Frontiers in nonequilibrium physics 2024 2024/07/25

# Non-equilibrium physicsMany-body (equilibrium) physicsNon-reciprocalfrustrationphysics

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R. Hanai, Phys. Rev. X 14, 011029 (2024).





Ground state:  $x = \{x_i\}$  that minimizes the energy *E* 

Reciprocal coupling  $J_{ij} = J_{ji}$ 



### Non-reciprocally interacting systems

#### Ecological systems



http://animal.memozee.com/vie w.php?tid=3&did=3513



e.g., A. Pluchino, et al., Inter. J. Mod. Phys. C 16, 515 (2005).





## Collective phenomena in non-reciprocal many-body systems

Odd ela

Biodiversity in ecosystems



B. Kerr. et al., Nature 2002



C. Scheibn

0.0

 $v_o = 139.4 \text{ mm}/$ 

 $(\mathbf{b})_{-}(\mathbf{f})$  asymmetric

PHYSICAL REVIEW E

VOLUME 49, NUL Det dhichi, et al., Physan Rev 194 2020

#### Spatially uniform traveling cellular patterns at a driven interface

PHYSICAL REVIEW E

(a)

Lihor Department of Physics, Memorial Univer

> We report on a study of asymi interface in the experimental system pattern appears via a supercritical pa symmetry and begins to drift with co the drifting pattern as a function of t increases with the square root of the asymmetry. This behavior is in acco however, with the predictions of a m spatial modes with wave numbers q

> PACS number(s): 47.54.+r, 47.20.K

#### I. INTRODUCTION

Stationary, one-dimensional patterns occu dynamical systems [1]. Typically, an initiall uniform system develops such a pattern, de a one-dimensional wave vector, when it is di ciently far out of equilibrium by the applica appropriate external forcing. An example fre periment to be discussed in this paper is show This figure shows video images of an oil-air which is initially straight. As the interface is

You, Basebuilden a one-dimensional pattern of meers FIG. 5. PaSaha, Aguootic and the sector of t translation in time. It is periodic in space the finite length of the experimental apparati is invariant under translation in the direction

Loos, Klapp, and Martynec, arXiv:2206.1051

ong-ranged

order in 2D

VOLUME 49, NUMBER 1

Spatially uniform traveling cellular patterns at a driven interface

Lihong Pan and John R. de Bruyn Department of Physics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada A1B 3X7 (Received 4 August 1993)

We report on a study of asymmetric, traveling patterns which develop at a driven fluid-air interface in the experimental system known as the printer's instability. We find that the traveling pattern appears via a supercritical parity-breaking transition, at which the pattern loses its reflection symmetry and begins to drift with constant speed. From measurements of the degree of asymmetry of the drifting pattern as a function of the experimental control parameter, we find that the asymmetry increases with the square root of the control parameter, and that the drift velocity is linear in the asymmetry. This behavior is in accord with recent theoretical predictions. Our results do not agree, however, with the predictions of a model of the parity-breaking transition involving the coupling of spatial modes with wave numbers q and 2q.

PACS number(s): 47.54.+r, 47.20.Ky, 68.10.Gw

#### I. INTRODUCTION

Stationary, one-dimensional patterns occur in many dynamical systems [1]. Typically, an initially spatially uniform system develops such a pattern, described by a one-dimensional wave vector, when it is driven sufficiently far out of equilibrium by the application of an appropriate external forcing. An example from the ex-2021 figure shows video images of an oil-air interface, which is initially straight. As the interface is driven out try properties. Since it is stationary, it is invariant under translation in time. It is periodic in space (neglecting

#### propagates with constant velocity.

The transition from the stationary, symmetric of Fig. 1(a) to the traveling, asymmetric pattern 1(b) is an example of a parity-breaking bifurcation a bifurcation was postulated by Coullet et al. explanation for phenomena observed in other exp tal systems, to be discussed below. Parity-break shown to be one of ten possible generic secondar bilities of stationary one-dimensional patterns h let and Iooss [3]. Parity-breaking bifurcations cently been the subject of a substantial amount retical work [2,4-19]. Experimentally, both loca gions of broken parity, which propagate throug tionary background pattern, and extended broke traveling-wave states have been observed in seve oratory systems [20-35]. While the system we st tere the came not true of a

#### SPATIALLY UNIFORM TRAVELING CELLULAR 1

face at the front of the apparatus was monitored with a charge-coupled-device (CCD) video camera and monitor, and data were recorded on a VCR or stored on a personal computer using a video frame grabber. Images of the interface presented in this paper have been contrast enhanced, but are otherwise unprocessed. For quantitative analysis of the interface shape, the interface height as a function of x was extracted from video images by having the computer trace along the path of darkest pixels from a given starting point.

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In the experiments reported here, the minimum width of the gap between the cylinders was 0.5 mm, set with the micrometer screws. The stability of the stationary fingering pattern observed when only one cylinder rotated was very sensitive to the parallelism of the cylinder axes; this fact was used to optimize the cylinder alignment. From the way in which the stationary pattern appeared at its

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## Non-reciprocal interaction ≈Frustration physics



## Non-reciprocal friendship ≈Frustration physics



### Non-reciprocal friendship: source of frustration



## **Geometrical** frustration

#### -Geometrically frustrated system

A system that cannot satisfy all the constitutes' "desire" to minimize all interaction energies.



No configuration can make all spins happy

Accidental degeneracy induces exotic phenomena in geometrical frustrated systems

Order-by-disorder



Villain, et al., J. Physique (1980)

Spin glass

(Image from <a href="https://scglass.uchicago.edu/">https://scglass.uchicago.edu/</a> )

Quantum/Classical spin liquid







R. Mossener and J. T. Chalker, PRL 1998 T. Imai and Y. Lee, Physics Today 2016



(Image from Wikipedia)

Villain, et al., J. Physique (1980)

Accidental degeneracy: Not protected by symmetry nor topology



(Example) <u>Antiferromagnetic XY spins on a pyrochlore lattice</u>



Disordered state

Long-ranged order

## Geometrical vs Non-reciprocal frustration

#### **Geometrical** frustration

Accidental degeneracy of ground state



Non-reciprocal frustration

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Energy cannot be defined ...

May not even converge to a static state...

## Geometrical vs Non-reciprocal frustration

#### **Geometrical** frustration

## Accidental degeneracy of ground state



#### Non-reciprocal frustration

"Accidental degeneracy" of orbits



Dynamical counterpart of order-bydisorder and spin glass(-like) state occurs!

This talk: order by disorder

## Dissipative XY spin dynamics

$$\dot{\theta}_i = \sum_j J_{ij} \sin(\theta_j - \theta_i)$$

Here, couplings are non-reciprocal in general  $J_{ij} \neq J_{ji}$ 

## Dissipative XY spin dynamics

> Reciprocal case  $(J_{ij} = J_{ji})$  : Energy minimization problem

$$\dot{\theta}_i = -\frac{\partial E(\theta)}{\partial \theta_i}$$
 with  $E(\theta) = -\sum_{i,j} J_{ij} \cos(\theta_j - \theta_i)$ 

Potential with geometrically frustration



Accidentally degenerate ground states



> Orbits



## "Accidental degeneracy" of orbits

> Anti-symmetric case  $(J_{ij} = -J_{ji})$ 



Conservation of phase volume = **Non-dissipative** dynamics



Marginal





Common feature between geometrically frustrated and non-reciprocal system: Marginal orbits!

## Order-by-disorder phenomena in geometrically frustrated systems

Zero temperature

Finite temperature



## Order-by-disorder phenomena in geometrically frustrated systems

Zero temperature

➤ Finite temperature

## Q: Does the same physics occur in **non-reciprocally** interacting systems?



> Dynamics (T > 0):

 $\dot{\theta} = -\frac{\partial F}{\partial \theta} = \frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial \theta}$ Entropic force

Entropic force selects the orbit! = order by disorder



$$\dot{\theta}_{i}^{a} = \sum_{b} \sum_{i=1}^{N_{b}} \frac{j_{ab}}{N_{b}} \sin(\theta_{j}^{b} - \theta_{i}^{a})$$

$$A \qquad B$$

$$\overset{\text{``Accidental}}{\text{degeneracy''}} \text{parameterized by} \qquad A \qquad find a bound on the second seco$$

Macroscopic spin (No noise)

$$\dot{\phi}_a = \sum_b j_{ab} \sin(\phi_b - \phi_a)$$



Macroscopic spin (No noise)

$$\dot{\phi}_a = \sum_b j_{ab} \sin(\phi_b - \phi_a)$$



 $\Delta \phi$ -dependent renormalized coupling





## Order-by-disorder phenomena $j_{AB} = -j_{BA}$

#### <u>No noise</u>

With noise



#### Noise-induced non-reciprocal phase transition $j_{AB} > -j_{BA}$ $\Delta \dot{\phi} = \left[ -2j_{+} + \frac{j_{0}j_{-}^{2}\sigma^{2}}{2} \frac{\cos \Delta \phi}{(j_{0}^{2} - j_{-}^{2}\cos^{2}\Delta \phi)^{2}} \right] \sin \Delta \phi \qquad j_{+} = j_{AB} + j_{BA} > 0$ "Entropic" force "Energetics" favoring $\Delta \phi = \pm \frac{\pi}{2}$ favoring $\Delta \phi = 0$ Z<sub>2</sub> symmetric Z<sub>2</sub>broken 0.50 TT 0.25 Noise induced ∆φ/π 0.00 symmetry breaking! $j_{AB} = 0.35$ -0.25See also, $j_{BA} = -0.25$ Fruchart\*, RH\*, Littlewood, Vitelli, Nature 2021 -0.50↓ 0.0 0.4 0.2 0.6 Noise strength $\sigma$

RH, PRX2024

 $j_{AA} = j_{BB} = j_0$ 

## Summary

 Pointed out a direct analogy between geometrical and non-reciprocal frustration

![](_page_28_Figure_2.jpeg)

RH, Phys. Rev. X **14**, 011029 (2024). RH and Weis, in preparation

### Derivation of renormalized coupling

Equation of motion

Order parameter dynamics in the absence of noise

$$\dot{\phi}_a = \sum_b j_{ab} \sin(\phi_b - \phi_a), \quad \bigstar$$

Marginal orbit (i.e. initial state dependent orbits) emerges when  $j_{ab} = -j_{ba}$ 

Fluctuation dynamics 
$$\delta \theta_i^a = \theta_i^a - \phi_a$$
  $\delta \dot{\theta}_i^a \approx -\sum_b j_{ab} \cos(\phi_a(t) - \phi_b(t)) \delta \theta_i^a + \eta_i^a$ .

Distribution function

$$\rho_i^a(t, \delta \theta_i^a; \phi(t)) = \frac{1}{\sqrt{\pi} w_a(t; \phi(t))} e^{-(\delta \theta_i^a)^2 / w_a^2(t; \phi(t))}$$

with configuration dependent width

$$w_{a}^{2}(t;\phi(t)) = 2\sigma \int_{0}^{t} d\tau e^{-2\int_{\tau}^{t} d\tau' \sum_{b} j_{ab} \cos(\phi_{a}(\tau') - \phi_{b}(\tau'))}$$

Especially when  $\Delta \phi_{ab} = \phi_a - \phi_b = \text{const.}$ ,  $w_a^2(\phi) = \frac{\sigma}{\sum_b j_{ab} \cos \Delta \phi_{ab}}$ 

### Derivation of renormalized coupling

Order parameter dynamics  $\psi_a = r_a e^{i\phi_a}$ 

$$\dot{\psi}_a = (\dot{r}_a + r_a i \dot{\phi}_a) e^{i\phi_a} = \frac{i}{N_a} \sum_{i=1}^{N_a} \dot{\theta}_i^a e^{i\theta_i^a}$$

$$\dot{\phi}_a = -\sum_b \frac{j_{ab}}{N_a} \sum_{i=1}^{N_a} \frac{r_b}{r_a} \sin(\theta_i^a - \phi_b) \cos(\theta_i^a - \phi_a) + \bar{\eta}_a$$
$$= -\sum_b j_{ab}^{\star}(\phi(t)) \sin(\phi_a - \phi_b) + \bar{\eta}_a$$

with renormalized coupling  $j_{ab}^{\star}(\phi(t)) = j_{ab} \frac{r_b(\phi(t))}{r_a(\phi(t))} \left\langle \cos^2 \delta \theta_i^a \right\rangle_{\phi(t)}$ 

where  $\langle \cdots \rangle_{\phi(t)} = \int d\delta \theta_i^a \rho_i^a (t, \delta \theta_i^a; \phi(t)) (\cdots)$ 

Distribution function  $\rho_i^a(t, \delta \theta_i^a; \phi(t)) = \frac{1}{\sqrt{\pi} w_a(t; \phi(t))} e^{-(\delta \theta_i^a)^2 / w_a^2(t; \phi(t))}$ 

Here, self-averaging  $\frac{1}{N_a} \sum_{i}^{N_a} (\cdots) = \langle \cdots \rangle_{\phi}$  is assumed.

Macroscopic noise strength

$$\langle \bar{\eta}_a(t)\bar{\eta}_b(t')\rangle\approx \frac{\sigma}{N_a}\delta_{ab}\delta(t-t') \longrightarrow 0 \; (N_a\to\infty)$$

## "Accidental degeneracy" of orbits

### [Proof]

Continuity equation: 
$$\frac{\partial \rho}{\partial t} = -\sum_{i} \frac{\partial (\rho \dot{\theta}_{i})}{\partial \theta_{i}} = -\sum_{i} \left[ \frac{\partial \rho}{\partial \theta_{i}} \dot{\theta}_{i} + \rho \frac{\partial \dot{\theta}}{\partial \theta_{i}} \right]$$

$$\begin{split} \sum_{i} \frac{\partial \dot{\theta}_{i}}{\partial \theta_{i}} &= \sum_{ij} \left[ J_{ij} \cos(\theta_{j} - \theta_{i}) \right] = 0 \\ \uparrow \\ J_{ij} &= -J_{ji} \\ \dot{\theta}_{i} &= \sum_{j} J_{ij} \sin(\theta_{j} - \theta_{i}) \\ \text{Therefore,} & \frac{\partial \rho}{\partial t} + \sum_{i} \frac{\partial \rho}{\partial \theta_{i}} \dot{\theta}_{i} = 0. \end{split}$$

XA similar theorem holds for non-reciprocally interacting particles and Heisenberg spins.