New Perspectives on Dense QCD Matter

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NSF
iTHEMS RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program

References:
[1] Y. Fujimoto, S. Reddy, PRD 109 (2024) (Editors' Suggestion), arXiv:2310.09427.
[2] Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL 129 (2022), arXiv:2207.06753.
[3] Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024), arXiv:2306.04304; arXiv:2410.22758.

January 29, 2025 - Nucleosynthesis and Evolution of Neutron Stars @ YITP, Kyoto U

Neutron stars: why do we study now?

Neutron star study is very old research field.
But, now is the most exciting period
because of…

- Recent advances in astrophysics

- **Recent advances in QCD — fundamental theory of the strong interaction**

Recent advances in QCD at high densities

- Higher-order computations of perturbative QCD (pQCD) EoS

Freedman,McLerran(1977); Baluni(1978); Kurkela,Romatschke,Vuorinen (2009);
Gorda,Säppi,Paatelainen,Seppänen,Österman,Schicho,Navarrete (2018-)

- Nuclear EoS from chiral effective field theory (χEFT)

Tews,Krüger,Hebeler,Schwenk(2013);Drischler,Furnstahl,Melendez,Philips(2020);
Keller,Hebeler,Schwenk(2022); … many others

- Lattice simulations of QCD at finite isospin density

Kogut,Sinclair (2002); NPLQCD collaboration (2007-);
Brandt,Chelnokov,Cuteri,Endrodi,… (2014-);

- Lattice simulations of two-color QCD at finite baryon density

e.g. Iida,Itou,Murakami,Suenaga (2024)

- Hadron-hadron interaction from the lattice QCD

HAL QCD collaboration (2006-)

- Hamiltonian lattice simulations of QCD in (1+1)-dimensions

Hayata,Hidaka,Nishimura (2023)

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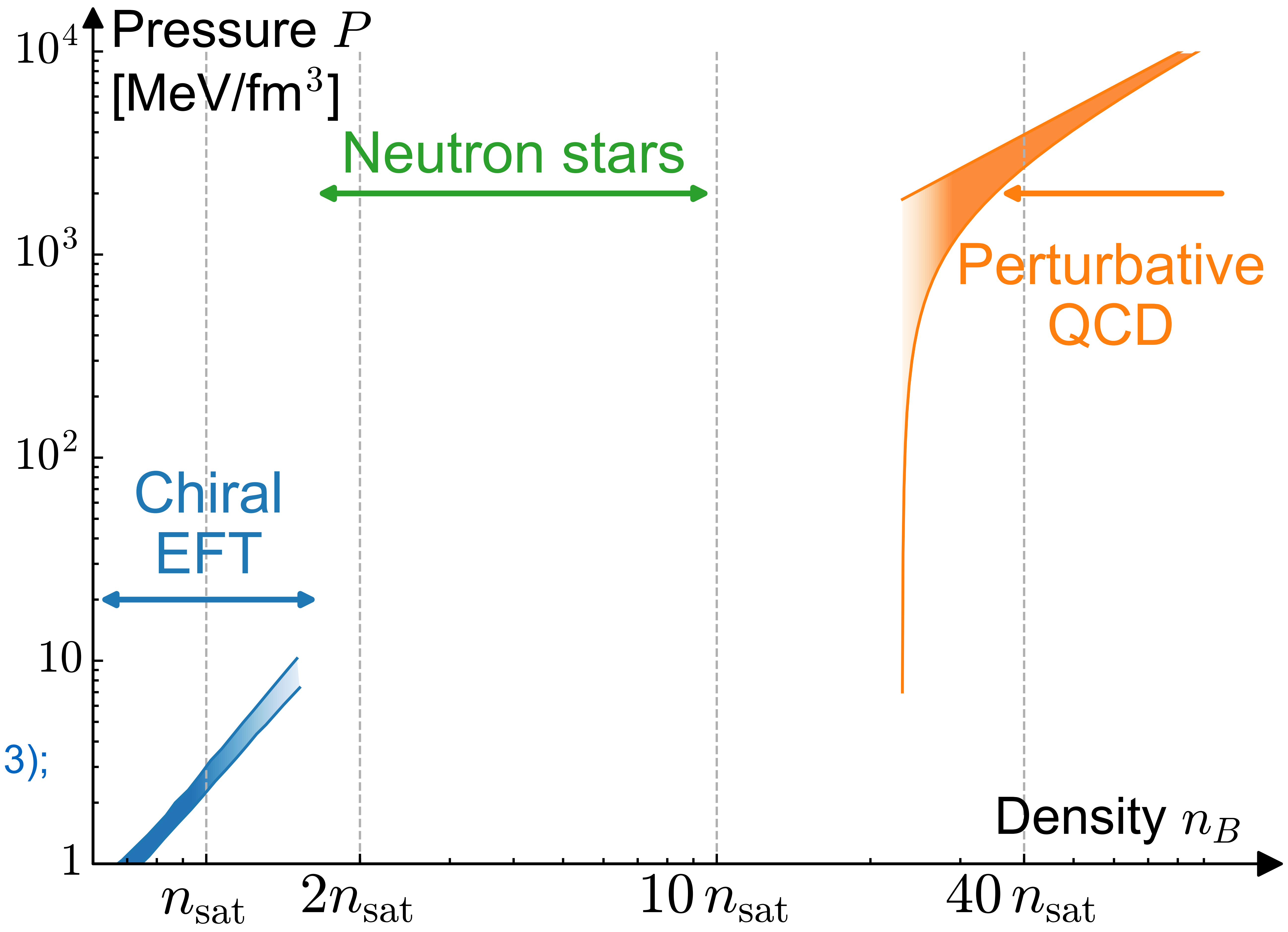

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Nuclear density: $n_{sat} = 0.16$ fm^{-3}

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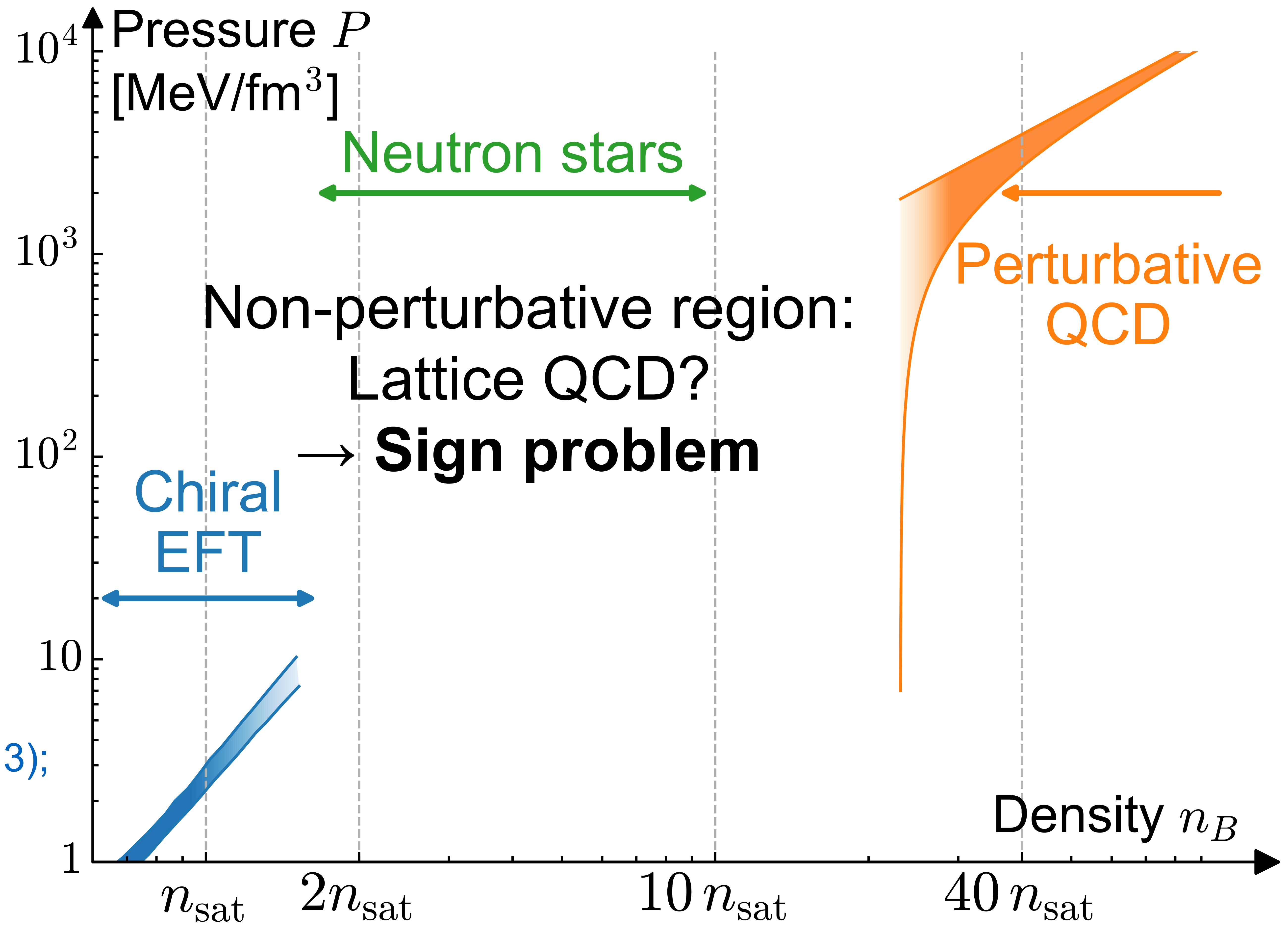

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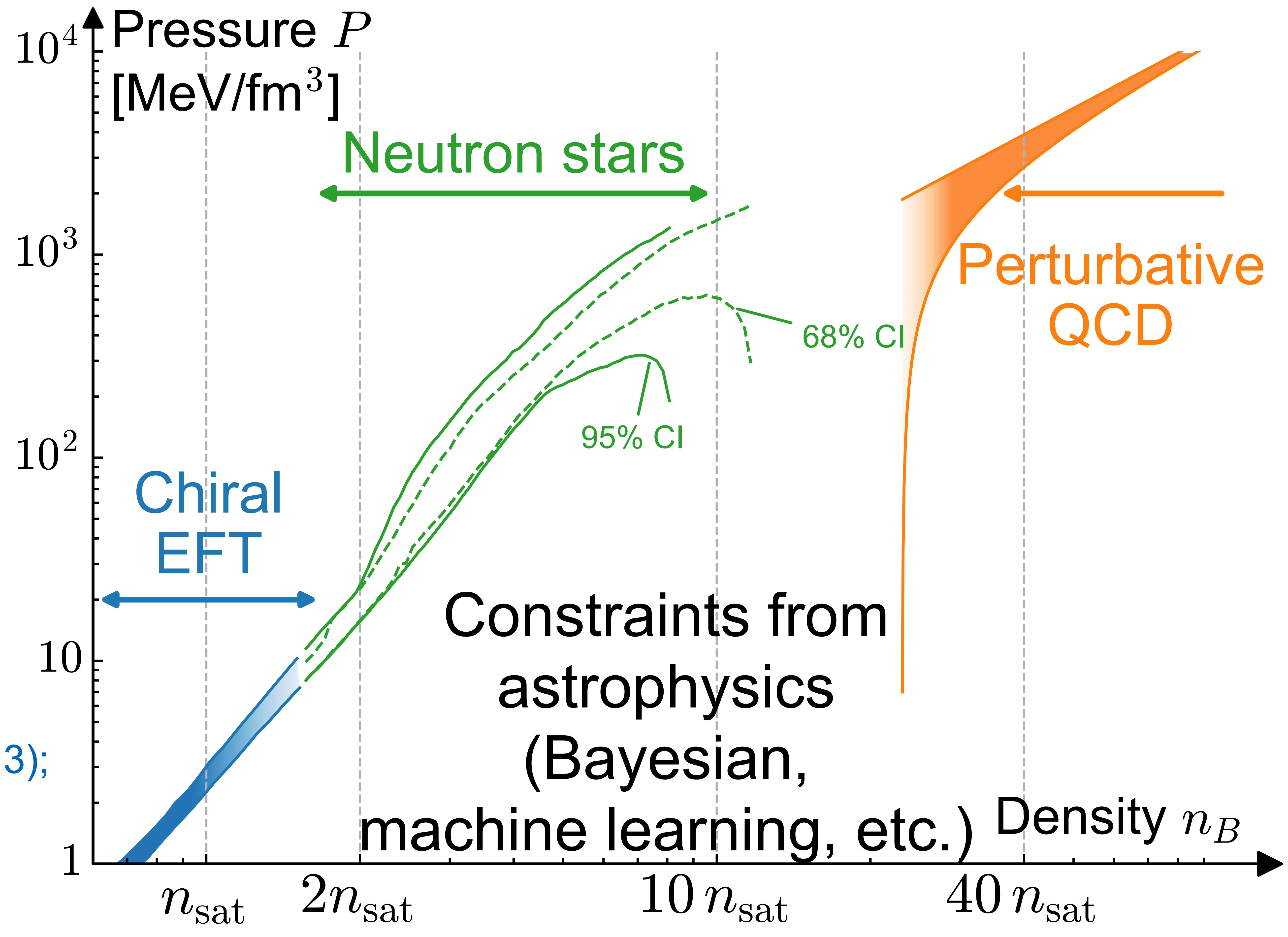

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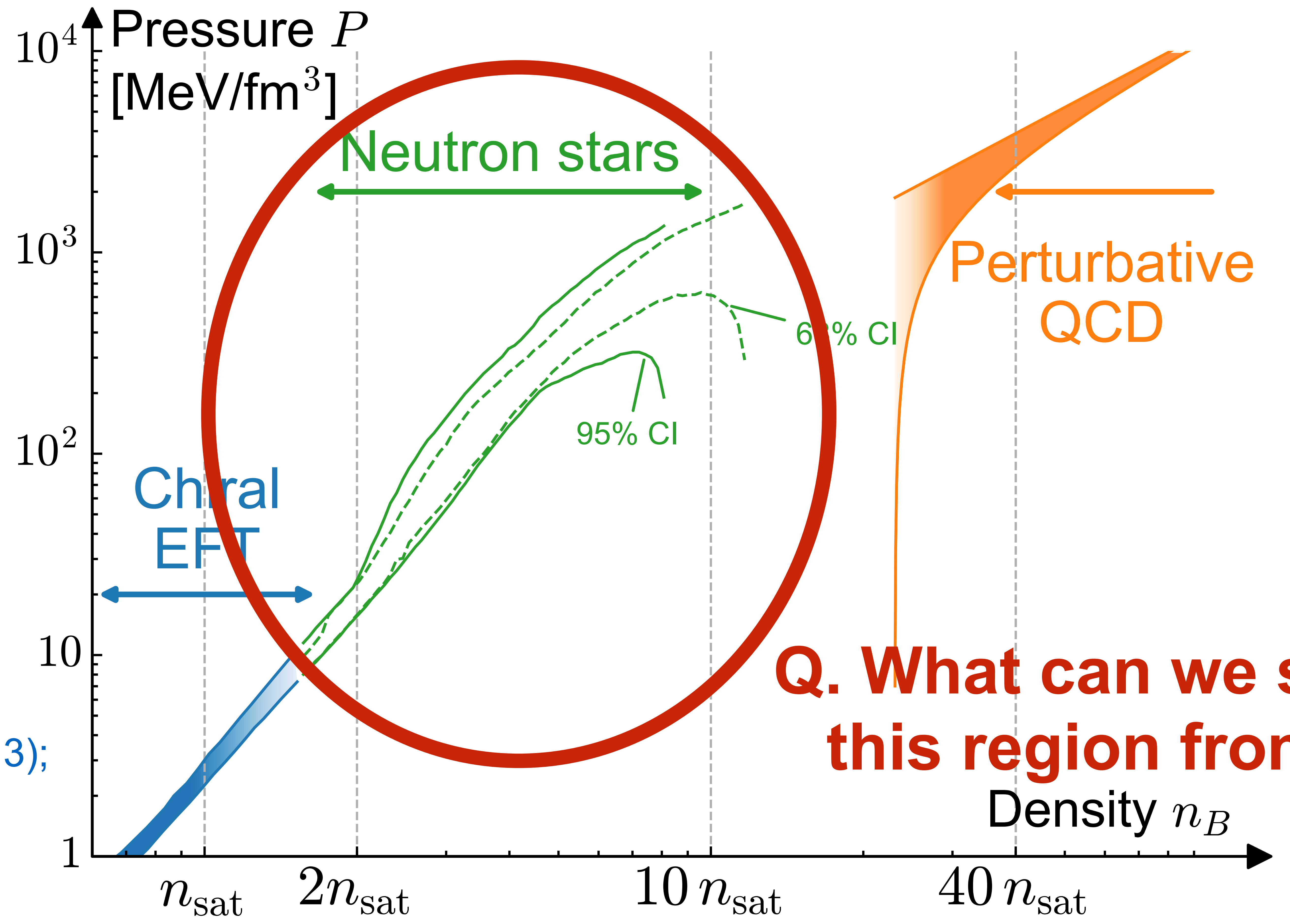

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Outline

1. Bounds on the EoS from QCD inequality and lattice data

2. Role of QCD in constraining the EoS

3. Inspiration from large-Nc QCD:

   Quarkyonic matter - duality between baryons and quarks

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QCD at finite isospin density

Alford,Kapustin,Wilczek (1999); Kogut,Sinclair (2002-);
Beane,Detmold,Savage et al. (2007-);
Endrodi et al. (2014-)…

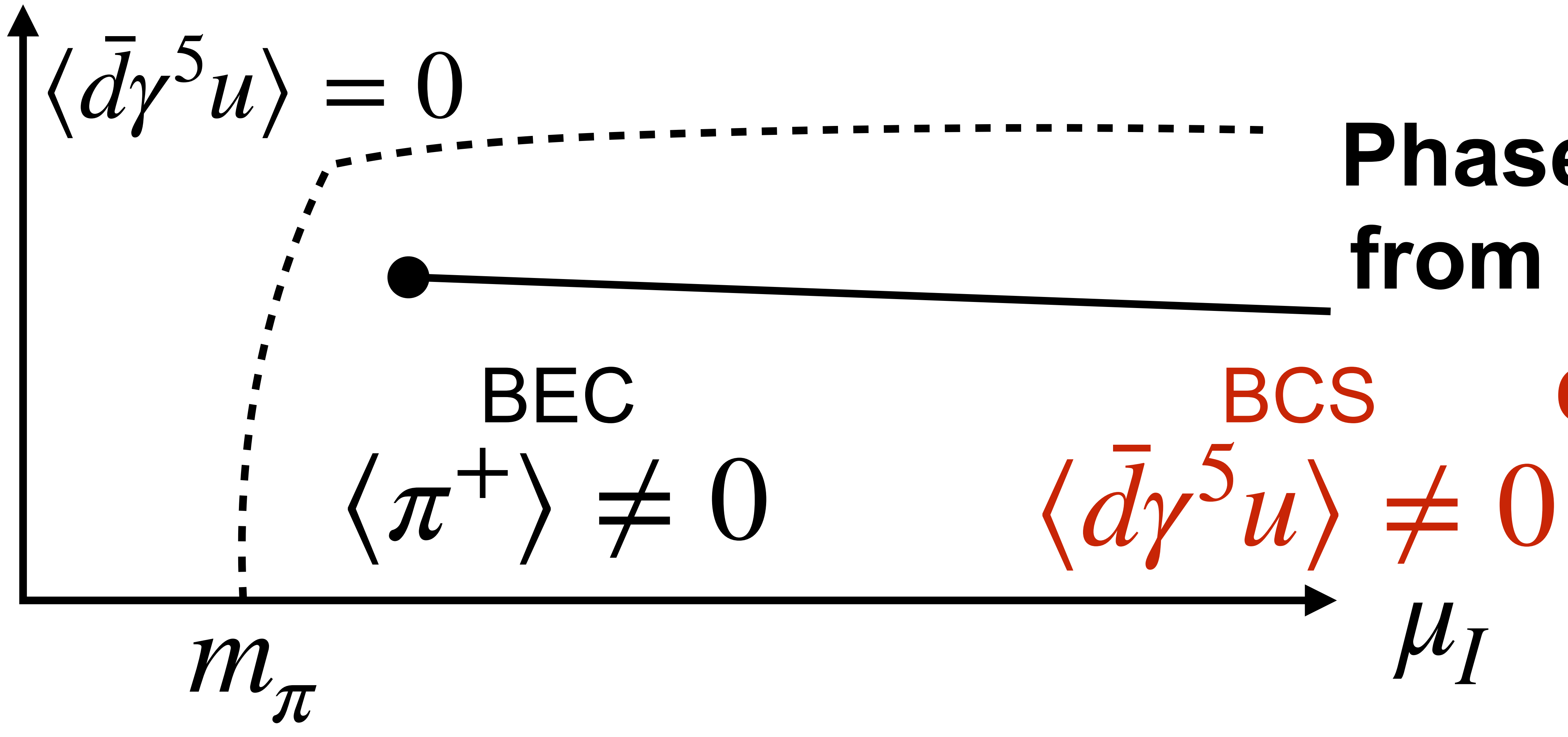
- **No sign problem** → EoS can be measured on the lattice!

- Isospin chemical potential (conjugate to isospin density I_3):

$$\mu_u = \frac{\mu_I}{2}, \quad \mu_d = -\frac{\mu_I}{2}$$ … Fermi surface of u & $\bar{d}$

- Phase structure:

Son,Stephanov (2000)



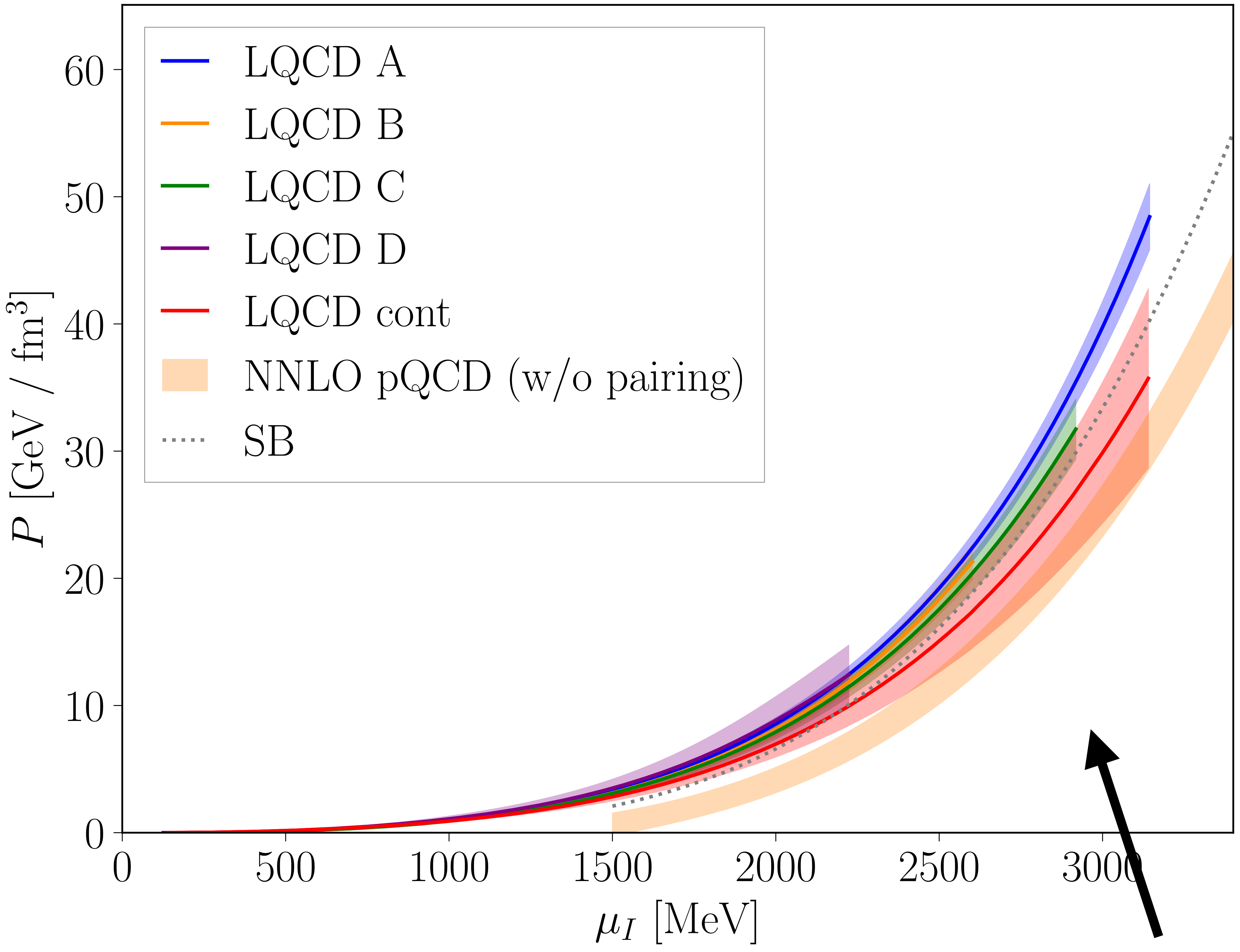
Phase structure is totally different from QCD at finite baryon density

QCD at finite isospin density

Recent impact:

Abbott et al. (NPLQCD) (2023, 24)

EoS is calculated up to $\mu_I \sim 3$ GeV by lattice QCD in the continuum limit



weak-coupling regime

What can we learn from this lattice data?

- Ground states of finite-μ_B QCD and finite-μ_I QCD are totally different
  $\rightarrow$ Naive comparison of EoS is meaningless

- There are a ways to utilize the finite-μ_I lattice data: **QCD inequality**

QCD inequality

Inequality among observables from path integrals Weingarten (1983); Witten (1983)

Inequality considered here:

QCD inequality for pressure $P \propto \log Z$:

$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Pressure of finite-μ_B QCD
(what we want to know)

Pressure of finite-μ_I QCD
(what we already know from lattice QCD)

NB: this is for symmetric nuclear matter

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

Robust bounds on the EoS

Fujimoto, Reddy (2023)

Lattice data: Abbott et al. (NPLQCD) (2023, 24)

From

Kurkela, Komoltsev (2021)

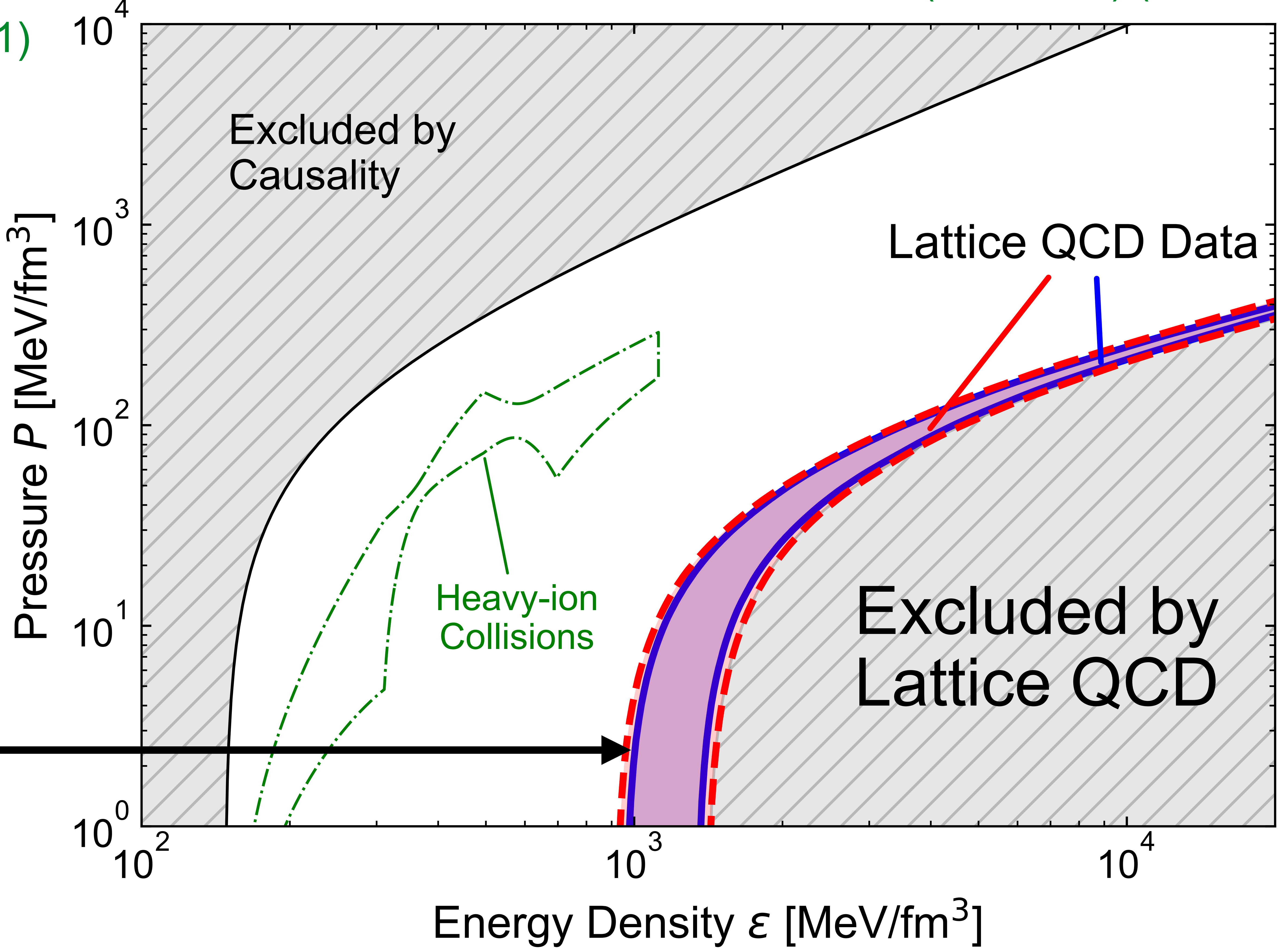
- **Causality:** $dn_B/d\mu_B > n_B/\mu_B$
- **Integral version of inequality:**

$$\int_{\mu_0}^{\mu_B} d\mu\, n_B(\mu) \leq P_I(2\mu_B/N_c),$$

$n_B(\mu_B)$ can be constrained

Then, from the relation

$\varepsilon = -P + \mu_B n_B$



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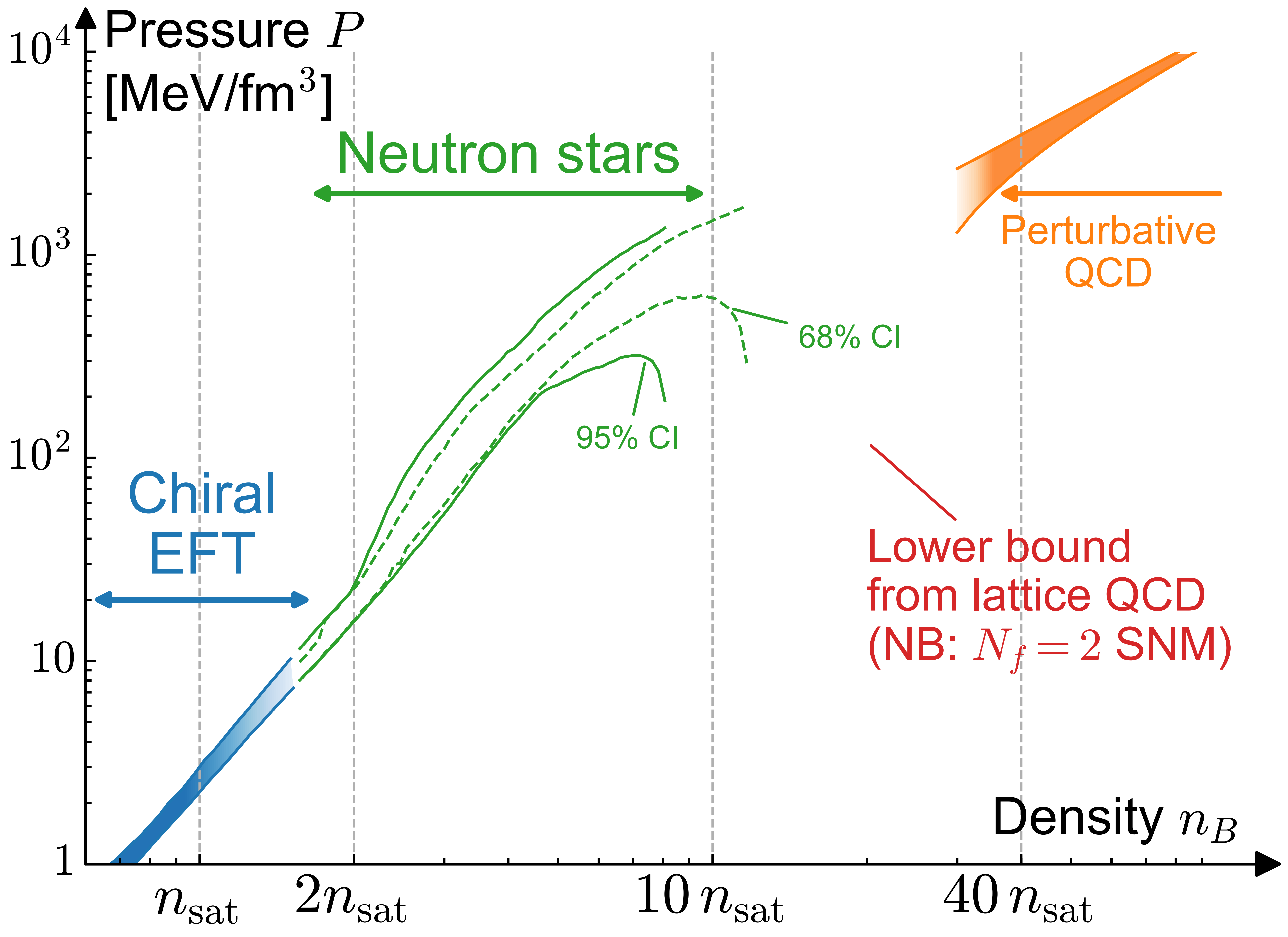

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3. Inspiration from large-Nc QCD:

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Role of QCD in constraining the EoS

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

- QCD input is useful in constraining EoS

- The pQCD calculations leads to the EoS: $P \approx \frac{1}{3}\varepsilon$

  … approximately conformal EoS

- The pQCD constraint requires all the EoS to approach this value

- Useful measure of conformality: **Trace anomaly** $\Delta = \frac{\varepsilon - 3P}{3\varepsilon}$

- Conformal EoS may be a signature of quark matter

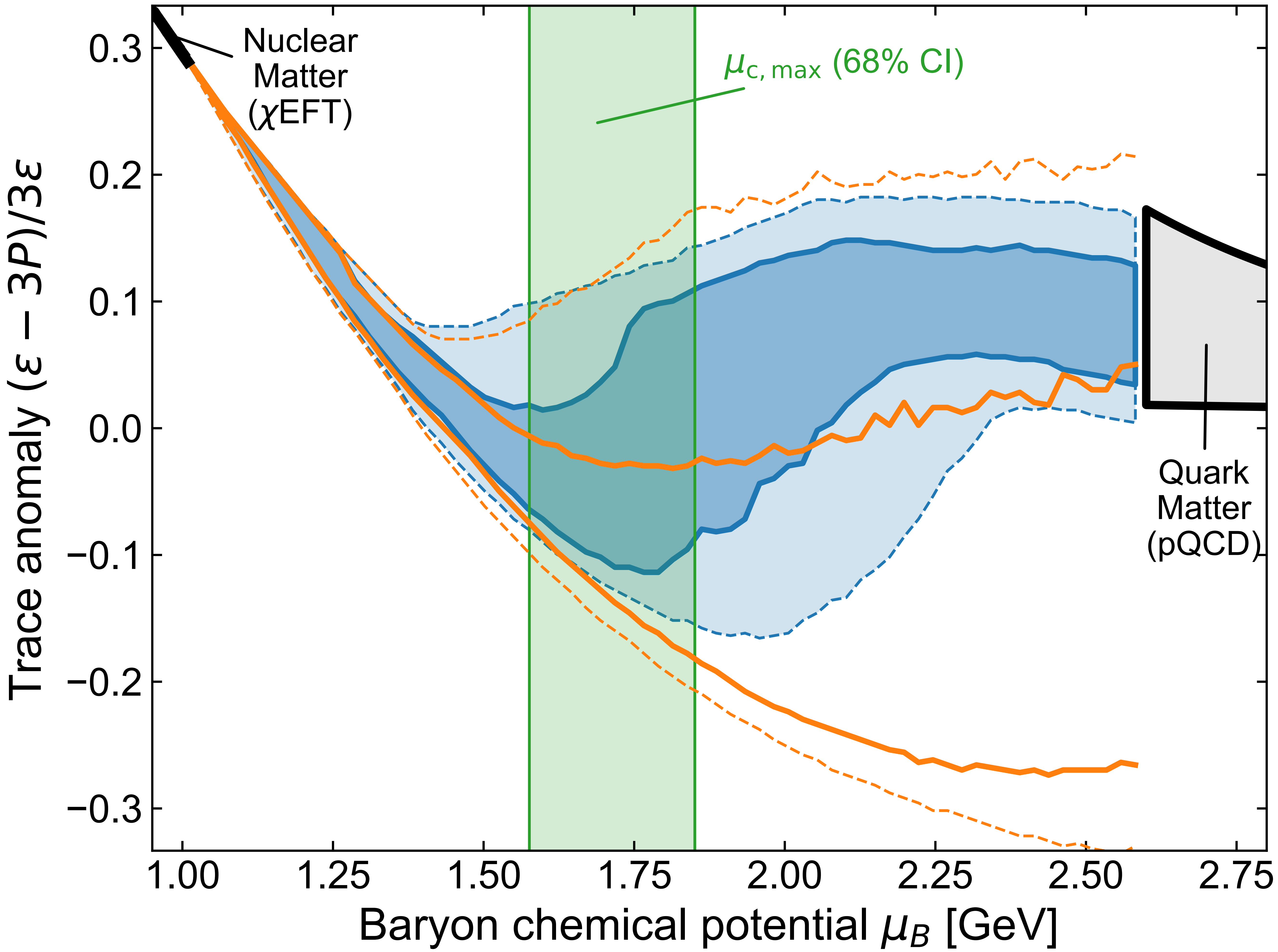
See also: Annala,Gorda,Hirvonen,Komoltsev,Kurkela,Nättilä,Vuorinen (2023); Komoltsev,Somasundaram,Gorda,Kurkela,Margueron,Tews (2023)

Behavior of the trace anomaly

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Trace anomaly $\Delta = \dfrac{\varepsilon - 3P}{3\varepsilon}$

1) Role of QCD
2) Approximately conformal EoS in NS



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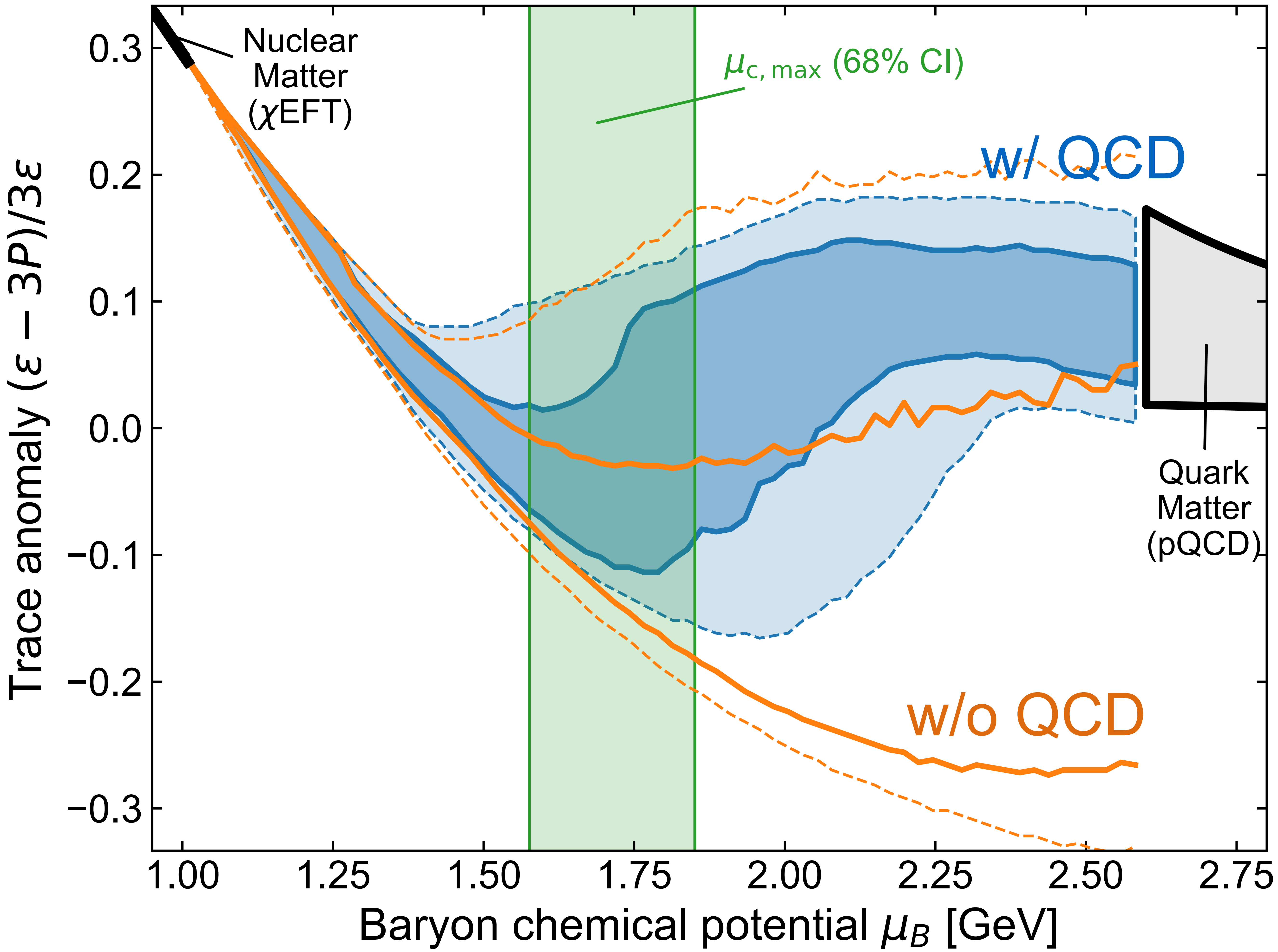
1) **Role of QCD**

   Difference at high density
   $\rightarrow$ QCD favors a soft EoS

2) Approximately conformal EoS in NS



Behavior of the trace anomaly

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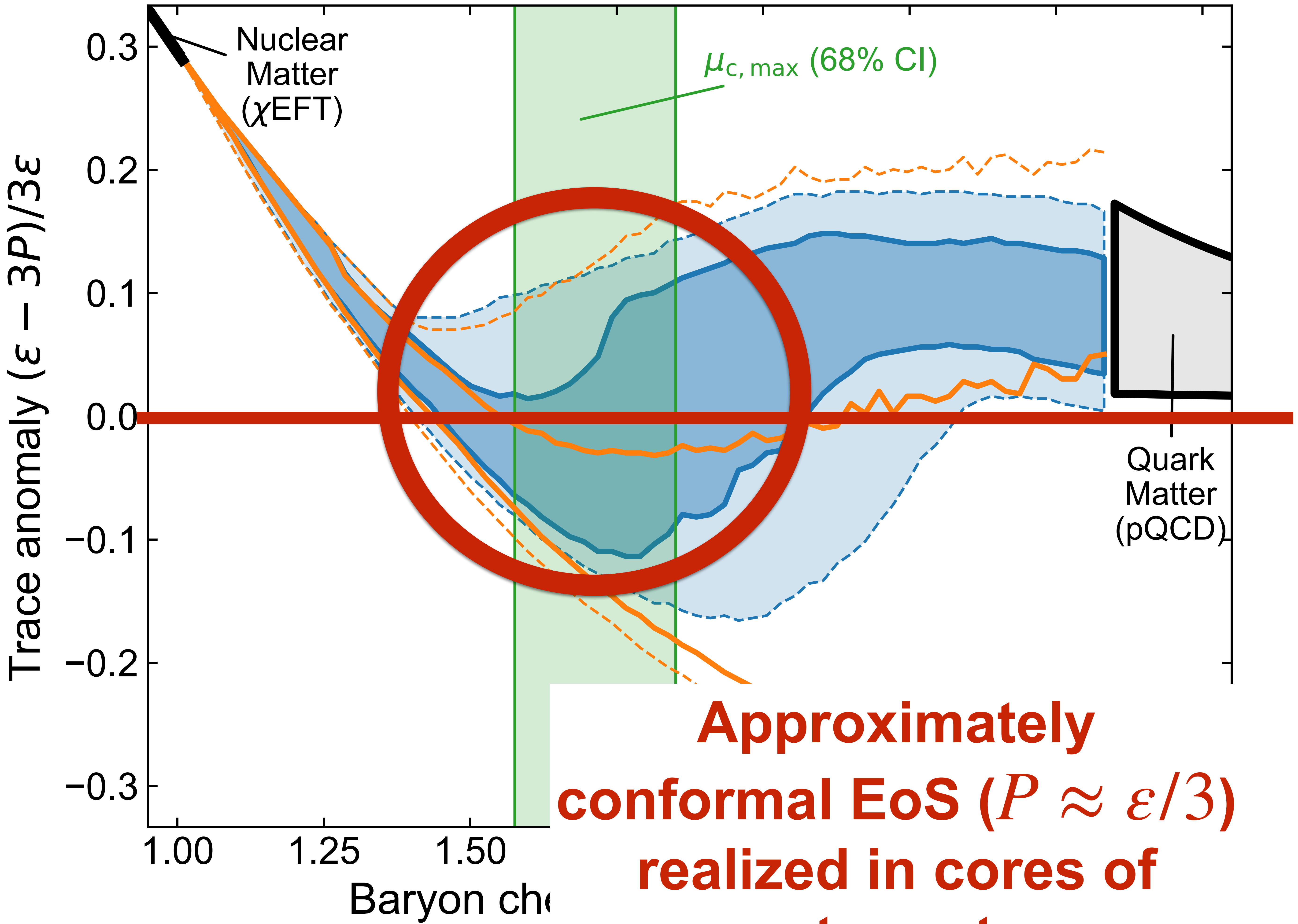
Trace anomaly $\Delta = \frac{\varepsilon - 3P}{3\varepsilon}$

1) Role of QCD
2) **Approximately conformal EoS in NS**

QCD favors conformal EoS around the NS core density
→ onset of quark matter?



Approximately conformal EoS ($P \approx \varepsilon/3$) realized in cores of neutron stars

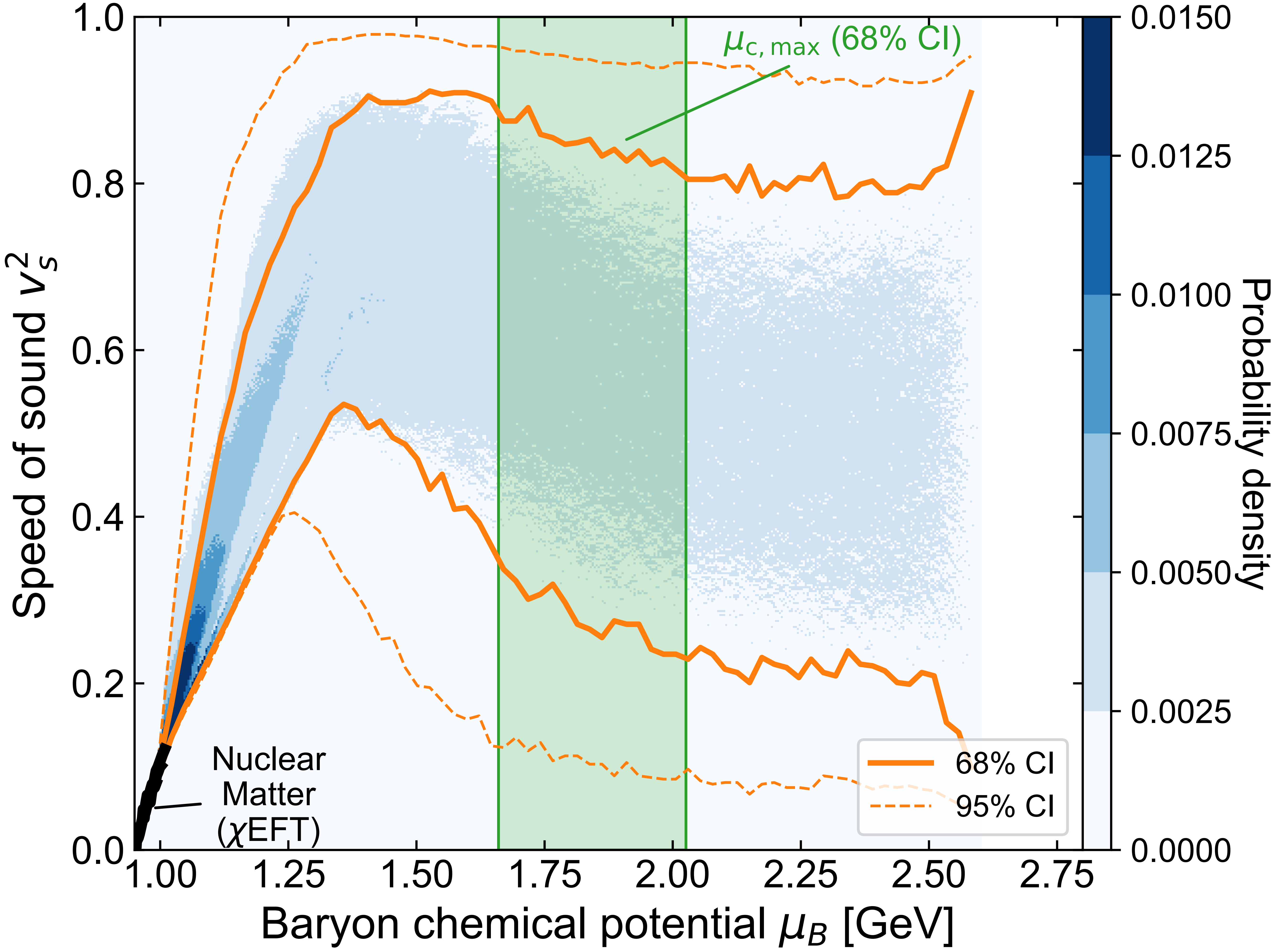
Effect of QCD: softening in EoS

Fujimoto,Fukushima,McLerran,Praszalowicz, PRL129 (2022)

Speed of sound:

$$v_s^2 = \frac{dP}{d\varepsilon}$$

Bayesian inference
w/o pQCD constraint



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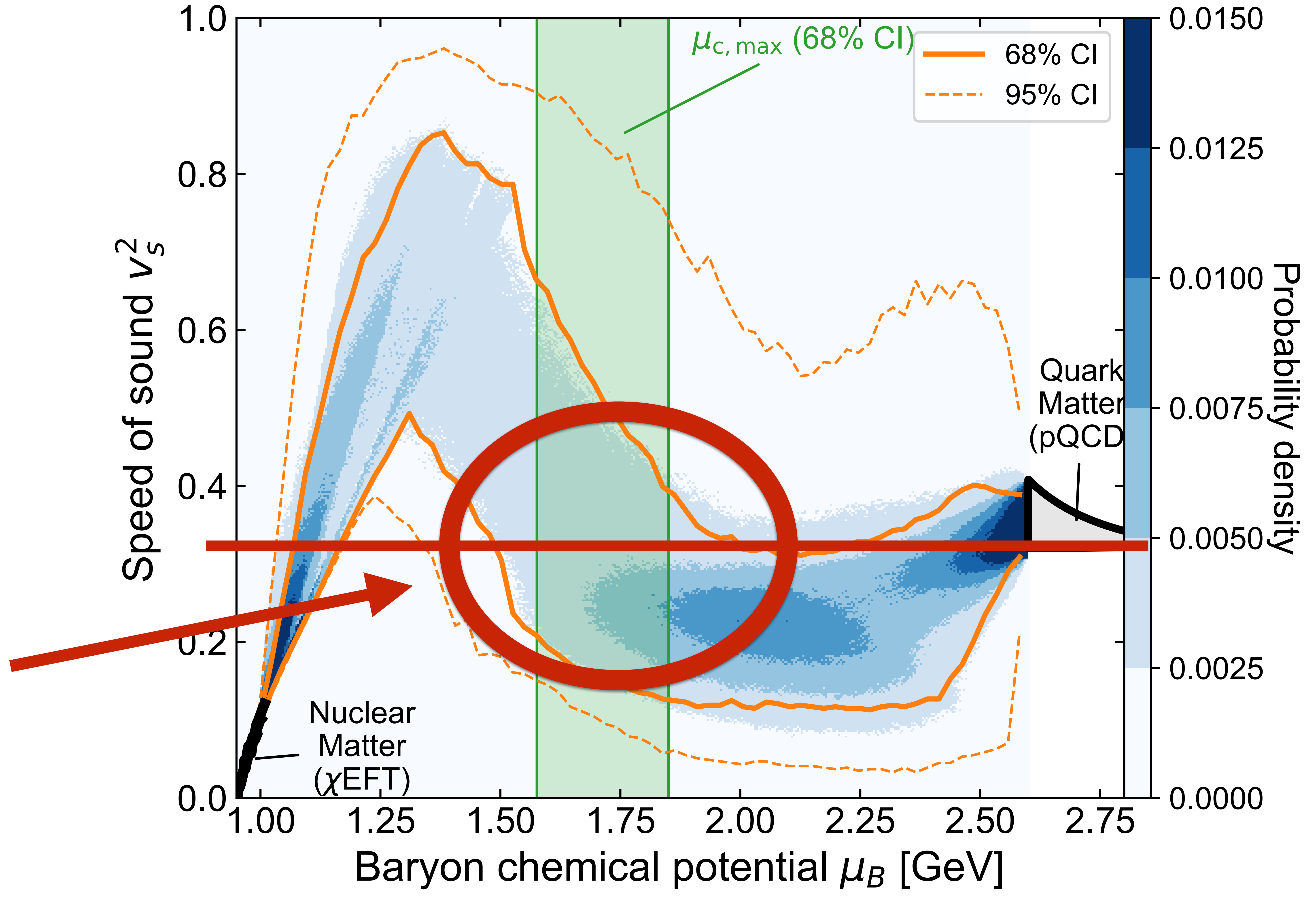

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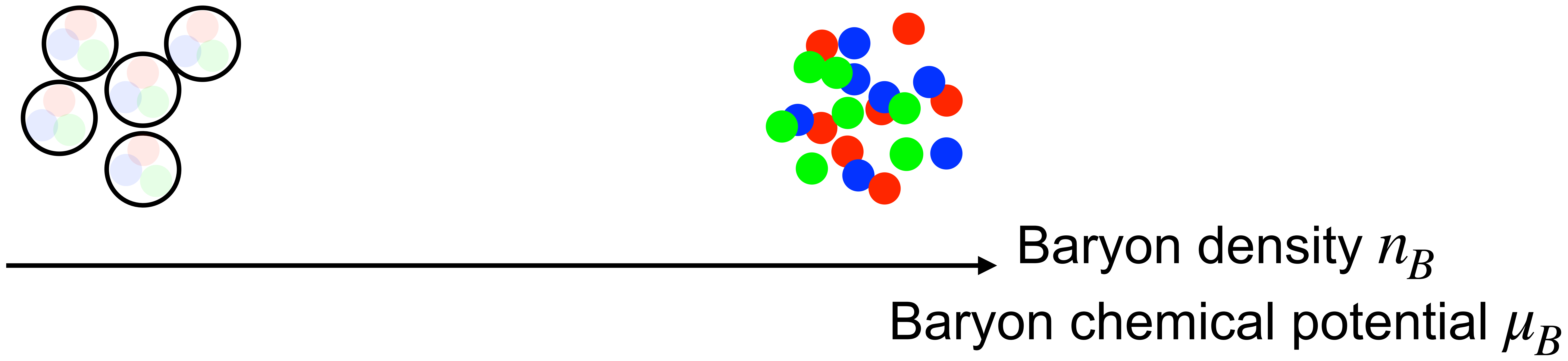
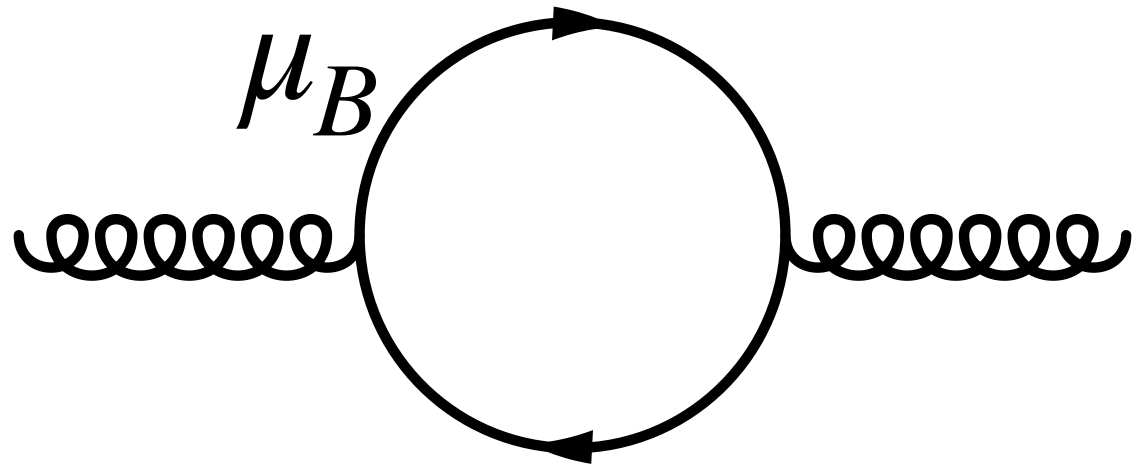
Quark deconfinement at high density

Collins & Perry (1974): **Naive picture of quark deconfinement**

In weak-coupling regime at high density, quarks liberate



This is led by screening of the confinement potential



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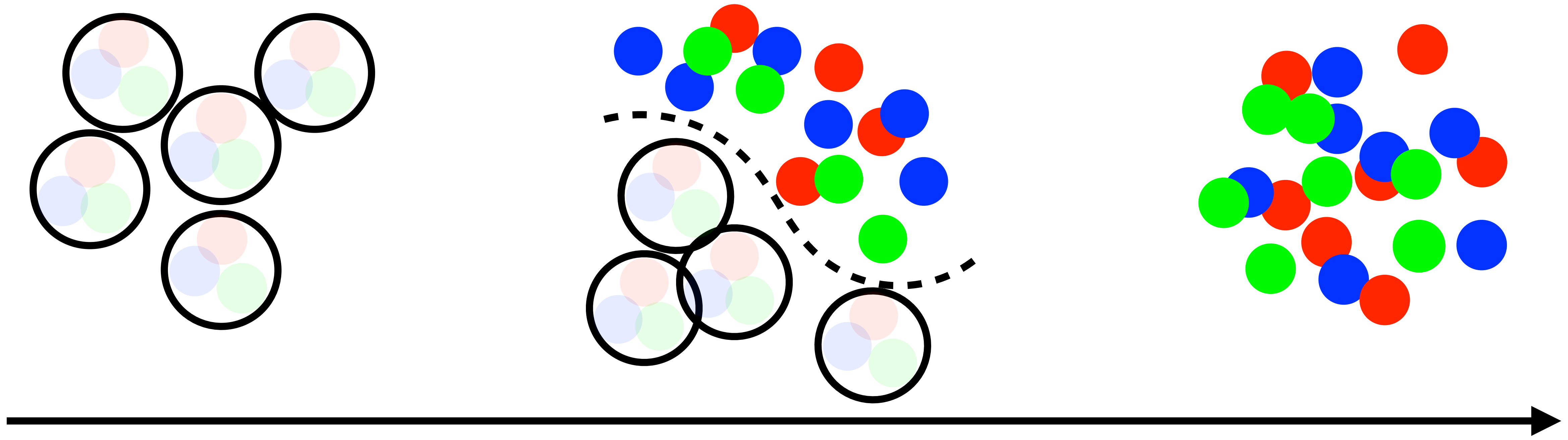


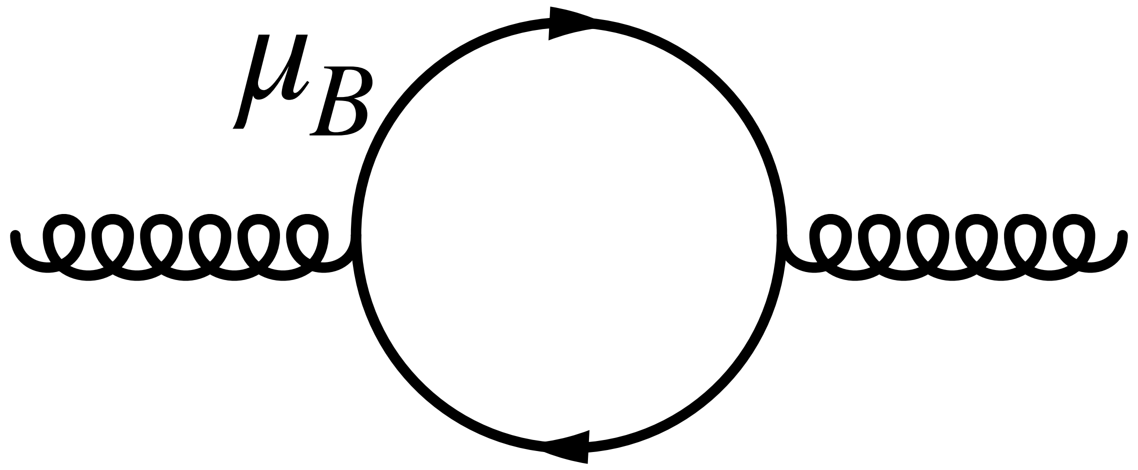
Baryon density n_B

Baryon chemical potential μ_B

1st-order phase transition (mixed phase)

This is assumed in many EoS construction in the neutron-star context

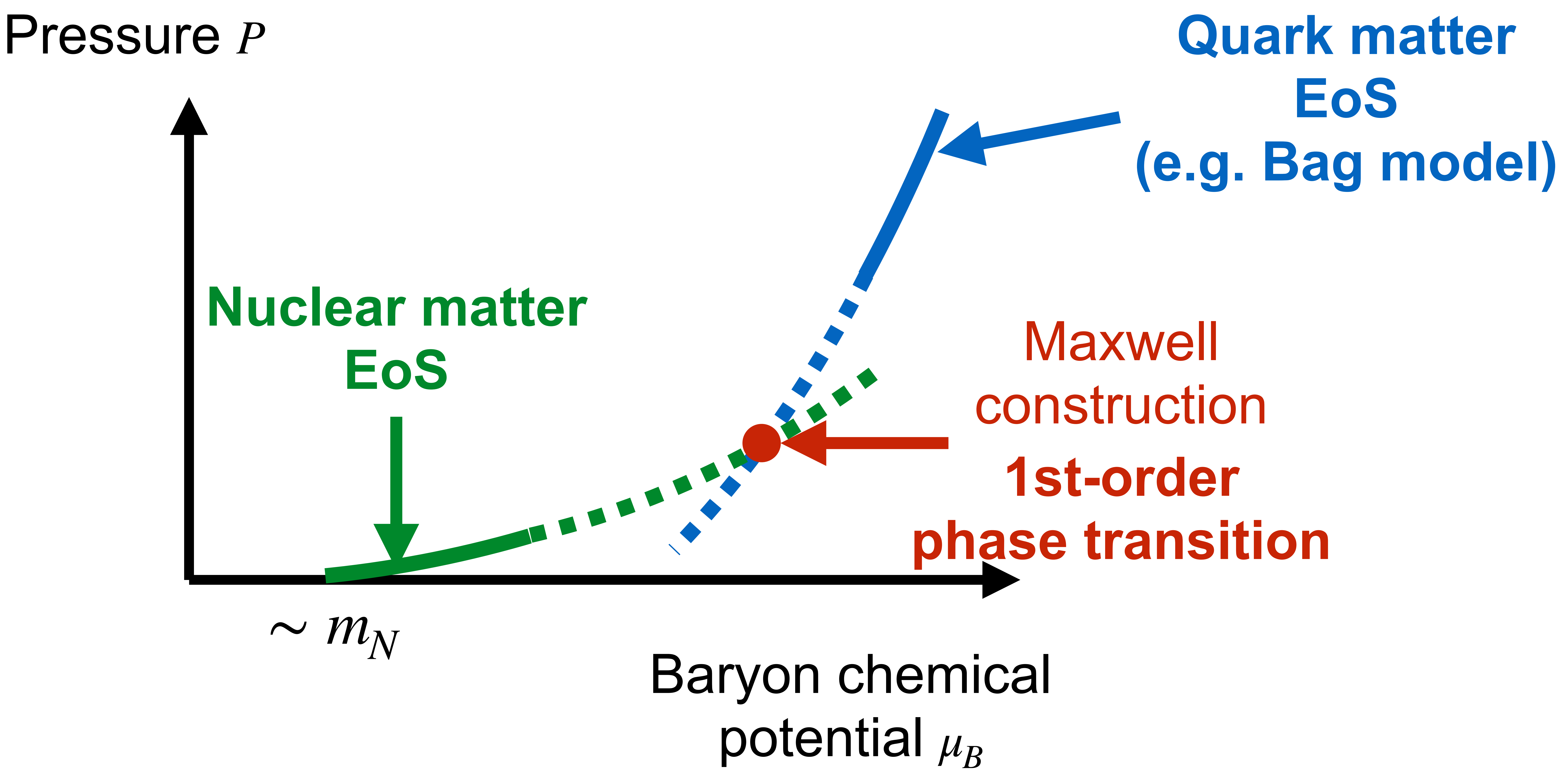
This is led by screening of the confinement potential



Quark deconfinement at high density

EoS corresponding to the conventional picture:

Baym,Chin (1976);
cf. Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2018)



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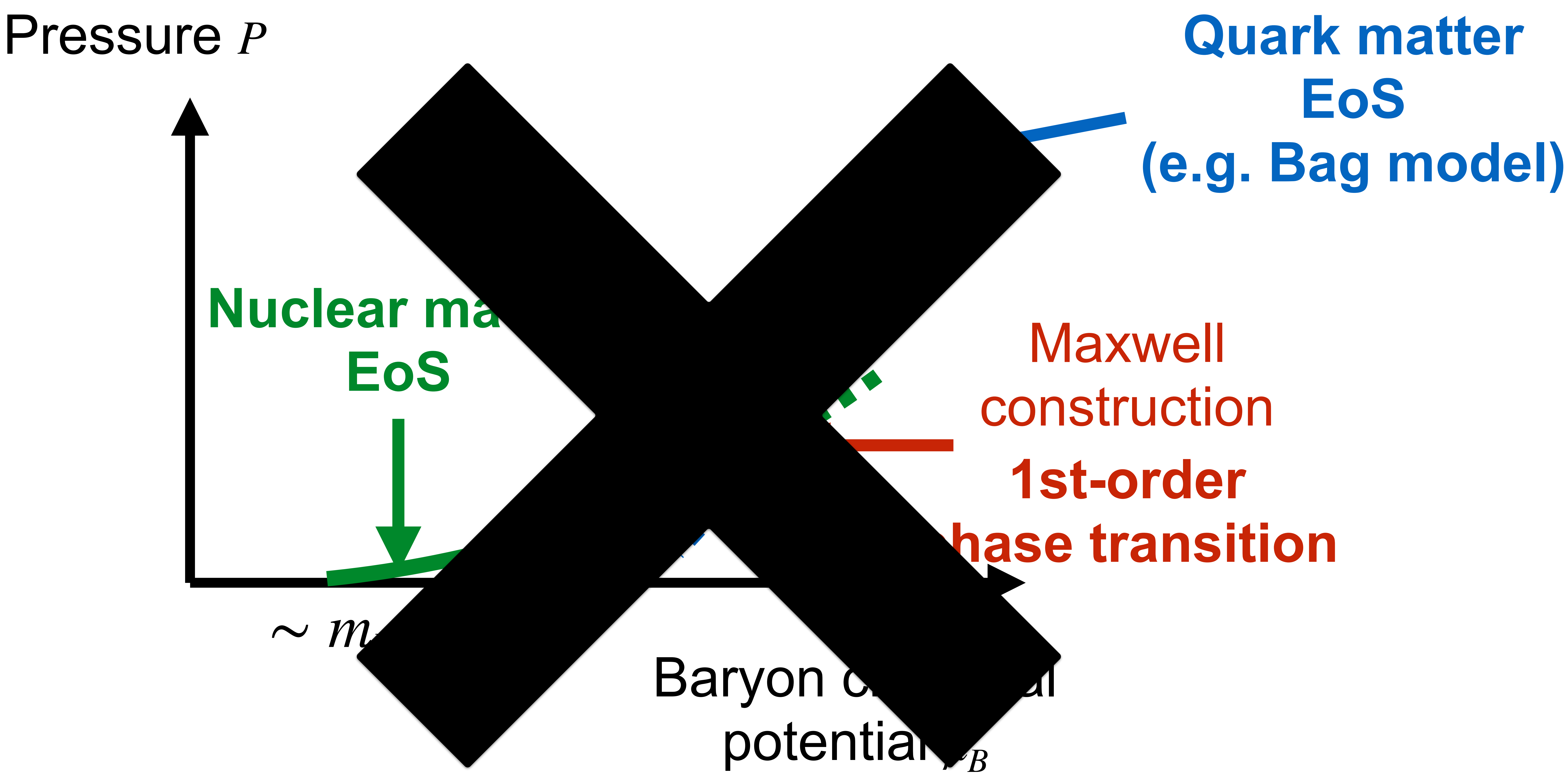
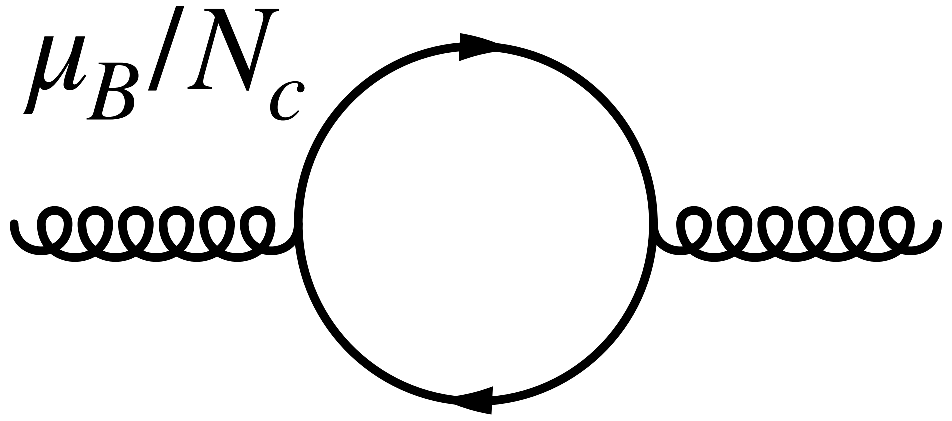


Quark deconfinement at high density

Deconfinement at high density may not be that simple…

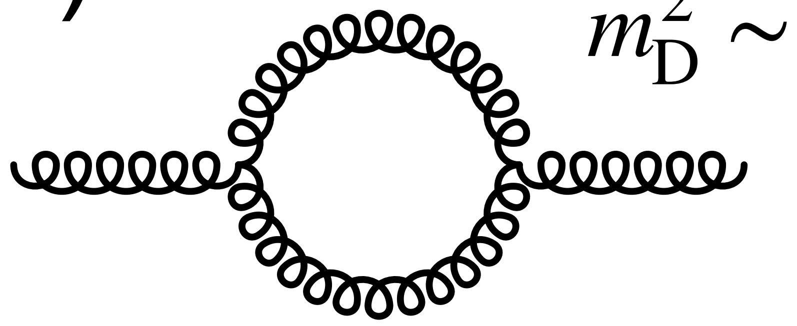
McLerran & Pisarski (2007): Quarks never deconfine in large-N_c QCD

$\mu = \mu_B / N_c$



$$m_D^2 \sim \frac{\lambda_{\text{'t Hooft}}\, \mu^2}{N_c} \to 0$$

cf)



$m_D^2 \sim g^2 N_c T^2$

$\sim \lambda_{\text{'t Hooft}}\, T^2$

… (de)confinement is never affected by quark medium!

Quark deconfinement at high density: duality

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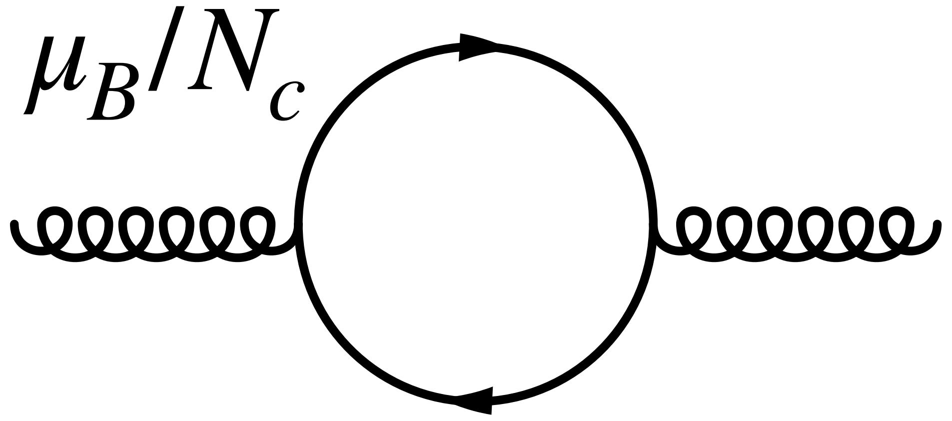

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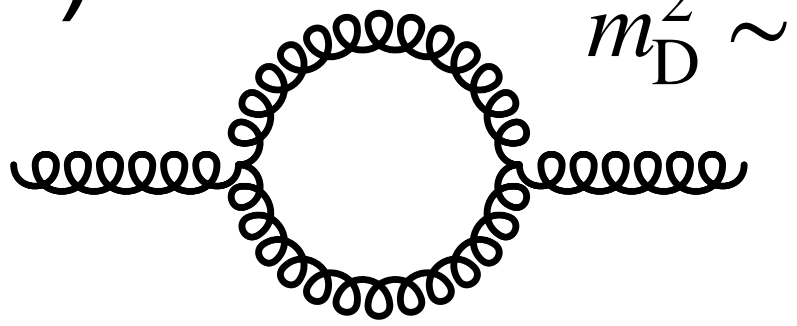

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Dense large-Nc QCD matter can be described either as

- **Confined baryons** (because confining interaction is never screened)
- **Quarks** (at densities where weak-coupling QCD is valid)

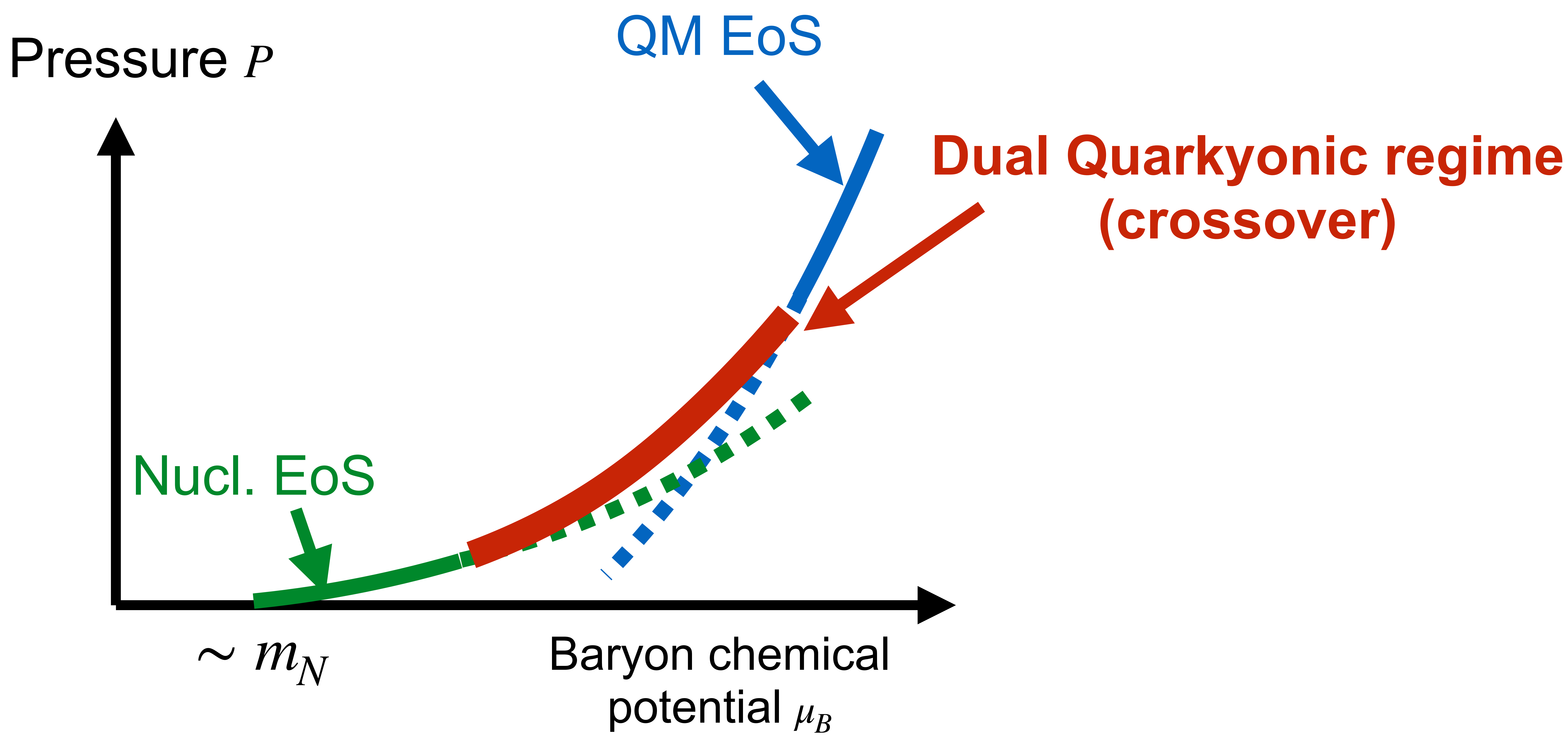
→ implies **duality** between quark and confined baryonic matter

Quark yonic

Quark deconfinement at high density: duality

Baym, Hatsuda, Kojo, Powell, Song, Takatsuka (2018);
McLerran, Reddy (2018)

EoS corresponding to the Quarkyonic picture:



Duality in Fermi gas model

Kojo (2021); Fujimoto,Kojo,McLerran, PRL 132 (2023)

Implement duality in Fermi gas model
(= simultaneous description in terms of baryons & quarks)

Fermi gas model w/ an explicit duality:

$$\varepsilon = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k) = \int_{\boldsymbol{k}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

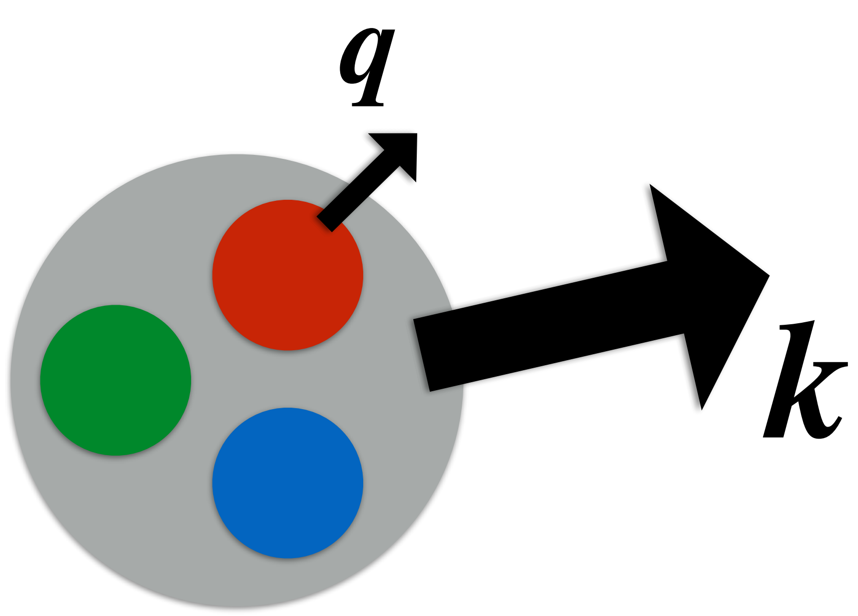
$$n_{\mathrm{B}} = \int_{\boldsymbol{k}} f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} f_{\mathrm{Q}}(q)$$

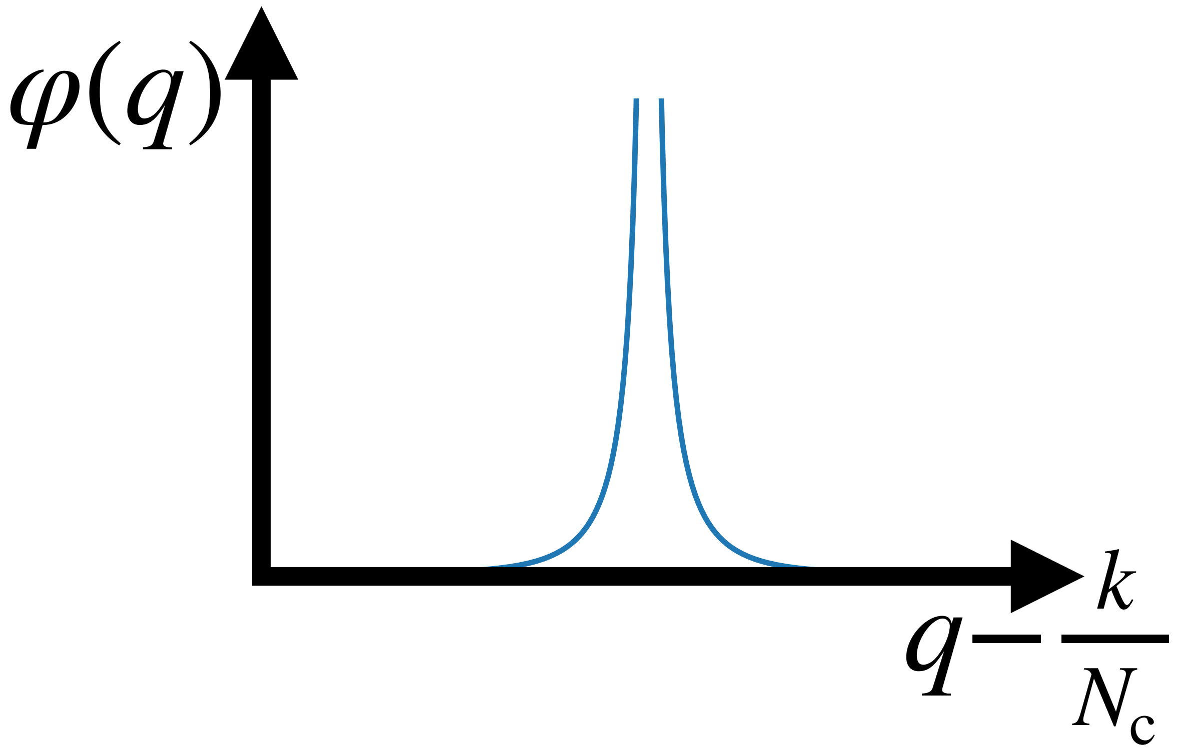
$0 \le f_{\mathrm{B,Q}} \le 1$: Pauli exclusion

$E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}$: ideal baryon dispersion relation

Modeling of confinement:

$$f_{\mathrm{Q}}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$





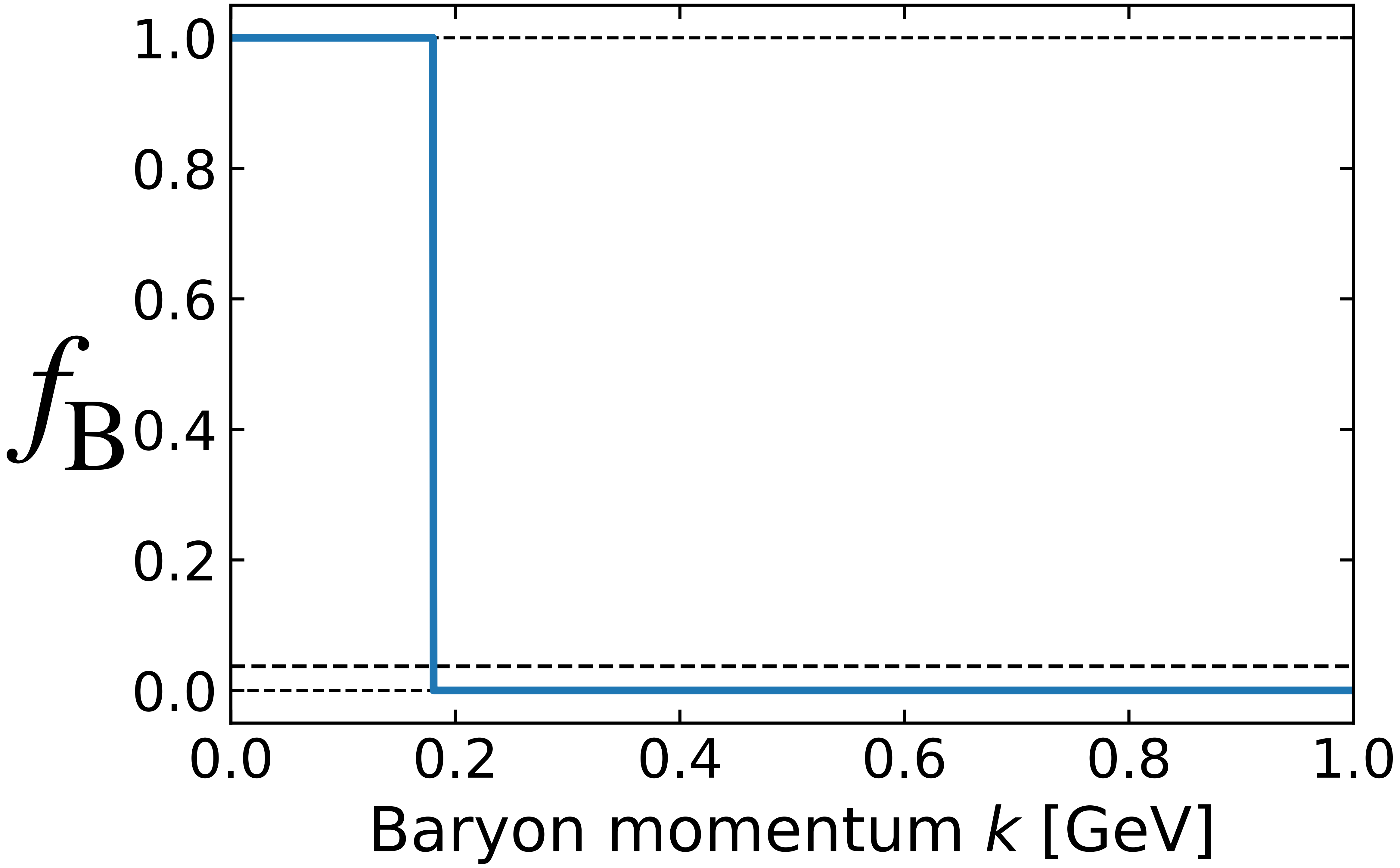
Ideal dual Quarkyonic model
→ **Find a solution for f_{B} and f_{Q} with minimum ε at a given n_B**

Solution of the dual model of Quarkyonic matter

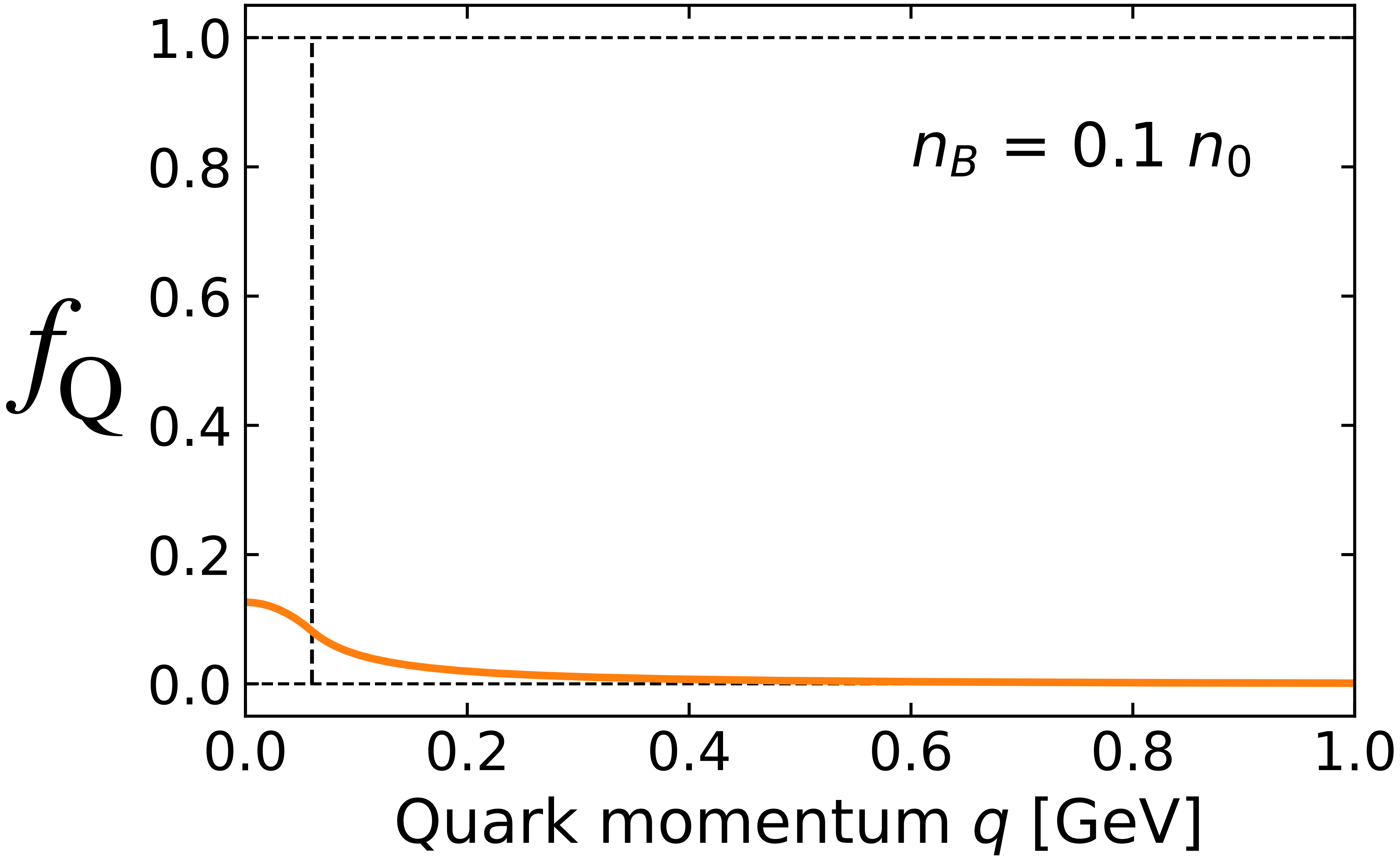
Kojo (2021); Fujimoto,Kojo,McLerran, PRL 132 (2023)

At low density…

Fermi-Dirac distribution for baryons



Quarks do not fill up the Fermi sea yet

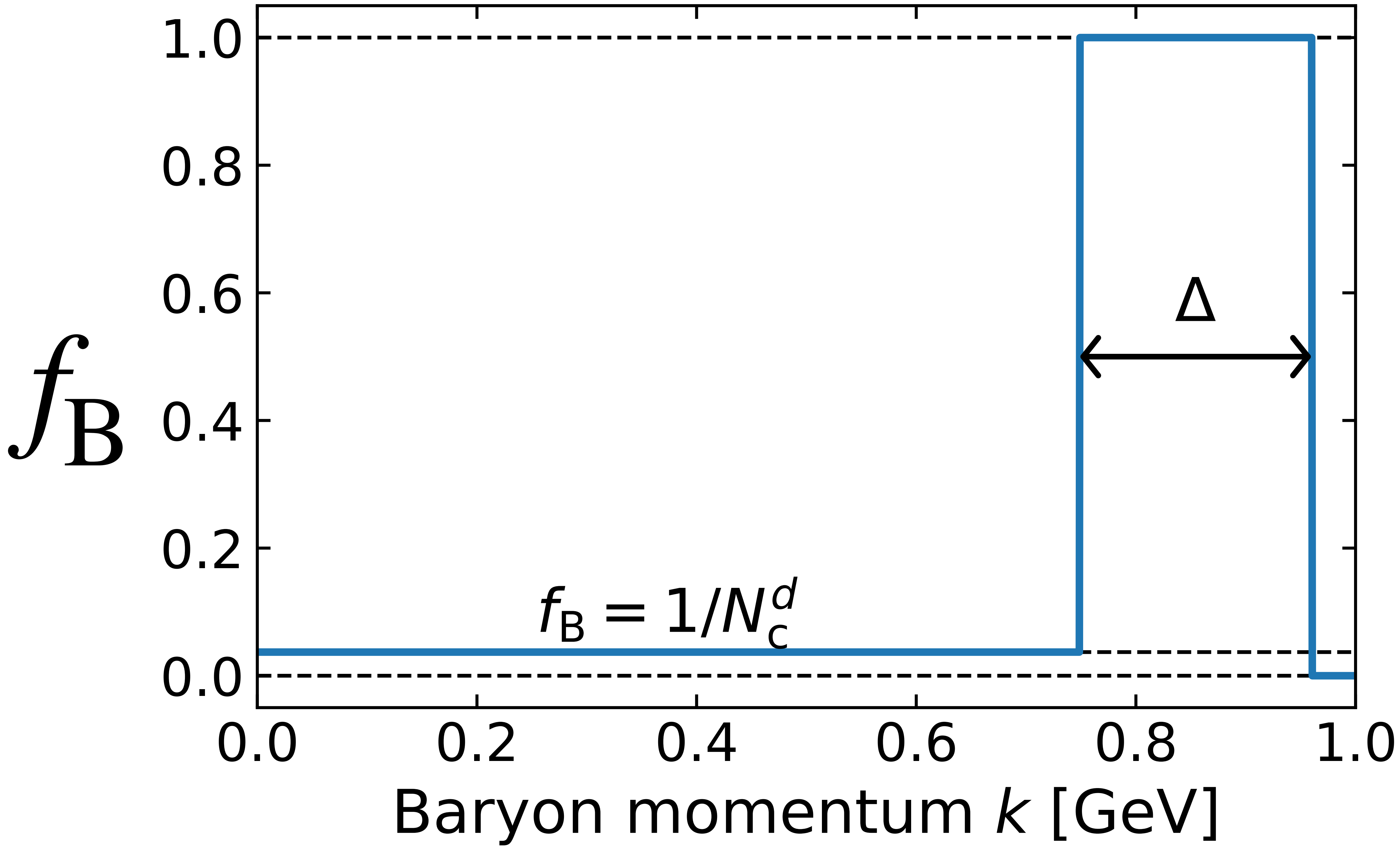


Solution of the dual model of Quarkyonic matter

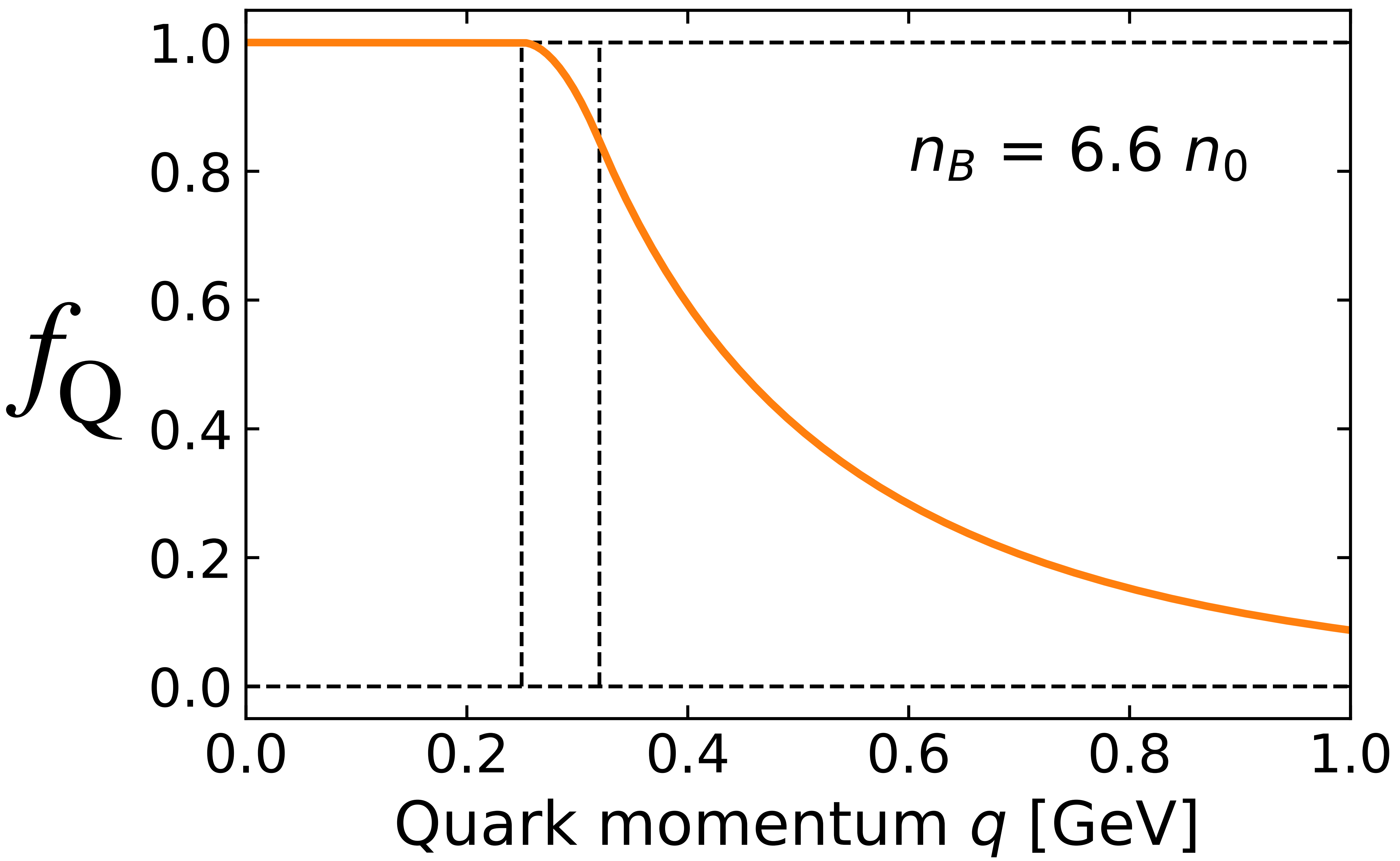
Fujimoto, Kojo, McLerran, PRL 132 (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



Quark obeys the FD distribution (with a tail from confinement)



… characteristic feature of Quarkyonic matter

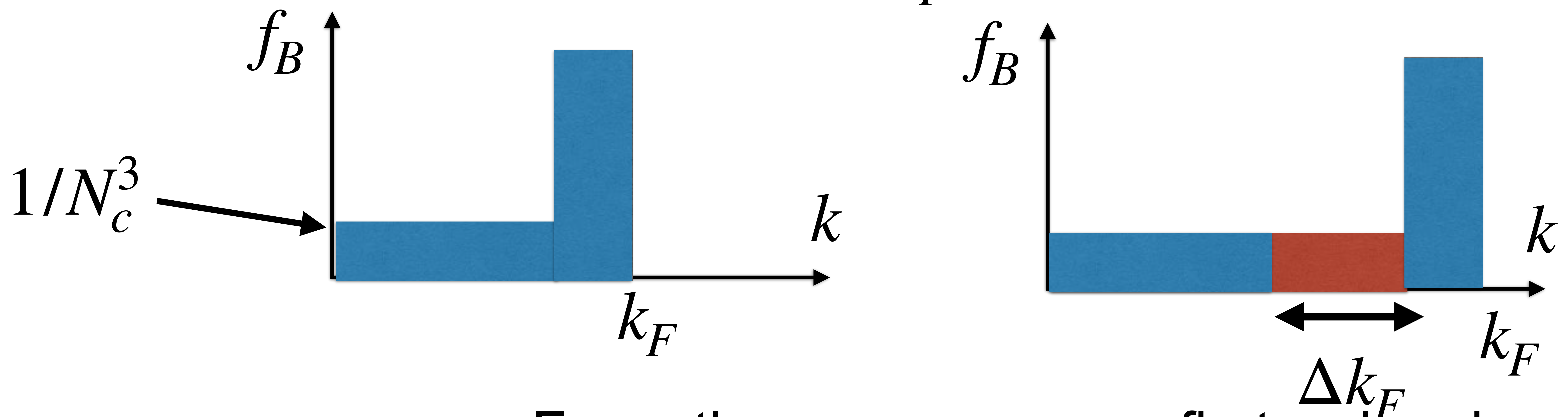
Favors crossover rather than 1st-order phase transition

Fujimoto,Kojo,McLerran (2023)

A partial occupation of available baryon phase space leads to **large sound speed:**

$$v_s^2 = \frac{n_\mathrm{B}}{\mu_\mathrm{B} dn_\mathrm{B}/d\mu_\mathrm{B}} \to \frac{\delta\mu_\mathrm{B}}{\mu_\mathrm{B}} \sim v_s^2 \frac{\delta n_\mathrm{B}}{n_\mathrm{B}}$$

If baryons have underoccupied state, the change in density is small while the change in Fermi energy ($\sim k_F$) is large



$\to$ Favor the crossover over first-order phase transition ($v_s^2 = 0$)

Summary

- **QCD at finite isospin density:** a useful nonperturbative piece of information on the lattice
- **QCD inequality:** one can put bound on the EoS of baryonic QCD from the isospin lattice-QCD
- **Role of QCD:** Useful in constraining neutron-star EoS. Favors approximately conformal EoS
- **Quarkyonic matter:** duality between baryons and weakly-coupled quarks from large-Nc
  → nontrivial modification in FD distribution, i.e., suppression in low-momentum baryonic states

Bonus materials

QCD inequality: derivation

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- **Dirac operator**: $\mathscr{D}(\mu) \equiv \gamma^\mu D_\mu + m - \mu\gamma^0$, **property**: $\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

- **QCD$_I$**: $Z_I(\mu_I) = \int [dA] \, \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_G} = \int [dA] \, \left| \det \mathscr{D}(\frac{\mu_I}{2}) \right|^2 e^{-S_G}$

u quark ↕ d quark ↕ (linking the $\mathscr{D}$ factors of QCD$_I$ and QCD$_B$)

charge conjugation symmetry $\mu_B \to -\mu_B$

- **QCD$_B$**: $Z_B(\mu_B) = \int [dA] \, \det \mathscr{D}(\frac{\mu_B}{N_c}) \det \mathscr{D}(\frac{\mu_B}{N_c}) \, e^{-S_G} = \int [dA] \, \mathrm{Re} \left[\det \mathscr{D}(\frac{\mu_B}{N_c}) \right]^2 e^{-S_G}$

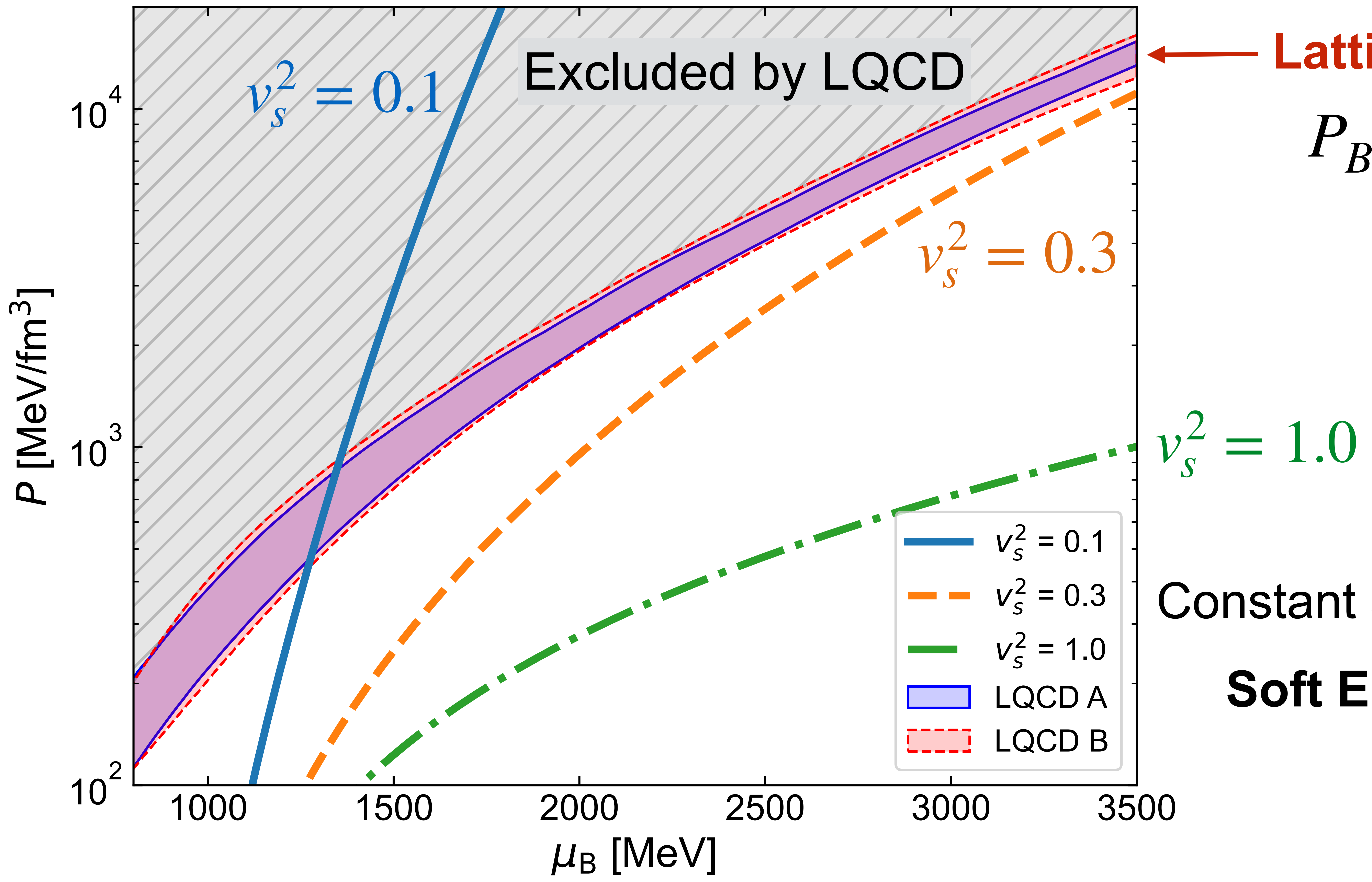
Note: this is **isospin symmetric** because there is no isospin imbalance

- From the relation $\mathrm{Re}\, z^2 \le |z^2| = |z|^2$:

$$Z_B(\mu_B) \le \int [dA] \, \left| \det \mathscr{D}(\frac{\mu_B}{N_c}) \right|^2 e^{-S_G} = Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Direct use of QCD inequality

Lattice data: Abbott et al. (2023); Fujimoto,Reddy (2023)



Lattice data: upper bound

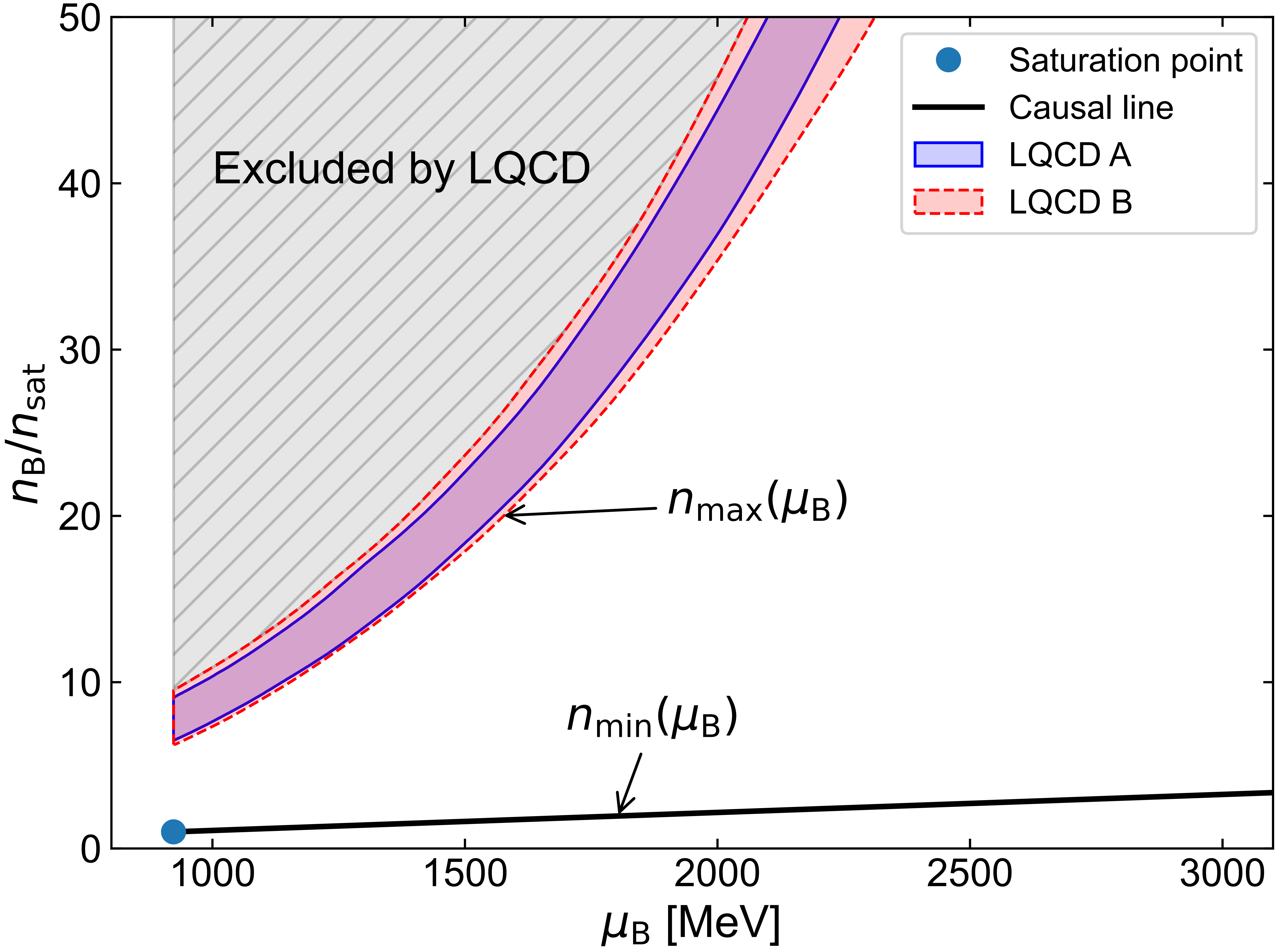
$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Constant sound speed EoS: $P(\varepsilon) \propto v_s^2 \varepsilon$

Soft EoS (smaller P at a given ε) is excluded

Bounds on $n_B(\mu_B)$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



Properties $n_B(\mu_B)$ must satisfy:

① Stability:

$$\frac{d^2P}{d\mu_B^2} \geq 0 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq 0$$

② Causality $v_s^2 \leq 1$:

$$v_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B} \leq 1 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq \frac{n_B}{\mu_B}$$

③ QCD inequality on the integral:

$$\int_{\mu_{\rm sat}}^{\mu_B} d\mu' \, n_B(\mu') \;\leq P_I\Big(\mu_I = \frac{2}{N_c}\mu_B\Big)$$

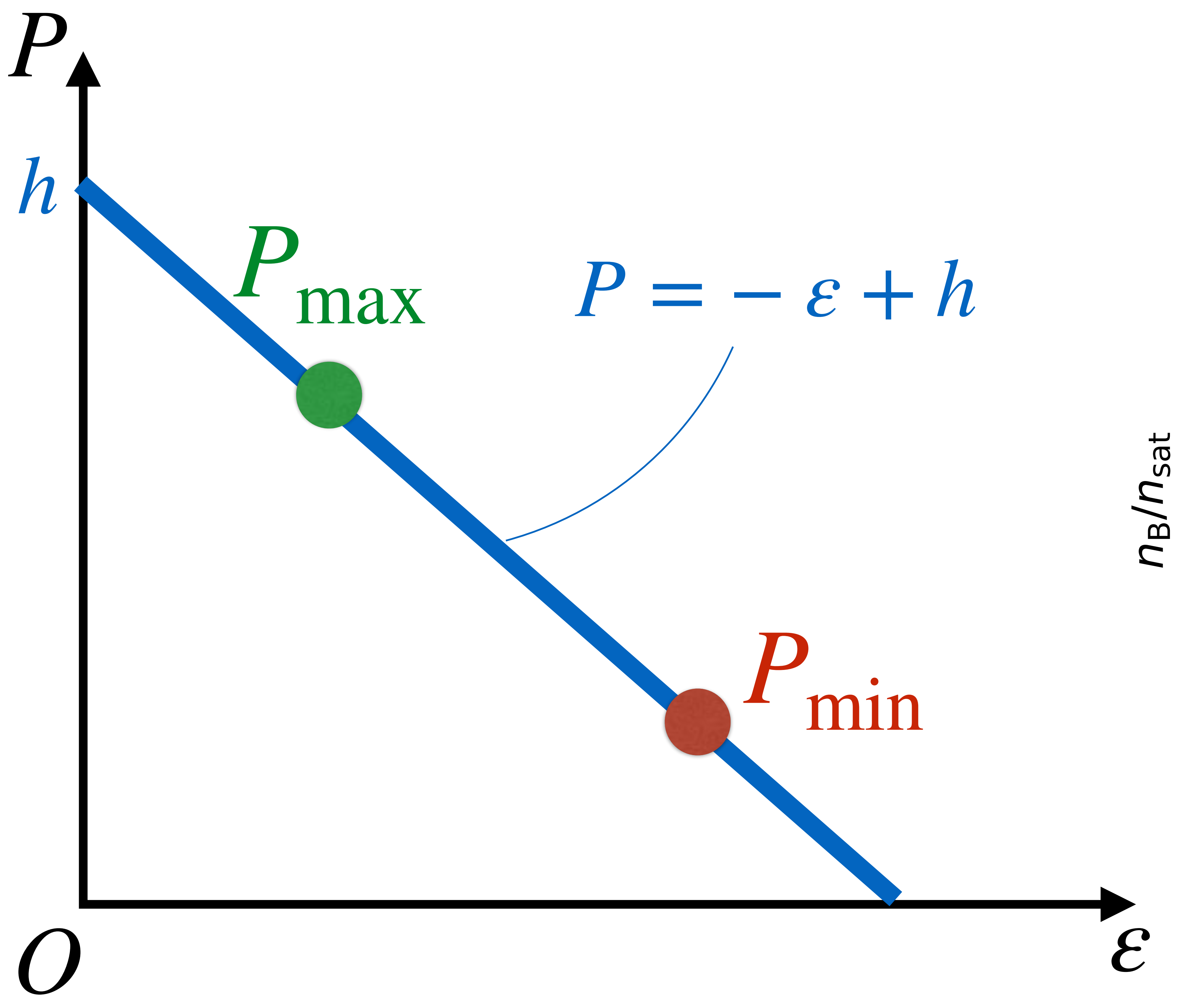
Lower bound of the integral must be specified

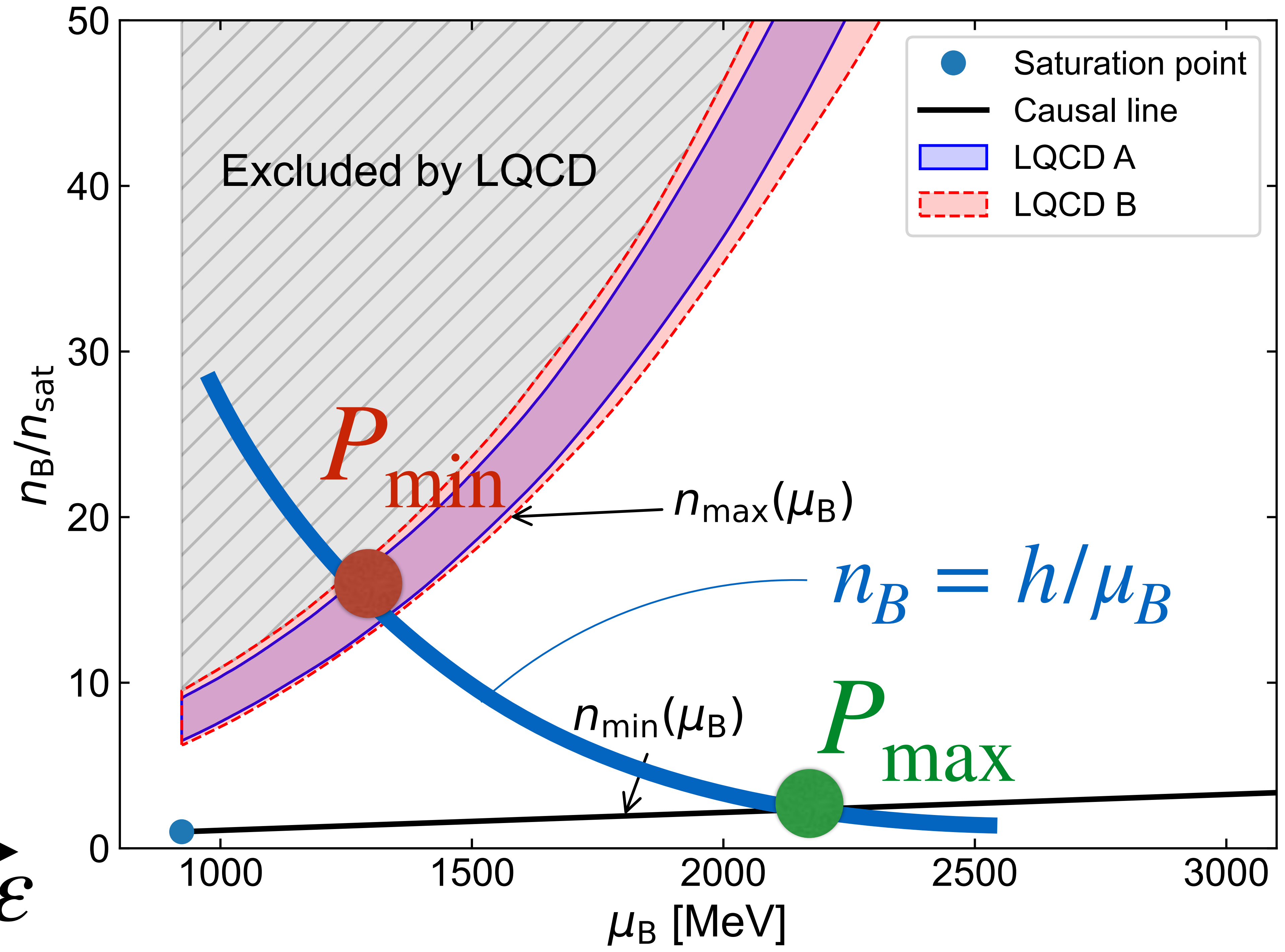
fix it to the **empirical saturation property**

Bounds on $P(\varepsilon)$

Isenthalpic line: $h = \mu_B n_B = \varepsilon + P = \text{const}$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



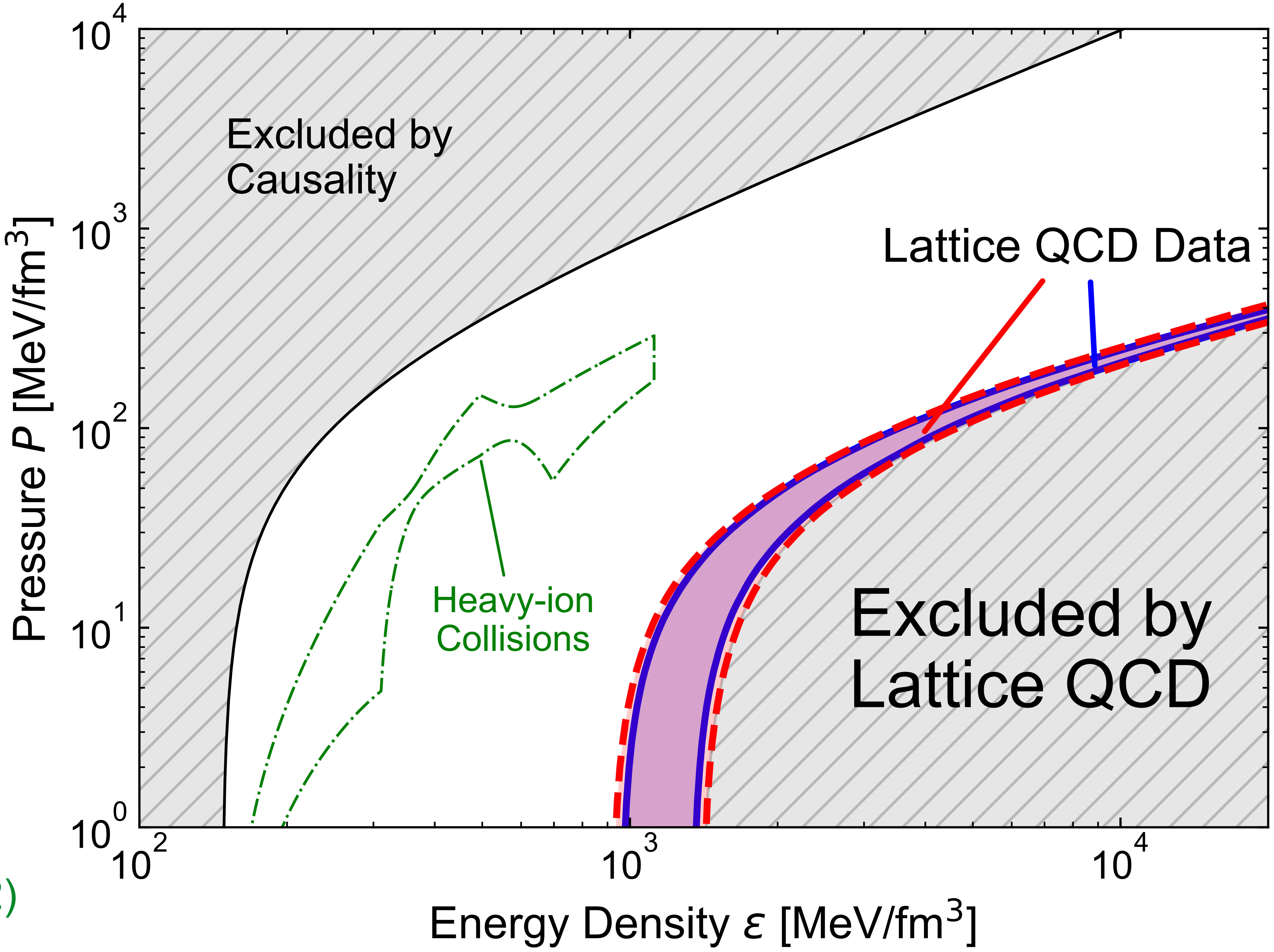


by changing value of h, the trajectories of $P_{\min}$ ($P_{\max}$)
gives the lower (upper) bound for $P(\varepsilon)$

Robust bounds on $P(\varepsilon)$

Fujimoto, Reddy (2023)

From the relation $\varepsilon = -P + \mu_B n_B$:



Soft EoS at large ε is excluded

Heavy-ion:
Oliinychenko et al.(2022)