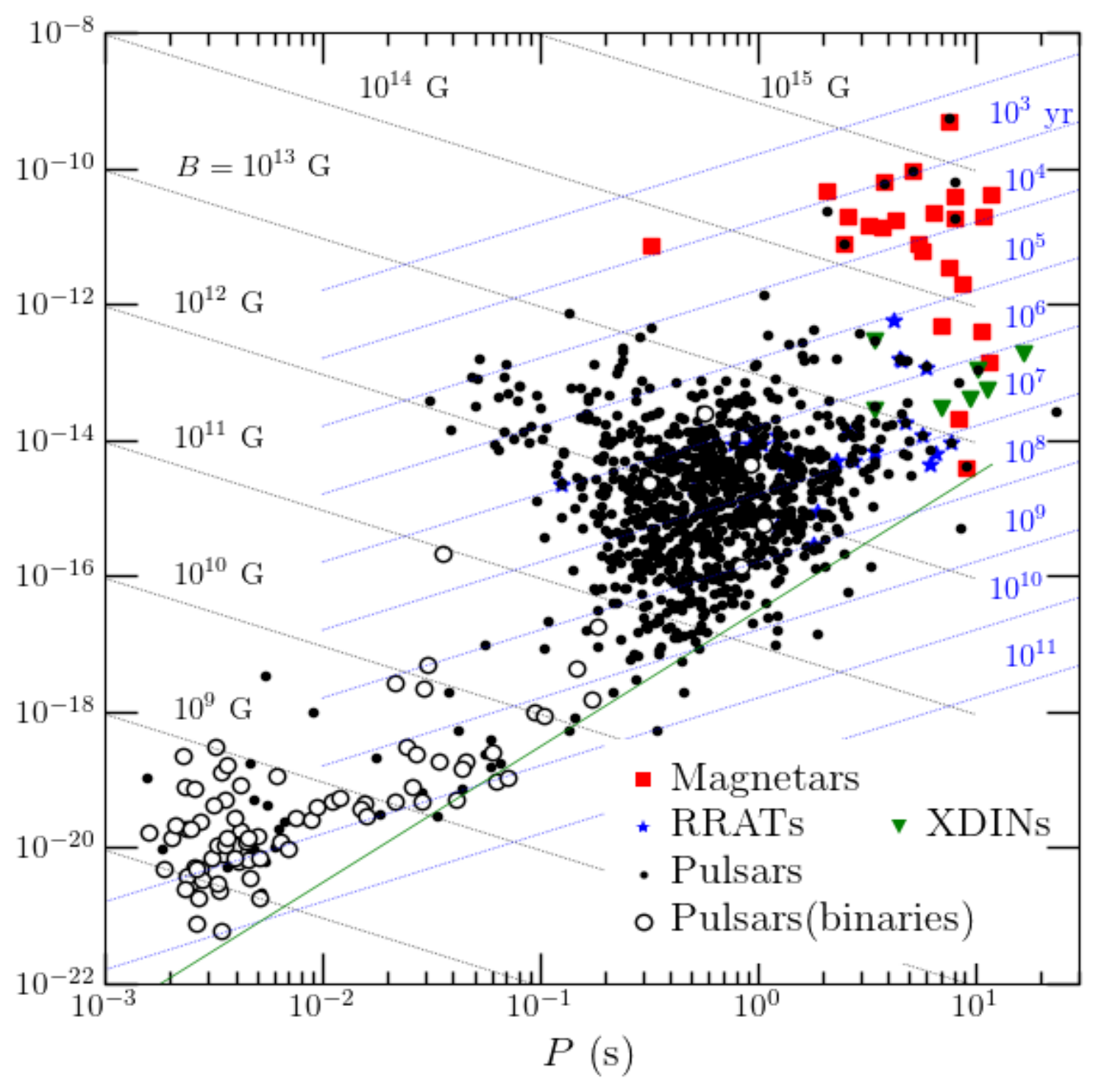
Constraining of Nuclear Matter Equation of States with Rotating Neutron Stars

Nuclear Theory Group   Kwon Hyukjin (M2)
Department of Physics, School of Science, Institute of Science Tokyo, Japan
Email : kwon.h.ab@m.titech.ac.jp, Supervisor : K. Sekizawa

1 Introduction

In nuclear physics, neutron stars provide a unique opportunity to overcome the limitations of terrestrial experiments in studying nuclear matter. They play a critical role in constraining the equation of states (EoS) of nuclear matter by evaluating which models and parameter sets of nuclear forces are suitable across various densities.



Traditionally, constraints on the EoS derived from neutron stars have been based on the **Tolman-Oppenheimer-Volkoff (TOV) equations**, which are analytic solutions of Einstein's equations under spherical symmetry.

However, neutron stars are rotating objects, and some observed pulsar signals, particularly from **millisecond pulsars**, exhibit extremely rapid rotation.

This study investigates the EoS constraints of nuclear matter under rotational effects by employing the **Komatsu-Eriguchi-Hachisu (KEH) method** **[Monthly Notice. Sup. 237, 355-379 (1989)]**, which numerically solves Einstein's equations assuming axial symmetry.

2 Methods

☐ Nuclear Interaction Models

◇ Skyrme interaction $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \alpha$, and, W_0 are parameters

$$V(\vec{r}_1, \vec{r}_2) = t_0(1 + x_0 P_\sigma)\delta(\vec{r})$$
Zero range term (Delta function)

$$+\frac{1}{2} t_1 (1 + x_1 P_\sigma)\left[\delta(\vec{r})\vec{P}'^2 + \vec{P}^2 \delta(\vec{r})\right]$$
$$+t_2(1 + x_2 P_\sigma)\vec{P}' \cdot \delta(\vec{r})\vec{P}$$
Momentum dependent term

$$+t_3 \rho^\alpha (1 + x_3 P_\sigma)\delta(\vec{r})$$
Density dependent term

$$+iW_0 \vec{\sigma} \cdot \left[\vec{P}' \times \delta(\vec{r})\vec{P}\right]$$
Spin-Orbit term

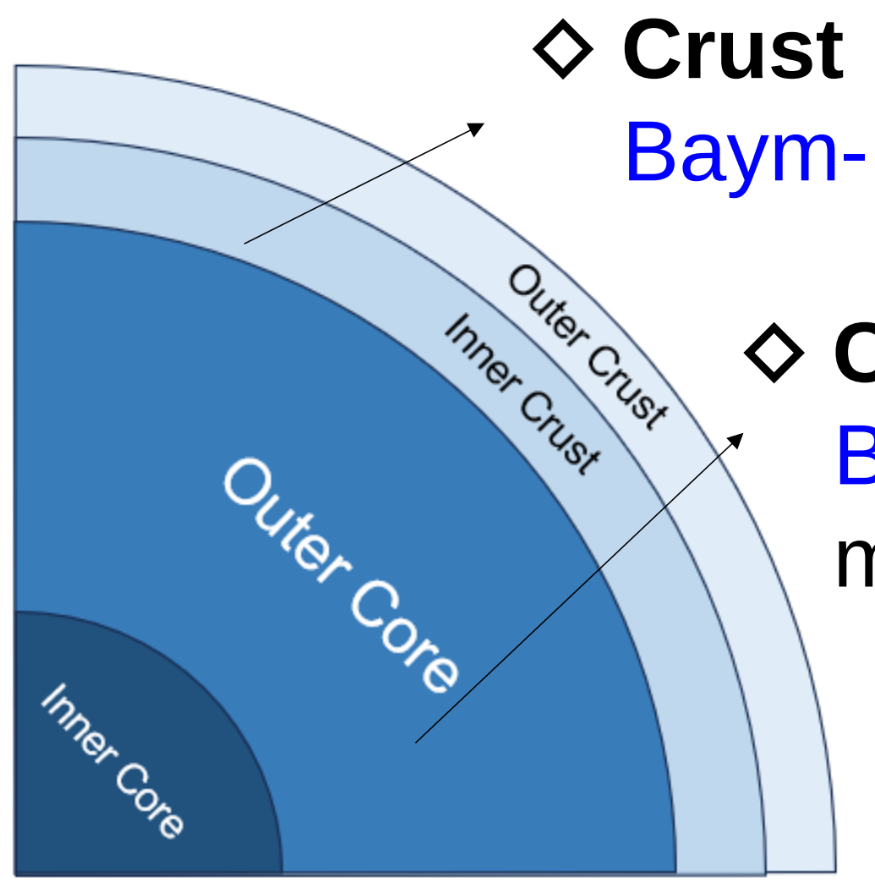
◇ Gogny interaction $W_i, B_i, H_i, M_i, t_3, x_3, \alpha, \mu_i$ and, W_0 are parameters

$$V(\vec{r}_1, \vec{r}_2) = \sum_{i=1}^{2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)\, e^{-\frac{(r_1 - r_2)^2}{\mu_i^2}}$$
Central term (Gaussian function)

$$+t_3 \rho^\alpha (1 + x_3 P_\sigma)\delta(\vec{r})$$
Density dependent term

$$+iW_0 \vec{\sigma} \cdot \left[\vec{P}' \times \delta(\vec{r})\vec{P}\right]$$
Spin-Orbit term

☐ Assumed Neutron Star Structure



◇ Crust
Baym-Pethick-Sutherland (BPS) EoS is used for Crust

◇ Core
Beta equilibrium and charge neutrality for $npe\mu$ matter are considered in the core region

$$\mu_n = \mu_p + \mu_e, \quad \mu_e = \mu_\mu \text{ (Beta equilibrium)}$$
$$n_p = n_e + n_\mu, \text{ (Charge neutrality)}$$

Calculated by Newton-Raphson Method

☐ Modeling of Neutron Stars

◇ Static Neutron Stars (TOV equation)

$$\frac{dP(r)}{dr} = -\frac{(\rho + P)[GM(r) + 4\pi G r^3 P]}{r[r - 2GM(R)]}$$

Solved by 4th Runge-Kutta Method

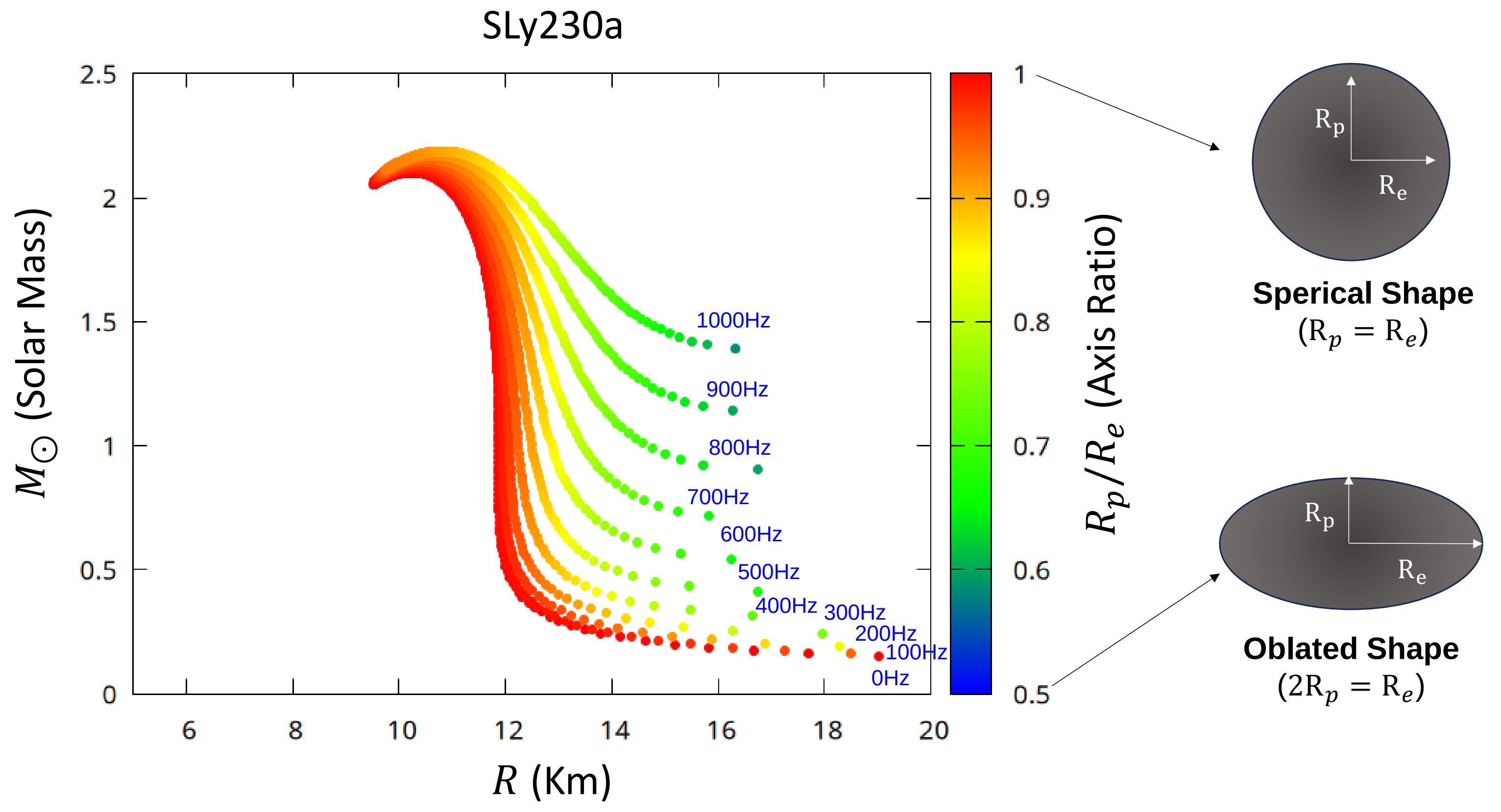
◇ Rotating Neutron Stars (KEH method)

$$\Delta\left[\rho e^{\gamma/2}\right] = S_\rho(r,\mu)$$
$$\left(\Delta + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial \mu}\right)\gamma e^{\gamma/2} = S_\gamma(r,\mu)$$
$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial \mu}\right)\omega e^{(\gamma-2\rho)/2} = S_\omega(r,\mu)$$

Solved by Self Consitent Field Iteration

3 Results

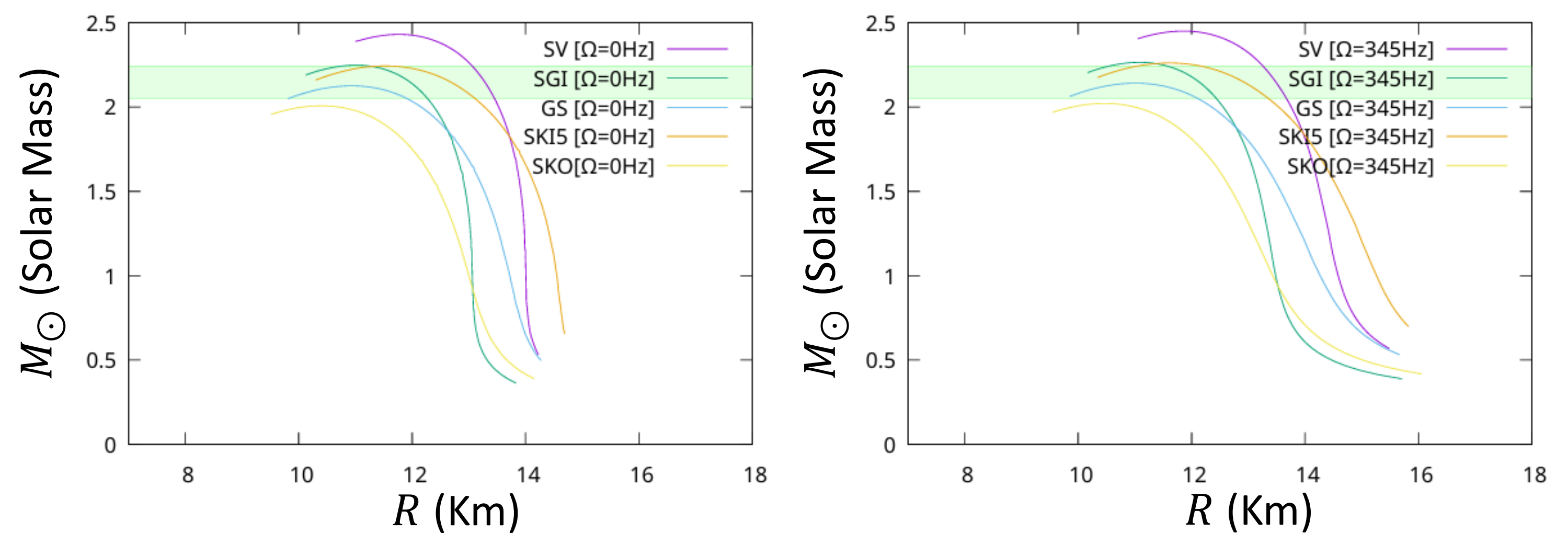
☐ Deformation of Rapidly-Rotating Neutron Stars



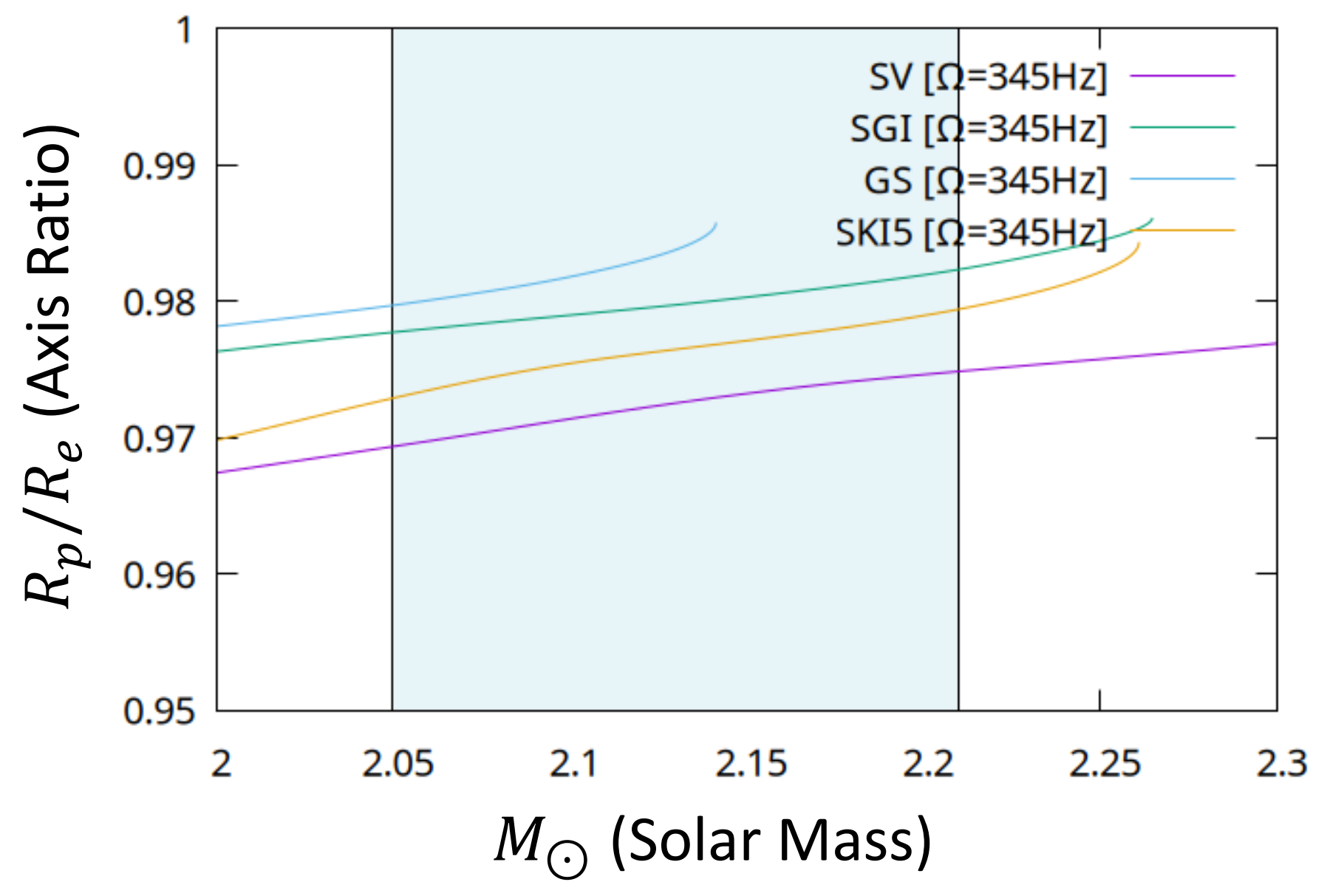
When considering the rotational effects of neutron stars, it can be observed that both their **mass and radius increase** as the rotational velocity rises. Additionally, for higher angular velocities relative to the central density, neutron stars exhibit significant **deformation**. This deformation affects not only the shape of the neutron star but also the distribution of density within its interior.

☐ Astrophysical Constraints ** PSR J0740+6620*

◇ Mass-Radius Relation



◇ Axis Ratio



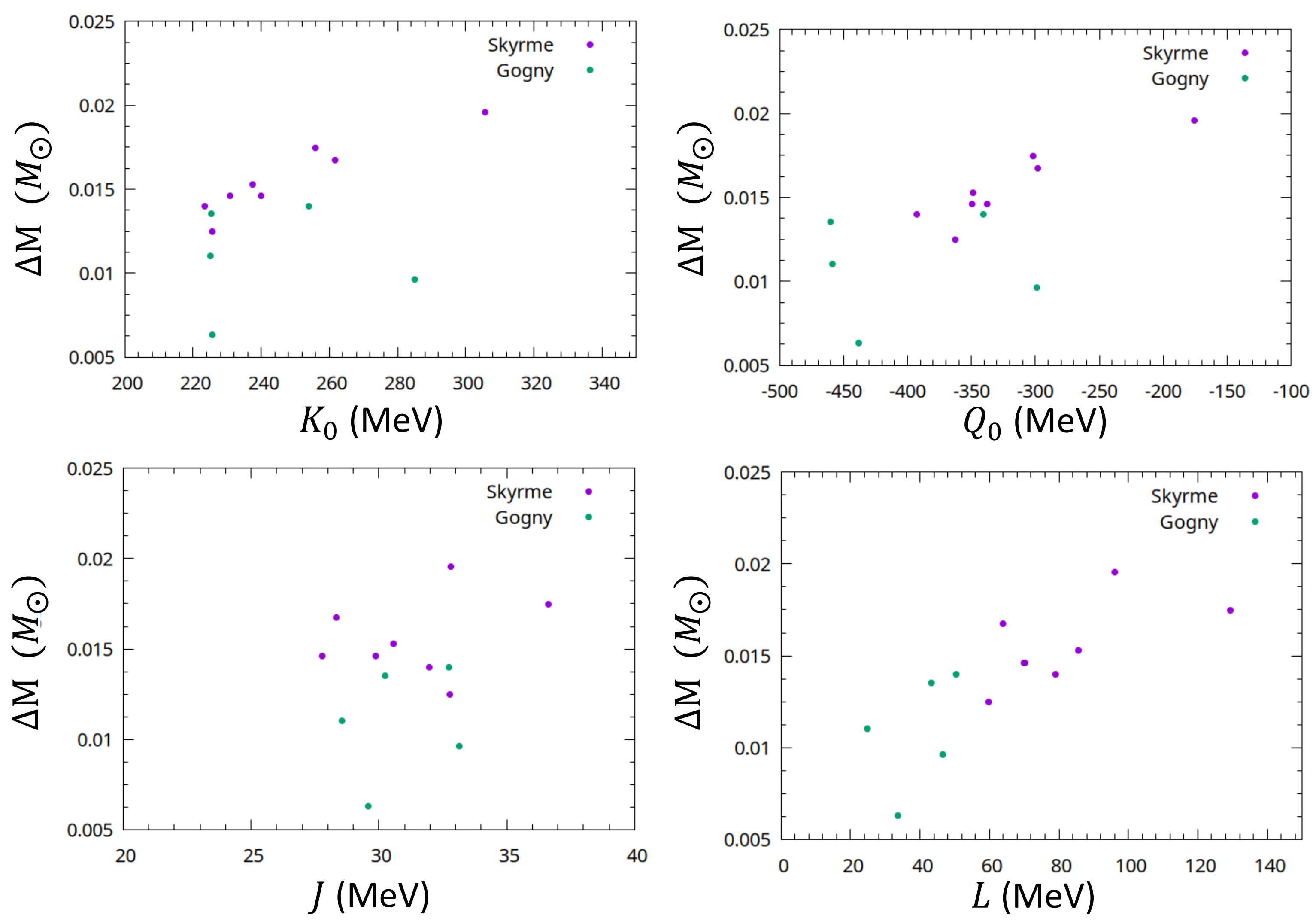
The **maximum mass** of a neutron star can be observed to **increase** due to rotational effects.

If the raito of the polar to equatorial radius of a neutron star can be accurately measured, it can be used a **new constraint** on the EoS.

☐ Laboratory Costraints vs Mass Differences

$$\Delta M = M_{\max}^{\mathrm{KEH}} - M_{\max}^{\mathrm{TOV}}$$

** PSR J0740+6620*



4 Conclusion

When pulsars rotation frequency is high ($\Omega \gtrsim 500\text{Hz}$), rotation effects can not be neglected. By comparing results with TOV and KEH, **I found a novel correlation between nuclear EoS parameters (K_0, Q_0, and L) with maximum mass difference ($\Delta M = M_{max}^{\mathrm{KEH}} - M_{\max}^{\mathrm{TOV}}$)**. This kind of analysis will serve a new way to better constraining nuclear matter EoS.