Nucleosynthesis and Evolution of Neutron Stars (XRB2025)

Compact Objects in Modified Gravity

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References

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Introduction

Compact objects are an interdisciplinary research subject in high-energy physics, and observations of compact objects allow us to test gravitational theory in a strong and non-perturbative gravitational field. In GR, solving the TOV equation, we can compute M-R relation of compact objects for given EOS, and observations of M-R relation can constrain EOS.

The modified gravity predicts the modified TOV equation and the different M-R relation, allowing us to distinguish the gravitational theories by the astrophysical observations. Thus, compact star physics is one of the phenomenological applications of the modified gravity theory.

F(R) Gravity

Scalaron Field

F(R) gravity theory is one of the modified gravity theories, and EH action is replaced by functional of scalar curvature F(R). F(R) gravity introduces a new scalar degree of freedom, scalaron.

F(R) gravity

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R)$$

$\Phi \equiv F_R(R)$ ➡

Scalar-Tensor description

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\Phi R - Y(\Phi)\right]$$

$$Y(\Phi) = R(\Phi)\Phi - F(R(\Phi))$$

Chameleon Mechanism

Scalaron field couples with trace of matter EMT tensor

$$\Box\Phi = \frac{dV_{\text{eff}}}{d\Phi} \qquad \frac{dV_{\text{eff}}(\Phi,T)}{d\Phi} \equiv \frac{1}{3}\left[\Phi R(\Phi) - 2Y(\Phi) + \kappa^2 T\right]$$

Scalaron effective mass is determined by distribution of ambient matter fields (= compact objects).

Effective mass

$$m_\Phi^2 \equiv \left.\frac{d^2V_{\text{eff}}}{d\Phi^2}\right|_{\Phi=\Phi_{\text{min}}} \qquad \frac{d^2V_{\text{eff}}}{d\Phi^2} = \frac{1}{3}\left[\frac{F_R(R)}{F_{RR}(R)} - R\right] \qquad \left.\frac{dV_{\text{eff}}}{d\Phi} = 0\right|_{\Phi=\Phi_{\text{min}}}$$

Modified TOV Equation

Static and Spherically Symmetric spacetime

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

We solve five equations for five functions: $(R, \nu, \lambda, \epsilon, P)$

Metric components

1. $$\lambda' = \frac{e^{2\lambda}\left[r^2\left(16\pi G\epsilon - F\right) + F_R\left(r^2R - 2\right)\right]}{2r\left(2F_R + rF_{RR}R'\right)} + \frac{2\left[F_R + r^2\left(F_{RR}R'' + F_{RRR}(R')^2\right) + 2rF_{RR}R'\right]}{2r\left(2F_R + rF_{RR}R'\right)}$$

2. $$\nu' = \frac{e^{2\lambda}\left[r^2\left(16\pi GP + F\right) - F_R\left(r^2R - 2\right)\right] - 2\left(F_R + 2rF_{RR}R'\right)}{2r\left(2F_R + rF_{RR}R'\right)}$$

Curvature as independent variable

3. $$R'' = \left[\frac{1}{r}\left(3\nu' - \lambda' + \frac{2}{r}\right) + e^{2\lambda}\left(\frac{1}{2}R - \frac{2}{r^2}\right)\right]\frac{F_R}{F_{RR}} + \left(\lambda' + \frac{1}{r}\right)R' - \frac{F_{RRR}}{F_{RR}}(R')^2$$

Matter contents

4. Conservation law: $0 = (\epsilon + P)\nu' + P'$ 5. EOS: $P = P(\epsilon)$

Numerical Results

Models

R-squared model $F(R) = R + \alpha R^2 \quad (\alpha > 0)$

Noninteger power (NIP) model $F(R) = R + aR^{1+\frac{1}{b}} \quad (a > 0, b > 1)$

Boundary Condition

Uniqueness of Schwarzschild sol. is absent in F(R) gravity. However, assuming asymptotical flatness, scalaron field $\Phi \to 1$ at $r \to \infty$ in vacuum for above two models, corresponding to GR limit $F(R) \to R$. We impose boundary condition away from surface of compact object.

M-R Relations

M-R curves show clockwise rotations.

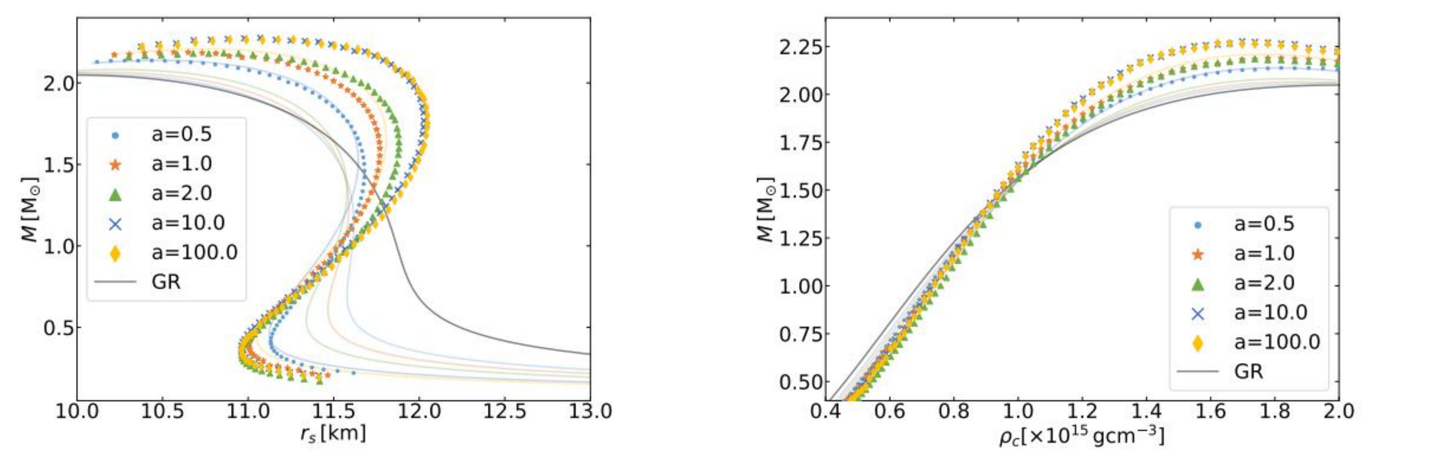

Fig.1 M-R relation (left) and M-central density relation (right) for Sly EOS. Black curves are GR results for reference. Colored points represent results in NIP model (b=2), and colored curves represent results in R-squared model (b→1, a = α).

Scalaron Distribution

Compact objects in F(R) gravity generally have scalar hair.

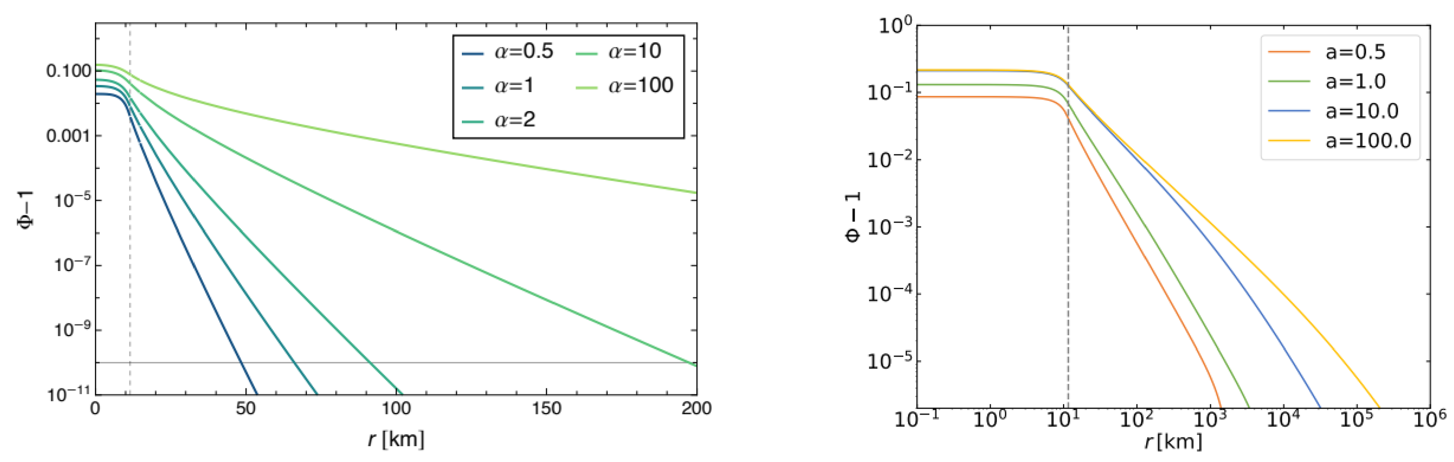

Fig.2 Scalaron field around compact object in R-squared model (left) and NIP model (right). Vertical dashed line represents surface of compact object.

Conclusion

- M-R relations in two models of F(R) gravity show large maximal mass and deviation from GR prediction in low-mass region
- Scalaron field takes a nearly constant value inside compact object and decreases outside compact object.

Degeneracy between F(R) function and EOS

- If M-R relation (= density profile) and EOS are given, we can solve TOV equation w.r.t. F(R) function.
- Thus, we can construct model of F(R) gravity which explains observed M-R relation for arbitrary EOS.
- We need more measurements other than M-R relation.

Prospects for Future Works

- Tidal deformation (perturbed TOV equation)
  --- Scalar-mode of perturbation ~ scalar mode of GW
- Thermal evolution (cooling or time-evolution)
  --- Decay of scalaron to neutrino and photon
- Correction to EOS
  --- New interactions induced from scalaron coupling