BARONET: A Nuclear Network Geared Towards Coupling with Hydrodynamics Simulations

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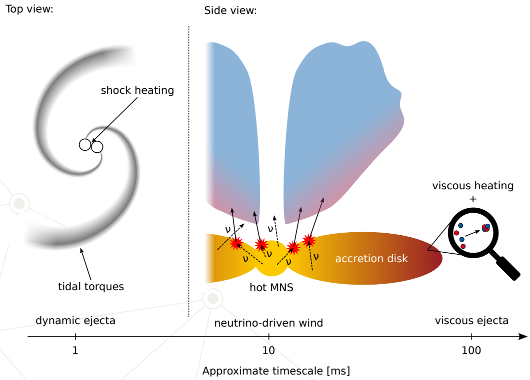
with Albino Perego, Luca Maggioni, Li Shifang

Nucleosynthesis and Evolution of Neutron Stars - Kyoto - 28/01/2025

Background & Motivation

Matter is ejected via a variety of channels:



Ejecta of BNS mergers are hot, fast expanding and very neutron rich.

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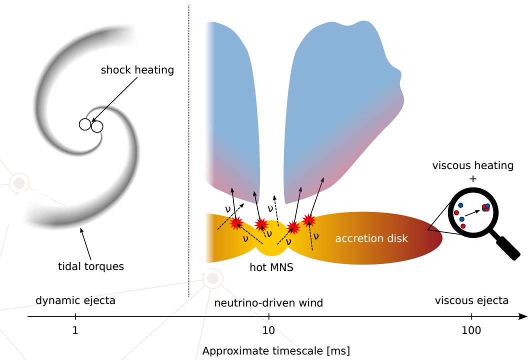
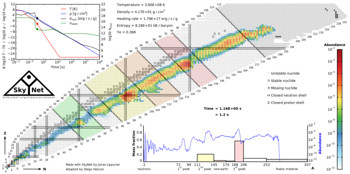


Ejecta of BNS mergers are hot, fast expanding and very neutron rich.

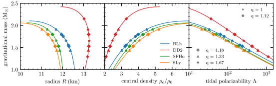
↓

Onset of (strong) r-process nucleosynthesis



Numerical simulations and observables

EOS of dense matter



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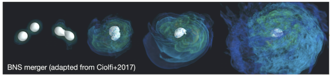
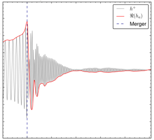
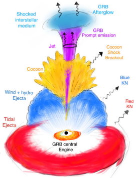
Hydrodynamical simulations



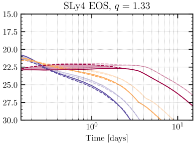
Gravitational waves



sGRB



Kilonovae



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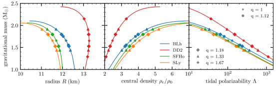

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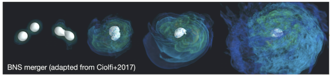

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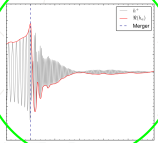

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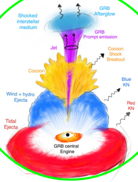

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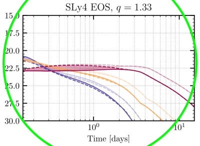

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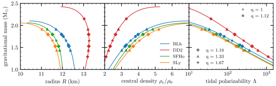

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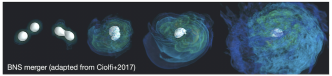

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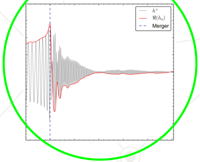

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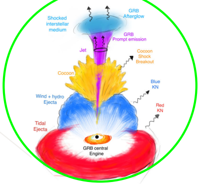
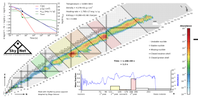


r-process nucleosynthesis



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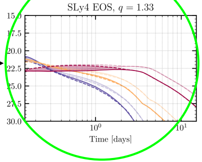

Nuclear networks: post-processing vs. in situ

A nuclear reaction network:

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} = \sum_j \lambda_j Y_j + \sum_{jk} \lambda_{jk} Y_j Y_k + \cdots$$

Usual coupling to hydro simulations:

↓

Extract initial Y_e, s, T along with history of ρ + homologous expansion. Assume NSE at start.

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Post-process with nuclear network.

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Post-process with nuclear network.

This method overly simplifies the density evolution and neglects the influence of the nuclear heating on the dynamics.

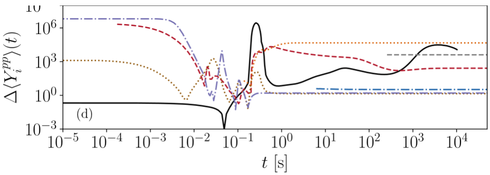

Image from *[Magistrelli et al., 2024]*

Proper coupling *in situ* to a long-lived simulation reveals significant discrepancies with the post processing approach.

Reduced networks

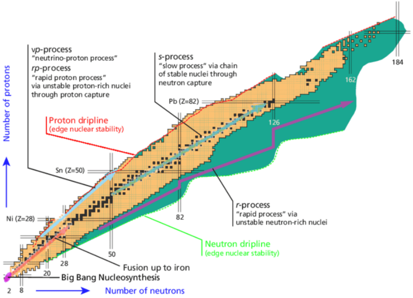

Image adapted from Martinez-Pinedo

The (strong) r-process runs through all nuclei between the valley of stability and the neutron drip line, for a total of $\sim$ 7000/8000 DoF.

A typical hydrodynamics simulation has (several) 10^7 DoF. Coupling to a nuclear network would result in 10^{11} DoF.

☛ **Infeasible!**

Either simplify the hydro simulation (cf. previous slide) or *simplify the nuclear network*, reducing the number of DoF necessary.

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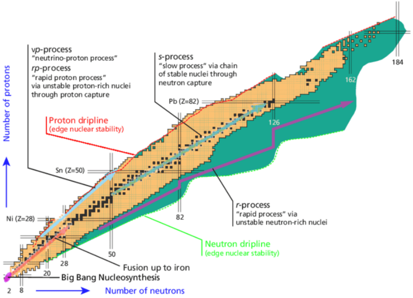

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$\downarrow$

The **BARONET** code
(BetA flow ReactiOn NETwork)

Techniques

$(\mathrm{N},\gamma)\leftrightarrow(\gamma,\mathrm{N})$ EQUILIBRIUM AND BETA FLOW

Original DoF: $Y_{A,Z} \to Y_{A,Z} = Y_Z P_{A,Z}$, where $Y_Z = \sum_A Y_{A,Z}$ and $P_{A,Z} = Y_{A,Z}/Y_Z$.

$$(A, Z) + n \longleftrightarrow (A + 1, Z) + \gamma$$

Valid until n_n is high enough, i.e. up to *neutron freeze out* (NFO).

$$\mu_{A,Z} + \mu_n = \mu_{A+1,Z}$$

$$\mu_{A,Z} = m_{A,Z} + k_B T \log\left[\frac{n_b Y_{A,Z}}{G(T)_{A,Z}}\left(\frac{2\pi\hbar^2}{m_{A,Z} k_B T}\right)^{3/2}\right]$$

$(n,\gamma) \leftrightarrow (\gamma,n)$ EQUILIBRIUM AND BETA FLOW

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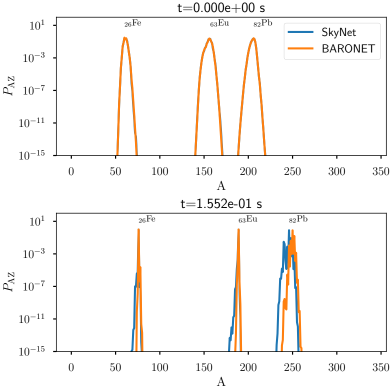
$$\downarrow$$

$$\frac{P_{A+1,Z}}{P_{A,Z}} = \frac{1}{2} n_n \frac{G(T)_{A+1,Z}}{G(T)_{A,Z}} \left(\frac{2\pi\hbar^2}{m_b k_B T}\frac{A+1}{A}\right)^{3/2} \exp\left(\frac{S_{n\,(A+1,Z)}}{k_B T}\right)$$

👉 the $P_{A,Z}$ are known analytically, only the Y_Z must be evolved.



BETA FLOW

Y_Z evolve by reactions that change Z, but not A

$\rightarrow \beta^-$ **decays**

$$\frac{dY_{A,Z}}{dt} = -Y_{A,Z} \sum_{i=0}^{3} \lambda^i_{A,Z} + \sum_{i=0}^{3} \sum_{A} Y_{A+i,Z-1} \lambda^i_{A+i,Z}$$

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$$\frac{dY_Z}{dt} = -Y_Z\sum_{A} P_{A,Z}\sum_{i=0}^{3}\lambda^i_{A,Z}$$
$$+Y_{Z-1}\sum_{i=0}^{3}\sum_{A} P_{A+i,Z-1}\lambda^i_{A+i,Z}$$

Need to sum over the $P_{A,Z}$, which can be computed analytically.

Evolution of the neutron fraction:

$$\frac{dY_n}{dt} = -Y_n\lambda^0_n$$
$$+Y_Z\sum_{i=1}^{3}\sum_{A} iP_{A,Z}\lambda^i_{A,Z}$$
$$+\frac{1}{\tau}(\chi_{tot}-1)Y_n$$

where $\chi_{tot} = \sum_{A,Z} A Y_Z P_{A,Z} = 1$ and $\tau \simeq 10^{-6}$ seconds.

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Evolution of the neutron fraction:

$$\frac{dY_n}{dt} = -Y_n\lambda^0_n + Y_Z\sum_{i=1}^{3}\sum_A iP_{A,Z}\lambda^i_{A,Z} - \frac{1}{\tau}(\chi_{tot} - 1)Y_n$$

where $\chi_{tot} = \sum_{A,Z} AY_Z P_{A,Z} = 1$ and $\tau \simeq 10^{-6}$ seconds.

The damping term is an effective way to recover missing reactions and ensure mass conservation.

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This ODE system is **non-stiff** and admits **large(-ish) timesteps**.

Post-NFO phase

(n,γ)$\leftrightarrow$(γ, n) equilibrium valid until NFO, e.g. $Y_{\mathrm{n}}/Y_{\mathrm{seed}} \sim 1$ ($Y_{\mathrm{seed}} = \sum_{i\neq \mathrm{n}} Y_i$). What to do beyond this point?

Keeping only β^- decays, one can write:

$$\frac{\mathrm{d}Y_{\mathrm{A,Z}}}{\mathrm{d}t} = \mathbf{M}Y_{\mathrm{A,Z}}$$

with explicit solution

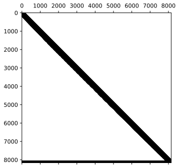
$$Y_{\mathrm{A,Z}}(t) = \exp(t\mathbf{M})Y_{\mathrm{A,Z}}|_{\mathrm{initial}}$$

since $\mathbf{M}$ is time-independent.

"Initial data" for this formula is easily expressed as

$$Y_{\mathrm{A,Z}}|_{\mathrm{initial}} = Y_{\mathrm{Z}}|_{\mathrm{NFO}}P_{\mathrm{A,Z}}|_{\mathrm{NFO}}$$

but the $P_{\mathrm{A,Z}}|_{\mathrm{NFO}}$ are easily computed on the fly by storing T_{NFO} and n_{n} **only**.



Sparsity pattern of $\mathbf{M}$

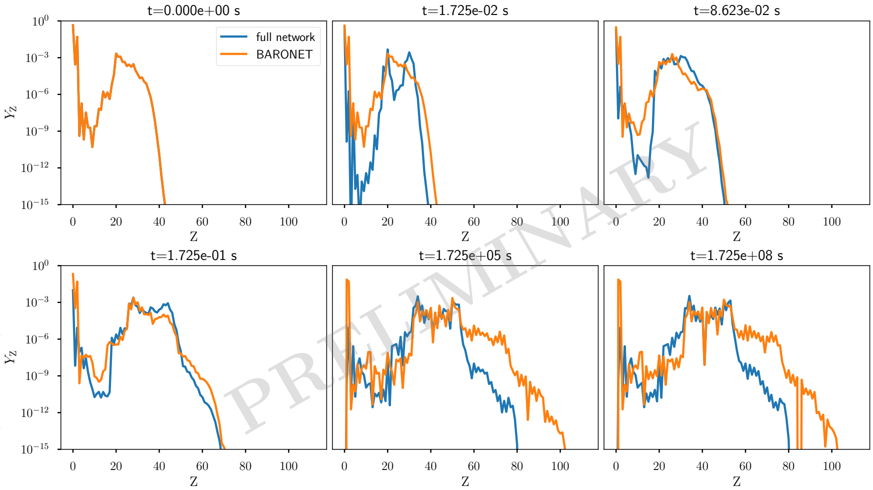
$\exp(t\mathbf{M})$ is not trivial to compute.

It is currently implemented as

$$\mathbf{M} = \mathbf{V}\Lambda\mathbf{V}^{-1} \rightarrow \exp(t\mathbf{M}) = \mathbf{V}\exp(t\Lambda)\mathbf{V}^{-1}$$

Preliminary results

Results: elemental abundances for "weak" r-process



$Y_e = 0.25$. Comparison data generated with SkyNet *[Lippuner and Roberts, 2017]*

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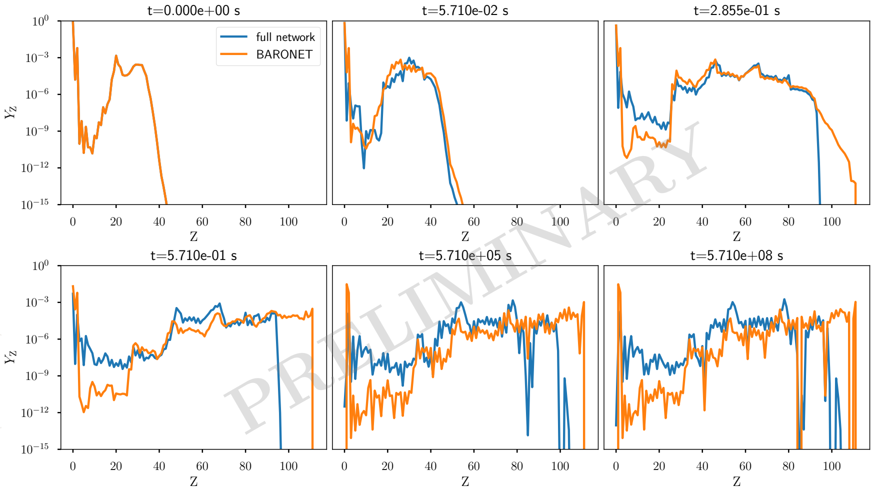
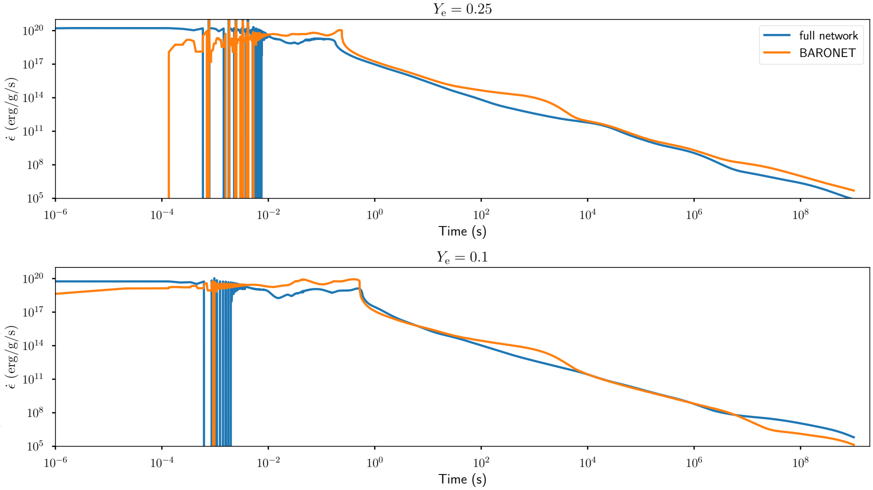


$Y_e = 0.1$. Comparison data generated with SkyNet *[Lippuner and Roberts, 2017]*

Results: nuclear heating rate



Heating rate as a function of time. Comparison data generated with SkyNet *[Lippuner and Roberts, 2017]*

Conclusions

- BARONET relies on dominant reactions to reduce the number of DoF to a few hundred pre-NFO
- post-NFO evolution coupled to hydro needs further simplification (impose functional form of $P_{\mathrm{A,Z}}$)

Ongoing work:

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- coupling to hydro simulations

Stay tuned...Thank you

References

References I

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