Statistics of stick-slip dynamics in contact line motion and dry friction

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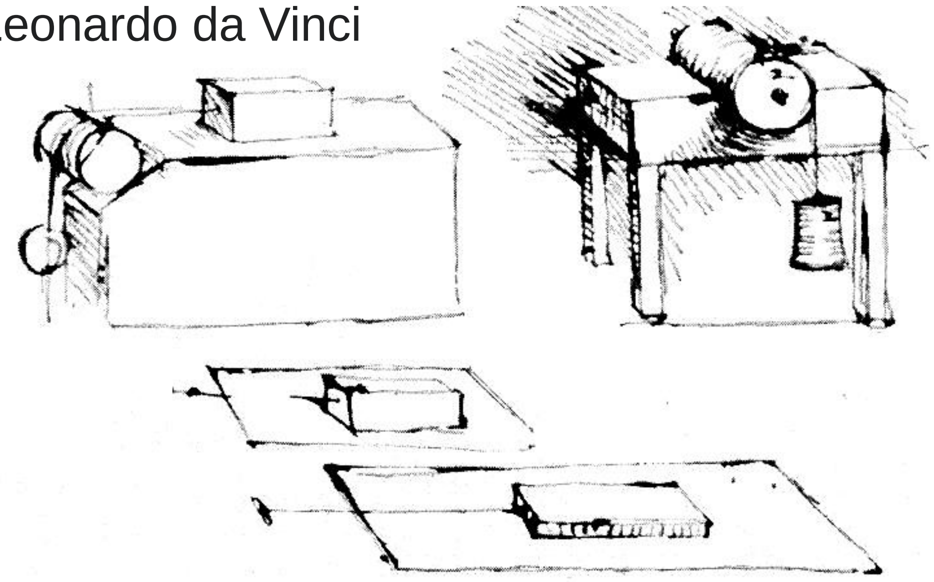
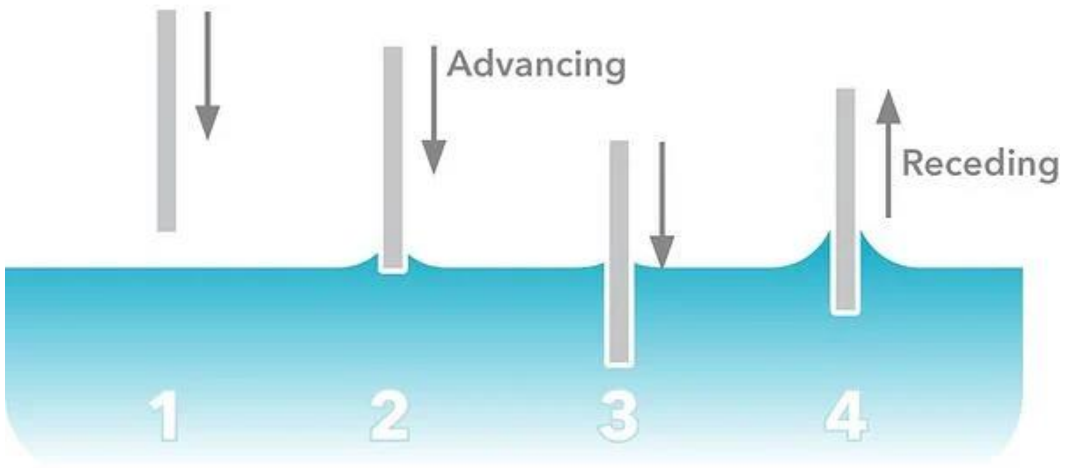
Leonardo da Vinci



https://en.wikipedia.org/wiki/Tribology



https://www.biolinscientific.com/measurements/dynamic-contact-angle

Statistical laws of stick-slip friction at mesoscale
Nature Communations (2023)
Caishan Yan and Penger Tong (HKUST)
Pik-Yin Lai (NCU)

Avalanches and extreme value statistics of a moving contact line, Phys Rev Lett (2024)
Caishan Yan, Dongshi Guan, Yin Wang, Penger Tong (HKUST)
Pik-Yin Lai (NCU)

Outline

- Introduction

1. Dry friction
2. Contact line motion

- Experiments on mesoscopic scale statistics of stick-slip dynamics

1. Contact line hysteresis
2. Mesoscopic friction

- Theory

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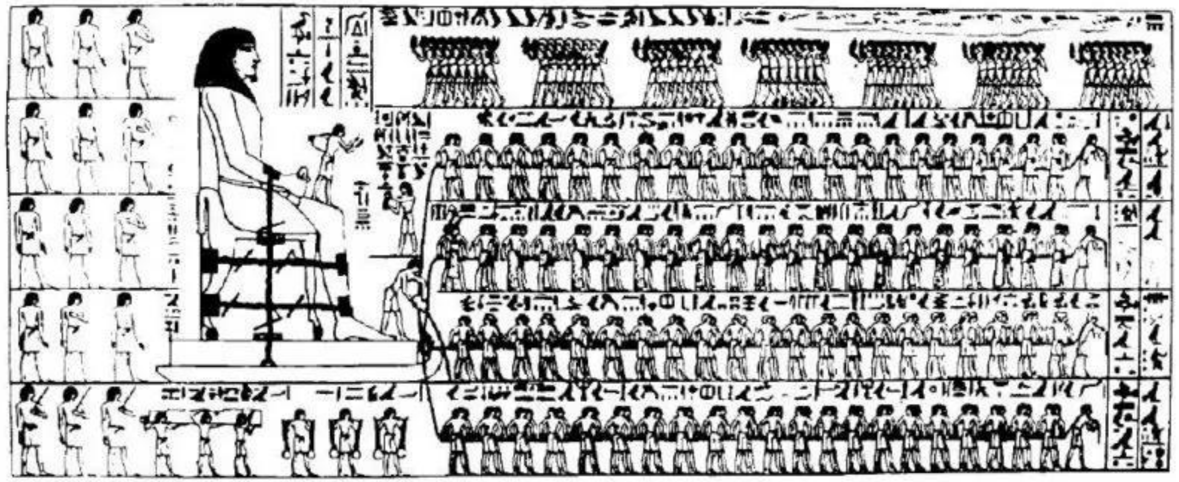
Introduction:
macroscopic, microscopic, and mesoscopic

Friction: 2400 BC and 21th century

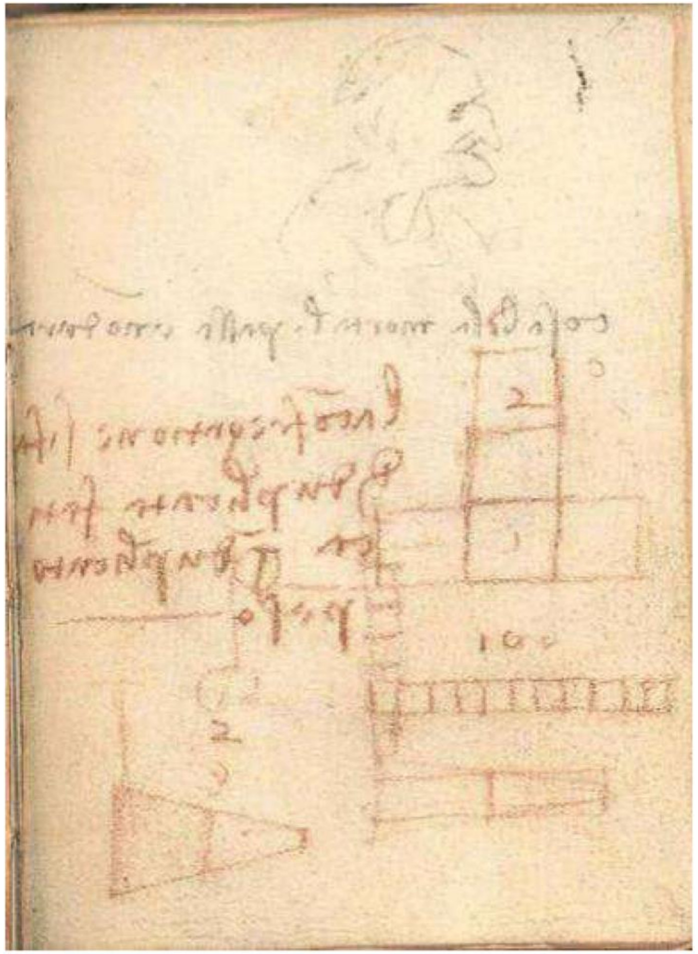


https://www.tribonet.org/wiki/tribology-history/:

slaves are dragging a large statue along sand or ground. One man, standing on the sledge supporting the statue, pours a liquid (oil/water) as a lubricant in order to reduce friction between sledge and ground/sand (2400 BC).

- https://www.youtube.com/watch?v=Upw6lvlEeV8
- 2024. July. 24 Gion Festival (7:20)

Leonardo da Vinci (1493)

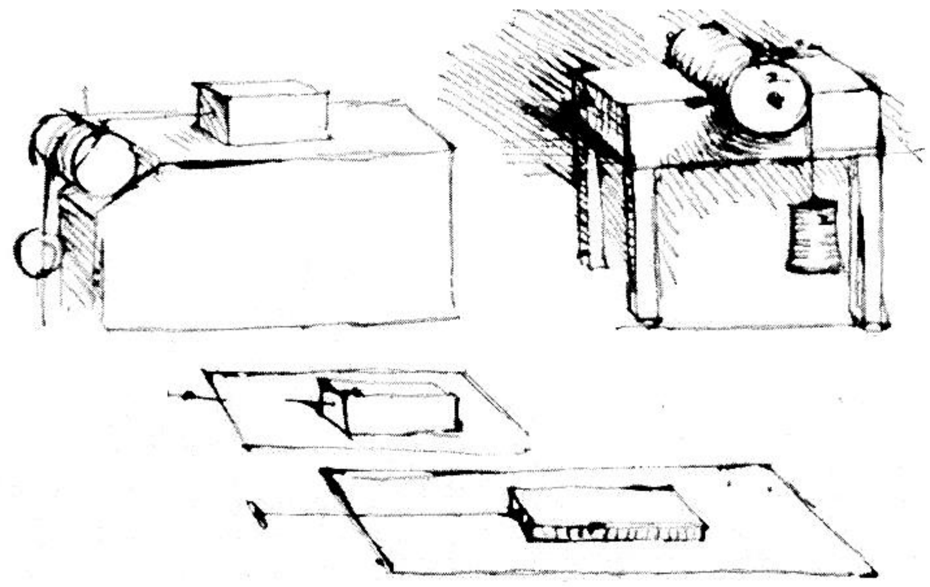


E. Blakemore, July 2016, Smithsonian

In a new study in the journal *Wear*, an engineer from the University of Cambridge describes how he found the artist's first writing on the laws of friction in a tiny notebook that dates from 1493 housed in the Victoria & Albert Museum in London. The text and accompanying sketches are apparently evidence of da Vinci's earliest experiments in friction.

Ironically, the doodle and text had previously been dismissed by art historians, who preferred to focus instead on a sketch of an old woman adjacent to the scribbles. The artists scribbled the quote "cosa bella mortal passa e non dura" (a line from Petrarch that means "mortal beauty passes and does not endure") beneath the sketch of the woman. But as long as da Vinci's notebooks keep revealing the depth of the master's brilliance, interest in their contents—both artistic and scientific—will never die.

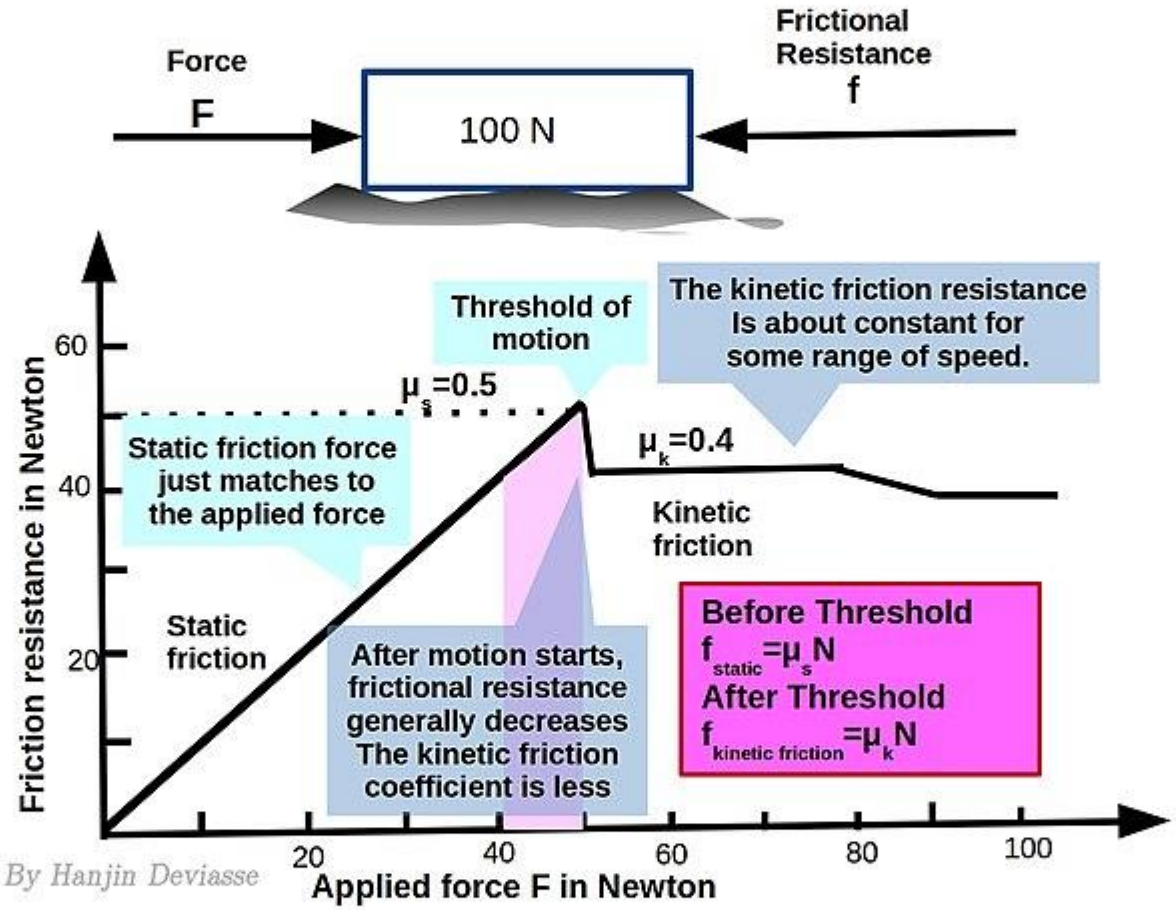
Leonardo da Vinci (1493)



Da Vinci:

- frictional force was independent of the apparent contact area
- observed that the force needed to overcome friction doubles as weight doubles. (https://en.wikipedia.org/wiki/Tribology)
- distinguished between rolling and sliding contacts and identified surface roughness as a factor in material movement.
- (https://www.popularmechanics.com/science/a31996381/what-is-tribology/)

Macroscopic studies of friction



- First Law (Amontons, close to 1700)

  Friction is independent of the apparent area of contact.

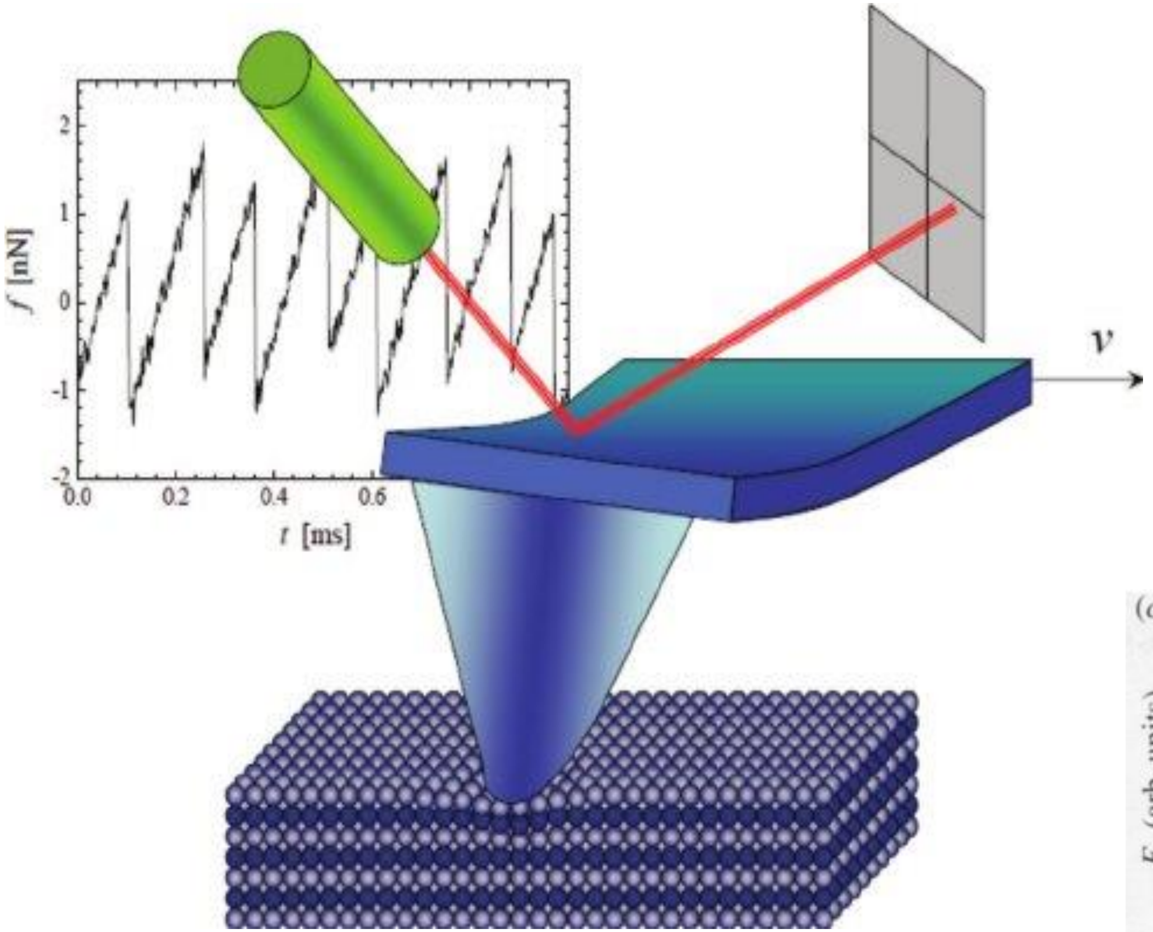
- Second Law (Amontons)

  The frictional force is directly proportional to the normal load.

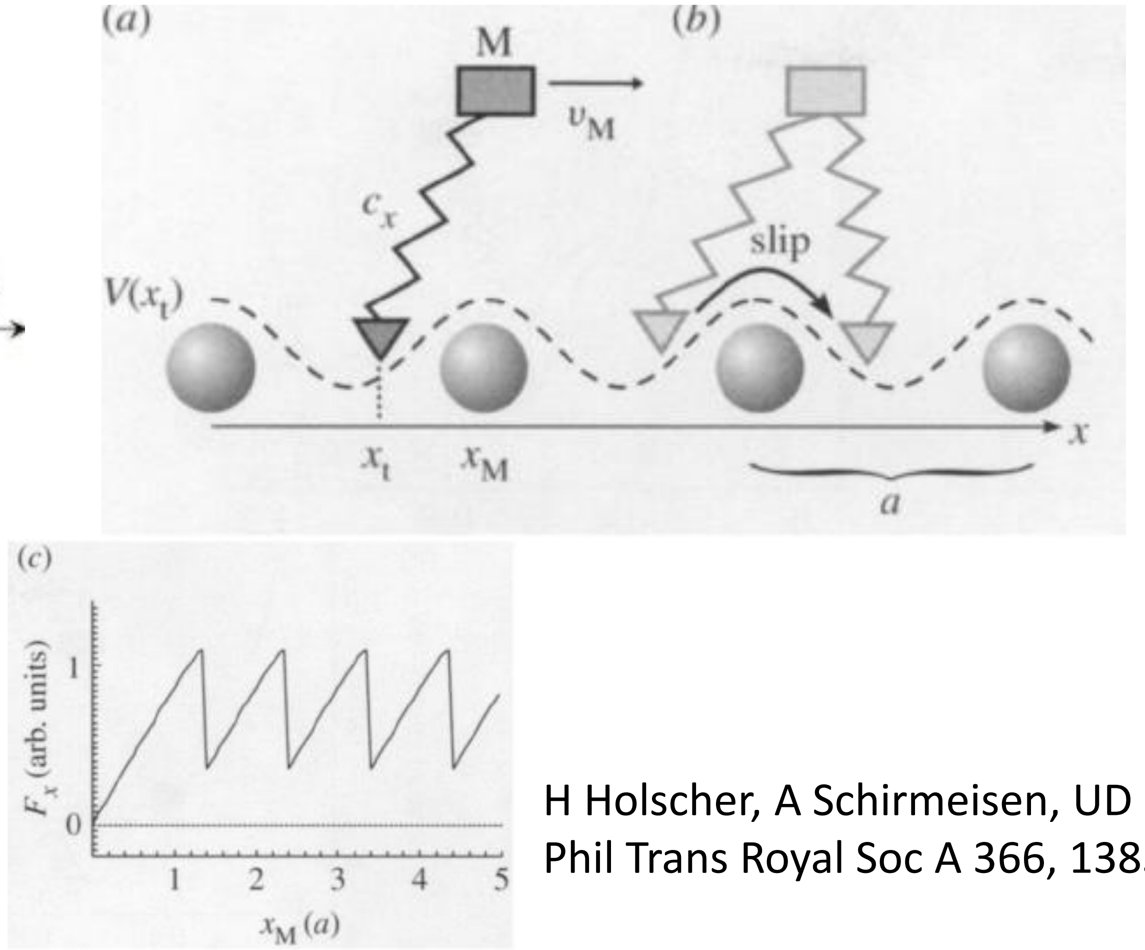
- Third Law (Coulomb)

  Dynamic friction is independent of the relative sliding speed.

Friction at atomic scale

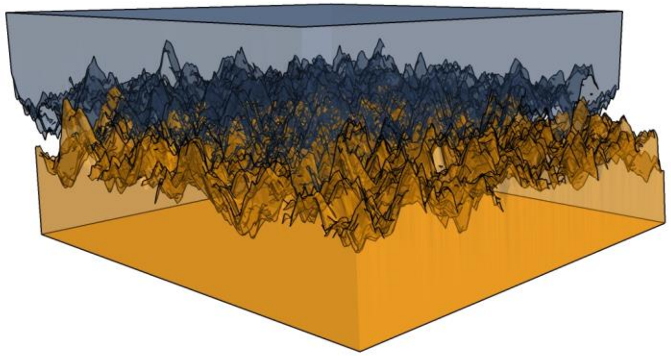


M Evstigneev, P Reimann,
Phys Rev B 87, 205441 (2013)



H Holscher, A Schirmeisen, UD Schwarz
Phil Trans Royal Soc A 366, 1383 (2008)

Friction at mesoscale



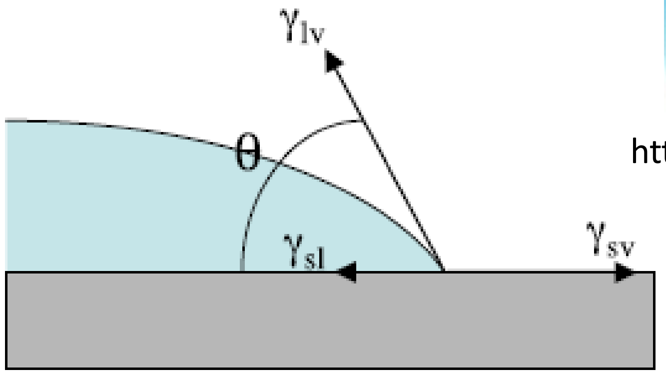
https://en.wikipedia.org/wiki/Friction#/media/File:Friction_between_surfaces.jpg

- Rough surfaces (Da Vinci)
- Many contact points
- Normal force affects the true contact area
- Sliding motion should show irregular stick slips
- Statistics of stick-slip events should be related to surface properties
- Statistics of stick-slip events may show universal characters

Contact angle and contact angle hysteresis: macroscopic picture

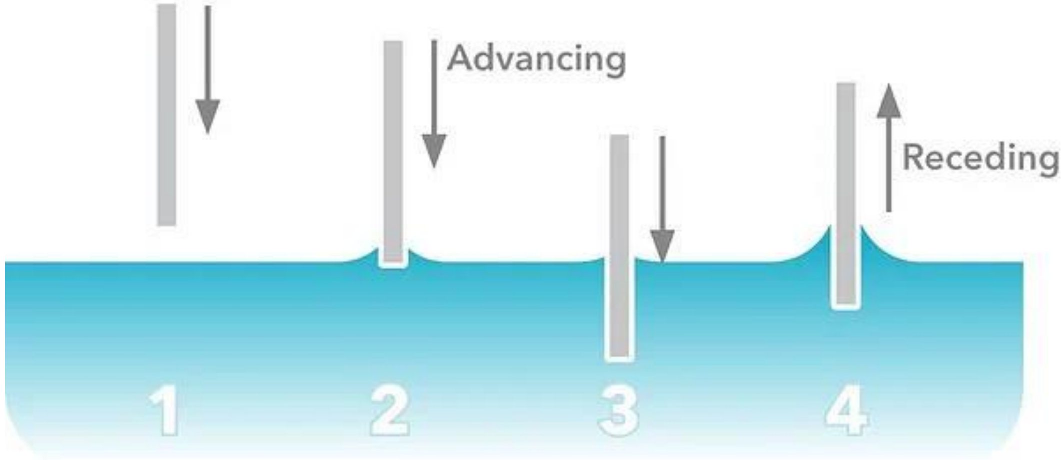
Classical idea: force balance

Thomas Young (1773-1829)

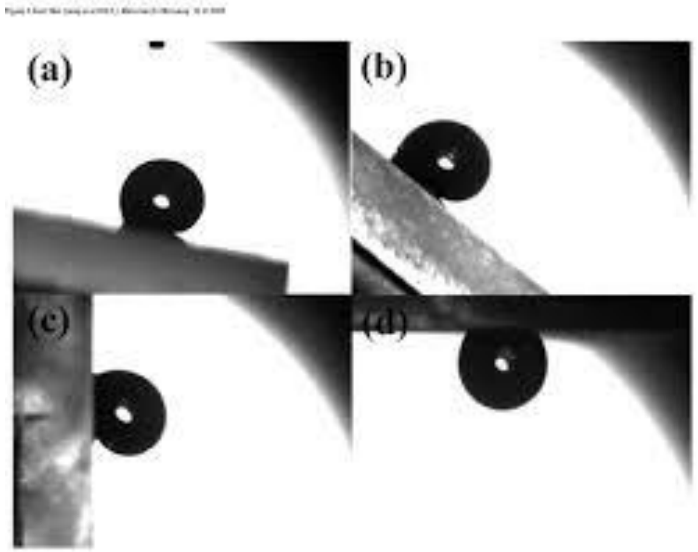


$$\gamma_{SV} = \gamma_{SL} + \gamma \cos \theta_{eq}$$

Advancing contact angle > receding contact angle



https://www.biolinscientific.com/measurements/dynamic-contact-angle



Jiang *et al* 2014 *J. Micromech. Microeng.*

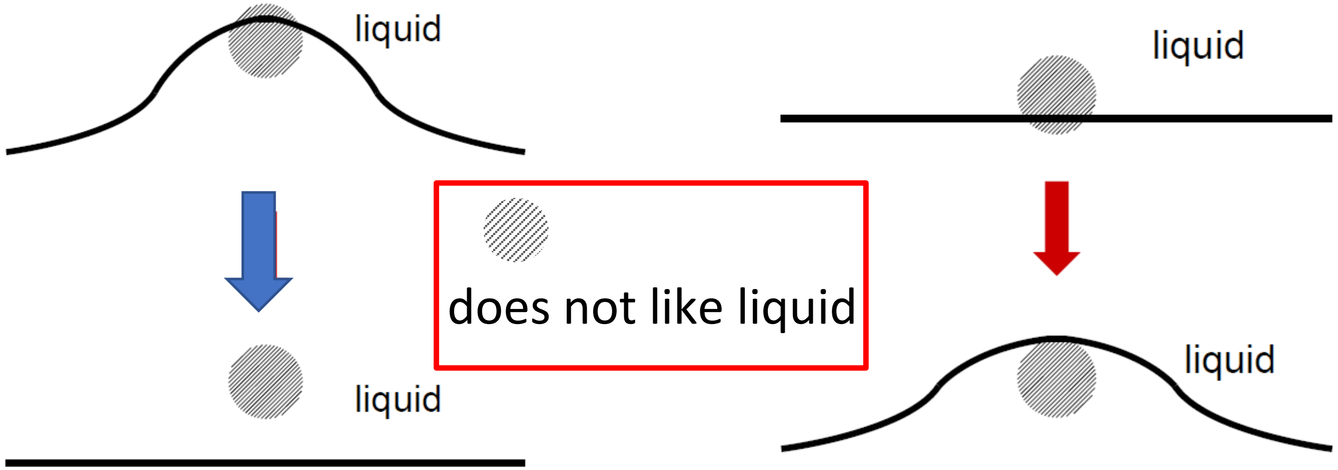
Contact angle hysteresis at single defect level

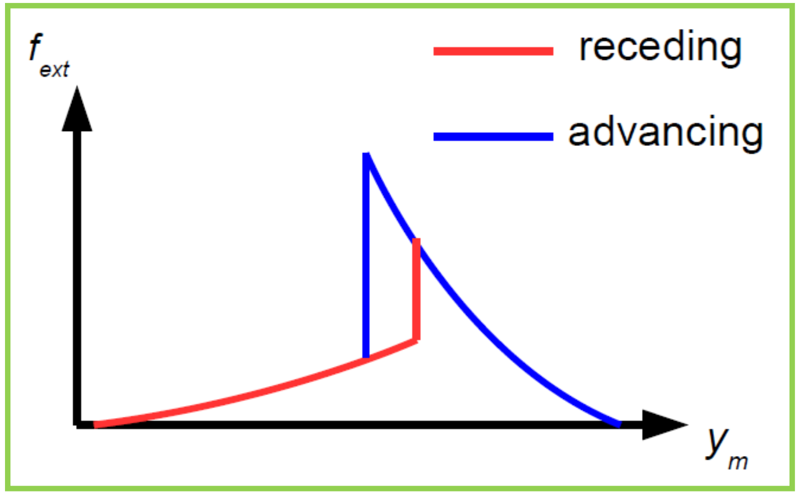
J-F Joanny and P.G. de Gennes JCP (1984), M O Robbins and J-F Joanny EPL (1987)

1. Defect force
2. Contact line elastic force
3. Force balance
4. Hysteresis

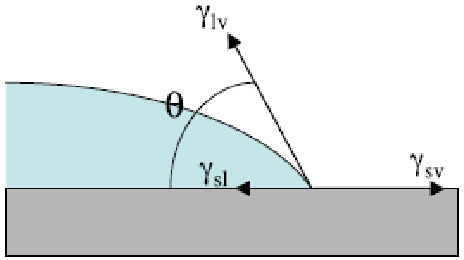
Leger and Joanny, Rep. Prog. Phys. (1992)

(a) an advancing experiment **(b) a receding experiment.**





Contact line motion at mesoscale



$$\gamma_{SV} = \gamma_{SL} + \gamma\ cos\theta + \text{“details”}$$

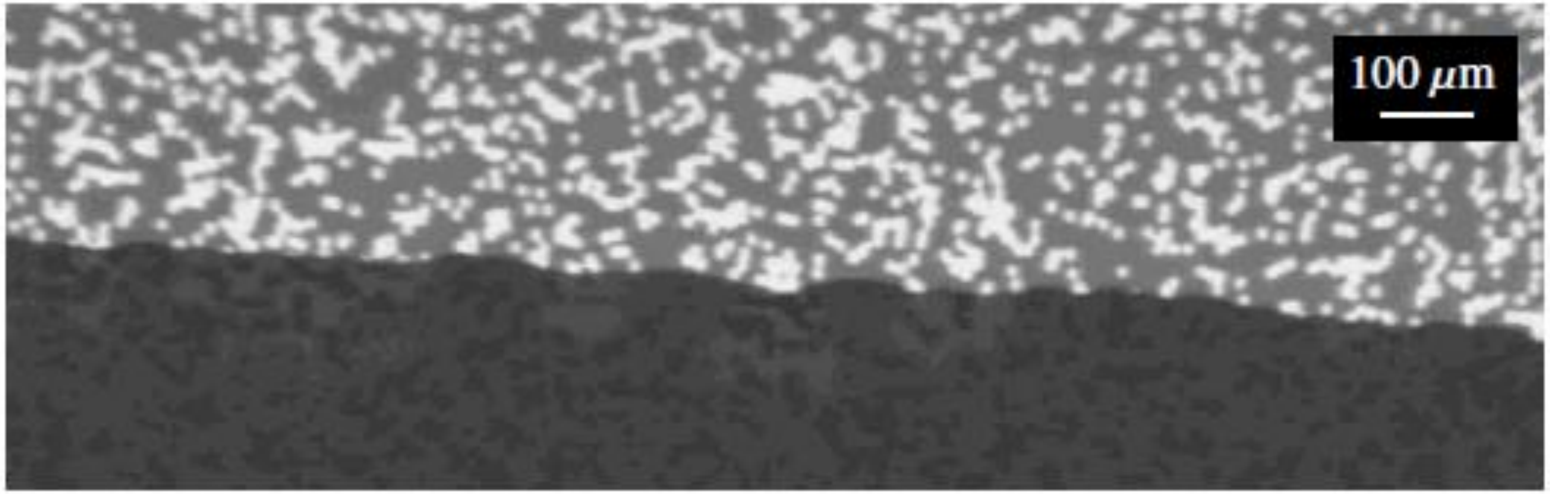
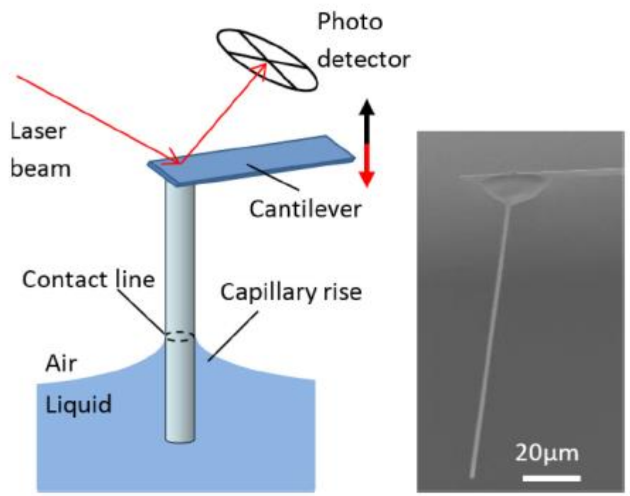


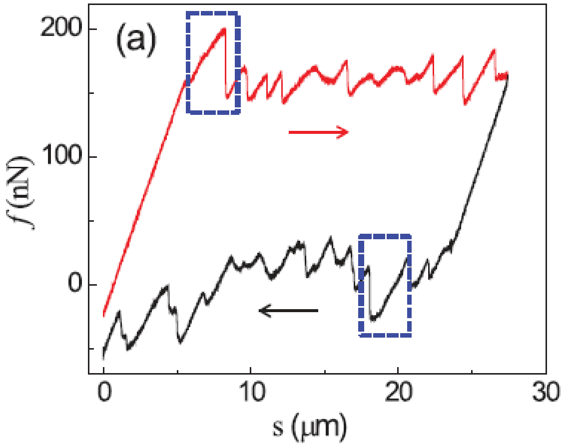
FIG. 9. The edge of a water drop receding on a disordered surface. The defects (size 10 μm) appear as white dots.

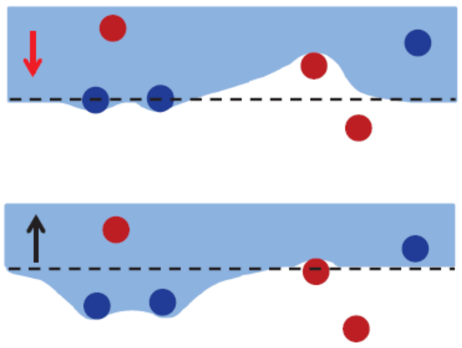
- Heterogeneous solid surface
- Contact line motion should show irregular stick slips
- Statistics of stick-slip events should be related to properties of solid surface
- Statistics of stick-slip events may show universal characters

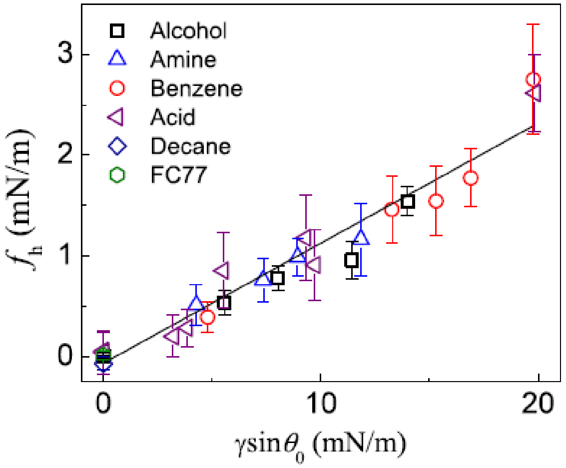
Contact angle hysteresis at mesoscale

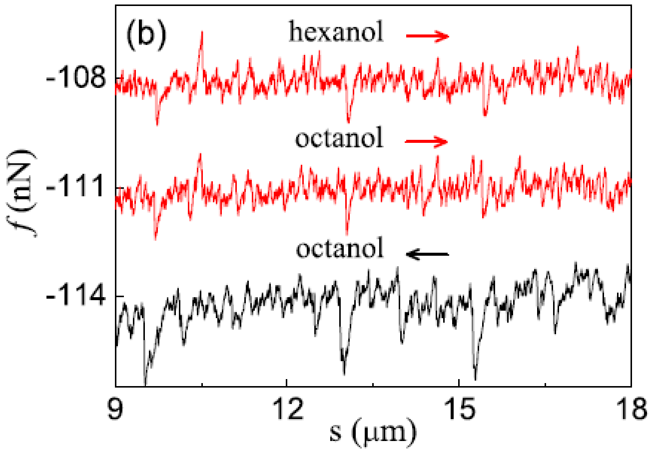
YJ Wang, S Guo, HY Chen, P Tong, PRE, 93, 052802 (2016)

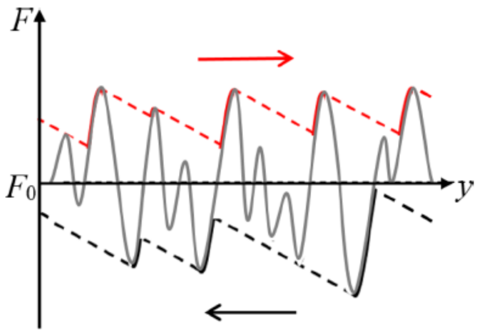












Question:
The statistics of contact line dynamics.

Outline

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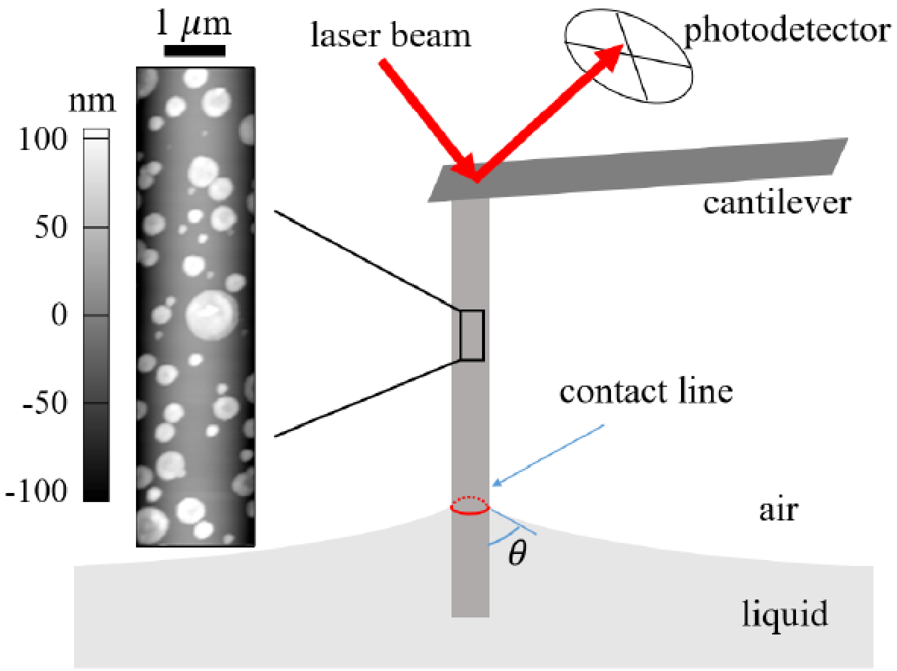
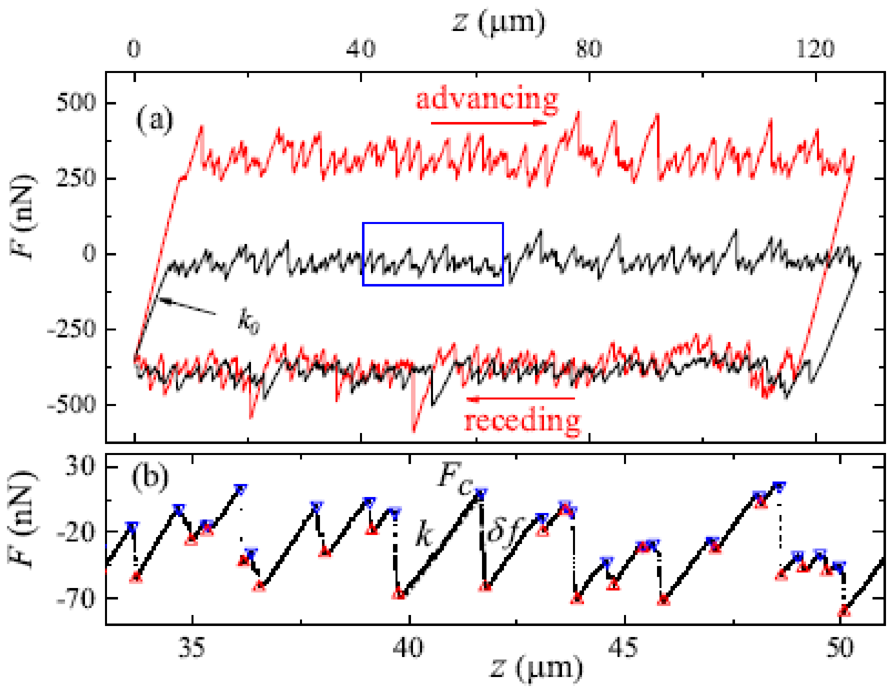
1. Contact line hysteresis
2. Mesoscopic friction

- Theory

Contact line hysteresis: experiment

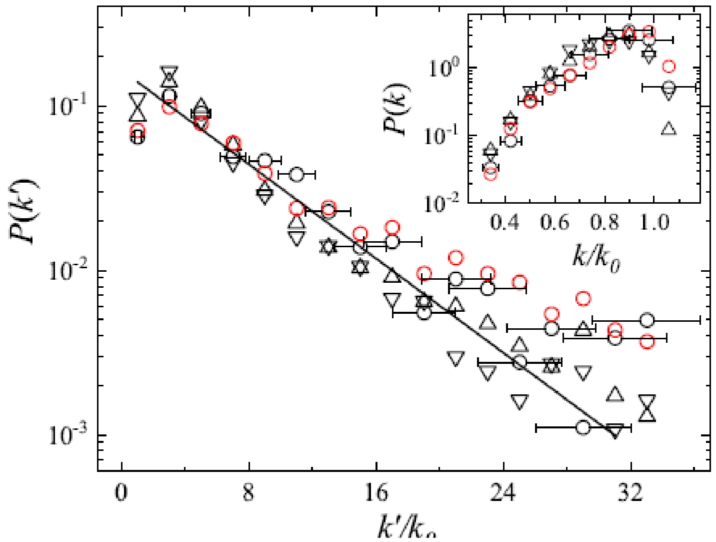
Caishan Yan, Dongshi Guan, Yin Wang, Penger Tong

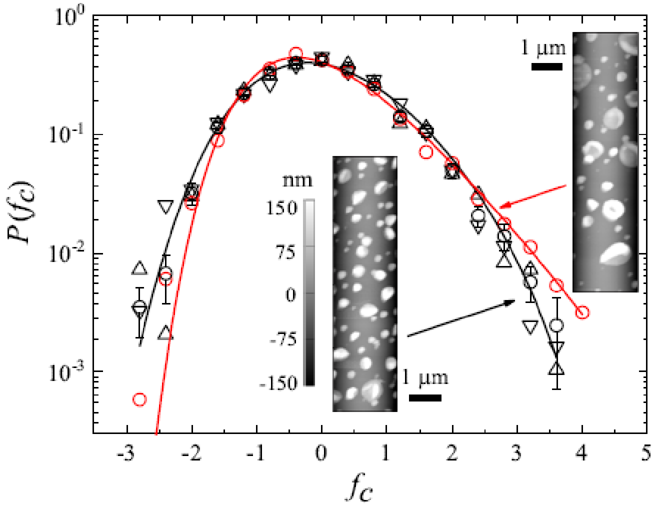


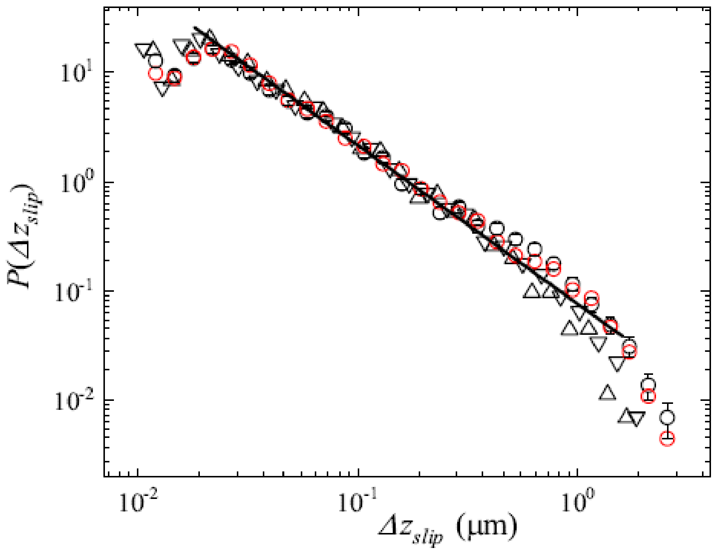


- Glass fiber diameter ~ 3μm, length ~ 600 μm, coated with a thin layer of propyltrichlorosilane (PTS).
- Liquid: water or ethylene-glycol, fiber moving speed = 0.62 μm/s
- Hysteresis, statistics of k, δf, and F_c.

Contact line hysteresis: data







Black: the same PTS-coated fiber in contact with three different liquids

Red: a different PTS-coated fiber in contact with the ethylene-glycol

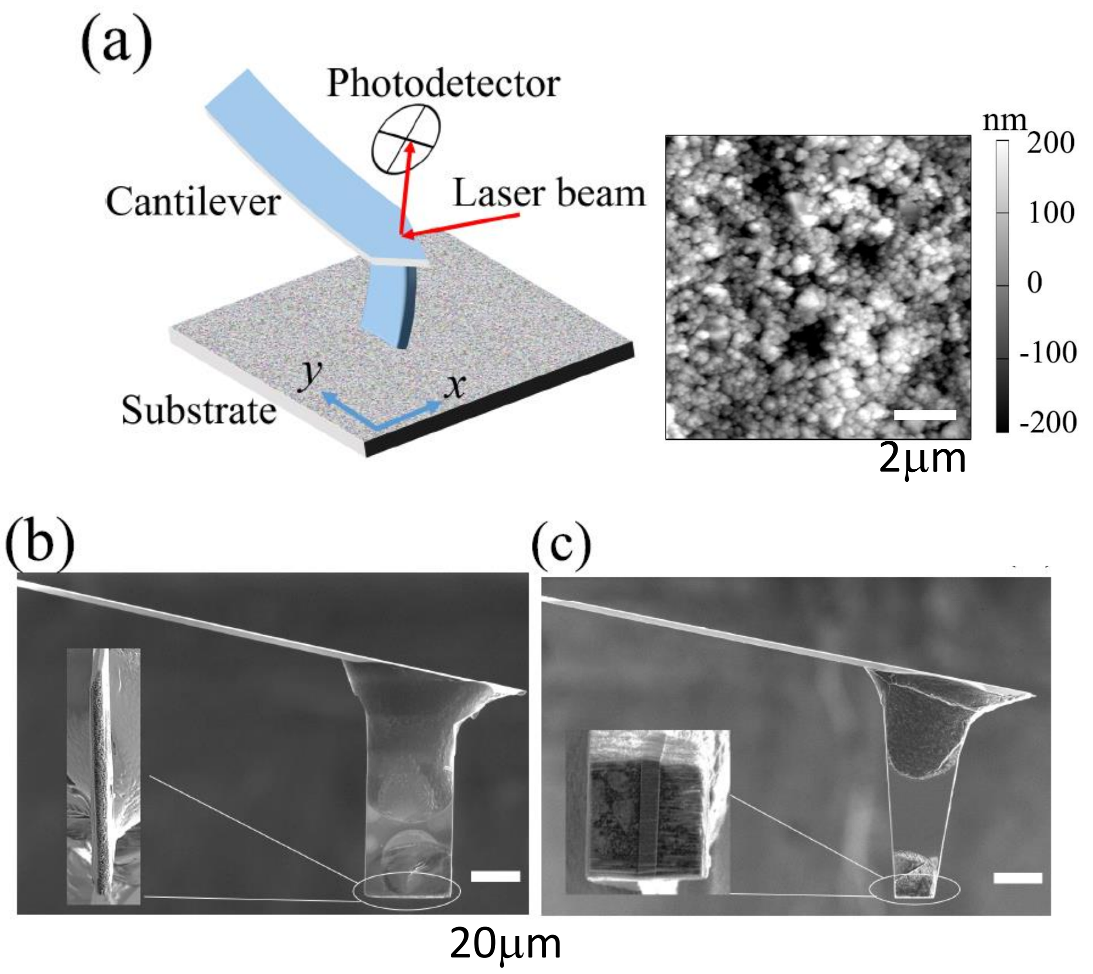
- Result depends on fiber, not liquid. (roughness, not chemical heterogeneity dominates stick-slip dynamics)
- Effective elastic constant k' of the contact line is exponentially distributed
- Avalanches size δf distribution: power law
- Max force F_c follows generalized extreme value distribution

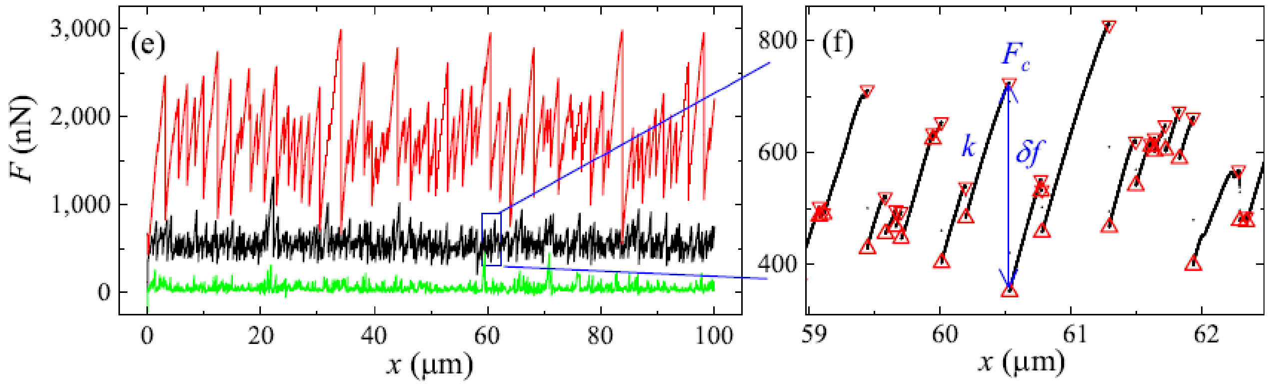
Question:

Similar statistics in other stick-slip dynamics?

Mesoscopic friction: experiment

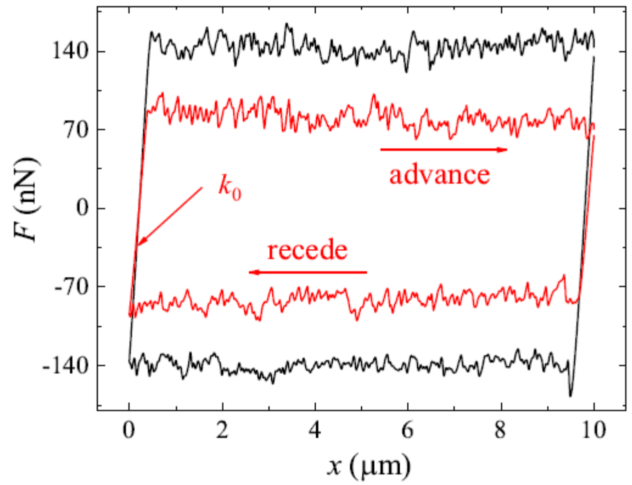
Caishan Yan and Penger Tong

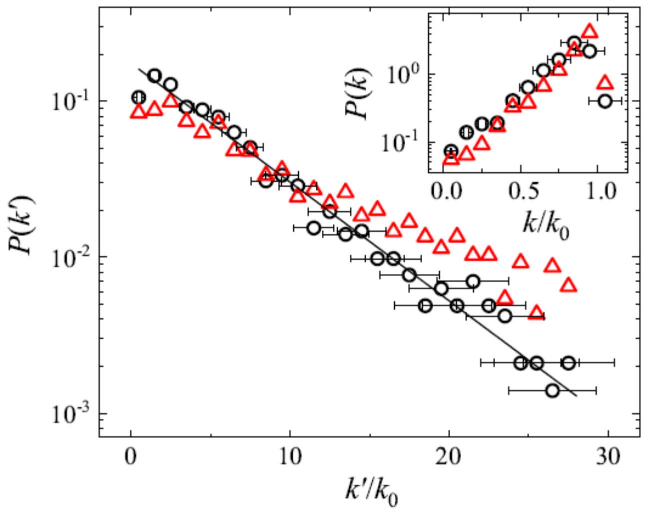


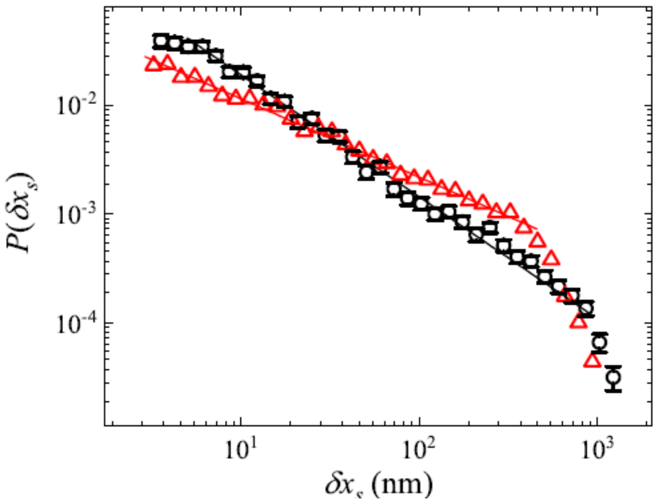


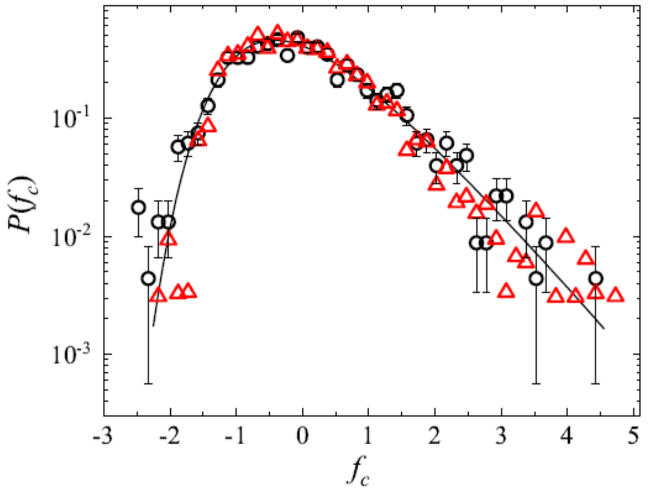
- Free end of hanging beam: enclosed by a layer of glue mixed with glass nanoparticles
- Substrate: ultra-fine silicon carbide sandpaper, average grain size ~ 100 nm
- Various normal force applied
- Statistics of k, δf, and F_c.

Mesoscopic friction: statistics









- Hysteresis
- Effective elastic constant k': exponentially distributed
- Avalanche size δf: power law
- Max force F_c follows generalized extreme value distribution.

(red: thin prob, black: square prob)

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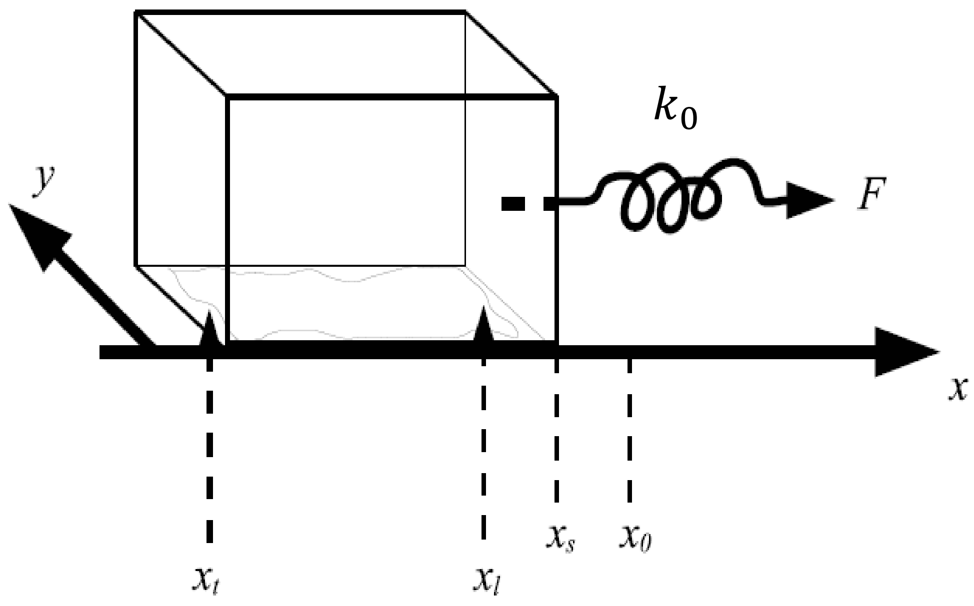
Theory: energy landscape, universality, and unanswered questions

Model "stick" in friction



Force balance: spring force = contact force

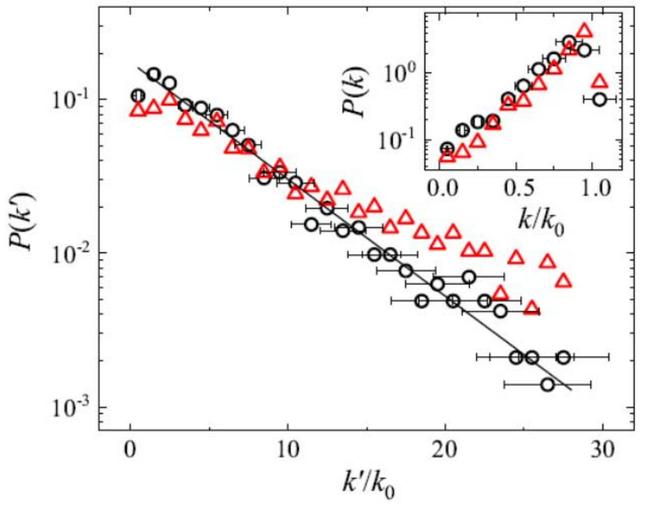
$$F(x_0) = k_0[x_0 - x_s(x_0)]$$

$$F(x_0) = \int dy \int_{x_t(y)}^{x_l(y)} dx\ \sigma_{xy}(x_0, x_s; x, y)$$

$$= \int dy\ \Gamma(x_0, x_s; x, y) \equiv h(x_s)$$

The slope of the force-displacement curve is related to the derivative of contact force

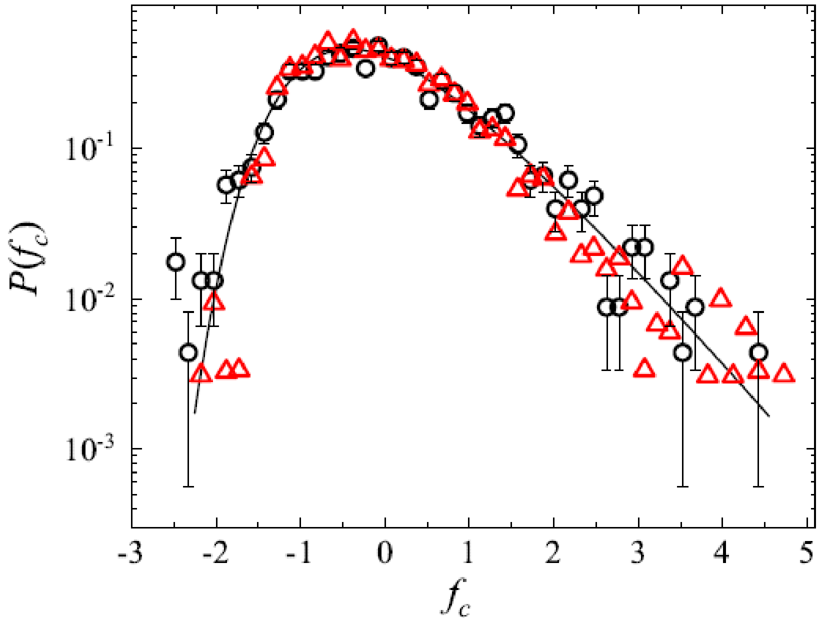
$$k = \frac{dF}{dx_0} = \frac{k_0 \frac{dh}{dx_s}}{k_0 + dh/dx_s}$$



- $\frac{dh}{dx_s} = k'$
- $p\left(\frac{k'}{k_0}\right) \sim e^{-b\frac{k'}{k_0}}$,
- black line: $b = 0.175 \pm 0.018$

The spring is soft compared to the surface.

Distribution of max force: solid friction



Dimensionless max force $f_c = \frac{F_c - \langle F_c \rangle}{\sigma_{F_c}}$

Generalized extreme value distribution (GEV) $P(z) = \frac{1}{\beta}(1+\xi z)^{-(1+1/\xi)} e^{-(1+\xi z)^{-1/\xi}}$

$$z = \frac{f_c - \mu}{\beta}$$

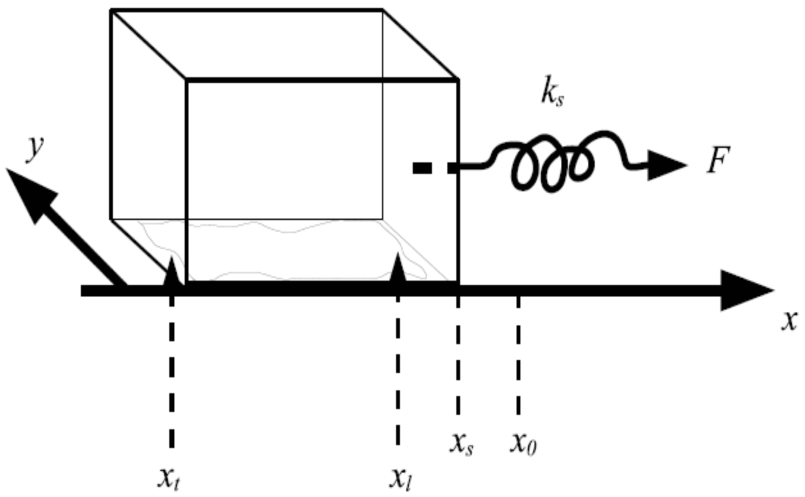
Scale parameter $\beta = \frac{|\xi|}{\sqrt{\Gamma(1-2\xi) - \Gamma^2(1-\xi)}}$

Location parameter $\mu = \frac{\beta[1-\Gamma(1-\xi)]}{\xi}$

Fitting parameter $\xi = -0.03 \pm 0.05$

For $\xi = 0$, the GEV is for independent variables with exponential tail at large z without upper bound.

The origin of generalized extreme value distribution -- solid friction

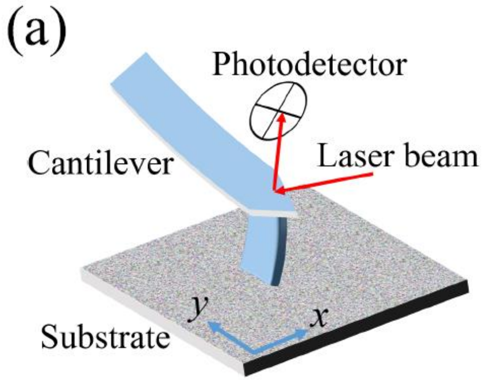


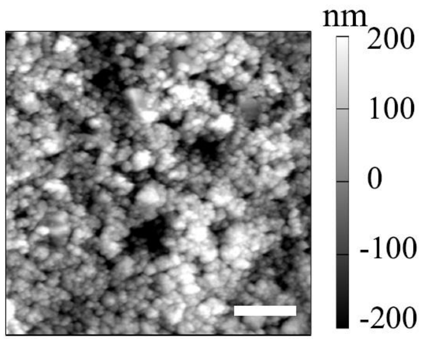
- l_d: the distance that the hanging beam needs to move in order to sample uncorrelated solid surface energy landscape.
- During the exp the handing beam has been dragged over a distance $\gg l_d$.
- At given x_0,

P($x_s = nl_d$)=P(beam is not pinned at $x_s < nl_d$) × P(pinning force at $nl_d > k_s(x_0 - nl_d)$)

- Suppose prob of local pinning force is $p(h) = (1 - h/h_M)^{b-1}$,

then $b = -(1 + \frac{1}{\xi})$ (Le Doussal and Wiese, Phys Rev E (2009))





2μm

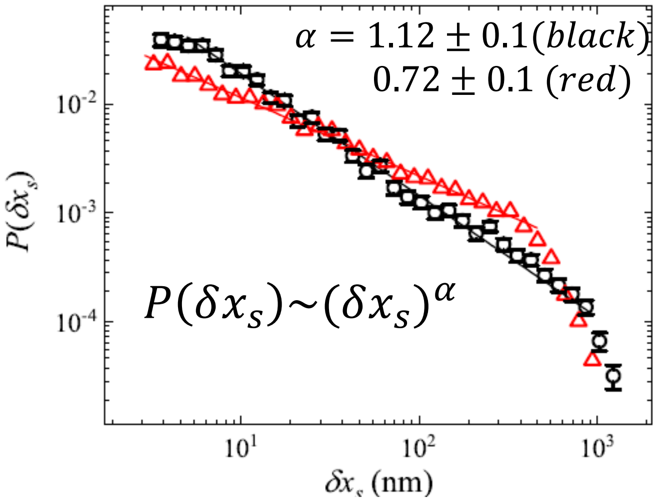
$$\frac{h_M^{-1}}{(k_0 l_d/h_M)^{1/(b+1)}} = -\frac{\xi}{\beta}\frac{1}{\sigma_{F_c}},$$

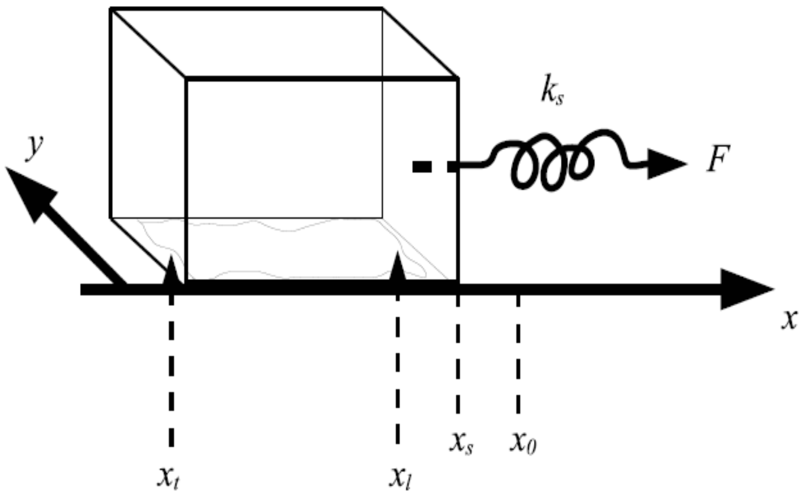
$$\frac{1}{(k_0 l_d/h_M)^{1/(b+1)}} = 1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c\rangle}{\sigma_{F_c}}\right)$$

$$P(F_c) = \frac{1}{\beta^2 \sigma_{F_c}}\left[1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c\rangle}{\sigma_{F_c}}\right) + \frac{\xi}{\beta}\frac{F_c}{\sigma_{F_c}}\right]^{-(1+1/\xi)} \times \exp\left\{-\left[1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c\rangle}{\sigma_{F_c}}\right) + \frac{\xi}{\beta}\frac{F_c}{\sigma_{F_c}}\right]^{-1/\xi}\right\}$$

Using $\langle F_c\rangle$, σ_{F_c}, ξ, β, and μ from the experiments, one can obtain the magnitudes of h_M and l_d, two key parameters in the model which characterize the properties of the fiber surfaces.

Avalanche size distribution – solid friction





- Friction force $F = \int dy \int_{x_t(y)}^{x_l(y)} \sigma_{xy}(x_0, x_s; x, y)\, dx \equiv \int dy\, \Gamma(x_0, x_s; y)$

- Dynamics of x_s (C Yan, PY Lai):

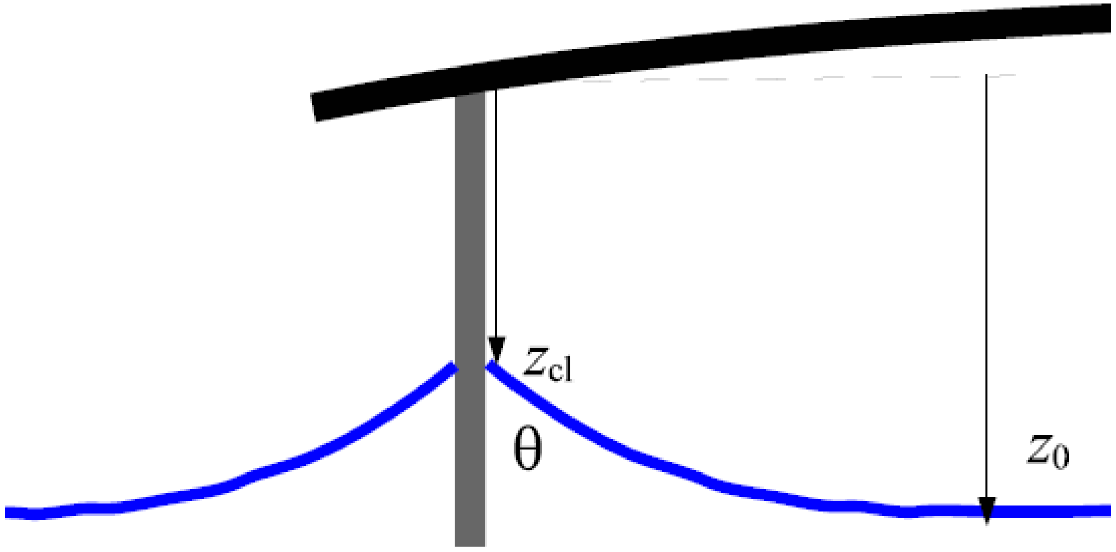
$\eta(x_s)$

$$m \frac{d^2 x_s}{dt^2} + \gamma \frac{dx_s}{dt} = k_0(x_0(t) - x_s) - \int dy\, \Gamma(x_0, x_s; y)$$

$$\langle(\eta(x_s) - \eta(x_s'))^2\rangle \sim |x_s - x_s'| \quad \text{when } |x_s - x_s'| < l_d$$

- Simulations (C Yen and PY Lai. See Lai's talk on July 4th): power law distribution for avalanches, exponent (< 1.2) increases with the strength of η.

Model “stick” in contact line dynamics



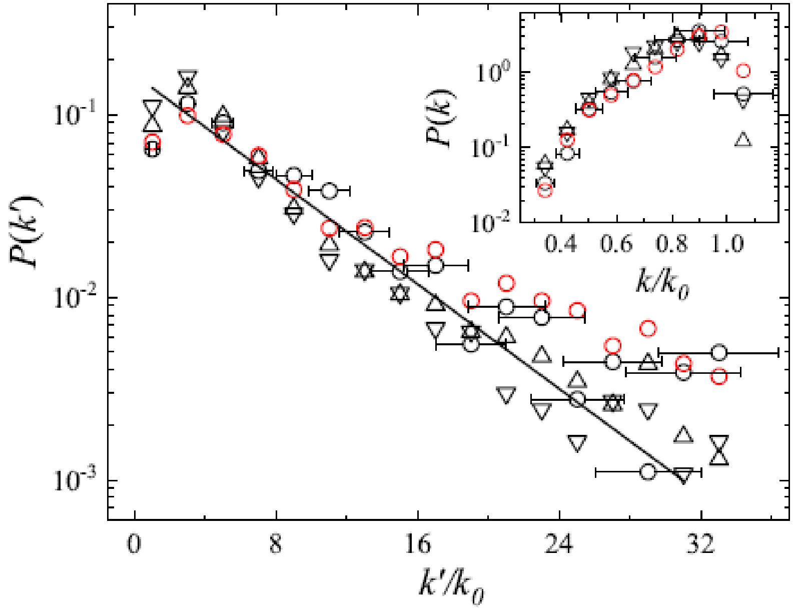
$$f(z_0) = k_0(z_0 - z_{\rm cl}) = \gamma \cos\theta_0 + h_e(z_{\rm cl})$$

Force due to roughness

$$h_e(z_{\rm cl}) = \frac{1}{\pi d}\int dx\ h(x,z)$$

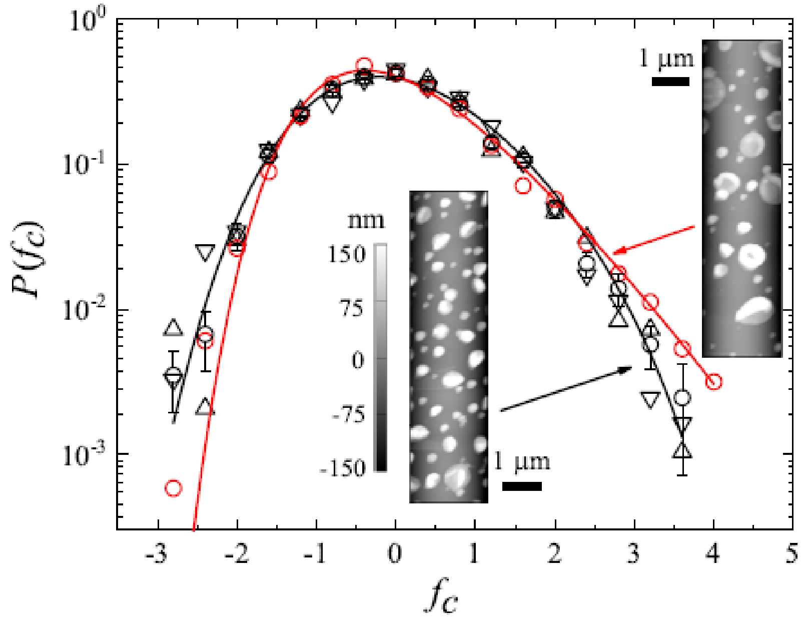
$$k_0\left(1 - \frac{dz_{\rm cl}}{dz_0}\right) dz_0 = h_e' \frac{dz_{\rm cl}}{dz_0} dz_0$$

$$k \equiv \frac{df}{dz_0} = \frac{\partial f}{\partial z_0} + \frac{\partial f}{\partial z_{\rm cl}}\frac{dz_{\rm cl}}{dz_0} = \frac{k_0 h_e'}{k_0 + h_e'}$$



- $h_e' = k'$
- $p\left(\frac{k'}{k_0}\right) \sim e^{-b\frac{k'}{k_0}}$
- $b = 0.165 \pm 0.015$

Distribution of max force: contact line motion



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$z = \frac{f_c - \mu}{\beta}$

Scale parameter $\beta = \frac{|\xi|}{\sqrt{\Gamma(1-2\xi) - \Gamma^2(1-\xi)}}$

Location parameter $\mu = \frac{\beta[1-\Gamma(1-\xi)]}{\xi}$

Fitting parameter $\xi = -0.17\ (black), -0.06\ (red)$

$$P(F_c) = \frac{1}{\beta^2 \sigma_{F_c}} \left[1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c \rangle}{\sigma_{F_c}}\right) + \frac{\xi}{\beta}\frac{F_c}{\sigma_{F_c}}\right]^{-(1+1/\xi)} \exp\left\{-\left[1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c \rangle}{\sigma_{F_c}}\right) + \frac{\xi}{\beta}\frac{F_c}{\sigma_{F_c}}\right]^{-1/\xi}\right\}.$$

Contact line motion: from generalized extreme value distribution to fiber surface property

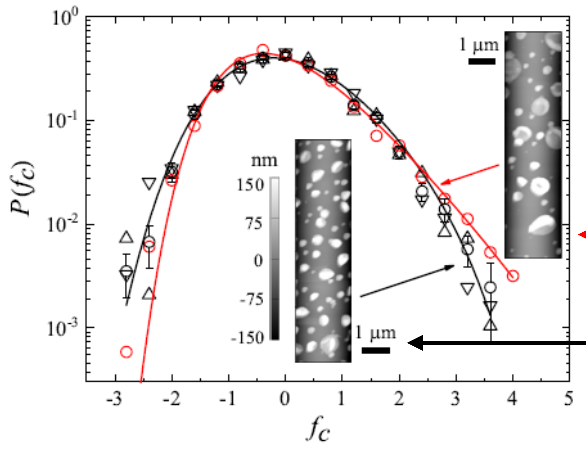
- l_d: the distance that the hanging beam needs to move in order to sample uncorrelated solid surface energy landscape.
- During the exp the handing beam has been dragged over a distance $\gg\ l_d$.
- At given z_0,

P($z_{cl} = nl_d$) = P(beam is not pinned at $z_{cl} < nl_d$) × P(pinning force at nl_d > $k_s(z_{cl} - nl_d)$)

- Suppose pinning force distribution is $p(h) = (1 - h/h_M)^{b-1}$, then $b = -(1 + \frac{1}{\xi})$ (Le Doussal and Wiese, Phys Rev E (2009))

$$\frac{h_M^{-1}}{(k_0 l_d/h_M)^{1/(b+1)}} = -\frac{\xi}{\beta}\frac{1}{\sigma_{F_c}},$$
$$\frac{1}{(k_0 l_d/h_M)^{1/(b+1)}} = 1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c\rangle}{\sigma_{F_c}}\right)$$

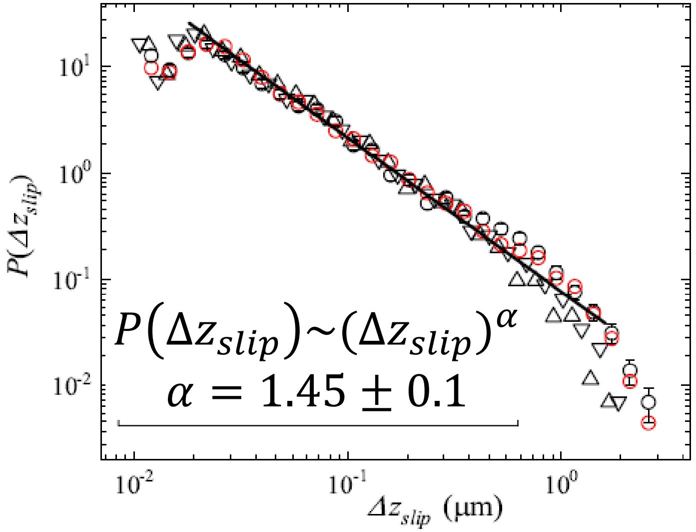
$$P(F_c) = \frac{1}{\beta^2 \sigma_{F_c}}\left[1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c\rangle}{\sigma_{F_c}}\right) + \frac{\xi}{\beta}\frac{F_c}{\sigma_{F_c}}\right]^{-(1+1/\xi)} \times \exp\left\{-\left[1 - \frac{\xi}{\beta}\left(\mu + \frac{\langle F_c\rangle}{\sigma_{F_c}}\right) + \frac{\xi}{\beta}\frac{F_c}{\sigma_{F_c}}\right]^{-1/\xi}\right\}$$

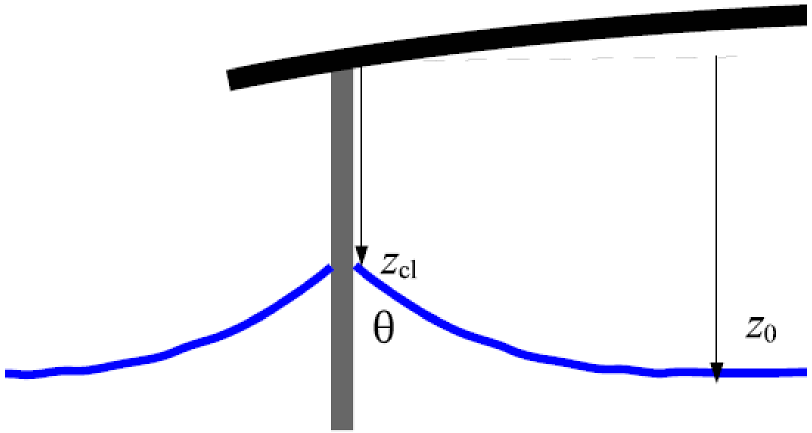


Fitting parameter $\xi = -0.17\ (black), -0.06\ (red)$

Local force max $(F_c)_M = (f_c)_M \sigma_{F_c} + \langle F_c\rangle \cong 359\ nN\ (red), 29\ nN\ (black)$.

Avalanche size distribution: contact line motion





Contact line dynamics: overdamped.

$$\varsigma \frac{\partial z(x,t)}{\partial t} = \int K(x-x')[z(x',t)-z(x,t)]dx + k_0[z_0(t)-z_{cl}(t)] - [\gamma\cos\theta_0 + h(x,z(x,t)]$$

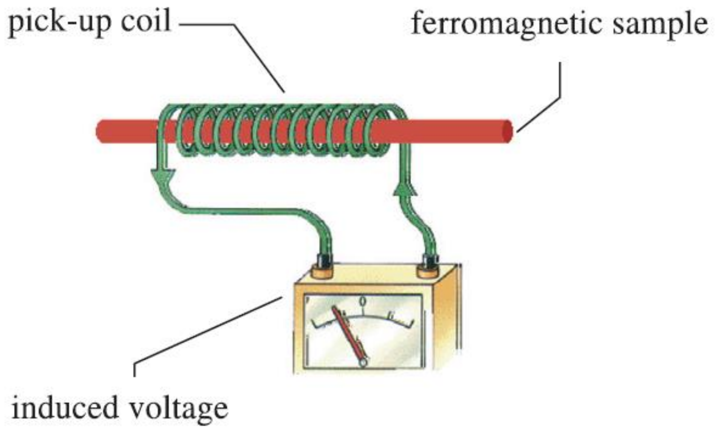
Mean field approximation $$\varsigma \frac{\partial z(x,t)}{\partial t} = J[z_{cl}(t)-z(x,t)] + k_0[z_0(t)-z_{cl}(t)] - [\gamma\cos\theta_0 + h(x,z(x,t)]$$

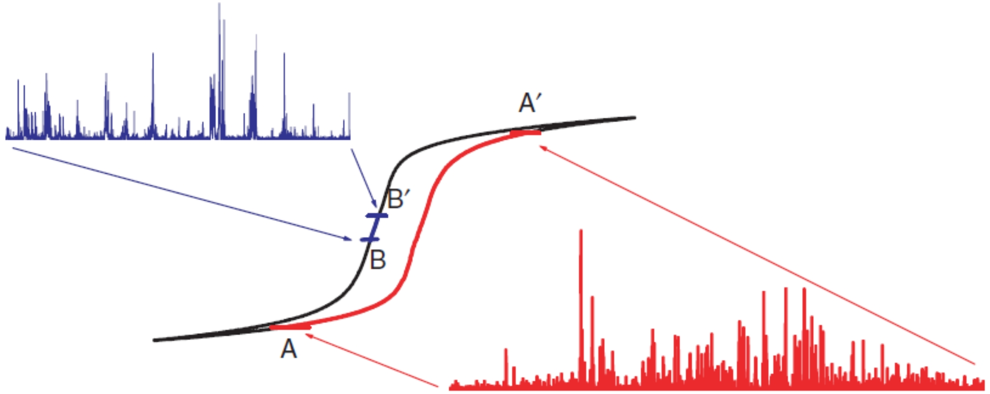
Average over x: $\varsigma \frac{\partial z_{cl}}{\partial t} = k_0[z_0(t)-z_{cl}] - [\gamma\cos\theta_0 + h_e(z_{cl})]$, h_e: Brownian correlated at lengths smaller than l_d.

The same form as the ABBM model for the Barkhausen noise in the interface dynamics of a magnetic domain.
Exponent of slip length distribution = 1.5 (Alessandro, Beatrice, Bertotti, and Montorsi, J. Appl. Phys., 68, 2901, 1990)

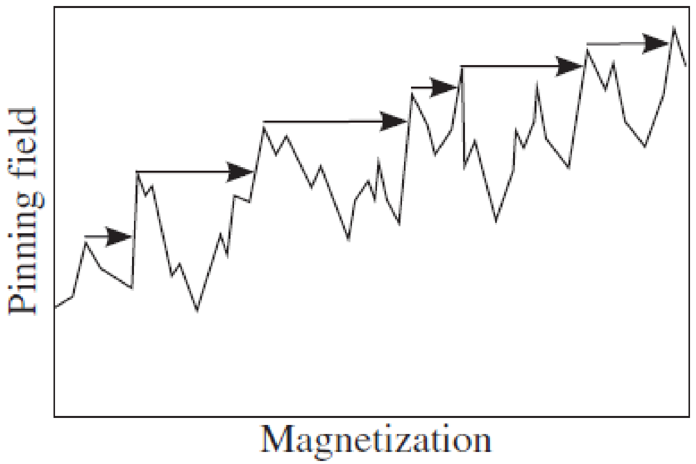
The Barkhausen noise

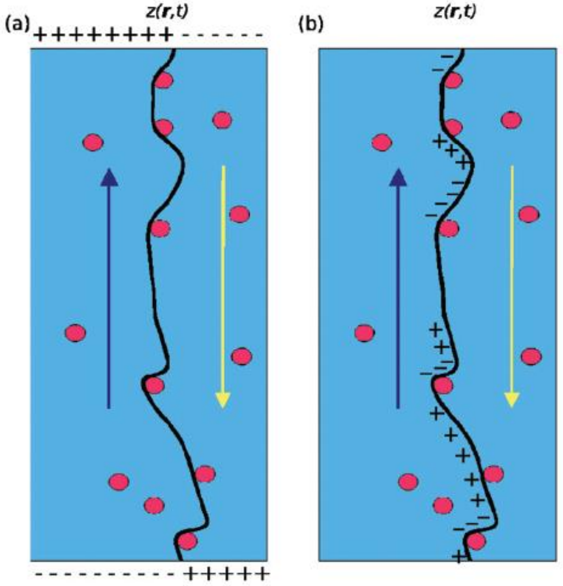
F. Colaiori, Adv. Phys., 57, 287 (2008)





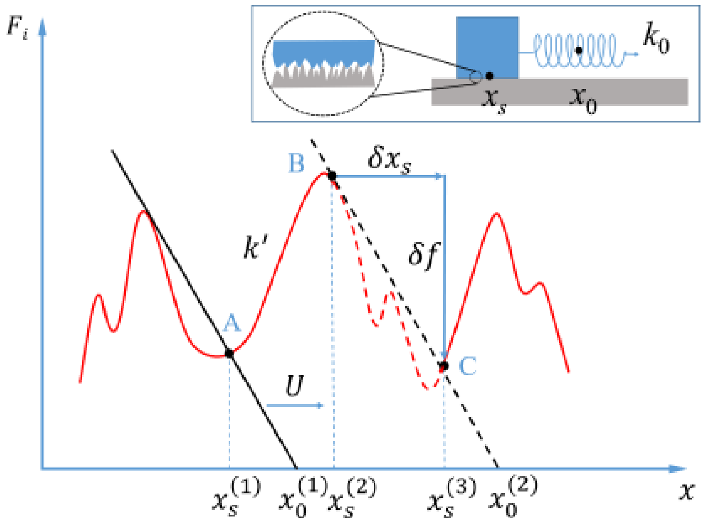
Hysteresis loop of a $Fe_{85}B_{15}$ amorphous alloy under moderate tensile stress

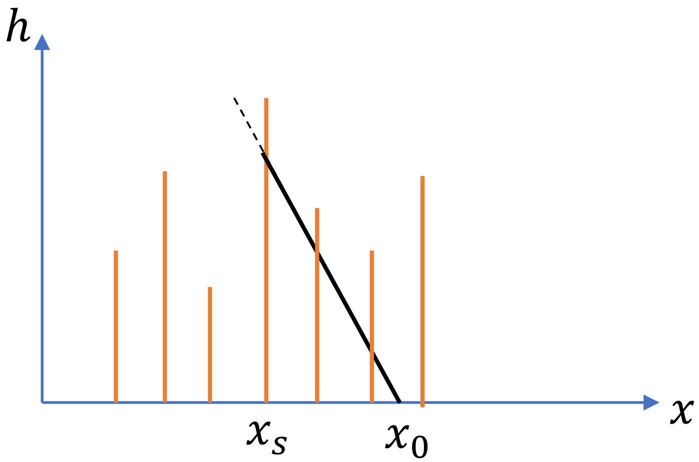




- Dynamics of magnetization $\rightarrow$ domain interface dynamics
- Amorphous material $\rightarrow$ random pinning sites
- Magnetic dipoles associated with interfaces and surfaces
- Long-range interaction $\rightarrow$ mean field approximation

Physical picture of stick-slip statistics





- $x_0^{(1)} \rightarrow x_0^{(2)}$: stick, force balance, effective spring constant detected.
- $x_0^{(2)}$: slip, avalanche size distribution depends on spatial correlation of surface heterogeneity, ABBM universality class (contact line), a different universality class due to inertia (friction). Typical slip length < l_d.
- Extreme value statistics: extracting max pinning force h, and pinning force correlation length l_d is possible.
- Spatial derivative of h,
  spatial correlation of h,
  distribution of h,
  and l_d can be obtained from exp data.

Conclusion

- Mesoscopic exp provide useful statistics of stick-slip dynamics not revealed by macro or micro exp.
- Roughness of the surfaces should be related to the statistics of max force, avalanche size, and effective elastic constant.
- Mean field theory for avalanche dynamics seems to hold for both contact line motion and solid friction. Inertia is important for solid friction.
- Future work: detailed comparison between surface landscape and stick-slip statistics.

Thank you for your attention.

Aside: generalized extreme value distribution

- Consider a set of random variables $\{x_1, x_2, \dots, x_N\}$, the distribution of x, $p(x)$, is known.
- Extreme value distribution: distribution of $M = \max(x_1, x_2, \dots, x_N) =?$

Derivation: For identically distributed independent variables,

1. Prob that M is not greater than m is $Q_N(m) = \left[1 - \int_m^\infty p(x)dx\right]^N$
2. Prob distribution of M is $f(M) = \left[\frac{dQ_N}{dm}\right]_{m=M}$
3. $f(M)$ has universal form.