Complex-valued problem on FRG analysis of relativistic BEC #14

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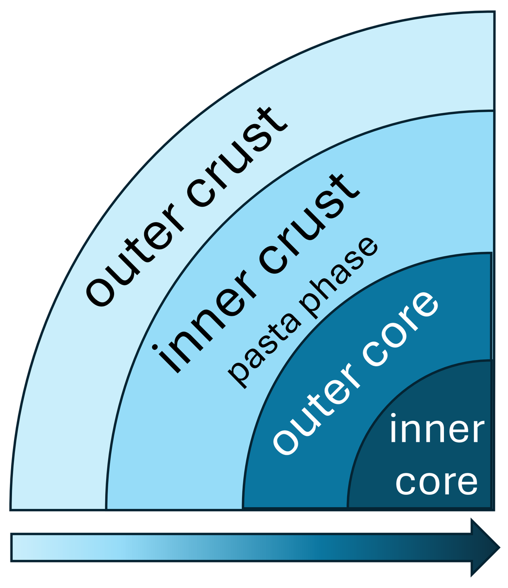
Introduction - relativistic Bose-Einstein Condensates (BEC) -



Electron chemical potential μ_e increases with density ρ.

In the outer core, ($\rho \approx \rho_{\text{saturation}}$)
$\mu_e \approx 100\text{MeV} \approx$ free pion mass
S. L. Shapiro et al. (1983)

Pion-BEC affects key properties of neutron stars:

- softening equation of state
- enhancing cooling rates via neutrino emission

Analyses of relativistic BEC in the complex scalar theory

$$\mathcal{L} = |\partial_\mu \varphi|^2 + m^2|\varphi|^2 + \lambda|\varphi|^4$$

- mean-field approximation (MFA) J. I. Kapusta (1981) ➡ including no effects of quantum fluctuations
- Functional Renormalization Group (FRG) L. F. Palfares (2012) ➡ **complex-valued problem** E. E. Svanes et al. (2011), O(N) model

Purpose: FRG analysis of the complex scalar theory as a basic step for studying pion-BEC

Formalism - Functional Renormalization Group (FRG) -

- FRG calculates effective actions $\Gamma[\langle\phi\rangle]$ non-perturbatively.

$$\Gamma[\langle\phi\rangle] \coloneqq -\log \int \mathcal{D}\phi \, e^{-S+\int d^4x j\phi} + \int d^4x \, j\langle\phi\rangle$$

$$\int \mathcal{D}\phi = \prod_{q=0}^{\Lambda} \int_{-\infty}^{+\infty} d\phi_q \quad \text{integrations over momentum modes } \phi_q$$

- An IR-regulator R_k is introduced in Γ. $\to \Gamma_k$

Γ_k follows the **Wetterich flow equation**:

$$\frac{\partial \Gamma_k}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial R_k}{\partial k} \frac{1}{\Gamma_k^{(2)} + R_k}\right] = \frac{1}{2} \bigcirc \quad \text{C. Wetterich (1993)}$$

$$\frac{\partial \Gamma_k^{(2)}}{\partial k} = [\Gamma_k^{(3)}\text{–}\Gamma_k^{(3)} \text{ loop}] - \frac{1}{2}[\Gamma_k^{(4)} \text{ loop}] \qquad \partial_k \Gamma_k^{(n)} = f_n\left(\Gamma_k^{(n+1)}, \Gamma_k^{(n+2)}\right)$$

$\Gamma_{k=\Lambda} = S$ (classical) : initial condition
$\Gamma_{k\to 0} = \Gamma$ (quantum) : solution

- Derivative Expansion

$$\Gamma_k = \sum_{n=0}^{\infty} g_{2n}\left[\langle\phi\rangle, \left(\partial_\mu\langle\phi\rangle\right)^{2n}\right] \to g_0 + g_2$$

- Local Potential Approximation
$Z_k(\partial_\mu\langle\phi\rangle)^2 \to 1 \cdot (\partial_\mu\langle\phi\rangle)^2$ in g_2

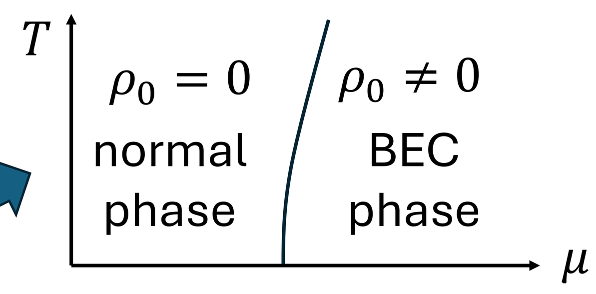
- Model - the complex scalar theory -

$$\Gamma_k = \int_0^\beta d\tau \int d^3x \left[\nabla\varphi \cdot \nabla\varphi^* + (\partial_\tau - \mu)\varphi(\partial_\tau + \mu)\varphi^* + V_k(\rho)\right] \qquad \rho := |\varphi|^2$$

$\Gamma_k \to \beta V \cdot U_k(\rho)$: effective potential ($\varphi(x) \to \varphi$)

- thermodynamics in FRG

order parameter (BEC^2): $\rho_0(T,\mu) := \underset{\rho}{\mathrm{argmin}}\, U_{k\to 0}(\rho)$



$\Omega(T,\mu) = U_{k\to 0}(\rho_0)$ grand-canonical potential ➡ pressure: $p = -\Omega$, number density: $n = -\partial\Omega/\partial\mu$, entropy: $s = -\partial\Omega/\partial\mu$, energy density: $\varepsilon = -p + \mu n + Ts$

- flow equation in $R_k(p) = (k^2 - |\vec{p}|^2)\cdot\theta(k^2 - |\vec{p}|^2)$ D.F. Litim (2001)

$$\frac{\partial U_k}{\partial k} = \frac{k^4}{6\pi^2}\sum_{s=\pm} \frac{E_s^2 + \mu^2 - E_k^2}{E_s^2 - E_{-s}^2} \frac{1 + 2n_B(E_s)}{E_s} \qquad E_k^2 = k^2 + V_k' + \rho V_k'',\ n_B(E) = \left(e^{E/T} - 1\right)^{-1}$$

$$E_\pm = \left[E_k^2 + \mu^2 \pm \left\{4E_k^2\mu^2 + (\rho V_k'')^2\right\}^{1/2}\right]^{1/2}$$

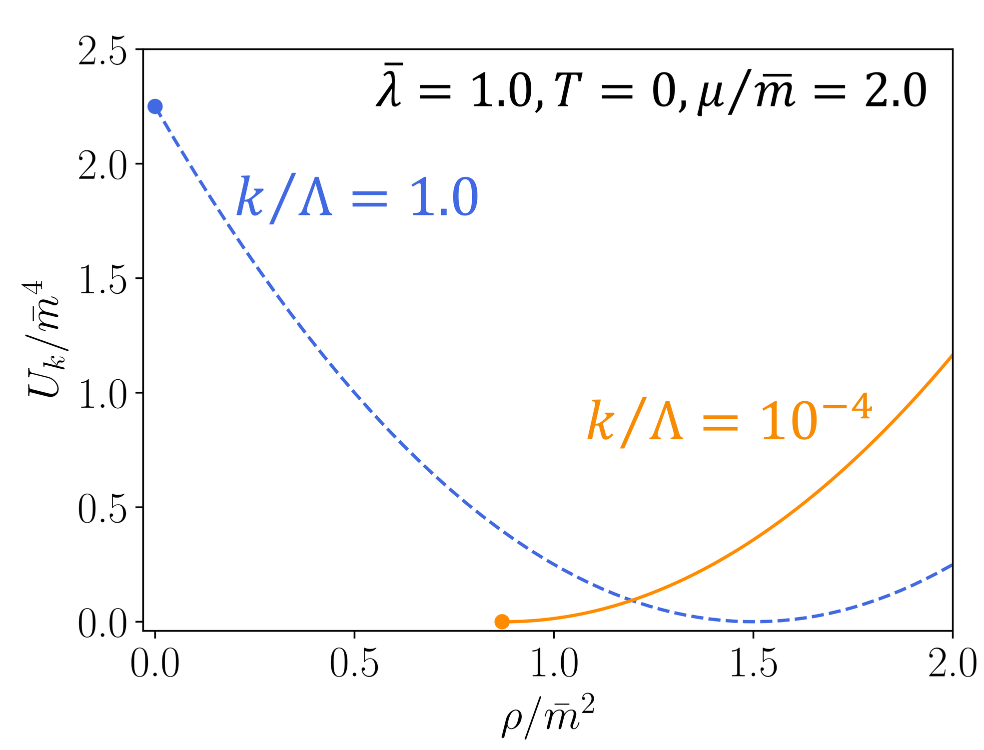
***E_-* takes imaginary values at some *ρ*-regions.** E. E. Svanes et al. (2011), consider only real part; L. F. Palfares (2012), restrict parameters

We require U_k to be real and finite.

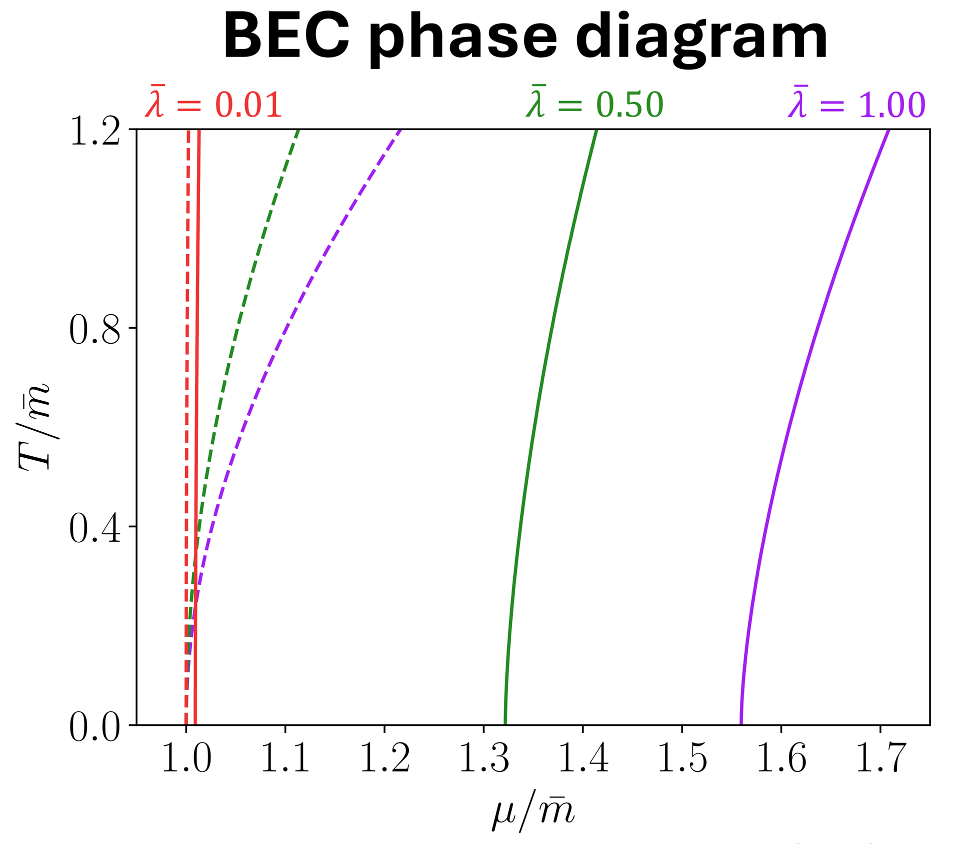
consider ρ-range where $E_-^2(\rho) > 0$ ➡ $\mu^2 < m^2_{\text{eff},k>0} \coloneqq k^2 + V'_{k>0}$
$\boldsymbol{\mu^2 \le m^2_{\text{eff},k\to 0}}$ ← Correspondence →

cf. condition in free theory
Ω_{free} is defined only if $\boldsymbol{\mu^2 \le m^2_{\text{bare}}}$.

Results



We can find real-valued ρ_0 in $\mu^2 < m^2_{\text{eff},k}(\rho)$.



FRG —— MFA - - - J. I. Kapusta (1981) MFA formula: $T_c^2 = 3(\mu_c^2 - \bar{m}^2)/\bar{\lambda}$

- For values of ρ smaller than the potential minimum "●", these ρ-points violate the condition, $\mu^2 > m^2_{\text{eff},k}(\rho)$.
- In a strong coupling regime ($\bar{\lambda} = 1.00$), quantum fluctuations strongly influence the behavior of the relativistic BEC.
- Regulator dependence...
The Litim R_k keeps poles away from relevant ρ-regions as much as possible. D.F. Litim (2001)

➡ The complex-valued problem would occur in other regulators.

Summary

The complex scalar theory in FRG suffers from the complex-valued problem.

- We developed the novel method and obtained real valued BEC.
- We found in a strong coupling regime, quantum fluctuations are important.

Outlooks:

- Application for models including pions
- Analyses of parameters that have not been explored before