Photon Vortex Generation and Photonuclear Reactions by Photon Vortex in Astronomical System

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Photon Vortex Generation in Strong Magnetic Field

T.M, T. Hayakawa, M.K.Cheoun, T.Kajino, PLB826, 136779 (2022)

Nuclear Photo-Reaction by Photon Vortex

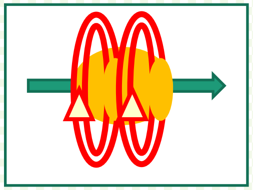
T.M., T. Hayakawa, M.K.Cheoun, T.Kajino, ApJ 975, 51 (2024)

§ 1 Introduction

z-Axis : Beam direction

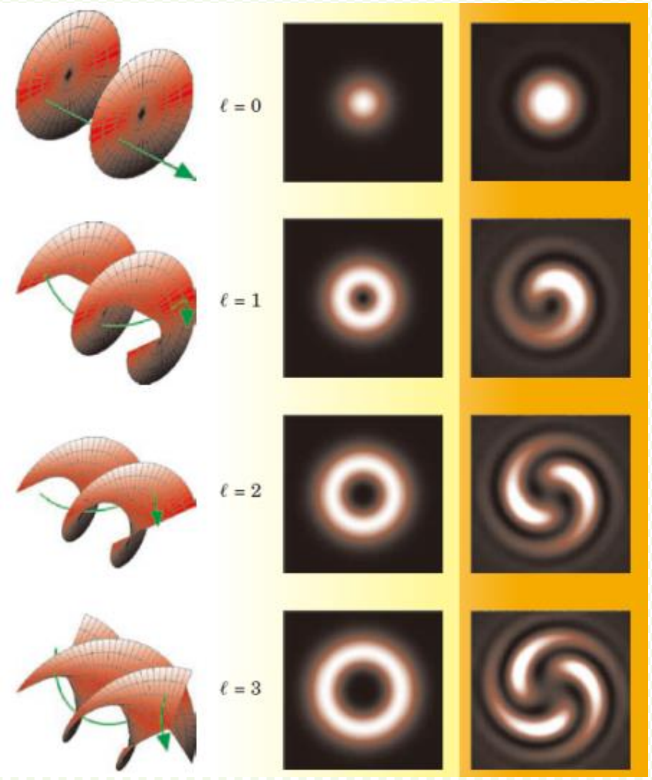
Photon Vortex

Eigen States of z-Comp. of Total Ang. Mom. (zTAM)

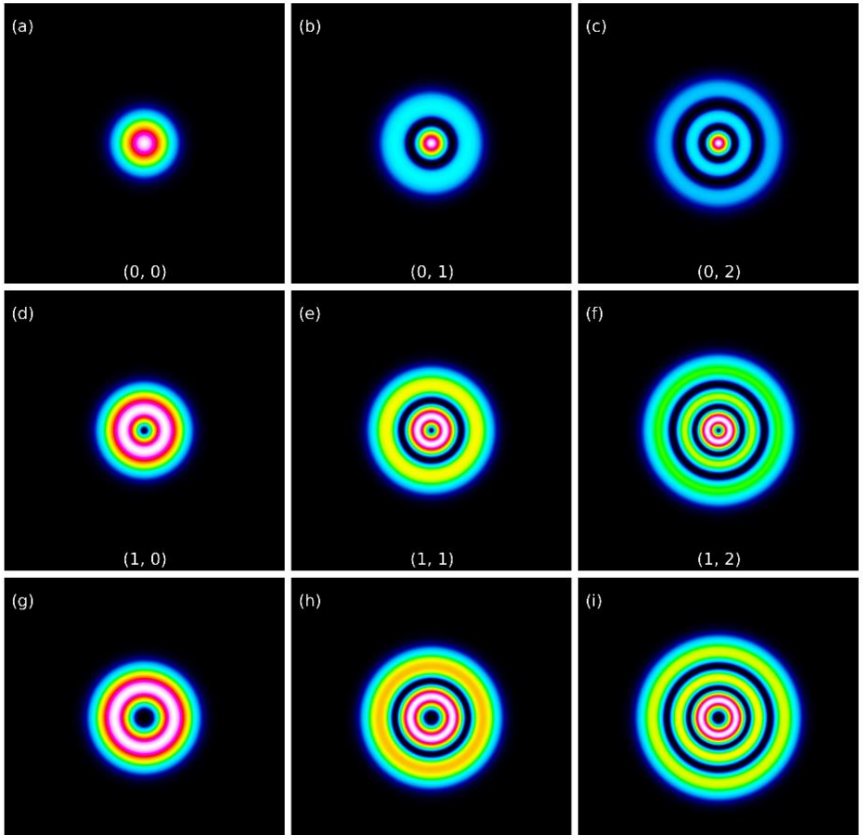


Optical Vortices L.Allen et al., PRA45, 8185 ('94)

Laguerre-Gaussian (LG) Mode



M. Padgett et al., Phys. Today 57 (2004) 35



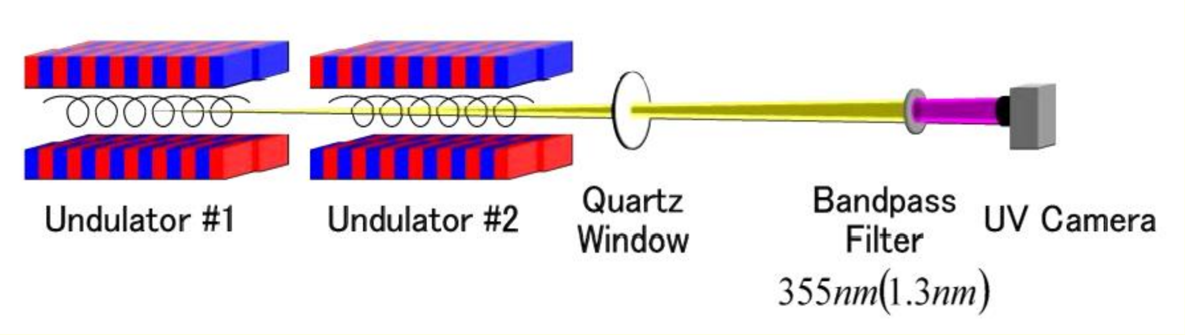
http://www.dataray.com/blog-m2-high-order-modes.html

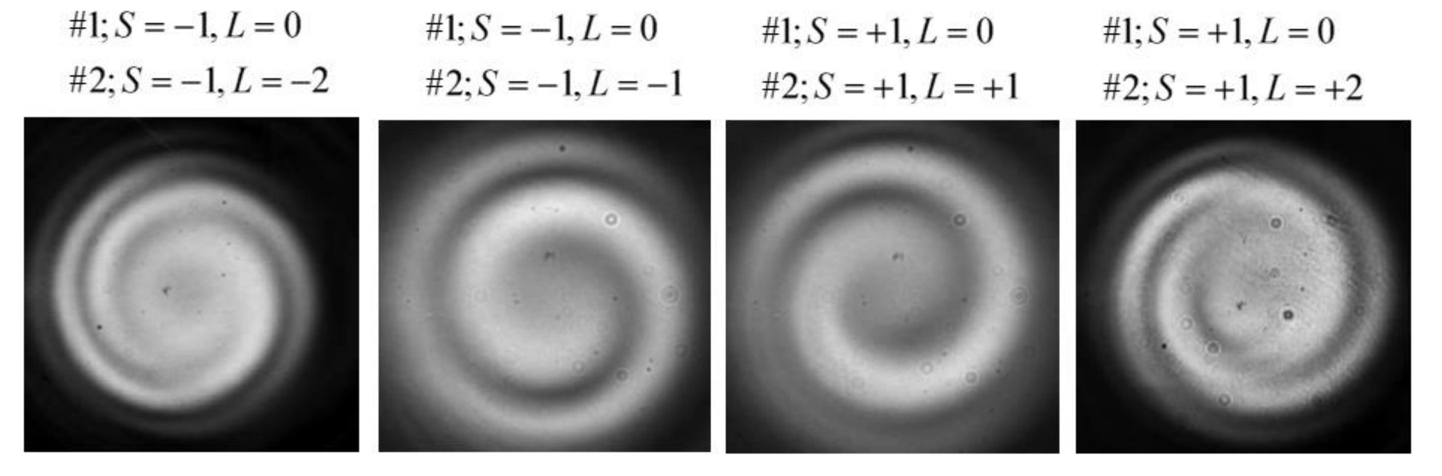
Optical Vortex Generation in Magnetic Field

Synchrotron Radiation from Electron in Magnetic Field

Generation of Light Vortex

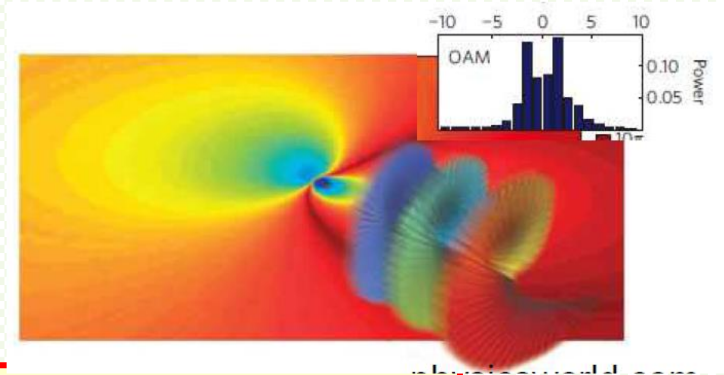
M. Katoh et al., PRL 118, 094801 (17), Sci. Rep. 7, 6130 (‘17)





Exp : S. Matsuba et al., doi:10.18429/JACoW-IPAC2019-TUPRB037

Photon Vortex in Nature



Strong Gravity (BH) ⇒ Optical Vortex

F. Tamburini et al., Nat.-Phys., Vol.7, 195 (`11),

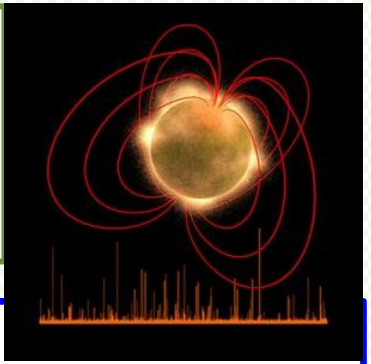
MNRAS **492,** L22 ('20)

Strong Magnetic Field Generating **Photon Vortex** (γ-Ray)

T.M, T. Hayakawa,. M.K.Cheoun, T.Kajino, PLB826, 136779 ('22)

Magnetar

- Strong Mag. Field $B \sim 10^{14-15}$ G (normal $B \sim 10^{12-13}$ G)
- Emitting High Energy γ **Soft Gamma Repeater (SGR)**



Photon Vortex carrying zTAM

⇒ **Different Multipole** of Giant Resonances, Zhi-Wei Lu et al., PRL 131, 202502 ('23)

Photon Vortex generation

⇒ Change of **Nuclear Reaction in Astronomical System?**

T.M., T. Hayakawa, M.K.Cheoun, T.Kajino, ApJ 975, 51 (2024)

nature COMMUNICATIONS

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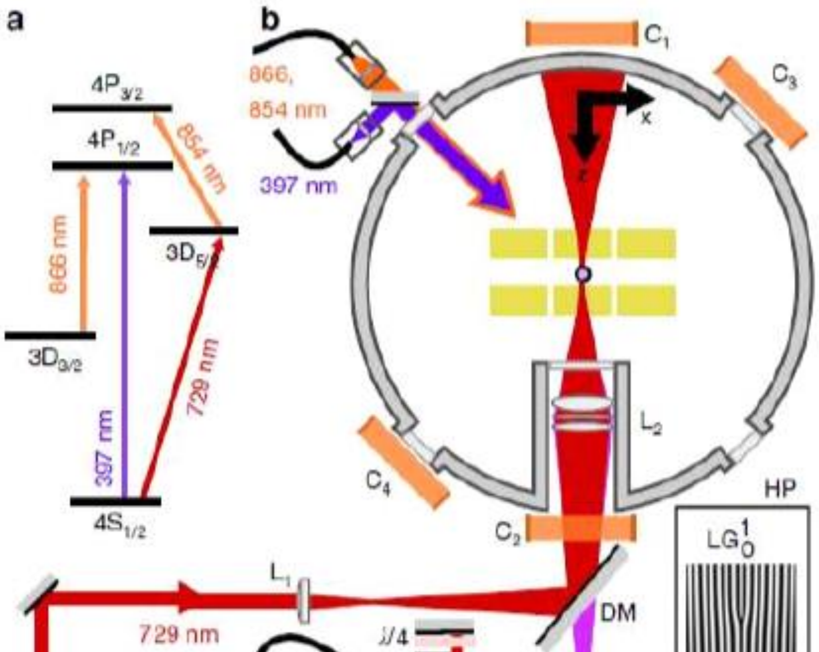
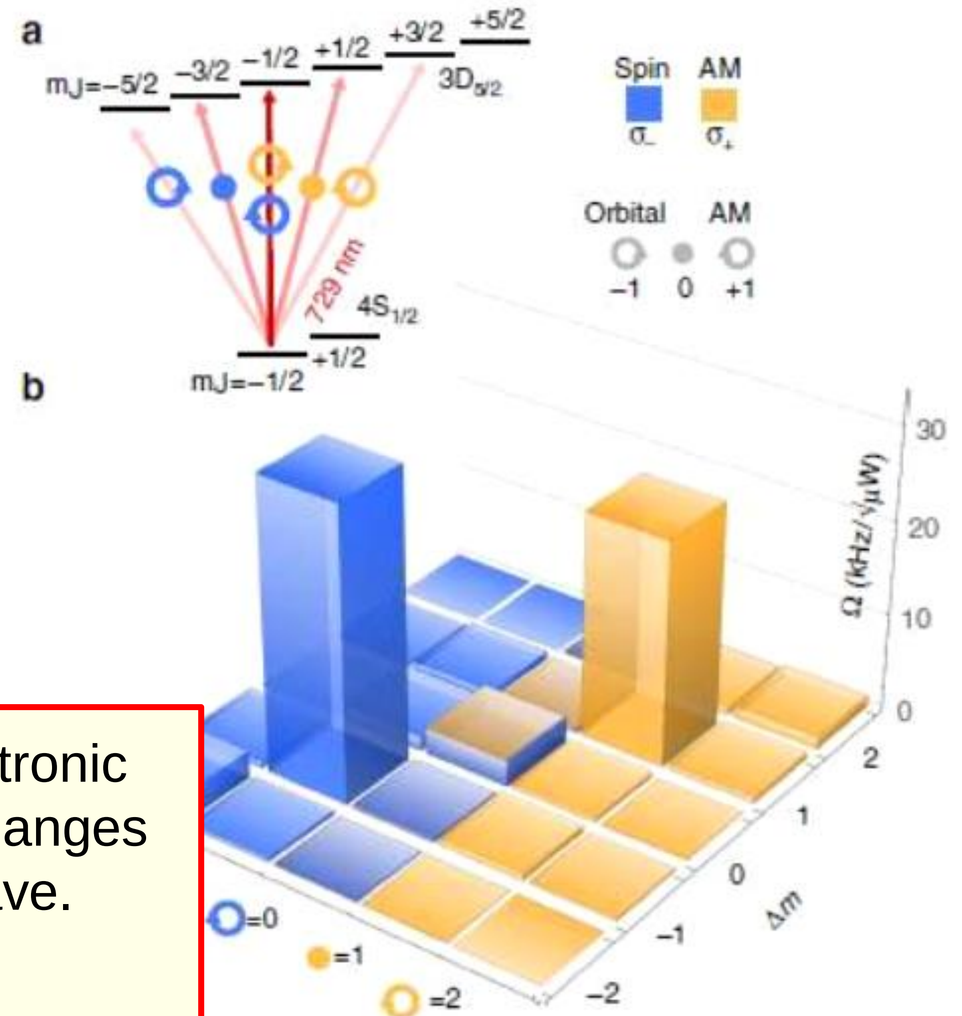
OPEN

Transfer of optical orbital angular momentum to a bound electron

Christian T. Schmiegelow[1,†], Jonas Schulz[1], Henning Kaufmann[1], Thomas Ruster[1], Ulrich G. Poschinger[1] & Ferdinand Schmidt-Kaler[1]



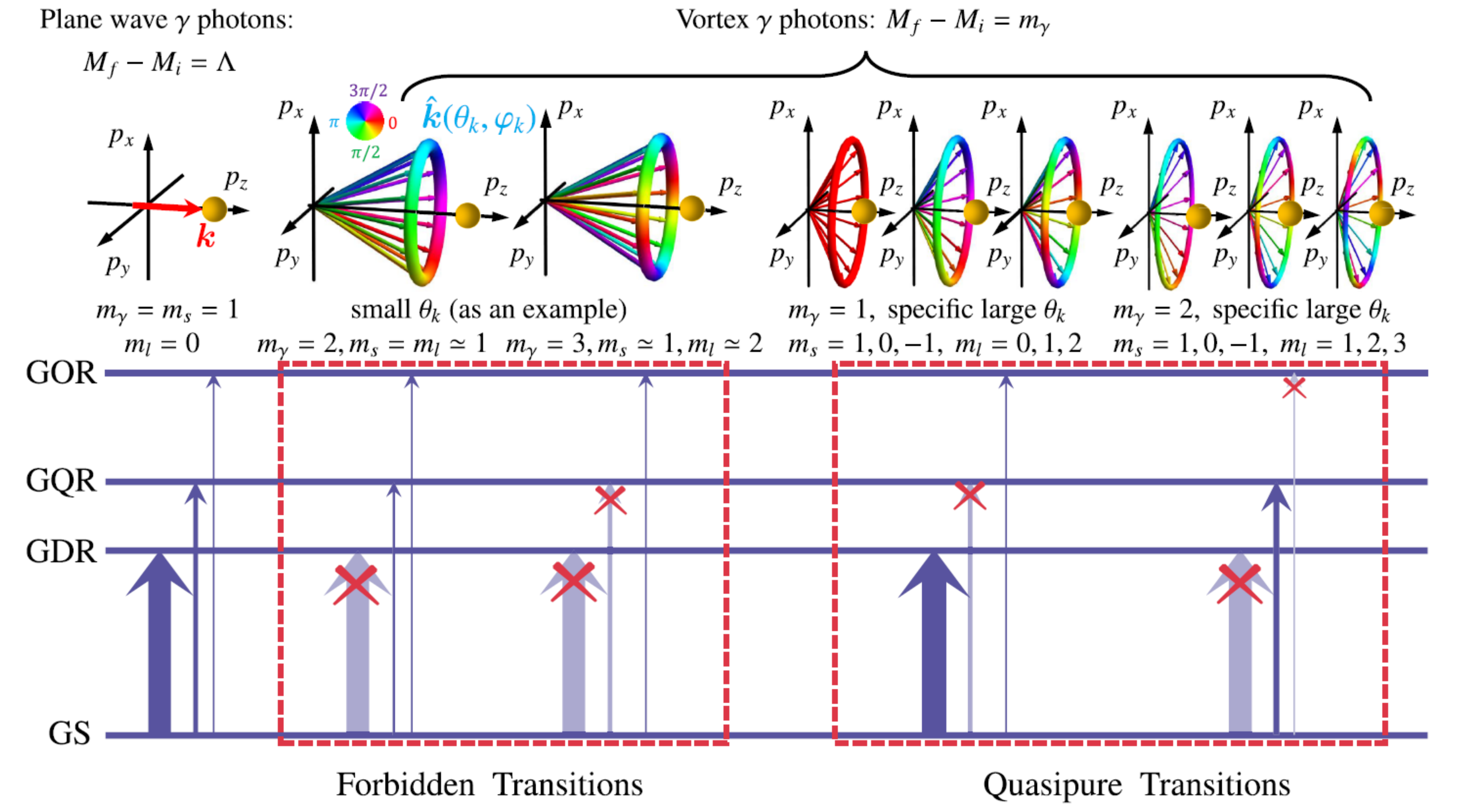


The selection rule for atomic electronic excitation by photon absorption changes when the photon is a vortex wave.

One Event

Giant Resonance States excited by Bessel Wave Photons

Zhi-Wei Lu, et al. Phys. Rev. Lett. 131, 202502 (2023)



Changing Selection Rule ⇒ Nuclear Synthesis (?)

§2　Photon Vortex Generation in Strong Magnetic Field

In Strong Magnetic Field,　$\boldsymbol{B} = B\hat{\boldsymbol{z}}$

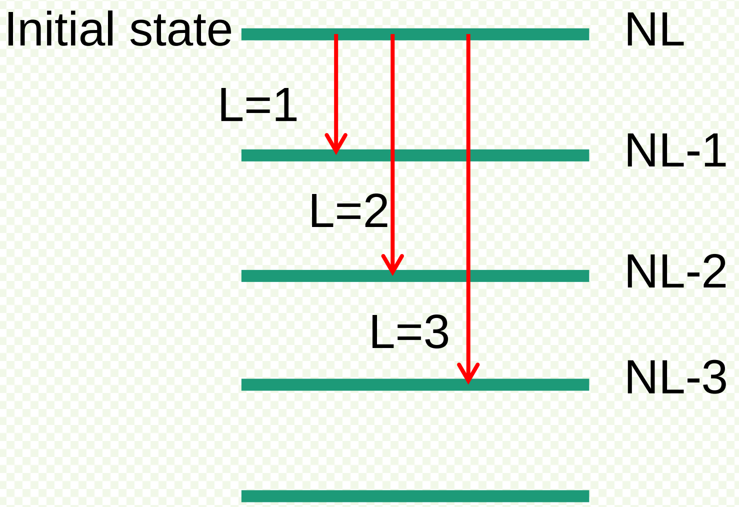
Electrons ⋯ Helical Motions → at Landau Level States

Eigen States of z-Component of Total Ang. Mom. and Momentum

zTAM　　　　p_z

Transition between different Landau Levels

→ producing One Photon : Eigen States of zTAM and $\boldsymbol{p_z}$



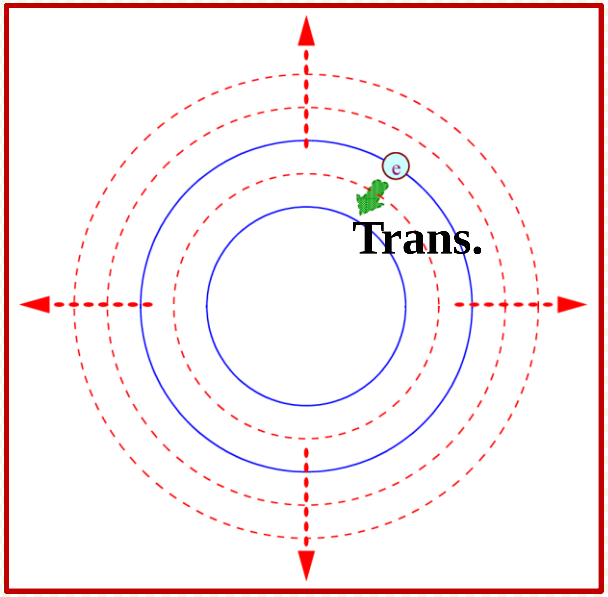
Photon Vortex (Bessel Wave)

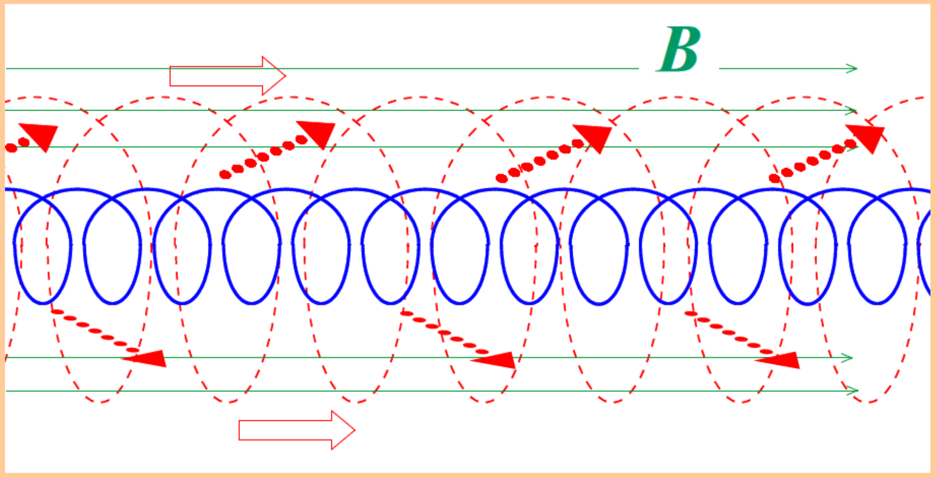
Synchrotron Radiation in Quantum Process

Electron in Strong Magnetic Field
are at Landau Levels
Eigen States of zOAM

High Speed Electron
Helical Motion along z-Direction
Eigen States of L_z & p_z // B

Mag. Fld. $\boldsymbol{B} = B\hat{\mathbf{z}}$





Transition of Electron
⇒ One-Photon Emission
Eigen States of OAM
Photon is ***Cylindrical Wave***

Lorentz Trans.

Emitted Photon:
Eigen States of : L_z、p_z
⇒ ***Bessel Wave***
(Vortex Wave)

§ 2-1 Bessel Wave

Gauge : $A_0=0,\ \boldsymbol{\nabla}\cdot\boldsymbol{A}=0$

z-Comp. of Momentum	q_z	Eigen State
zTAM	K	

$$\boldsymbol{A}(\boldsymbol{r}) \approx \boldsymbol{\epsilon}_h J_L(q_T r)e^{iL\phi}e^{ip_z z} \qquad \boldsymbol{\epsilon}_h = (1, \pm h)/\sqrt{2}\,, \quad K = L + h$$

$$\boldsymbol{A}_K^{(TM)} = \frac{1}{2e_q}e^{i(q_z z - e_q t)}\left[iq_z\left(\tilde{J}_{K+1} - \tilde{J}_{K-1}\right), q_z\left(\tilde{J}_{m+1} + \tilde{J}_{m-1}\right), 2q_T\tilde{J}_K\right],$$

$$\boldsymbol{A}_K^{(TE)} = \frac{1}{2}e^{i(q_z z - e_q t)}\left[i\left(\tilde{J}_{K+1} + \tilde{J}_{K-1}\right), \left(\tilde{J}_{K+1} - \tilde{J}_{K-1}\right), 0\right].$$

$$\tilde{J}_L(\boldsymbol{r}_T) = J_L(q_T r_T)e^{iL\phi}, \quad K = J_i - J_f, \quad e_q^2 = q_z^2 + q_T^2 = q_z^2 + q_T^2 + q_x^2, \quad e_q = E_i - E_f$$

K : z-Comp of Total Ang, Mom. (zTAM)

$|q_z| \gg q_T \Rightarrow$ Bessel Wave (BW) （zOAM: $K-h$, Spi: h ）

$|q_z| \ll q_T \Rightarrow$ **Cylindrical Wave** (zOAM: K , Polarized in z-Dir.)

Two Waves are connected in Lorentz Transformation

§ 2 -2 Electron Wave Function in Magnetic Field

Mag. Fld. $\boldsymbol{B} = (0, 0, B)$, $\boldsymbol{A} = \frac{B}{2}(-y, x\ \ 0)$ **Symmetry Gauge**

Wave Function

$(L \geq 0)$

$$\psi(\boldsymbol{r}) = \begin{bmatrix} G_n^{L-1}\left(\frac{\boldsymbol{r}_T}{\sqrt{2}}\right) e^{ip_z z} \\ G_n^{L}\left(\frac{\boldsymbol{r}_T}{\sqrt{2}}\right) e^{ip_z z} \end{bmatrix} , \quad \boldsymbol{r} = (\boldsymbol{r}_T, z) = (x, y, z),$$

$$G_n^L(\boldsymbol{r}_T) = \sqrt{\frac{n!}{\pi(n+|L|)!}} e^{iL\phi} r^{|L|} e^{-r_T^2/2} \mathcal{L}_p^{|L|}(r_T^2)$$

Associated Laguerre Function

2D HO W.F.

$$E = \sqrt{p_z^2 + 2eB\hbar^2\left(n + \frac{L+|L|)}{2}\right) + m_e^2 c^2} = \sqrt{p_z^2 + 2eBN_L + m_e^2 c^2}$$

L : z-comp of OAM (zOAM), ***n*** : Node number in *xy*-plane

Decay Width of Electron

$$\Gamma_e = \frac{e^2}{8\pi^2} \sum_{f,\alpha} \int \frac{dq_z dq_T q_T}{|\boldsymbol{q}|} \delta(E_i - E_f - |\boldsymbol{q}|) \left| \int d\boldsymbol{r} \overline{\psi}_f(\boldsymbol{r}) \not{A}_\alpha^{(\alpha)*}(\boldsymbol{r}) \psi_i(\boldsymbol{r}) \right|^2$$

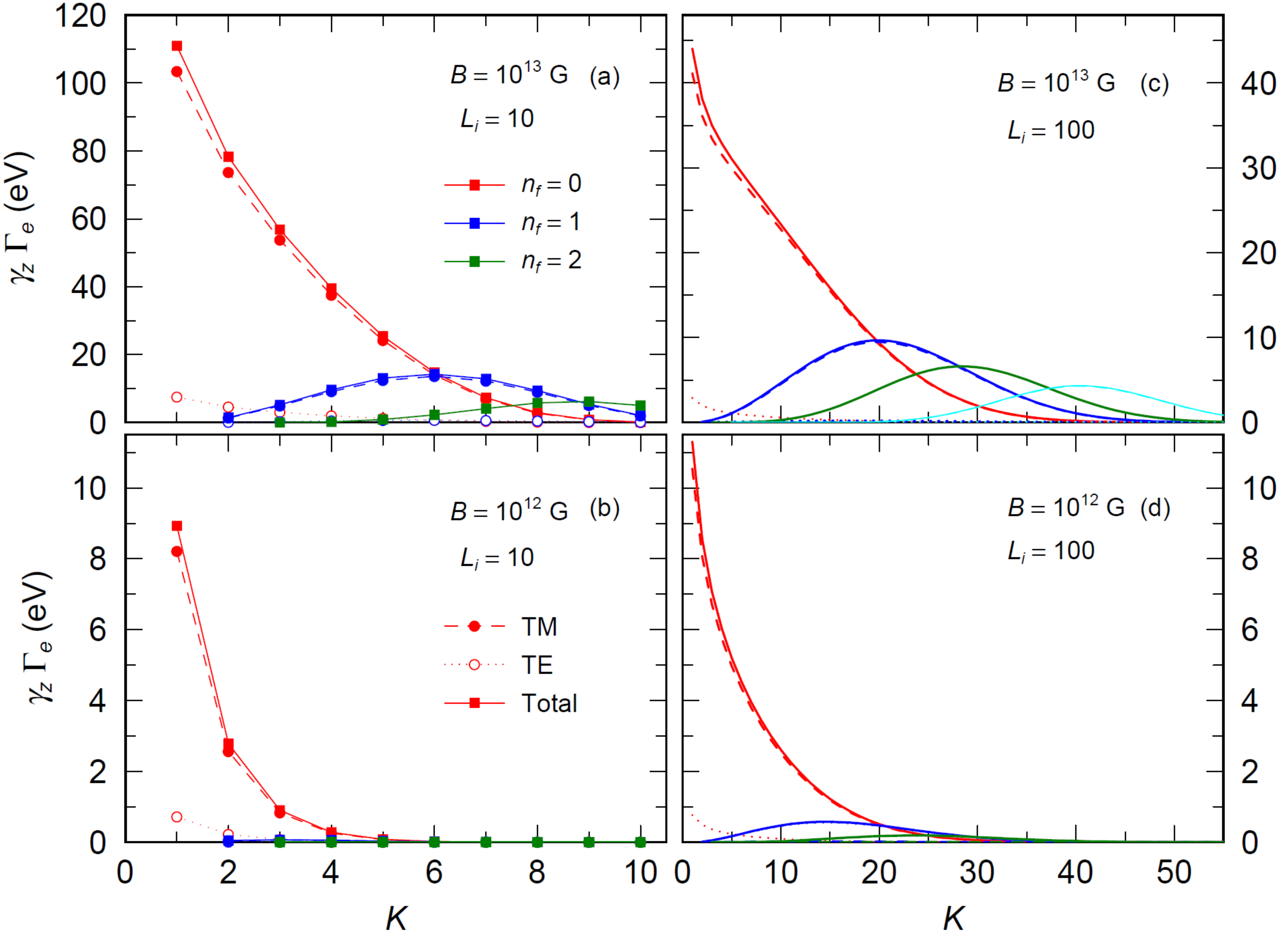
$$= \frac{\alpha_e}{2\pi} \sum_{f,\alpha} \int dq_z \left| \int d\boldsymbol{r} \overline{\psi}_f(\boldsymbol{r}) \not{A}^{(\alpha)*}(\boldsymbol{r}) \psi_i(\boldsymbol{r}) \right|^2$$

⇒ Emission Probability

§ 2-3 Results

T.M, et al., PLB826, 136779 (22)

Decay Width ∝ Transition Prob



***K*=*1* : Largest**

***K* ≥ 2 (PhV)**

not Small

TM mode: dominant

***n_f* ≥ 1**

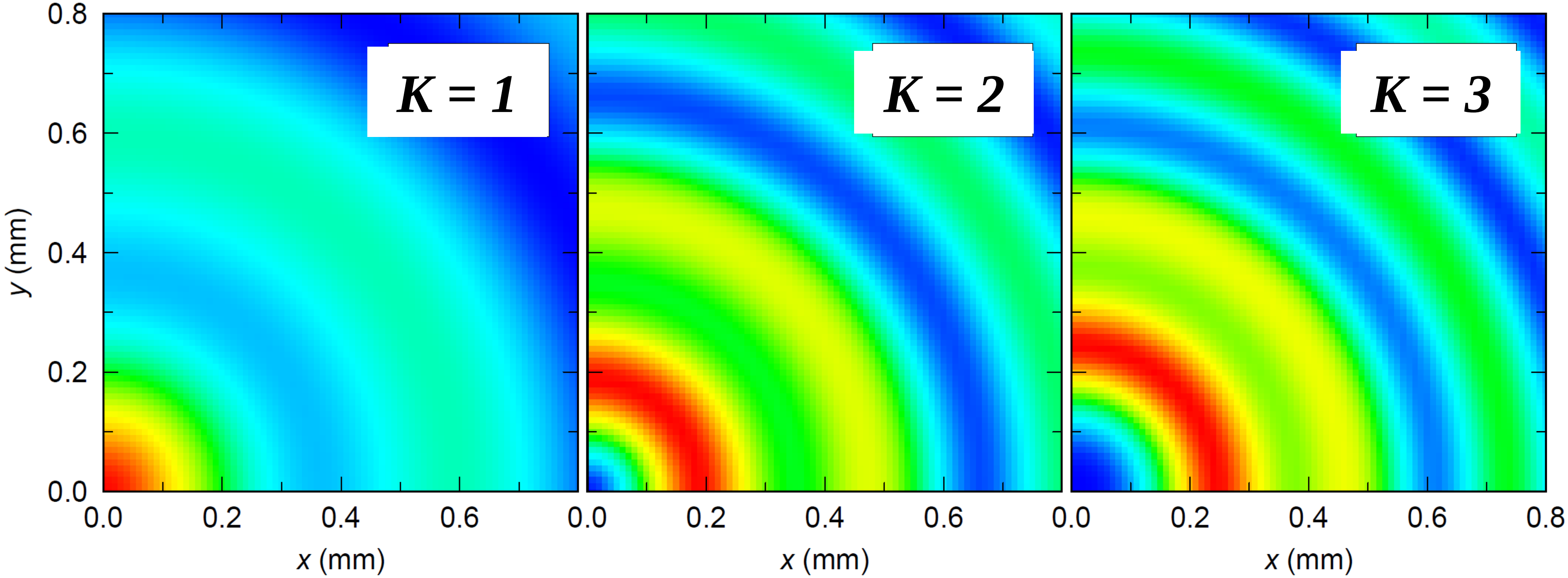
Small except Large K

$K = J_i - J_f$: zTAM of Photon $K = 1$: Fundamental,

(z-Comp of Total Ang. Mom.)   ≥ 2 : **Harmonic ⇒ Photon Vortex**

Areal Photon Density

$$\rho_\gamma(r_T) = N_\gamma^{(TE)} \left|A^{(TE)}\right|^2 + N_\gamma^{(TM)} \left|A^{(TM)}\right|^2 .$$



Photon vortex generation by synchrotron radiation can be done in the laboratory

T.M. et al., PRR 5, 043289 (2023)

$B = 10^6$ G, $L_i \sim 10^6$ ($R_L \sim 10$ μm) ⟶ Photon Energy ~ 1 − 100 eV

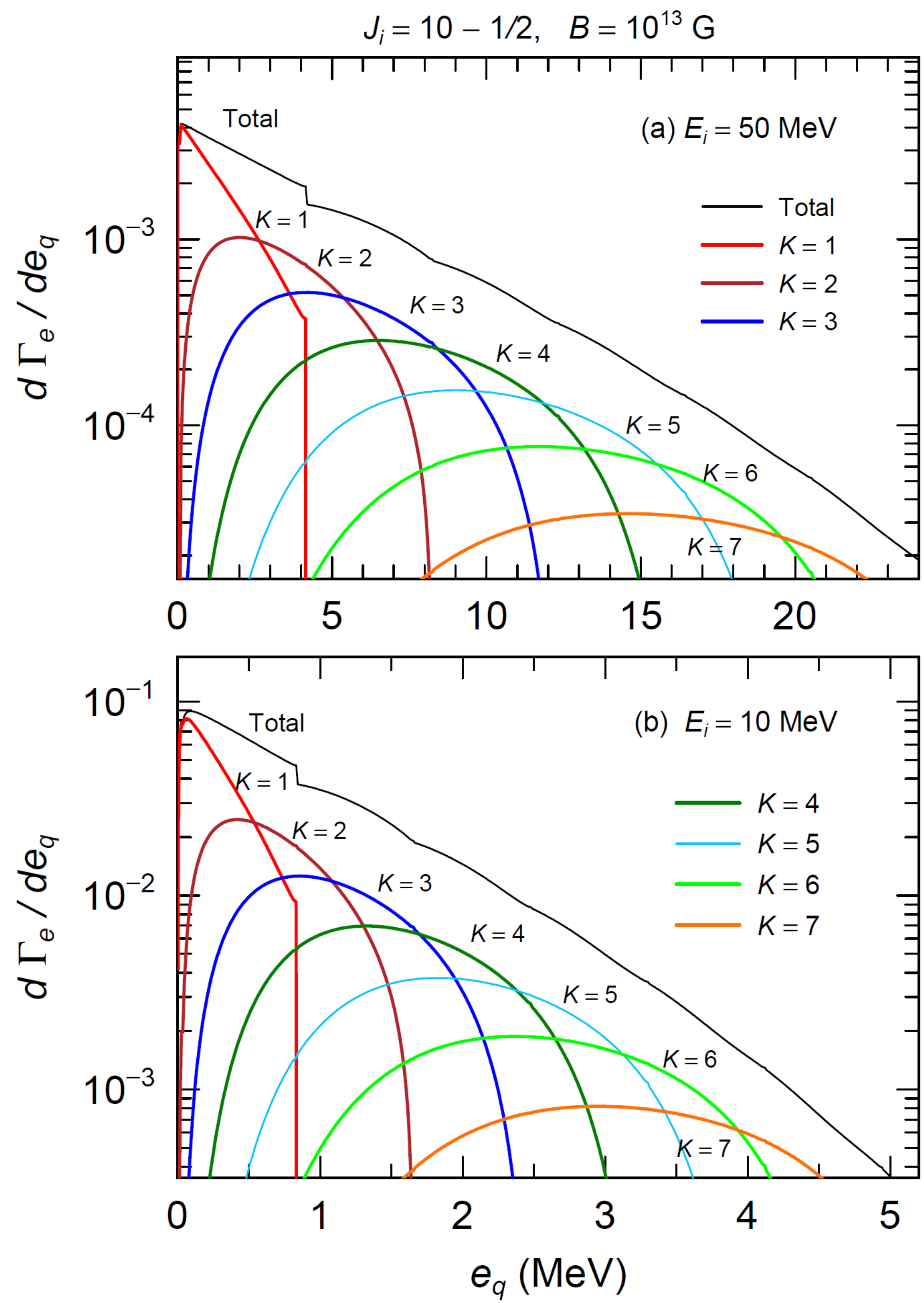
Photon Energy Spectrum

$$\frac{d\Gamma_e}{dq_z} = \frac{\alpha_e}{2\pi}\left|\int d\boldsymbol{r}\overline{\psi}_f(\boldsymbol{r})\not{A}^{(\alpha)*}(\boldsymbol{r})\psi_i(\boldsymbol{r})\right|^2$$

K = 1: Fundamental Largest
(L_z = 0 and 2 are Mixed)

$K \geq 2$: Harmonic **not Small**
(Photon Vortex)

As zTAM increases
⇓
Photon Energy increases



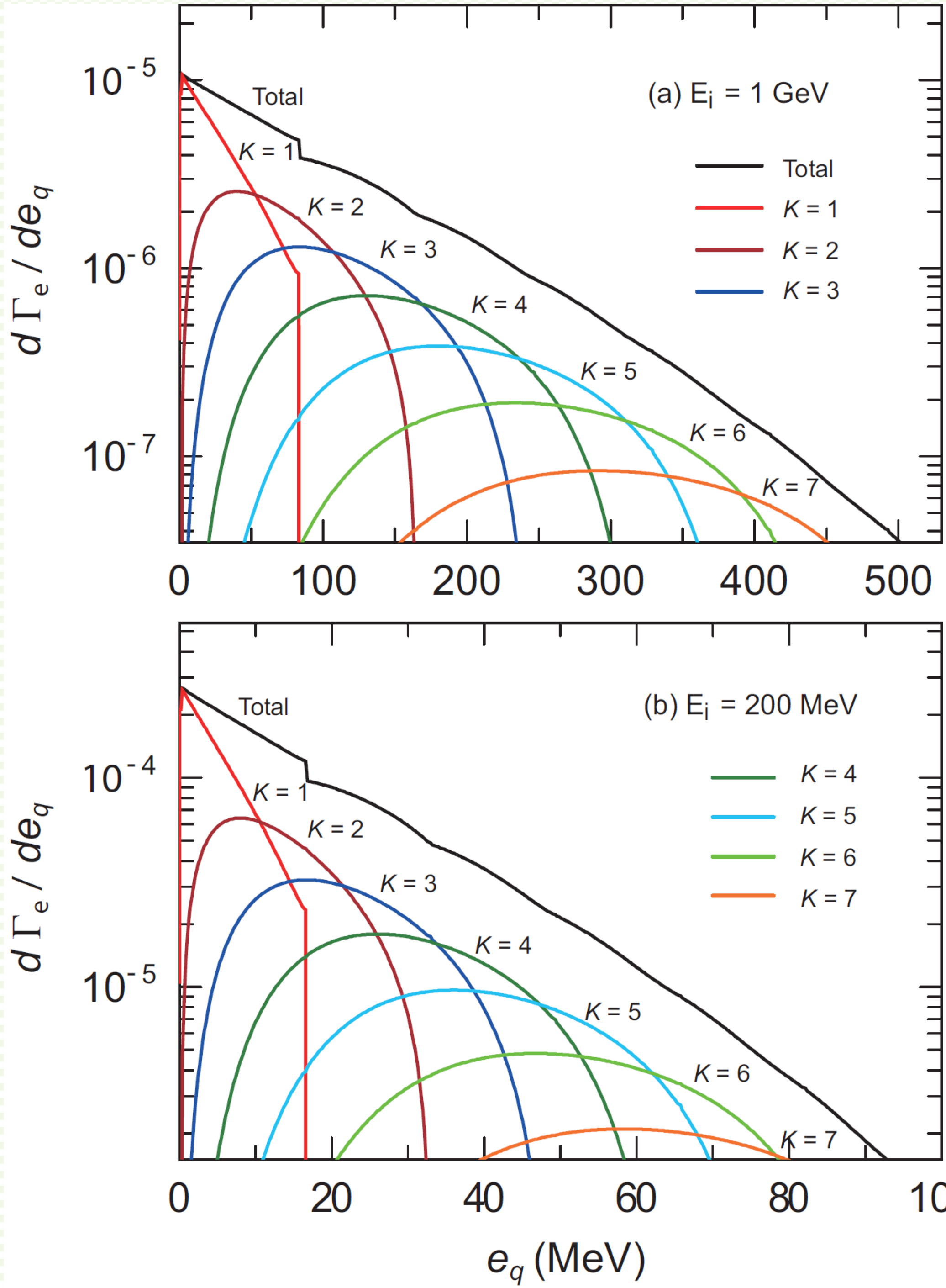
Photon Energy Spectrum

$$\frac{d\Gamma_e}{dq_z} = \frac{\alpha_e}{2\pi} \left| \int d\boldsymbol{r} \overline{\psi}_f(\boldsymbol{r}) \not{A}^{(\alpha)*}(\boldsymbol{r}) \psi_i(\boldsymbol{r}) \right|^2$$

As zTAM increases

⇓

Photon Energy increases



§ 3 Nuclear Photoreaction with Photon Vortex

Photon Vortex carrying zTAM

Its Interaction with Matter is different from that of Plane Wave Photon

Change of **Selection Rule** for **H-atom** A. Afanasev et al., PRA 88, 033841 (3)

$n + p \rightarrow \gamma + d, \quad \gamma + d \rightarrow n + p$ J. Phys. G **45** 055102

Different Multipole of Giant Resonances, Zhi-Wei Lu et al., PRL 131, 202502 (23)

Photon Vortex in Super Novae

Photo-Absorption Reaction : Selection Rule is changed (?)

zTAM ($J_z \geq 2$) + OAM ($\perp$ Beam Dir.) = Total AM ($J \geq 2$)

***E1* Transition does not occur?** Influencing to Nuclear Synthesis？

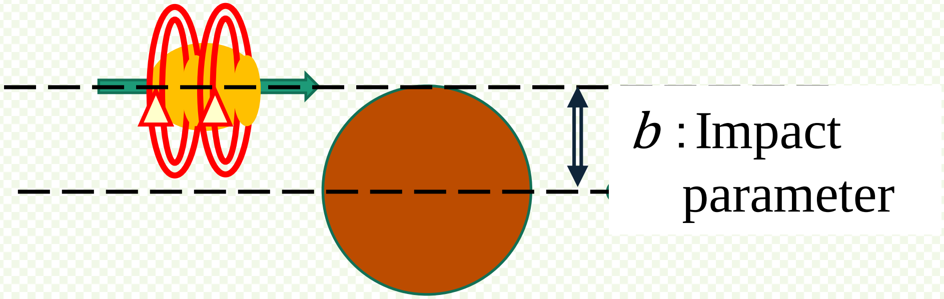
Y. Taira,at al., Sci. Rep. 7, 5018 (2017).

In Previous Works, $b = 0$,

or Small b

In Nature, No Restriction

⇒ Integrating Results Over b



§ 3-1 Transition Strength with Plane Wave (PW)

$$\boldsymbol{e}_h e^{ie_q z} = \sum_{J=1}^{\infty} \sqrt{2\pi(2J+1)}(i)^J \left[h\boldsymbol{T}^{mag}_{Jh} + \boldsymbol{T}^{el}_{Jh}\right]$$

$$\boldsymbol{e}_h = -h(1, ih, 0)/\sqrt{2},$$

$$\boldsymbol{T}^{mag}_{JM} = j_J(e_q r)\mathcal{Y}^h_{JJ1}(\Omega_r)$$

$$\boldsymbol{T}^{el}_{JM} = -\frac{i}{e_q}\boldsymbol{\nabla} \times \left[j_J(e_q r)\mathcal{Y}^h_{JJ1}(\Omega_r)\right]$$

Trans. Amp.

$$\hat{T}^{\kappa}_{JM} \equiv \int d^3r \hat{\boldsymbol{J}} \cdot \boldsymbol{T}^{\kappa}_{JM},$$

Current Op.

$$T^{\kappa}_{JM} \equiv < J, M, \kappa|\hat{T}^{\kappa}_{JM}|0 >$$

Excited States with (J, M)

Parity : κ

$$\boldsymbol{T}^{\kappa}_{JM} = h\boldsymbol{T}^{mag}_{JM} \quad (\kappa = (-)^J), \qquad \boldsymbol{T}^{\kappa}_{JM} = \boldsymbol{T}^{el}_{JM} \quad (\kappa = (-)^{J+1}).$$

$$T^{\kappa}_{JM} =< 00JM|JM >< J, \kappa|| \int d^3r \hat{\boldsymbol{J}} \cdot \boldsymbol{T}^{\kappa}_{JM}||0 >=< J, \kappa||\hat{T}^{\kappa}_{JM}||0 >$$

Transition Probability

$$\mathcal{P}^{(0)}_{J\kappa} = \left|< J, h, \kappa| \int d^3r \left(\hat{\boldsymbol{J}} \cdot \boldsymbol{e}_h\right) e^{ie_q z}|0, 0, + >\right|^2 = 2\pi\|T^{\kappa}_{Jh}\|^2$$

Only for $M = h = \pm 1$ (helicity)

Calculation in Nuclei

§ 3-2 Transition Strength with Bessel Wave

Photon Wave Func. Bessel Wave

$$\boldsymbol{A}_K^h(\boldsymbol{r}) = e^{iq_z z}\left[\frac{i(e_q + hq_z)}{2e_q}\tilde{J}_{K-1}\boldsymbol{e}_+ + \frac{i(e_q - hq_z)}{2e_q}\tilde{J}_{K+1}\boldsymbol{e}_{-1} + \frac{hq_T}{\sqrt{2}e_q}\tilde{J}_K\boldsymbol{e}_0\right]$$

$$\tilde{J}_M(q_T\boldsymbol{r}_T) = J_M(q_T r_T)e^{iM\phi}$$

Fourier Transformation

$$\boldsymbol{A}_K^h(\boldsymbol{p}) = \int d^3r e^{-i\mathbf{p}\cdot\mathbf{r}}\boldsymbol{A}_K^h(\boldsymbol{r}) = \frac{(2\pi)^2}{q_T}\delta(p_z - q_z)\delta(p_T - q_T)(-i)^{K-h}e^{i(K-h)\phi_p}\boldsymbol{e}(\boldsymbol{p}, h)$$

$$\boldsymbol{e}(\boldsymbol{p}, h) = \frac{(1 + h\cos\theta_p)}{2}e^{-i\phi_p}\boldsymbol{e}_+ + \frac{(1 - h\cos\theta_p)}{2}e^{i\phi_p}\boldsymbol{e}_- + \frac{h\sin\theta_q}{\sqrt{2}}\boldsymbol{e}_0,$$

Polarization Vector $\perp \boldsymbol{p}$

Multipole Expansion

$$\boldsymbol{e}(\boldsymbol{p}, h)e^{i\mathbf{q}\cdot\mathbf{r}} = \sum_{JM,\kappa}\sqrt{2\pi(2J+1)}(i)^J\mathcal{D}^J_{M,h}(\phi_p, \theta_p, 0)\boldsymbol{T}^\kappa_{JM},$$

Wigner D-Function

$$\mathcal{D}^J_{M,h}(\phi_p, \theta_p, 0) = \exp(-iM\phi_p)d_{Mh}(\theta_p)$$

Excitation to Various $\boldsymbol{M}$

Wigner d-Matrix

Transition Strength with Bessel Wave 2

Shifting Central Axis of BW with b

$$\mathcal{A}^{Kh}_{JM\kappa}(b) = < J, M, \kappa| \int d^3r \hat{\boldsymbol{J}}(\boldsymbol{r}) \cdot \boldsymbol{A}^h_K(\boldsymbol{r}-\boldsymbol{b})|00 >$$
$$= \int \frac{d^3p}{(2\pi)^3} e^{-i\mathbf{p}\cdot\mathbf{b}} \frac{(2\pi)^2}{q_T} \delta(p_z - q_z)\delta(p_T - q_T)(i)^{J-K+h} e^{i(K-h-M)\phi_p}$$
$$\times \sqrt{2\pi(2J+1)} d^J_{M,h}(\theta_p) \| T^\kappa_J \|.$$

Transition Probability at fixed b

$$P^{Kh}_{JM\kappa}(b) = \left| \left\langle J,\, M,\, \kappa \right| \int d^3r \hat{\boldsymbol{J}}_{\rm nuc}(\boldsymbol{r}) \cdot \boldsymbol{A}^h_K(\boldsymbol{r}-\boldsymbol{b}) \left|0,\, 0,\, +\right\rangle \right|^2$$
$$= 2\pi\, |d^J_{M,h}(\theta_q)|^2 \,[J_{M-K}(q_T b)]^2 \,\|T^\kappa_J\|^2.$$

Ratio between PW and BW

$$R_K(b) = \sum_{M=-J}^{J} \frac{P^{Kh}_{JM\kappa}(b)}{\mathcal{P}^{(0)}_{J\kappa}} = \sum_{M=-J}^{J} |d^J_{M,h}(\theta_q)|^2 \,[J_{M-K}(q_T b)]^2.$$

Actual Transition Calculation is not Needed

Transition Strength with Bessel Wave 3

Integrating over Impact Parameters

$$\mathcal{P}^{Kh}_{JM\kappa} = \frac{1}{S_T} \int d\boldsymbol{b} \left| \int d^3 r < J, M, \kappa | \hat{\boldsymbol{J}}(\boldsymbol{r}) \cdot \boldsymbol{A}^h_K(\boldsymbol{r} - \boldsymbol{b}) | 0 > \right|^2$$

S_T: Cross-Section in System

$$S_T = \frac{2\pi}{q_T} \delta(p_T - q_T)$$

$$\mathcal{P}^{Kh}_{JM\kappa} = 2\pi \left| d^J_{M,h}(\theta_q) \right|^2 \| T^\kappa_J \|^2.$$

$$\int dr\, r J_n(qr) J_n(pr) = \frac{1}{q} \varepsilon(p - q)$$

$$\mathcal{P}^K_{J\kappa} = \sum_M \mathcal{P}^{Kh}_{JM\kappa} = 2\pi \| T^\kappa_J \|^2.$$

Same as that in PW

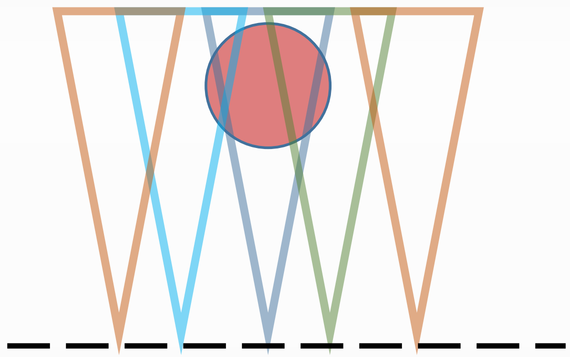
No BW Effect for Total Excitation Probability

Transition by BW Photons

Trans. Prob. : Averaging Over Impact Parameter (IP)

⇒ Same Results of PW

In Nature Selection Rule is not Changed



Changing Selection Rule is Observed for Atom（1event）

Restriction for Impact Parameter in Laboratory?

Experimental Projects of Gamma-ray Vortex GenrationCan

Can we get New Information by restricting IP in Experiments?

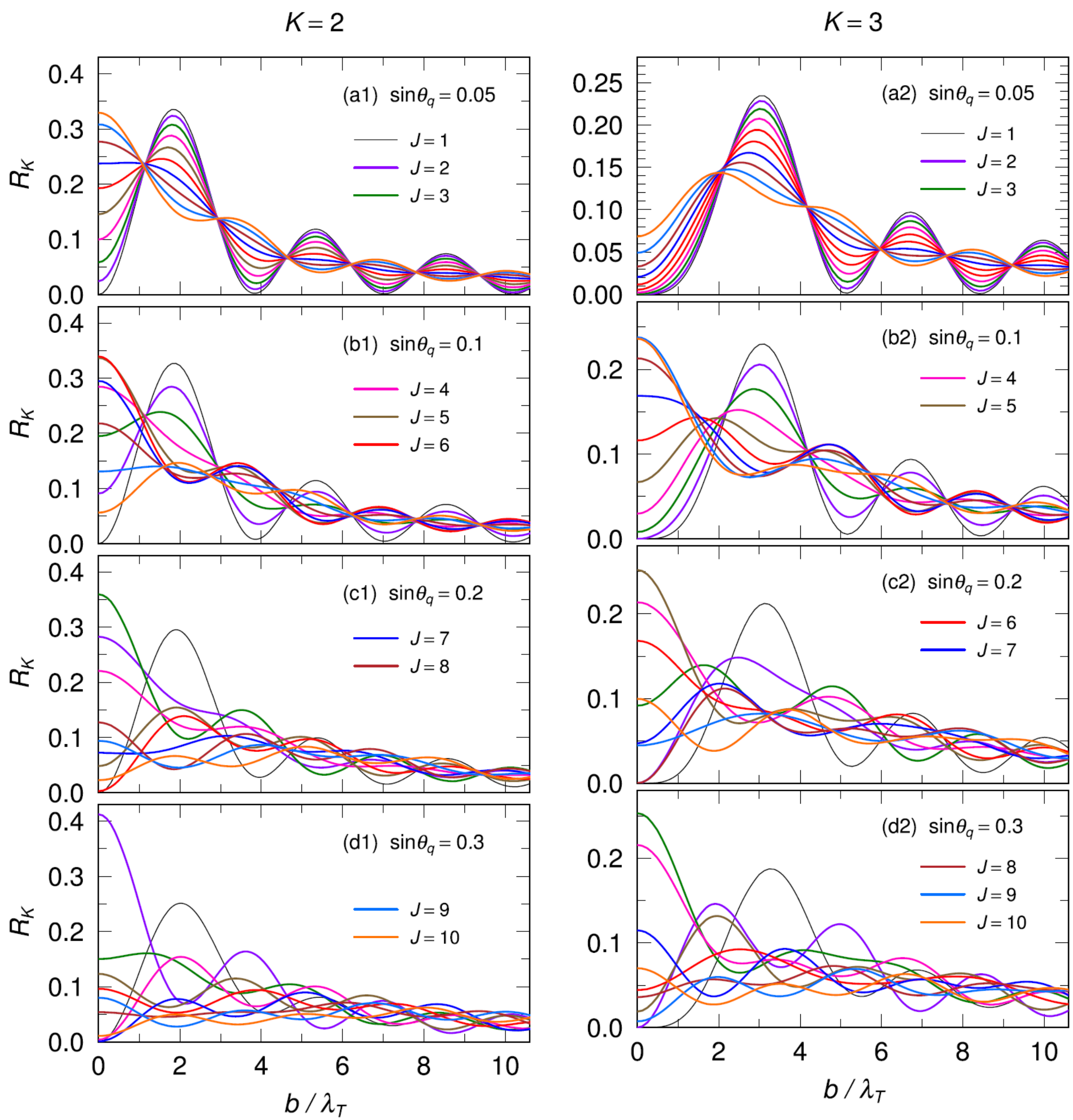
Impact Parameter (IP) Dependence

$$\frac{\text{Exc. Prob. In BW}}{\text{Exc. Prob. In PW}}$$

$$\sin\theta_q = q_T/|\boldsymbol{q}|$$

This value is fixed for BW

In small b, Contrs. from $J < K$ are small

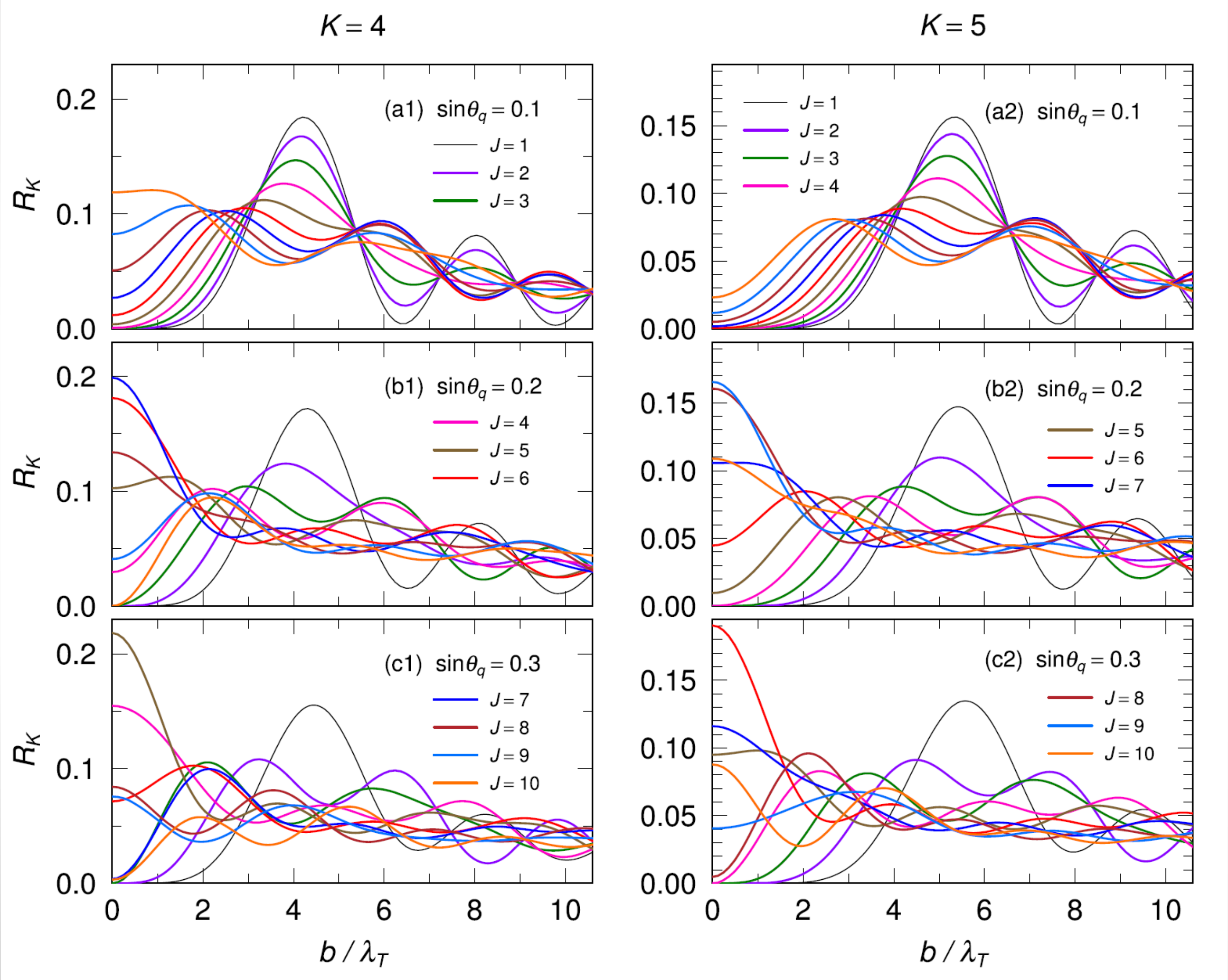


Impact Parameter (IP) Dependence

Orderly
for small θ_q

Chaos
for large θ_q

Special
Regularities are
Hard to see.



§ 4 Summary

Electron in Strong Mag. Fld. is in Eigen State of a **Landau Level**
Eigen State of zTAM

Trans. Between two Landau Levels → 1-Photon Emission
→ **Bessel Wave** (γ-Ray Vortex with $L_z \geq 1$)

Harmonic Photons with zTAM $K \geq 2$ ($L_z \geq 1$) are Emitted
to Direction of Magnetic Field (Arctic or Antarctic)
in Different Energy Region

Synchrotron Radiation
Linear Polarization in the Dir. perpendicular to Mag. Fld.
Circular Polarization to the Dir. parallel to Mag. Fld.
+ **Vortex Wave** in Strong Magnetic Field

This phenomena can be examined in Laboratory

T.M. et al., Phys. Rev. Res. **5**, 043289 (2023)

Nuclear Photoreaction with Bessel Wave

In Nature, Selection Rule is Not Changed

IP cannot be controlled → Same in PW

BW → Various J_z, PW → Only $J_z = \pm 1$

Different Angular Distribution for Emitted Particles

Changing Selection Rule is observed for Atom with LG Wave, whose intensity distribution is concentrated around the symmetry axis.

In Laboratory, BW cannot be created.

Controlling Width of LG Wave ↔ Controlling IP ?

Contributions from $(J \geq K)$ are Large

We expect to find a method to observe high AM excitations that are difficult to observe with PW.

Thank You!