Collective advantages in finite-time thermodynamics

Martí Perarnau-Llobet

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Swiss National Science Foundation

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Finite-time Landauer principle beyond weak coupling

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Collective Advantages in Finite-Time Thermodynamics

Alberto Rolandi, Paolo Abiuso, and Martí Perarnau-Llobet
Phys. Rev. Lett. **131**, 210401 – Published 22 November 2023

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ABSTRACT

A central task in finite-time thermodynamics is to minimize the excess or dissipated work W_{diss} when manipulating the state of a system immersed in a thermal bath. We consider this task for an N-body system whose constituents are identical and uncorrelated at the beginning and end of the process. In the regime of slow but finite-time processes, we show that W_{diss} can be dramatically reduced by considering collective protocols in which interactions are suitably created along the protocol. This can even lead to a sublinear growth of W_{diss} with N: $W_{\mathrm{diss}} \propto N^x$ with $x < 1$; to be contrasted to the expected $W_{\mathrm{diss}} \propto N$ satisfied in any noninteracting protocol. We derive the fundamental limits to such collective advantages and show that $x = 0$ is in principle possible; however, it requires long-range interactions. We explore collective processes with spin models featuring two-body interactions and achieve noticeable gains under realistic levels of control in simple interaction architectures. As an application of these results, we focus on the erasure of information in finite time and prove a faster convergence to Landauer's bound.

Collective advantages in physics

Phase transitions, entanglement, superradiance…

The outcome of a task is improved when performed globally on a collection of systems than when realized on each system individually.

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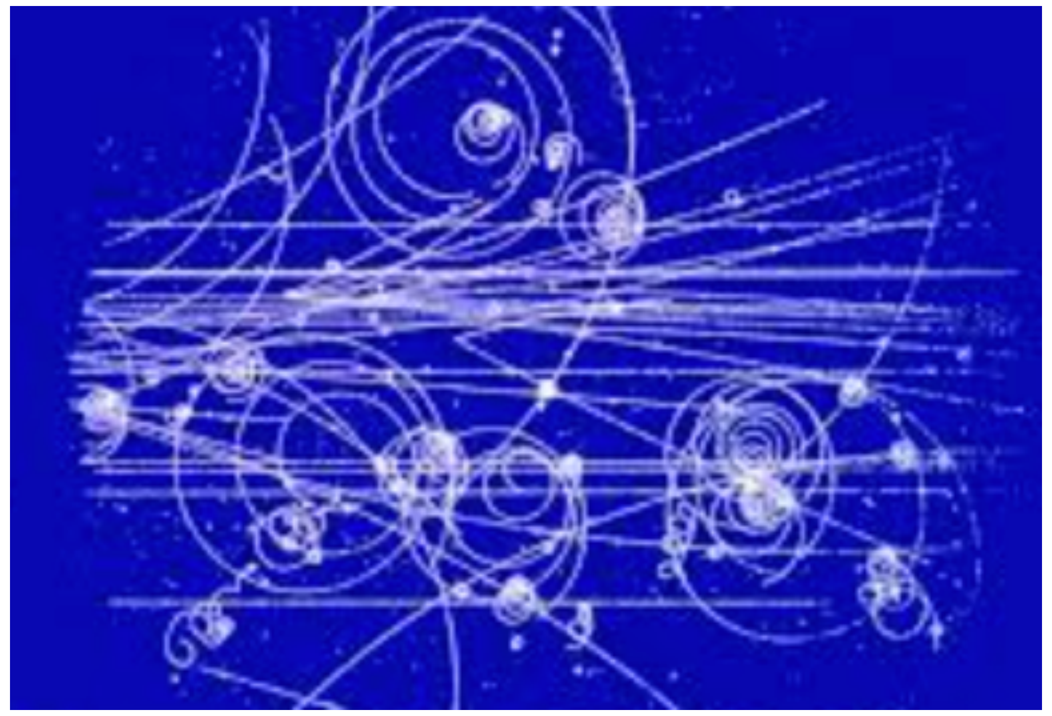
Measurements



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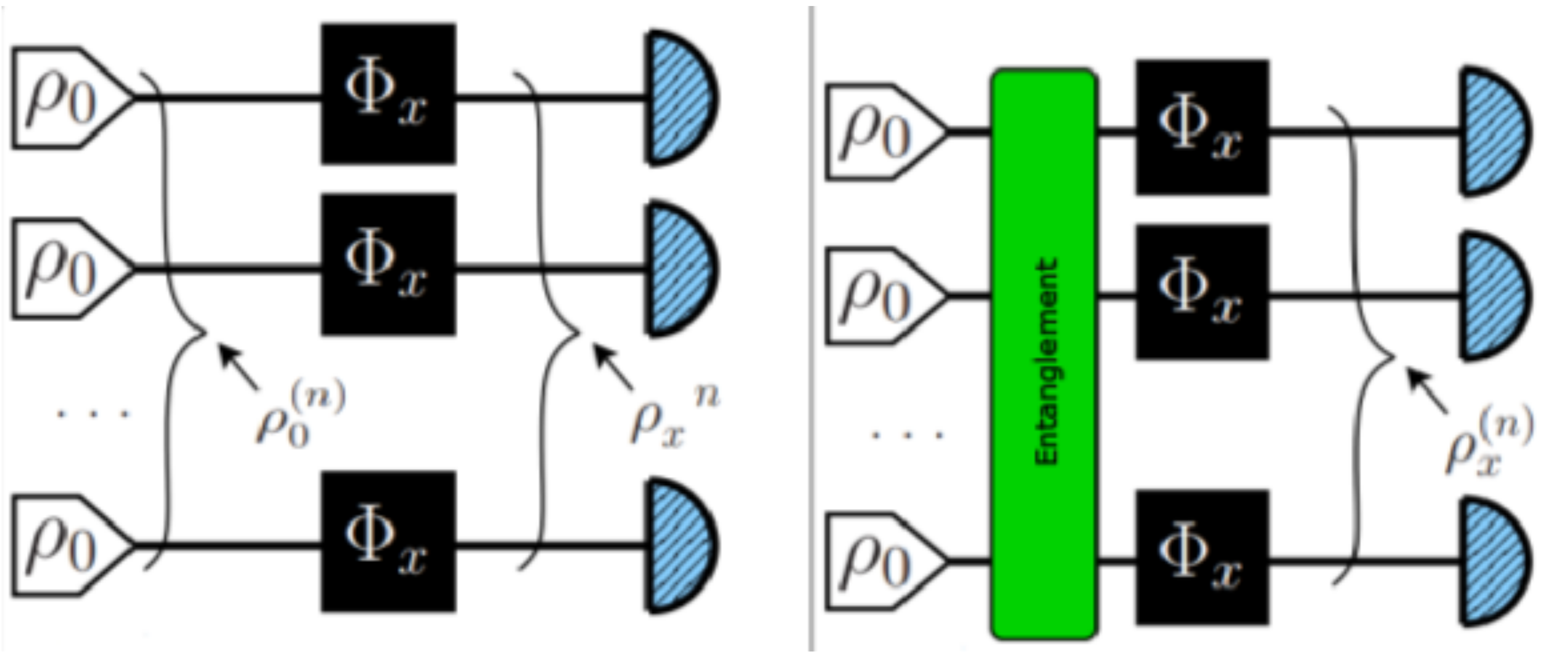
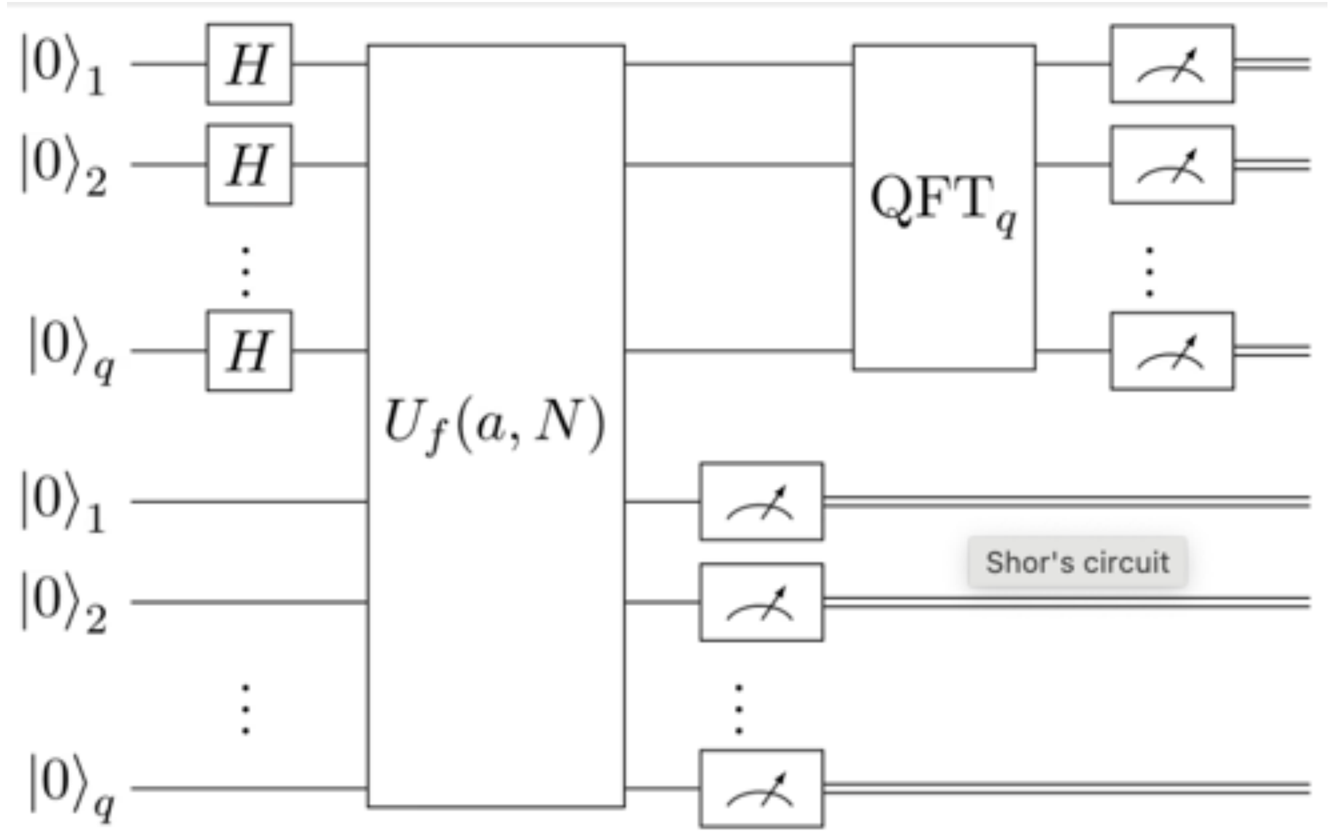
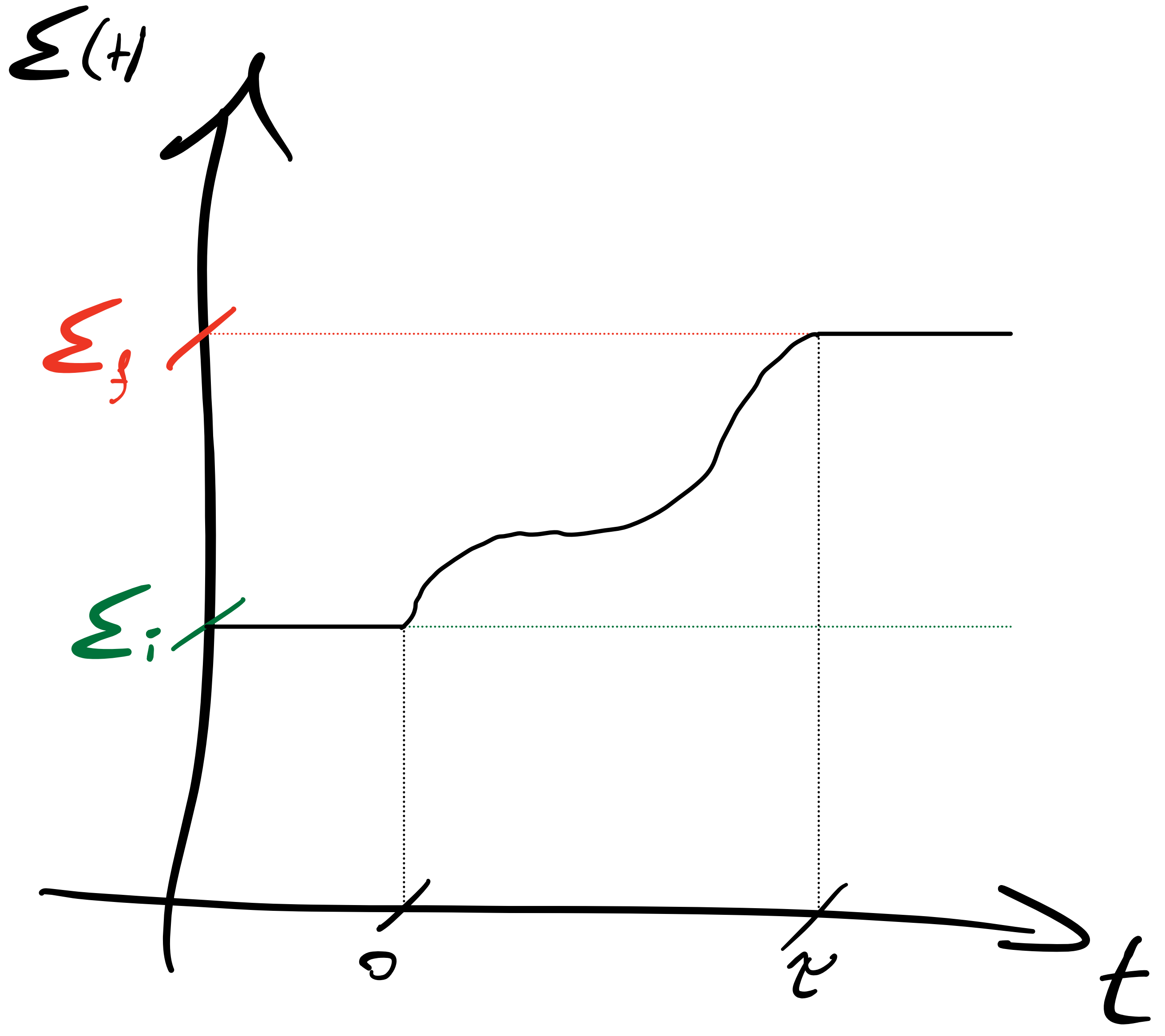
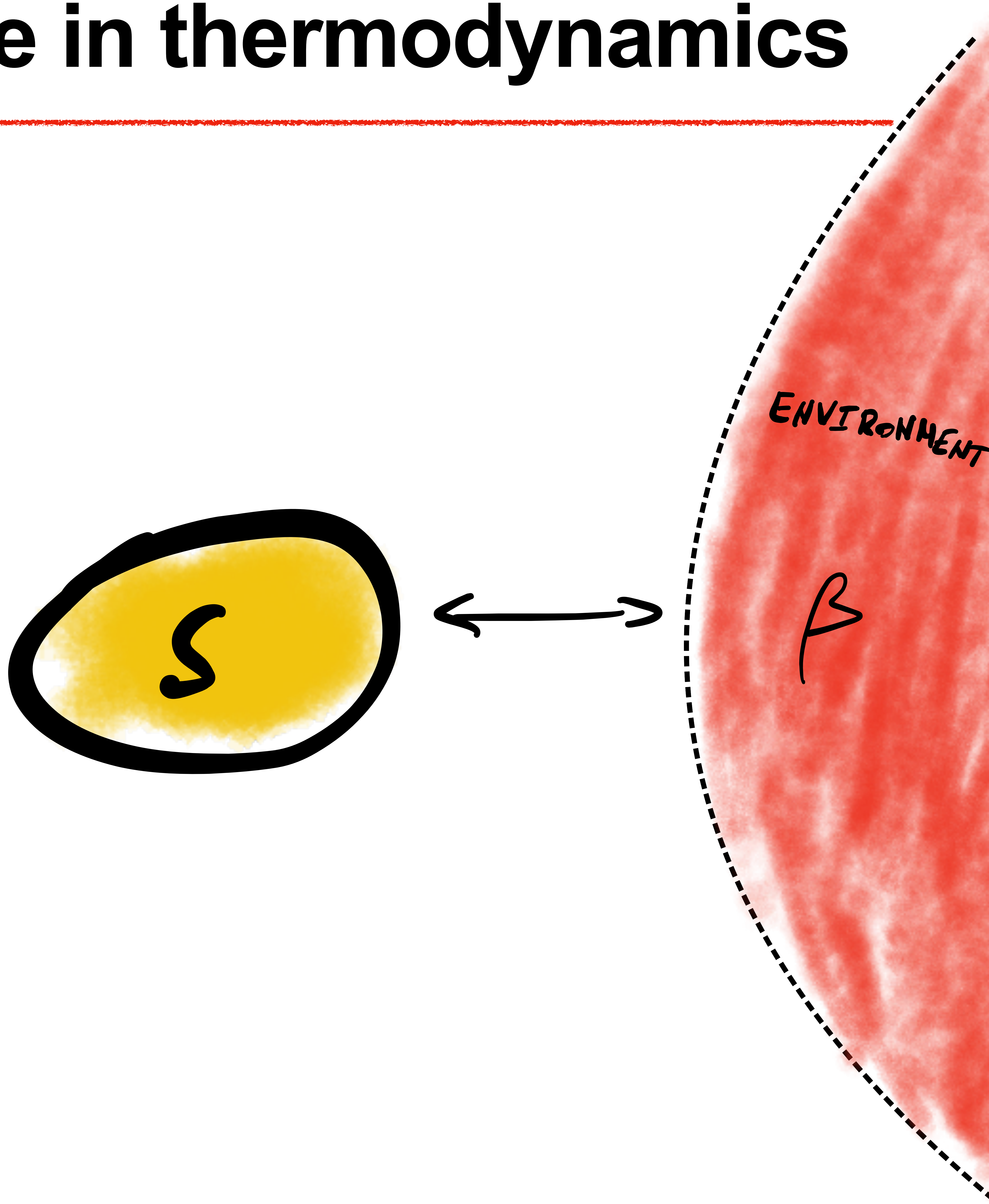


Computation



This talk: Collective advantage in thermodynamics





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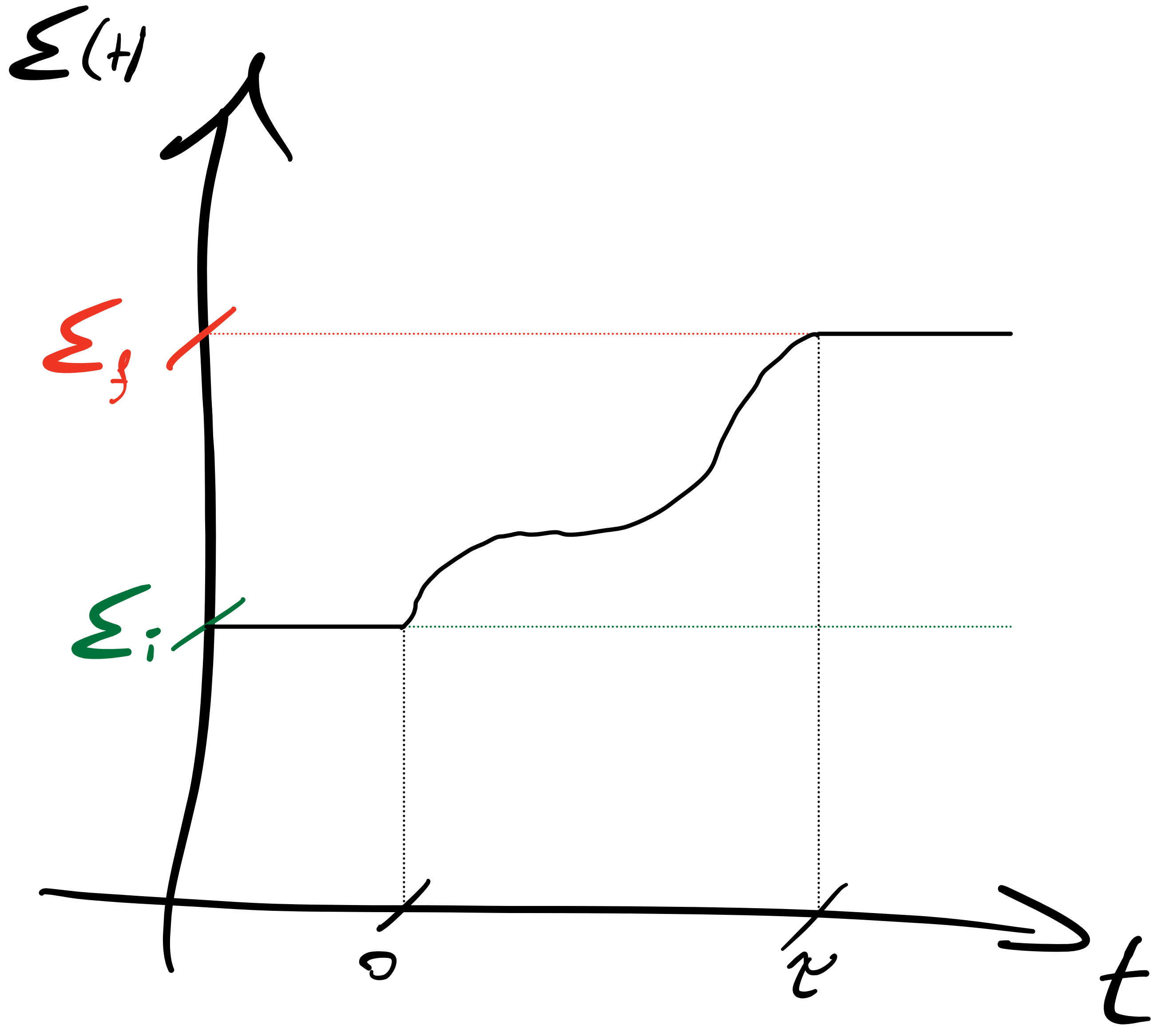
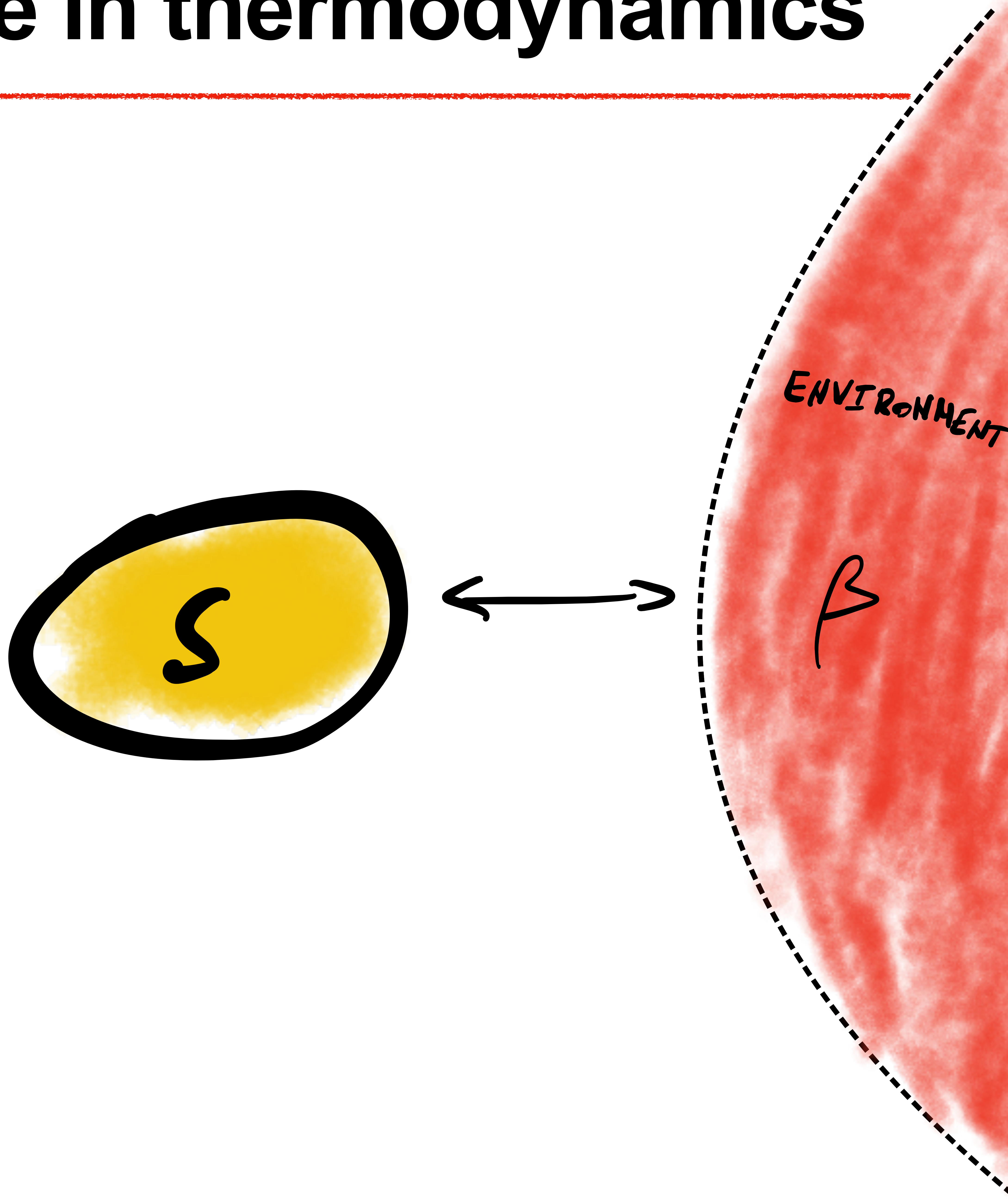
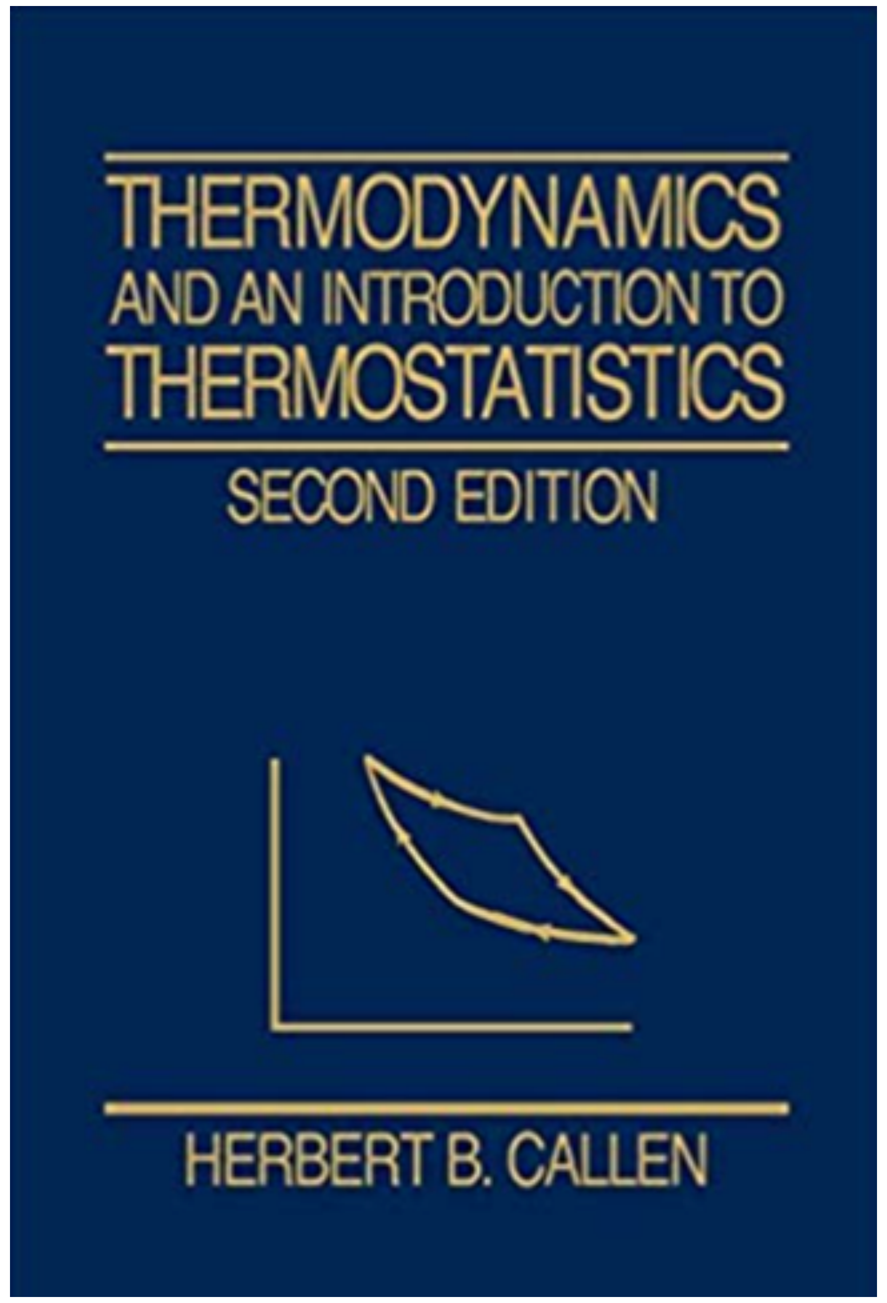


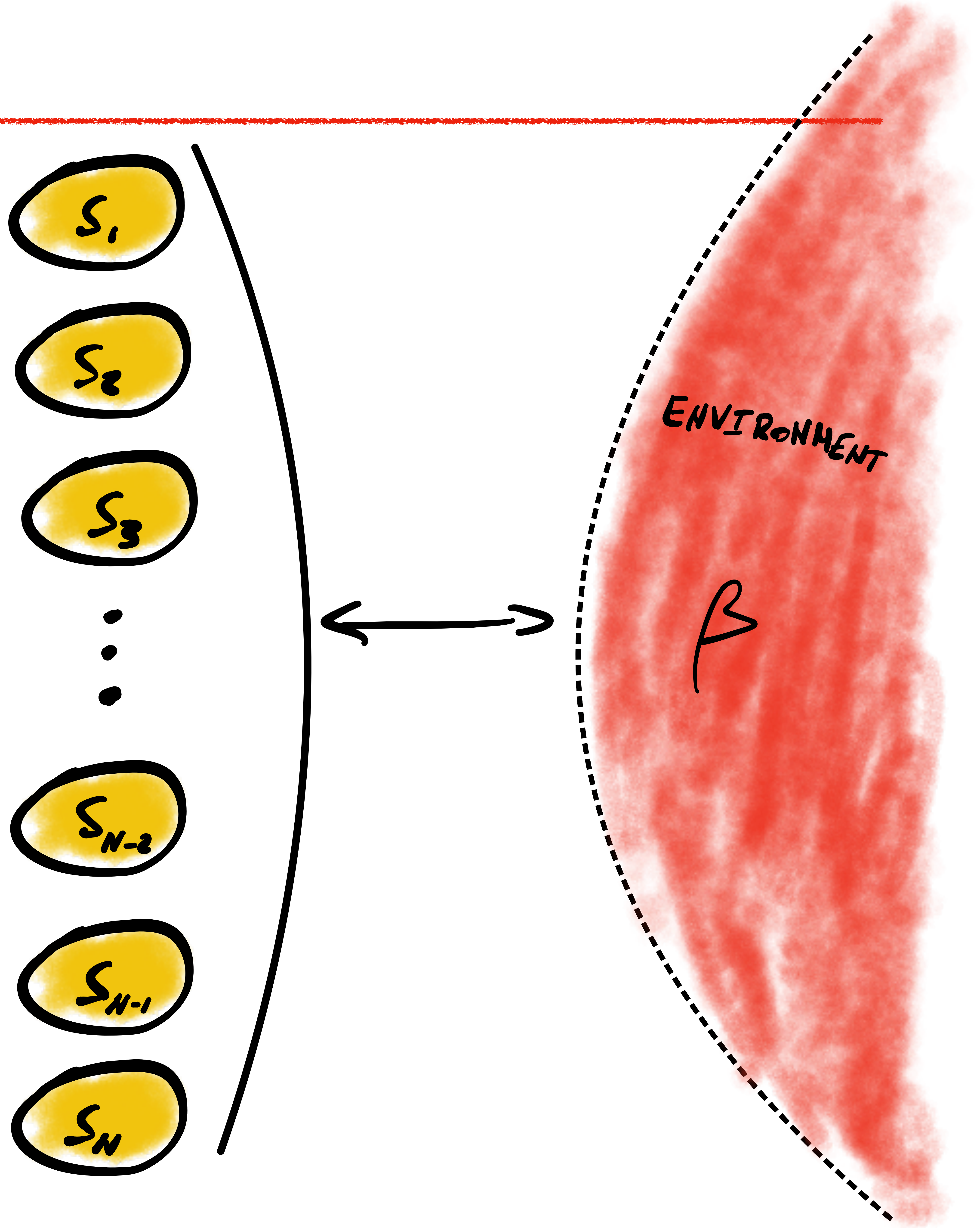




$$W = \Delta F + W_{diss}$$

Sublinear dissipation?

$$W = \Delta F + W_{diss}$$

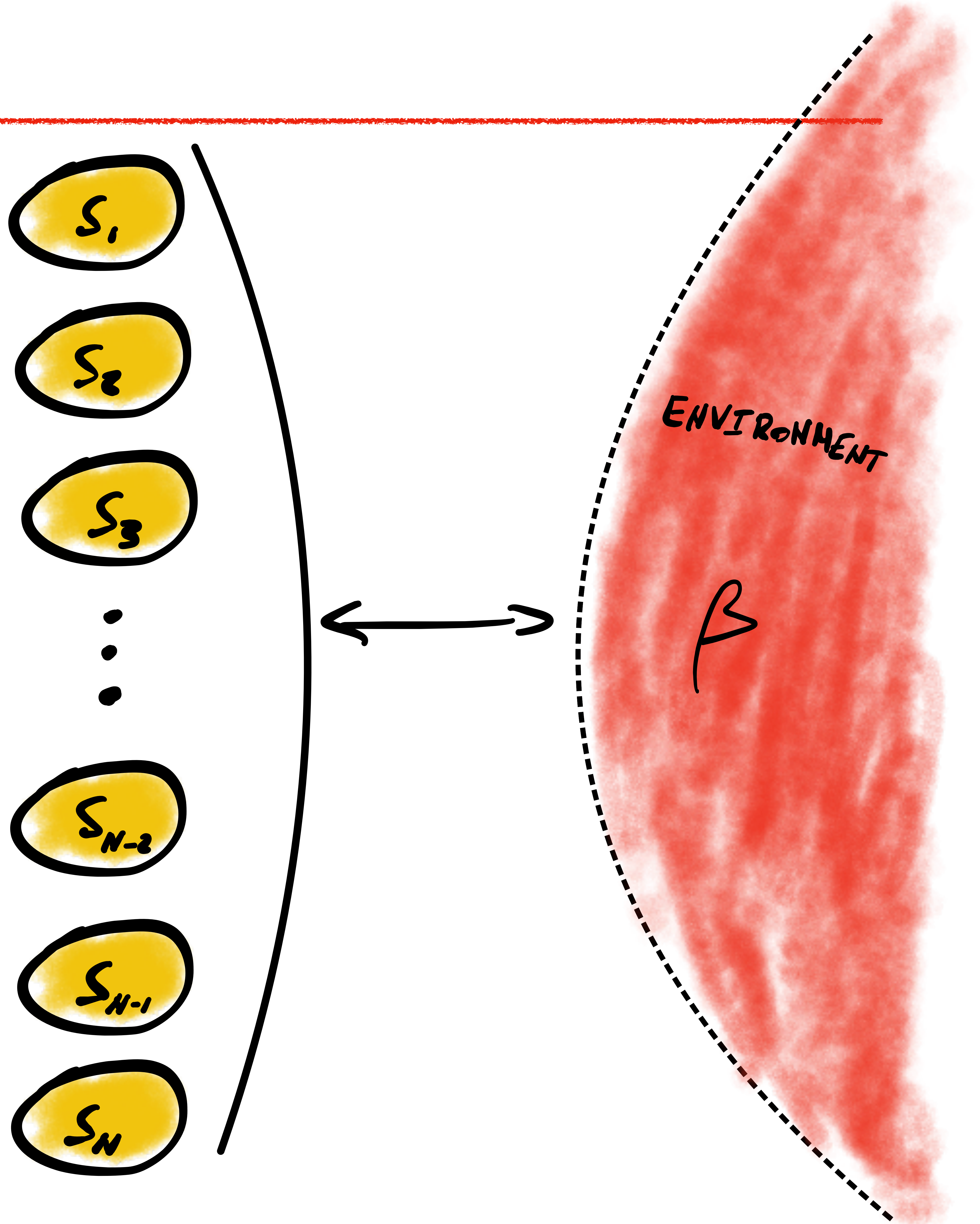


Sublinear dissipation?

$$W = \Delta F + W_{diss}$$

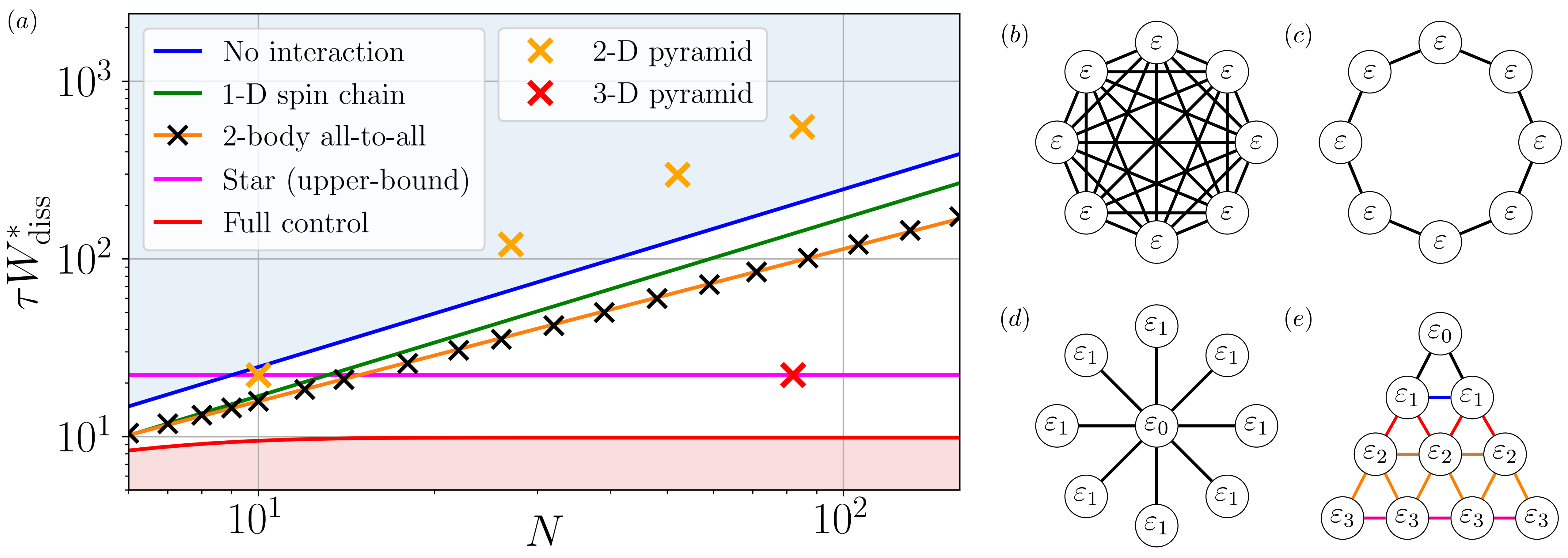
Extensive

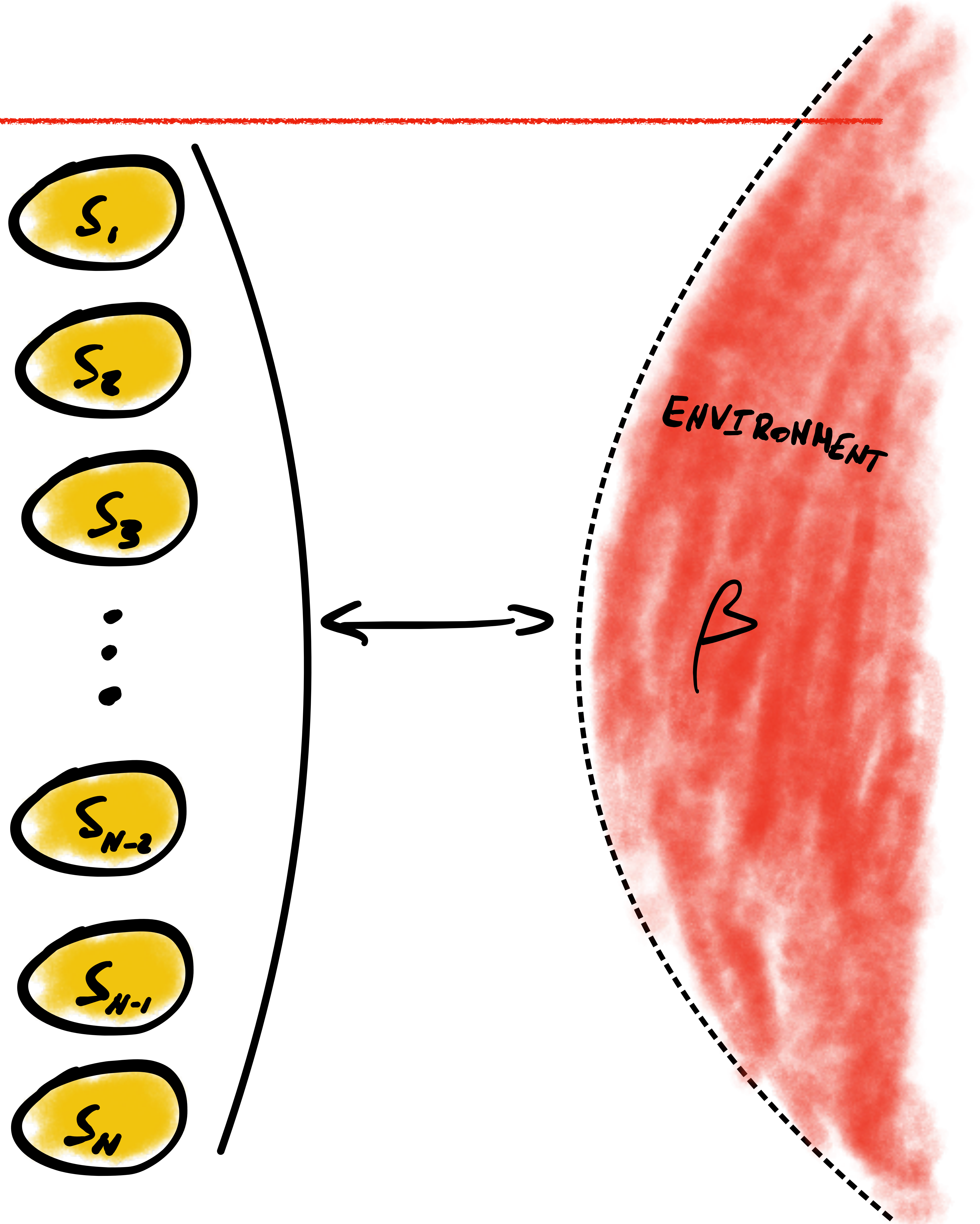
?



Sublinear dissipation?

$$W = \Delta F + W_{diss}$$





Collective advantages in thermodynamics

Quantum batteries

PHYSICAL REVIEW LETTERS 128, 140501 (2022)

Editors' Suggestion | Featured in Physics

Quantum Charging Advantage Cannot Be Extensive without Global Operations

Ju-Yeon Gyhm,[1,2,*] Dominik Šafránek,[1,†,‡] and Dario Rosa[1,4,§]
[1]*Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Republic of Korea*
[2]*Department of Physics and Astronomy, Seoul National University, 1 Gwanak-ro, Seoul 08826, Korea*

(Received 13 August 2021; accepted 8 February 2022; published 4 April 2022)

Quantum batteries are devices made from quantum states, which store and release energy in a fast and efficient manner, thus offering numerous possibilities in future technological applications. They offer a

RL 118, 150601 (2017) PHYSICAL REVIEW LETTERS week ending 14 APRIL 2017

Enhancing the Charging Power of Quantum Batteries

Francesco Campaioli,[1,*] Felix A. Pollock,[1] Felix C. Binder,[2] Lucas Céleri,[3] John Goold,[4] Sai Vinjanampathy,[5,6] and Kavan Modi[1,†]
[1]*School of Physics and Astronomy, Monash University, Victoria 3800, Australia*
[2]*School of Physical & Mathematical Sciences, Nanyang Technological University, 637371 Singapore, Singapore*
[3]*Instituto de Física, Universidade Federal de Goiás, Caixa Postal 131, 74001-970, Goiânia, Brazil*
[4]*The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste 34151, Italy*

Thermal engines

Open Access | Published: 20 June 2016

The power of a critical heat engine

Michele Campisi & Rosario Fazio

Nature Communications 7, Article number: 11895 (2016) | Cite this article

PHYSICAL REVIEW LETTERS 127, 190604 (2021)

Editors' Suggestion | Featured in Physics

Superconducting-like Heat Current: Effective Cancellation of Current-Dissipation Trade-Off by Quantum Coherence

Hiroyasu Tajima
Graduate School of Informatics and Engineering, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo 182-8585, Japan

DOI: 10.1103/PhysRevLett.127.190604

Quantum transport

PHYSICAL REVIEW LETTERS 120, 090601 (2018)

Thermodynamic Bounds on Precision in Ballistic Multiterminal Transport

Kay Brandner,[1] Taro Hanazato,[2] and Keiji Saito[2]
[1]*Department of Applied Physics, Aalto University, 00076 Aalto, Finland*
[2]*Department of Physics, Keio University, 3-14-1 Hiyoshi, Yokohama 223-8522, Japan*

(Received 10 October 2017; revised manuscript received 28 December 2017; published 2 March 2018)

For classical ballistic transport in a multiterminal geometry, we derive a universal trade-off relation between total dissipation and the precision, at which particles are extracted from individual reservoirs. Remarkably, this bound becomes significantly weaker in the presence of a magnetic field breaking time-reversal symmetry. By working out an explicit model for chiral transport enforced by a strong magnetic field, we show that our bounds are tight. Beyond the classical regime, we find that, in quantum systems far from equilibrium, the correlated exchange of particles makes it possible to exponentially reduce the thermodynamic cost of precision.

DOI: 10.1103/PhysRevLett.120.090601

PHYSICAL REVIEW B 104, 045424 (2021)

Broadband frequency filters with quantum dot chains

Tilmann Ehrlich[1] and Gernot Schaller[1,2,*]
[1]*Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstr. 36, 10623 Berlin, Germany*
[2]*Helmholtz-Zentrum Dresden-Rossendorf, Bautzner Landstraße 400, 01328 Dresden, Germany*

(Received 12 March 2021; revised 11 June 2021; accepted 12 July 2021; published 22 July 2021)

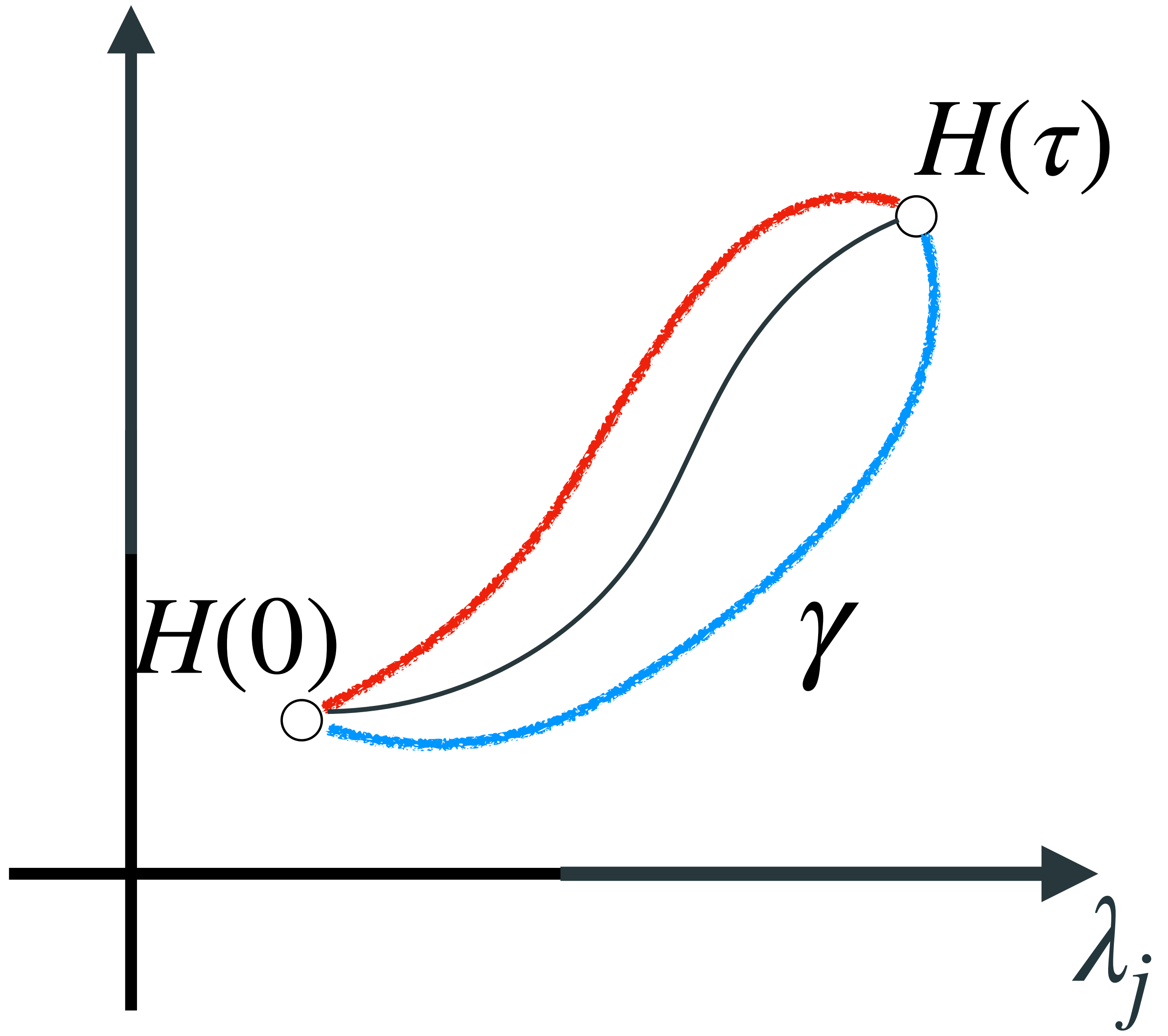
Two-terminal electronic transport systems with a rectangular transmission can violate standard thermodynamic uncertainty relations. This is possible beyond the linear response regime and for parameters that are not accessible with rate equations obeying detailed balance. Looser bounds originating from fluctuation theorem symmetries alone remain respected. We demonstrate that optimal finite-sized quantum dot chains can implement rectangular transmission functions with high accuracy and discuss the resulting violations of standard thermodynamic uncertainty relations as well as heat engine performance.

DOI: 10.1103/PhysRevB.104.045424

Collective phenomena $\rightleftarrows$ Thermodynamics

Thermodynamic geometry

A framework for optimising finite-time thermodynamic processes



$$\beta W_{diss} \geq \frac{1}{\tau} L^2$$

Historical perspective

Entropy (Basel). 2020 Aug; 22(8): 908.
Published online 2020 Aug 18. doi: 10.3390/e22080908

How It All Began

R. Stephen Berry,[1] Peter Salamon,[2] and Bjarne Andresen[3,*]

1. R. Stephen Berry

Go to:

The topic—or field—of finite time thermodynamics has an interesting history. Its stimulus was a far cry from a motivation to do basic science. Its real origins began when I moved to The University of Chicago in 1964. I had thought I was prepared to adapt to the Chicago environment, but it turned out otherwise. At that time, Chicago was a very smoky, dirty, even smelly city. Each morning, windowsills had new layers of fine grit that had drifted in from the outside during the night. I found myself angry that my new city could have such terrible air pollution. I was sufficiently troubled that I wrote a letter to then-Mayor Richard Daley, which began, "Dear Mayor Daley, You live like a pig!" I went on to say that I had heard that the City of

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As a freshman graduate student looking for an advisor to work with, I was looking for one to underwrite a project exploring the differential geometry of thermodynamics. When I approached Steve Berry, he responded with a question, "While you're at it, can you put time in?" It was spring of 1973 and I had found my mentor.

The notion seemed intriguing. Having been raised in the deeply structuralist traditions prevailing in mathematics departments in the 1970s, it seemed likely to me that understanding the mathematical structure of thermodynamics would enable us to see how this structure might accommodate time. I had studied the differential geometrical framework of classical mechanics and knew how time dependence changes the symplectic structure on the manifold of configurations into a contact structure. I thought there was a good chance of finding something similar for thermodynamics.

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Metric geometry of equilibrium thermodynamics

F. Weinhold

J. Chem. Phys. 63, 2479–2483 (1975)
https://doi.org/10.1063/1.431689

Tools

It is shown that the principal empirical laws of equilibrium thermodynamics can be brought into correspondence with the mathematical axioms of an abstract metric space. This formal correspondence permits one to associate with the thermodynamic formalism a geometrical aspect, with intrinsic metric structure, which is distinct from that arising from graphical representations of equilibrium surfaces in phase space.

$$\beta W_{diss} \geq \frac{1}{\tau} L^2$$

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VOLUME 51, NUMBER 13 PHYSICAL REVIEW LETTERS 26 SEPTEMBER 1983

Thermodynamic Length and Dissipated Availability

Peter Salamon
Department of Mathematical Sciences, San Diego State University, San Diego, California 92182

and

R. Stephen Berry
Department of Chemistry and The James Franck Institute, The University of Chicago, Chicago, Illinois 60637

(Received 10 January 1983)

New expressions for the availability dissipated in a finite-time endoreversible process are found by use of Weinhold's metric on equilibrium states of a thermodynamic system. In particular, the dissipated availability is given by the square of the length of the corresponding curve, times a mean relaxation time, divided by the total time of the process. The results extend to local thermodynamic equilibrium if instead of length one uses distance (length of the shortest curve) between initial and final states.

PACS numbers: 05.70.-a

$$\beta W_{diss} \geq \frac{1}{\tau} L^2$$

Historical perspective

From (macroscopic) thermodynamics to stochastic thermodynamics:

Thermodynamic Metrics and Optimal Paths

David A. Sivak and Gavin E. Crooks
Phys. Rev. Lett. **108**, 190602 – Published 8 May 2012

$$\beta W_{diss} \geq \frac{1}{\tau} L^2$$

Historical perspective

From (macroscopic) thermodynamics to stochastic thermodynamics:

Thermodynamic Metrics and Optimal Paths

David A. Sivak and Gavin E. Crooks
Phys. Rev. Lett. **108**, 190602 – Published 8 May 2012

From stochastic thermodynamics to quantum thermodynamics:

Thermodynamic length for far-from-equilibrium quantum systems

Sebastian Deffner and Eric Lutz
Phys. Rev. E **87**, 022143 – Published 28 February 2013

Thermodynamic length in open quantum systems

Matteo Scandi and Martí Perarnau-Llobet

Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany

Published:	2019-10-24, volume 3, page 197
Eprint:	arXiv:1810.05583v5
Doi:	https://doi.org/10.22331/q-2019-10-24-197
Citation:	Quantum 3, 197 (2019).

$$\beta W_{diss} \geq \frac{1}{\tau} L^2$$

… and many more

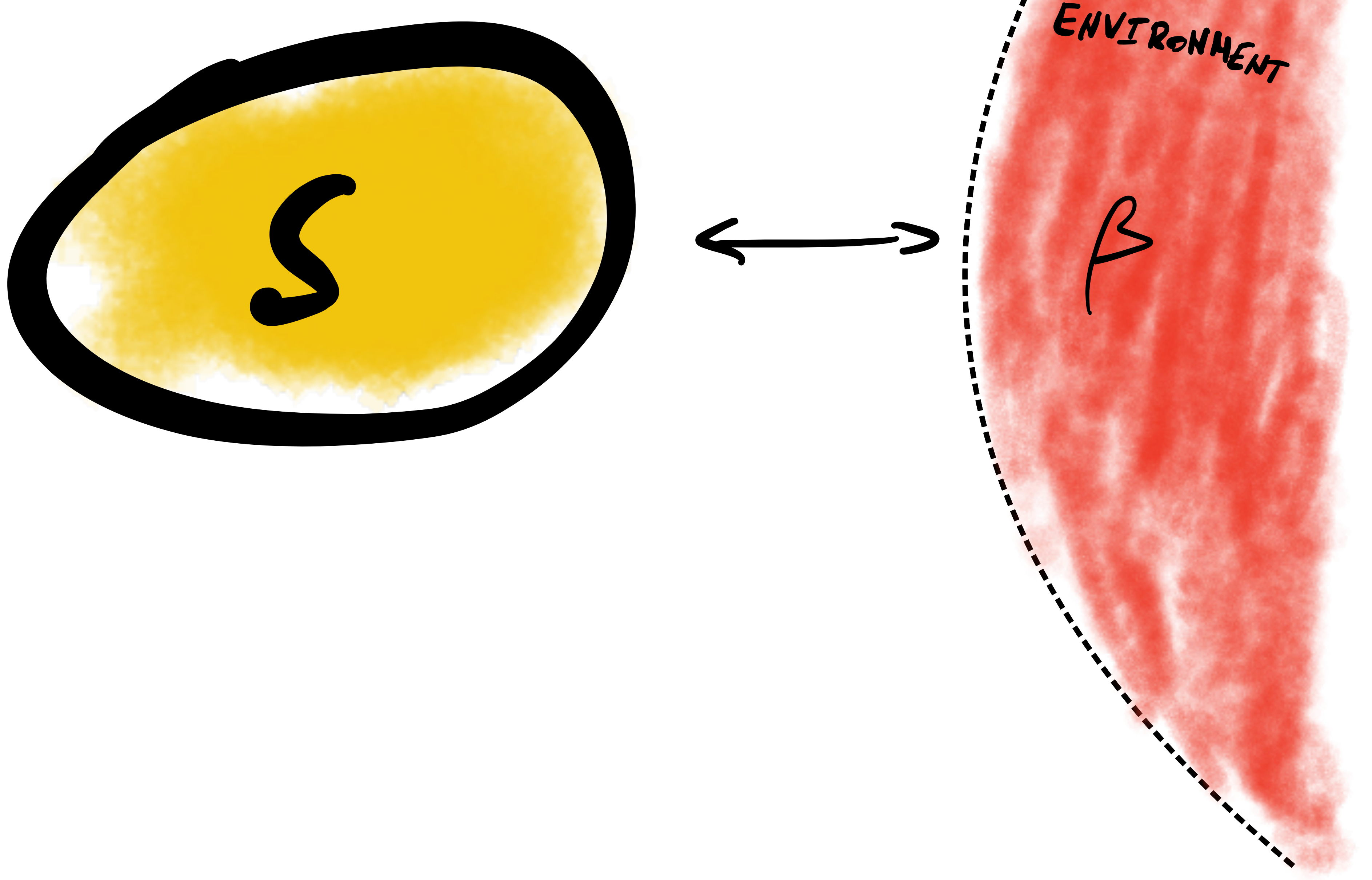
General definitions

$$E = \mathrm{Tr}[\hat{H}(t)\hat{\rho}(t)]$$

$$dE = \underbrace{\mathrm{Tr}[\hat{H}'(t)\hat{\rho}(t)]dt}_{\delta W} + \underbrace{\mathrm{Tr}[\hat{H}(t)\hat{\rho}'(t)]dt}_{\delta Q}$$

$$\hat{H}(0) \longrightarrow \hat{H}(\tau)$$

$$t : 0 \to \tau$$



$$\hat{\rho} \otimes \frac{e^{-\beta \hat{H}_B}}{\mathrm{Tr}[e^{-\beta \hat{H}_B}]} \longrightarrow \hat{U}\hat{\rho} \otimes \frac{e^{-\beta \hat{H}_B}}{\mathrm{Tr}[e^{-\beta \hat{H}_B}]}\hat{U}^{\dagger}$$

General framework for geometric thermo

$$\frac{d}{dt}\hat{\rho}(t) = i[\hat{\rho}(t), \hat{H}(t)] + \mathscr{D}_t[\hat{\rho}(t)]$$

$$\hat{\rho}_{th}(t) = \frac{e^{-\beta \hat{H}(t)}}{Z}$$

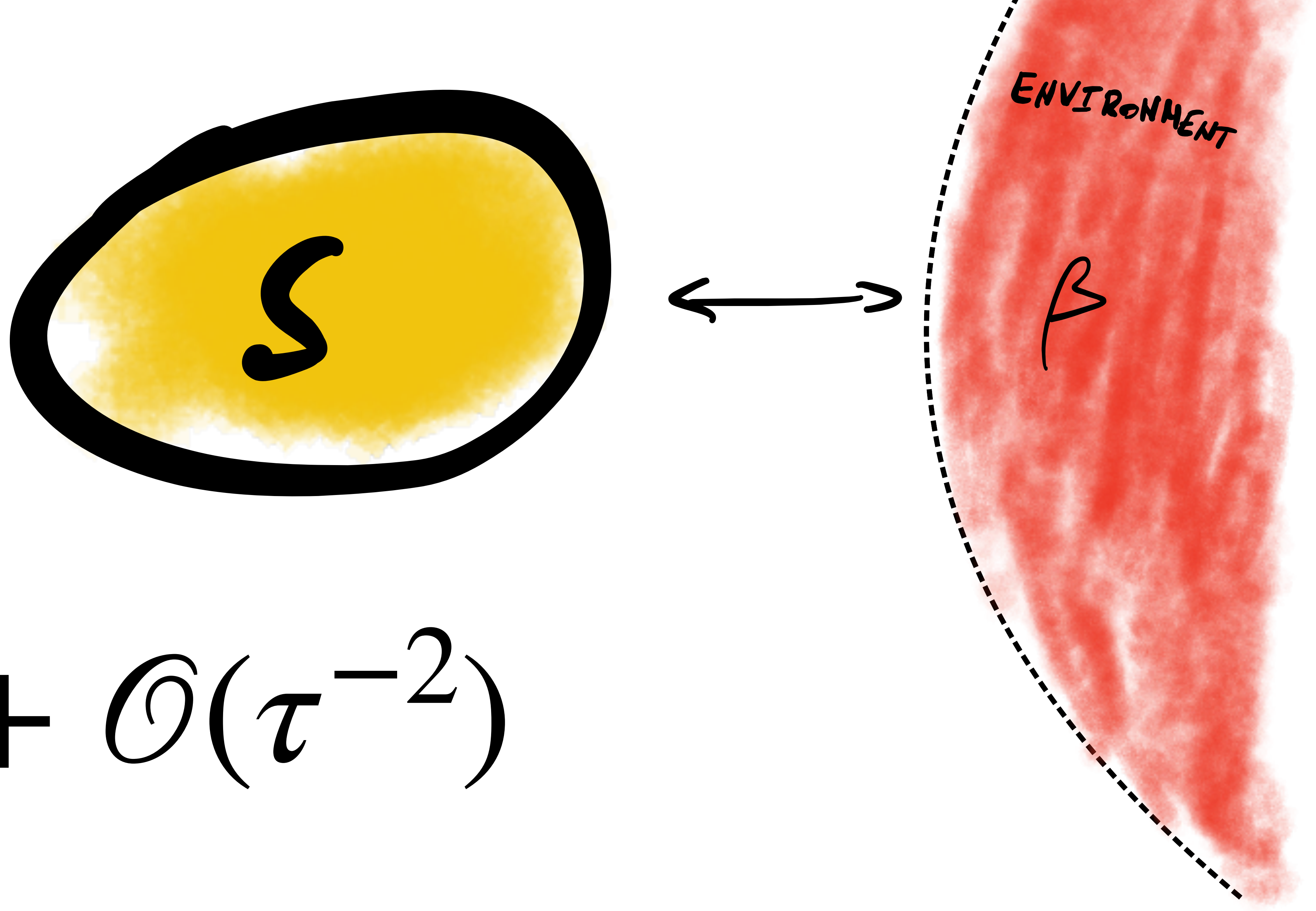
$$\hat{\rho}(t) = \hat{\rho}_{th}(t) + \frac{1}{\tau}\hat{\rho}^{(1)}(t) + \mathscr{O}(\tau^{-2})$$

$\hat{H}(0) \longrightarrow \hat{H}(\tau)$

- Thermalization
- Slow driving

$$W = \int_0^\tau dt\, \mathrm{Tr}[\hat{\rho}(t)\hat{H}'(t)]$$

$$W = \Delta F + W_{diss}$$

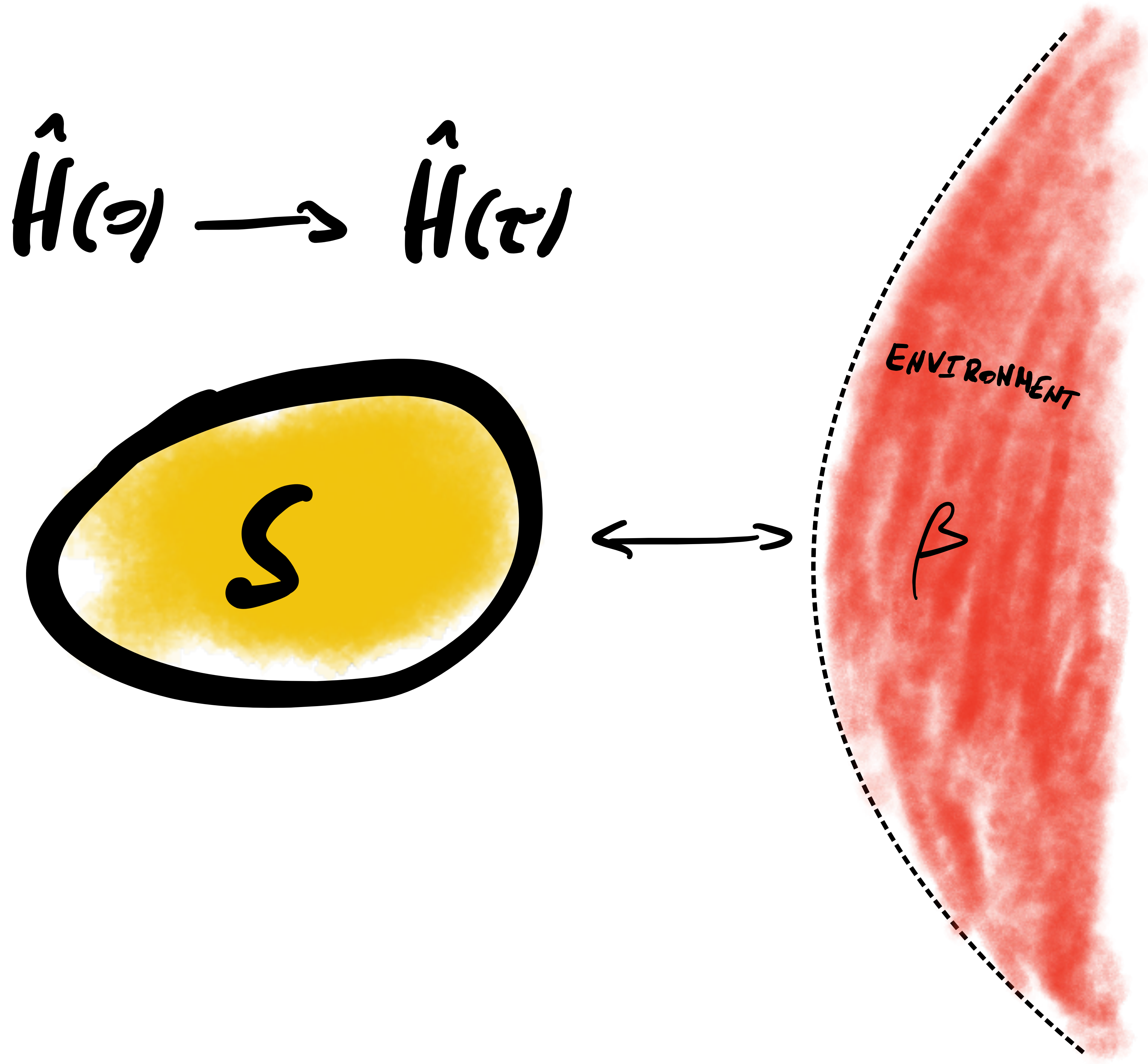


$$W_{diss} = \frac{1}{\tau}\int_0^\tau dt\, \mathrm{Tr}[\hat{\rho}^{(1)}(t)\hat{H}'(t)] + \mathscr{O}(\tau^{-2})$$

Thermodynamic geometry

$$\hat{H}(t) = \sum_k \lambda_k(t)\hat{X}_k$$

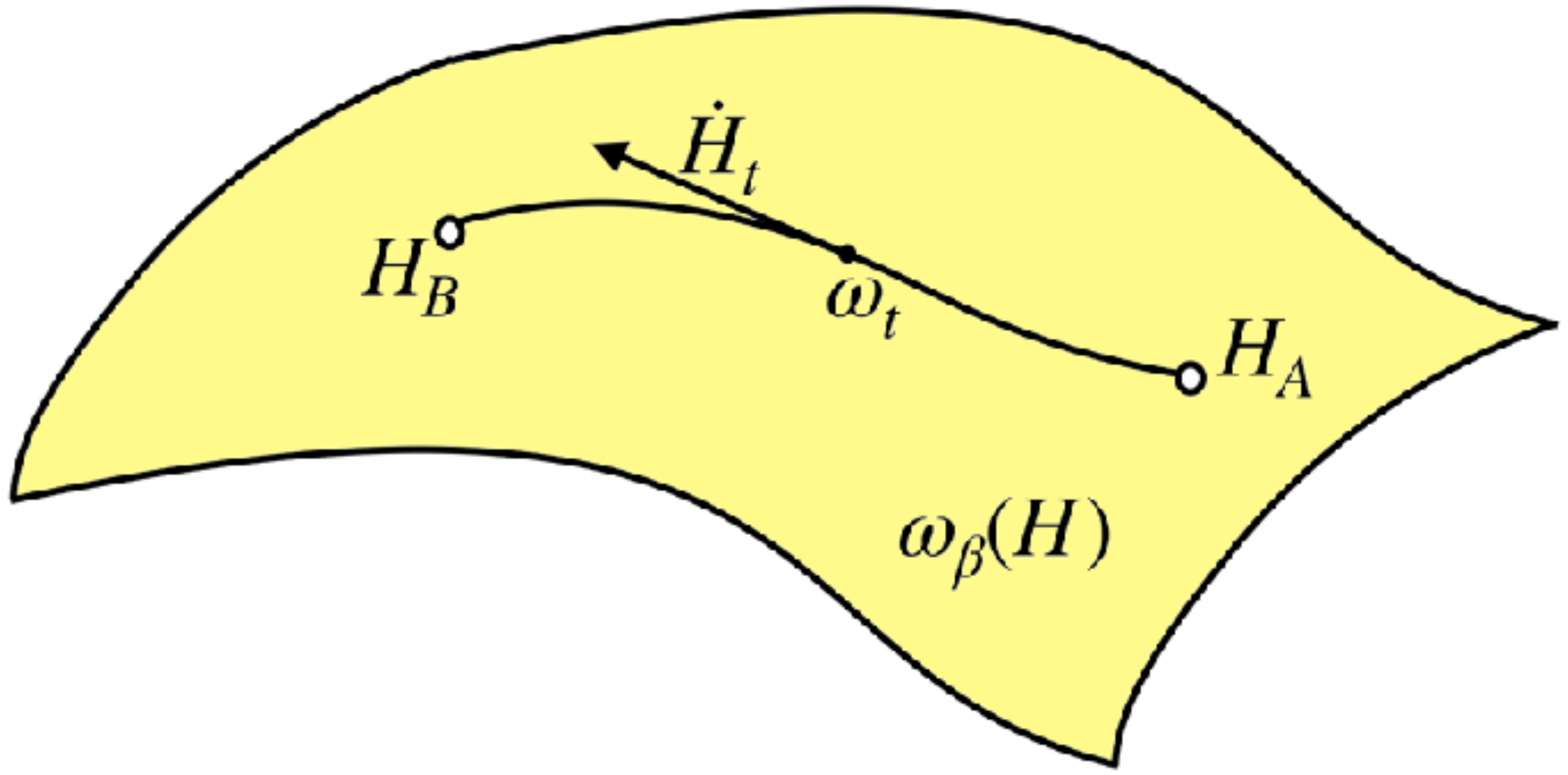
$$W_{diss} = k_B T \int_0^{\tau} dt\ \dot{\lambda}^i(t)\dot{\lambda}^j(t)\, g_{ij}$$



Thermodynamic length

$$W_{diss} = k_B T \int_0^\tau dt\ \dot{\lambda}^i(t)\dot{\lambda}^j(t)\, g_{ij}$$

$$L[\lambda] = \int_0^\tau dt \sqrt{\dot{\lambda}^i(t)\dot{\lambda}^j(t)\, g_{ij}}$$



Thermodynamic length

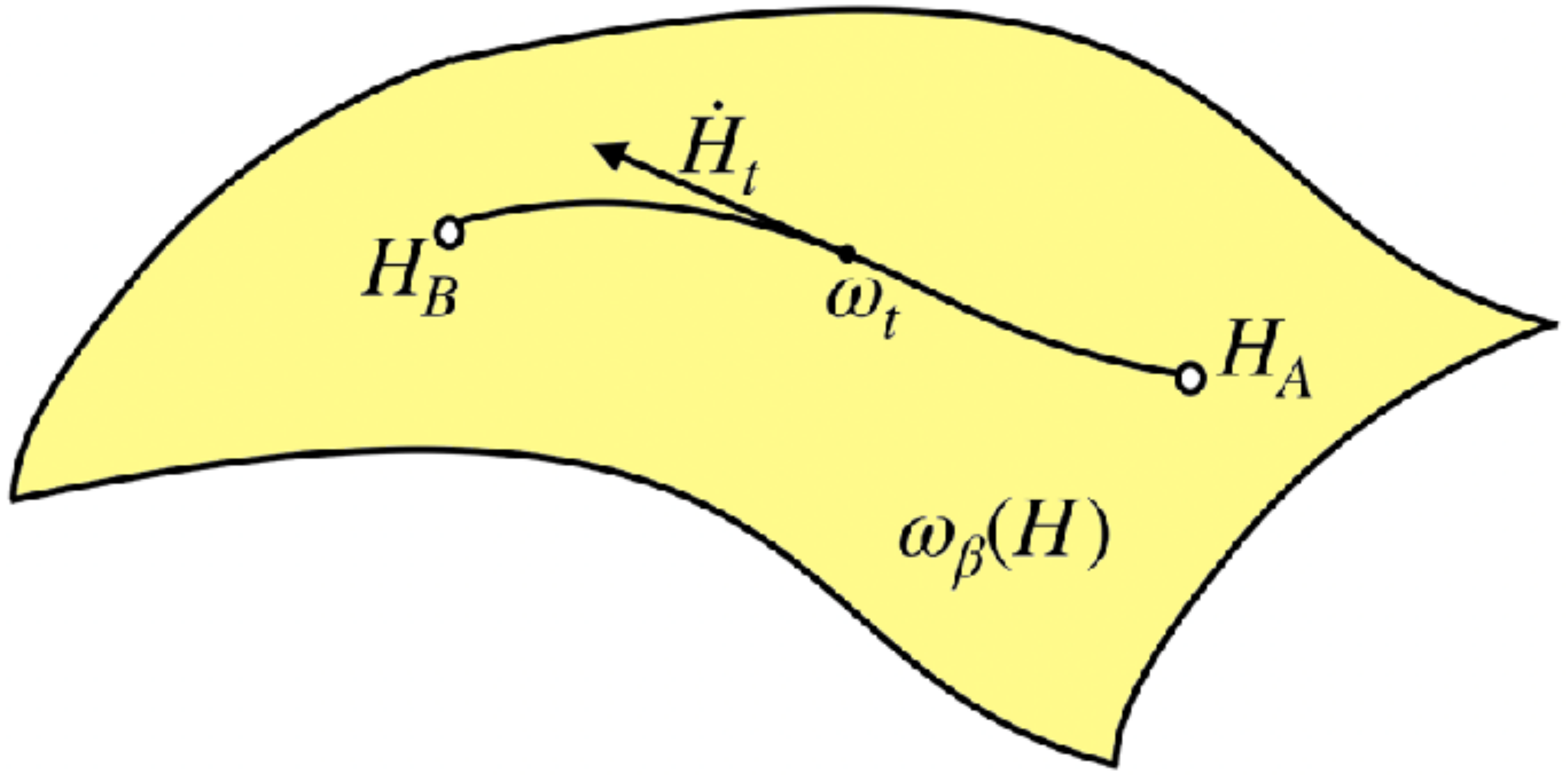
$$W_{diss} = k_B T \int_0^\tau dt\ \dot{\lambda}^i(t)\dot{\lambda}^j(t)\, g_{ij}$$

$$\boxed{\beta W_{diss} \geq \frac{1}{\tau} L^2}$$

$$L[\lambda] = \int_0^\tau dt \sqrt{\dot{\lambda}^i(t)\dot{\lambda}^j(t)\, g_{ij}}$$

Principle I

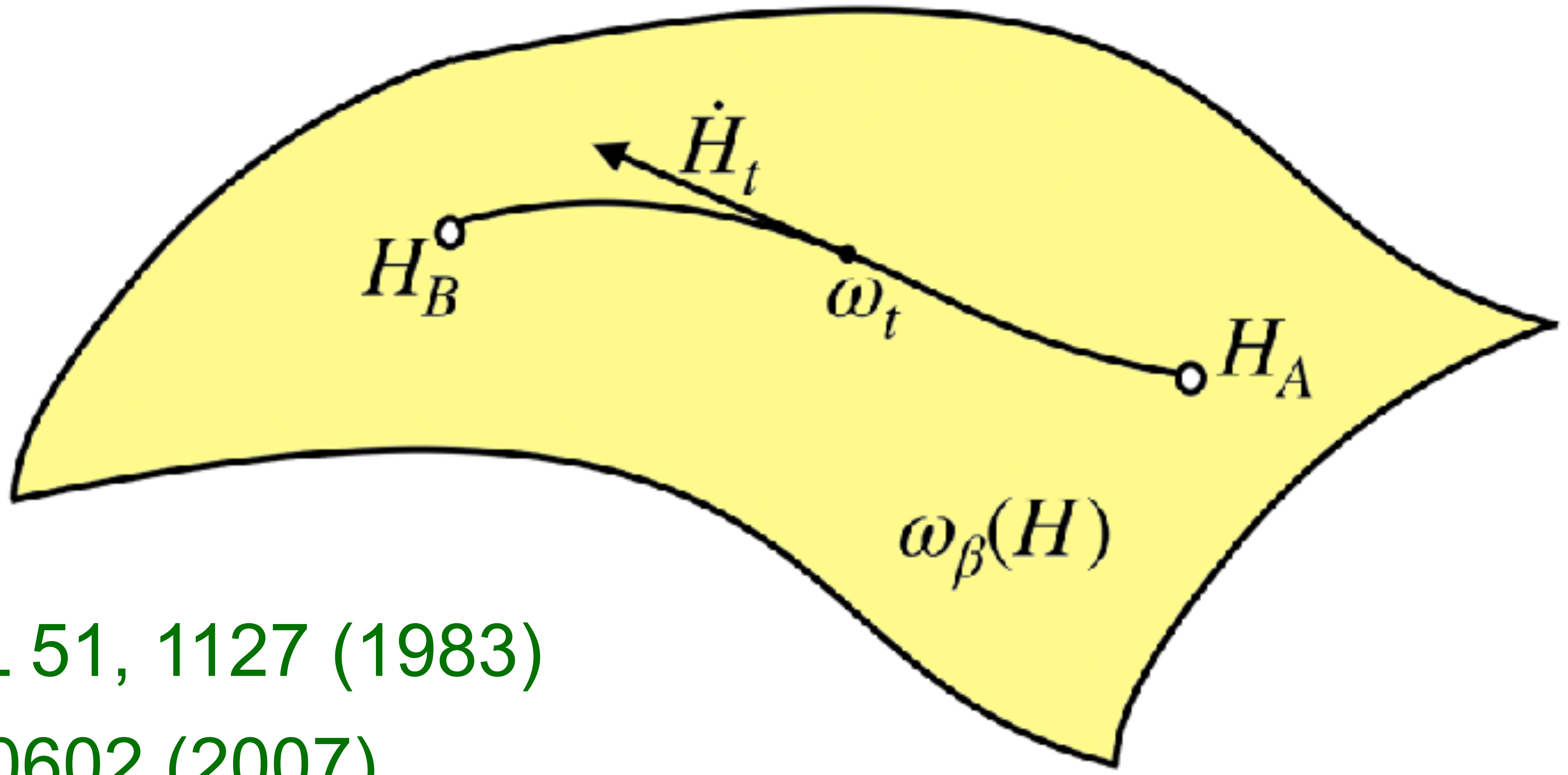
Optimal thermodynamic protocols have constant dissipation rate



Thermodynamic length

$$W_{diss} = k_B T \int_0^\tau dt\ \dot\lambda^i(t)\dot\lambda^j(t)\, g_{ij}$$

$$L[\lambda] = \int_0^\tau dt \sqrt{\dot\lambda^i(t)\dot\lambda^j(t)\, g_{ij}}$$



$$\boxed{\beta W_{diss} \geq \frac{L^2}{\tau} \geq \frac{\mathbb{L}^2}{\tau}}$$

Principle I

Optimal thermodynamic protocols have constant dissipation rate

Principle II

Optimal protocols are geodesics

$$\ddot\lambda^i + \Gamma^i_{jk}\dot\lambda^j\dot\lambda^k = 0$$

$$\Gamma^i_{jk} = \frac{1}{2} g^{il}\left(\partial_k g_{jl} + \partial_j g_{kl} - \partial_l g_{jk}\right)$$

- Salamon, Berry PRL 51, 1127 (1983)
- Crooks, PRL 99, 100602 (2007).
- Scandi, M. P.-L, Quantum 3, 197 (2019).
- …

Thermodynamic metric

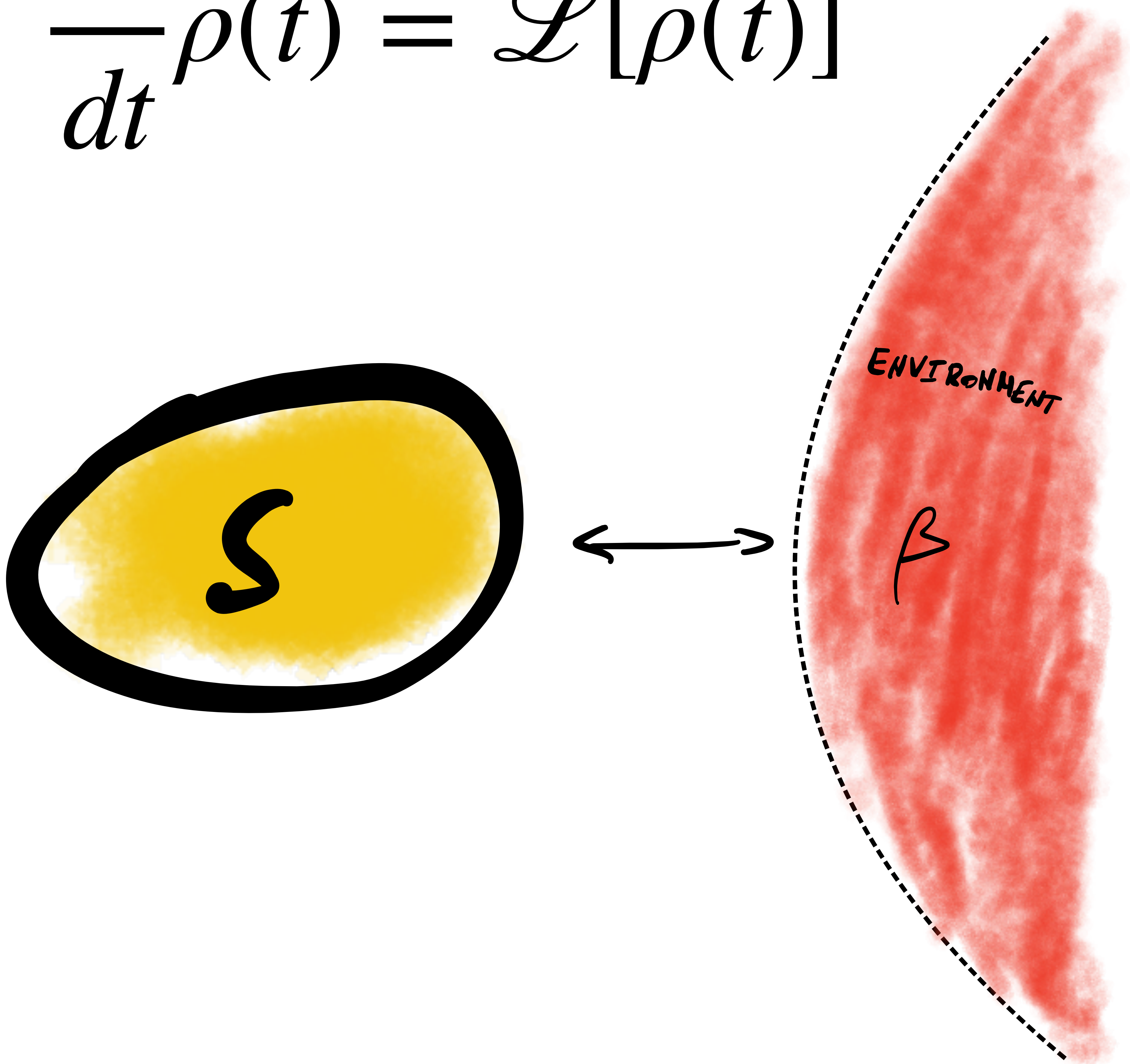
Single relaxation timescale

$$\hat{\rho}_{th}(t) = \frac{e^{-\beta \hat{H}(t)}}{Z}$$

$$\frac{d}{dt}\hat{\rho}(t) = \mathscr{L}[\hat{\rho}(t)]$$

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{\tau_{\mathrm{eq}}}\left(\hat{\rho}_{\mathrm{th}}(t) - \hat{\rho}(t)\right)$$

$$\boxed{g_{ij} = \tau_{eq}\frac{\partial^2 \ln Z}{\partial\lambda_i \partial\lambda_j}}$$



Thermodynamic metric

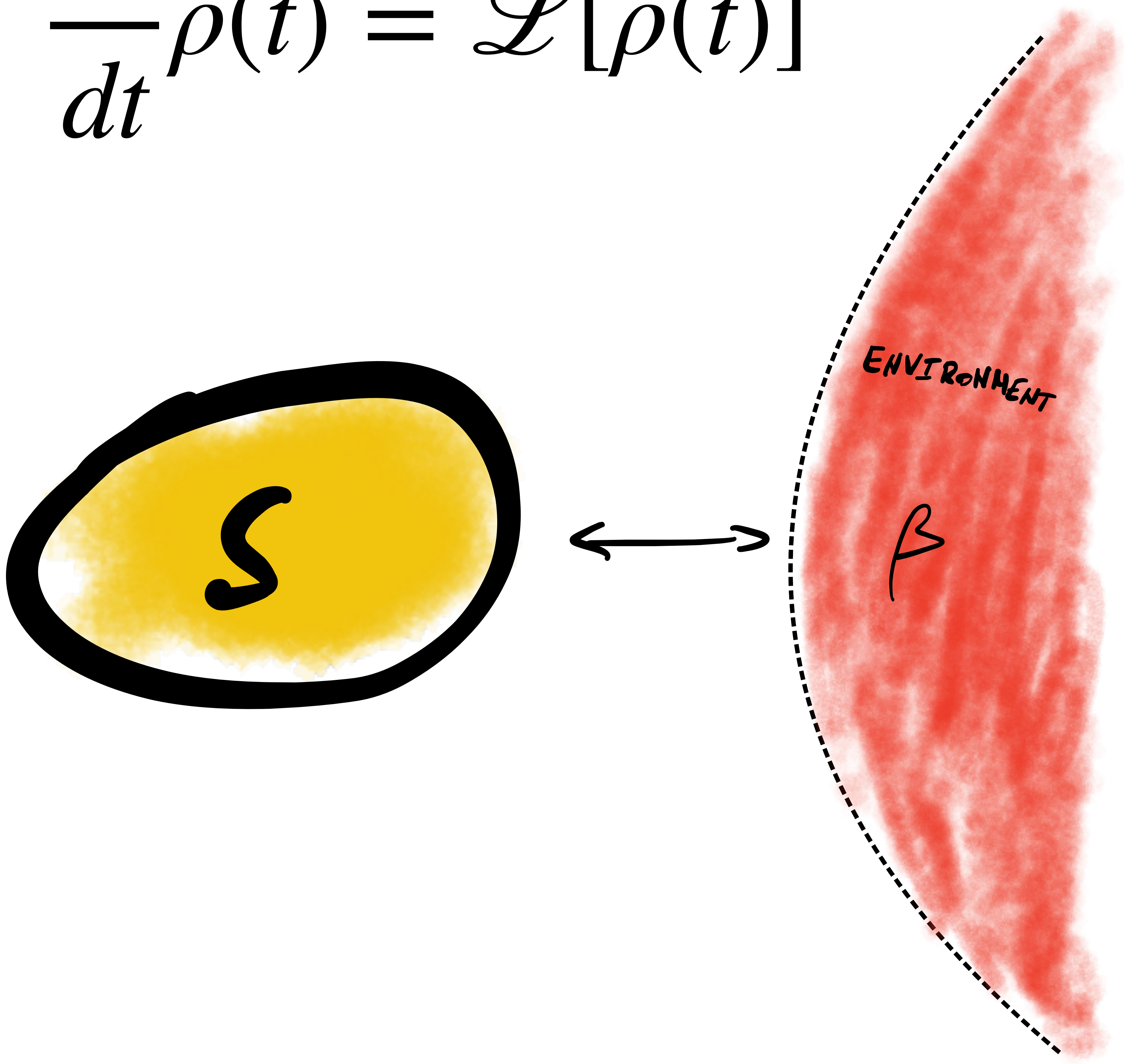
Multiple timescales

$$\hat{H}(t) = \sum_k \lambda_k(t)\hat{X}_k$$

$$\frac{d}{dt}\langle \hat{X}_k \rangle_{\rho(t)} = \frac{1}{\tau_k}\left(\langle \hat{X} \rangle_{\rho_{\text{th}}(t)} - \langle \hat{X} \rangle_{\rho(t)}\right)$$

$$\boxed{g_{ij} = \frac{\tau_i + \tau_j}{2}\frac{\partial^2 \ln Z}{\partial\lambda_i \partial\lambda_j}}$$

$$\frac{d}{dt}\hat{\rho}(t) = \mathscr{L}[\hat{\rho}(t)]$$

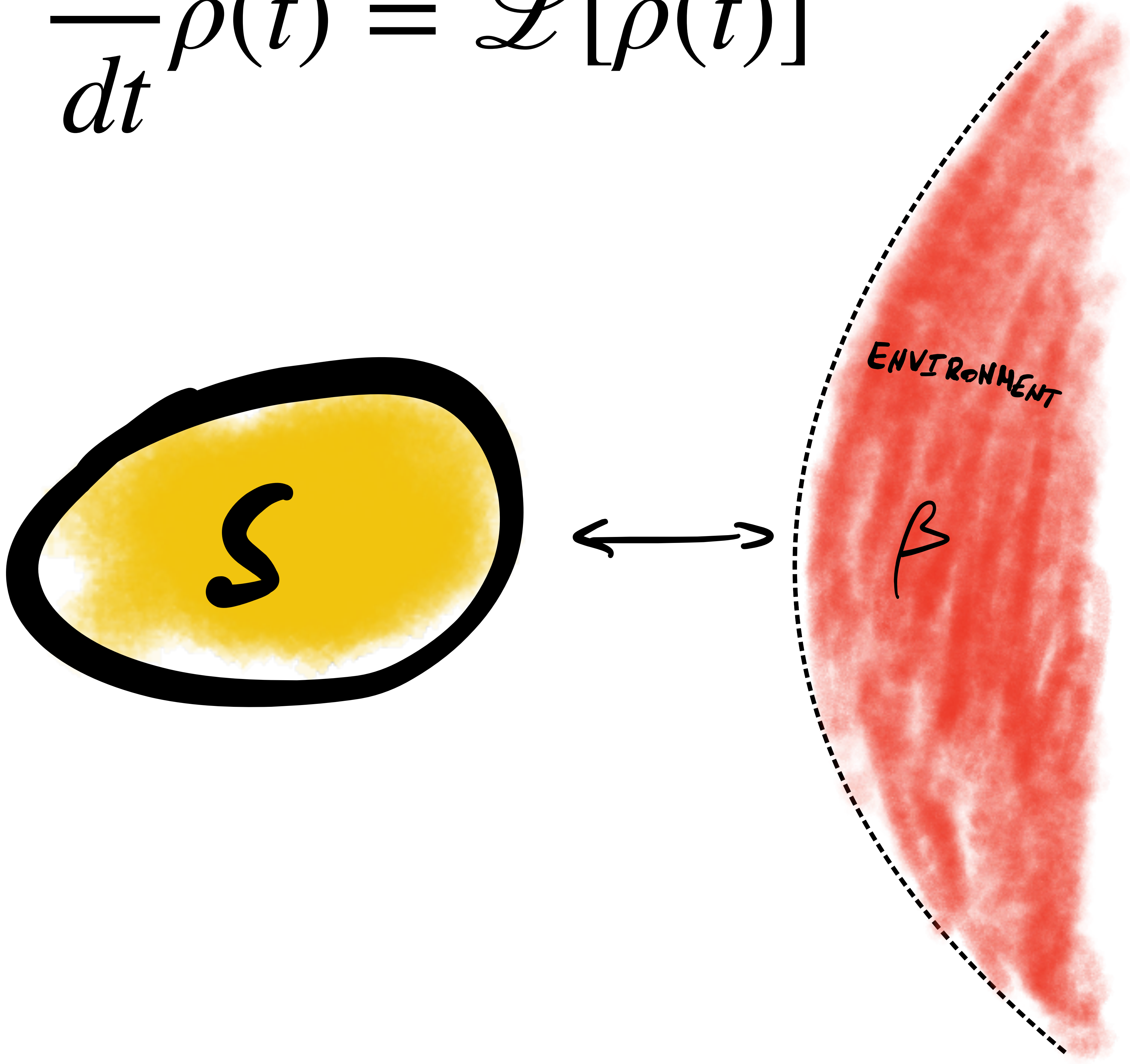


Thermodynamic metric

General case

$$\hat{H}(t) = \sum_k \lambda_k(t)\hat{X}_k$$

$$\frac{d}{dt}\hat{\rho}(t) = \mathscr{L}[\hat{\rho}(t)]$$



$$\mathscr{L}[Y_a] = \frac{Y_a}{\tau_a} \qquad X_k = \sum_a U_{ia} Y_a$$

$$g_{ij} = \boxed{\sum_{a,b} U_{ia}U_{ib}\frac{\tau_a+\tau_b}{2}}\boxed{\frac{\partial^2 \ln Z}{\partial\lambda^a\partial\lambda^b}}$$

Timescales

Thermodynamic metric
(Fisher information)

Finite-time Landauer erasure (at strong coupling)

An application of thermodynamic geometry

See also:

Finite-Time Landauer Principle
Karel Proesmans, Jannik Ehrich, and John Bechhoefer
Phys. Rev. Lett. **125**, 100602 – Published 3 September 2020

Finite-Time Quantum Landauer Principle and Quantum Coherence
Tan Van Vu and Keiji Saito
Phys. Rev. Lett. **128**, 010602 – Published 4 January 2022

Thermodynamic Unification of Optimal Transport: Thermodynamic Uncertainty Relation, Minimum Dissipation, and Thermodynamic Speed Limits
Tan Van Vu and Keiji Saito
Phys. Rev. X **13**, 011013 – Published 3 February 2023

Finite-time erasing of information stored in fermionic bits
Giovanni Diana, G. Baris Bagci, and Massimiliano Esposito
Phys. Rev. E **87**, 012111 – Published 11 January 2013

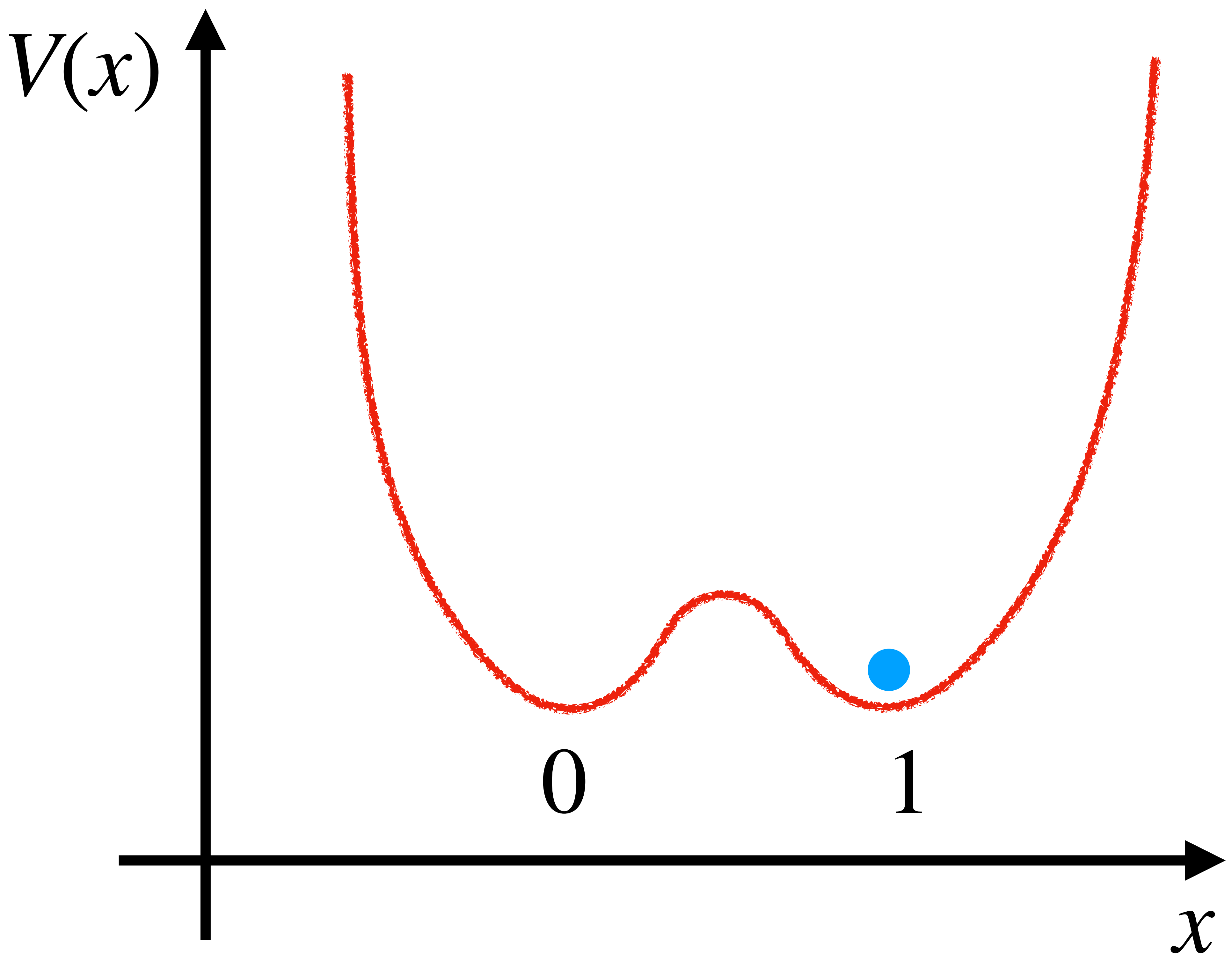
Geometrical Bounds of the Irreversibility in Markovian Systems
Tan Van Vu and Yoshihiko Hasegawa
Phys. Rev. Lett. **126**, 010601 – Published 4 January 2021

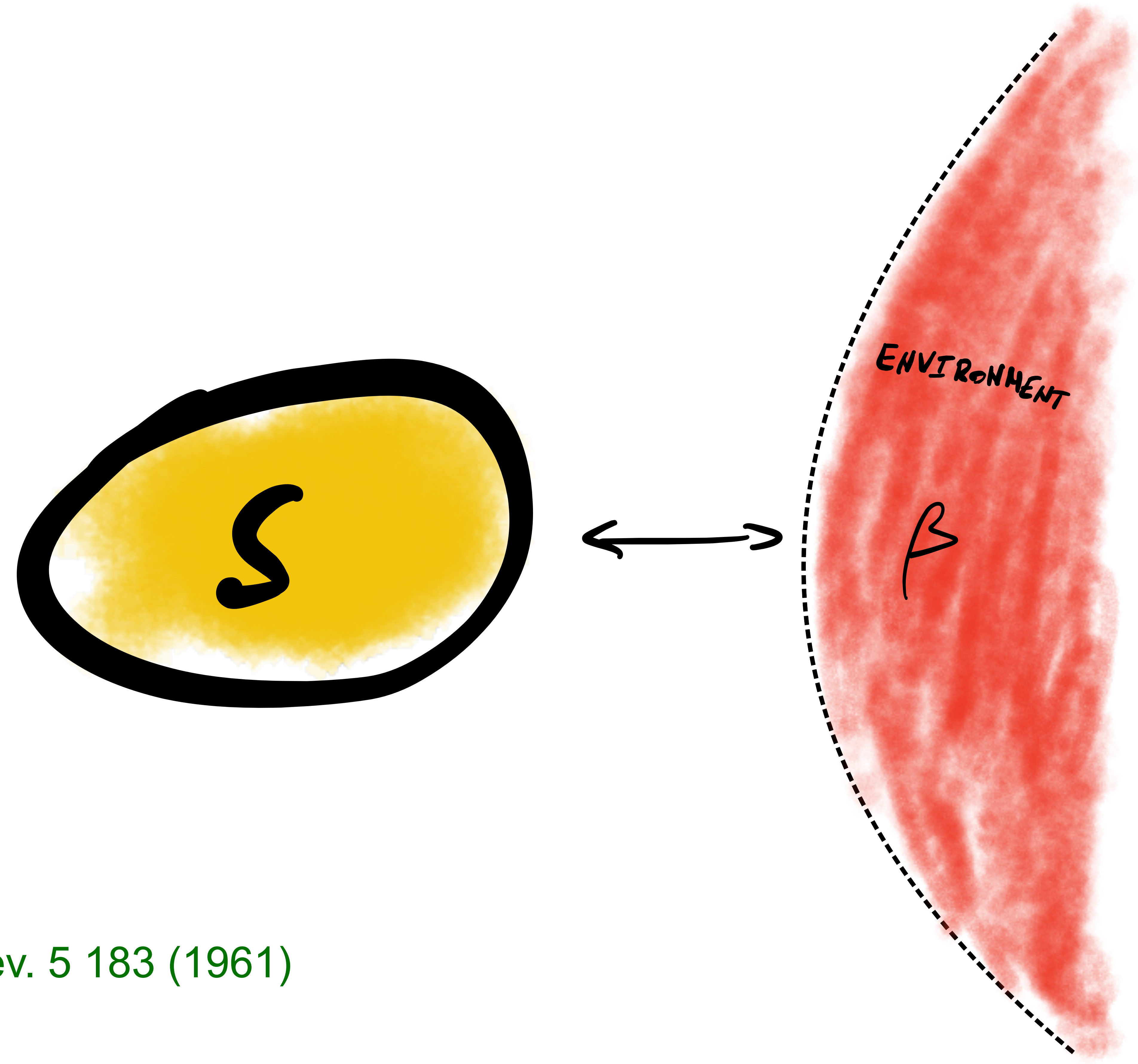
Universal Bound on Energy Cost of Bit Reset in Finite Time
Yi-Zheng Zhen, Dario Egloff, Kavan Modi, and Oscar Dahlsten
Phys. Rev. Lett. **127**, 190602 – Published 1 November 2021

Speed Limit for a Highly Irreversible Process and Tight Finite-Time Landauer's Bound
Jae Sung Lee, Sangyun Lee, Hyukjoon Kwon, and Hyunggyu Park
Phys. Rev. Lett. **129**, 120603 – Published 13 September 2022

… and many more

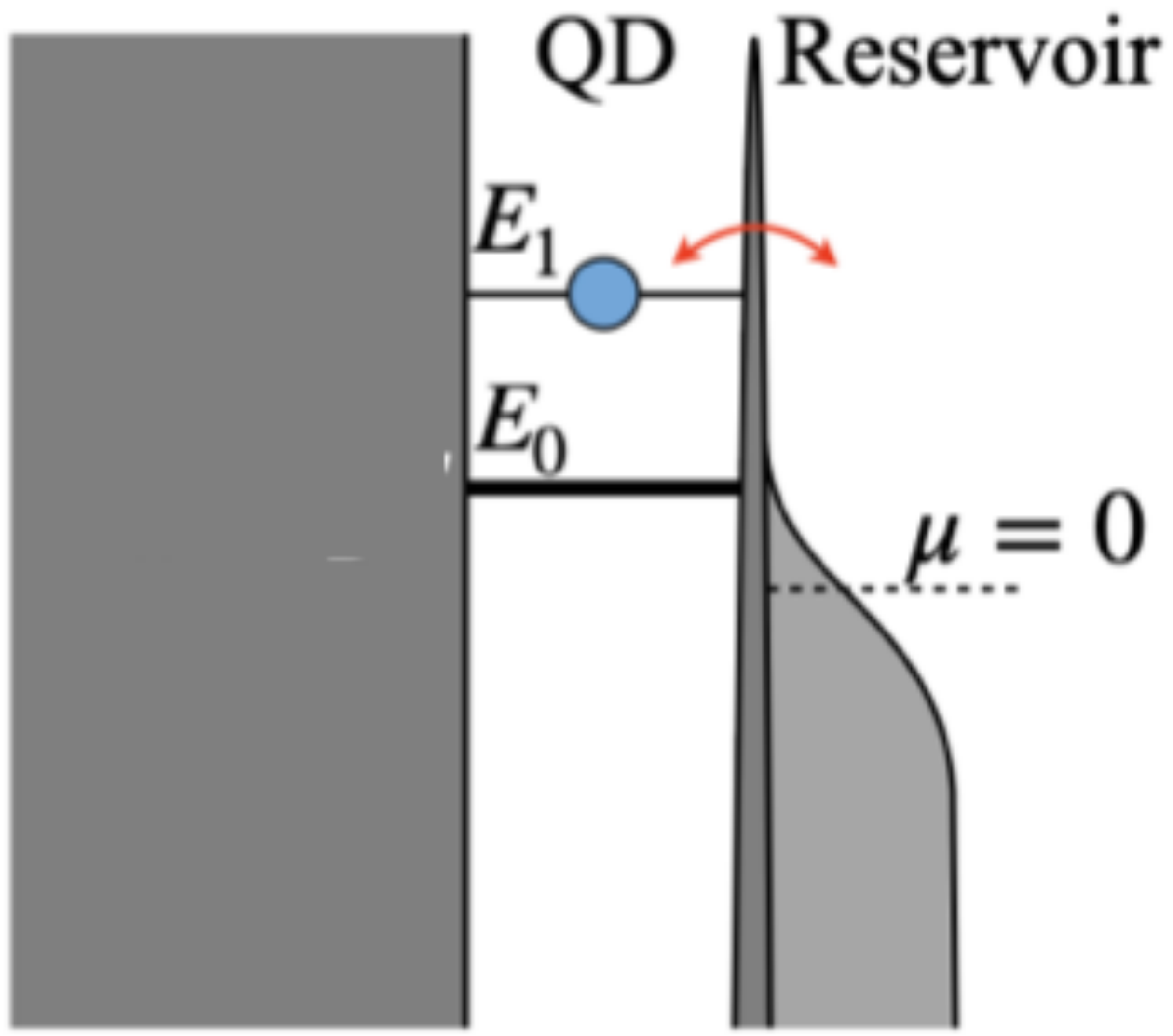
Application: Information erasure in finite time





Landauer: IBM J. Res. Dev. 5 183 (1961)

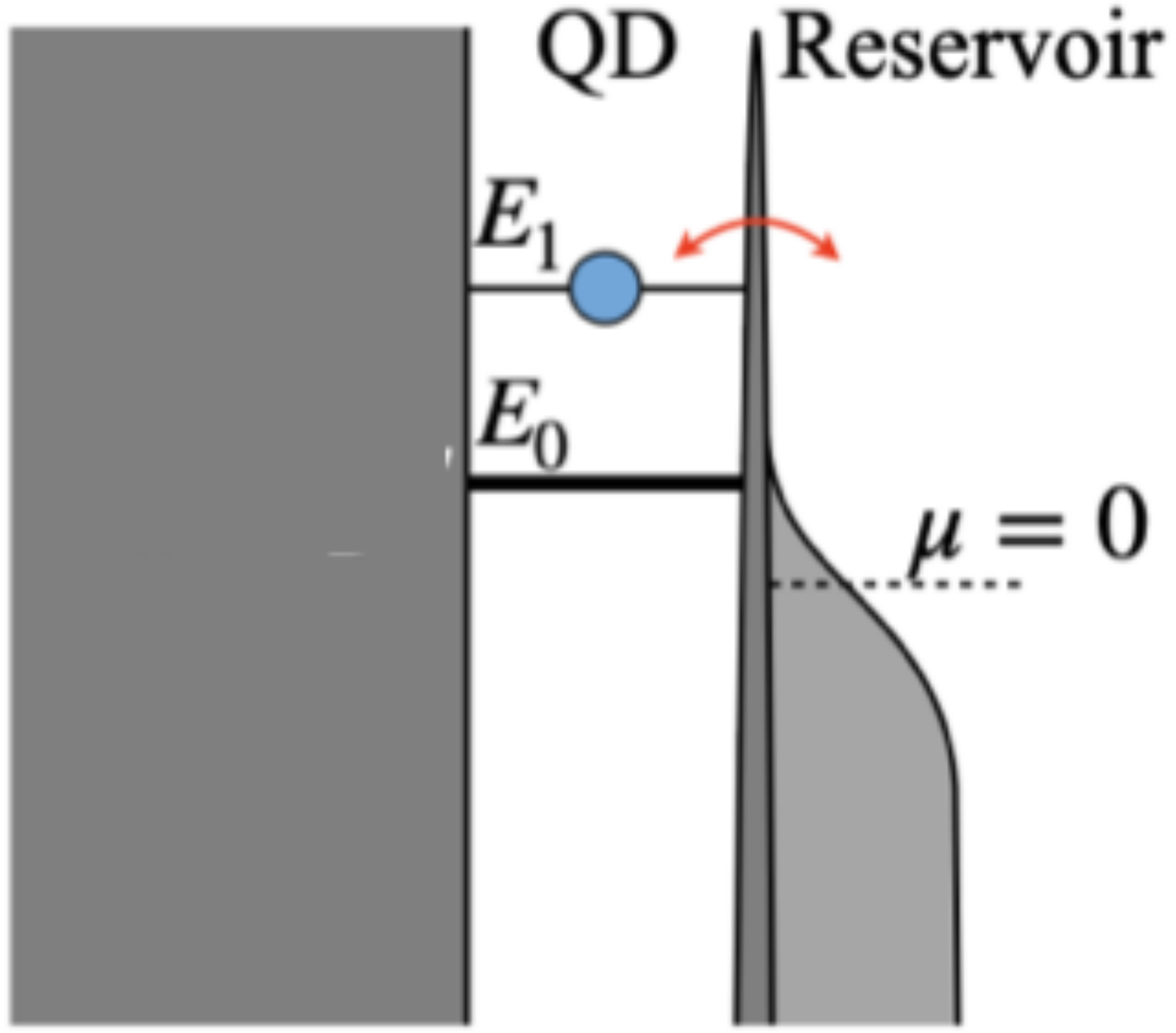
Erasure of for a single-level quantum dot



$$\dot{p}(t) = \Gamma\left(p_{\mathrm{eq}}(t) - p(t)\right)$$

$$p_{\mathrm{eq}} = \frac{e^{-\beta E_1}}{e^{-\beta E_0} + e^{-\beta E_1}}$$

Erasure of for a single-level quantum dot



$$\dot{p}(t) = \Gamma\left(p_{\mathrm{eq}}(t) - p(t)\right)$$

$$p_{\mathrm{eq}} = \frac{e^{-\beta E_1}}{e^{-\beta E_0} + e^{-\beta E_1}}$$

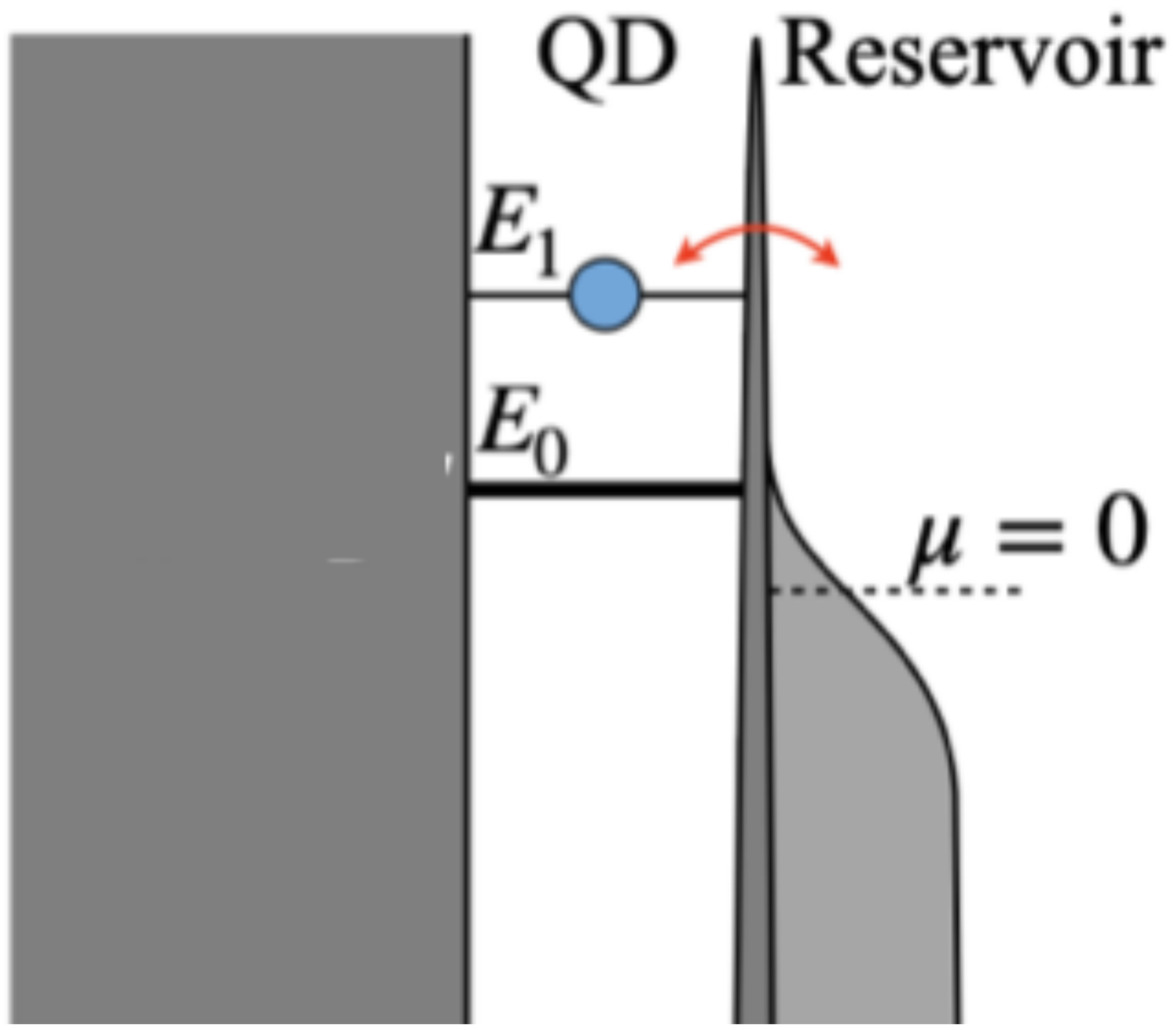
Metric:

$$g(t) = \frac{1}{2 + 2\cosh(E(t) - \log 2)}$$

Geodesic:

$$E_1(t) = 2\log\left(\sqrt{2}\cot\left(\frac{\pi}{4} + B\frac{t}{\tau}\right)\right)$$

Erasure of for a single-level quantum dot



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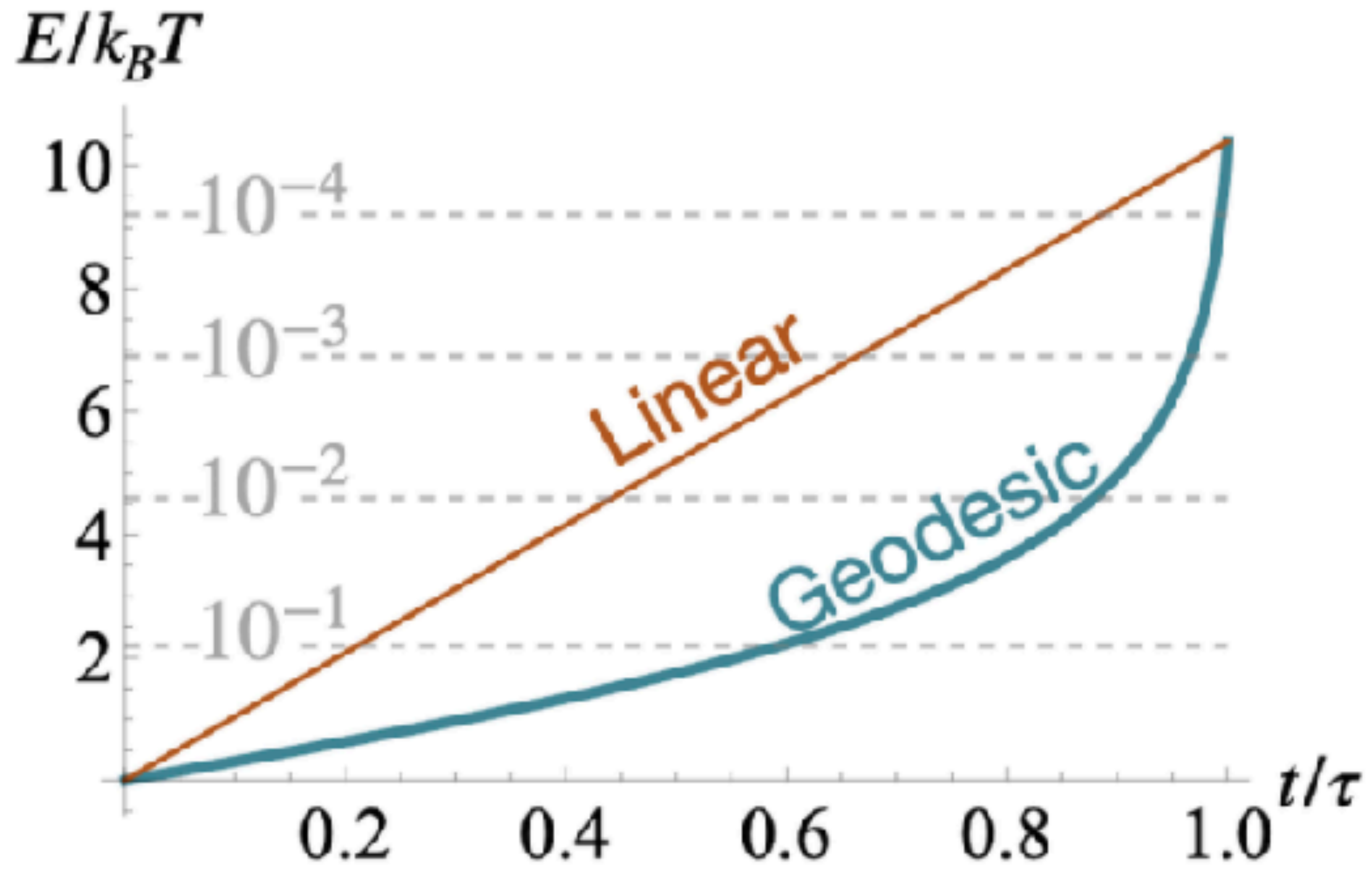
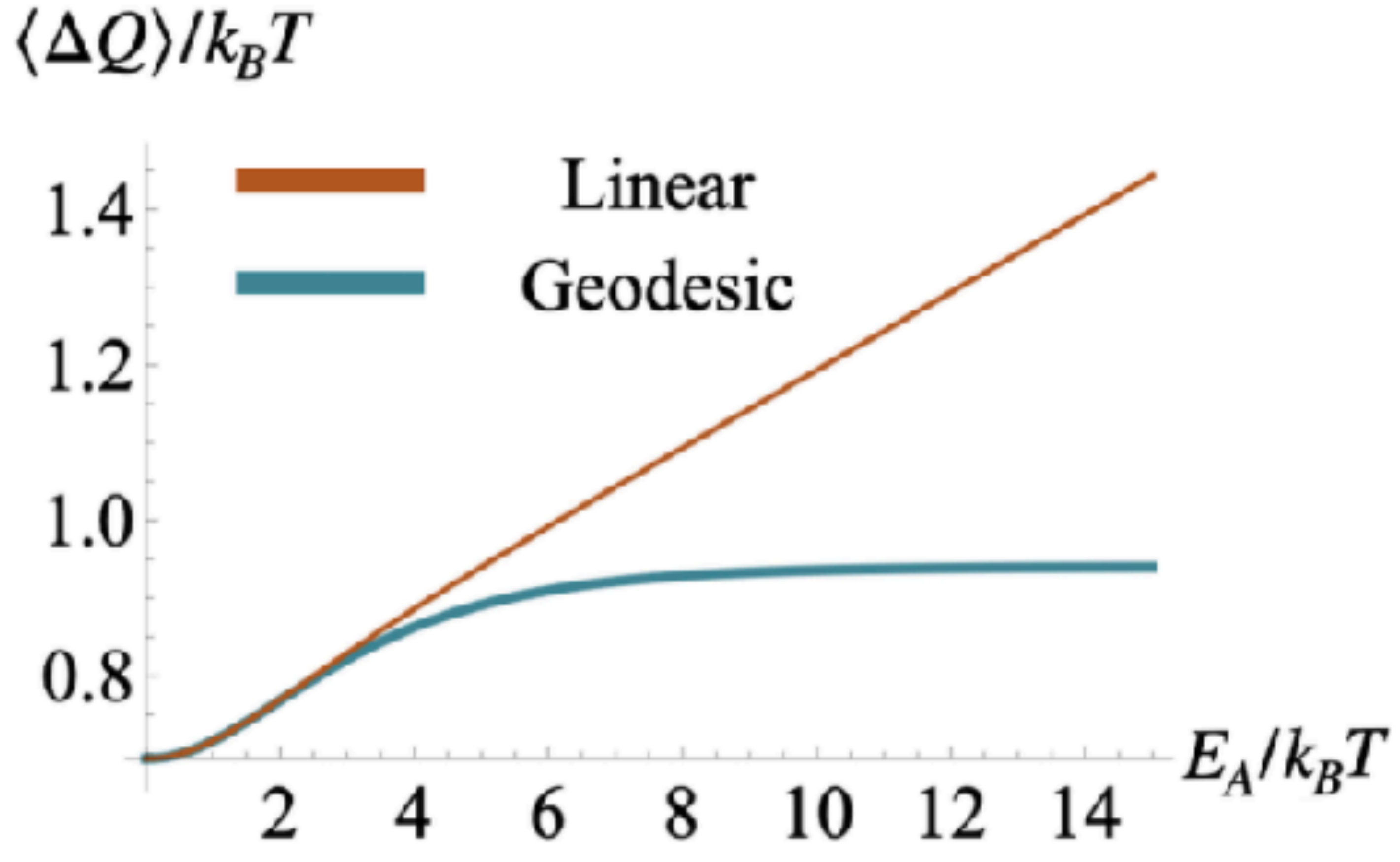


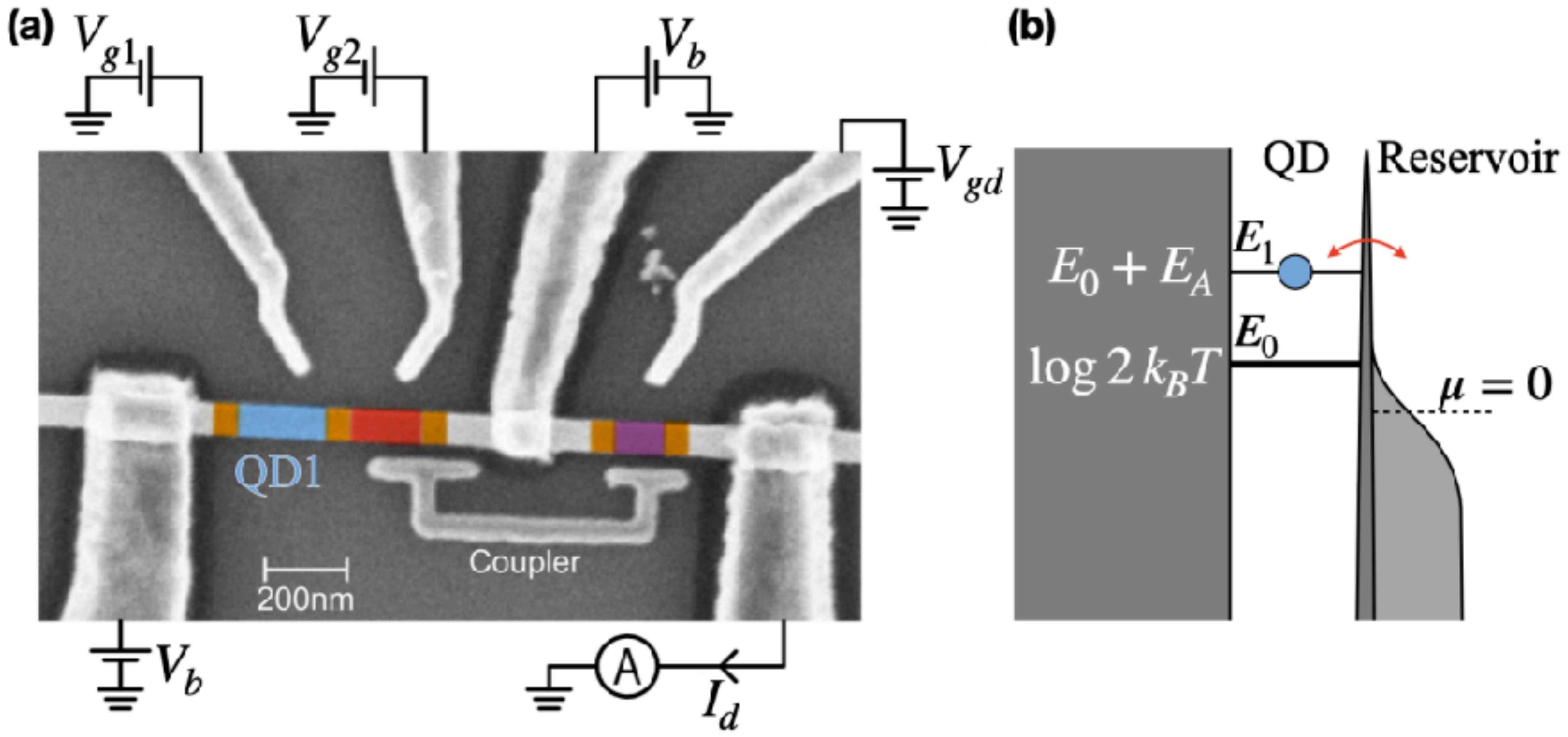


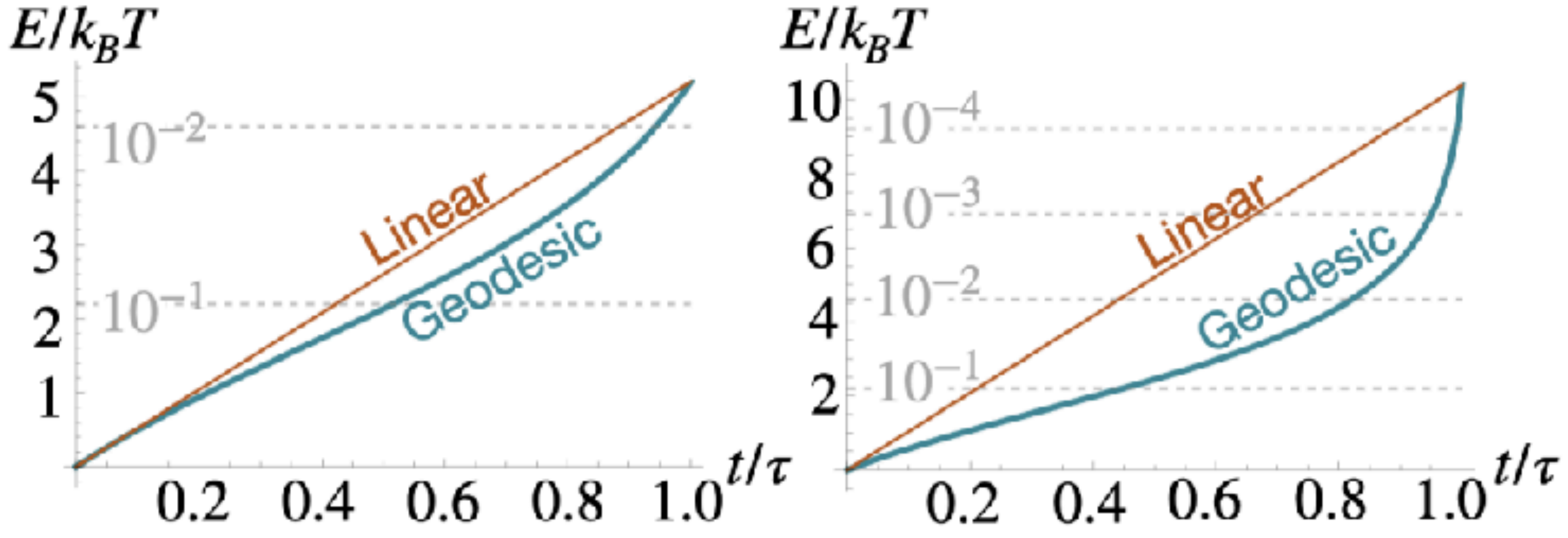
$$\boxed{W_{\mathrm{diss}} \geq \frac{k_B T \pi^2}{4\Gamma\tau}}$$

M. Scandi, M. P.-L., Quantum 3, 197 (2019)

Erasure of for a single-level quantum dot

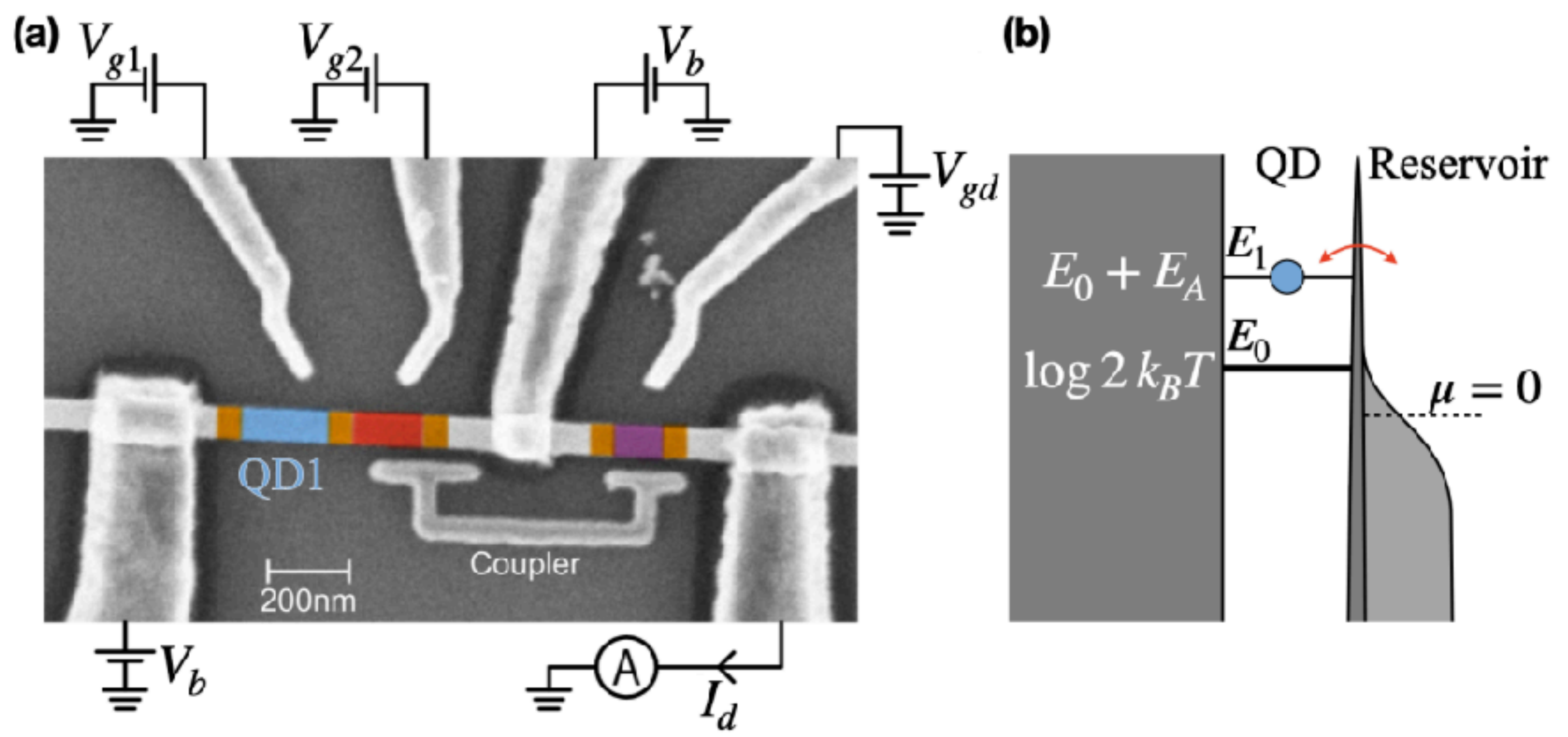
Scandi, Barker, Lehmann, Dick, Maisi, M. P.-L., PRL129, 270601 (2022)

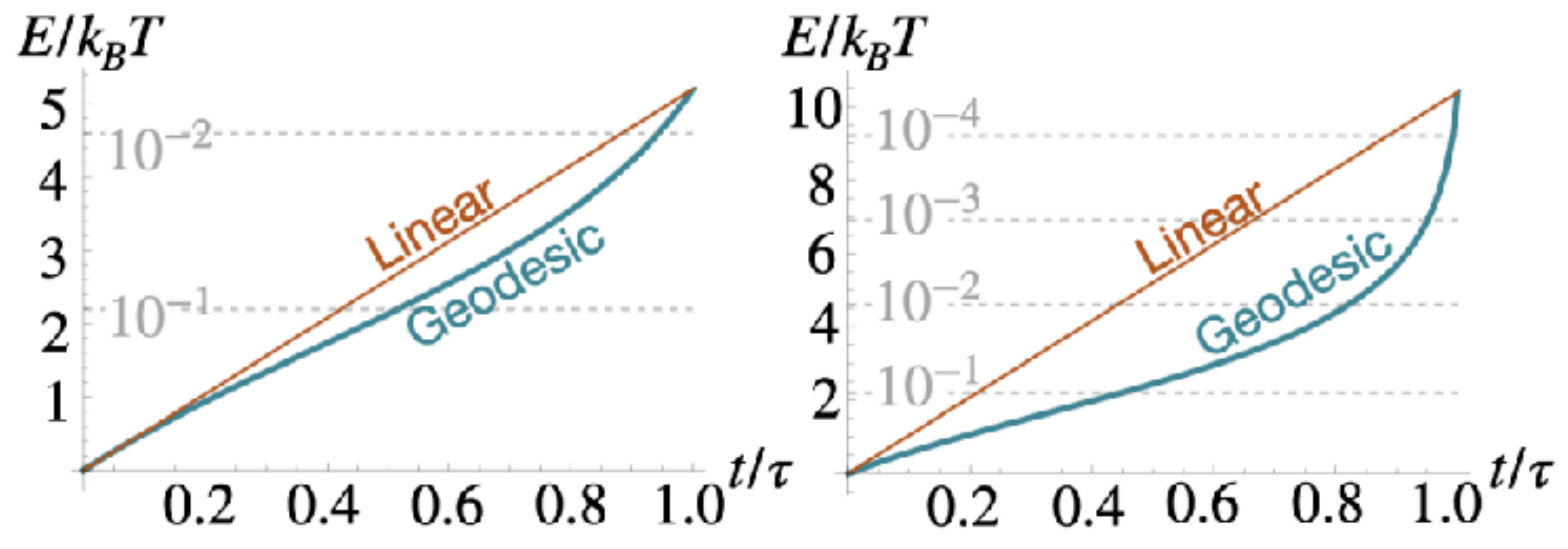


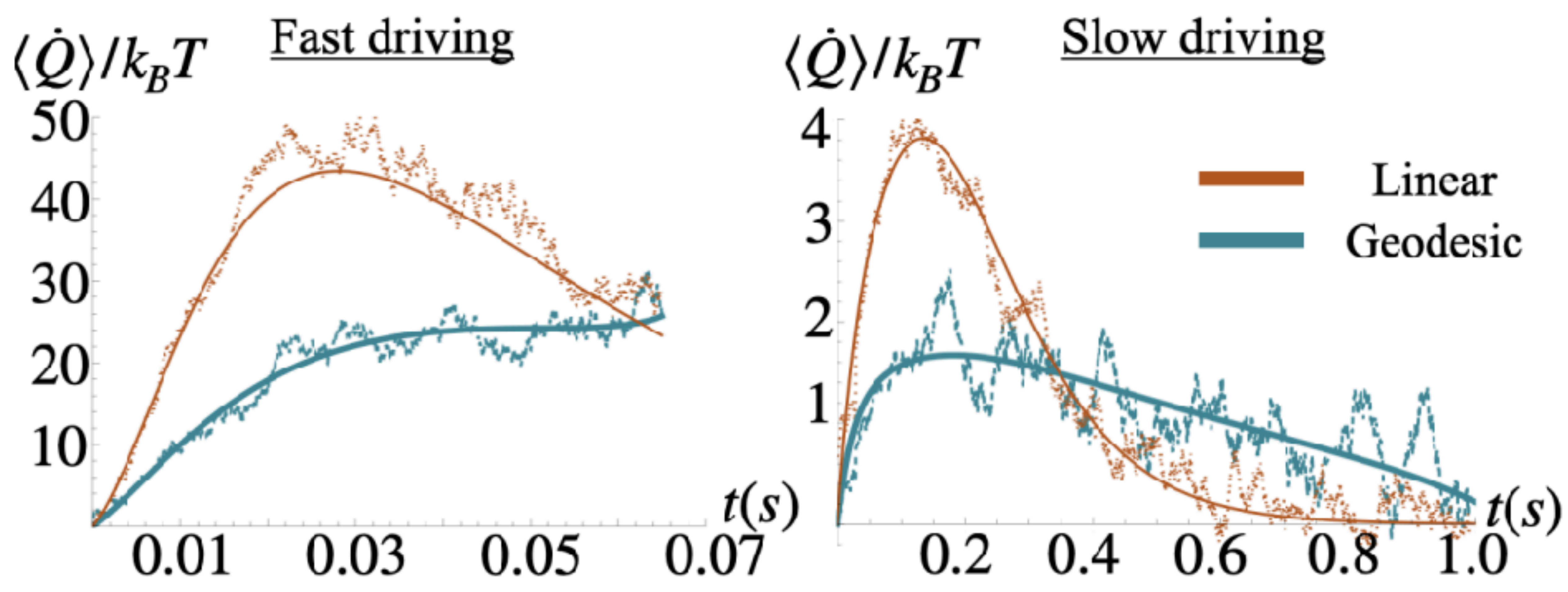


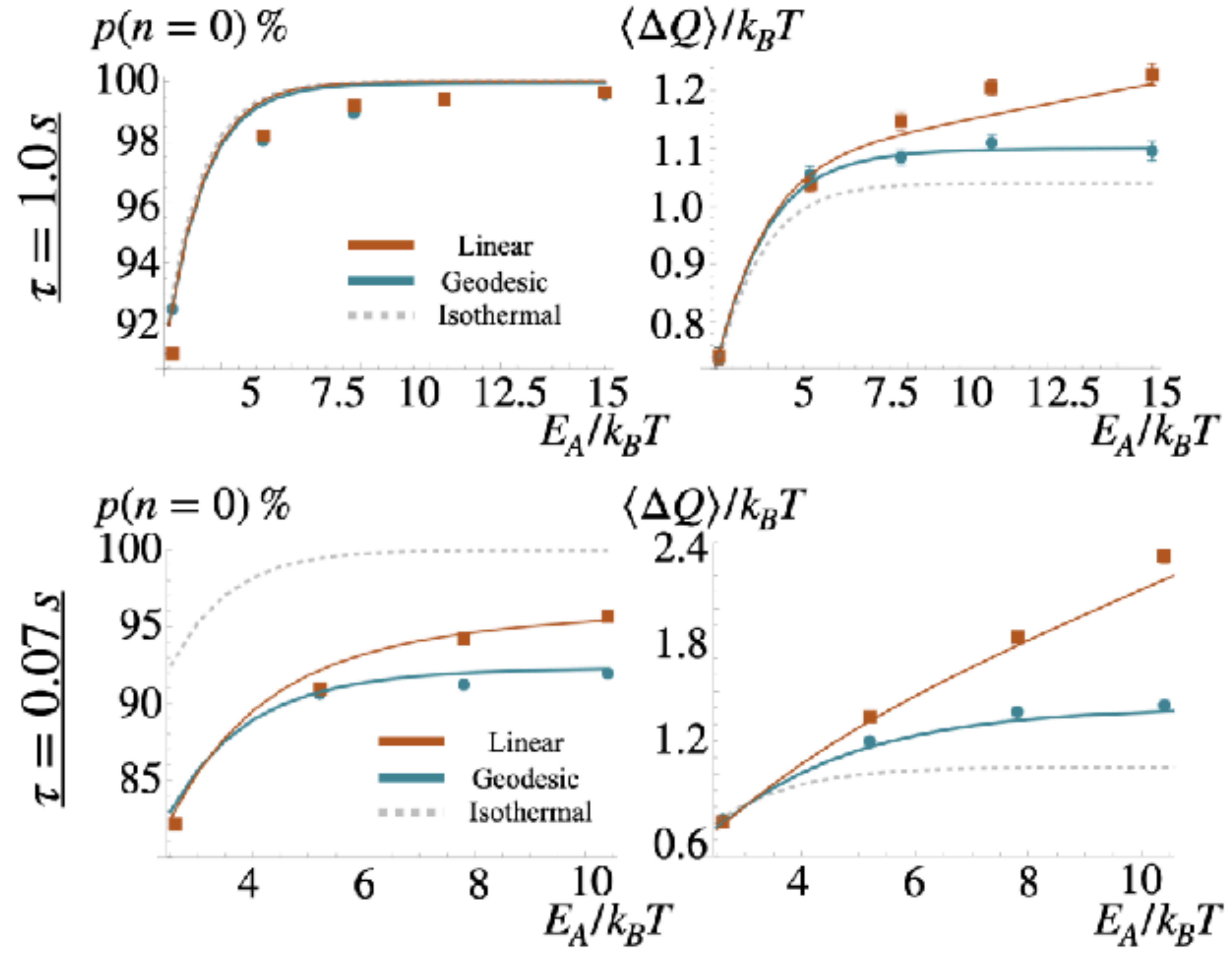
Erasure of for a single-level quantum dot

Scandi, Barker, Lehmann, Dick, Maisi, M. P.-L., PRL129, 270601 (2022)





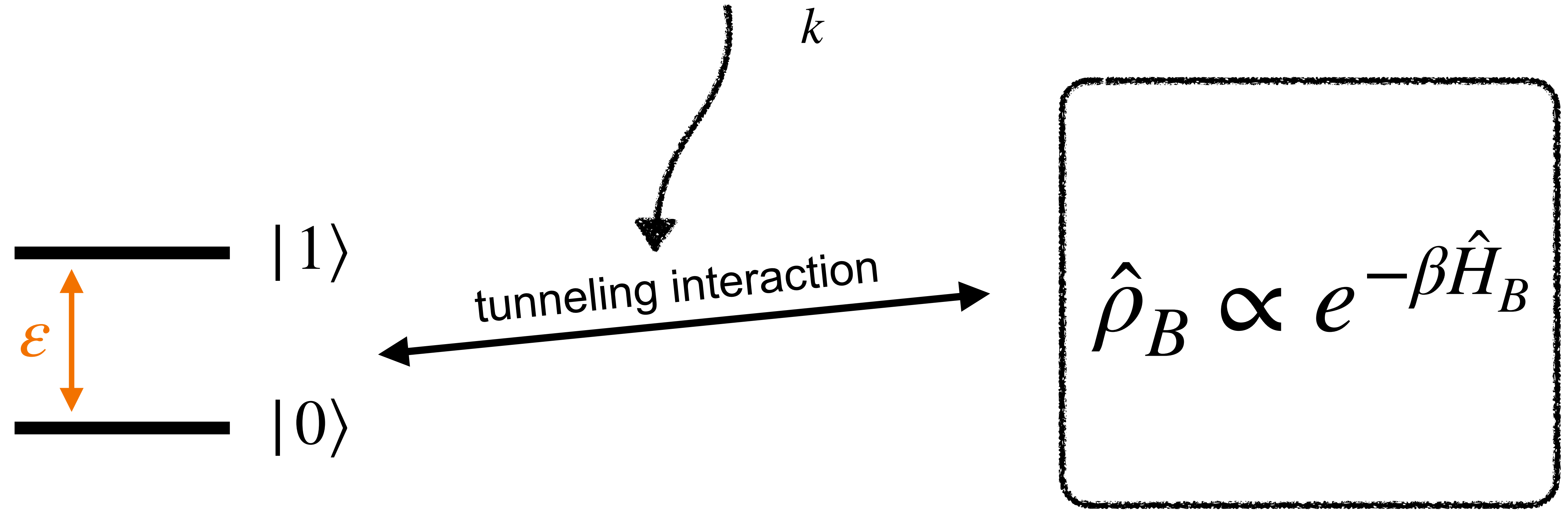




Finite-time erasure at strong coupling

Rolandi, M. P.-L., Quantum 7, 1161 (2023)

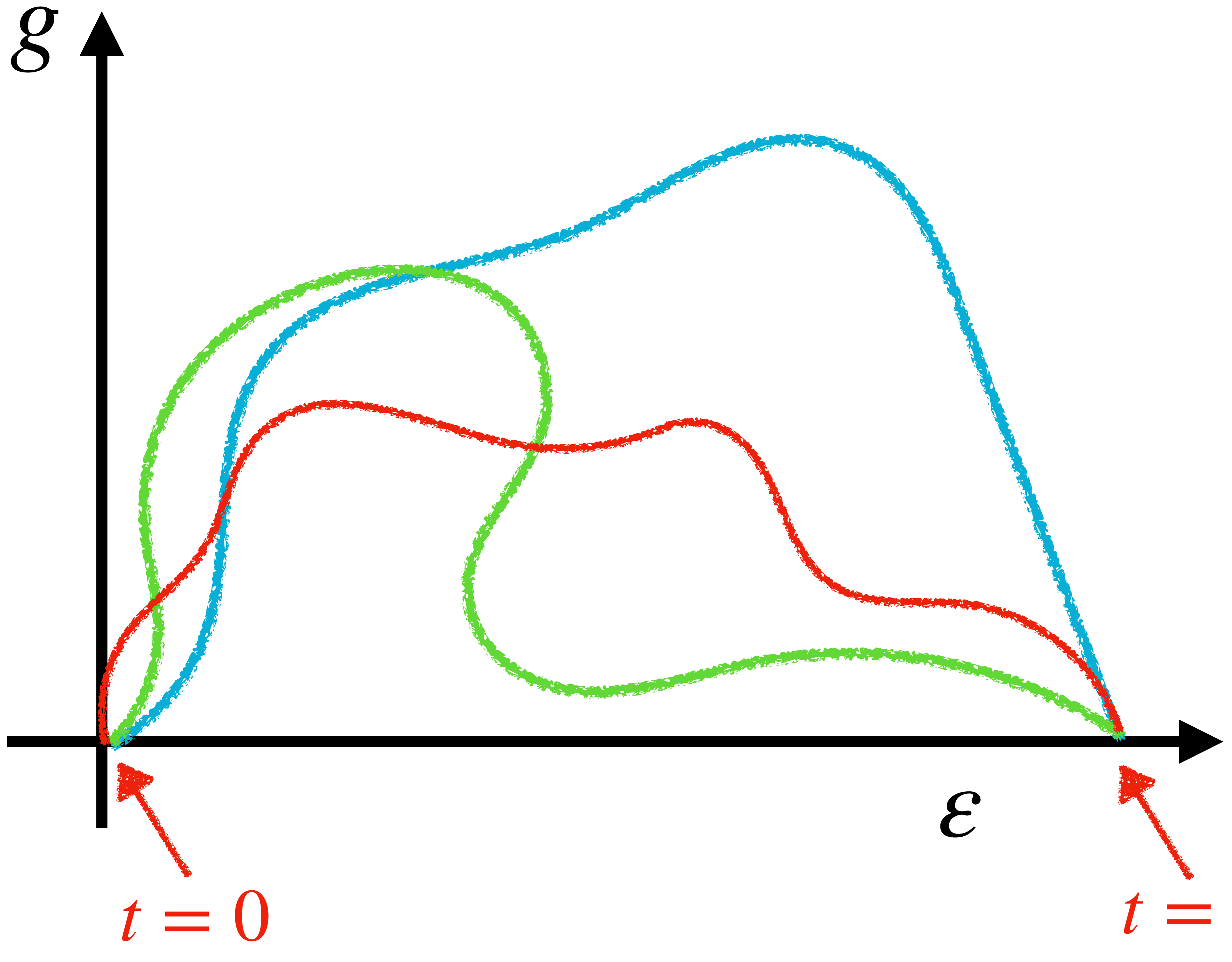
$$\hat{H}_{SB} = g(t)\hat{V} = g(t)\sum_k \lambda_k \hat{a}^\dagger \hat{b}_k + h.c.$$



$$\hat{H}_S = \varepsilon(t)\hat{a}^\dagger \hat{a}$$

$$\hat{H}_B = \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k$$

possible protocols for erasure



We take the continuum limit and wide-band limit:

$$\mathfrak{J}(\omega) = 2\pi \sum_k |\lambda_k|^2 \delta(\omega - \omega_k)$$

$$\mathfrak{J}(\omega) = \frac{\Lambda^2}{\Lambda^2 + \omega^2}$$

Finite-time erasure at strong coupling

Rolandi, M. P.-L., Quantum 7, 1161 (2023)

Reversible contribution

Dissipated work

$$W = \Delta F + k_B T \Sigma$$

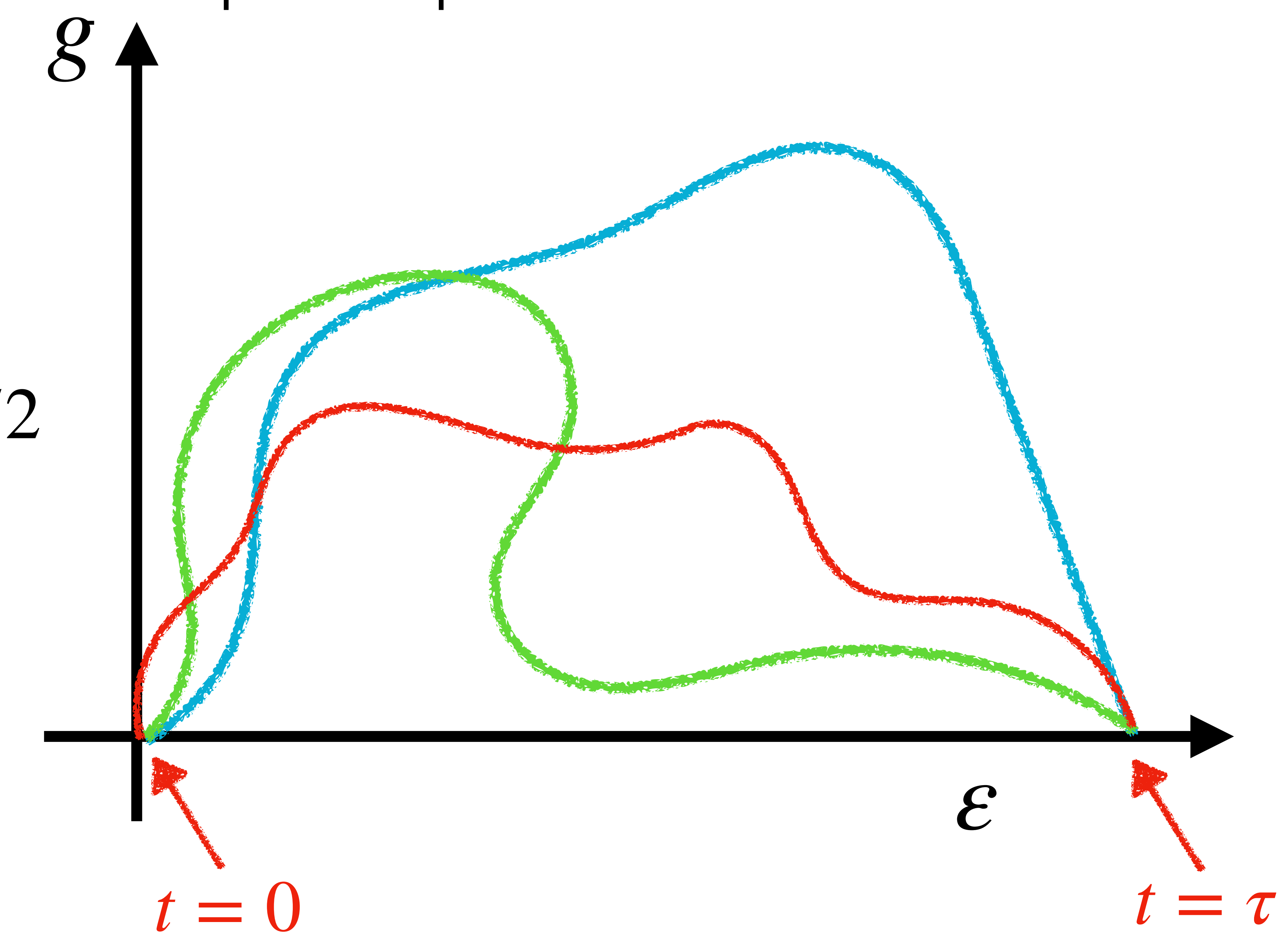
$$\boxed{\Sigma = \frac{1}{\tau}\int_\gamma \sum_{ij} \dot{\lambda}_i m_{ij} \dot{\lambda}_j + \mathcal{O}\left(\frac{1}{\tau^2 \Gamma^2}\right)}$$

$\mu := g^2/2$

$$\vec{\lambda} := (\varepsilon, \mu)^T$$

$$\Gamma := \frac{2}{\hbar \tau}\int_0^\tau dt\, \mu(t)$$

possible protocols for erasure



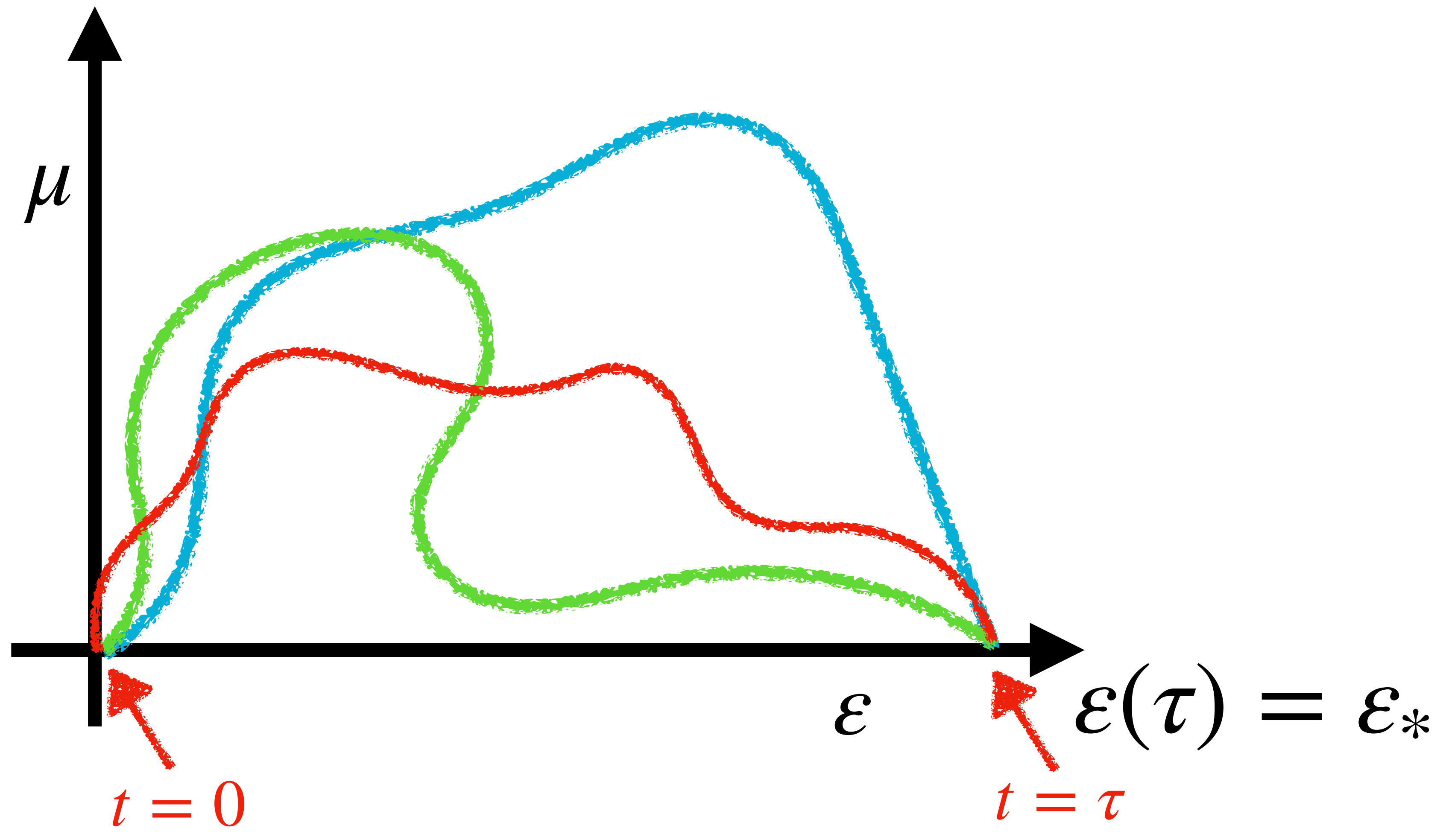
$$m = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{(1 + e^{\beta(\varepsilon - \omega)})^{-1}}{(\mu^2 + \omega^2)^3} \begin{pmatrix} 4\omega\mu^2 & \mu(\mu^2 - 3\omega^2) \\ \mu(\mu^2 - 3\omega^2) & 2\omega(\omega^2 - \mu^2) \end{pmatrix}$$

- ✔ Metric (smooth, symmetric, positive-definite)
- ✔ Valid for arbitrarily strong coupling

High-temperature limit

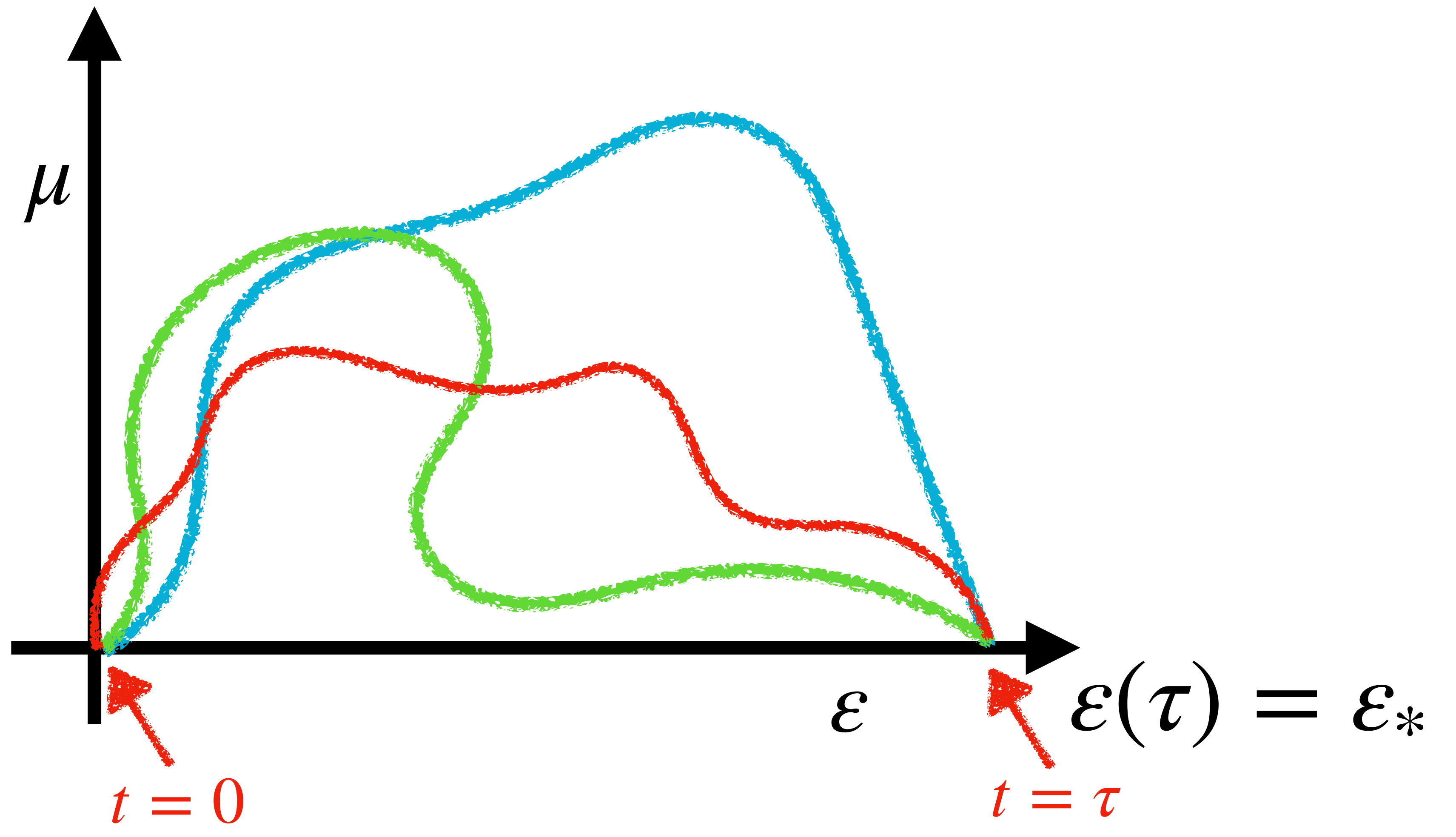
$\beta\varepsilon \ll 1$ and $\beta\mu \ll 1$

$$\boxed{m_{HT} = \frac{\hbar\beta}{8\mu}\mathrm{Id}}$$



High-temperature limit $\quad \beta\varepsilon \ll 1$ and $\beta\mu \ll 1$

$$\boxed{m_{HT} = \frac{\hbar\beta}{8\mu}\mathrm{Id}}$$



$$\varepsilon(t) = \varepsilon_* \left(t/\tau - \frac{\sin(2\pi t/\tau)}{2\pi} \right)$$

$$\mu(t) = \frac{\varepsilon_*}{\pi} \sin(\pi t/\tau)^2$$

$$\boxed{k_B T \Sigma_{\min} = \frac{\pi\hbar\beta\varepsilon_*}{2\tau}}$$

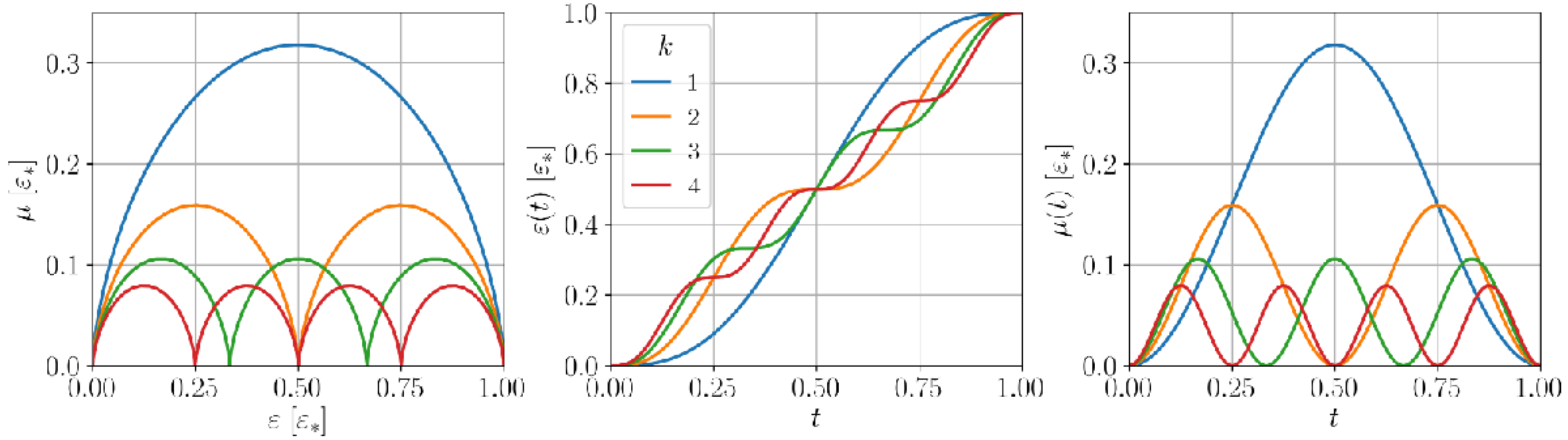


FIG. 2. Parametrization of $\mu(t)$ and $\varepsilon(t)$ described by eq. (D6) for multiple values of k. Shown in the parameter space (left) and as a function of time (centre and right).

Comparison with weak coupling results

$$\varepsilon(0) = 0, \quad \beta\varepsilon(\tau) \gg 1,$$
$$\mu(0) = \mu(\tau) = 0$$

Weak coupling

$$\boxed{\Sigma_{\min} = \frac{\pi^2}{8\mu\tau}}$$

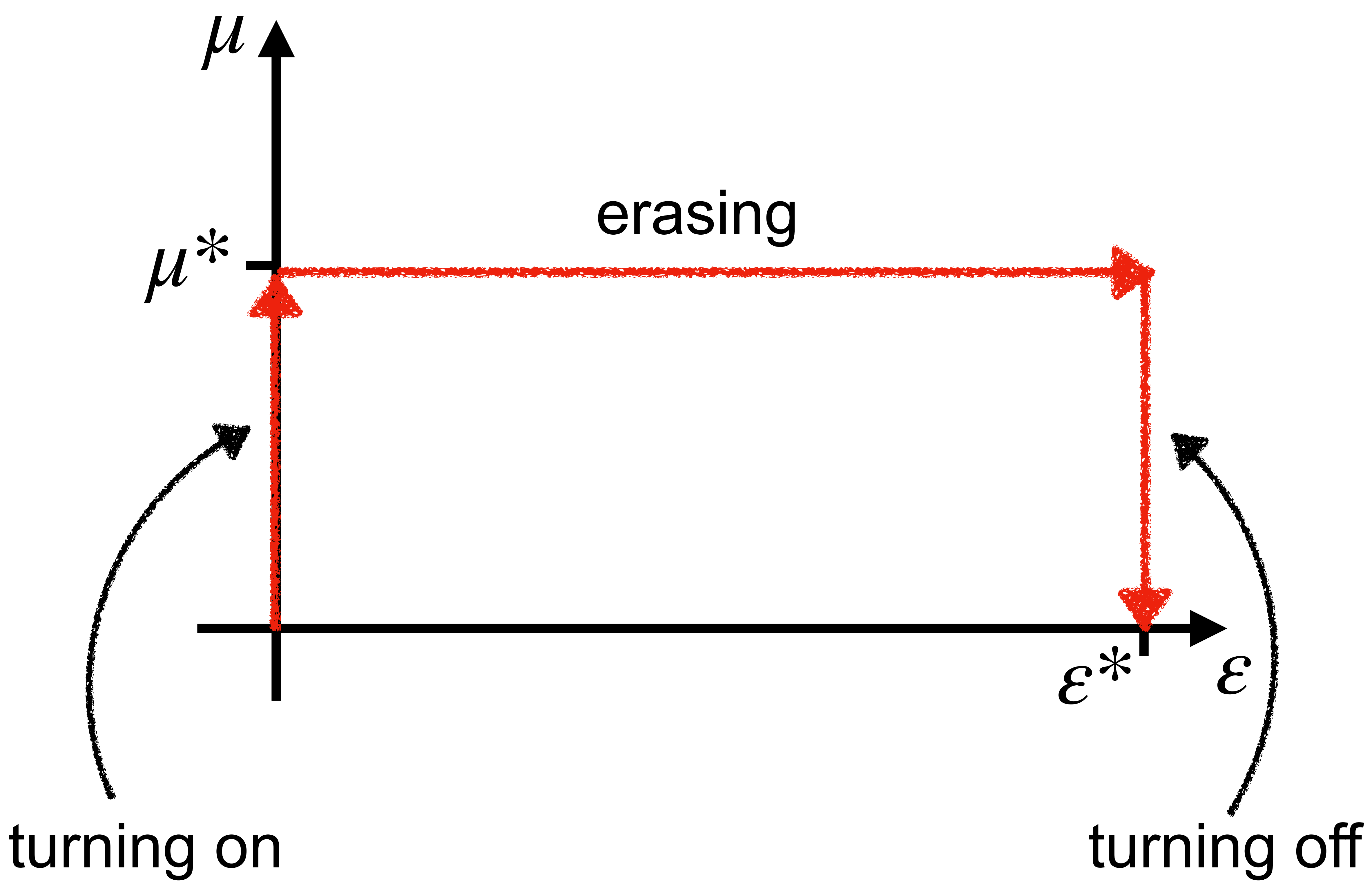
Arbitrary coupling

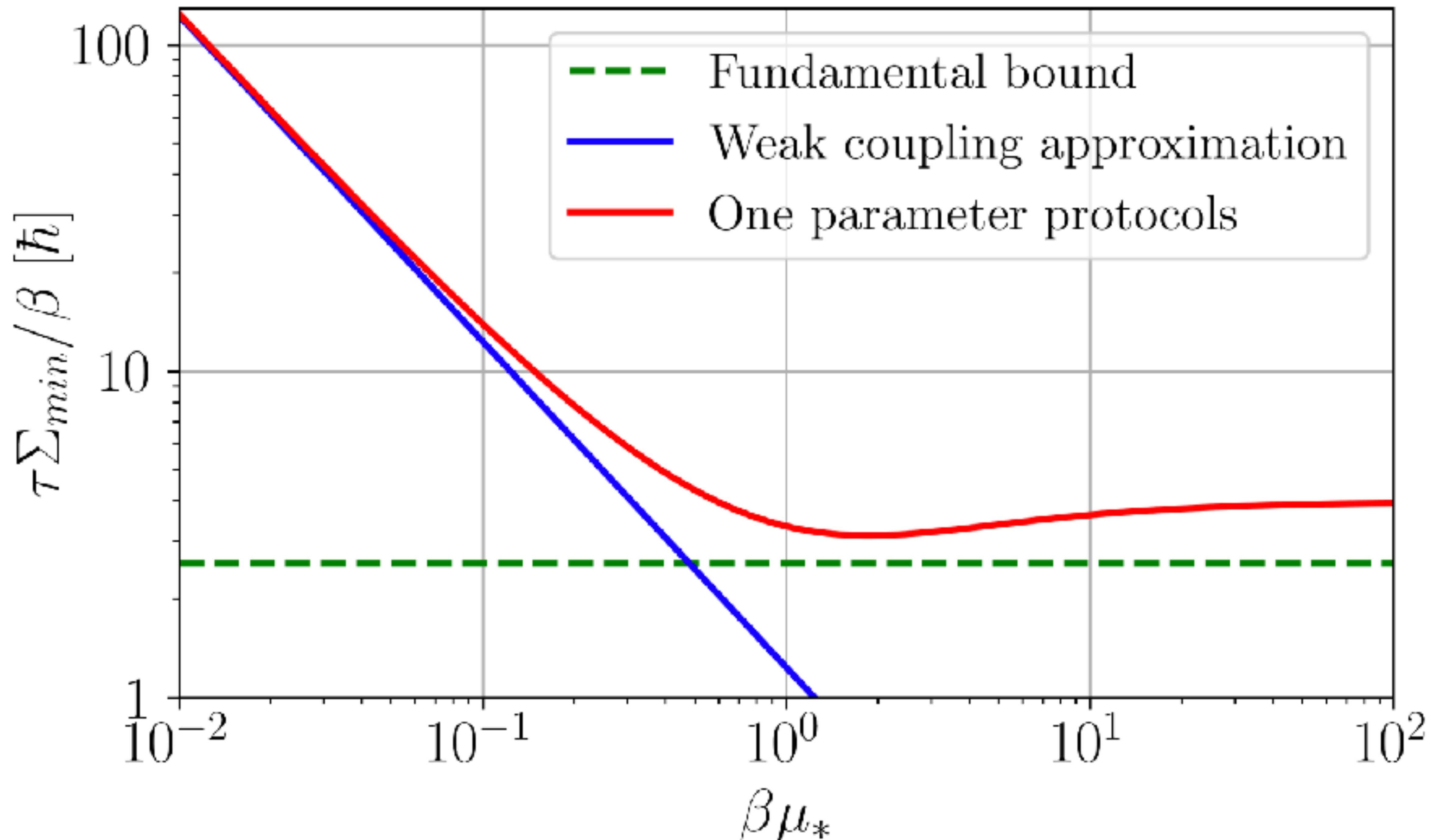
$$\boxed{\Sigma_{\min} = \frac{\boxed{\beta\hbar}}{\tau} a}$$

Planckian time

$$a \approx 2.57946$$

Rolandi, M. P.-L.,
Quantum 7, 1161 (2023)





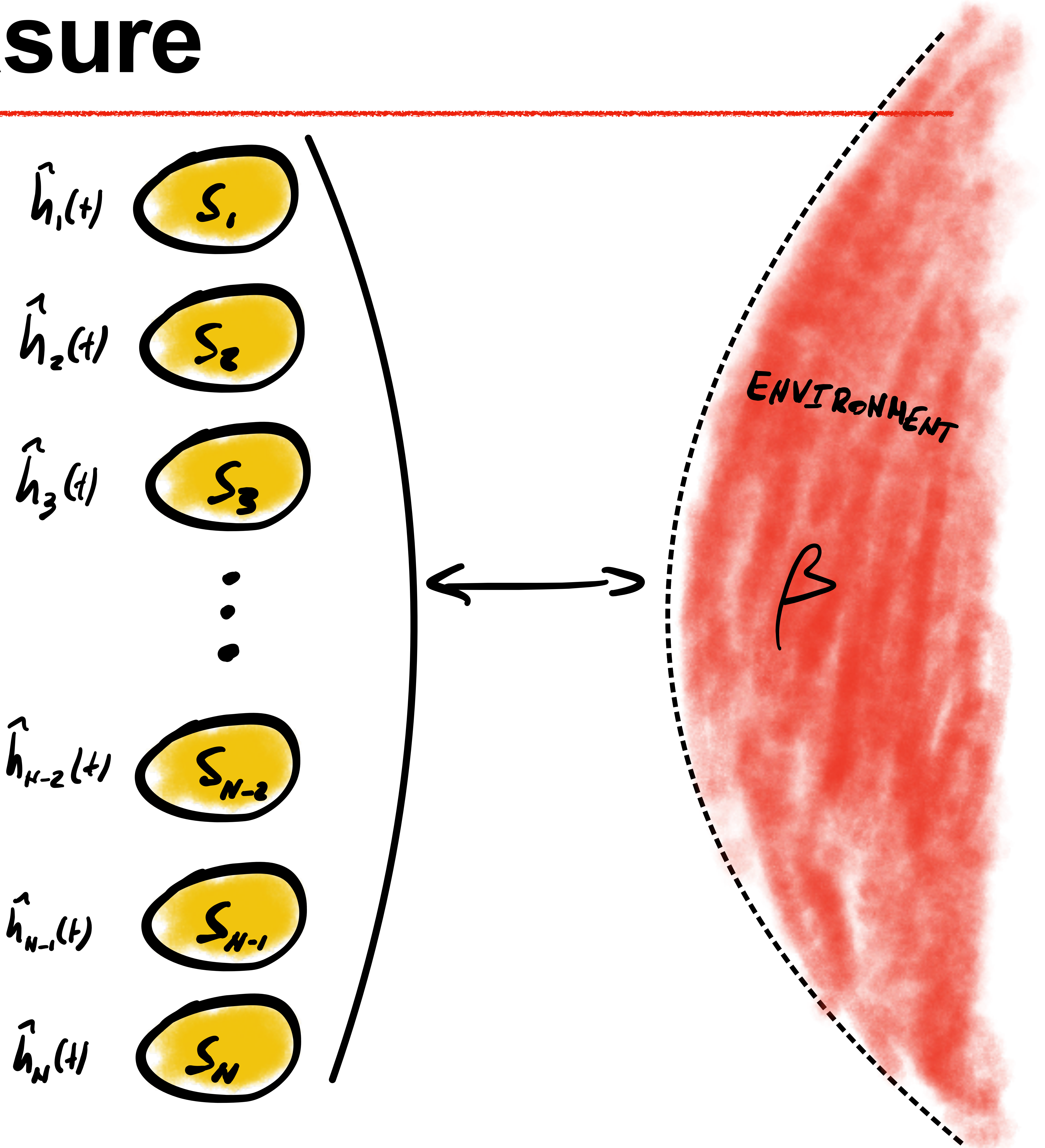
Collective advantages in finite-time thermodynamics

Collective Landauer erasure

$$\hat{H}(t) = \hat{H}_0(t) + \hat{V}(t)$$

$$\hat{H}_0(t) = \sum_{k=1}^{N} \hat{h}_k(t)$$

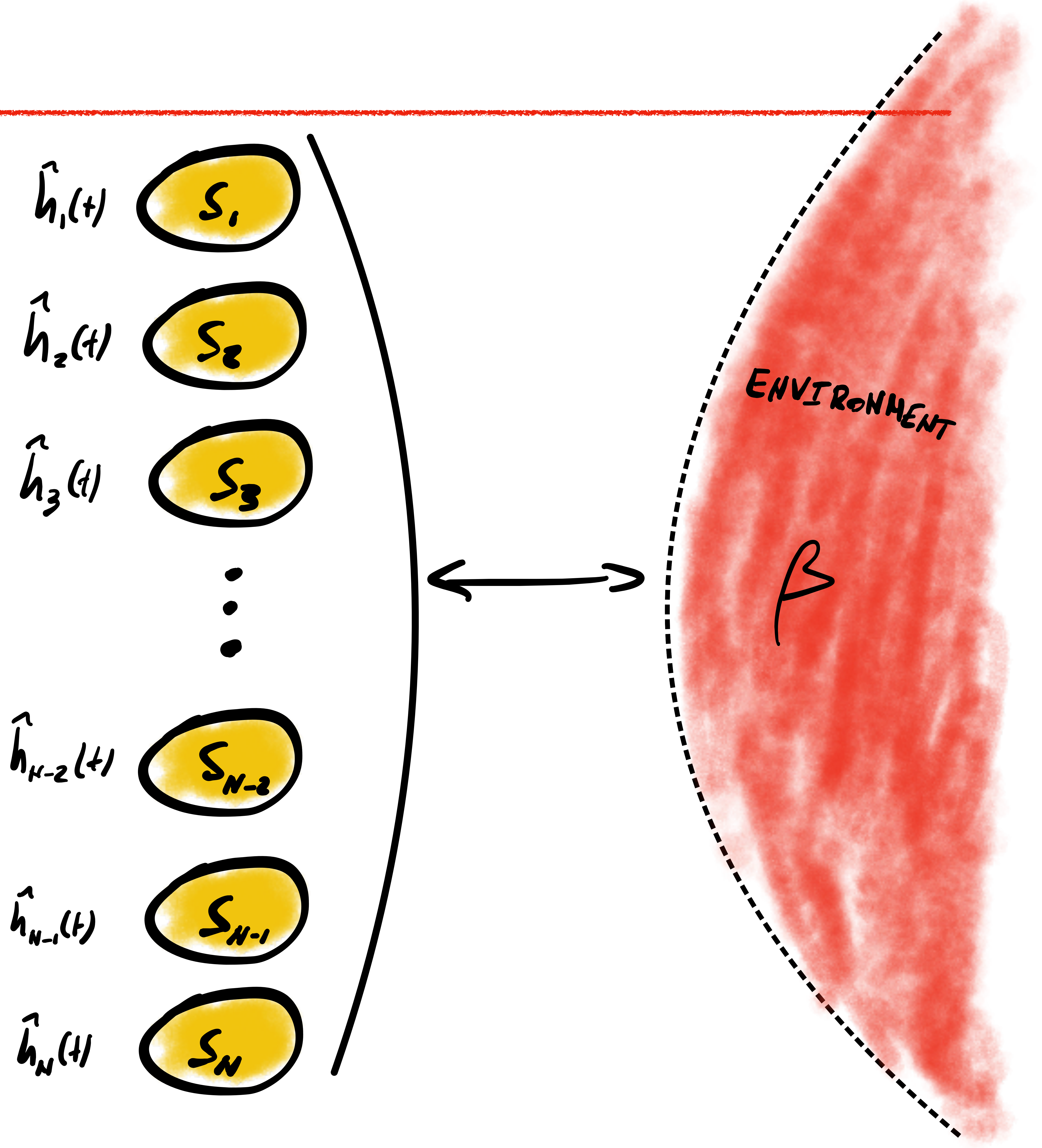
$$\hat{V}(0) = \hat{V}(\tau) = 0$$



Local limit

$$\hat{H}(t) = \hat{H}_0(t) + \cancel{\hat{V}(t)}$$

$$\beta W_{diss}^{local} = N\frac{\pi^2}{4\tau}$$



Fundamental limit

→ Full control

$$\hat{V} \;—\; \left(\begin{array}{l} \hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)},\ \hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(3)},\ \hat{\sigma}_z^{(2)}\hat{\sigma}_z^{(3)},\ \ldots \\ \hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\hat{\sigma}_z^{(3)},\ \ldots \\ \quad\vdots \\ \hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\hat{\sigma}_z^{(3)}\ldots\hat{\sigma}_z^{(N)} \end{array} \right.$$

Fundamental limit

$$\hat{H}(t) = \sum_k \lambda_k(t)\hat{X}_k \qquad g_{ij} = \tau_{eq}\frac{\partial^2 \ln Z}{\partial\lambda^i\partial\lambda^j}$$

$$x_i := \sqrt{\frac{e^{-\beta\varepsilon_i}}{Z}} \longrightarrow ds^2 = \sum_i dx_i^2$$

→ Geodesics are great circles

$$\min_\lambda \beta W_{diss} \leq \frac{\pi^2}{\tau}$$

Fundamental limit

$$\hat{H}(t) = \sum_k \lambda_k(t)\hat{X}_k \qquad g_{ij} = \tau_{eq}\frac{\partial^2 \ln Z}{\partial\lambda^i\partial\lambda^j}$$

$$x_i := \sqrt{\frac{e^{-\beta\varepsilon_i}}{Z}} \longrightarrow ds^2 = \sum_i dx_i^2$$

More generally,

$$\min_\lambda \beta W_{diss} = \frac{4}{\tau}\arccos^2 \mathrm{Tr}\left[\sqrt{\hat{\rho}_{th}(0)}\sqrt{\hat{\rho}_{th}(\tau)}\right]$$

Abiuso, Miller, M. P.-L., Scandi, Entropy 22(10), 1076 (2020)

Fundamental limit

- Geodesics are great circles

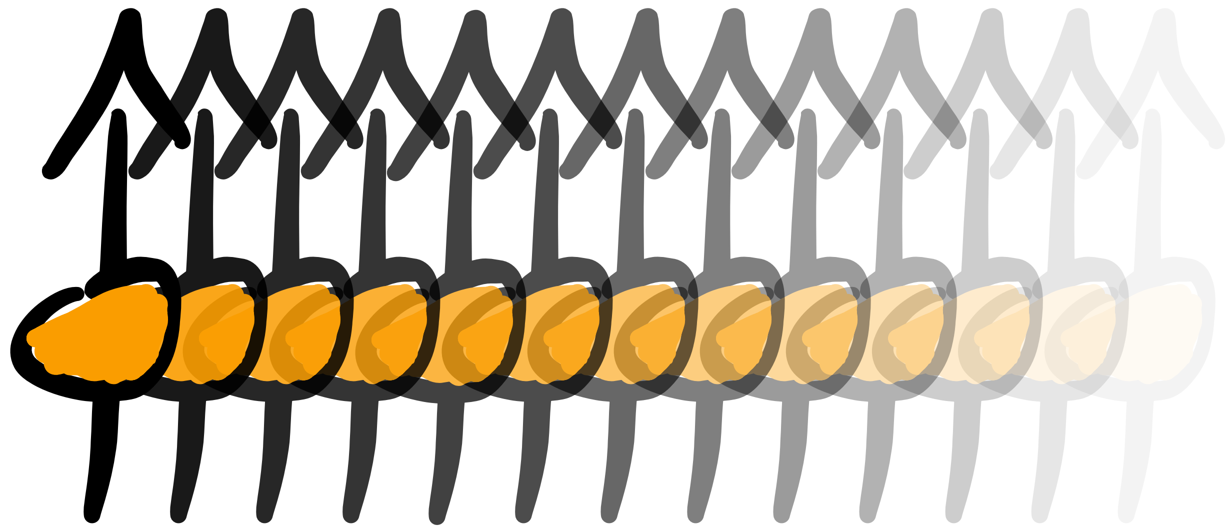
$$\min_{\lambda} \beta W_{diss} = \frac{4}{\tau} \arccos^2 \mathrm{Tr}\left[\sqrt{\hat{\rho}_{th}(0)}\sqrt{\hat{\rho}_{th}(\tau)}\right]$$

$$\hat{H}(t) = -2\log\left[\sin\left(\frac{L(\tau - t)}{\tau}\right)\sqrt{\hat{\rho}_{th}(0)} + \sin\left(\frac{Lt}{\tau}\right)\sqrt{\hat{\rho}_{th}(\tau)}\right]$$

- Every order of interaction is needed

Local vs global erasure

$$\hat{H} = \varepsilon \hat{\sigma}_z$$



$$\varepsilon : 0 \longrightarrow \infty$$

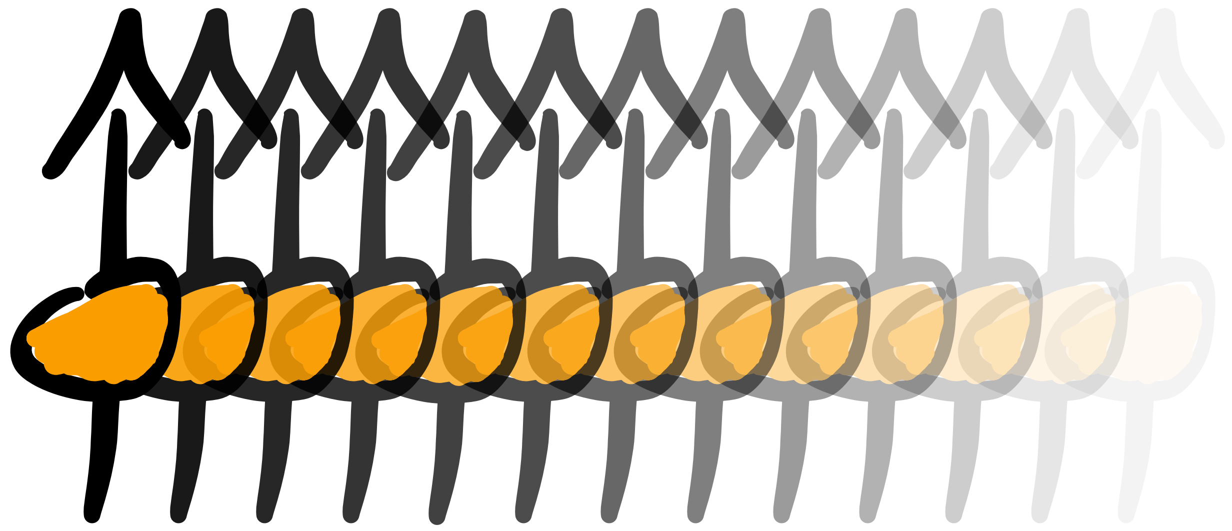
$$W = k_B T N \ln 2 + W_{diss}$$

$$\beta W_{diss}^{local} = N \frac{\pi^2}{4\tau}$$

$$\beta W_{diss}^{global} = \frac{\pi^2}{\tau}$$

Collective bit reset

$$\hat{H} = \varepsilon \hat{\sigma}_z$$



$$W^{global}_{qubit} = k_B T \ln 2 + \frac{k_B T \pi^2}{\tau N} + \mathcal{O}(\tau^{-2})$$

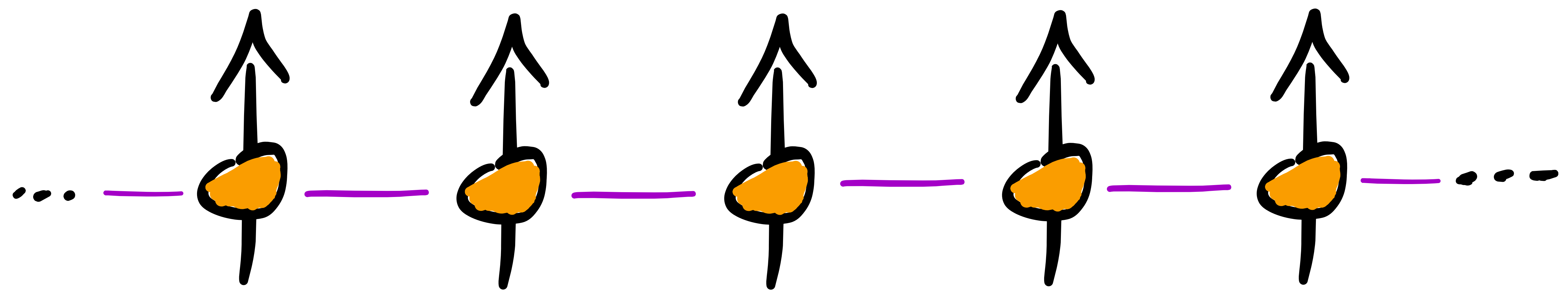
More realistic control

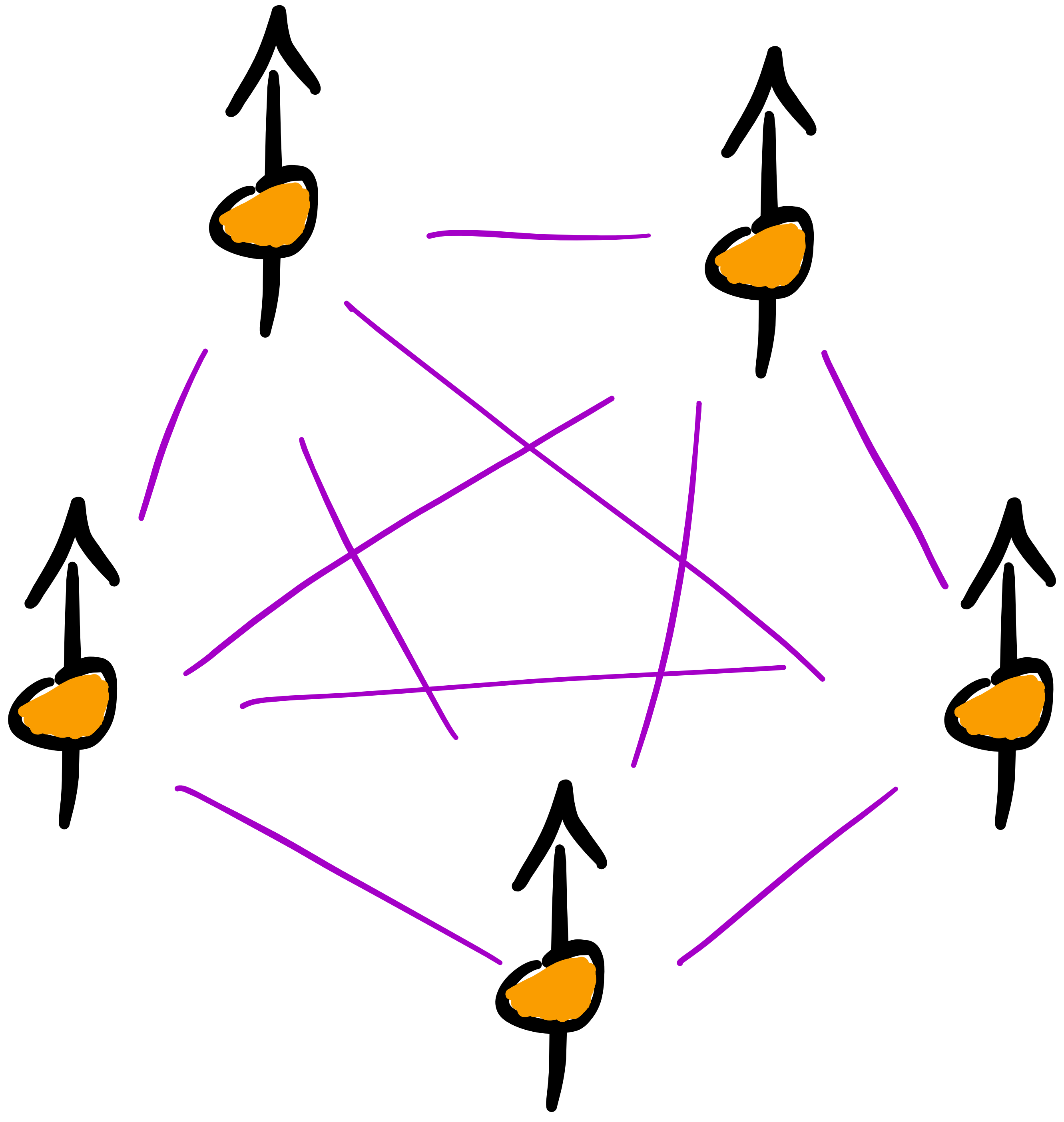
→ Restrict to two-body interaction with few control parameters

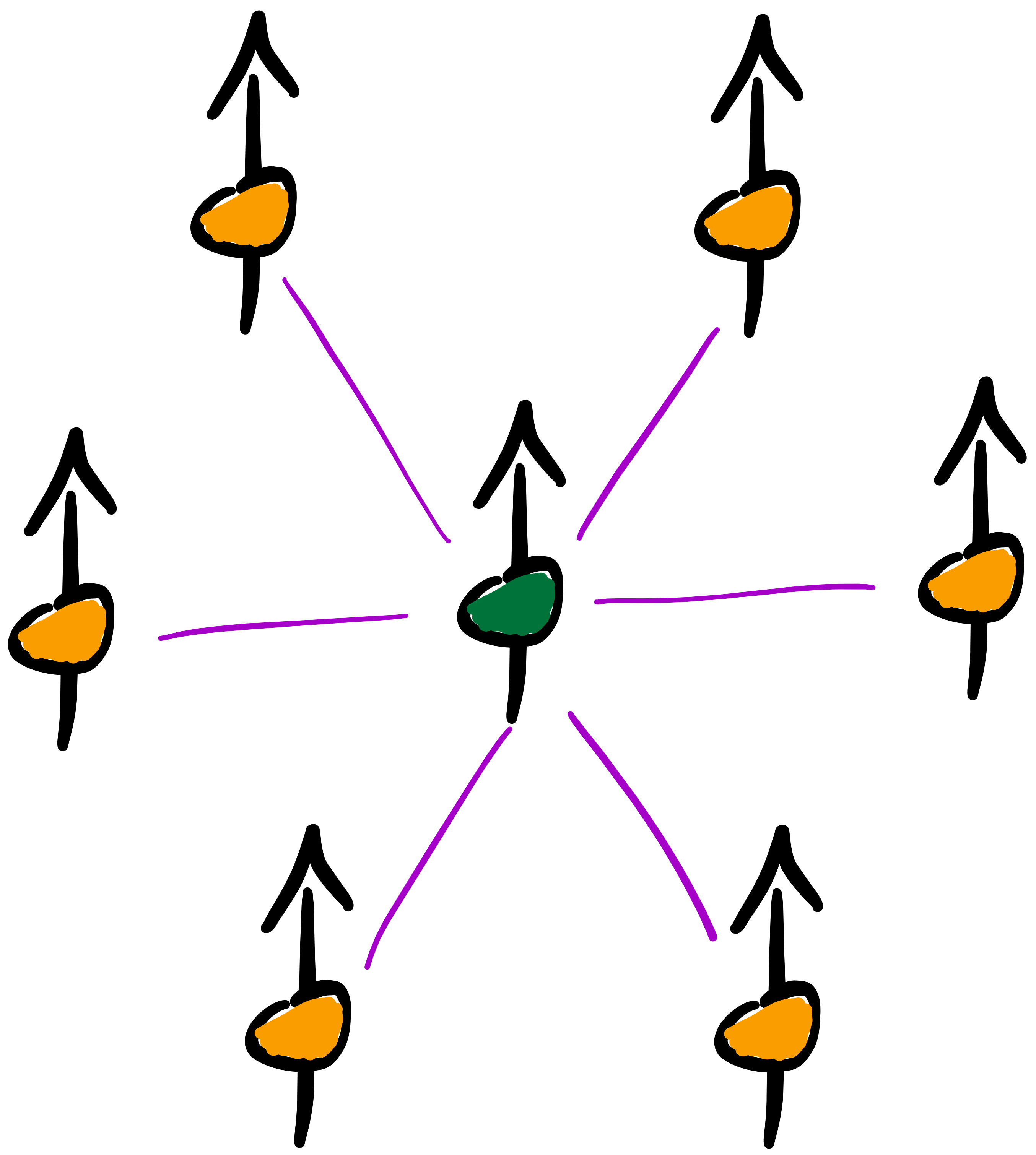
$$\hat{V} \;—\; \left(\begin{array}{l} \hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)},\ \hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(3)},\ \hat{\sigma}_z^{(2)}\hat{\sigma}_z^{(3)},\ \ldots \\ \cancel{\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\hat{\sigma}_z^{(3)},\ \ldots} \\ \cancel{\vdots} \\ \cancel{\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}\hat{\sigma}_z^{(3)}\cdots\hat{\sigma}_z^{(N)}} \end{array} \right.$$

More realistic control

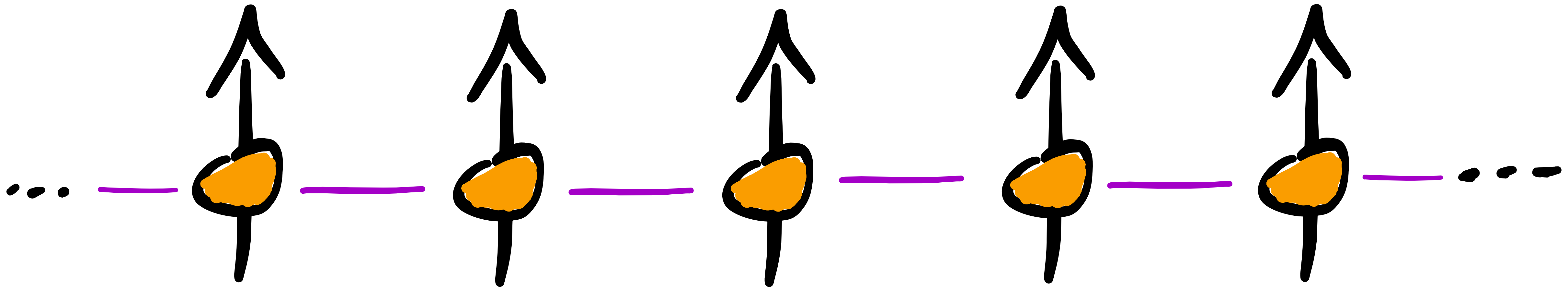
$$\hat H(t) = \epsilon_0(t)\,\hat\sigma_z^{(0)} + \epsilon(t)\sum_{i=1}^{N}\hat\sigma_z^{(i)} + J(t)\sum_{i=1}^{N-1}\hat\sigma_z^{(i)}\hat\sigma_z^{(i+1)} + J(t)\sum_{i\neq j}^{N}\hat\sigma_z^{(i)}\hat\sigma_z^{(j)} + J(t)\sum_{i}\hat\sigma_z^{(i)}\hat\sigma_z^{(0)}$$







Erasure on a 1D spin chain



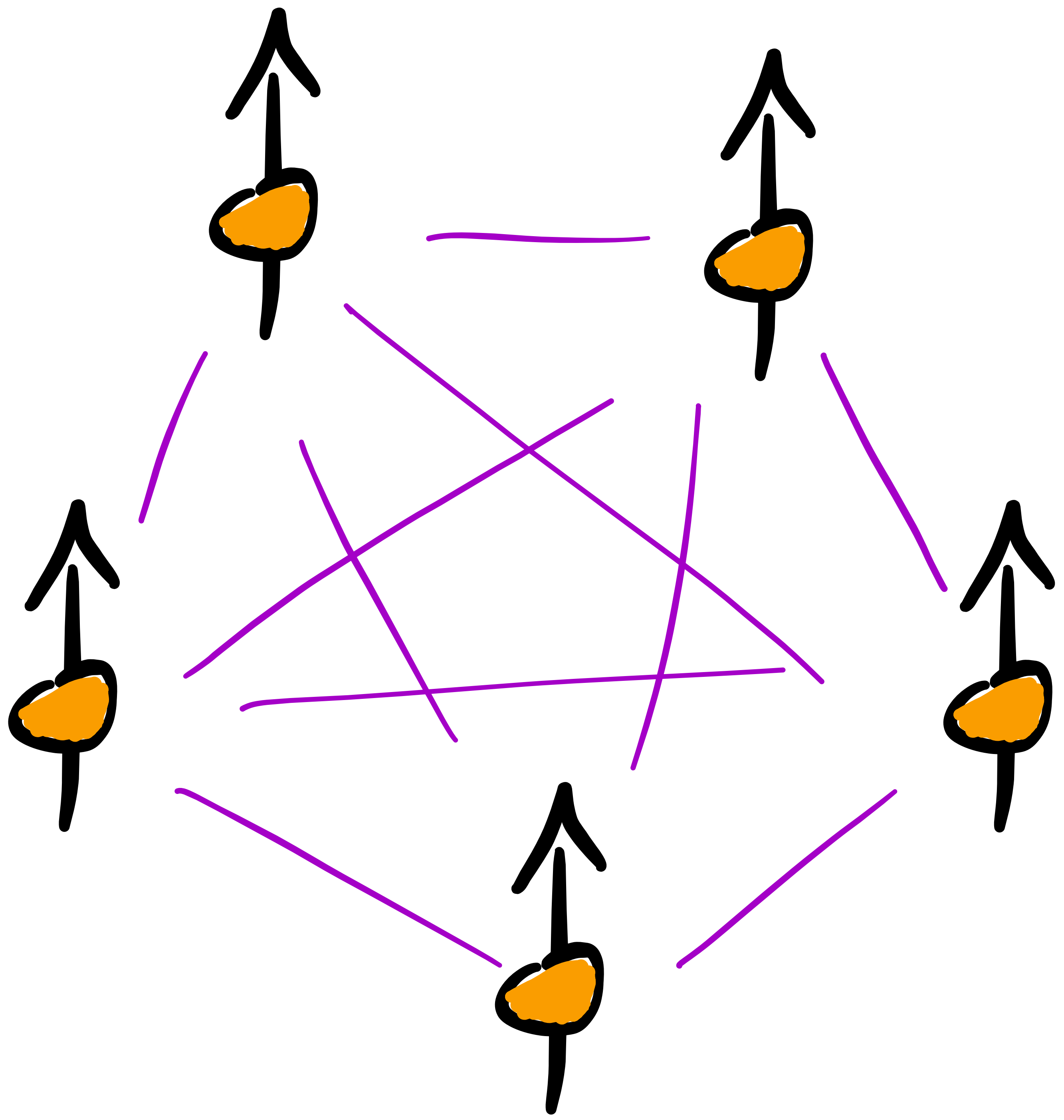
$$\hat{H}(t) = \varepsilon(t)\sum_{i=1}^{N}\hat{\sigma}_z^{(i)} + J(t)\sum_{i=1}^{N}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(i+1)}$$

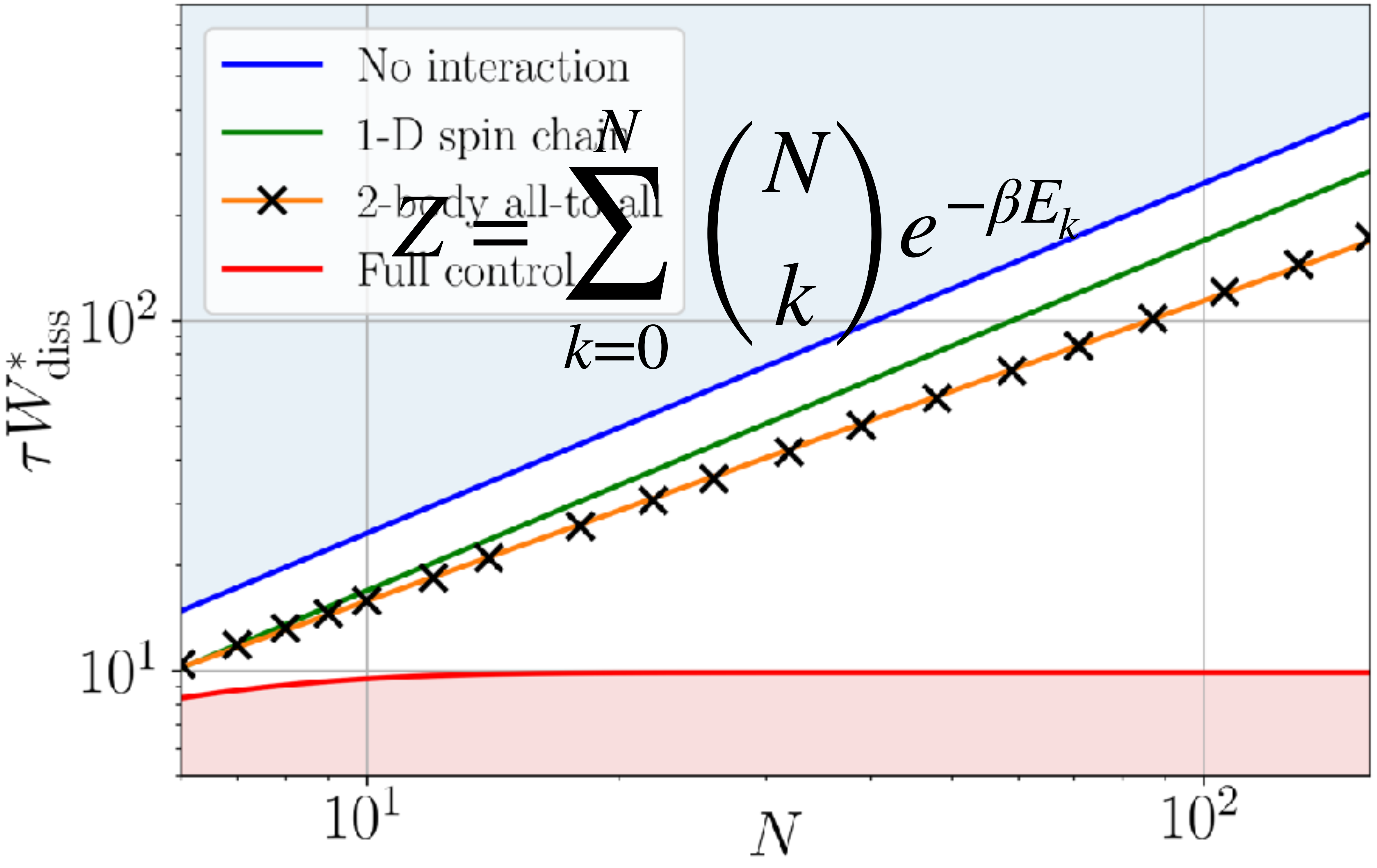
$$\ln Z = -N\beta J + N\ln\left[\cosh\beta\varepsilon + \sqrt{\sinh^2\beta\varepsilon + e^{4\beta J}}\right]$$

$$\longrightarrow \quad g_{ij} \propto N \quad \longrightarrow \quad W_{diss} \propto N$$

Erasure on a all-to-all model

$$\hat{H}(t) = \varepsilon(t)\sum_{i=1}^{N} \hat{\sigma}_z^{(i)} + J(t)\sum_{i>j}^{N} \hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}$$



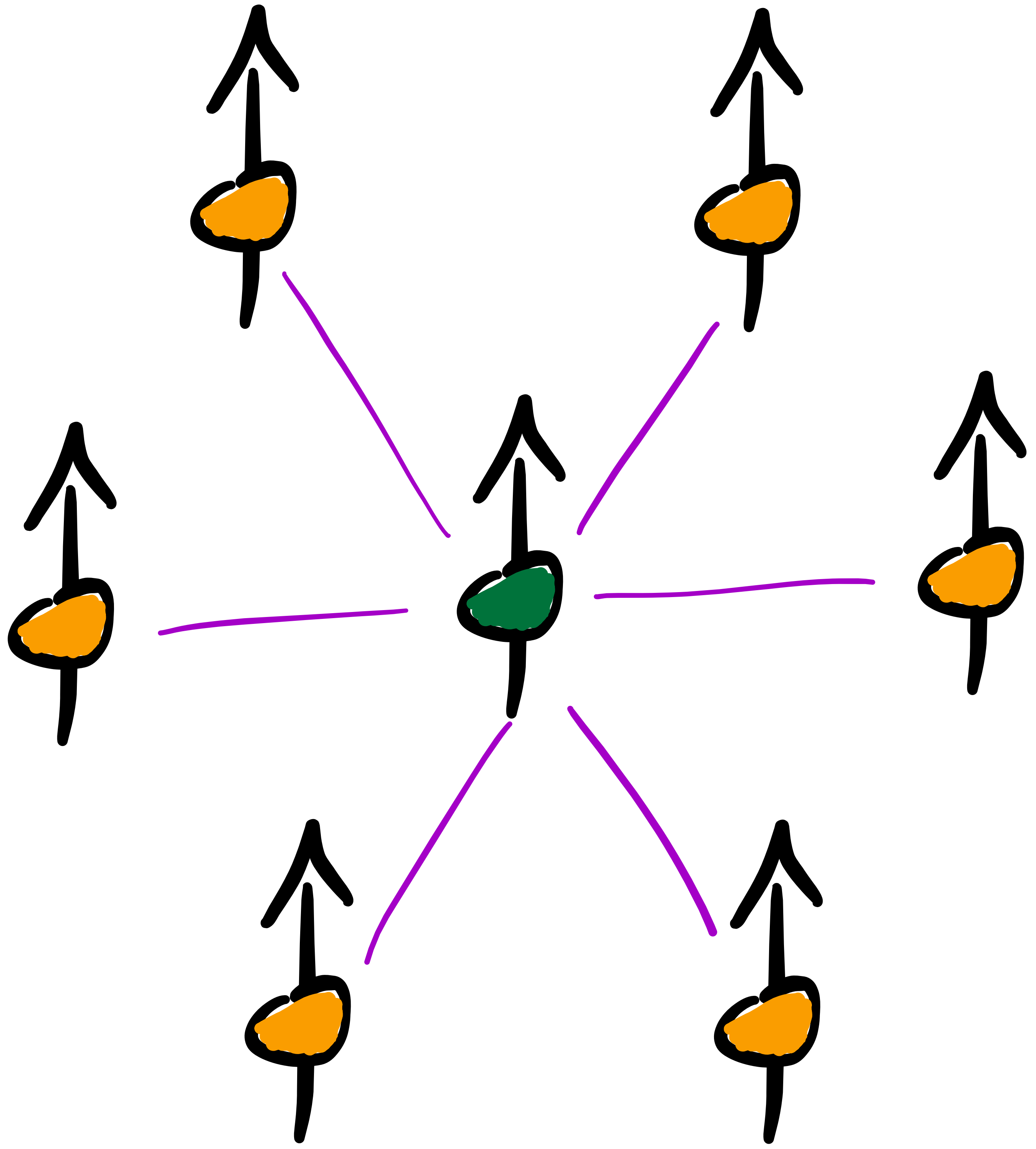


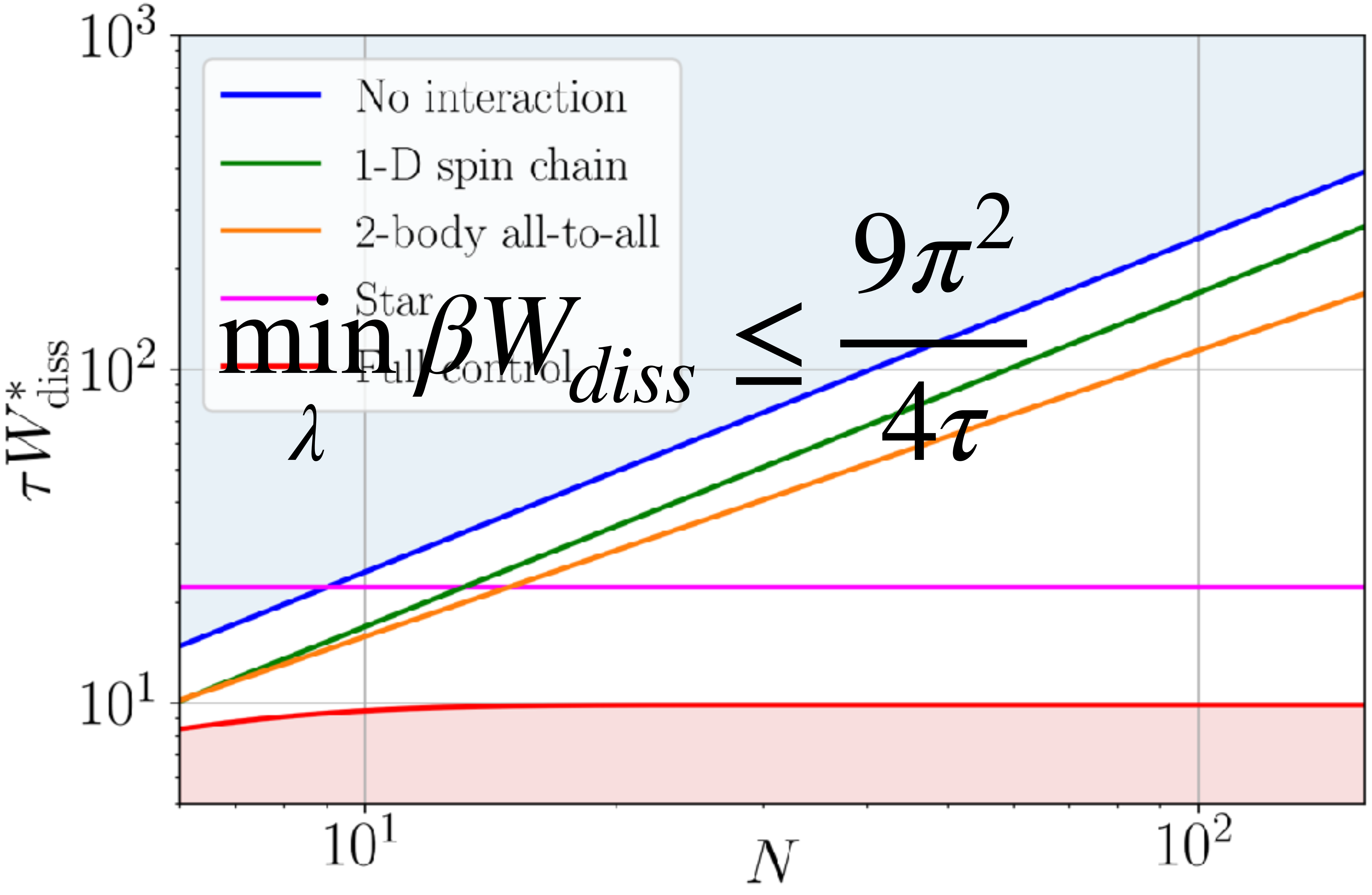
$$Z = \sum_{k=0}^{N} \binom{N}{k} e^{-\beta E_k}$$

$W_{diss} \sim N^{0.85}$

Erasure on the star model

$$\hat{H}(t) = \varepsilon_0(t)\hat{\sigma}_z^{(0)} + \sum_{i=1}^{N-1} \varepsilon_1(t)\hat{\sigma}_z^{(i)} + J(t)\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(0)}$$

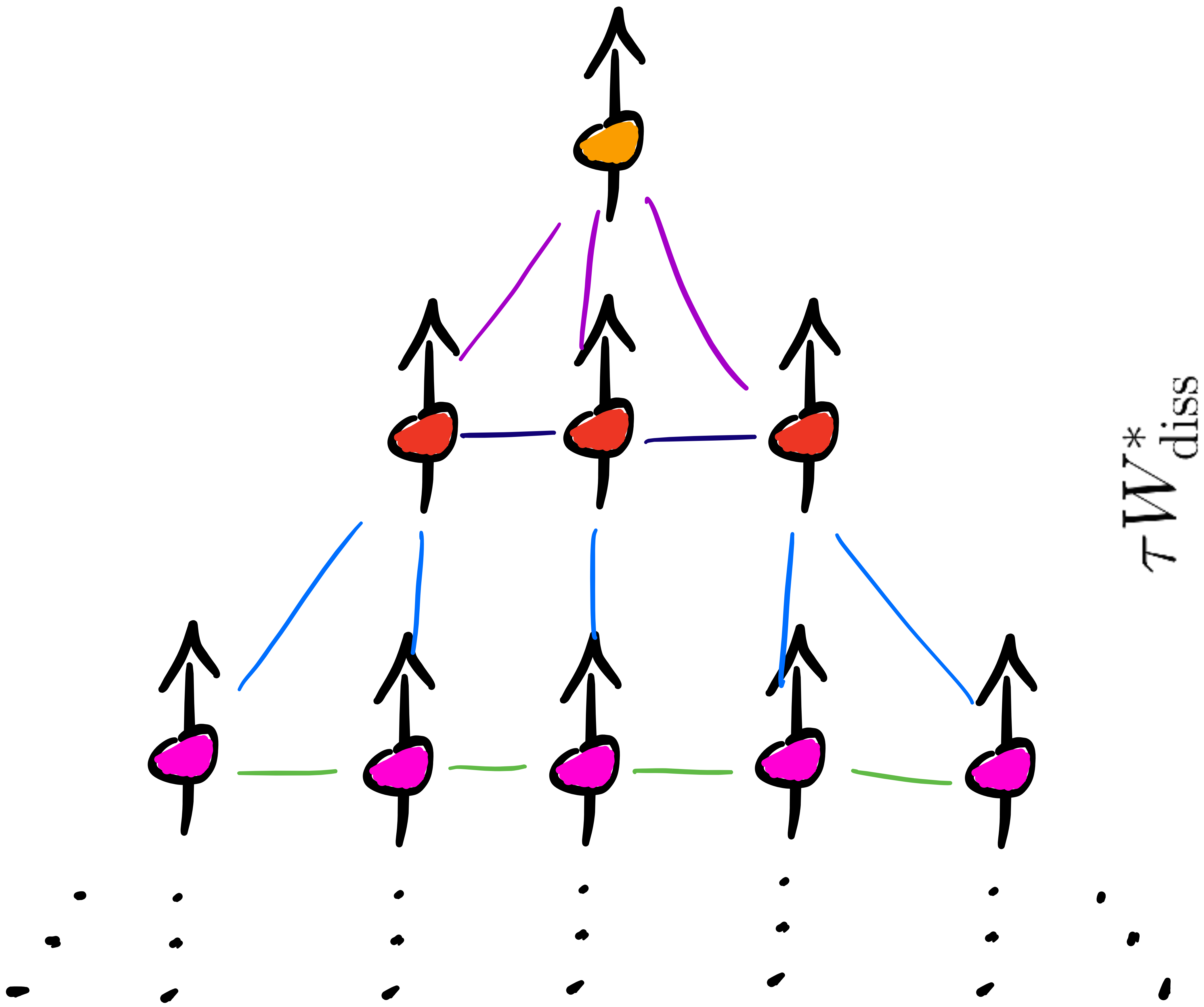


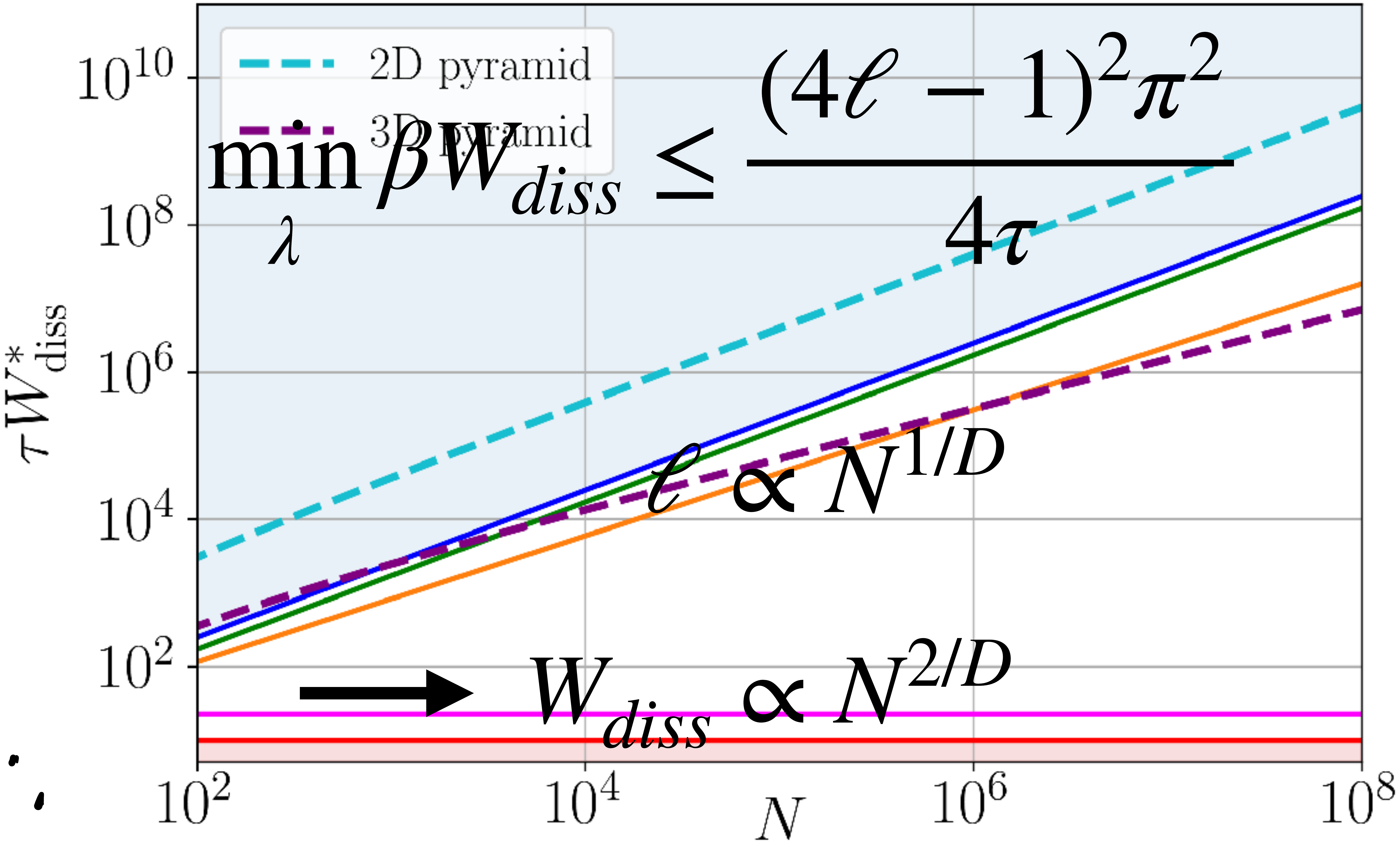


$$\min_{\lambda} \beta W_{diss} \leq \frac{9\pi^2}{4\tau}$$

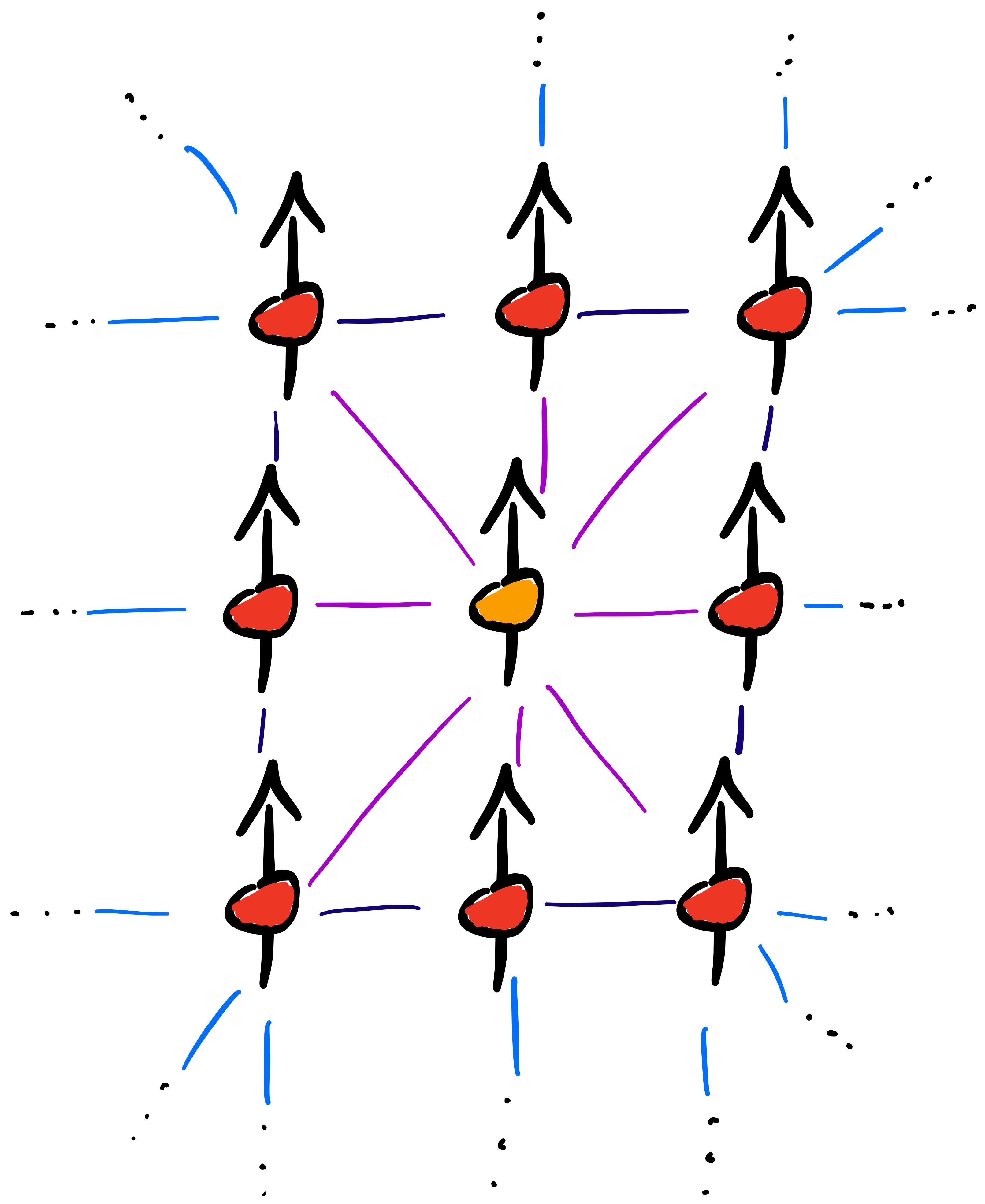
Erasure on the pyramid model (short range)

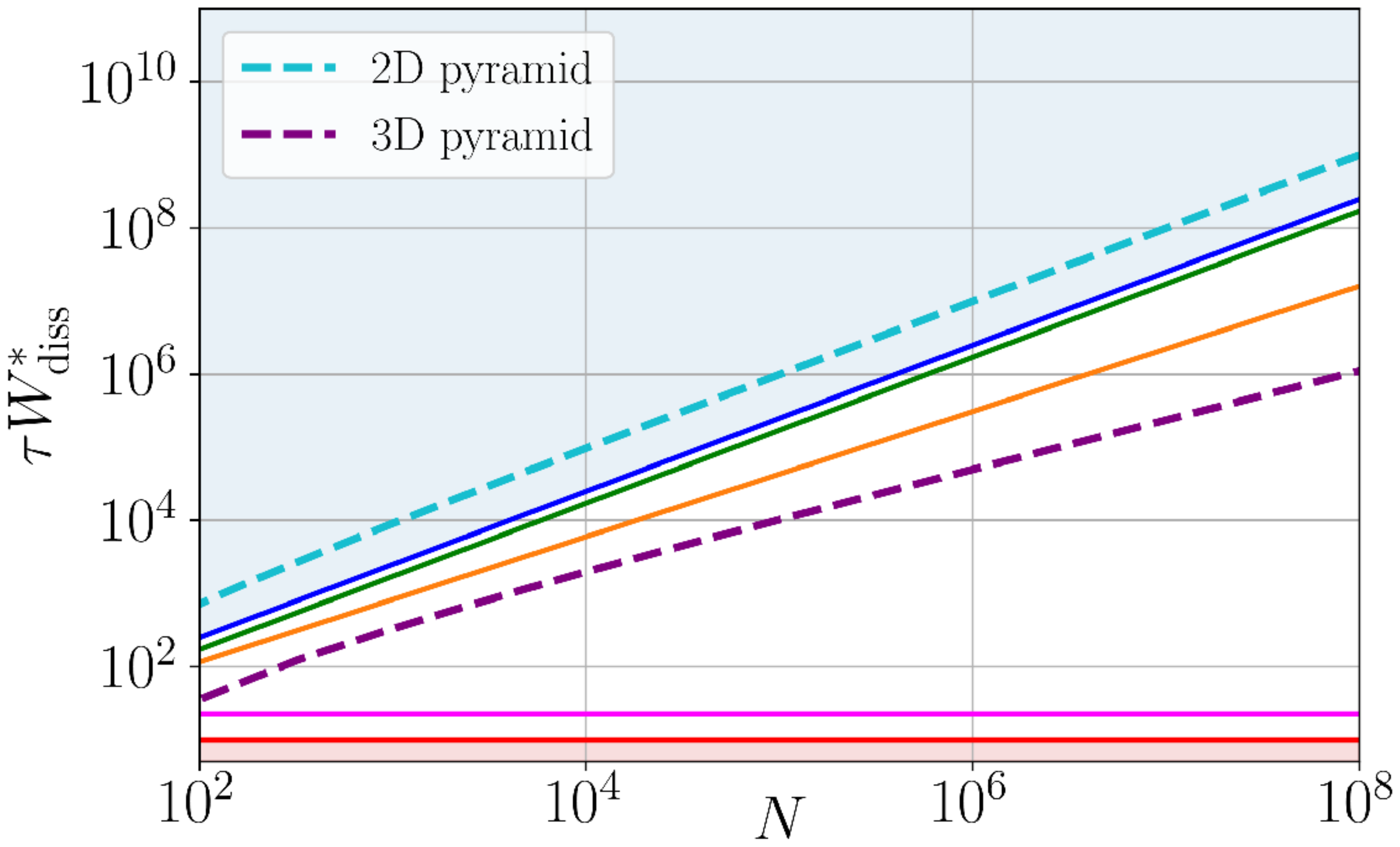
→ Same as star model, but in ℓ layers





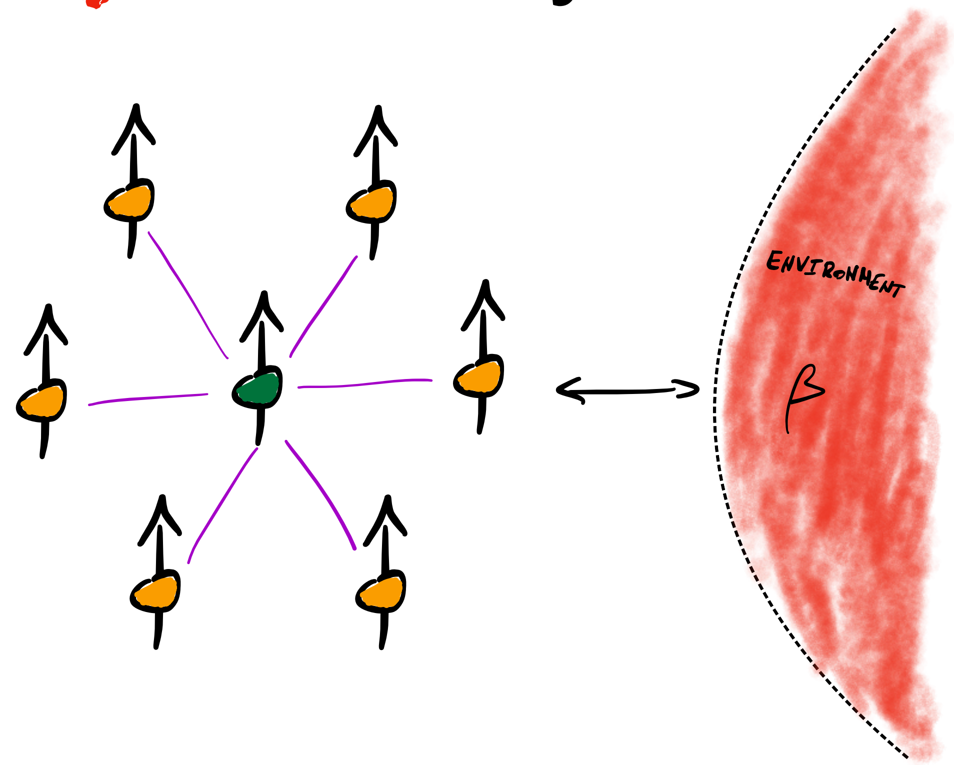
Erasure on the pyramid model (short range)





Outlook

- Beyond trivial models of thermalization



$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{\tau_{\mathrm{eq}}}(\hat{\rho}_{\mathrm{eq}}(t) - \hat{\rho}(t)] \longrightarrow \frac{d}{dt}\hat{\rho}(t) = \mathscr{L}[\hat{\rho}(t)]$$

- Beyond linear-response (slow driving)

$$W_{qubit} = k_B T \ln 2 + \frac{k_B T \pi^2}{\tau N} + \mathcal{O}(\tau^{-2})$$

- Beyond weak coupling

- Implementations

$$H = e_A a^\dagger a + e_B b^\dagger b + J(a^\dagger b + b^\dagger a) + U a^\dagger a b^\dagger b$$

Conclusion

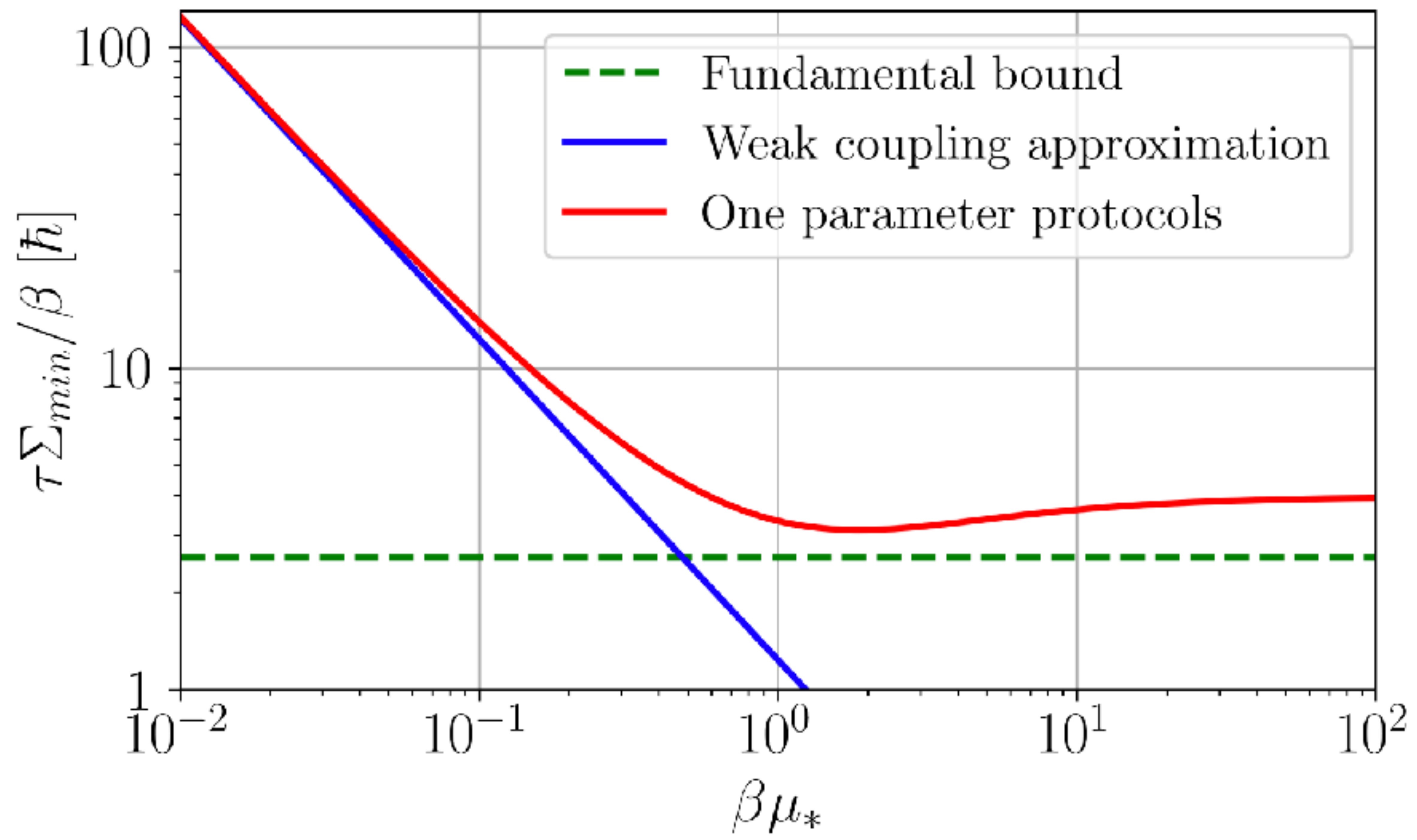
- Quantum thermodynamic geometry for many-body driven systems
- Finite-time Landauer erasure at strong coupling

Weak coupling

$$\boxed{k_B T \Sigma_{\min} = \frac{k_B T \pi^2}{4\Gamma\tau}}$$

Arbitrary coupling

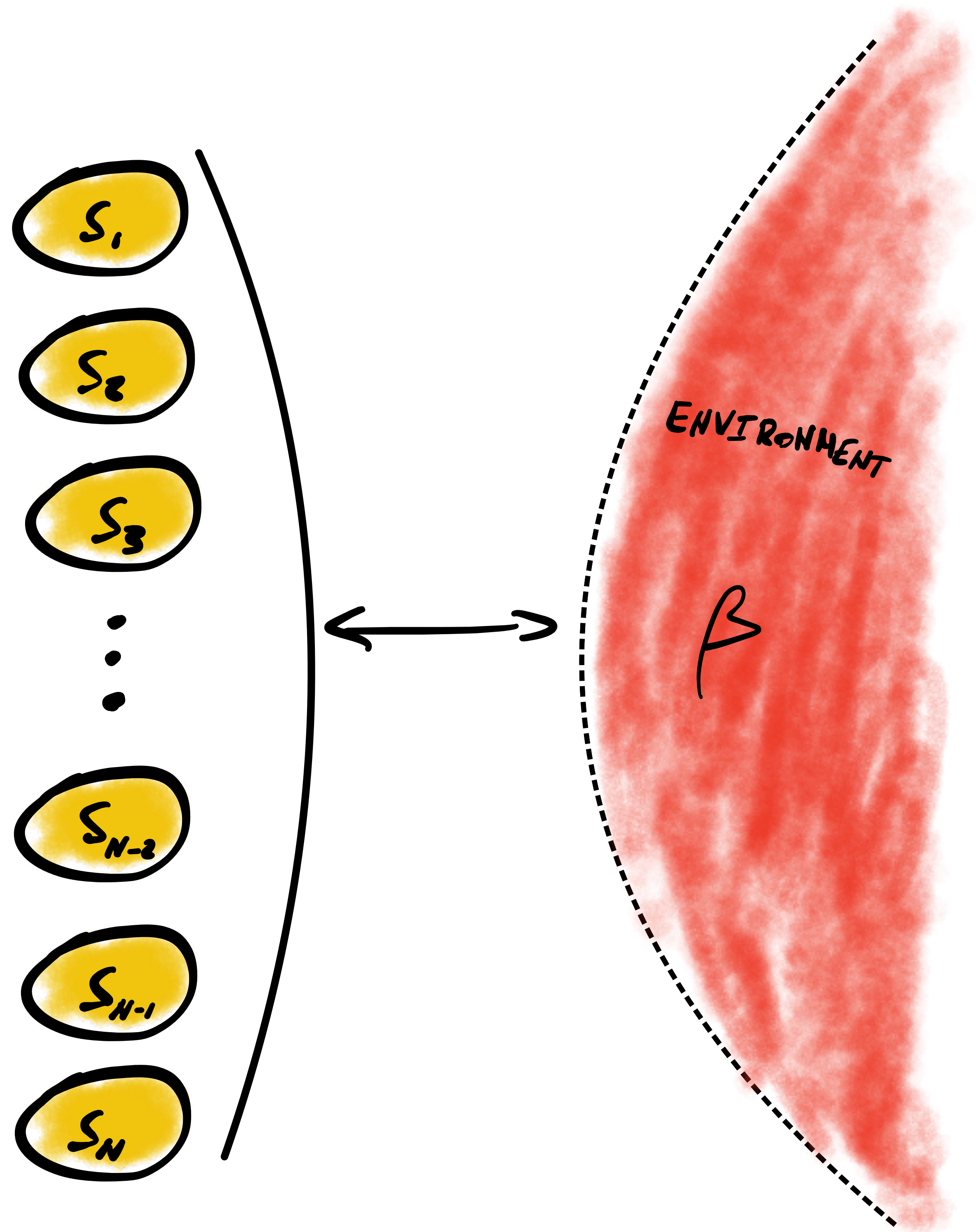
$$\boxed{k_B T \Sigma_{\min} = \frac{\hbar a}{\tau}}$$



Conclusion

- Quantum thermodynamic geometry for many-body driven systems
- Collective thermodynamic advantage

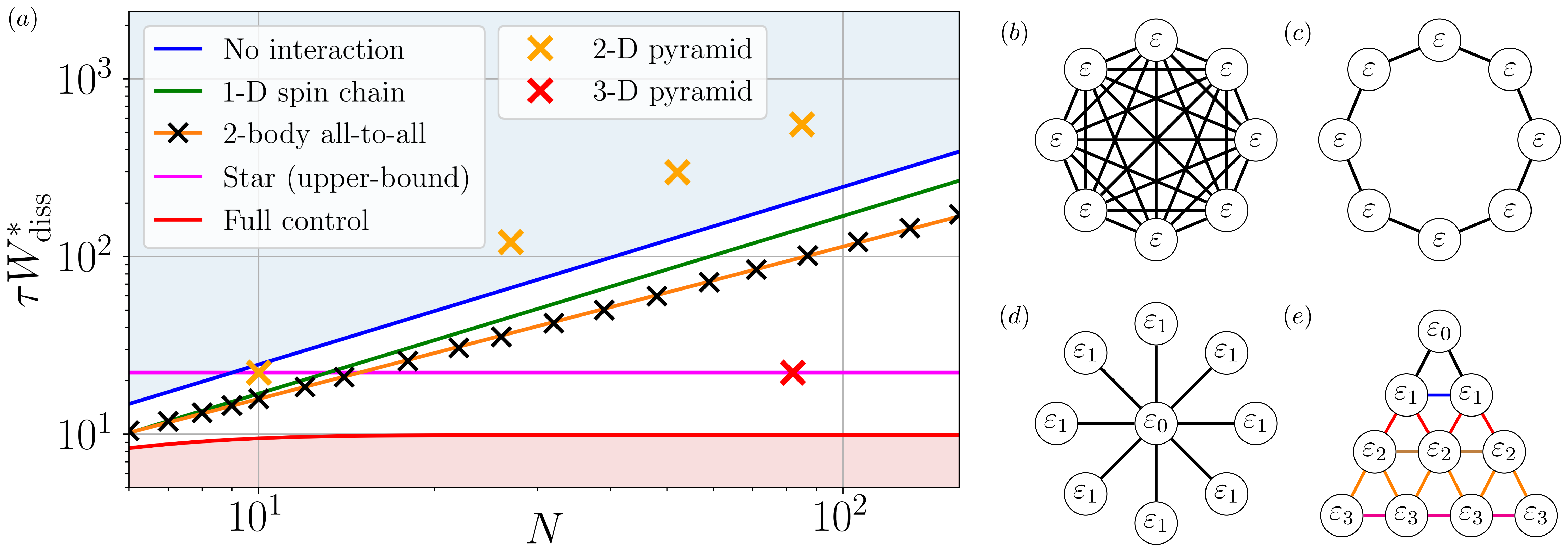
$$W = \Delta F + W_{diss}$$



Conclusion

- Quantum thermodynamic geometry for many-body driven systems
- Collective thermodynamic advantage
- Sub-linear scaling of dissipation with two-body interactions

$$W = \Delta F + W_{diss}$$



Summary

Thermodynamic geometry in open quantum systems

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Collective effects in finite-time thermodynamics

Collective advantages in finite-time thermodynamics
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