(Multiple) **Quantum Mpemba effect**
[exceptional points and complex eigenvalues]

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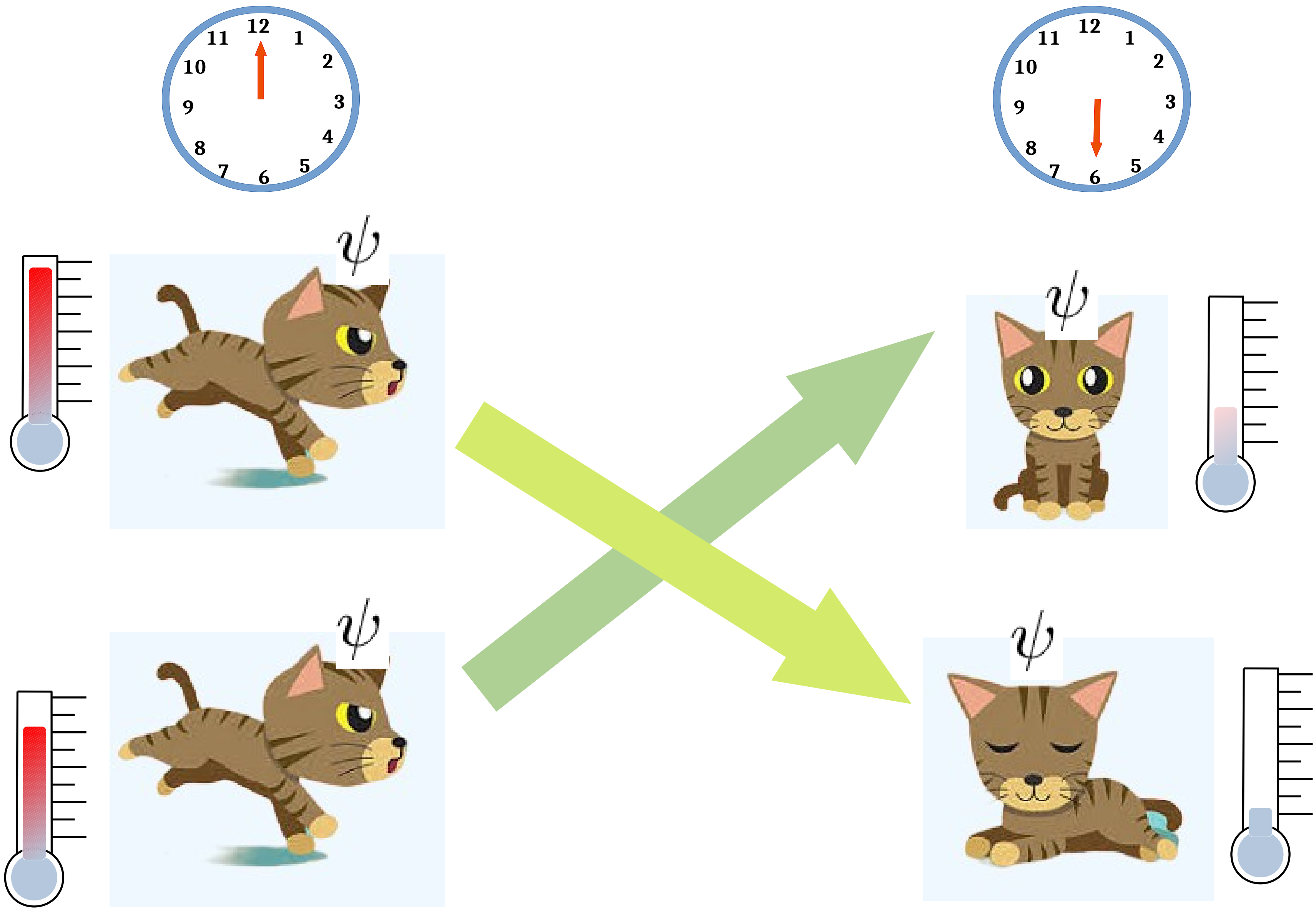
References:

1. **A.K.C**, S. Takada, H. Hayakawa, PRL 131, 080402 (2023) [Editors' Suggestion]
2. **A.K.C**, S. Takada, H. Hayakawa, arXiv:2311.01347 [to be published in PRA]

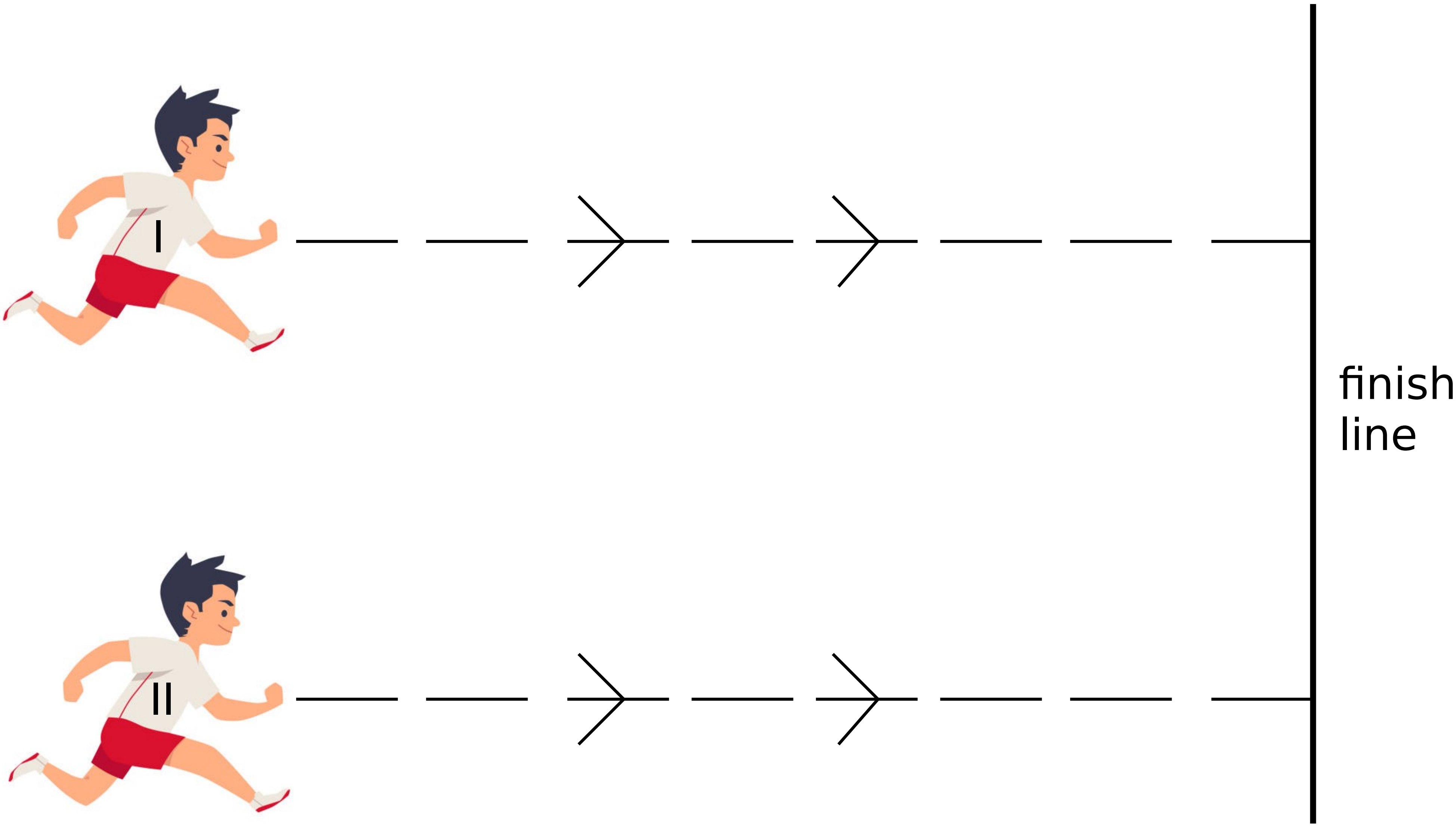
What are we going to show ?



A quantum system (cat) can cool faster when starting from initially hotter rather than colder temperature.

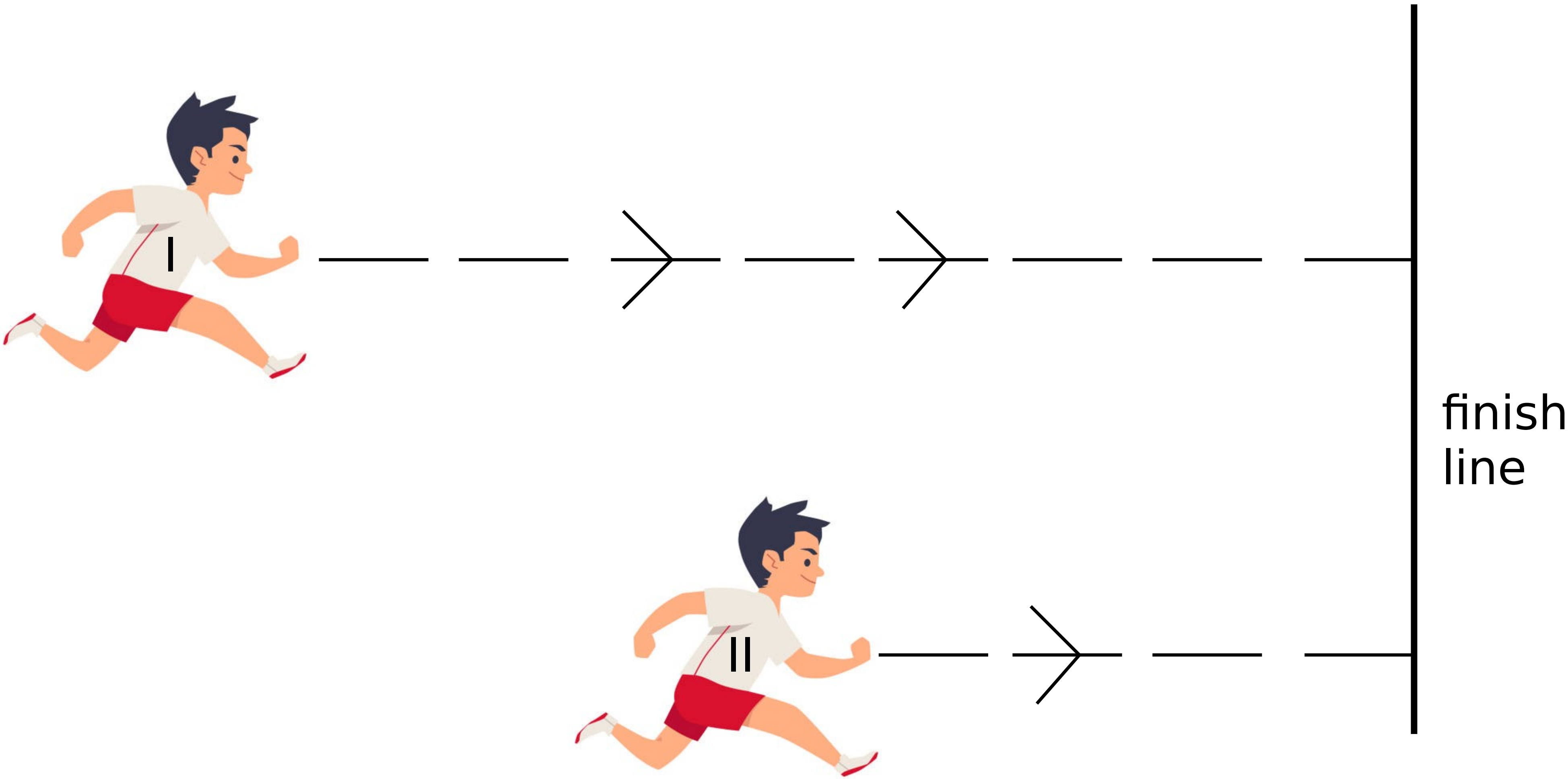
How do we achieve such anomalous relaxation ?

Two regular persons start the race from same distance



Who will reach the finishing line faster ? ⇨ Hard to tell !

Two regular persons start the race from different distances

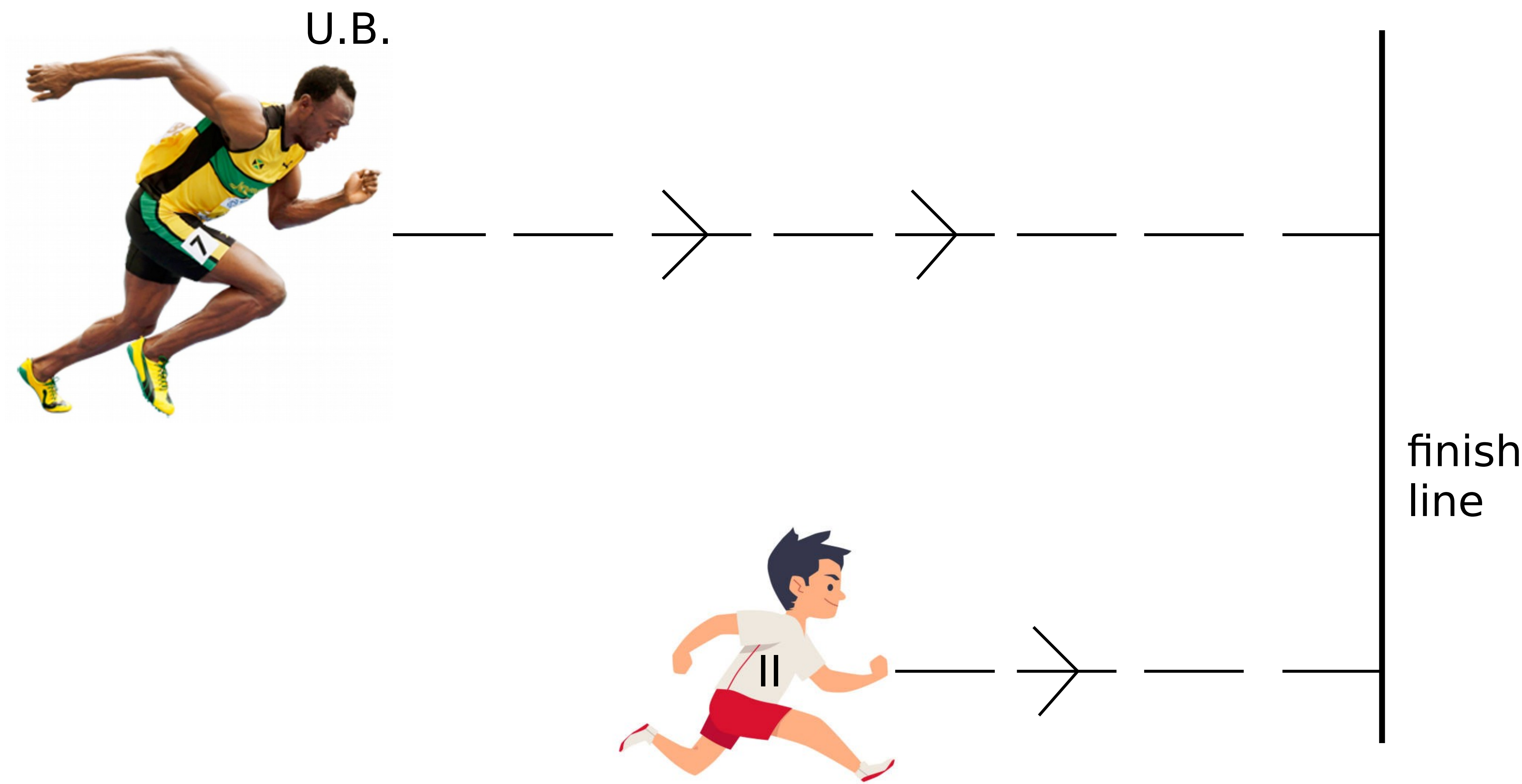


Who will reach the finishing line faster ?

runner II wins ⇨ normal expectation

runner I wins ⇨ **Mpemba effect** (surprising, how ?)

Replace person I by Usain Bolt



if U.B. wins ⇒ **Mpemba effect**

But now, Mpemba effect is not so surprising because **initial state** of U.B. is **specially prepared** to win the race

We try to find **initial conditions**
that lead us to **Mpemba effect in quantum systems**

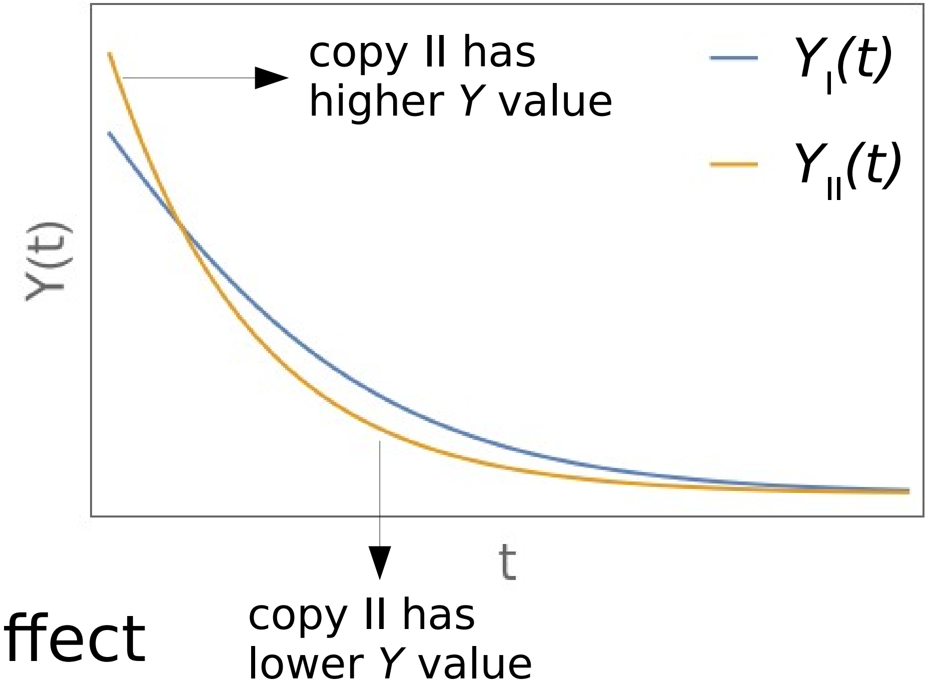
How do we detect (quantum) Mpemba effect

"crossing of trajectories" indicates Mpemba effect

Observable: Y

$Y_{I}(t=0) < Y_{II}(t=0)$

$Y_{I}(ss) = Y_{II}(ss)$



If, at intermediate time:

$Y_{I}(t) > Y_{II}(t) \Longrightarrow$ Mpemba effect

QUANTUM MPEMBA EFFECT (QMPE)

(background & motivations, results)

VIEWPOINT

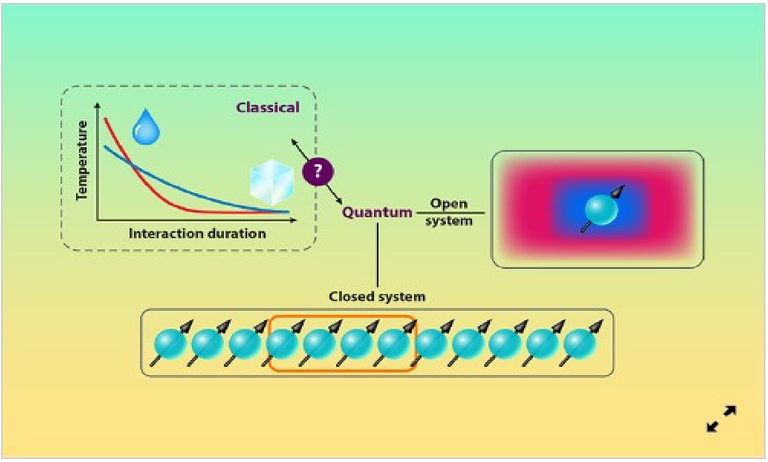
PDF Version

Exploring Quantum Mpemba Effects

Ulrich Warring
Institute of Physics, University of Freiburg, Freiburg, Germany

July 1, 2024 • *Physics* 17, 105

In the Mpemba effect, a warm liquid freezes faster than a cold one. Three studies investigate quantum versions of this effect, challenging our understanding of quantum thermodynamics.



APS/Carin Cain

Figure 1: (Top left) Under specific conditions, hot water (red curve) can freeze faster than cold water (blue curve) when interacting with an external environment. This classical phenomenon is known as the Mpemba effect. (Right) Aharony Shapira and colleagues studied an inverse quantum Mpemba-like effect in an open quantum system, consisting of a single cold trapped ion, interacting with a warm external environment [3]. (Bottom) Joshi and colleagues studied a quantum Mpemba-like effect in subsystems of a closed quantum system, consisting of 12 interacting trapped ions [4]. Lastly, Rylands and colleagues theoretically studied the microscopic mechanisms driving quantum Mpemba-like effects in closed quantum systems [5]. A remaining question is how to establish a link between these classical and quantum phenomena.

Observing the Quantum Mpemba Effect in Quantum Simulations
Lata Kh. Joshi, Johannes Franke, Aniket Rath, Filiberto Ares, Sara Murciano, Florian Kranzl, Rainer Blatt, Peter Zoller, Benoît Vermersch, Pasquale Calabrese, Christian F. Roos, and Manoj K. Joshi
Phys. Rev. Lett. 133, 010402 (2024)
Published July 1, 2024
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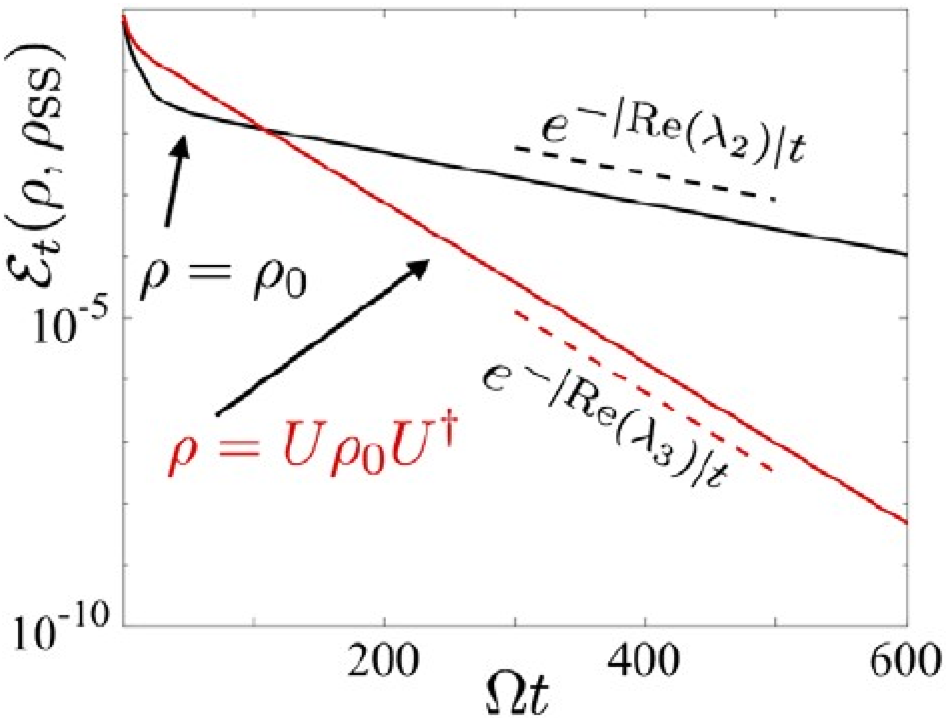
Inverse Mpemba Effect Demonstrated on a Single Trapped Ion Qubit
Shahaf Aharony Shapira, Yotam Shapira, Jovan Markov, Gianluca Teza, Nitzan Akerman, Oren Raz, and Roee Ozeri
Phys. Rev. Lett. 133, 010403 (2024)
Published July 1, 2024
Read PDF

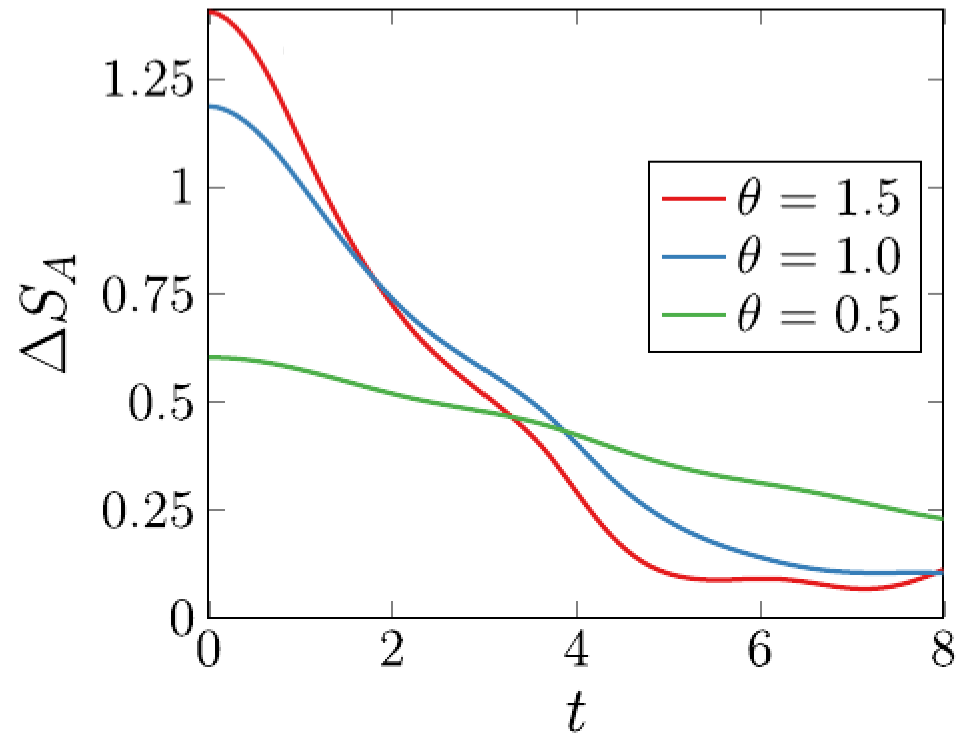
Microscopic Origin of the Quantum Mpemba Effect in Integrable Systems
Colin Rylands, Katja Klobas, Filiberto Ares, Pasquale Calabrese, Sara Murciano, and Bruno Bertini
Phys. Rev. Lett. 133, 010401 (2024)
Published July 1, 2024

Quantum Mpemba effect (QMPE)

vastly unexplored field



Distance from equilibrium in dissipative Dicke model



Entangle asymmetry in XXZ spin chain

[Carollo, Lasanta and Lesanovsky, PRL 127, 060401 (2021)]

[Ares, Murciano and Calabrese, Nature Communications 14, 2036 (2023)]

Our motivations

Analysis of temperature missing

⇩

Question: What about "thermal" Quantum Mpemba effect ?
(*closer to original classical Mpemba effect*)

Question: Relation between in "thermal"QMPE
and "distance" QMPE ?

Answer: Part A [quantum dot connected to reservoirs]

Question: Possibility of "multiple crossings" between
trajectories, i.e. "multiple" QMPE ?

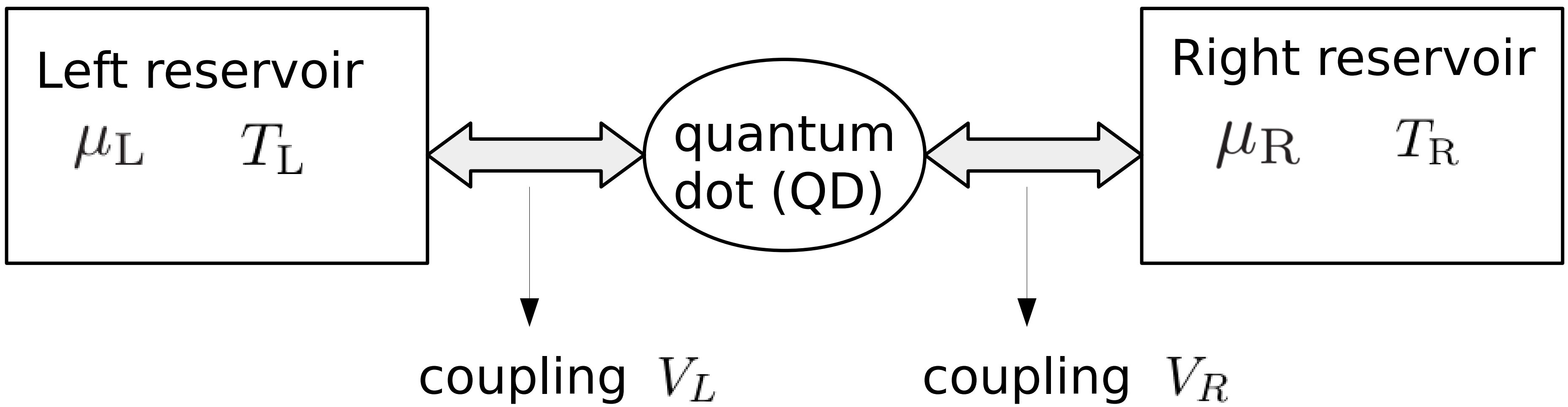
Answer: Part B [two level driven dissipative system]

Part A: quantum dot connected to reservoirs

[A. K. Chatterjee, S. Takada, and H. Hayakawa, PRL 131, 080402 (2023)]

[Editors' Suggestion]

Quantum dot coupled to two reservoirs



QD states: $\uparrow\downarrow$, $\uparrow$, $\downarrow$, vacant

Total Hamiltonian: $\hat{H}_{\rm tot} = \hat{H}_{\rm s} + \hat{H}_{\rm r} + \hat{H}_{\rm int}$

- $\hat{H}_{\rm s}$: System Hamiltonian
- $\hat{H}_{\rm r}$: Reservoir Hamiltonian
- $\hat{H}_{\rm int}$: System-reservoirs interaction

Anderson model:

$$\hat{H}_{\rm s} = \sum_{\sigma} \epsilon_0 \hat{d}_\sigma^\dagger \hat{d}_\sigma \quad + \quad U \hat{n}_\uparrow \hat{n}_\downarrow$$

$$\hat{H}_{\rm r} = \sum_{\gamma,k,\sigma} \epsilon_k \hat{a}^\dagger_{\gamma,k,\sigma} \hat{a}_{\gamma,k,\sigma}$$

$$\hat{H}_{\rm int} \;=\; \sum_{\gamma,k,\sigma} V_\gamma \hat{d}_\sigma^\dagger \hat{a}_{\gamma,k,\sigma} + \text{h.c.}$$

ϵ_0: energy of electron in quantum dot

ϵ_k: energy of electron corresponding to wave number k in reservoirs

U: electron-electron interaction in quantum dot

V_L, V_R: coupling strength between quantum dot and reservoirs

$\hat{d}^\dagger, \hat{d}$: creation and annihilation operators in quantum dot

$\hat{a}^\dagger, \hat{a}$: creation and annihilation operators in reservoirs

$\hat{n}$: number operator $(= \hat{d}^\dagger \hat{d})$

γ: reservoir indices L, R $\qquad$ σ: up-spin $(\uparrow)$ or down-spin $(\downarrow)$

Quantum Master equation:

$$\frac{d}{d\tau}\hat{\rho} = \hat{K}\hat{\rho}$$

$\hat{\rho}$: four possible states: ↑↓, ↑, ↓, vacant

Transition matrix:

$$\hat{K} = \begin{pmatrix} -2f_-^{(1)} & f_+^{(1)} & f_+^{(1)} & 0 \\ f_-^{(1)} & -f_-^{(0)} - f_+^{(1)} & 0 & f_+^{(0)} \\ f_-^{(1)} & 0 & -f_-^{(0)} - f_+^{(1)} & f_+^{(0)} \\ 0 & f_-^{(0)} & f_-^{(0)} & -2f_+^{(0)} \end{pmatrix}$$

where

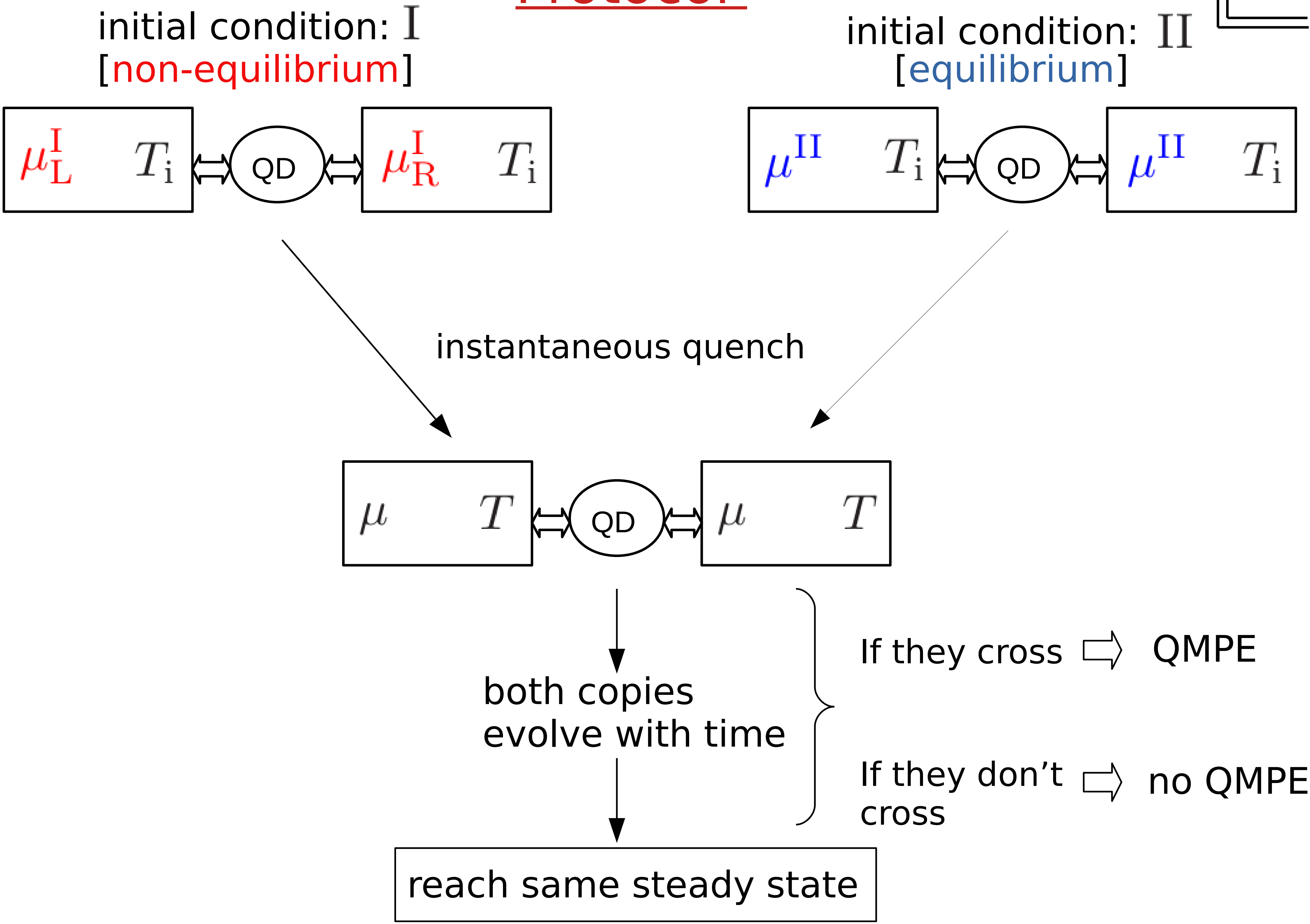
$$f_+^{(j)} := f_{\mathrm{L}}^{(j)}(\mu_{\mathrm{L}}, U, \epsilon_0, T) + f_{\mathrm{R}}^{(j)}(\mu_{\mathrm{R}}, U, \epsilon_0, T), \quad j = 0, 1$$

$$f_-^{(j)} = 2 - f_+^{(j)}$$

$$f_\gamma^{(j)}(\mu_\gamma, U, \epsilon_0, T) := \frac{1}{1 + e^{(\epsilon_0 + jU - \mu_\gamma)/T}}$$: Fermi-Dirac distribution

$\gamma = \mathrm{L}, \mathrm{R}$

Protocol


initial condition: I
[non-equilibrium]
initial condition: II
[equilibrium]
μ_L^I T_i
QD
μ_R^I T_i
μ^{II} T_i
QD
μ^{II} T_i
instantaneous quench
μ T
QD
μ T
both copies
evolve with time
If they cross
QMPE
If they don't cross
no QMPE
reach same steady state

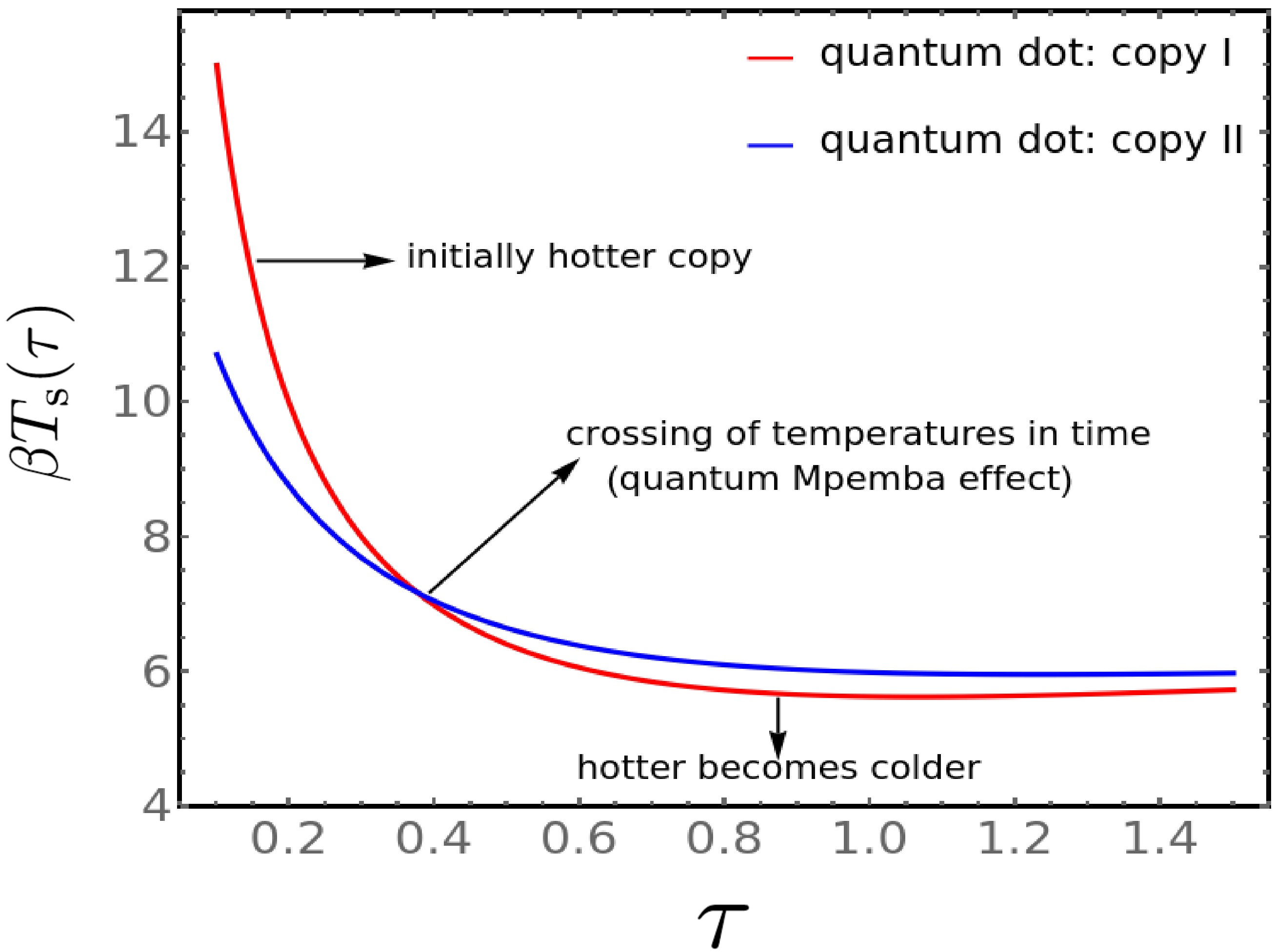
Thermal Quantum Mpemba effect

Temperature: $$T_s(\tau) := \frac{\partial E_s(\tau)}{\partial S_{vN}(\tau)} = \frac{\partial E_s(\tau)}{\partial \tau} \Big/ \frac{\partial S_{vN}(\tau)}{\partial \tau}$$

$$E_s(\tau) = \mathrm{Tr}[\hat{\rho}(\tau)\hat{H}_s]$$

$$S_{vN}(\tau) = -\sum_\alpha \rho_\alpha(\tau)\ln(\rho_\alpha(\tau))$$

$$\beta = 1/T$$



$$\beta\epsilon_0 = 2.0, \beta U = 1.25, \beta\mu_L^I = 4.5, \beta\mu_R^I = 1.0, \beta\mu^{II} = 2.43, \beta T_i = 1.15, \beta\mu = 2.0.$$

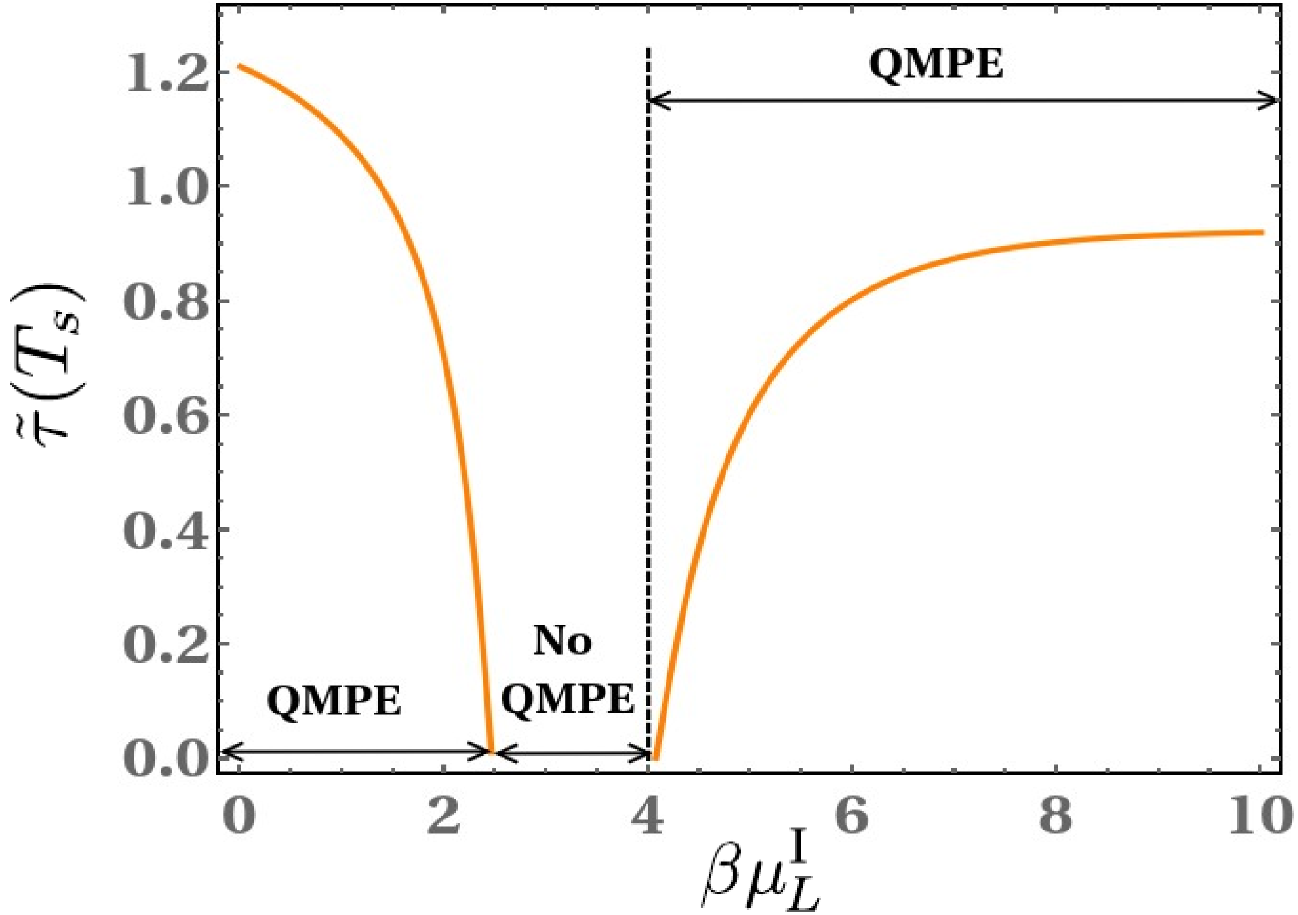
Crossing time: indicator of QMPE

$\Delta T_{\rm s} := T_{\rm s}^{\rm I} - T_{\rm s}^{\rm II}$

crossing time: $\tilde{\tau}(T_{\rm s})$: solution of $\Delta T_{\rm S} = 0$

$0 < \tilde{\tau}(T_{\rm s}) < \infty$: thermal QMPE

$\tilde{\tau}(T_{\rm s}) \to \infty$: no QMPE

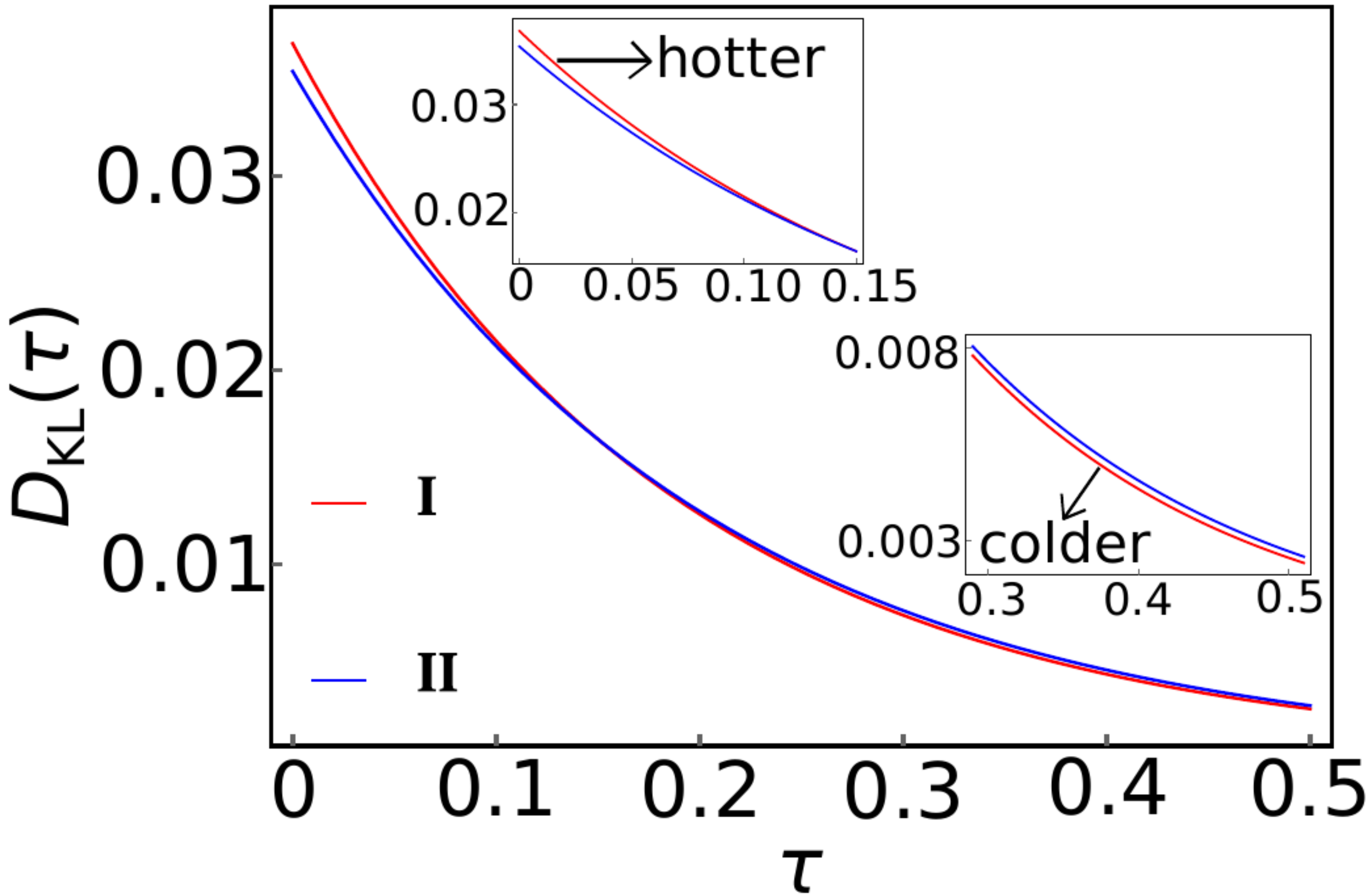


⇨ thermal QMPE is rather generic than occasional

QMPE in Kullback-Liebler divergence

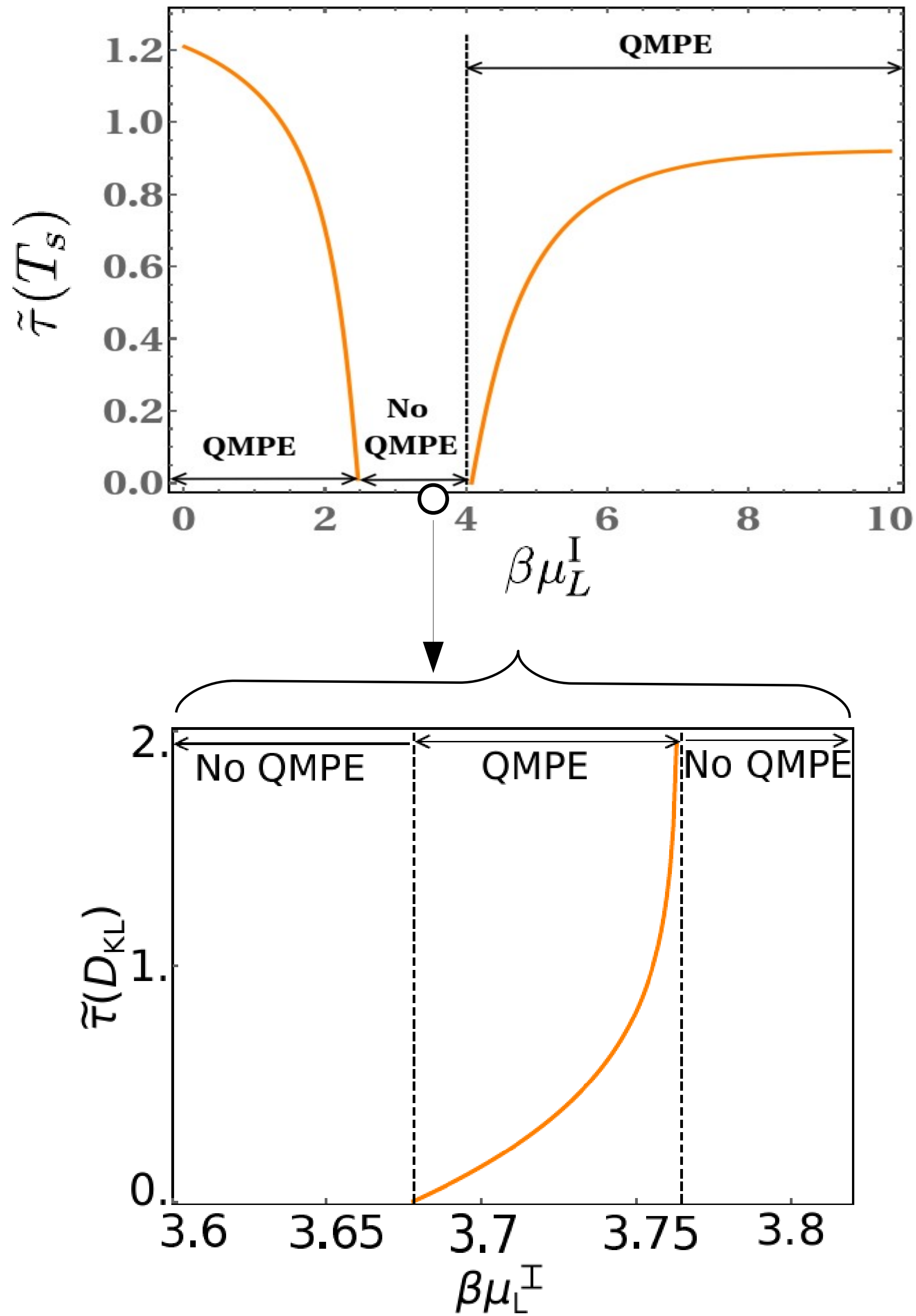
measures "Distance from steady state"

$$D_{\mathrm{KL}}(\tau) = \mathrm{Tr}[\hat{\rho}(\tau)\ (\ln\hat{\rho}(\tau) - \ln\hat{\rho}_{\mathrm{ss}})]$$



$$\beta\epsilon_0 = 2.0, \beta U = 1.25, \beta\mu_{\mathrm{R}}^{\mathrm{I}} = 1.0, \beta\mu^{\mathrm{II}} = 2.43, \beta T_{\mathrm{i}} = 1.15, \beta\mu = 2.0$$

Thermal QMPE vs Distance QMPE

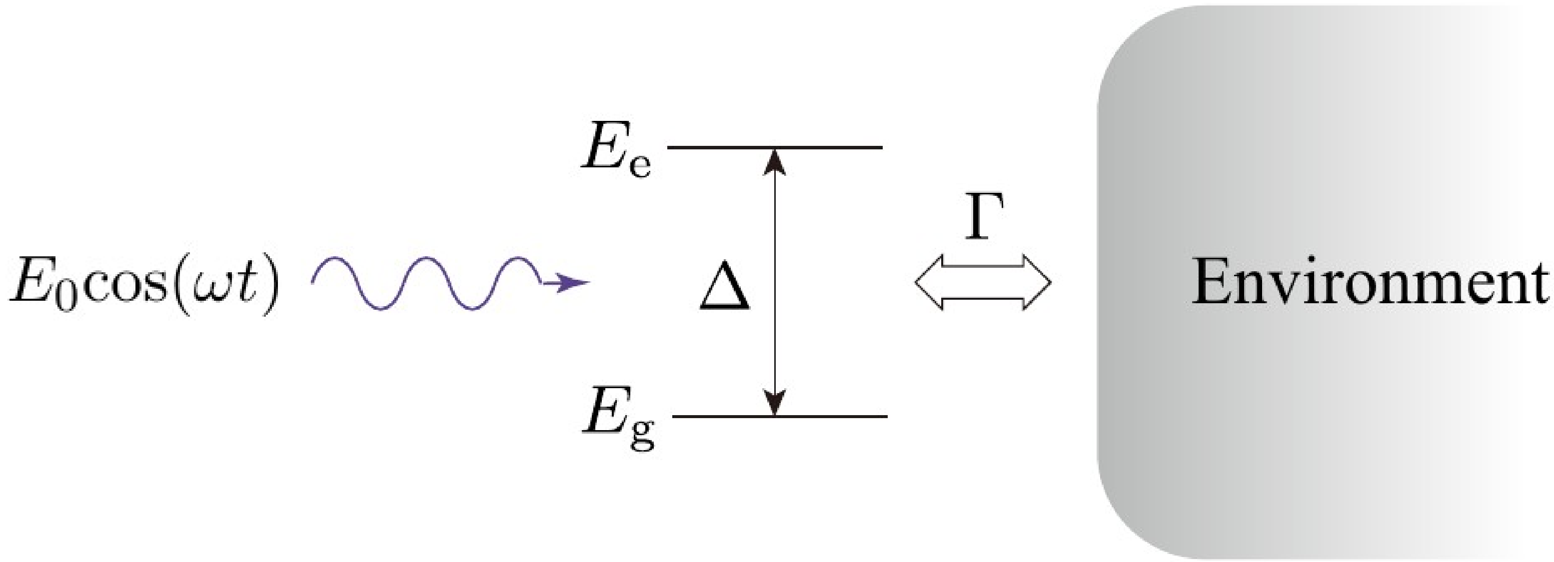


Distance function *may not* act as an alternative indicator for thermal QMPE

Part B: two-level driven dissipative system [exceptional points & oscillations]

[A. K. Chatterjee, S. Takada, and H. Hayakawa, arXiv:2311.01347
(to be published in Physical Review A)]

A driven dissipative two-level quantum system



E_g: ground state energy

E_e: excited state energy

Γ: dissipative coupling with the environment

E_0: driving amplitude of the electric field

ω: driving frequency of the electric field

D: electric dipole moment

$\Delta = E_e - E_g$

[N. Hatano, Molecular Physics 117, 2121 (2019)]

Density matrix, Lindbladian, time evolution

density matrix:

$$\hat{\rho}(t) = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}$$

recast in column vector form

$$|\hat{\rho}(t)\rangle = \begin{pmatrix} \rho_{eg}(t) \\ \rho_{ge}(t) \\ \rho_{ee}(t) \\ \rho_{gg}(t) \end{pmatrix}$$

has **non-zero off-diagonal elements**

Time evolution:

$$i\frac{d}{dt}|\widehat{\rho}(t)\rangle = \widehat{K}|\widehat{\rho}(t)\rangle$$

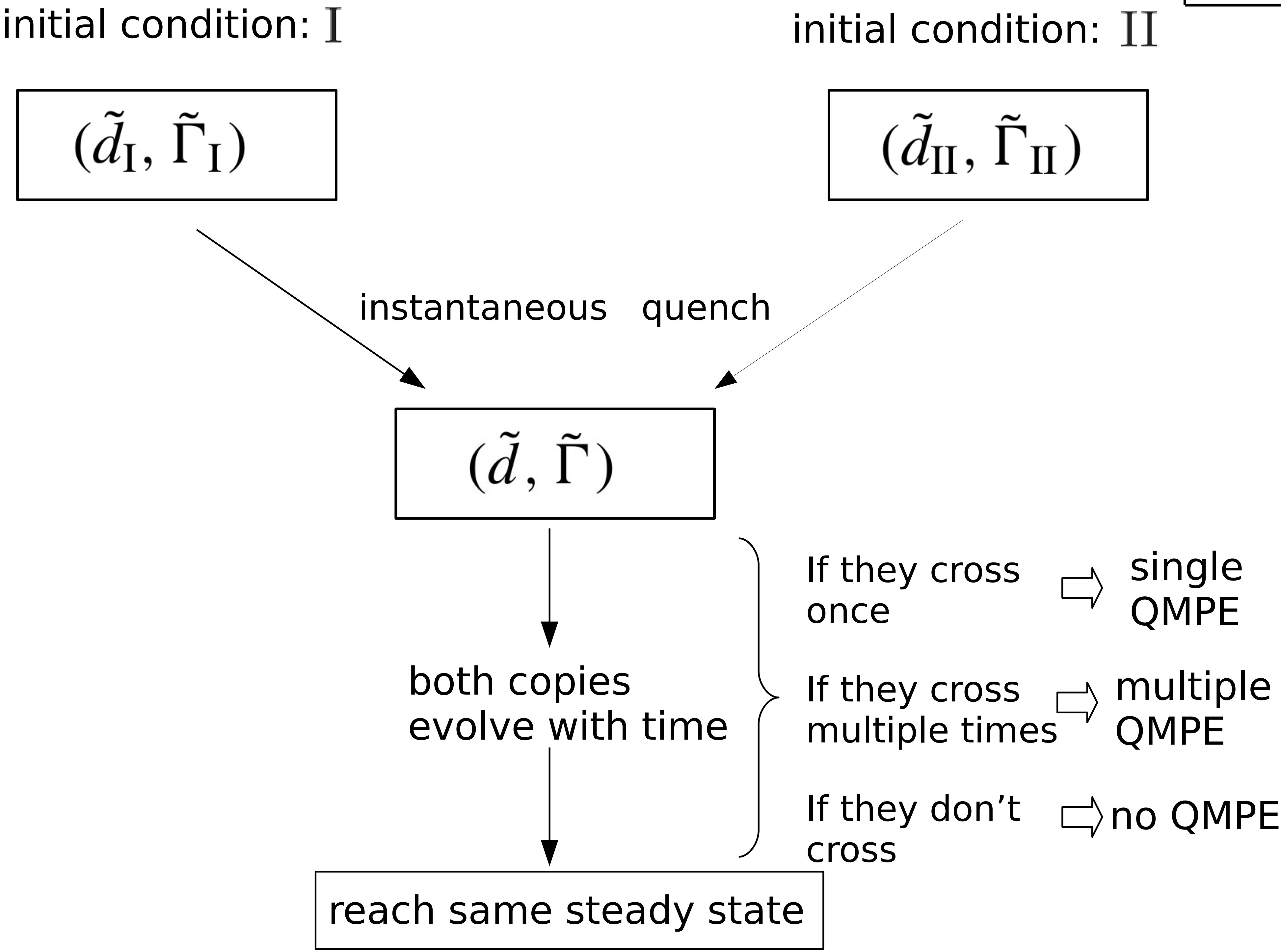
Lindbladian:

$$\widehat{K} = \begin{pmatrix} 1 - i\tilde{\Gamma}/2 & 0 & -\tilde{d}/2 & \tilde{d}/2 \\ 0 & -1 - i\tilde{\Gamma}/2 & \tilde{d}/2 & -\tilde{d}/2 \\ -\tilde{d}/2 & \tilde{d}/2 & -i\tilde{\Gamma} & 0 \\ \tilde{d}/2 & -\tilde{d}/2 & i\tilde{\Gamma} & 0 \end{pmatrix}$$

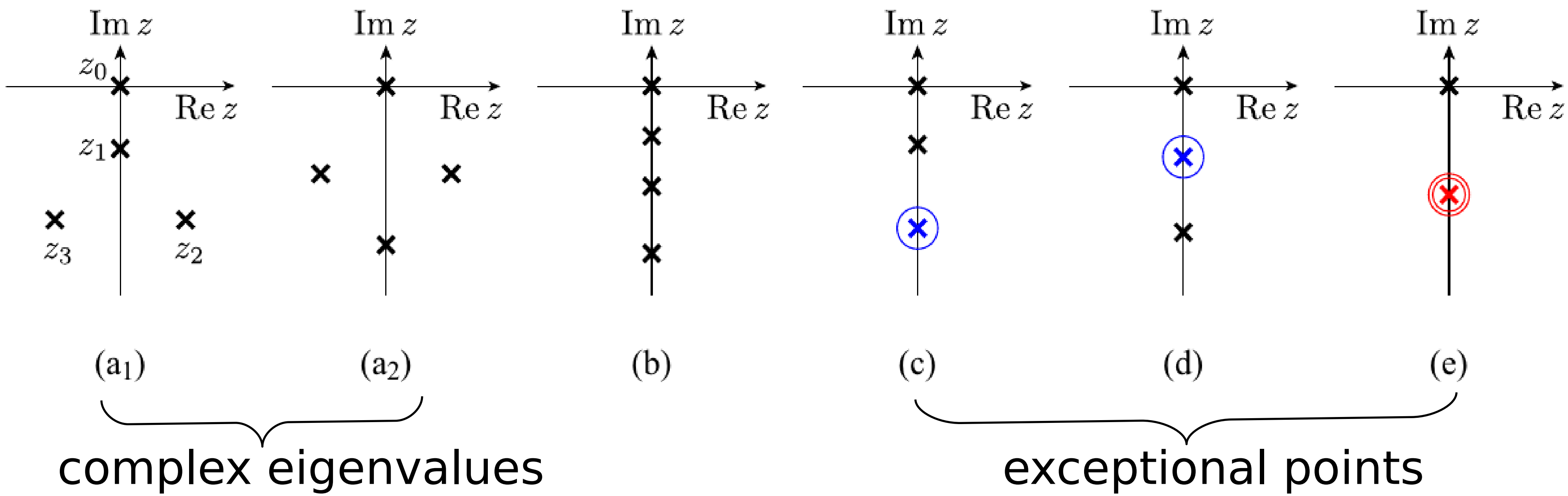
Two free parameters: $\tilde{d} = d/\delta$ and $\tilde{\Gamma} = \Gamma/\delta$

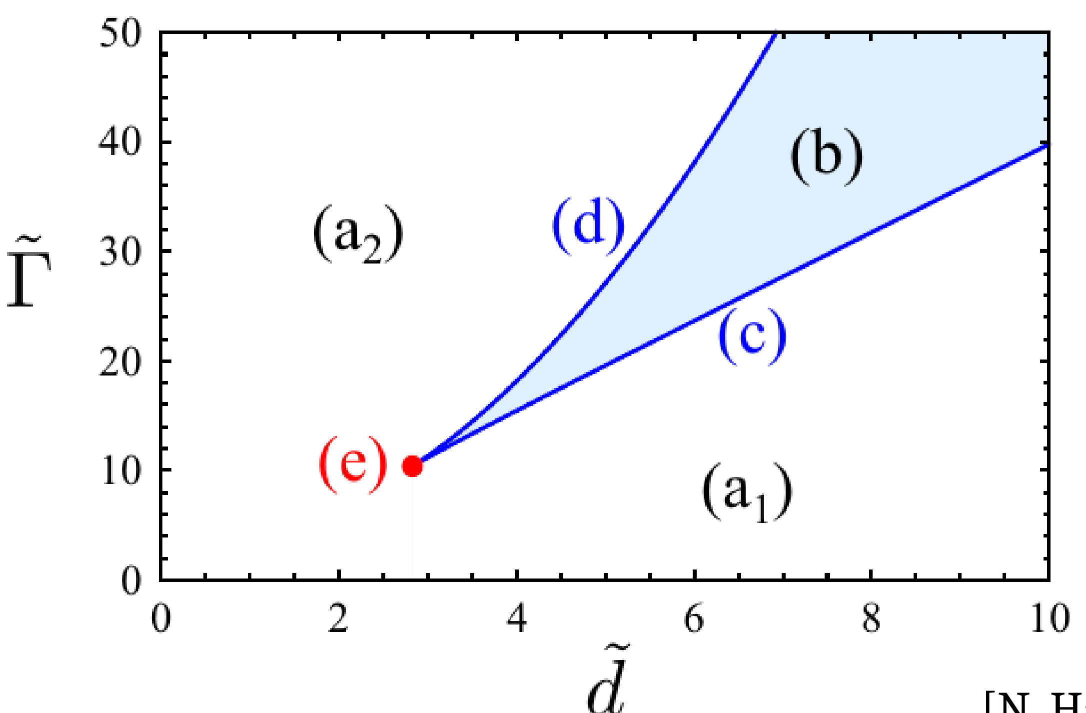
where $d = DE_0$ and $\delta = \Delta - \omega = E_{\mathrm{e}} - E_{\mathrm{g}} - \omega$

Protocol


initial condition: I
initial condition: II
$(\tilde{d}_{\mathrm{I}}, \tilde{\Gamma}_{\mathrm{I}})$
$(\tilde{d}_{\mathrm{II}}, \tilde{\Gamma}_{\mathrm{II}})$
instantaneous quench
$(\tilde{d}, \tilde{\Gamma})$
both copies evolve with time
If they cross once
single QMPE
If they cross multiple times
multiple QMPE
If they don't cross
no QMPE
reach same steady state

Eigenvalue distribution in the parameter space





consider parameter values corresponding to *exceptional points* and *complex eigenvalues*

⇓

analyze QMPE

[N. Hatano, Molecular Physics 117, 2121 (2019)]

Region (d): second order exceptional points

2nd order EP: Two eigenvalues of $\widehat{K}$ become equal and their eigenvectors coalesce

consequence: complete diagonalization of $\widehat{K}$ not possible

Jordan normal form: $\widehat{\Lambda} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\mathrm{i}\lambda_2 & 0 & 0 \\ 0 & 0 & -\mathrm{i}\lambda_3 & 1 \\ 0 & 0 & 0 & -\mathrm{i}\lambda_3 \end{pmatrix}$ → off-diagonal "*defect*"

$(\widehat{\Lambda} = S^{-1}\widehat{K}S)$

Effect on density matrix time evolution:

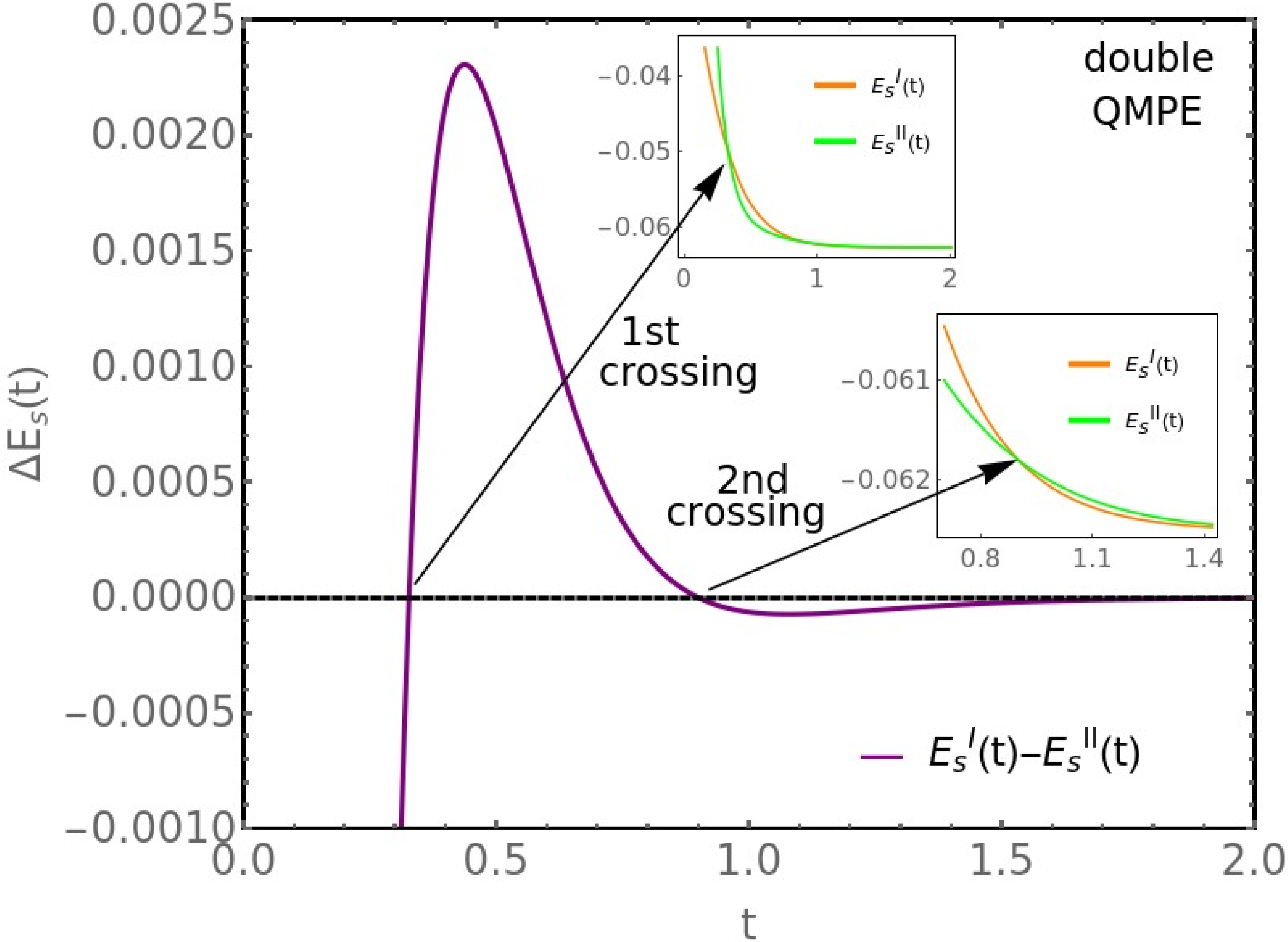
$$\rho_j(t) = \sum_{k=1}^{4}\sum_{m=1}^{4} e^{-\lambda_k t} S_{jk} S^{-1}_{km} \rho_m(0) - \mathrm{i}\, t\, e^{-\lambda_3 t} S_{j3} \sum_{m=1}^{4} S^{-1}_{4m} \rho_m(0)$$

Extra algebraic "*t*" dependence

(Double) QMPE in energy

$$E(t) = \mathrm{Tr}[\hat{\rho}(t)H] = 1 - \rho_4(t) + \frac{\tilde{d}}{2}[\rho_1(t) + \rho_2(t)]$$

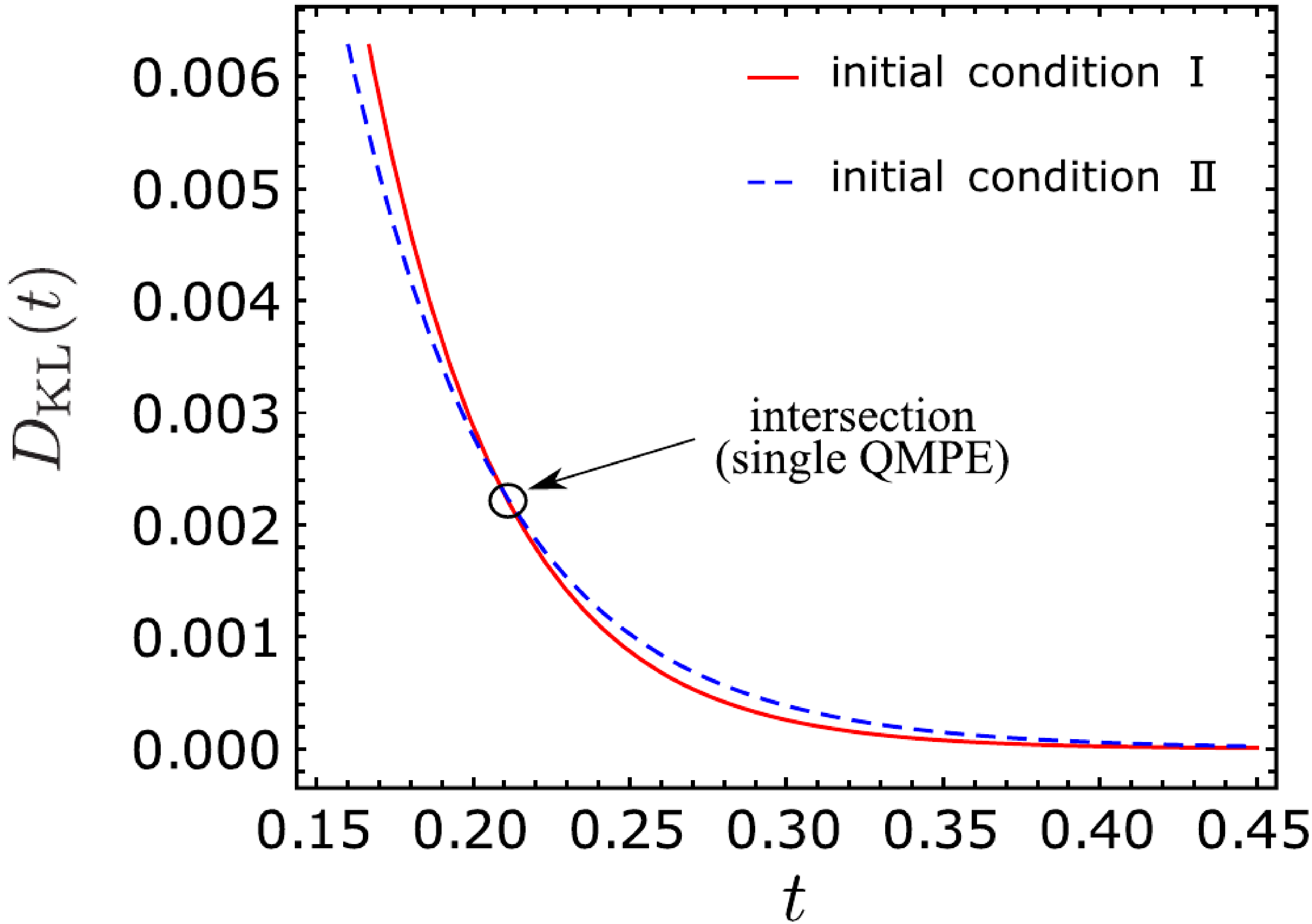
$$\Delta E(t) = -e^{-\lambda_2 t}[\gamma_1 e^{-(\lambda_4-\lambda_2)t} + t\,\gamma_2 + \gamma_3]$$



$$\tilde{d}_{\mathrm{I}} = 11.0, \tilde{d}_{\mathrm{II}} = 8.5, \quad \tilde{d} = 4.0, \tilde{\Gamma} = \sqrt{(568 + 64\sqrt{2})/2}, \tilde{\Gamma}_{\mathrm{I}} = \tilde{\Gamma}_{\mathrm{II}} = 25.0$$

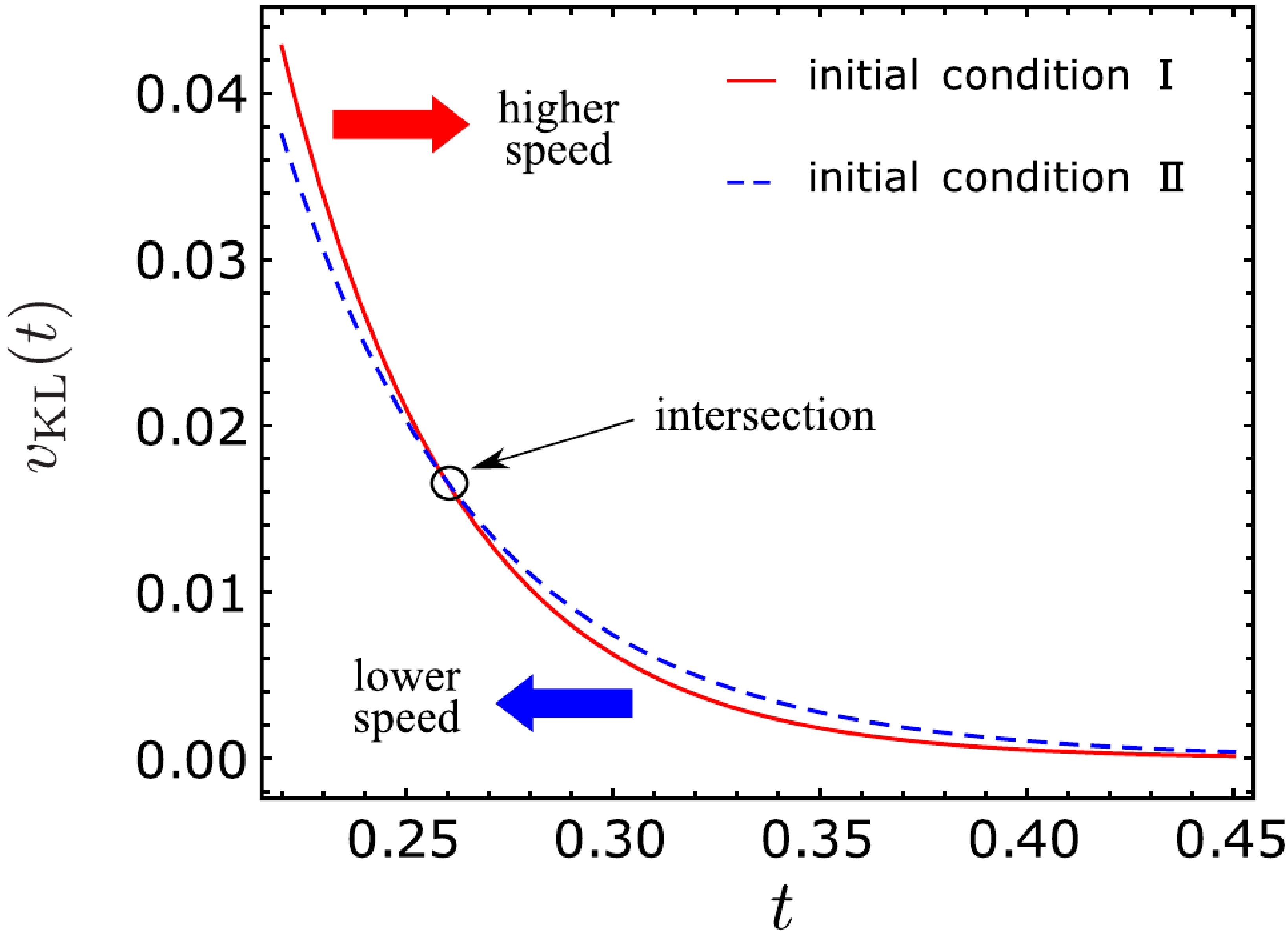
(Single) QMPE in KL divergence

$$D_{\mathrm{KL}}(t) := \mathrm{Tr}[\hat{\rho}(t)\{\ln[\hat{\rho}(t)] - \ln(\hat{\rho}_{\mathrm{ss}})\}]$$



(Single) QMPE in relaxation speed

$$v_{\mathrm{KL}}(t) = -\frac{\partial D_{\mathrm{KL}}(t)}{\partial t}$$



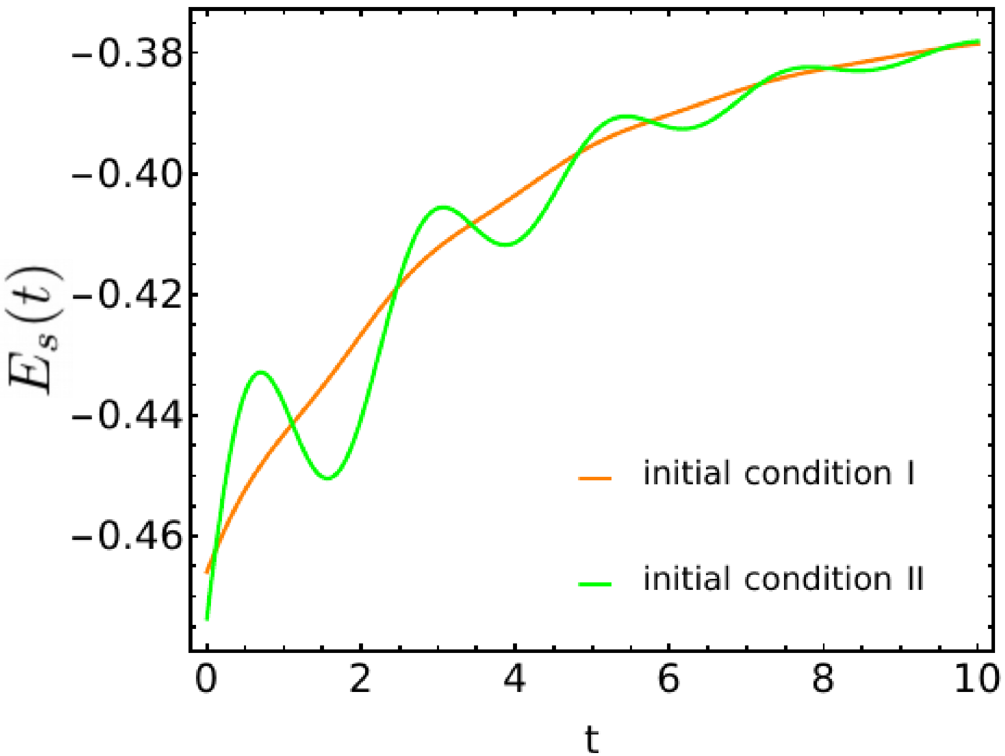
Region (a_1): oscillations in QMPE: *energy*

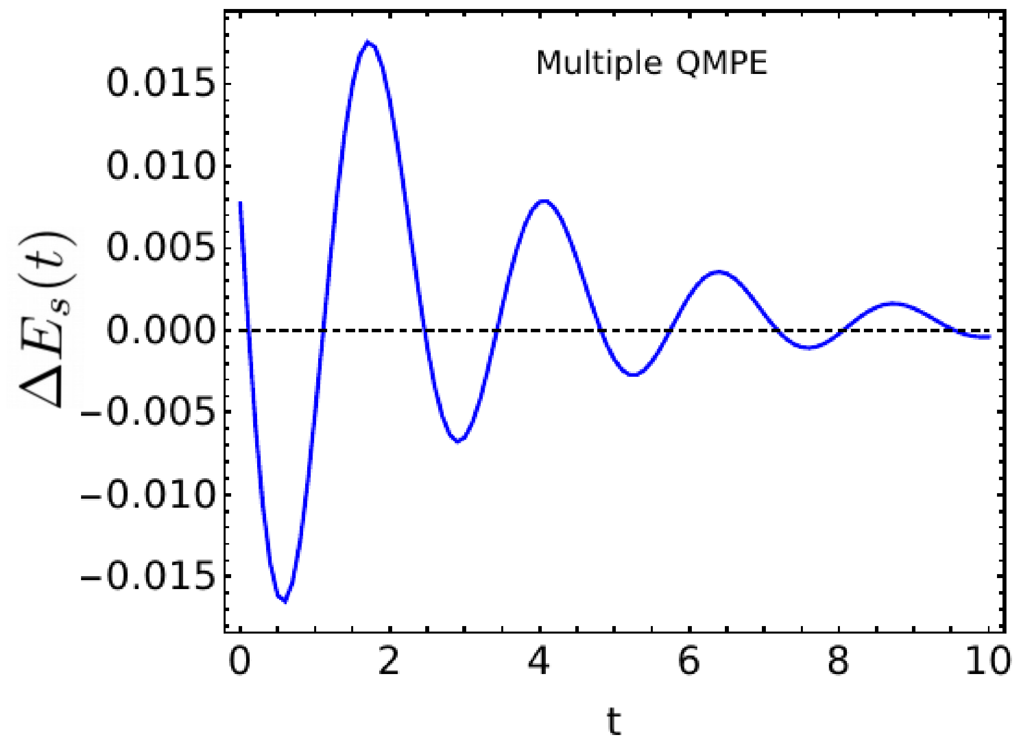
(effect of complex eigenvalues)

$$\Delta E_s(t) = E_s^{\mathrm{I}}(t) - E_s^{\mathrm{II}}(t) = e^{-\lambda_2 t}\nu_1 \left[\nu_2 + e^{-(\lambda_3^{\mathrm{re}}-\lambda_2)t}\cos(\lambda_3^{\mathrm{im}}t+\phi)\right]$$

$$\tilde{d}_{\mathrm{I}} = 2.1, \tilde{d}_{\mathrm{II}} = 0.5, \tilde{d} = 2.5, \tilde{\Gamma}_{\mathrm{I}} = \tilde{\Gamma}_{\mathrm{II}} = \tilde{\Gamma} = 0.5$$

$$\lambda_2 = 0.28,\ \lambda_3 = 0.36 + 2.69\mathrm{i},\ \lambda_3 = 0.36 - 2.69\mathrm{i}$$

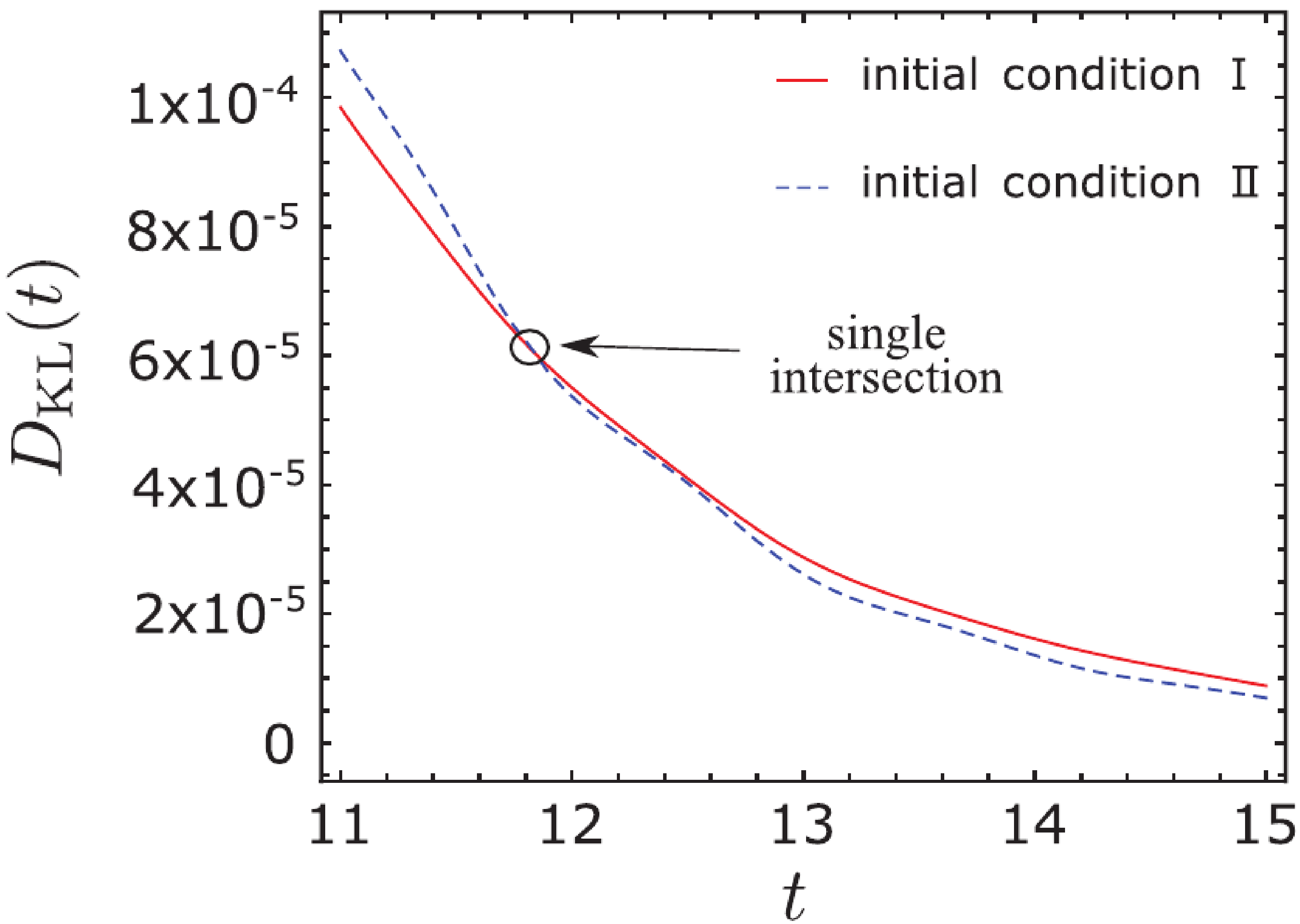




multiple crossing of two copies ⇨ multiple QMPE

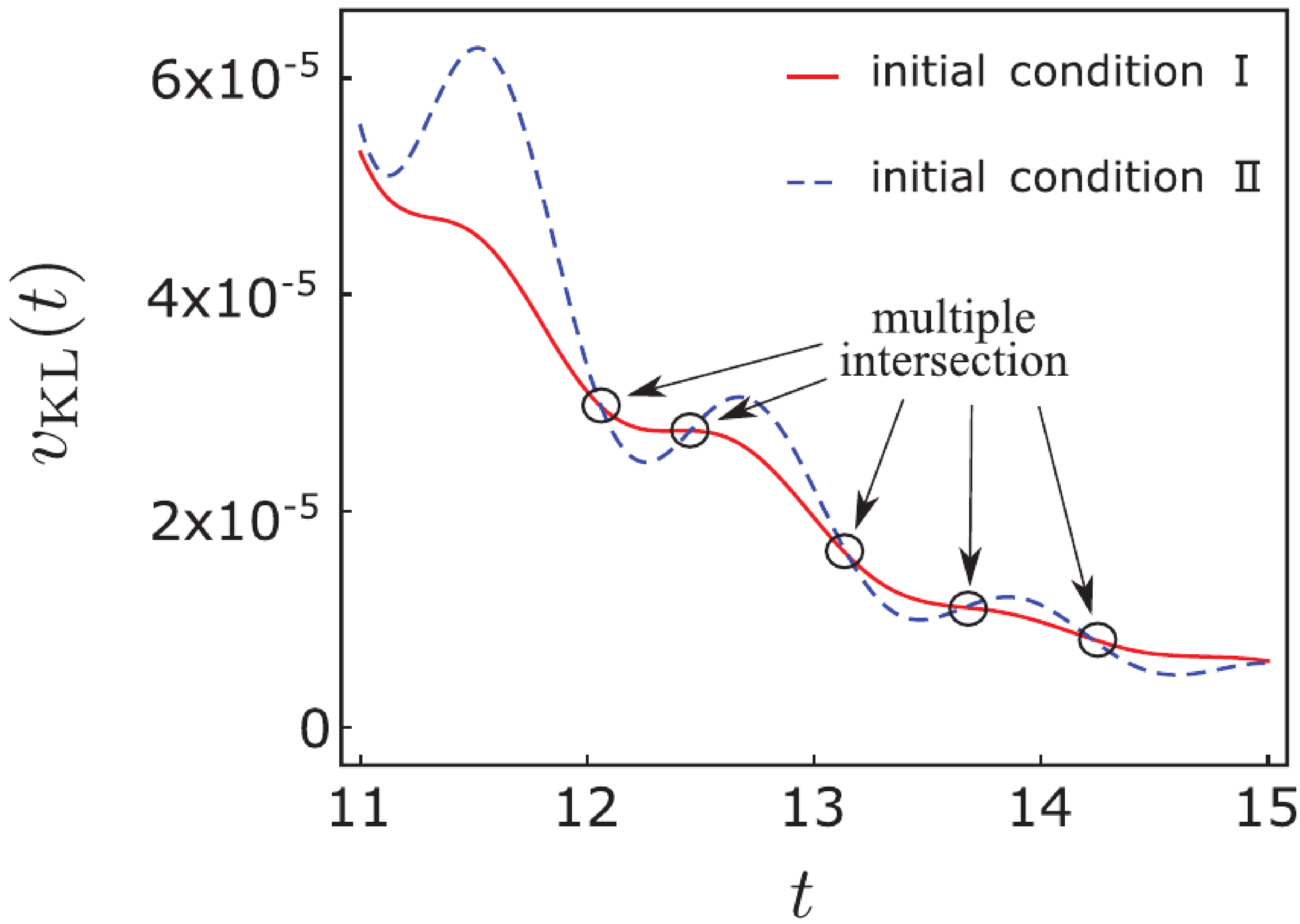
(Single) QMPE in KL divergence

$$D_{\mathrm{KL}}(t) := \mathrm{Tr}[\hat{\rho}(t)\{\ln[\hat{\rho}(t)] - \ln(\hat{\rho}_{\mathrm{ss}})\}]$$



(Multiple) QMPE in relaxation speed

$$v_{\mathrm{KL}}(t) = -\frac{\partial D_{\mathrm{KL}}(t)}{\partial t}$$



Summary

- A quantum system can cool faster when it starts from hotter initial temperature rather than colder initial temperature. This is the *thermal quantum Mpemba effect*.

  (demonstrated in a single level quantum dot with reservoirs)

- We achieve QMPE by controlling initial conditions before quench.

- Thermal QMPE does not necessarily imply distance QMPE and vice versa.

- We observe multiple QMPE in energy, relaxation speed at exceptional points and complex eigenvalues. But, single QMPE in distance.

[**Ref:** Amit Kumar Chatterjee, Satoshi Takada, and Hisao Hayakawa, PRL 131, 080402 (2023)
Amit Kumar Chatterjee, Satoshi Takada, and Hisao Hayakawa, arXiv:2311. 01347
(to be published in PRA)]

Future directions

- Quantum Mpemba effect bears the imprint of initial conditions. This is a kind of *quantum memory effect*. Further studies of its connection to other memory effects.

- Quantum Mpemba effect route to "faster relaxation" in quantum systems. Systematic control of QMPE can initiate speed up for computation and other processes of interest.

- Establishing thermal QMPE as a generic phenomenon in quantum system and finding its general mechanism. Relation to thermalization in integrable and non-integrable systems.

- QMPE in many body systems, corresponding mechanisms.

- Physical understanding (i.e. reasons) behind QMPE. (initial conditions ? quantum fluctuations ? Entanglement ?....)

THANK YOU