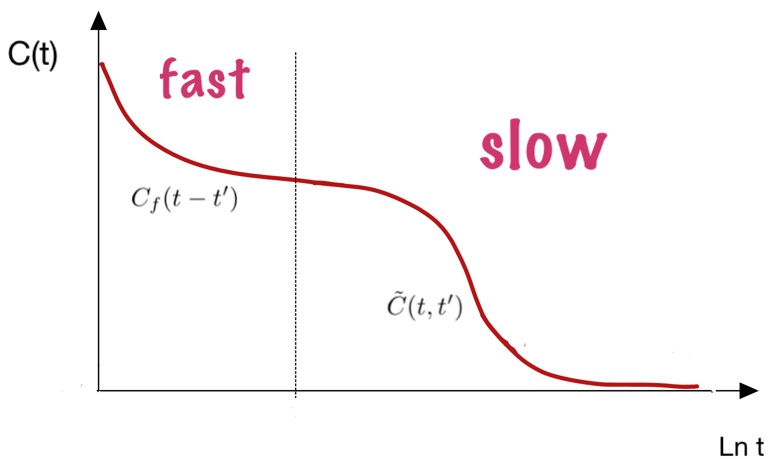
V= { number of badly coloured links}

ORDER PARAMETERS

$$C(t,t') = \frac{1}{N}\sum_i \langle q_i(t) q_i(t') \rangle \ ,$$

$$R(t,t') = \frac{1}{N}\sum_i \langle q_i(t) \eta_i(t') \rangle \quad , \quad \chi(t,t') = \int_{t'}^{t} ds \; R(t,s)$$

$$D(t,t') = \frac{1}{N}\sum_i \langle \eta_i(t) \eta_i(t') \rangle - 2T_s \delta(t-t') \ ,$$



- **Aging:**

$$C(t,t') = C_f(t-t') + \tilde{C}(t,t') \quad \text{for example} = C_f(t-t') + \mathcal{C}\left(\frac{t-t'}{t'}\right)$$

- **Slowly evolving disorder:**

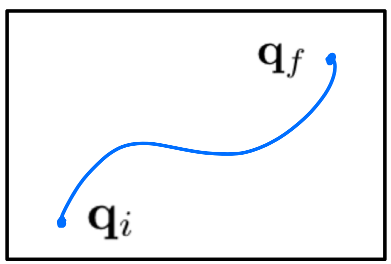
$$\langle J_{i_1,\dots,i_p}(t) J_{i_1,\dots,i_p}(t') \rangle = p!/2N^{p-1} e^{-\frac{t-t'}{\tau}}$$

- **Weakly driven system:**

$$\dot{q}_i(t) = \text{ usual terms } + \frac{1}{\tau} \sum_j A_{ij} q_j$$

$$C(t,t') = C_f(t-t') + \tilde{C}(t,t') \quad \text{for example} = C_f(t-t') + \mathcal{C}\left(\frac{t-t'}{\tau}\right)$$

- **Breaking causality:**



in particular

$$\mathbf{q}_i = \mathbf{q}_f$$

τ

EXACT EQUATIONS

$$\mathbf{Q} = \begin{bmatrix} R & C \\ D & R^t \end{bmatrix}(t,t') \quad , \quad \boldsymbol{\delta} = \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix}(t-t') \quad , \quad \boldsymbol{\lambda} = \begin{bmatrix} \lambda & 0 \\ -\hat{\lambda} & \lambda \end{bmatrix}(t) \quad , \quad \mathbf{U} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\Sigma} = p \begin{bmatrix} (p-1)C^{p-1}R & C^{p-1} \\ (p-1)C^{p-2}D + (p-1)(p-2)C^{p-3}R & (p-1)C^{p-1}R^t \end{bmatrix}(t,t')$$

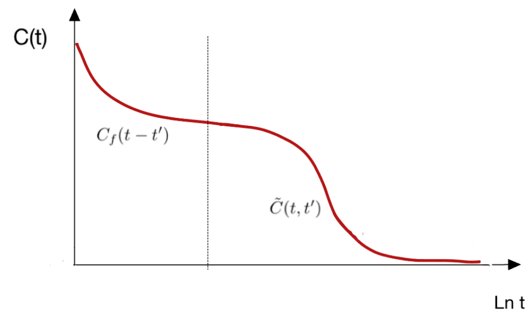
$$C(t,t') = \frac{1}{N}\sum_i \langle q_i(t) q_i(t') \rangle \ ,$$

$$R(t,t') = \frac{1}{N}\sum_i \langle q_i(t) \eta_i(t') \rangle \quad , \quad \chi(t,t') = \int_{t'}^{t} ds \; R(t,s)$$

$$D(t,t') = \frac{1}{N}\sum_i \langle \eta_i(t) \eta_i(t') \rangle - 2T_s \delta(t-t') \ ,$$

$$\left(\frac{d}{dt} - 2T\mathbf{U}\right)\mathbf{Q}(t,t') = \int ds \; \boldsymbol{\Sigma}(t,s)\mathbf{Q}(s,t') - \boldsymbol{\lambda}(t)\mathbf{Q}(t,t') + \boldsymbol{\delta}(t-t')$$

fast slow



fast slow

$$C(t,t') = C_f(t-t') + \tilde{C}\,(t,t')$$
$$R(t,t') = R'_f(t-t') + \tilde{R}\,(t,t')$$
$$D(t,t') = D_f(t-t') + \tilde{D}\,(t,t')$$

slow :

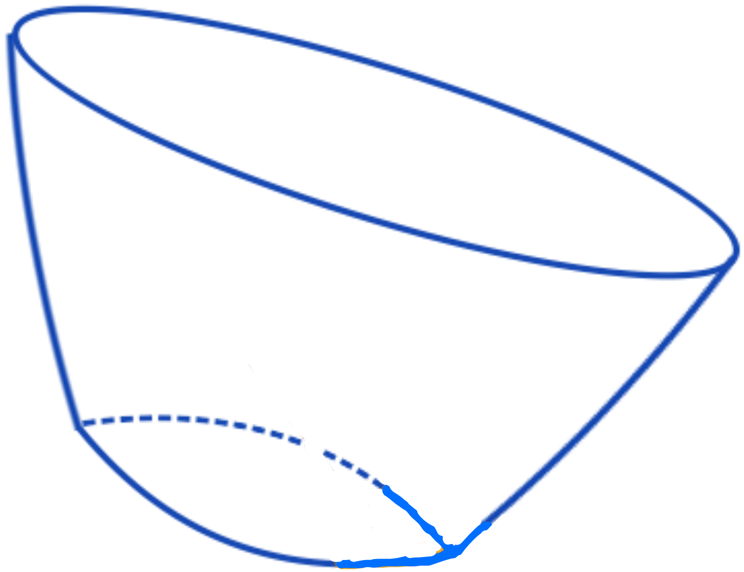
$$\left(\frac{d}{dt} - 2T\mathbf{U}\right)\mathbf{Q}(t,t') = \int ds\;\boldsymbol{\Sigma}(t,s)\mathbf{Q}(s,t') - \boldsymbol{\lambda}(t)\mathbf{Q}(t,t') + \boldsymbol{\delta}(t-t')$$

$$\tilde{C}(t,t') \to \tilde{C}(h(t),h(t'))\ ,$$
$$\tilde{R}(t,t') \to \dot{h}(t')\ \tilde{R}(h(t),h(t'))\ ,$$
$$\tilde{D}(t,t') \to \dot{h}(t)\dot{h}(t')\ \tilde{D}(h(t),h(t'))$$

and

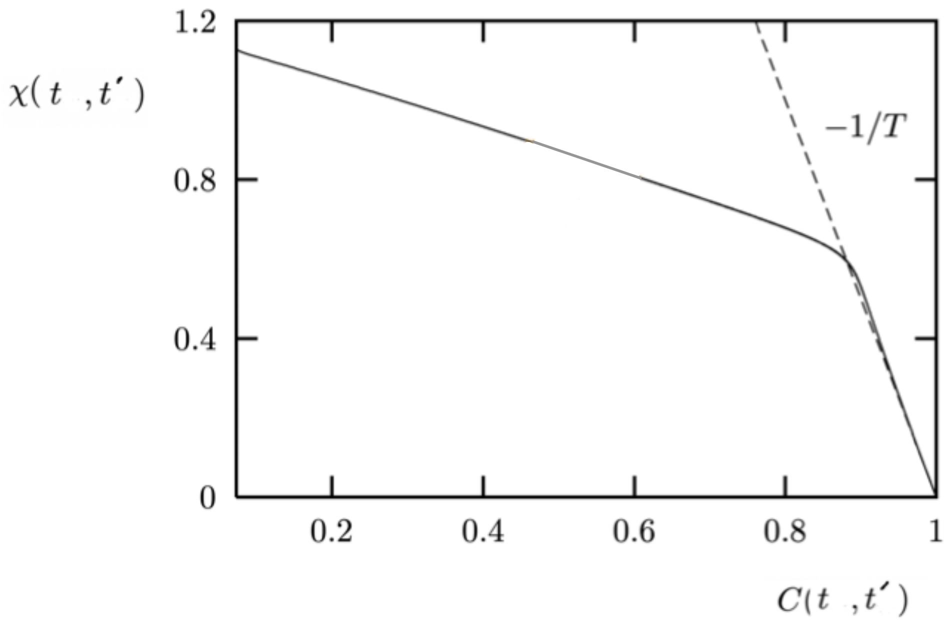
$$\lambda(t) \to \lambda(h(t))\ ,$$
$$\hat{\lambda}(t) \to \dot{h}(t)\ \hat{\lambda}(h(t))$$

REPARAMETRIZATION INVARIANCE!



a quasi-symmetry

response



correlation

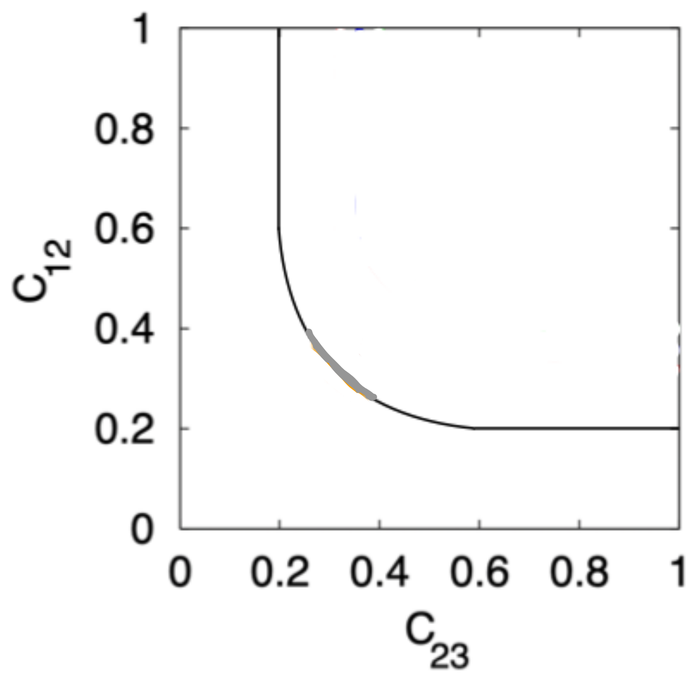
TTI but out of equilibrium?

The same!

Very drastic changes in time-dependence leave reparametrization-invariant plots unchanged

A REPARAMETRIZATION-INVARIANT PLOT

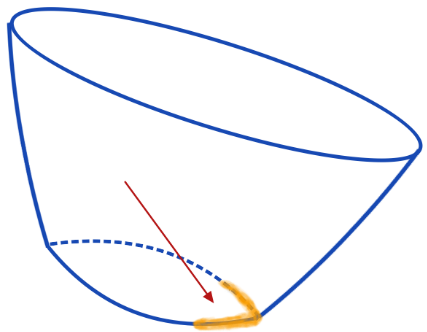
Another one:

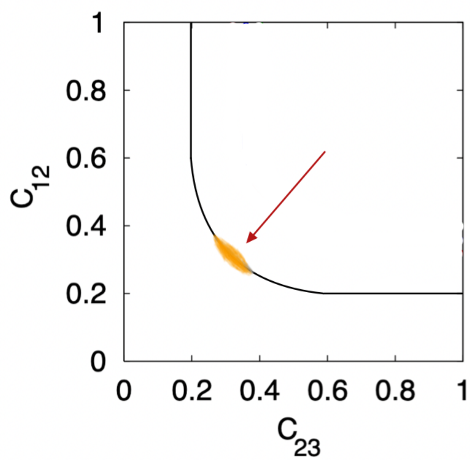


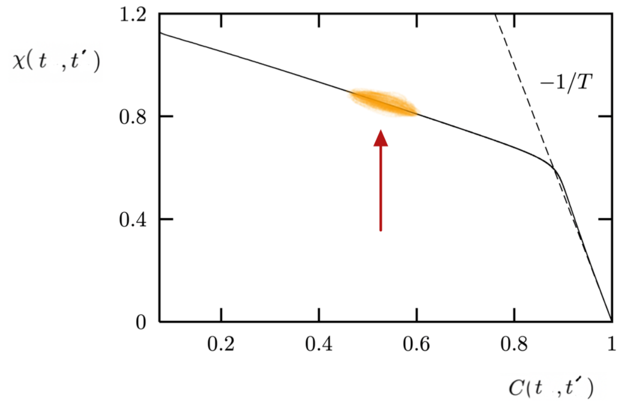
Very drastic changes in time-dependence leave reparametrization-invariant plots unchanged

Castillo, Chamon,Cugliandolo, Kennet:

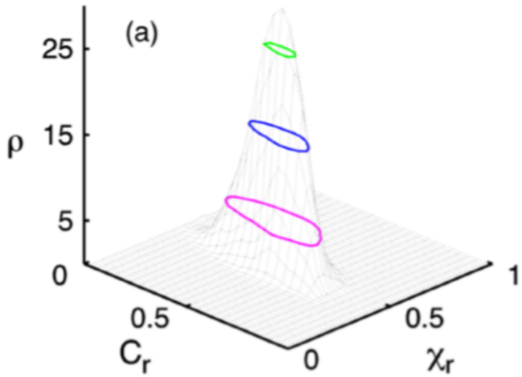
FLUCTUATIONS

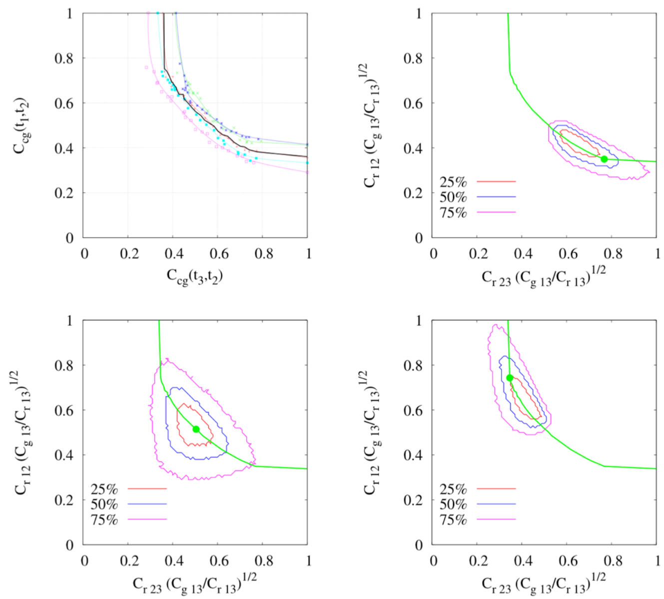






FLUCTUATIONS





Castillo, Chamon,Cugliandolo, Kennett

Parisi ansatz in finite dimension needs some form of reparametrization invariance

Otherwise the coupling of two ultrametric systems is not ultrametric

Easy to argue going through dynamics, I would like a direct proof!

Ghimenti, Van Wijland and JK

Are different algorithms (event driven molecular dynamics, parallel tempering, swap) time-reparemetrizations of one another?

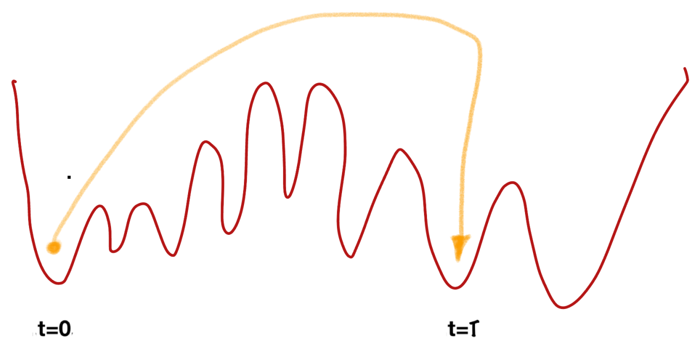
e.g. C(q,t) vs C(q’,t) universal?

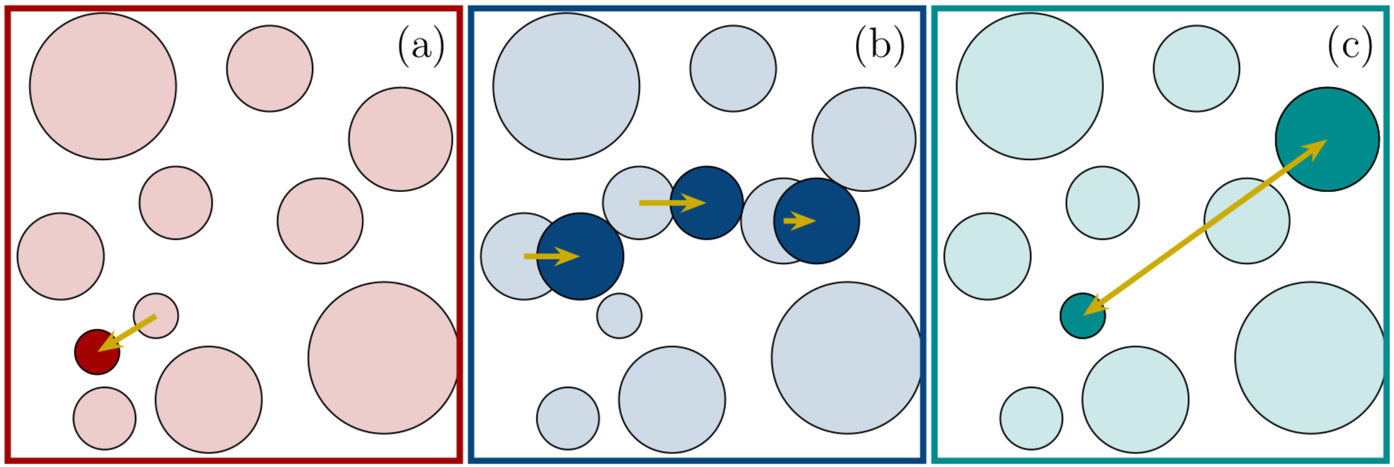
See also the papers from Till Böhmer et al, Jeppe Dyre et al

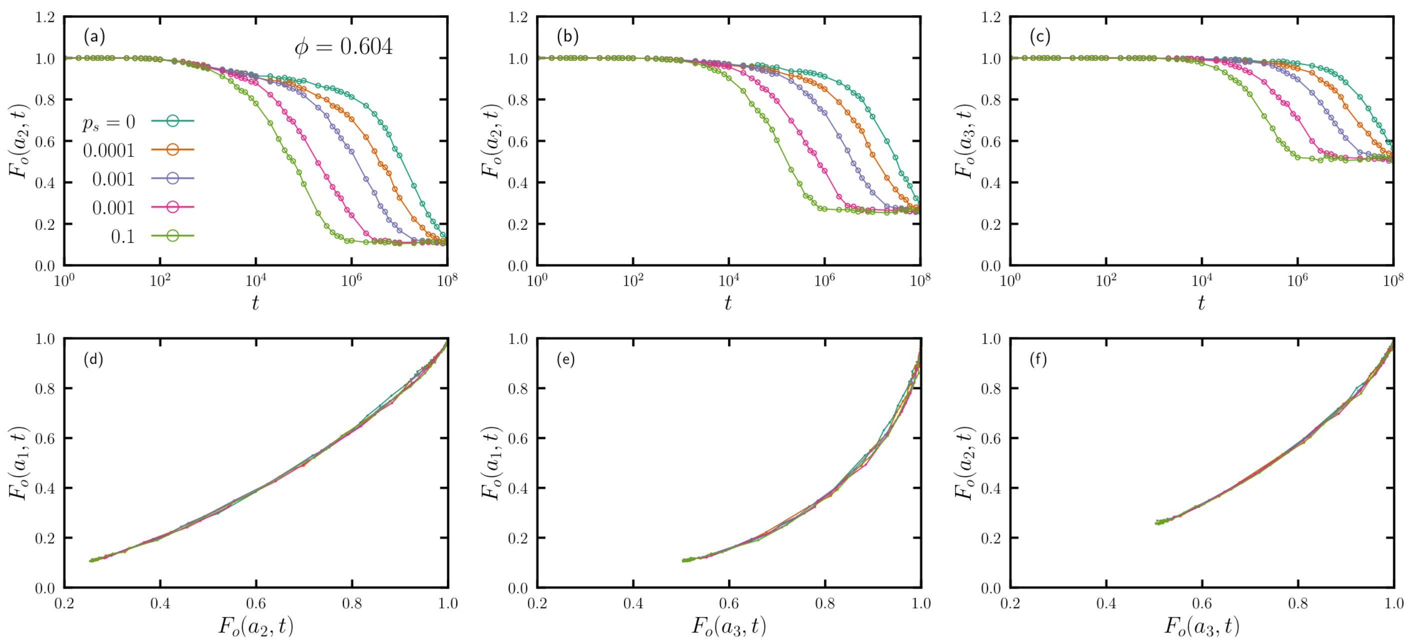
« material time »

Tommaso Rizzo

Barrier-crossings at different total times are reparametrizations of one another!




(a)
(b)
(c)


(a)
(b)
(c)
(d)
(e)
(f)
$\phi = 0.604$
$p_s = 0$
0.0001
0.001
0.001
0.1
t
$F_o(a_2, t)$
$F_o(a_3, t)$
$F_o(a_1, t)$

2. SYK

90's

→ Sachdev, Georges, Parcollet, … , Kitaev - Maldacena, Shenker and Stanford

$$H_{syk} = \sum_{1\leq i<j<k<l\leq N} J_{ijkl}\ \chi_i\chi_j\chi_k\chi_l\ ,$$

Majorana (K) or ordinary (SY) fermions

Using exactly the same methods as before

$$G(t) = \sum_i \langle T\chi_i(t)\chi_i(\hat{0})\rangle$$

$$\frac{\partial G(t_1 - t_2)}{\partial t_1} + J\int_0^\beta dt\ G(t_1, t)\ G(t, t_2)^3 = -\delta(t_1 - t_2)$$

And again, we may neglect the time derivative

$G(t) = \sum_i \langle T \chi_i(t) \chi_i(0) \rangle$

$$\frac{\partial G(t_1 - t_2)}{\partial t_1} + J \int_0^\beta dt\ G(t_1, t)\ G(t, t_2)^3 = -\delta(t_1 - t_2)\ .$$

$$G(t_1, t_2) \to |\dot{h}(t_1)\dot{h}(t_2)|^{1/4} G(h(t_1), h(t_2))$$

$$S[h] \propto \int D[h] \left\{ \frac{h'''}{h'} - \frac{3}{2}\left(\frac{h''}{h'}\right)^2 \right\}$$

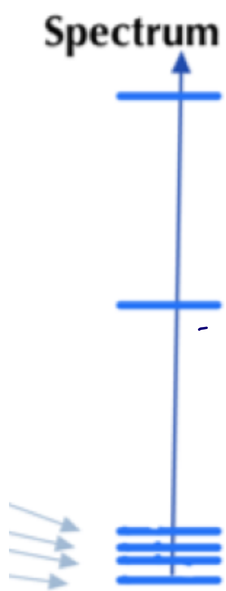
Maldacena, Shenker and Stanford

$$\boxed{\sum J_{ijkl}\chi_i\chi_j\chi_k\chi_l} \quad \rightarrow \quad \boxed{\int dt dt' \left\{ \frac{\partial G(t,t')}{\partial t} - \frac{J^2}{4} G(t,t')^4 \right\} + \mathrm{tr} \ln G} \quad \rightarrow \quad \boxed{\int D[h] \left\{ \frac{h'''}{h'} - \frac{3}{2} \left(\frac{h''}{h'} \right)^2 \right\}}$$

'QFT'   with ultraviolet   infrared='gravity'

Gravity is to QFT what dynamic heterogeneities are for (quantum or classical) glasses

The `black hole'



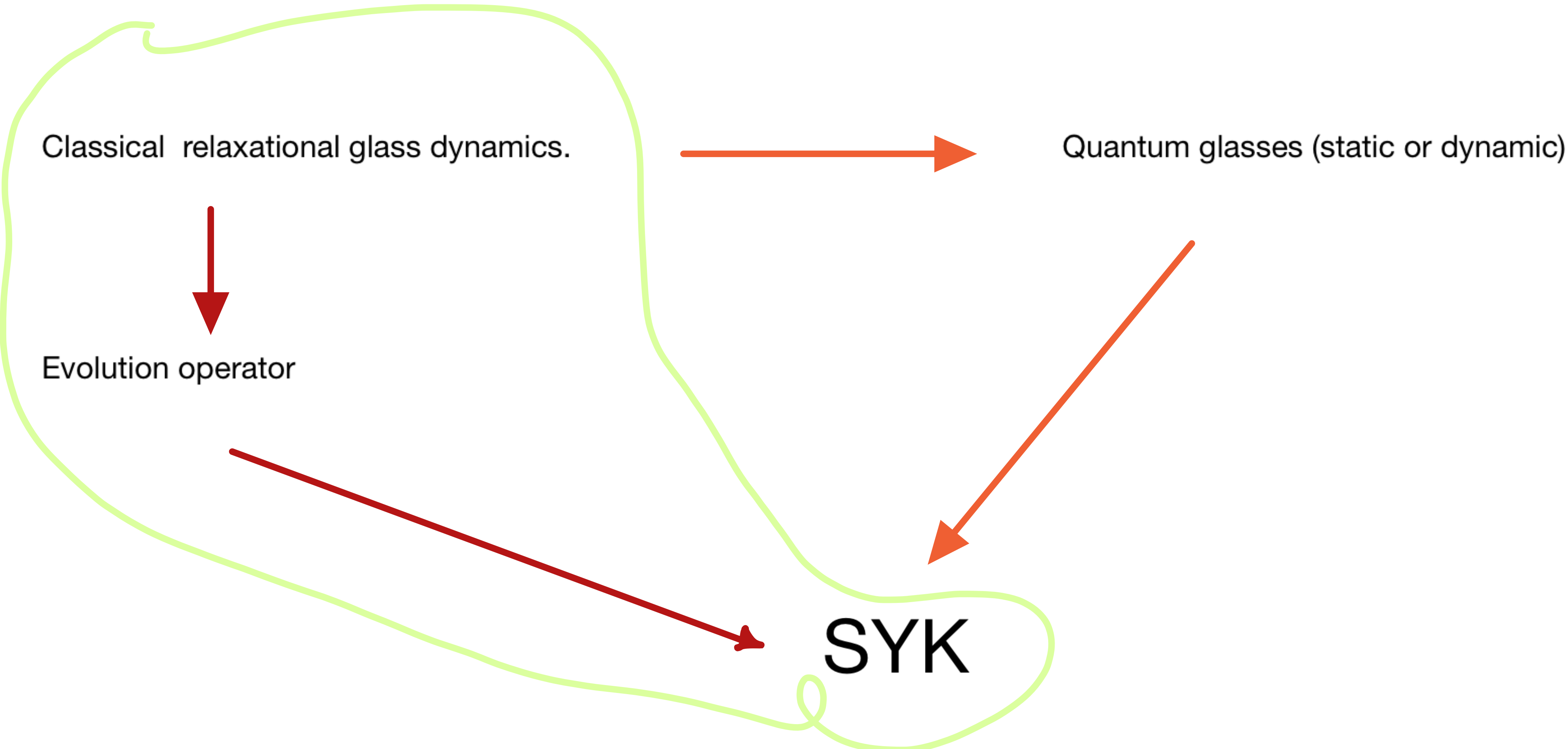
we have a model that is a metaphor of the

real situation, but to which we may

`ask questions' and obtain definite answers

3. THE BRIDGE

NOTA BENE



Not the only possibility!

Facoetti, Biroli, Kurchan, Reichman

EVOLUTION OPERATOR

$$\partial_t P_t(\mathbf{q}) = \sum_i \frac{\partial}{\partial q_i}\left[T_s \frac{\partial}{\partial q_i} + \frac{\partial V}{\partial q_i}\right] P_t(\mathbf{q}) \equiv -H_{\mathrm{FP}} P_t(\mathbf{q}).$$

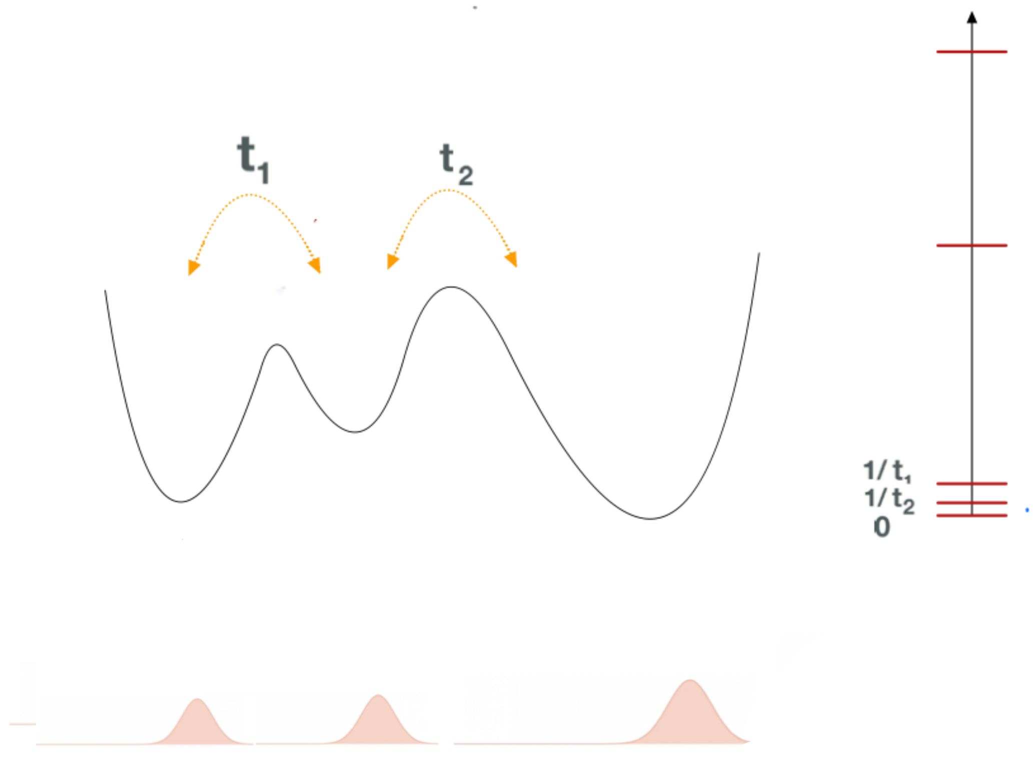
Hermitian

$$e^{V/T_s} H_{\mathrm{FP}} e^{-V/T_s} = H_{\mathrm{FP}}^{\dagger} \; .$$

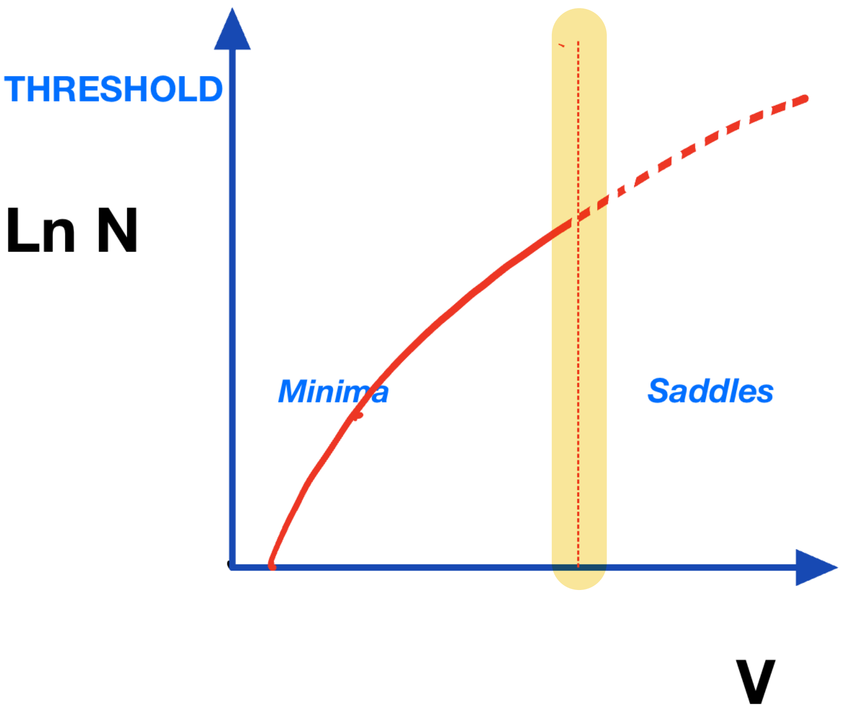
$$H = \frac{T_s}{2} e^{V/2T_s} H_{\mathrm{FP}} e^{-V/2T_s} = \sum_i \left[-\frac{T_s^2}{2}\frac{\partial^2}{\partial q_i^2} + \frac{1}{8}\left(\frac{\partial V}{\partial q_i}\right)^2 - \frac{T_s}{4}\frac{\partial^2 V}{\partial q_i^2}\right]$$

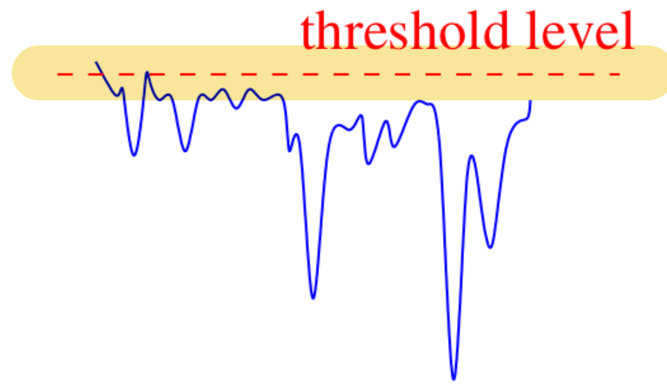
$$V_{\mathrm{eff}} = \frac{1}{8}\sum_i \left(\frac{\partial V}{\partial q_i}\right)^2 - \frac{T_s}{4}\sum_i \frac{\partial^2 V}{\partial q_i^2} \; .$$

METASTABILITY AND SPECTRUM



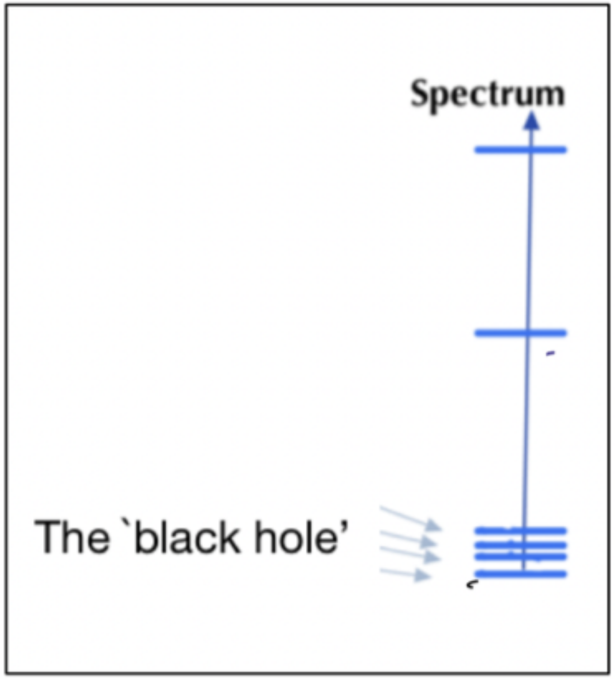
take it now to glasses

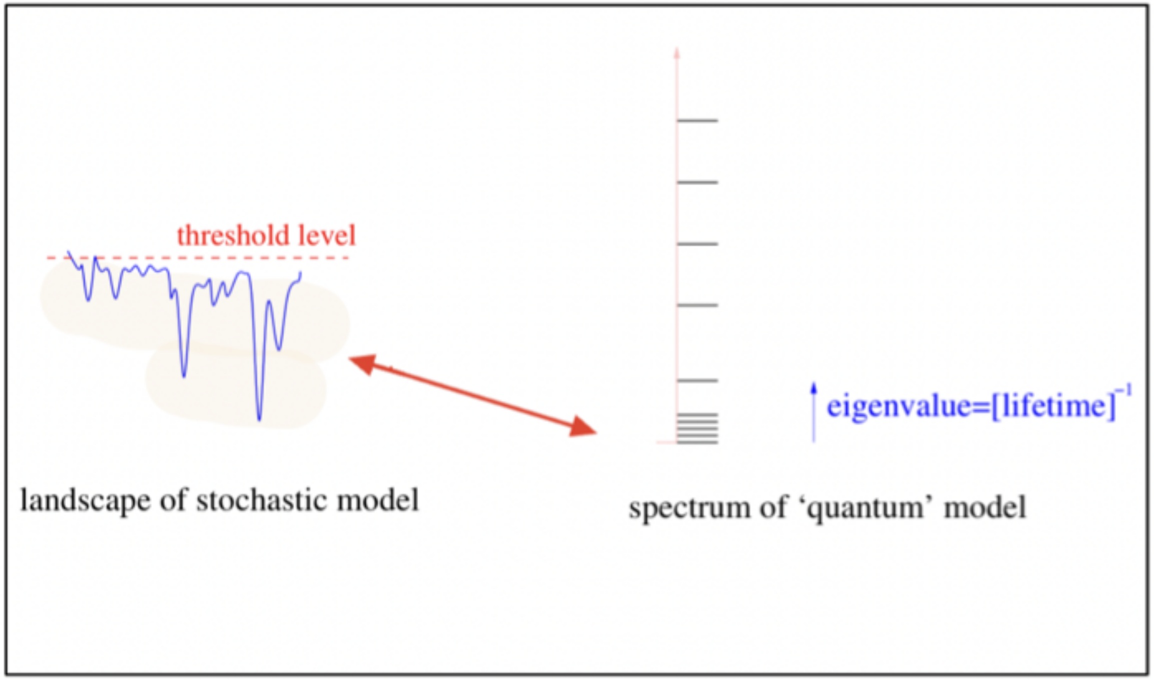




landscape of stochastic model

so, here it is


Spectrum
The `black hole'


threshold level
landscape of stochastic model
eigenvalue=[lifetime]$^{-1}$
spectrum of 'quantum' model

What is the meaning of this striking analogy?

What can glasses possibly have in common with Black Holes?

A tentative answer: reparametrization invariance

(which both glasses and SYK (and perhaps black holes) have

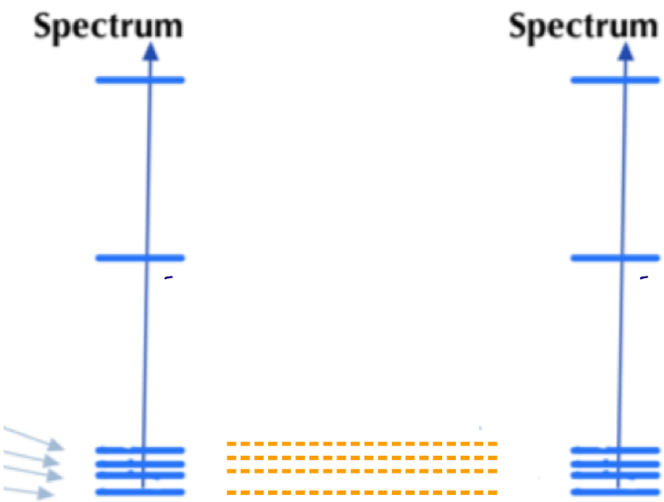
reflects the fact that systems have `meticulous dynamics': they completely equilibrate a neighborhood before moving on.

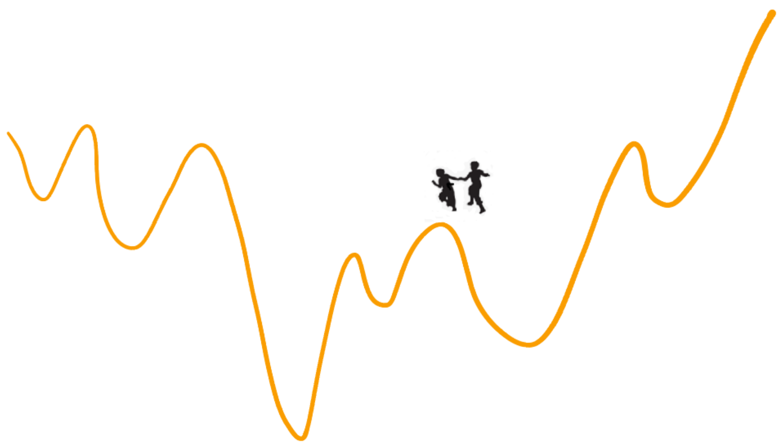
This is VERY non-generic.

Wormholes and coupled replicas

$$V^{(2)} = -\frac{1}{p!}\sum_{i_1,\ldots,i_p} J_{i_1,\ldots,i_p}\left\{q_{i_1}\ldots q_{i_p} + q'_{i_1}\ldots q'_{i_p}\right\} + \epsilon\sum_i q_i q'_i$$

Two weakly coupled, identical black holes

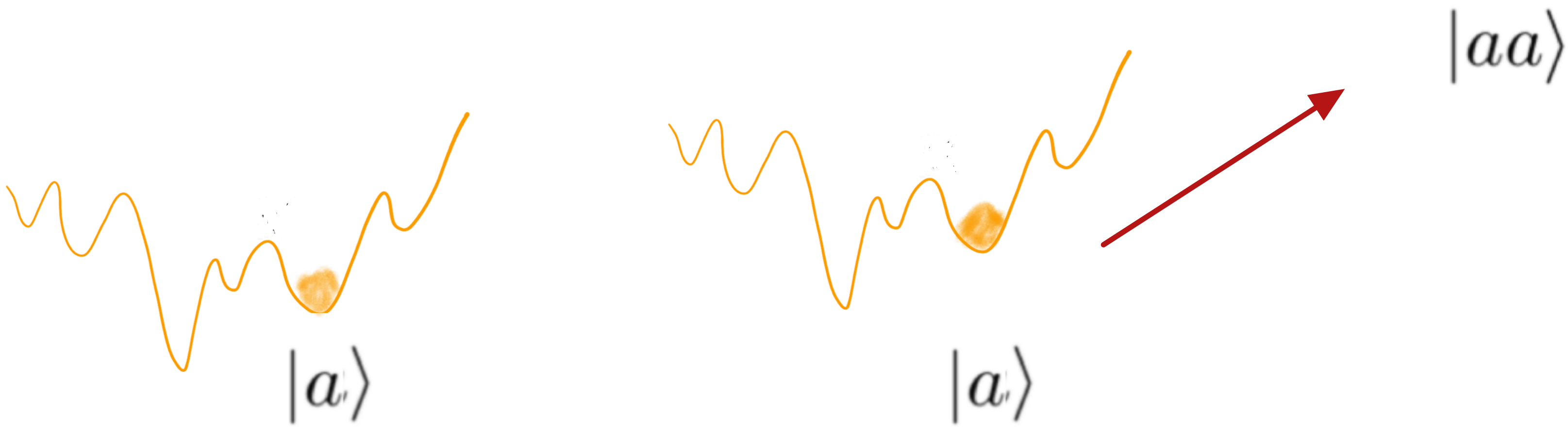




Two weakly coupled, identical glasses

THERMO-FIELD DOUBLE DESCRIBES EVOLUTIC

$$|TFD\rangle \propto \sum_a e^{-\beta_{\text{eff}} F(a)} |aa\rangle$$



4. PERSPECTIVES

Many-Level Wormhole?

$$C(t,t') = C_f(t-t') + \mathcal{C}_1\left(\frac{t-t'}{\tau^a}\right) + \mathcal{C}_2\left(\frac{t-t'}{\tau^b}\right)$$