Unraveling the Mpemba and Kovacs effects using the time-delayed Newton's cooling law

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Universidad de Extremadura,
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11 June, 2024

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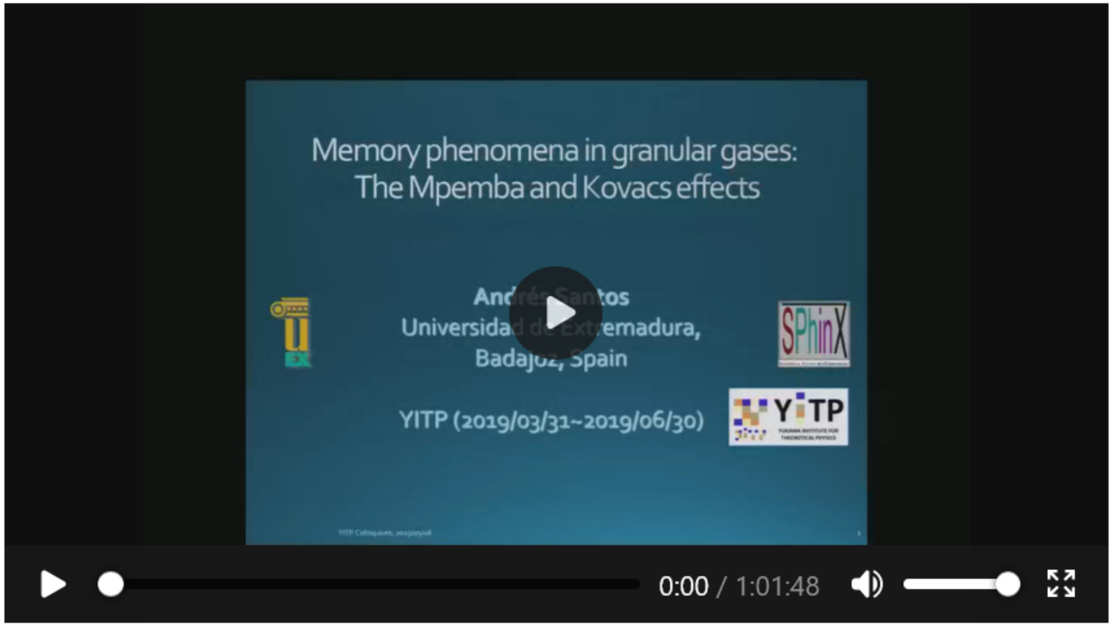
BACK

YITP Colloquium: Memory phenomena in granular gases: The Mpemba and Kovacs effects

Andres Santos (Universidad de Extremadura/ YITP, Kyoto University)

2019/05/08 15:30 --

Panasonic Auditorium, Yukawa Hall, Yukawa Institute, Kyoto U.



Outline

- The Mpemba effect
- The time-delayed Newton's cooling law
- Application to the Mpemba effect
- Application to the Kovacs effect
- Conclusions

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What is the Mpemba effect?

"Hot water <u>can</u> freeze faster than cold water"

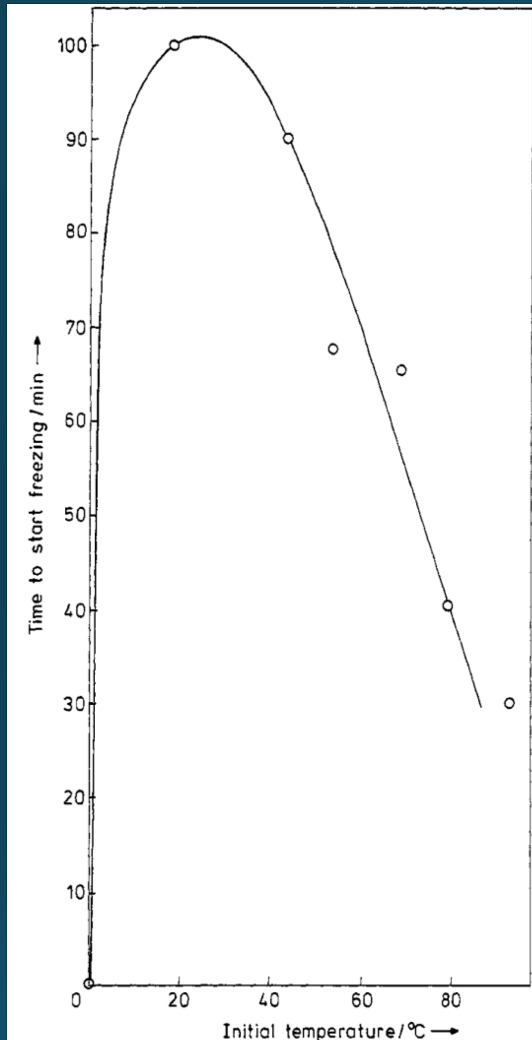
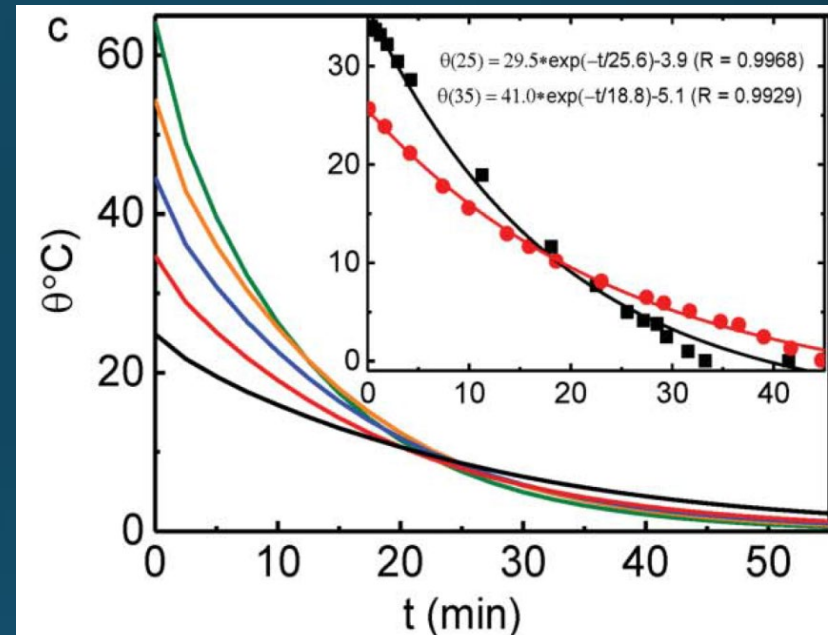


Figure 1 Plot of time for water to start freezing against initial temperature of water

E. B. Mpemba and D. G. Osborne, *Cool?*, Phys. Educ. **4**, 172 (1969)



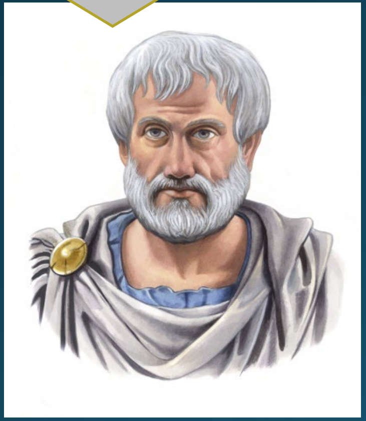
C. Q. Sun, *Behind the Mpemba paradox*, Temperature **2**, 38 (2015)

Erasto B. Mpemba and Denis G. Osborne in London (2013).

Erasto B. Mpemba talking at the TEDxDar event (Dar es Salaam, November 2011)

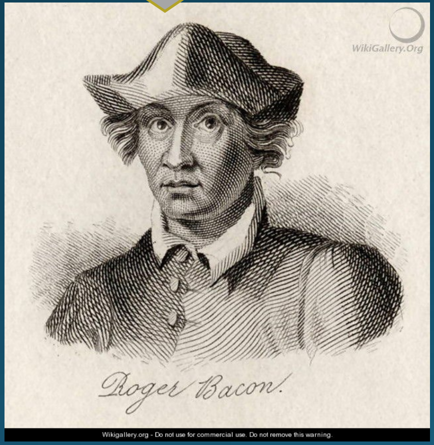
The problem had been around for millennia, with philosophers such as Aristotle, R. Bacon, G. Marliani, F. Bacon, and Descartes pondering over it.

The fact that the water has previously been warmed contributes to its freezing quickly; for so it cools sooner



Aristotle
(384–322 BC)

If cold water and hot water are poured on a cold place, as upon ice, the hot water freezes more quickly



Roger Bacon
(1214–1294)

Water slightly warm is more easily frozen than quite cold

Francis Bacon
(1561–1626)

Experience shows that water which has been kept for a long time on the fire freezes sooner than other water

René Descartes
(1596–1650)

- Scientists have suggested a number of theories (evaporation, dissolved gases, convection, supercooling, bonding of water molecules, …).
- No full consensus on whether or not the effect might be an ***artifact*** of the experimental procedures.

Questioning the Mpemba effect: hot water does not cool more quickly than cold

Henry C. Burridge[1,2] & Paul F. Linden[1]

The Mpemba effect is the name given to the assertion that it is quicker to cool water to a given temperature when the initial temperature is higher. This assertion seems counter-intuitive and yet references to the effect go back at least to the writings of Aristotle. Indeed, at first thought one might consider the effect to breach fundamental thermodynamic laws, but we show that this is not the case. We go on to examine the available evidence for the Mpemba effect and carry out our own experiments by cooling water in carefully controlled conditions. We conclude, somewhat sadly, that there is no evidence to support meaningful observations of the Mpemba effect.

Sci. Rep. **6**, 37665 (2016)

Mpemba-like effects observed in different types of systems

- Granular gases
- Inertial suspensions
- Spin glasses
- Carbon nanotube resonators
- Clathrate hydrates
- Markovian models
- Active systems
- Ising models
- Non-Markovian mean-field systems
- Colloids (active and passive)
- Polymer crystallization
- Quantum systems
- …

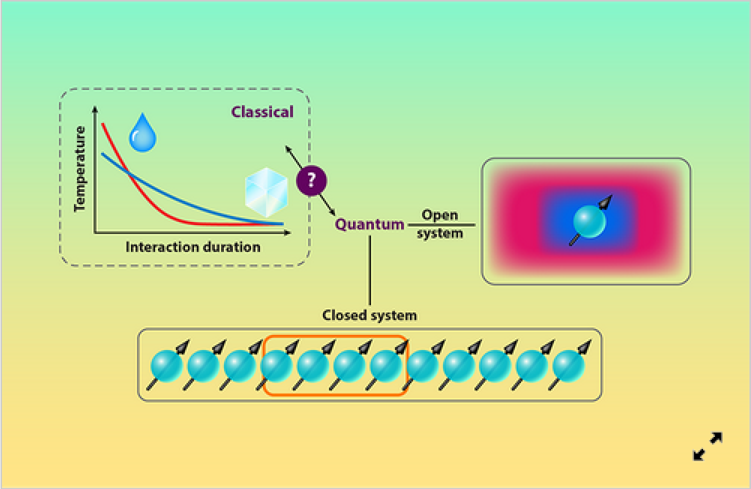
Exploring Quantum Mpemba Effects

Ulrich Warring

Institute of Physics, University of Freiburg, Freiburg, Germany

July 1, 2024 • *Physics* 17, 105

In the Mpemba effect, a warm liquid freezes faster than a cold one. Three studies investigate quantum versions of this effect, challenging our understanding of quantum thermodynamics.



APS/Carin Cain

Figure 1: (Top left) Under specific conditions, hot water (red curve) can freeze faster than cold water (blue curve) when interacting with an external environment. This classical phenomenon is known as the Mpemba effect. (Right) Aharony Shapira and colleagues studied an inverse quantum Mpemba-like effect in an open quantum system, consisting of a single cold trapped ion, interacting with a warm external environment [3]. (Bottom) Joshi and colleagues studied a quantum Mpemba-like effect in subsystems of a closed quantum system, consisting of 12 interacting trapped ions [4]. Lastly, Rylands and colleagues theoretically studied the microscopic mechanisms driving quantum Mpemba-like effects in closed quantum systems [5]. A remaining question is how to establish a link between these classical and quantum phenomena.

Under certain conditions, warm water can freeze faster than cold water. This phenomenon was named the Mpemba effect after Erasto Mpemba, a Tanzanian high schooler who described the effect in the 1960s [1]. The phenomenon

Observing the Quantum Mpemba Effect in Quantum Simulations

Lata Kh. Joshi, Johannes Franke, Aniket Rath, Filiberto Ares, Sara Murciano, Florian Kranzl, Rainer Blatt, Peter Zoller, Benoît Vermersch, Pasquale Calabrese, Christian F. Roos, and Manoj K. Joshi

Phys. Rev. Lett. 133, 010402 (2024)

Published July 1, 2024

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Inverse Mpemba Effect Demonstrated on a Single Trapped Ion Qubit

Shahaf Aharony Shapira, Yotam Shapira, Jovan Markov, Gianluca Teza, Nitzan Akerman, Oren Raz, and Roee Ozeri

Phys. Rev. Lett. 133, 010403 (2024)

Published July 1, 2024

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Microscopic Origin of the Quantum Mpemba Effect in Integrable Systems

Colin Rylands, Katja Klobas, Filiberto Ares, Pasquale Calabrese, Sara Murciano, and Bruno Bertini

Phys. Rev. Lett. 133, 010401 (2024)

Published July 1, 2024

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Exploring Quantum Mpemba Effects

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July 1, 2024 • *Physics* 17, 105

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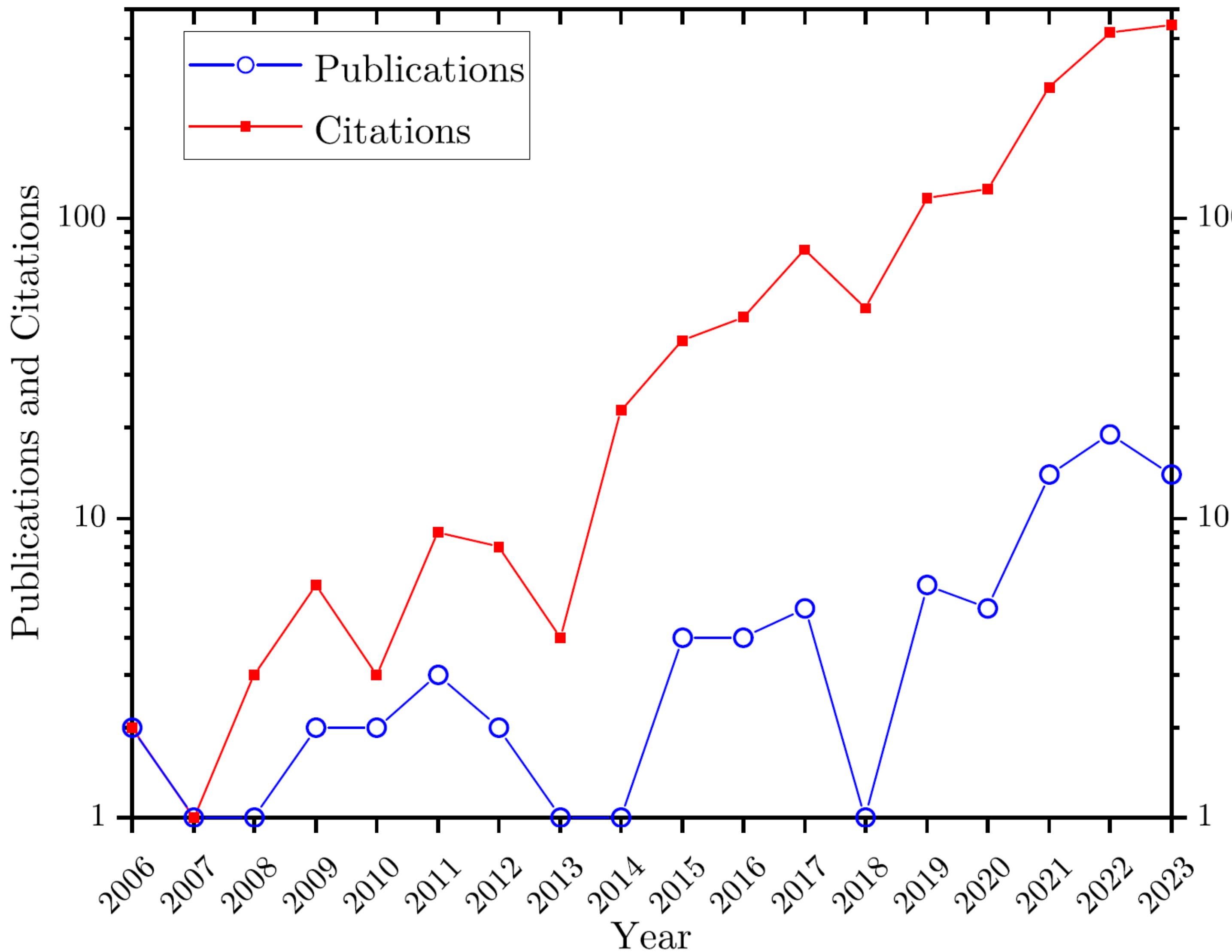

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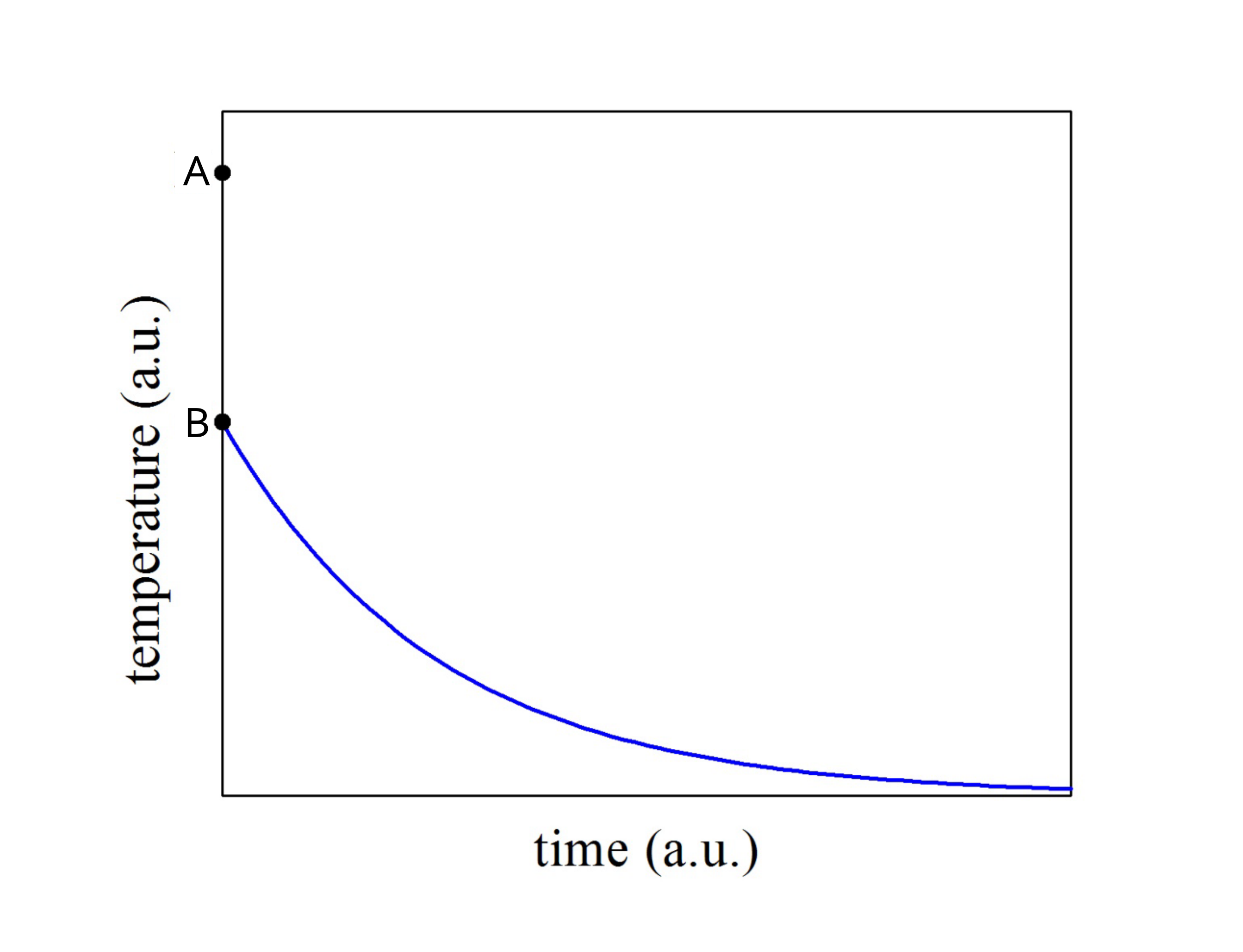
effect ...

Why is the effect paradoxical?



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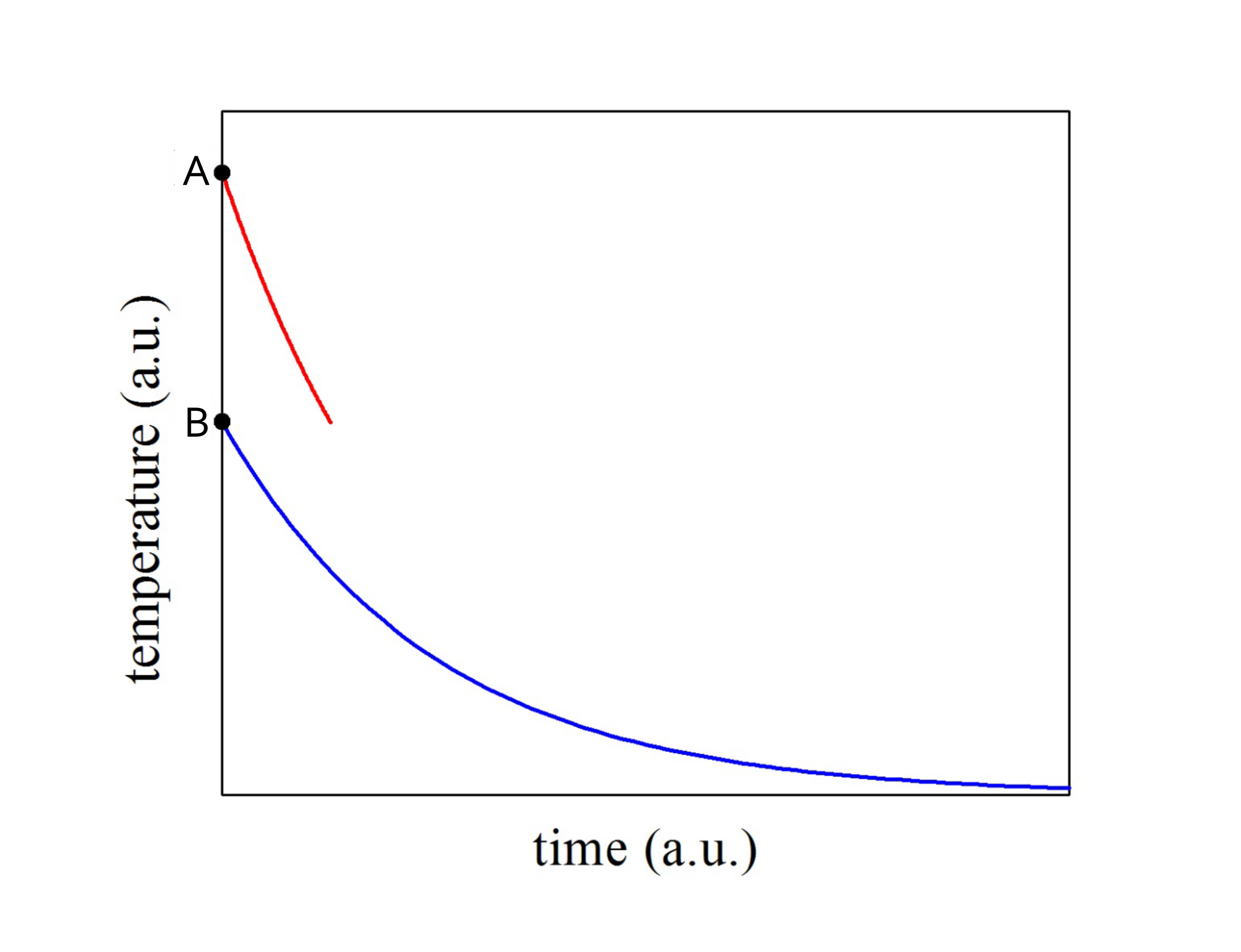

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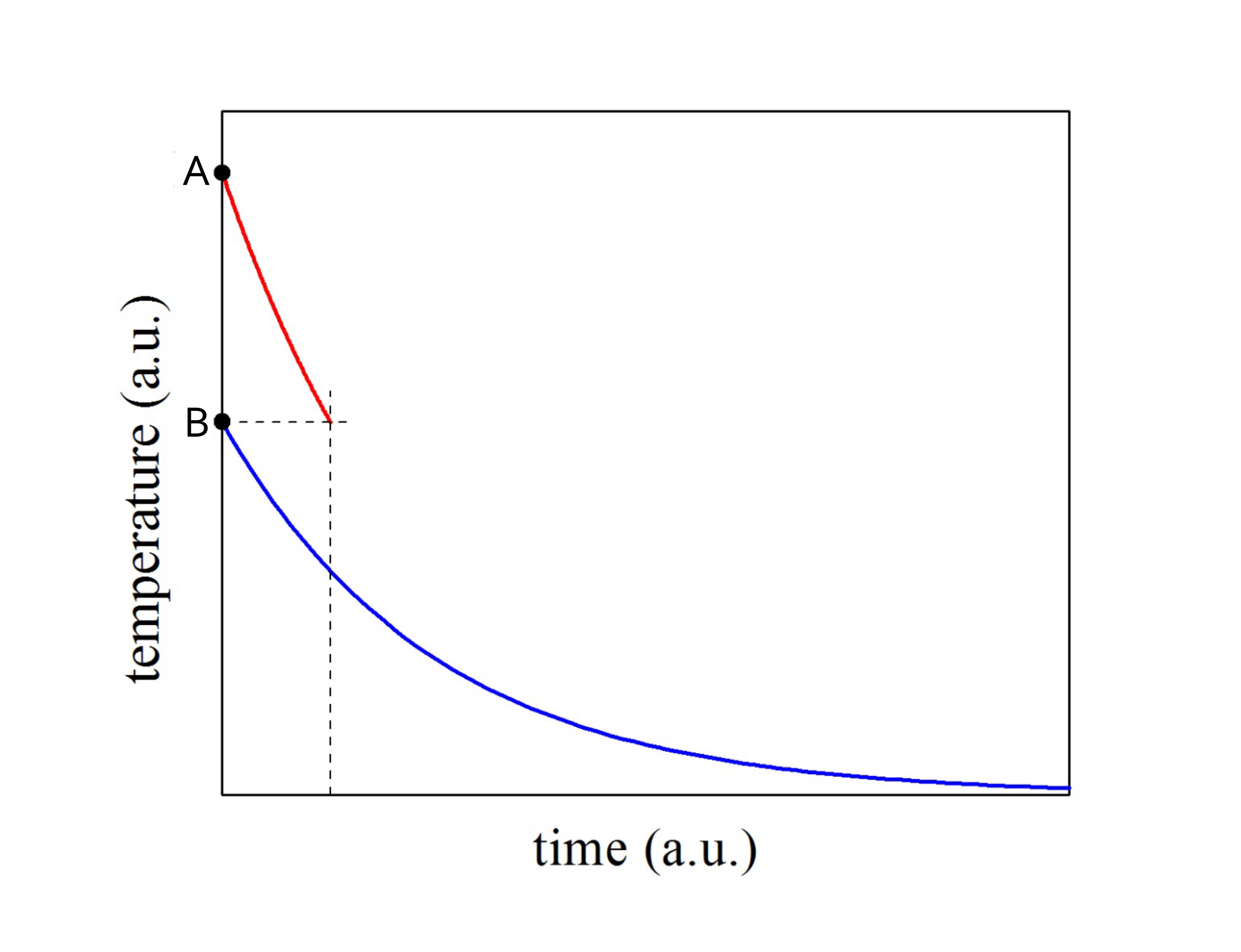

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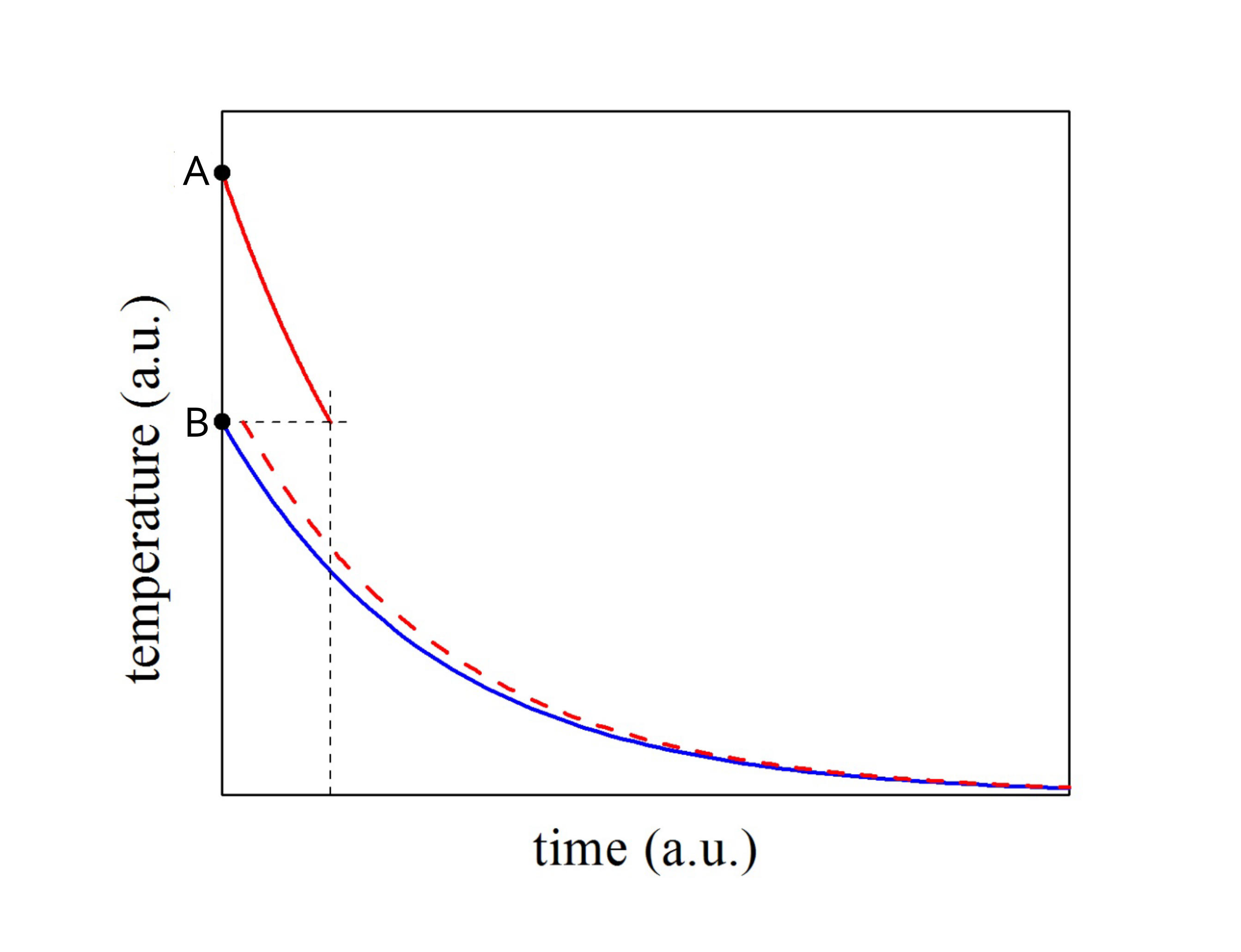

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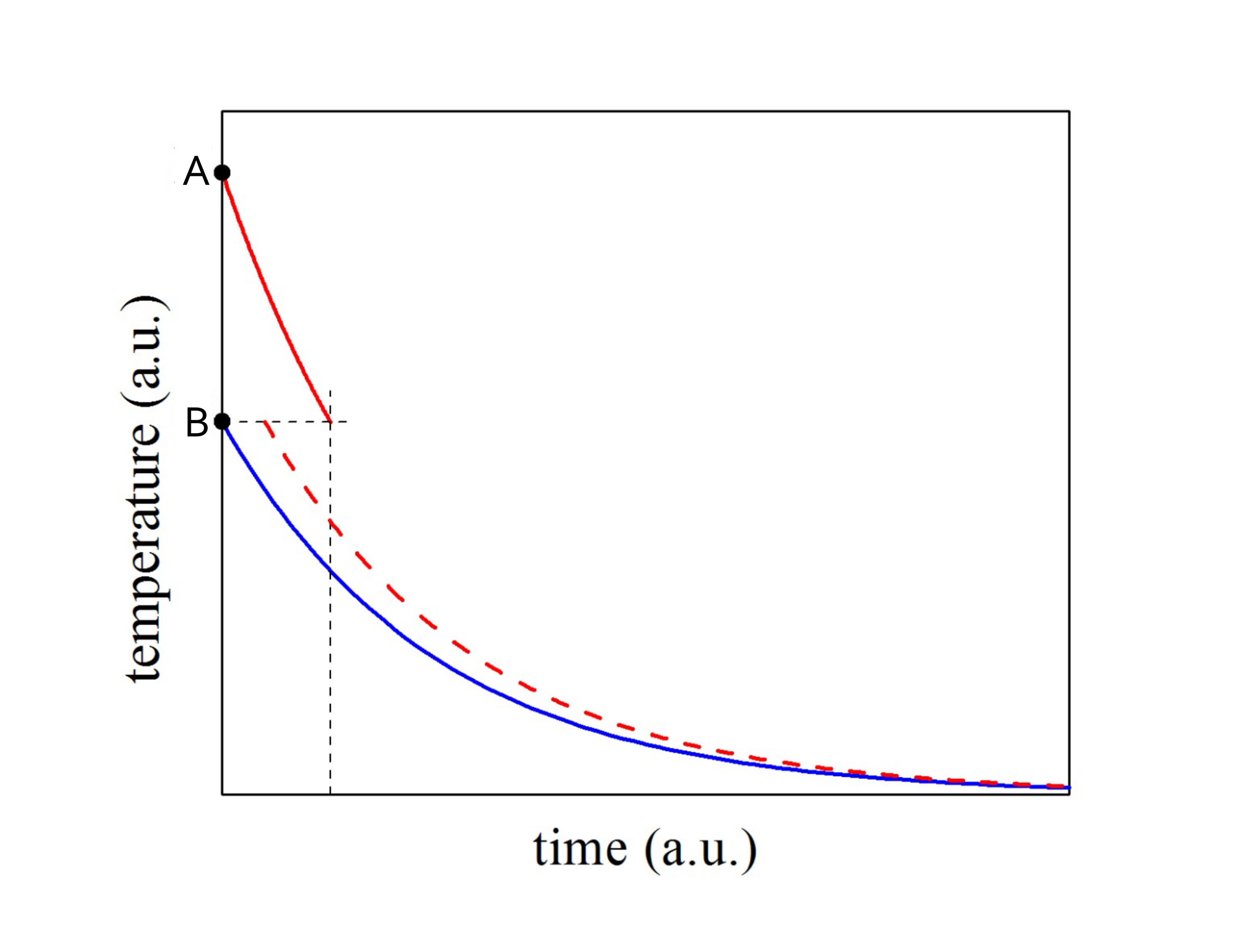

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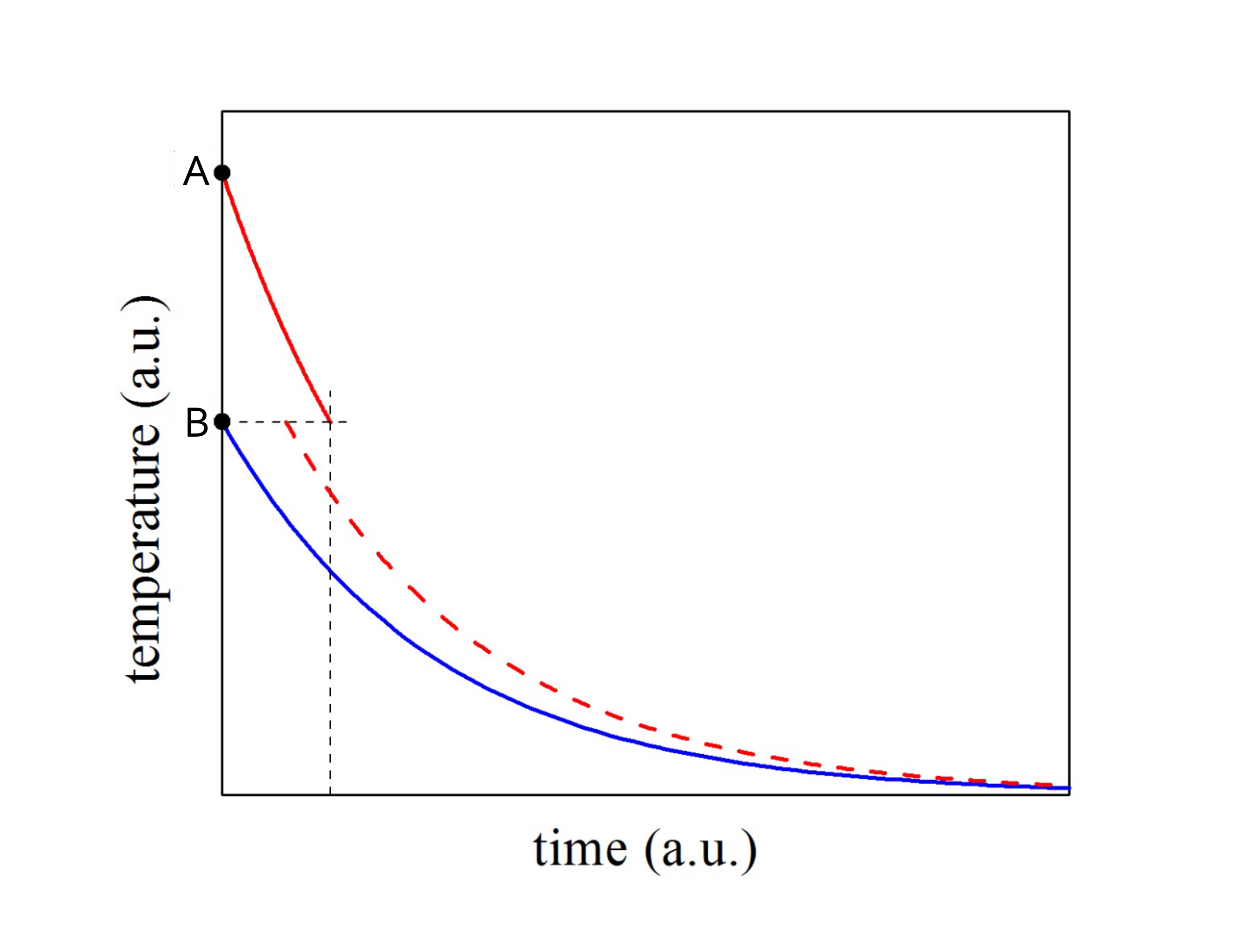

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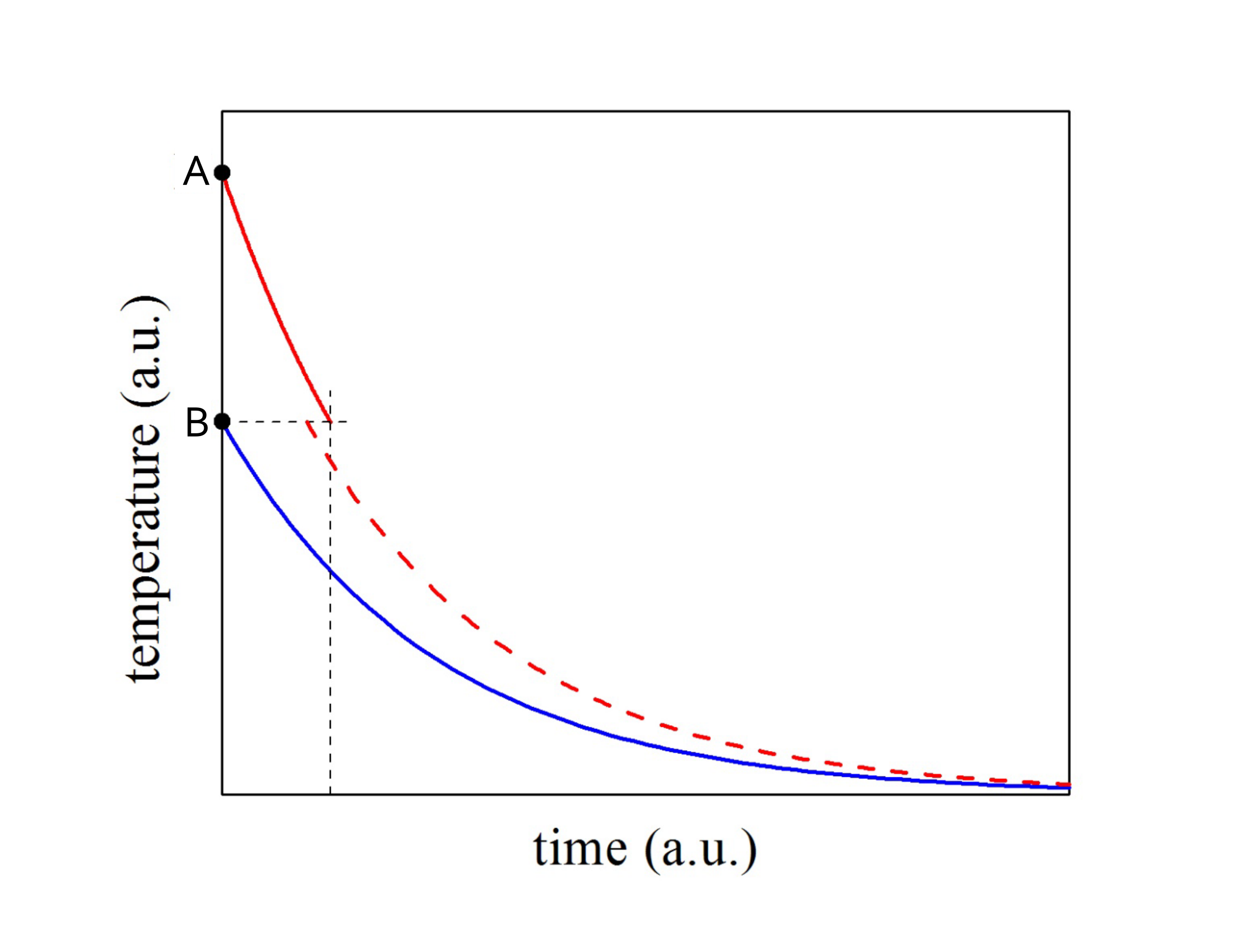

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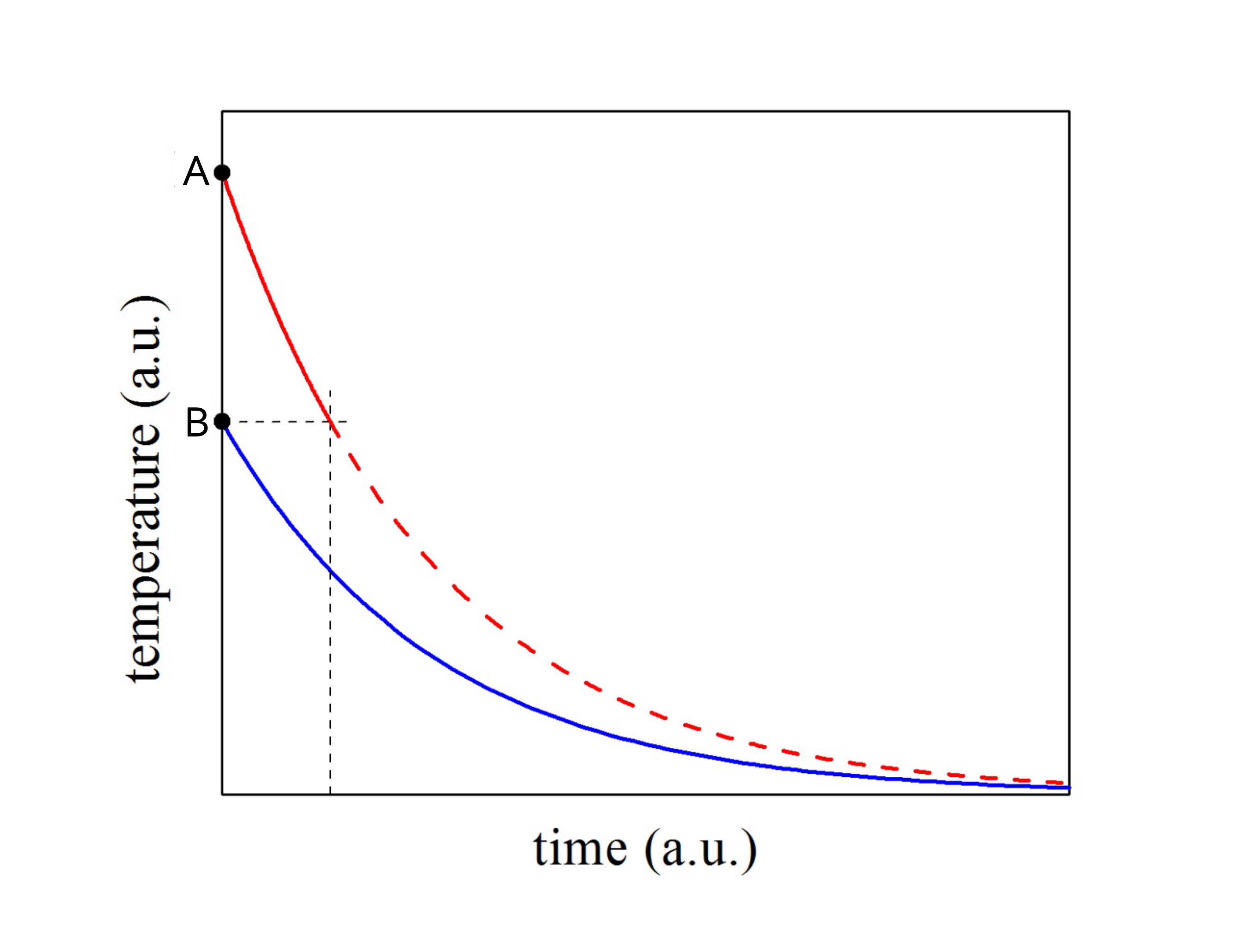

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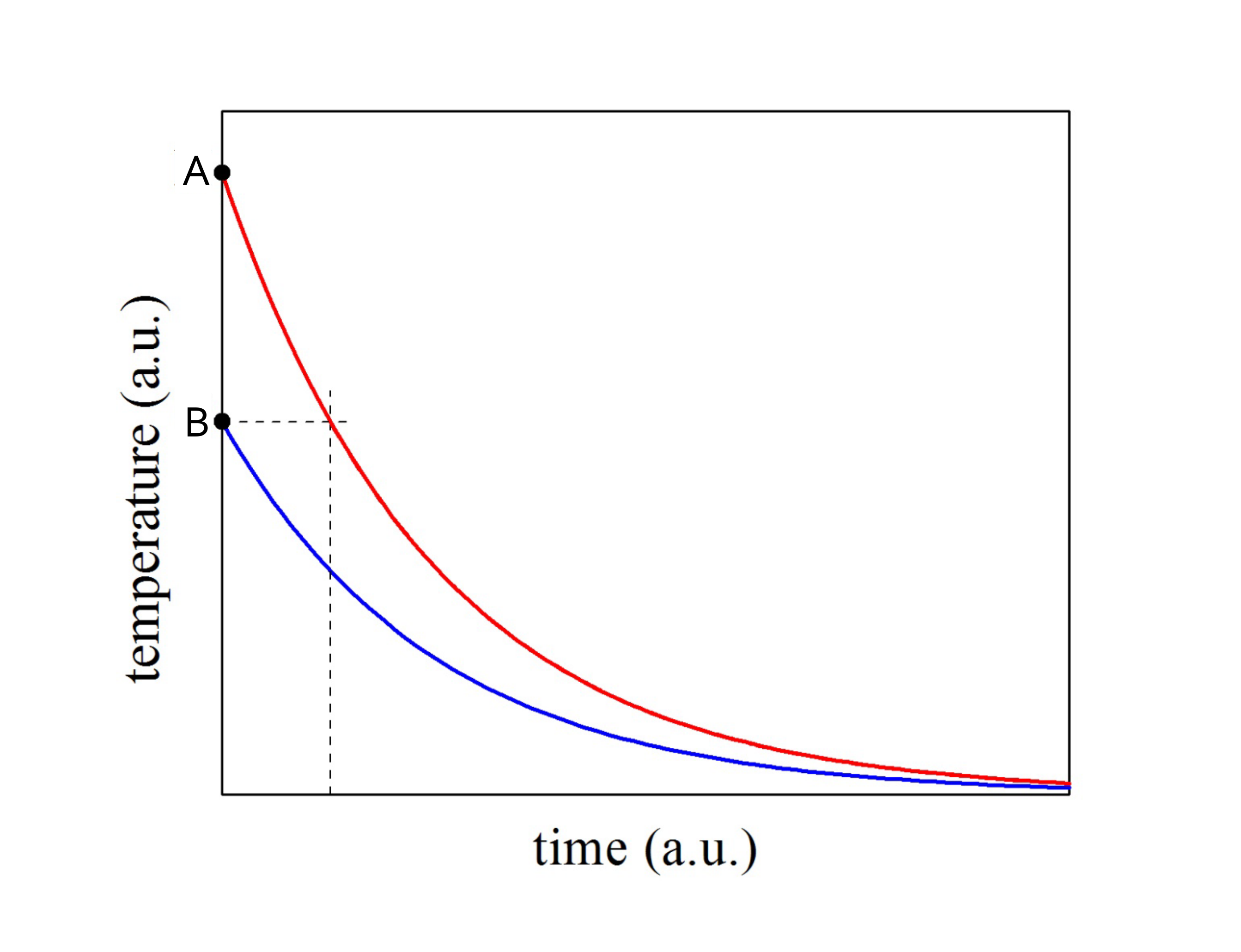

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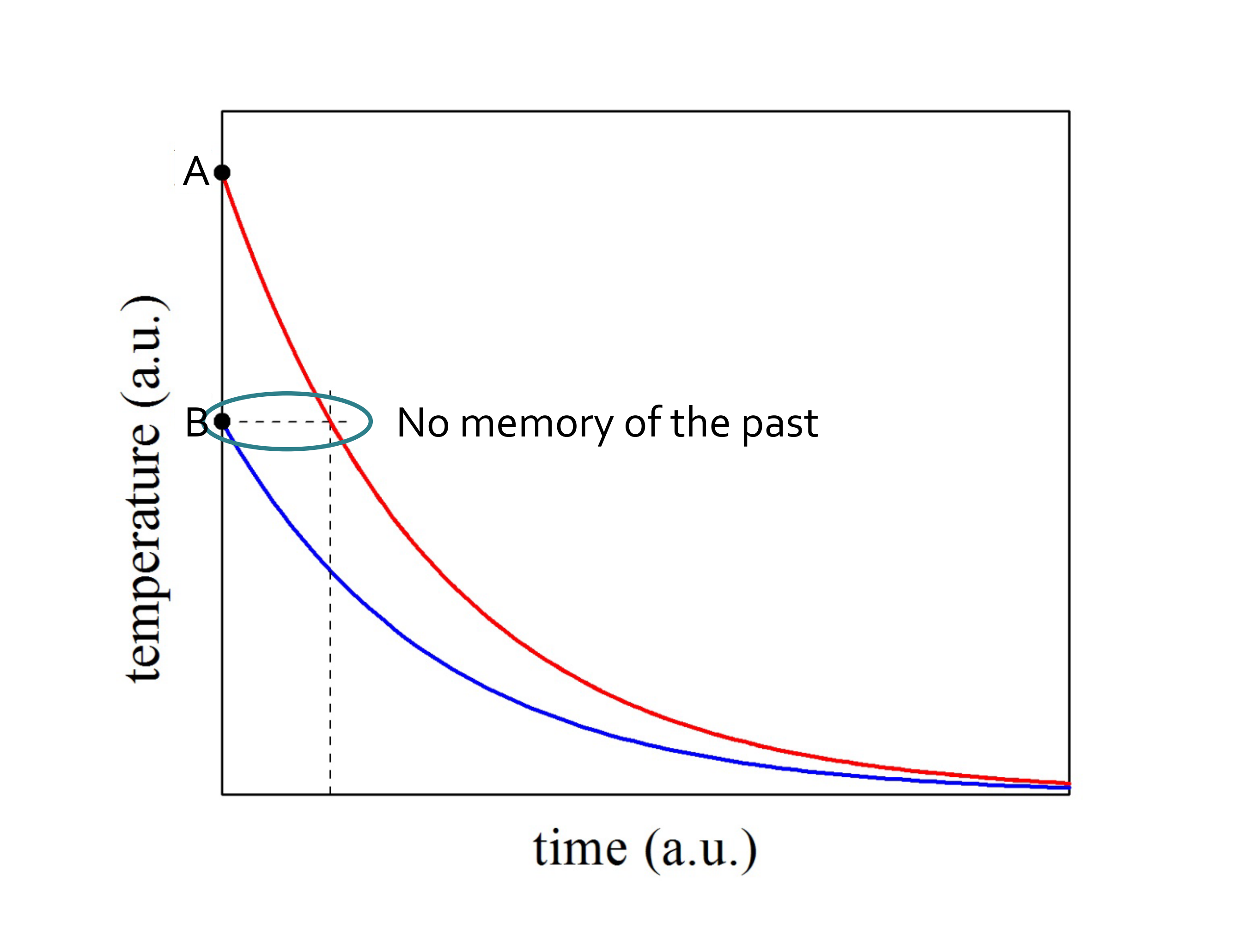

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What if a delay time is introduced?
(Simplest?) phenomenological model

Newton's law of cooling: $\dot{T}(t) = -\lambda[T(t) - T_\mathrm{b}]$

$$\Rightarrow T(t) = T_\mathrm{b} + e^{-\lambda t}(T_0 - T_\mathrm{b})$$

$$\Rightarrow \text{NO Mpemba effect}$$

A.S., Phys. Rev. E **109**, 044149 (2024)

$$\dot{T}(t) = -\lambda[T(t-\tau) - T_\mathrm{b}(t)]$$

τ : delay time (memory effects)

$(\lambda^{-1} = 1$: unit of time$)$

A brief digression: The time-delayed equation is equivalent to an infinite set of linear equations

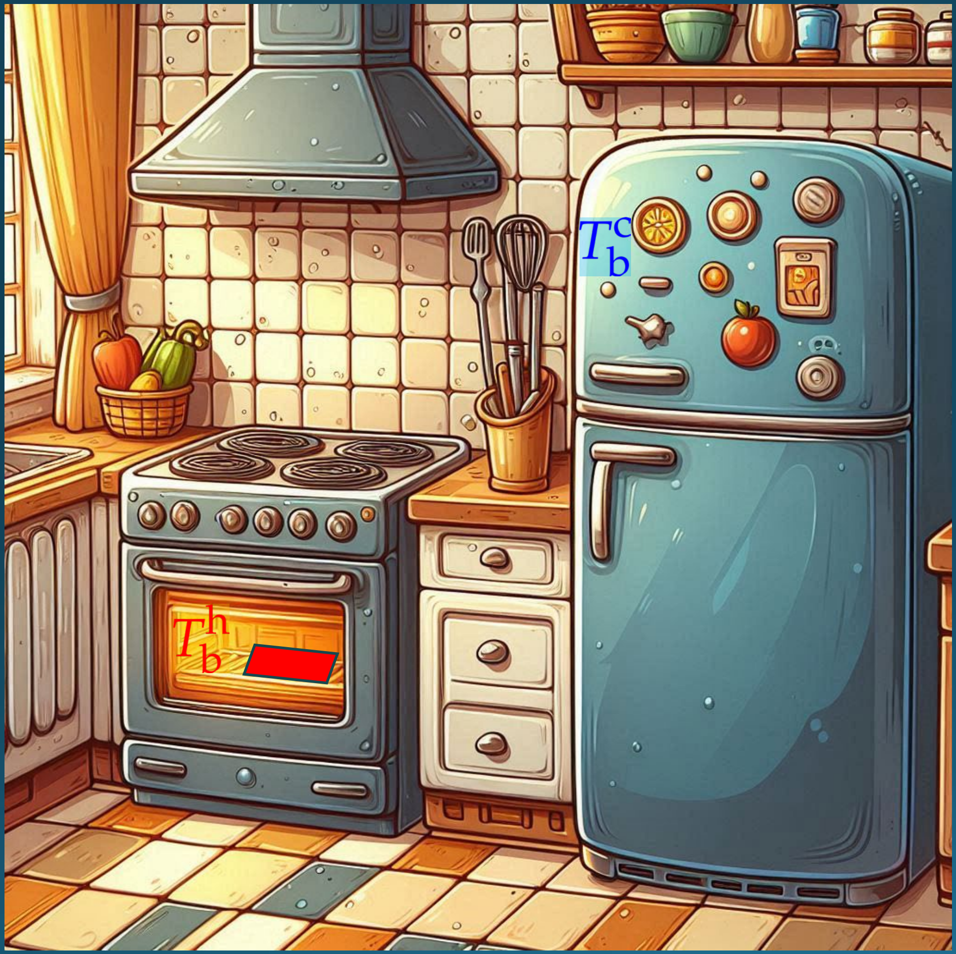
$$\boxed{\dot{T}(t) = -[T(t-\tau) - T_{\mathrm{b}}]}$$

Expand $T(t-\tau)$ in powers of τ and truncate at order n :

$$-(1-\tau)\dot{T}(t) = T(t) - T_{\mathrm{b}}(t) + \sum_{k=2}^{n} \frac{(-\tau)^k}{k!} T^{(k)}(t)$$

$$T^{(k)}(t) \equiv \left(\frac{d}{dt}\right)^k T(t)$$

Single-quench protocol



$$T_\mathrm{b}(t) = \begin{cases} T_\mathrm{b}^\mathrm{h}, & t < 0, \\ T_\mathrm{b}^\mathrm{c}, & t > 0. \end{cases}$$

Single-quench protocol

Laplace space:

$$\widetilde{T}(s) = T_{\mathrm{b}}^{\mathrm{c}} s^{-1} + \left(T_{\mathrm{b}}^{\mathrm{h}} - T_{\mathrm{b}}^{\mathrm{c}}\right) \widetilde{\mathcal{E}}(s), \quad \widetilde{\mathcal{E}}(s) \equiv s^{-1} - \frac{s^{-2}}{1 + s^{-1} e^{-s\tau}}$$

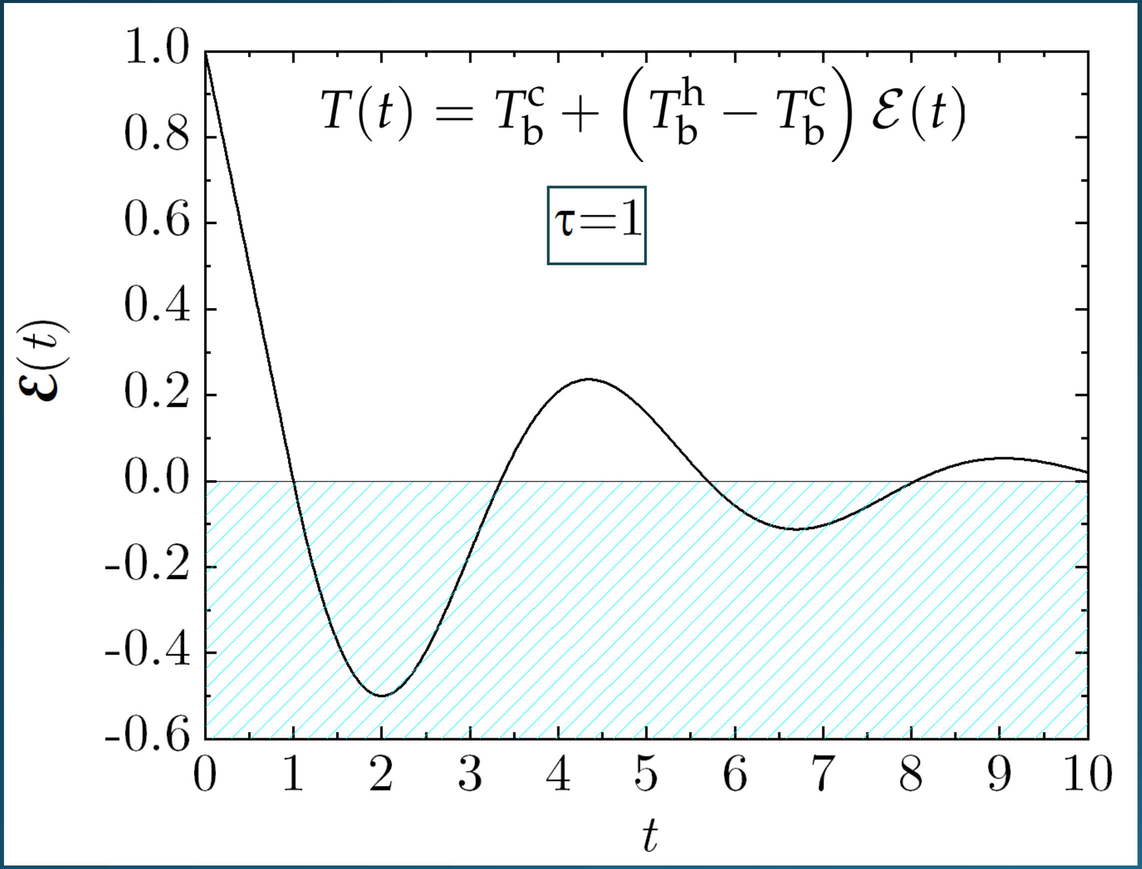
Real time:

$$T(t) = T_{\mathrm{b}}^{\mathrm{c}} + \left(T_{\mathrm{b}}^{\mathrm{h}} - T_{\mathrm{b}}^{\mathrm{c}}\right) \mathcal{E}(t), \quad \mathcal{E}(t) = 1 + \sum_{n=0}^{\lfloor t/\tau \rfloor} \frac{(n\tau - t)^{n+1}}{(n+1)!}$$

Quasi-exponential function:

$$\mathcal{E}(t) = \begin{cases} 1 - t, & 0 \le t \le \tau, \\ 1 - t + \frac{(\tau - t)^2}{2}, & \tau \le t \le 2\tau, \\ 1 - t + \frac{(\tau - t)^2}{2} + \frac{(2\tau - t)^3}{3!}, & 2\tau \le t \le 3\tau, \\ \ldots, & \ldots \end{cases}$$

Can the delay time take any value?



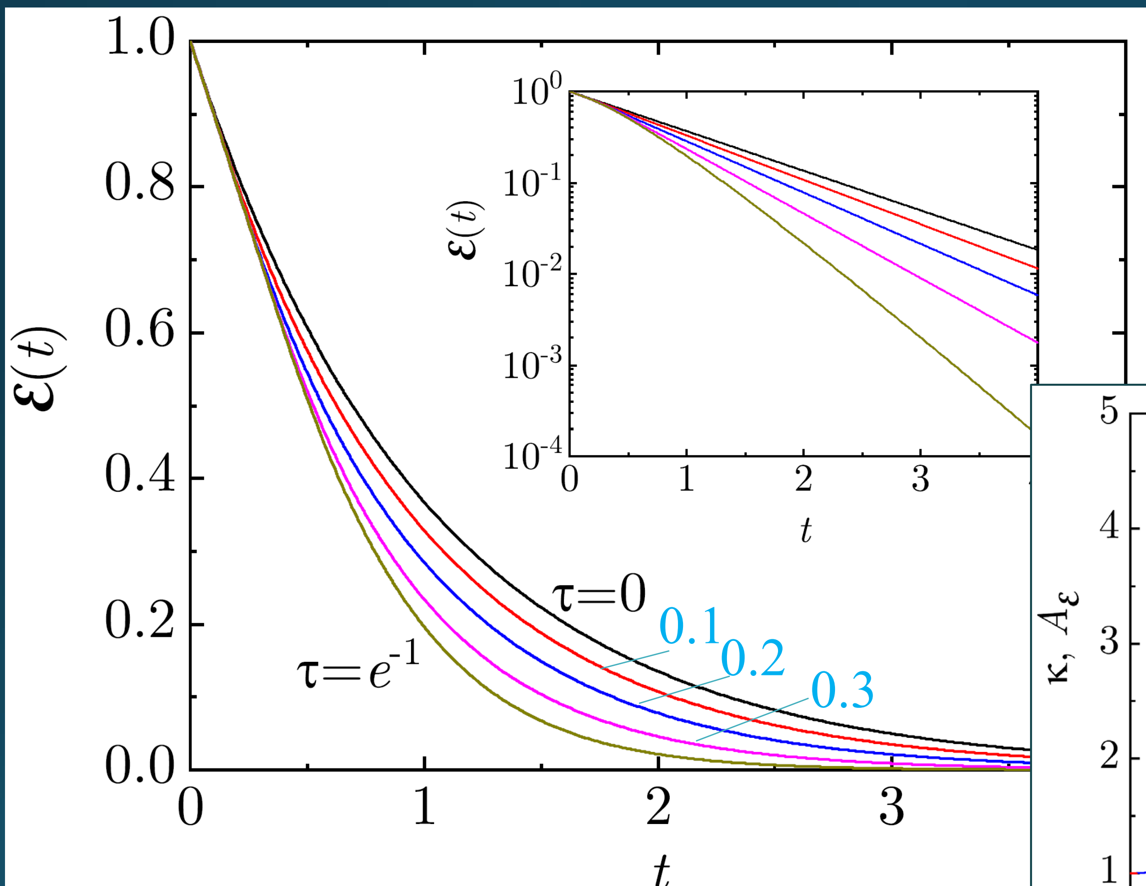
$T(t) > 0 \quad \forall t$

$\Rightarrow \mathcal{E}(t) > 0 \quad \forall t$

$\Rightarrow$ Dominant pole of $\widetilde{\mathcal{E}}(s)$ must be *real*

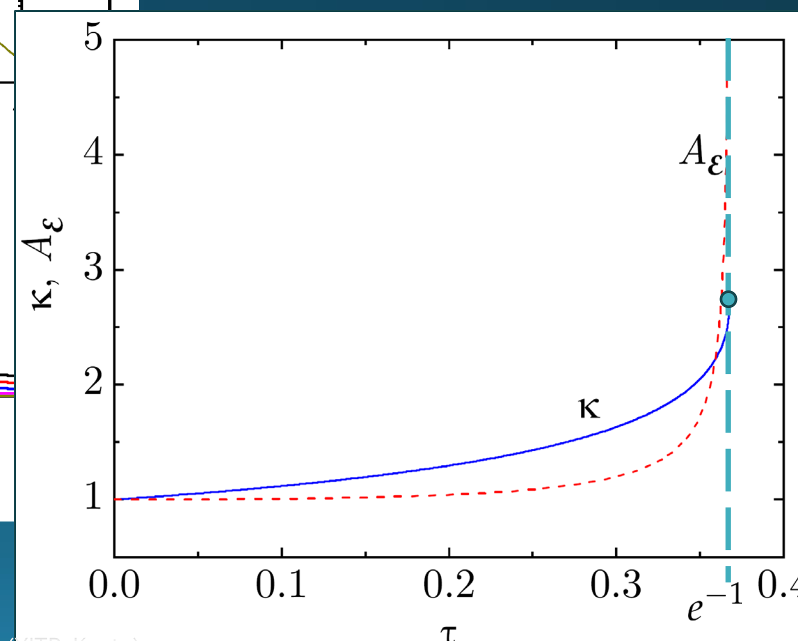
$\Rightarrow \tau \le \tau_{\max} = e^{-1} \simeq 0.368$

Can the delay time take any value?



$$\mathcal{E}(t \gg 1) \approx A_{\mathcal{E}} e^{-\kappa t},$$

$$\begin{cases} \kappa = -\tau^{-1} W_0(-\tau), \\ A_{\mathcal{E}} = \dfrac{\kappa^{-1}}{1-\kappa\tau}. \end{cases}$$



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Single-quench protocol.

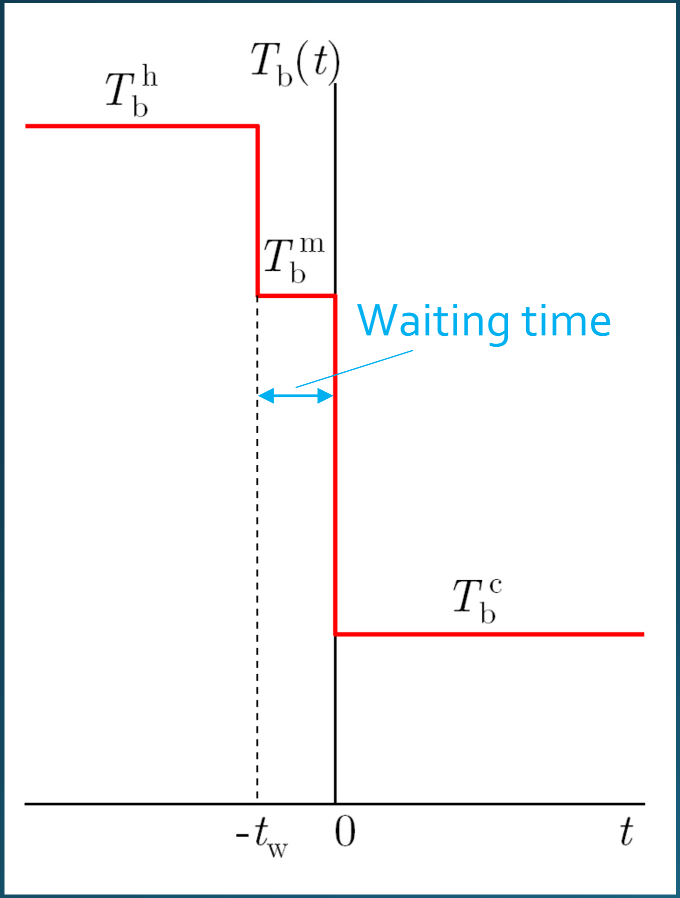
Two samples preheated at different temperatures $T^{\mathrm{h}}_{\mathrm{b,A}}, T^{\mathrm{h}}_{\mathrm{b,B}}$

$$T(t) = T^{\mathrm{c}}_{\mathrm{b}} + \left(T^{\mathrm{h}}_{\mathrm{b}} - T^{\mathrm{c}}_{\mathrm{b}}\right)\mathcal{E}(t)$$

$$\Rightarrow \boxed{T_{\mathrm{A}}(t) - T_{\mathrm{B}}(t) = (T^{\mathrm{h}}_{\mathrm{b,A}} - T^{\mathrm{h}}_{\mathrm{b,B}})\mathcal{E}(t)}$$

No Mpemba effect is posible with a *single*-quench protocol

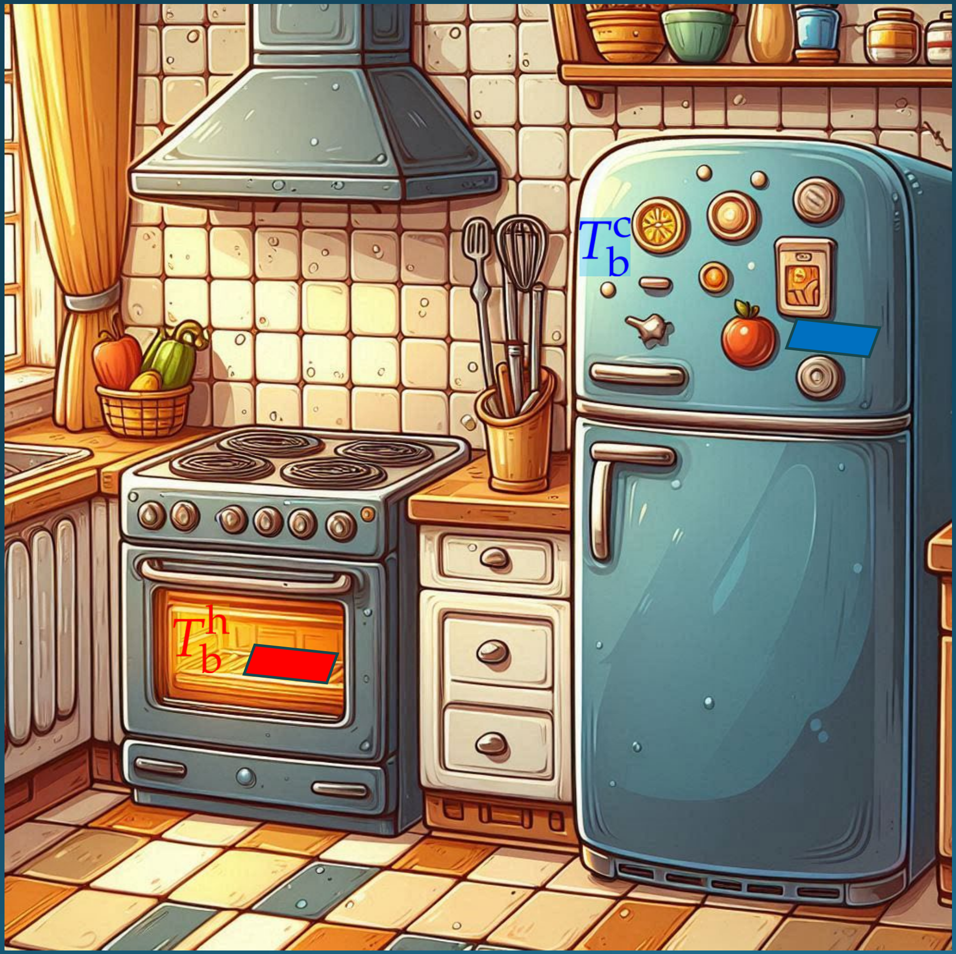
Double-quench protocol

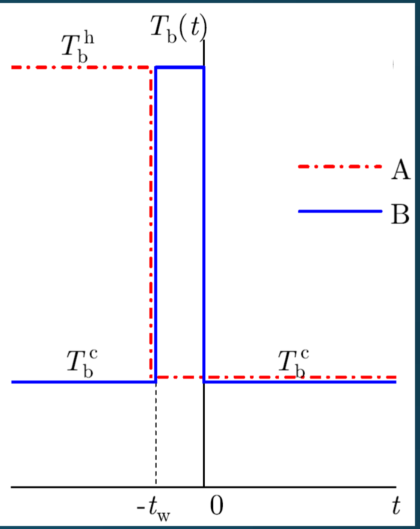


$$T_\mathrm{b}(t) = \begin{cases} T_\mathrm{b}^\mathrm{h}, & t < -t_\mathrm{w}, \\ T_\mathrm{b}^\mathrm{m}, & -t_\mathrm{w} < t < 0, \\ T_\mathrm{b}^\mathrm{c}, & t > 0. \end{cases}$$

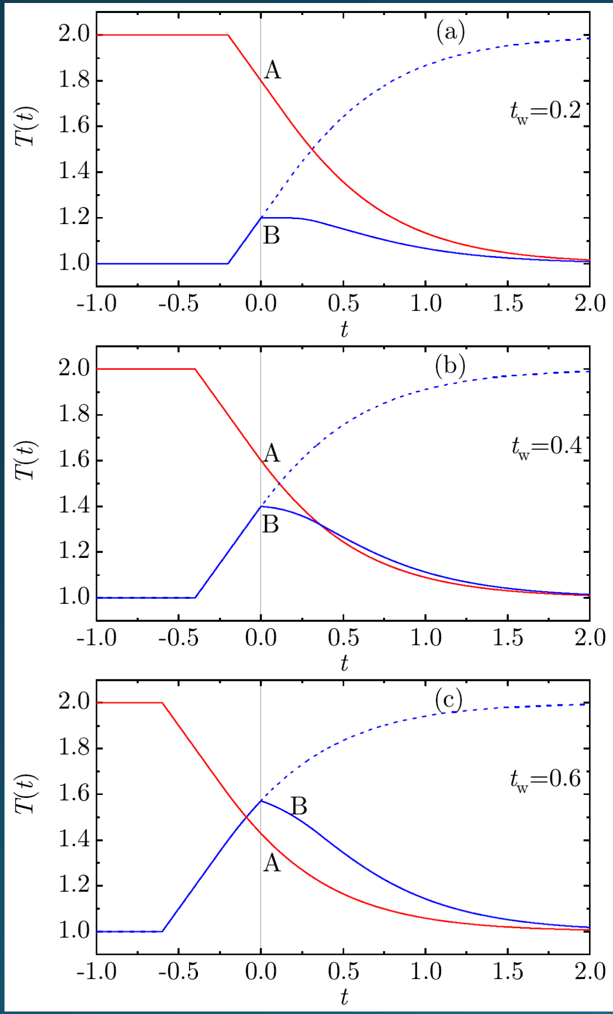
$$T(t) = T_\mathrm{b}^\mathrm{c} + (T_\mathrm{b}^\mathrm{h} - T_\mathrm{b}^\mathrm{m})\mathcal{E}(t + t_\mathrm{w}) + (T_\mathrm{b}^\mathrm{m} - T_\mathrm{b}^\mathrm{c})\mathcal{E}(t)$$

Protocol for the Mpemba effect





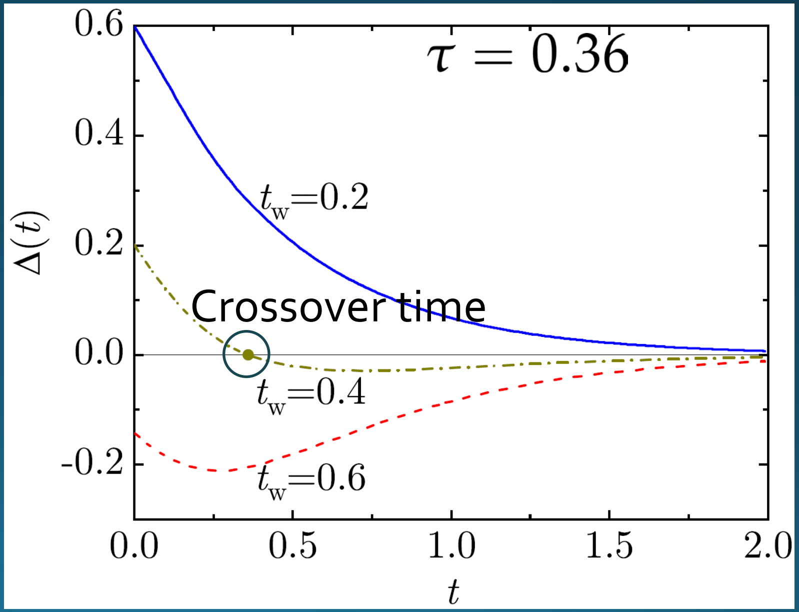
The influence of the waiting time



$\tau = 0.36$

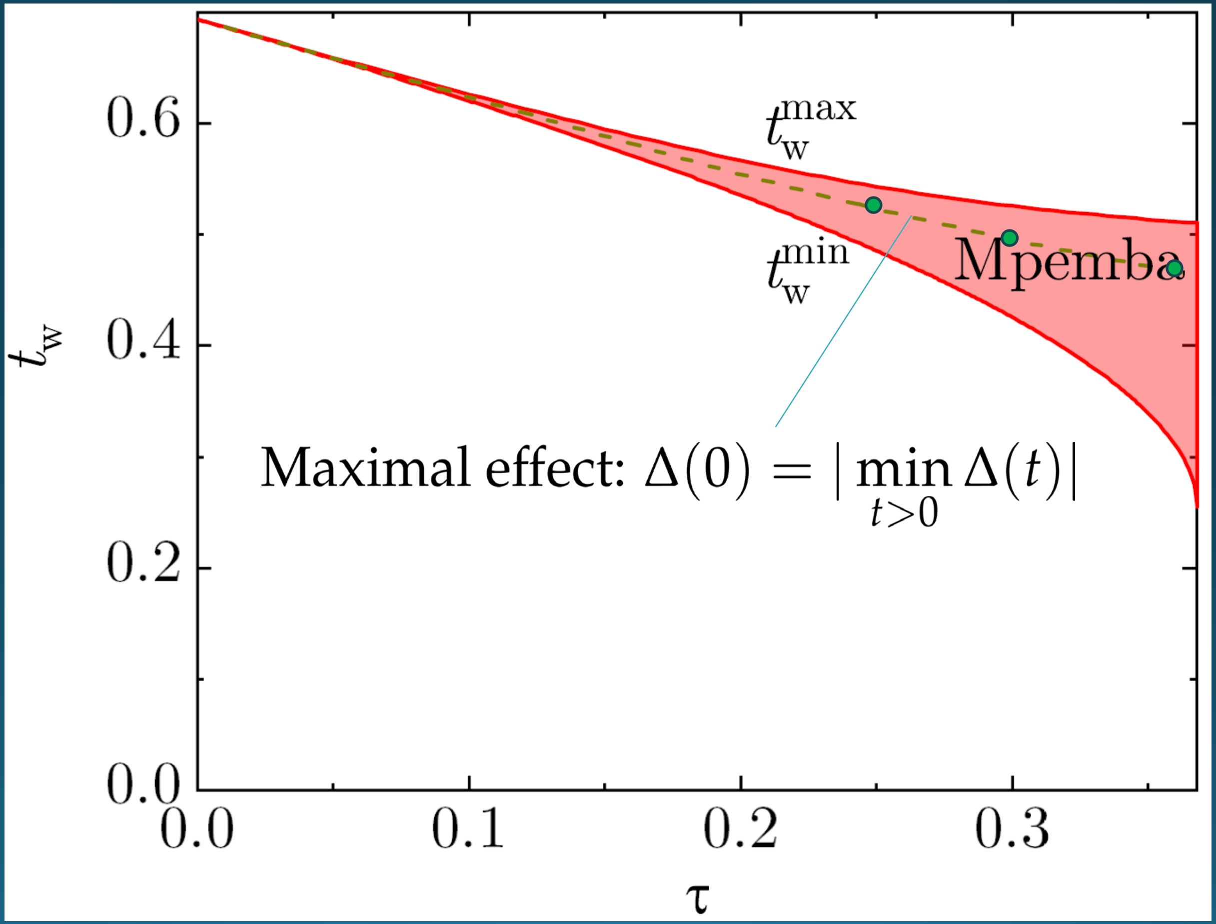
$$T_A(t) - T_B(t) = (T_b^h - T_b^c)\Delta(t), \quad t \geq 0$$

$\Delta(t) = 2\mathcal{E}(t + t_w) - \mathcal{E}(t)$: difference function

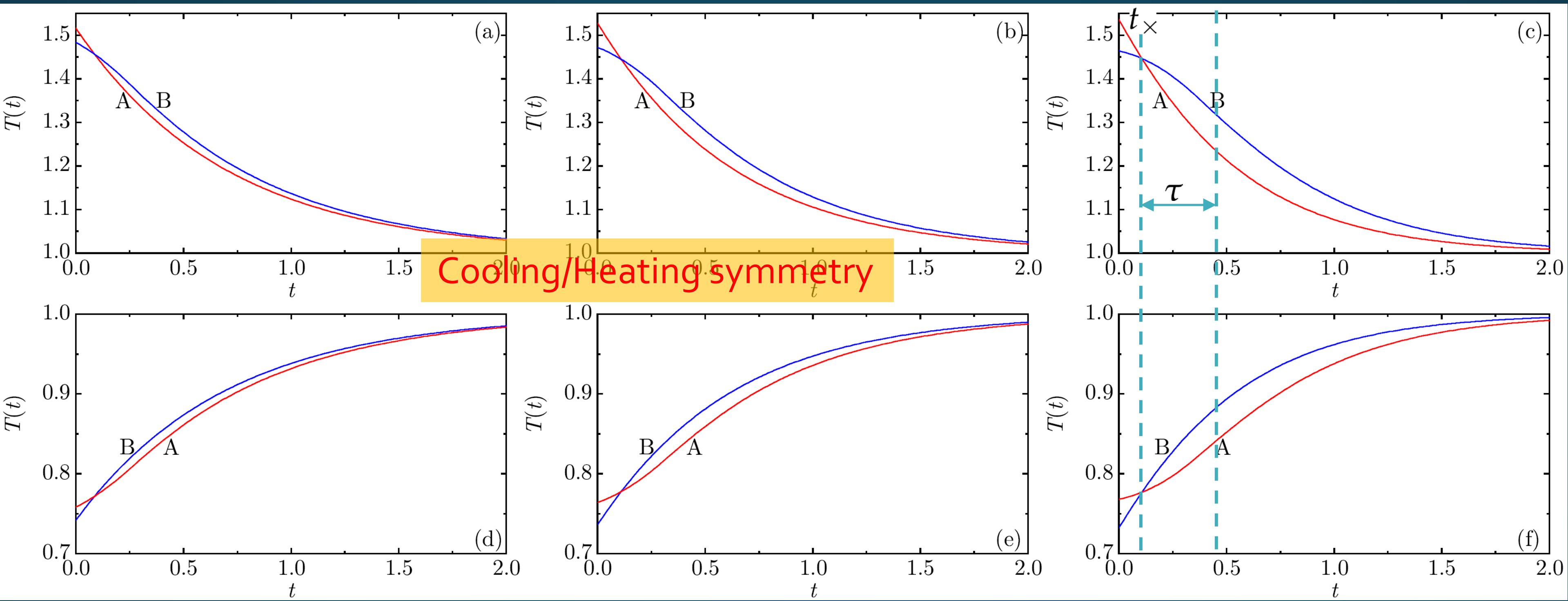


$$\text{Mpemba effect} \Rightarrow \begin{cases} \text{(i) } \Delta(0) > 0 & \Rightarrow t_\mathrm{w} < t_\mathrm{w}^\mathrm{max}, \quad \mathcal{E}(t_\mathrm{w}^\mathrm{max}) = \frac{1}{2} \\ \text{(ii) } \Delta(t \to \infty) < 0 & \Rightarrow t_\mathrm{w} > t_\mathrm{w}^\mathrm{min}, \quad t_\mathrm{w}^\mathrm{min} = \kappa^{-1} \ln 2 \end{cases}$$

Phase diagram

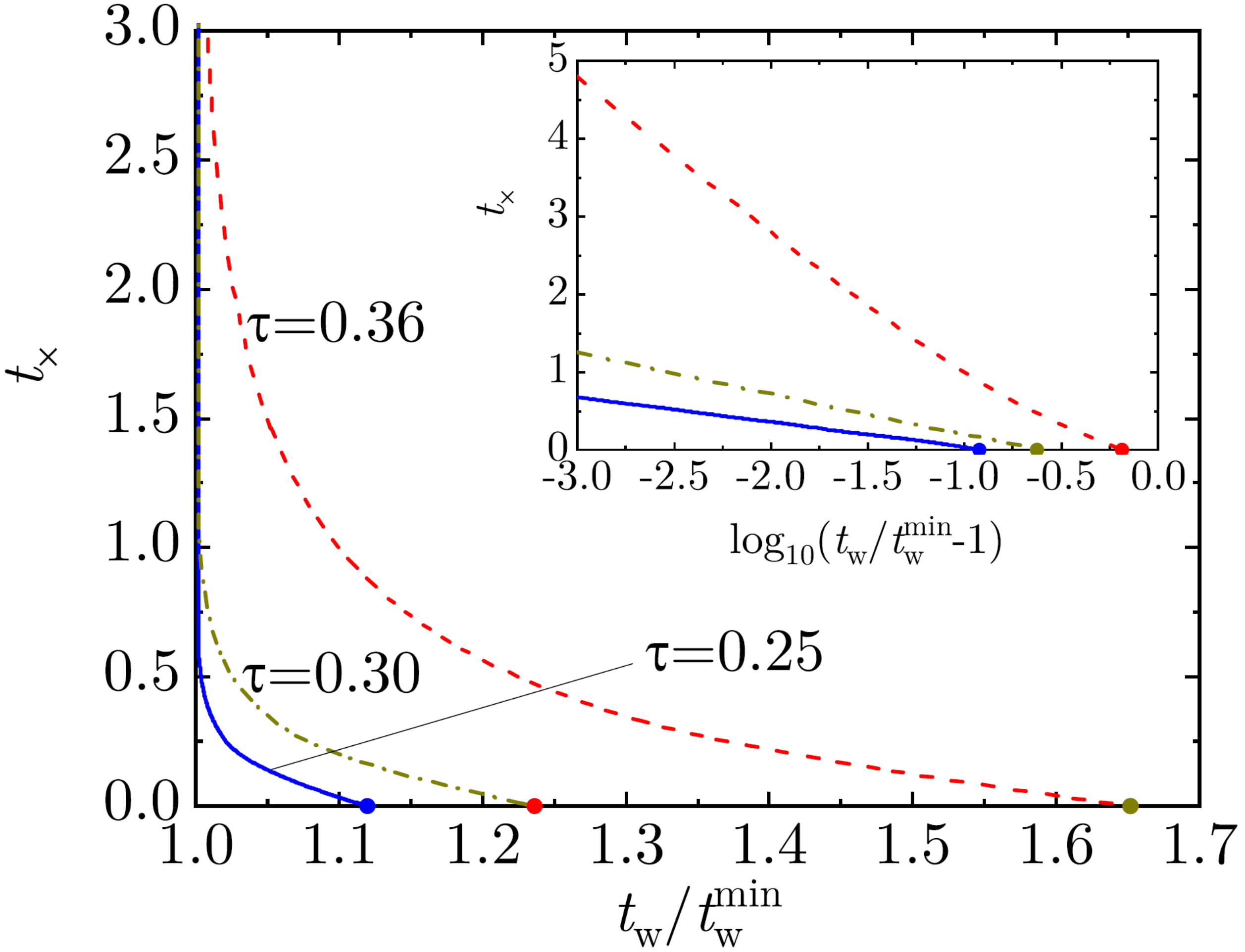


Illustrative examples



$\tau = 0.25, \quad t_{\mathrm{w}} = 0.52$ $\tau = 0.30, \quad t_{\mathrm{w}} = 0.49$ $\tau = 0.36, \quad t_{\mathrm{w}} = 0.47$

Crossover time: $\Delta(t_\times) = 0$



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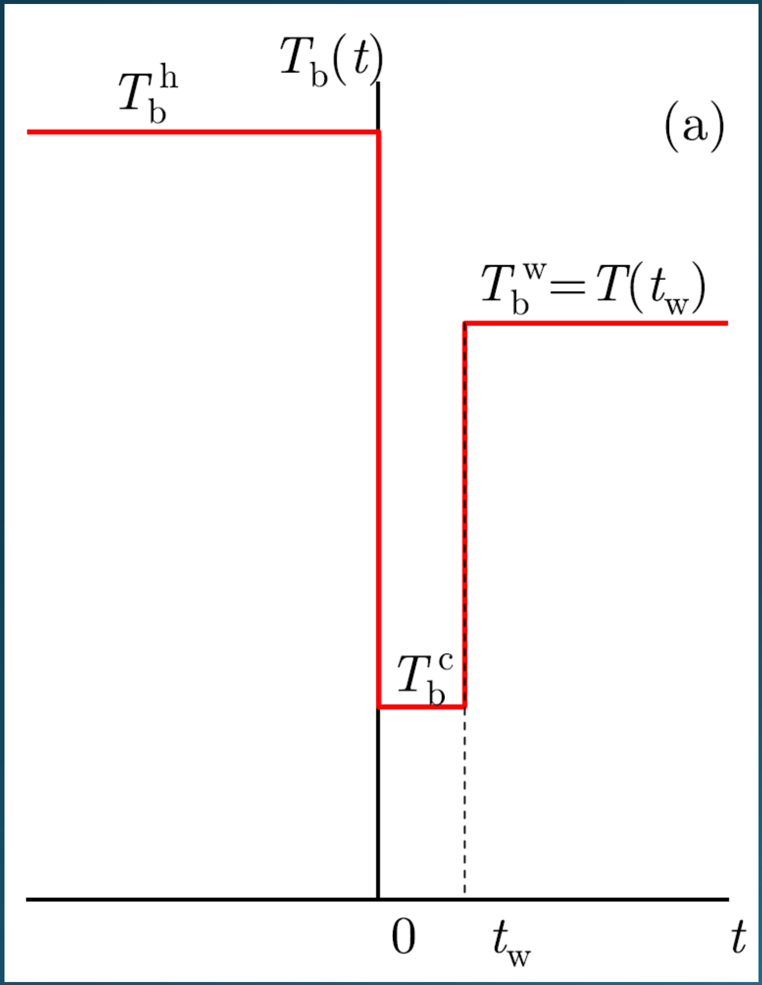
Kovacs effect



A. J. Kovacs, *Transition vitreuse dans les polymères amorphes. Étude phénoménologique*, Fortschr. Hochpolym.-Forsch. **3**, 394 (1963).

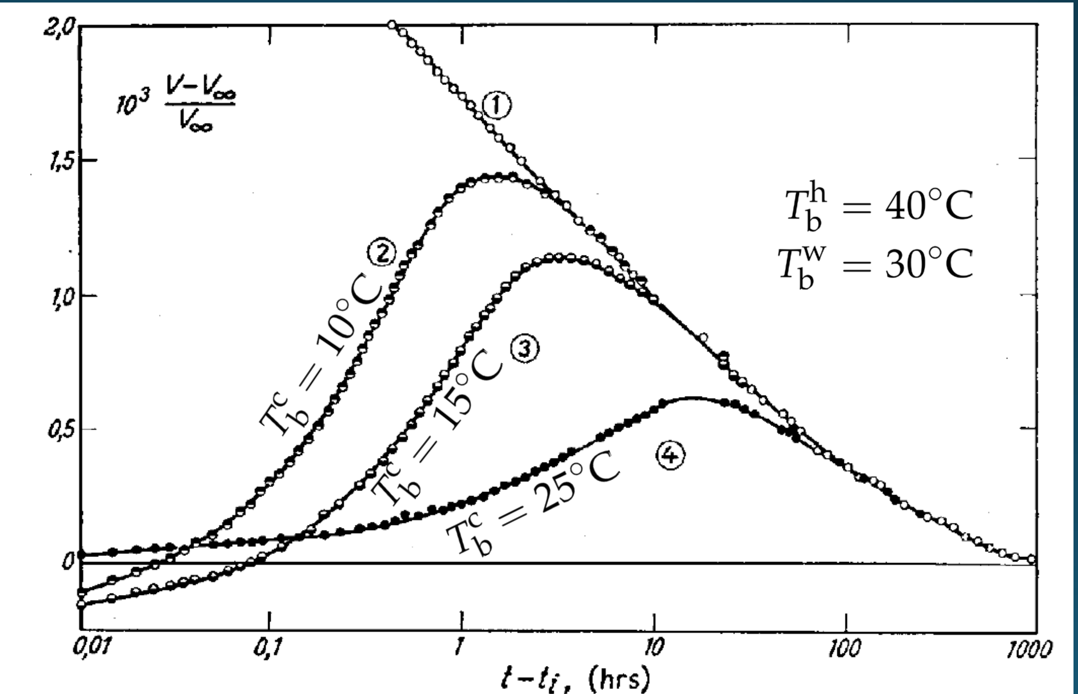
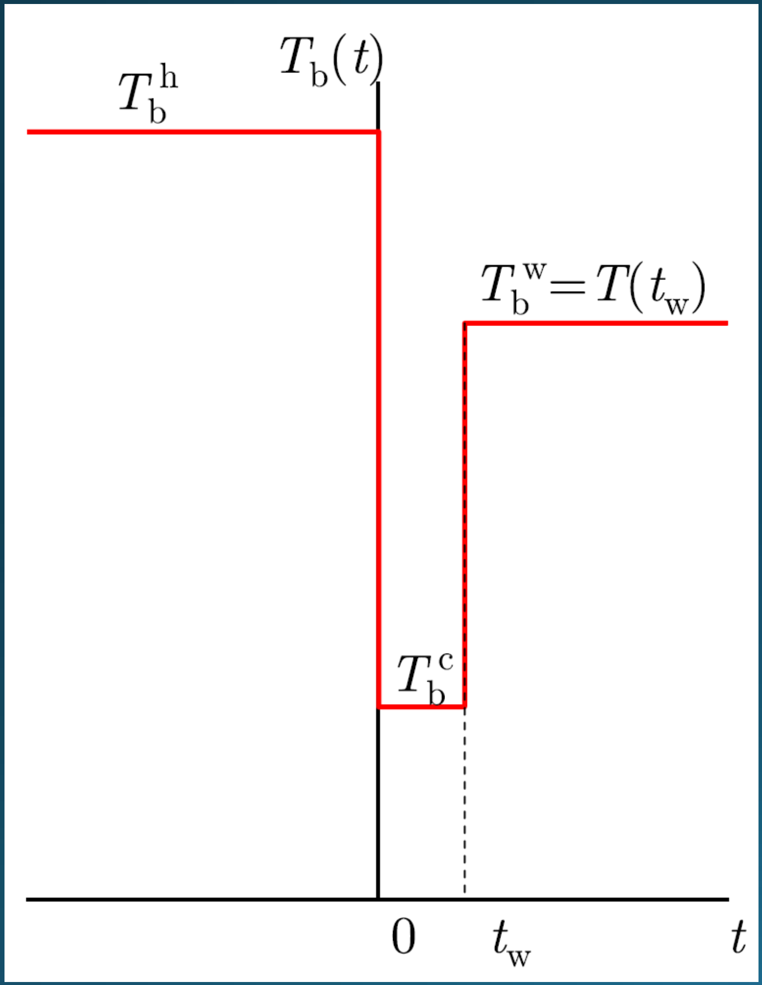


Fig. 24. Evolution isotherme du PVA II après trempe et réchauffement à 30° C (Tableau 8)

Kovacs effect



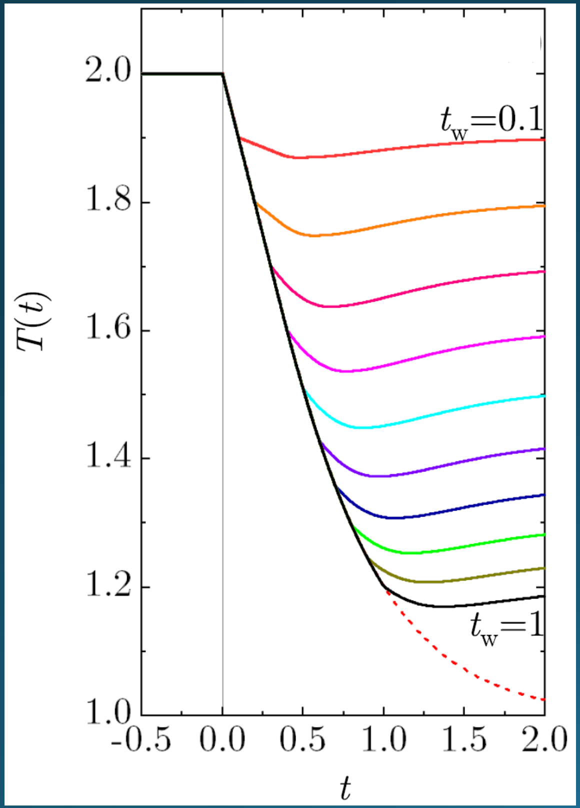
$$T_b(t) = \begin{cases} T_b^h, & t < 0, \\ T_b^c, & 0 << t < t_w, \\ T_b^w, & t > t_w. \end{cases}$$

$$T(t) = \begin{cases} T_b^c + (T_b^h - T_b^c)\mathcal{E}(t), & 0 \le t \le t_w \\ T_b^w + (T_b^c - T_b^w)\mathcal{E}(t - t_w) & \\ +(T_b^h - T_b^c)\mathcal{E}(t), & t \ge t_w \end{cases}$$

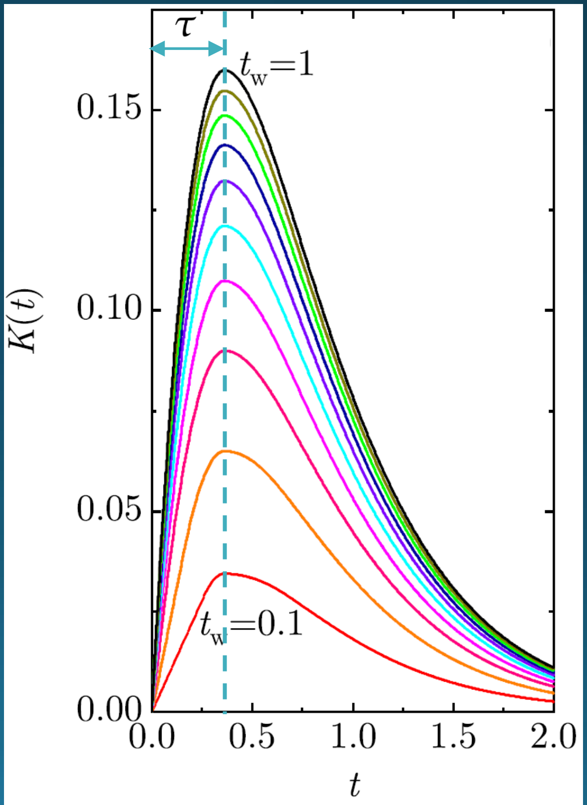
Kovacs hump function

$$T(t) - T_b^w = -(T_b^w - T_b^c)K(t - t_w)$$

$$K(t) = \mathcal{E}(t) - \frac{\mathcal{E}(t + t_w)}{\mathcal{E}(t_w)}$$



$$\tau = 0.36$$



Kovacs hump function

$$T(t) - T_{\mathrm{b}}^{\mathrm{w}} = -(T_{\mathrm{b}}^{\mathrm{w}} - T_{\mathrm{b}}^{\mathrm{c}})K(t - t_{\mathrm{w}})$$

$$K(t) = \mathcal{E}(t) - \frac{\mathcal{E}(t + t_{\mathrm{w}})}{\mathcal{E}(t_{\mathrm{w}})}$$

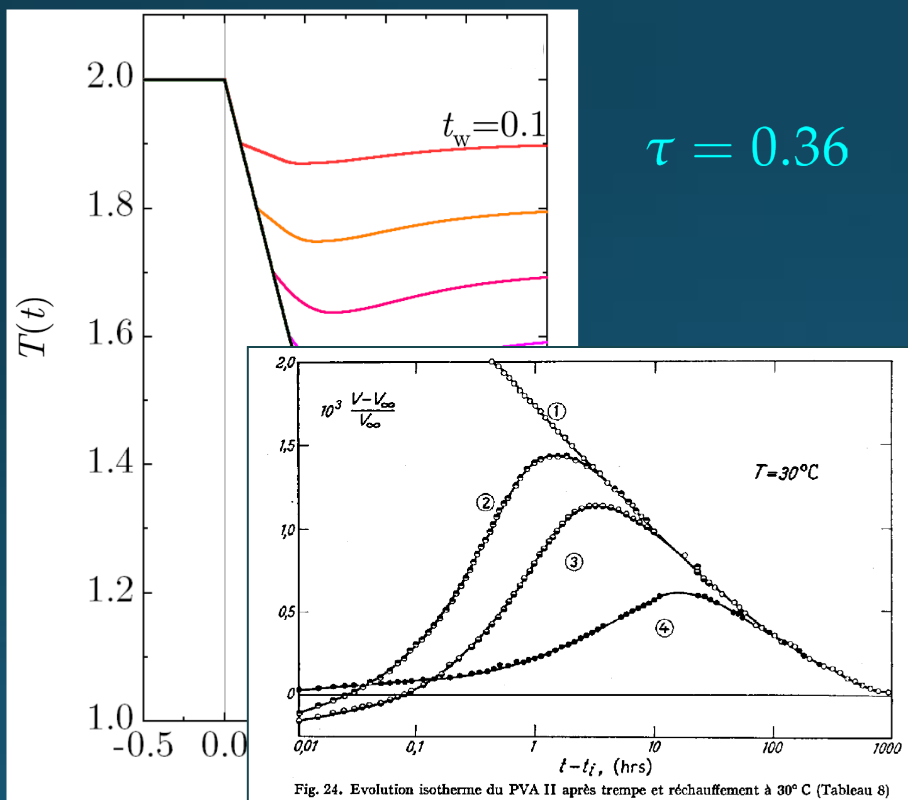
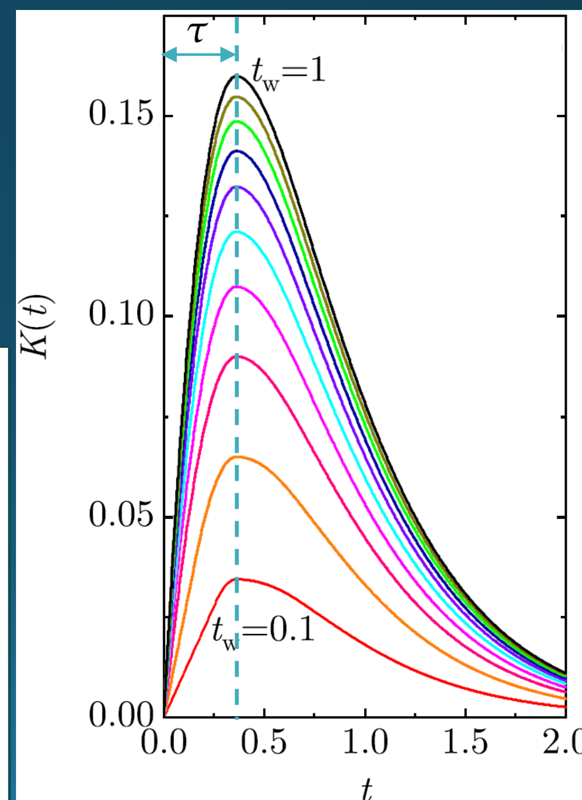


Fig. 24. Evolution isotherme du PVA II après trempe et réchauffement à 30° C (Tableau 8)

$$\tau = 0.36$$



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- The time-delayed Newton's cooling law provides a simple yet powerful framework to explain the Mpemba and Kovacs effects, highlighting the importance of memory phenomena in thermal processes.
- The presence and strength of both effects are determined within a two-dimensional parameter space (delay time and waiting time), independent of the thermal bath temperatures.
- These findings may offer a deeper understanding of thermal dynamics and could inform research in various fields, such as materials science and thermodynamics.
- Further studies could explore the specific physical mechanisms behind the delay time and investigate other systems where the Mpemba and Kovacs effects might occur.


Thank you for your attention!


QUESTIONS