Toward Understanding Dense QCD Matter

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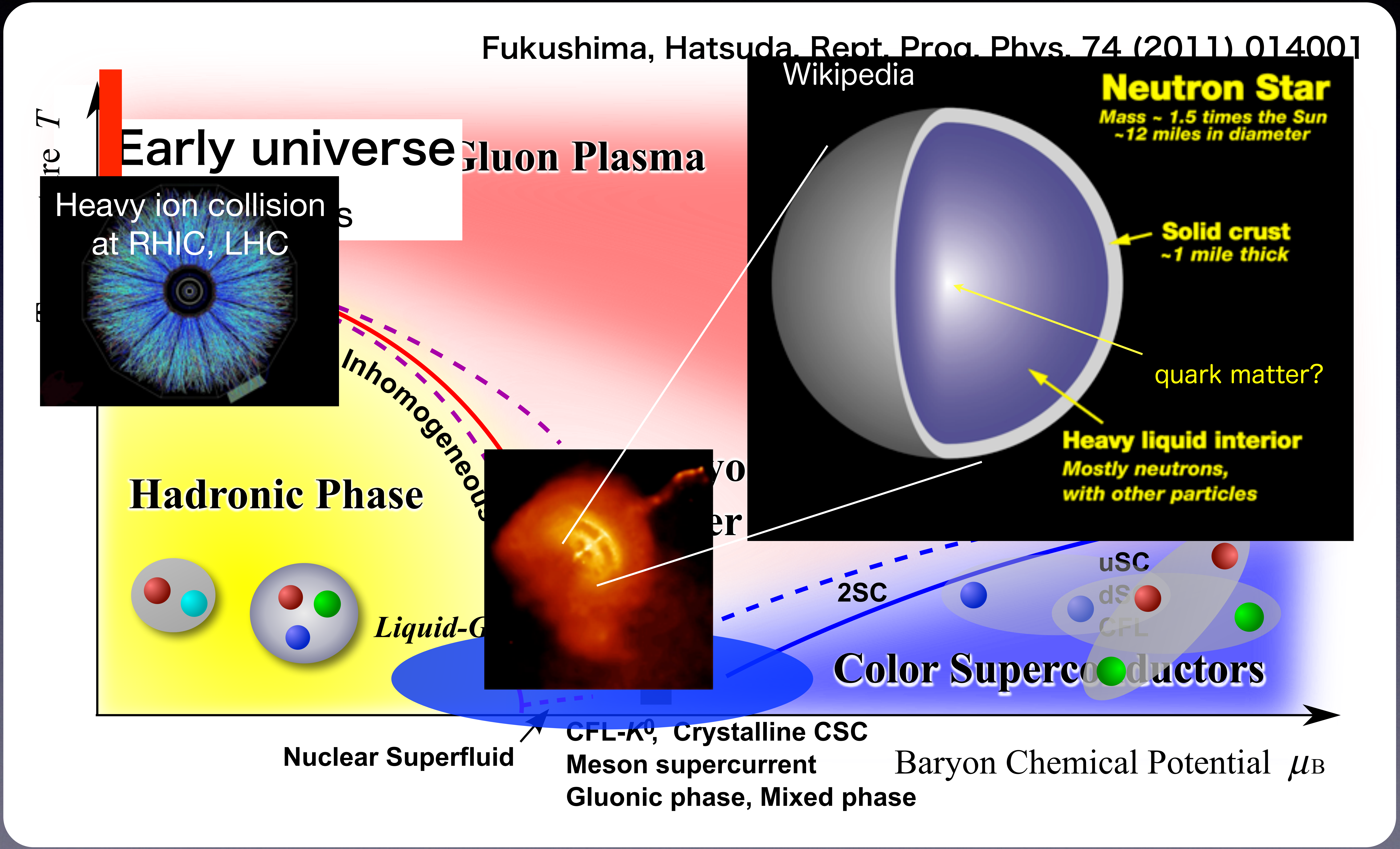
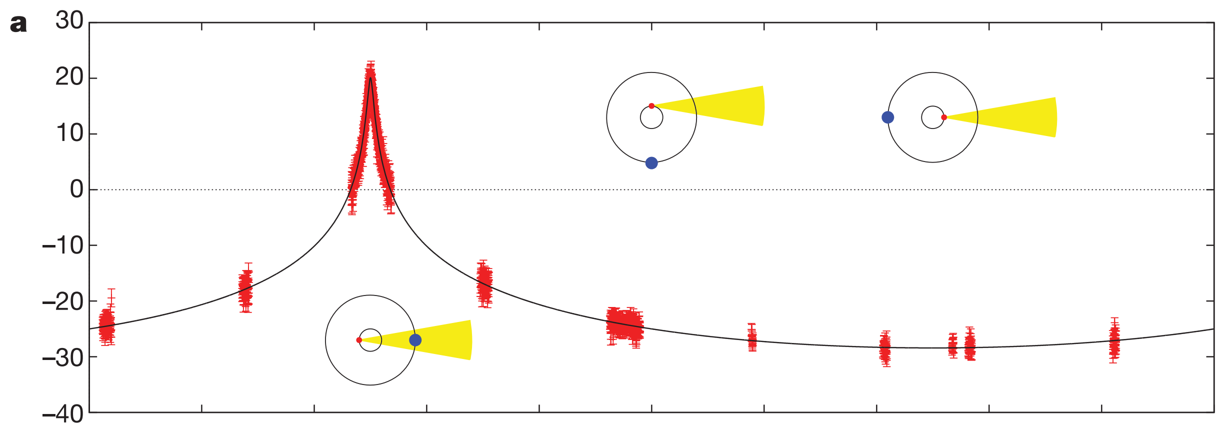
QCD phase diagram



Neutron Star Physics: Diverse Observations

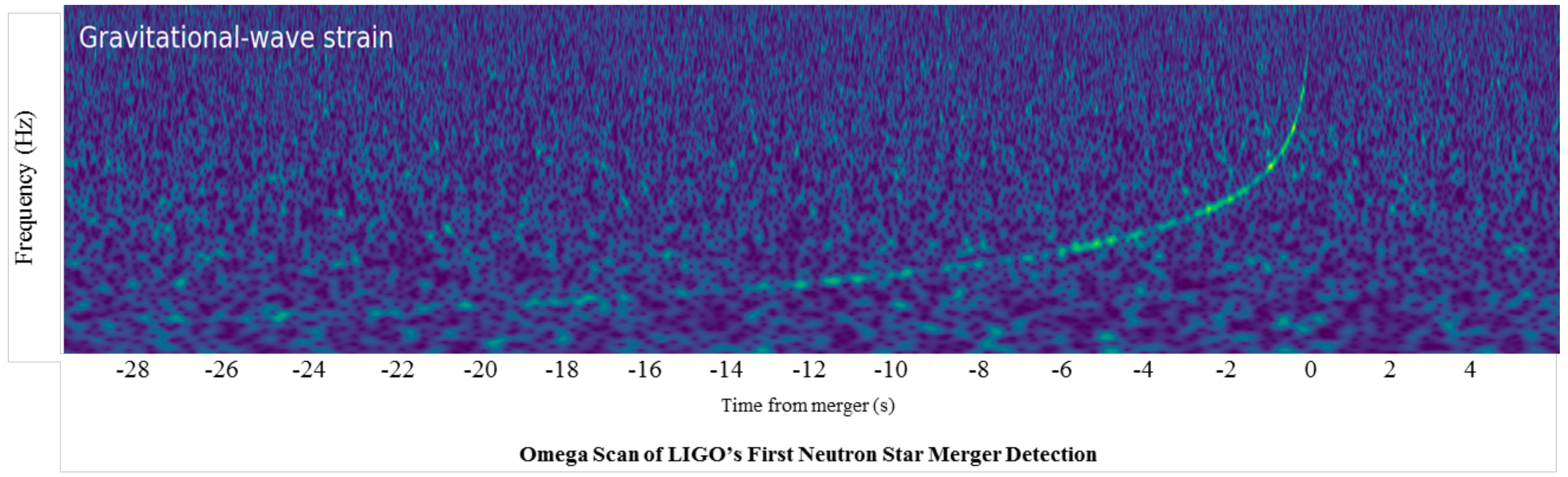
2010 Shapiro delay



Demorest et al, Nature 467 (2010) 1081

Two solar mass neutron stars

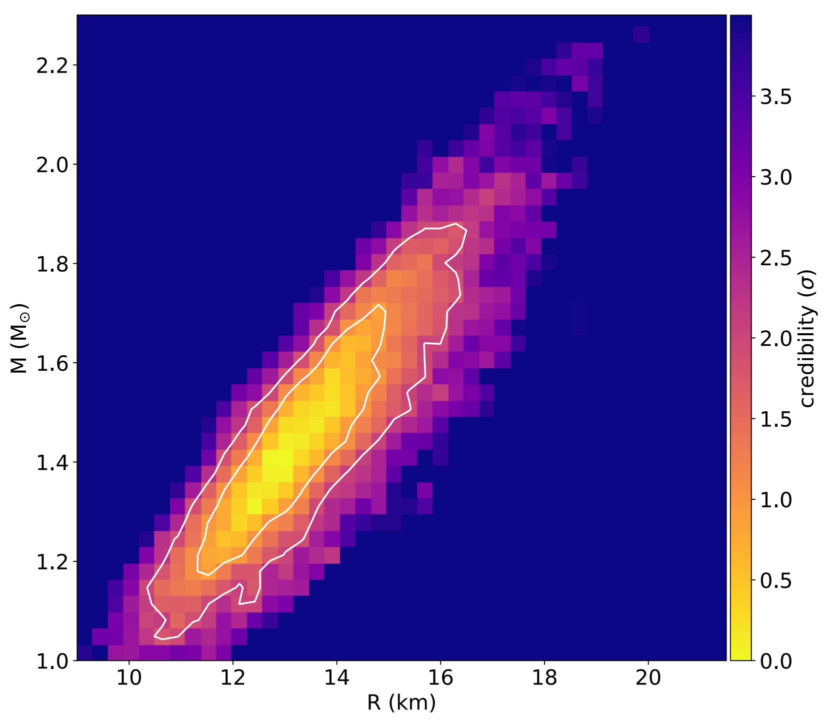
2017 LIGO / Virgo



Observations from Neutron Star Mergers

2019 NICER

Miller et al., APJ 887 (2019) L24



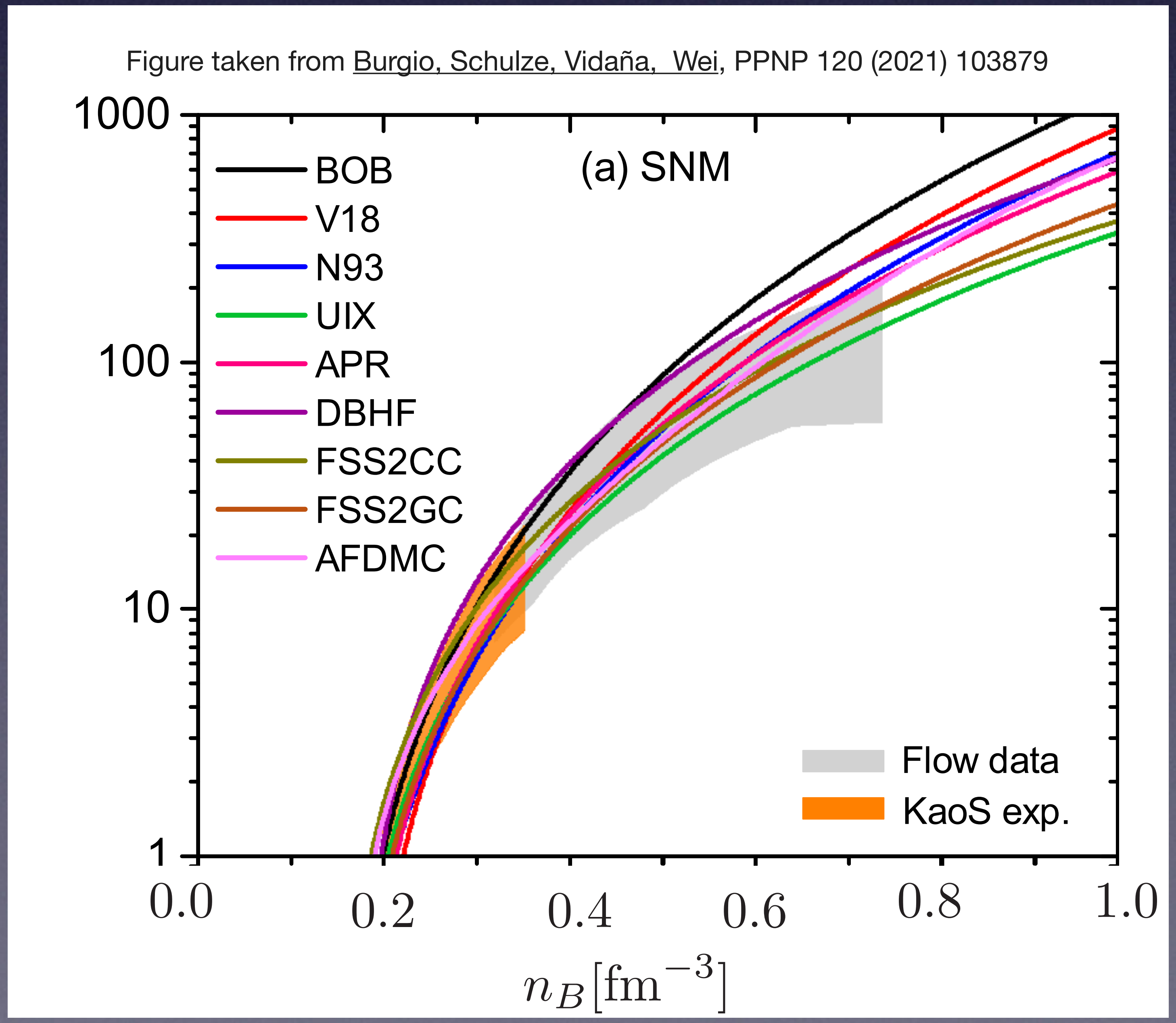
millisecond passer

Constraint of EOS from Observations

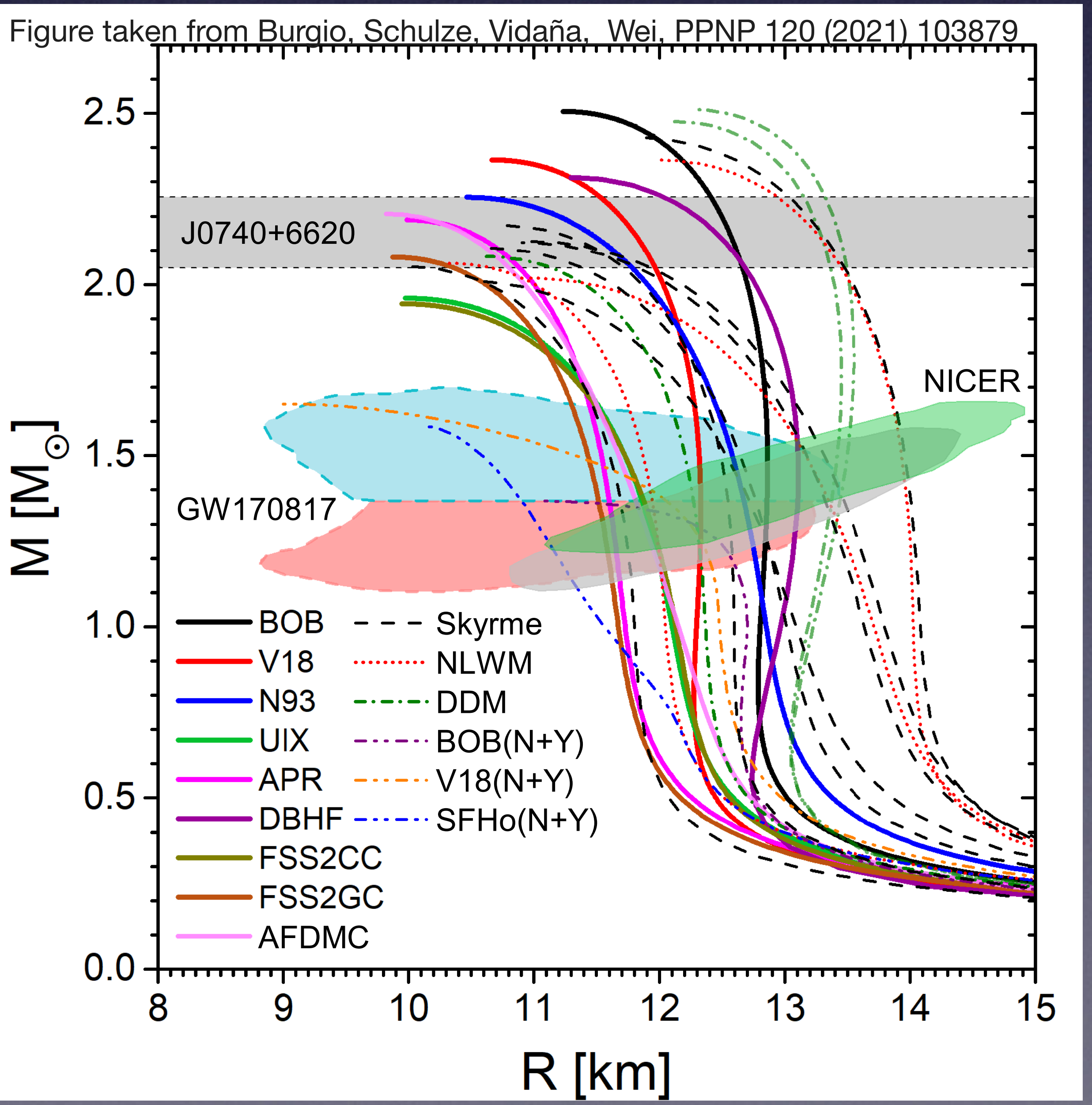
Low density: Nuclear Phys.

High density: pQCD

What is the Intermediate Density Regime?



TOV equations



Plan of this talk

- Basics of QCD
- Vacuum property
- QCD finite temperature
- QCD finite density
- Development of numerical simulations
- Summary

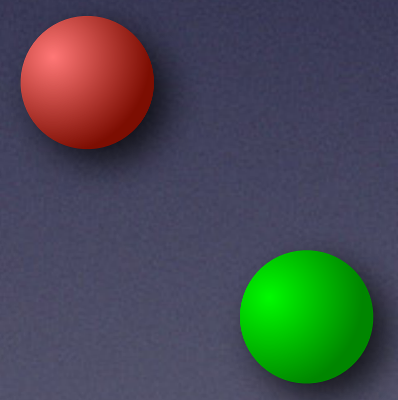
Quantum ChromoDynamics

SU(3) gauge theory

$$\mathscr{L} = \sum_q \left(\bar{q}_R D q_R + \bar{q}_L D q_L - m_q(\bar{q}_R q_L + \bar{q}_L q_R) \right) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

where $D = \gamma^\mu (i\partial_\mu + gA_\mu)$

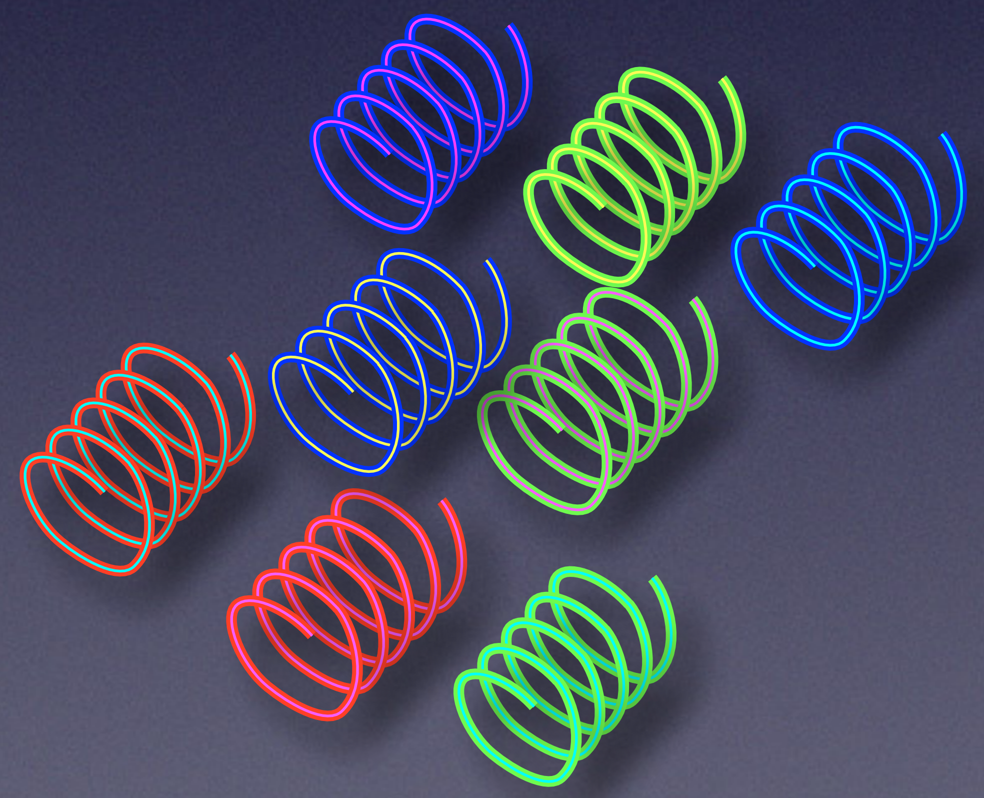
quarks



massive, spin *1/2*

Representation: *3* (R,G,B)

gluons



massless, spin *1*

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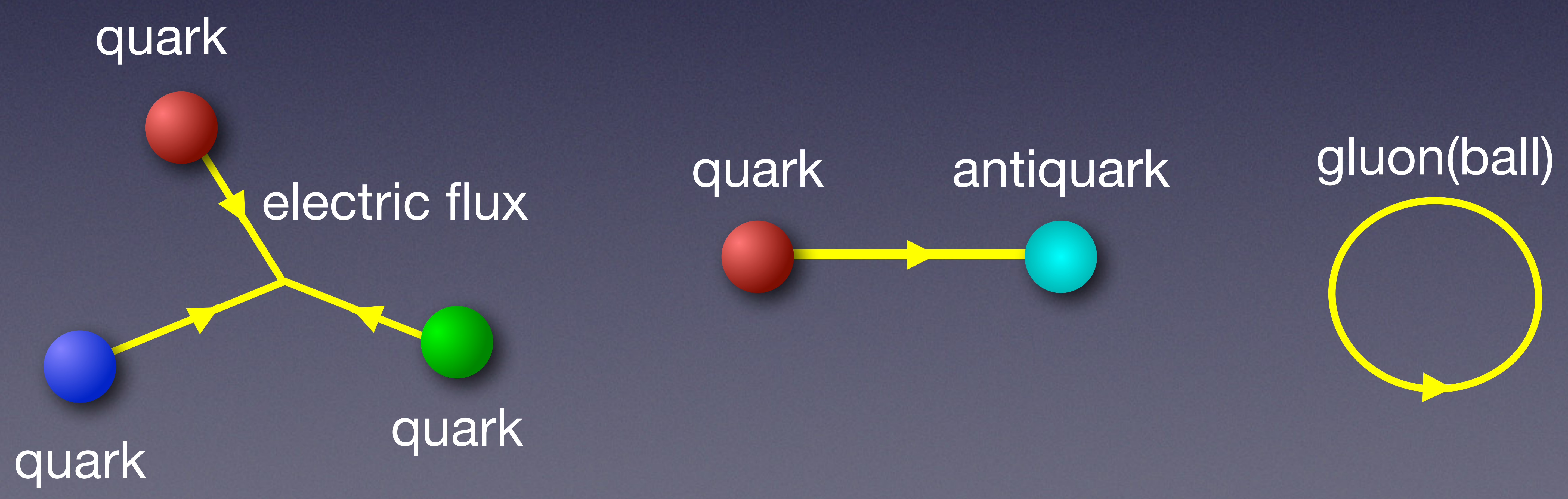
Property of gauge theory

Gauge symmetry:

Color RGB is not observable

Gauge symmetry cannot be spontaneously broken

Gauge invariant degrees of freedom



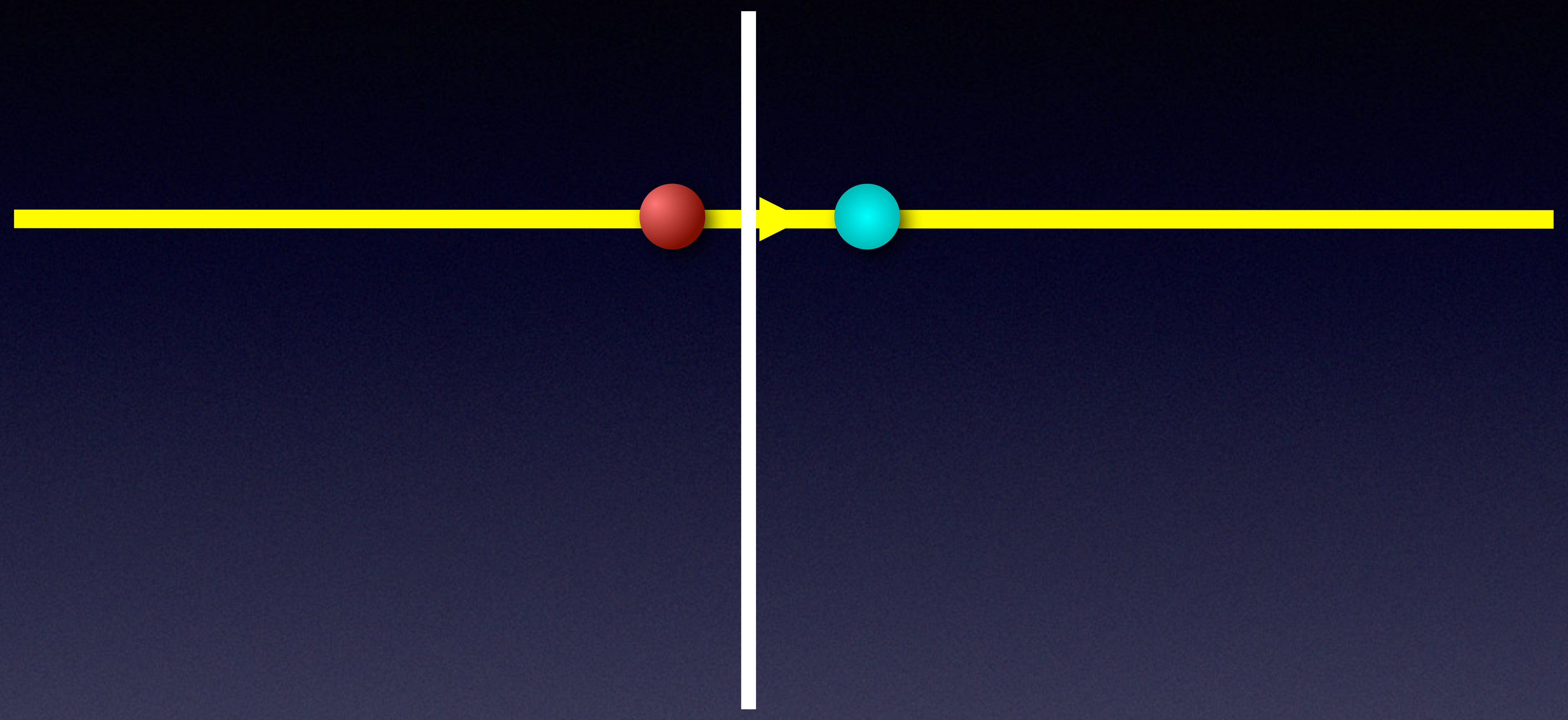
Possible phases of QCD

- Confinement phase
- Deconfinement phase
- Higgs phase (superconducting phase)
- Topological phase

  and .....

Deconfinement phase

Consider a pair of test charge



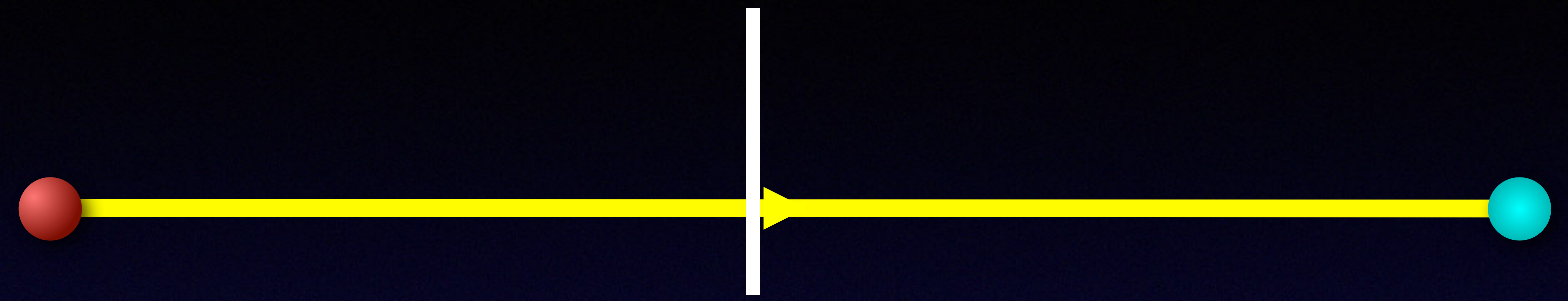
Color deconfinement

At long distance the energy is finite and
color flux exists between test charges.

(Topological ordered exhibits similar property)

Confinement phase

Consider a pair of test charge



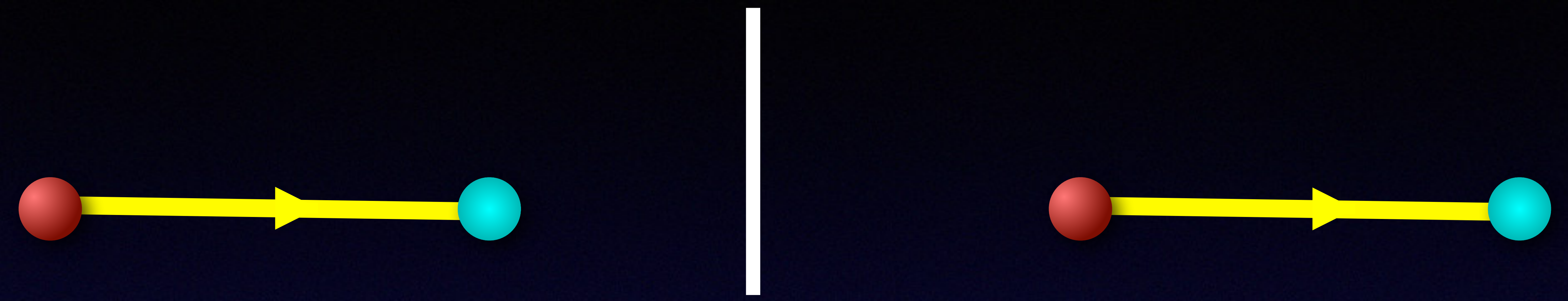
Confinement (narrow sense)

At long distance the energy diverges.

This is well-defined if $m_q \to \infty$

Confinement phase

Consider a pair of test charge



Confinement (narrow sense)

At large distances, the energy diverges.

This is well-defined in the limit $m_q \to \infty$

Confinement (broad sense)

At large distances, color flux between test charges vanishes.

This is the case for QCD.

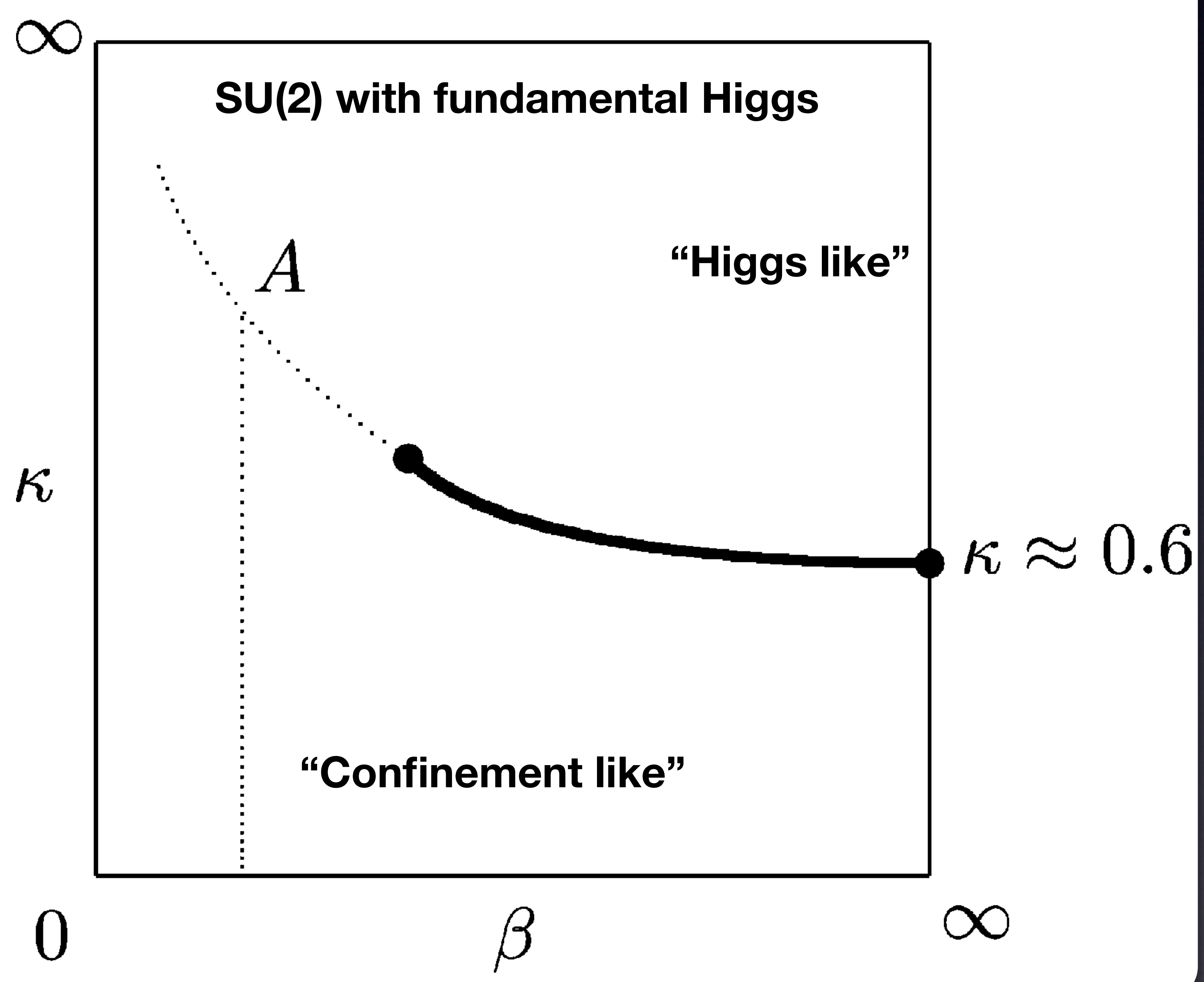
However, this cannot distinguish

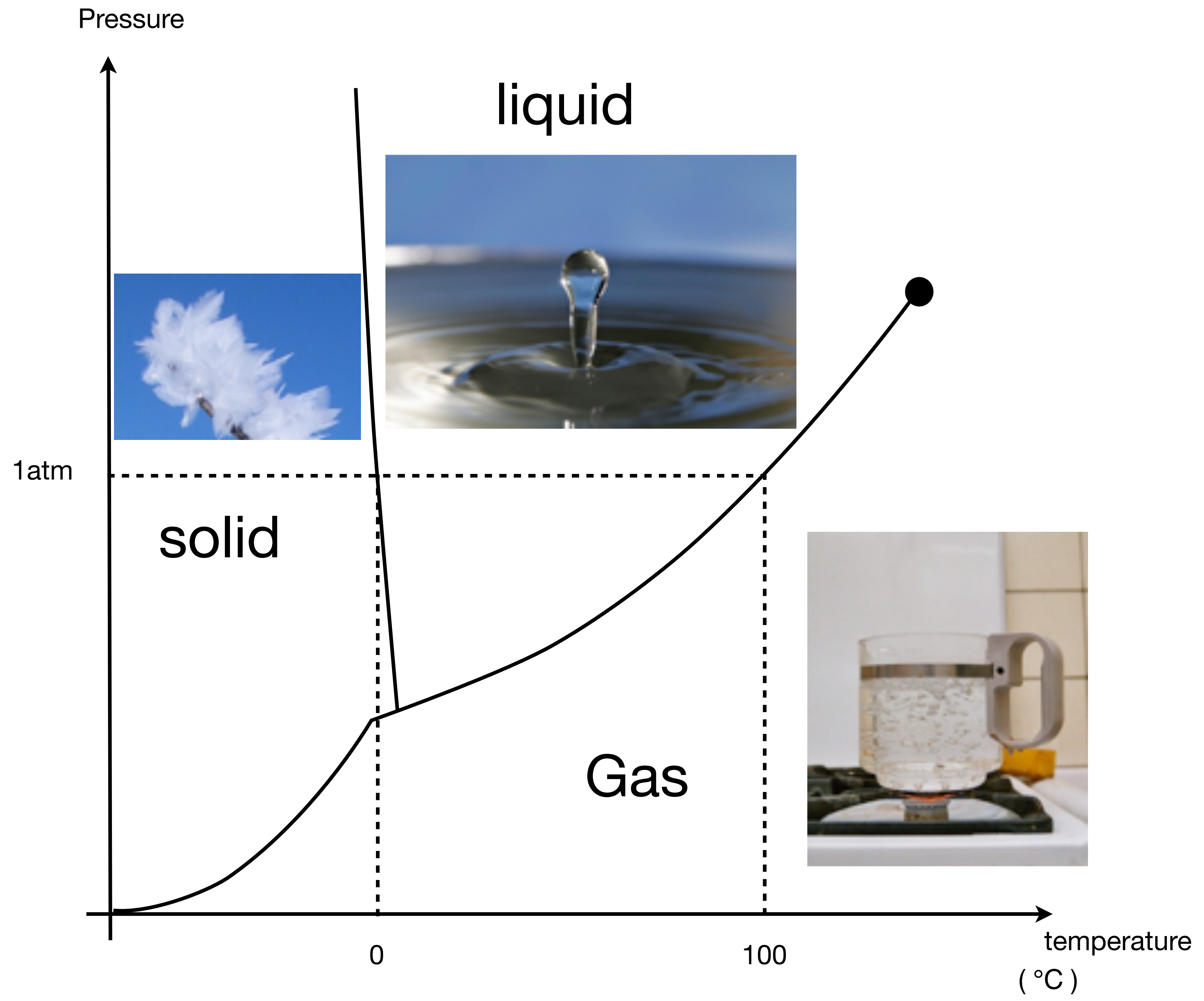
between **confinement** and **Higgs** phases

Confinement and Higgs are similar to liquid and gas

Physical property may be different between two phases, but it will be smoothly connected (Fradkin–Shenker ’79).

Bonati,. Cossu, D’Elia, Giacomo *NPB* 828, 39 (2010)





Chiral symmetry

At $m_q \to 0$

$$\mathscr{L} = \sum_q \left(\bar{q}_R D q_R + \bar{q}_L D q_L \right) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

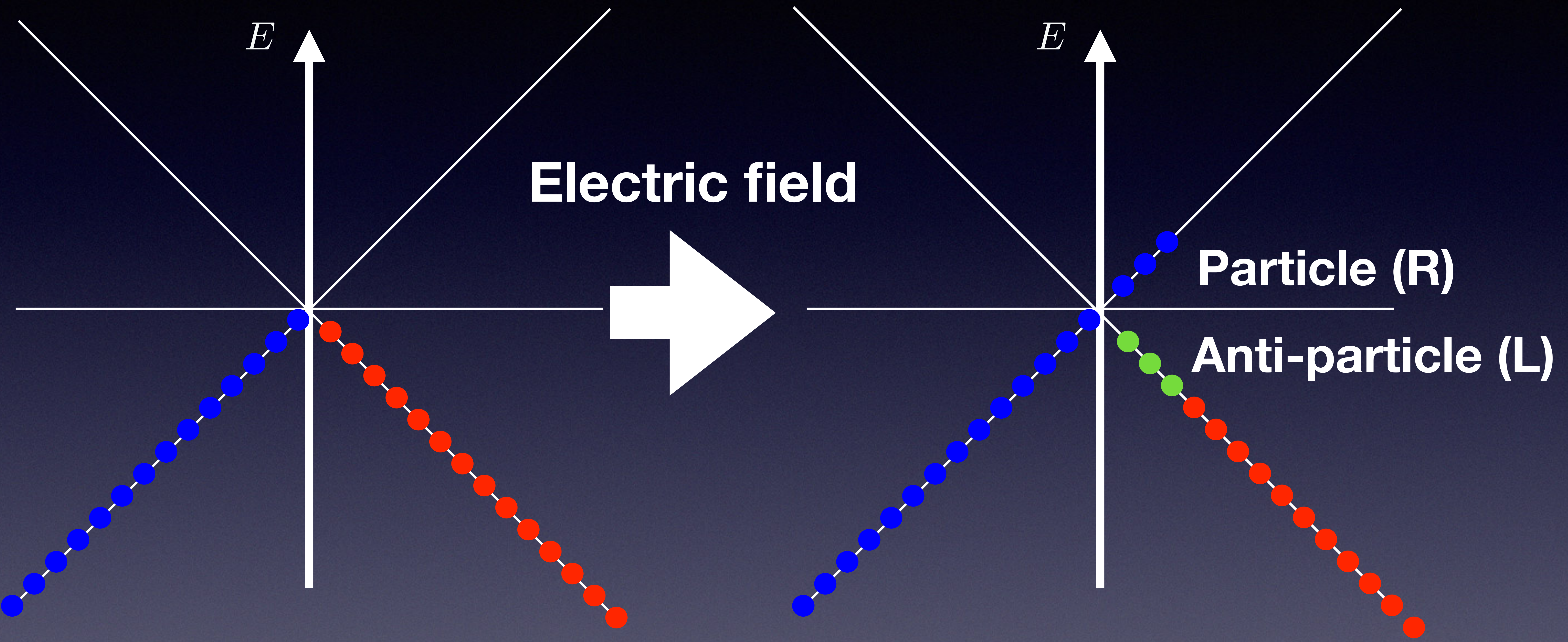
Global symmetry $SU(N_f)_R \times SU(N_f)_L \times U(1)_V \times U(1)_A$

$U(1)_A \to \mathbb{Z}_{2Nf}$ **: Explicit breaking by anomaly**

$SU(N_f)_R \times SU(N_f)_L \to SU(N_f)_V$ **:Spontaneous breaking**

Chiral anomaly

Ex) (1+1) Dirac Fermion



$$\Delta Q = \Delta Q_R + \Delta Q_L = 0$$

$$\Delta Q_5 = \Delta Q_R - \Delta Q_L = \frac{ET}{\pi}$$

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = \frac{E}{\pi}$$

(3+1)d

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu j_5^\mu = C\boldsymbol{E} \cdot \boldsymbol{B}$$

Vacuum property

Q: Why is the vacuum condensate scalar?

Vacuum: Lorentz invariant: $P_\mu|\Omega\rangle = 0 \;\; J_{\mu\nu}|\Omega\rangle = 0$

➡ **Only scalar~1, symmetric tensor~$\eta_{\mu\nu}$, four rank antisymmetric tensor ~$\epsilon_{\mu\nu\rho\sigma}$ and its combination can be condensed.**

Q: Is a parity odd vacuum possible?

No, parity cannot be broken in the QCD vacuum

(Vector-like gauge theories with $\theta = 0$)

Vafa-Witten theorem

Vafa, Witten, PRL53, 535 (1984)

derived from positivity of quark determinant $\det(D - m) > 0$

$D = -D^\dagger \;\; \gamma_5 D = -D\gamma_5$

Isospins are also unbroken

Vafa, Witten, Nucl. Phys. B234, 173 (1984)

Vacuum property

Q. How about chiral symmetry?

A. Chiral symmetry need to be spontaneously broken

Existence of chiral anomaly:

➡ **The vacuum can not be trivially gapped:**

The low-energy effective theory need to match the anomaly

't Hooft ('80)

Chiral symmetry breaking

Baryon is massless

Other symmetry breaking

Topological phase, CFT, …

Vacuum property

Baryon cannot be massless for $N_f = 3$

➡ **Chiral symmetry breaking**

Positivity of quark determinant implies

➡ **Baryons satisfy $m_B \geq \frac{3}{2} m_\pi$** Nussinov('83)

➡ **Baryons cannot be massless as long as $m_\pi \neq 0$**

Therefore, chiral symmetry breaking is natural

What happens at finite T and μ ?

At finite T

Order parameter (universally) vanishes at high T

➡ **Chiral symmetry restoration**

Order parameter vanishes $\langle \bar{q}q \rangle \to 0$

$SU(N_f)_R \times SU(N_f)_L$ become good quantum number

Degeneracy of spectra $(\sigma, \pi^a), (\rho^a, a_1^a), \cdots$

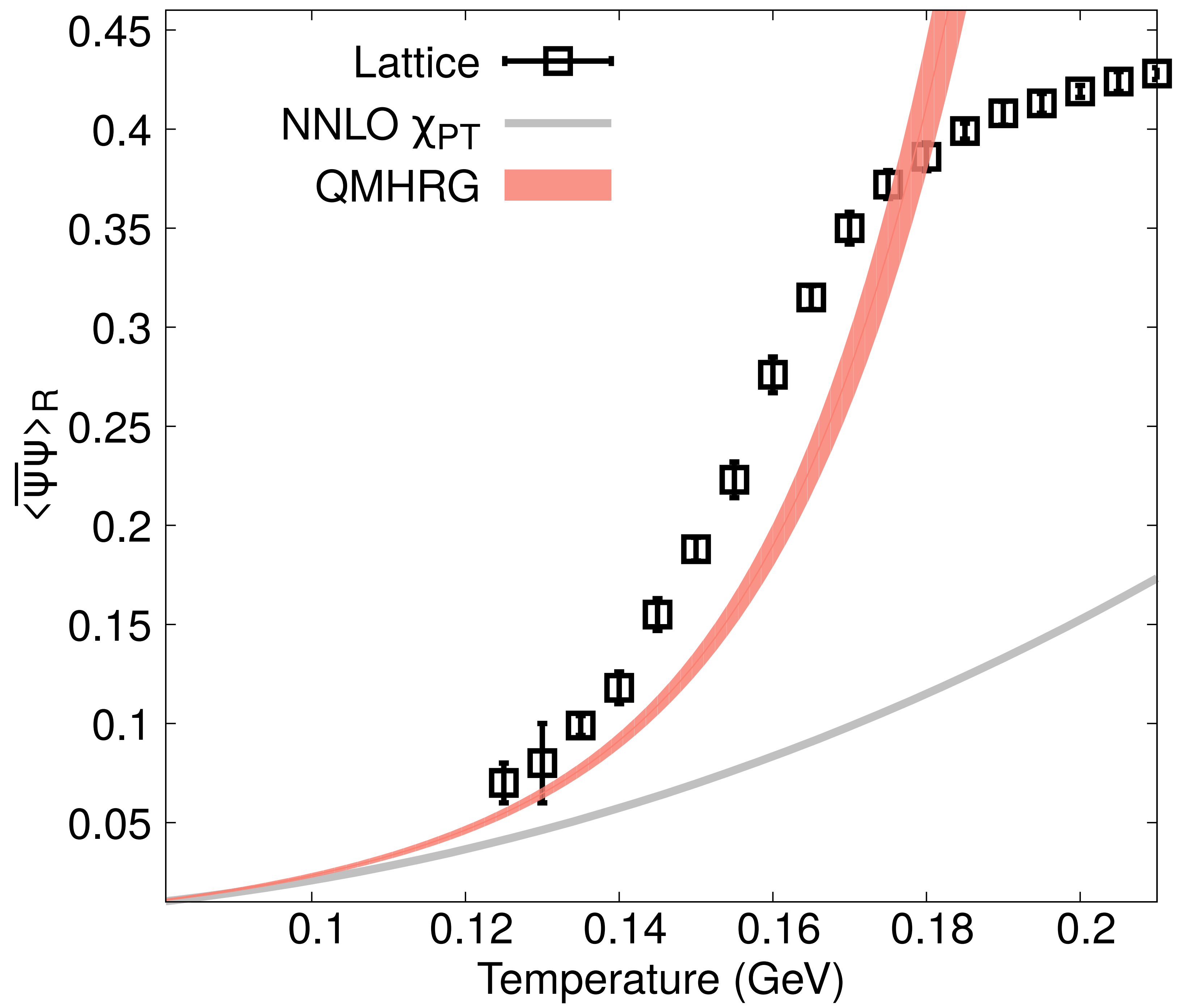
Anomaly matching does not impose a constraint
(Instead, it leads to chiral magnetic and vortical effects.)

Results from Lattice QCD

Chiral condensate

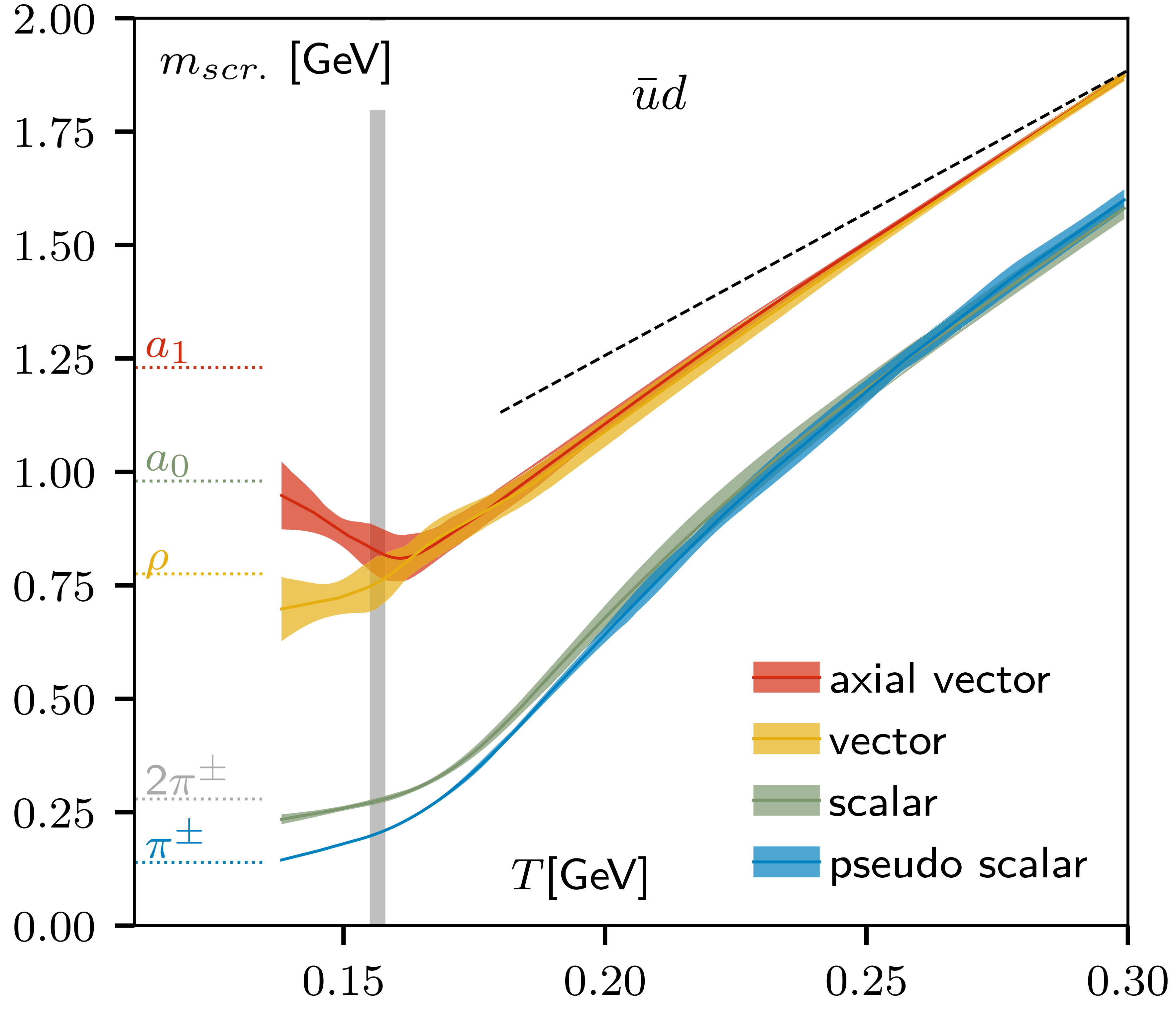
Biswas, Petreczky, Sharma, *PRC*106, 045203 (2022)

$$\langle\bar\psi\psi\rangle_R = -\frac{m_l}{m_\pi^4}\left[\langle\bar\psi\psi\rangle_{l,T} - \langle\bar\psi\psi\rangle_{l,0}\right]$$



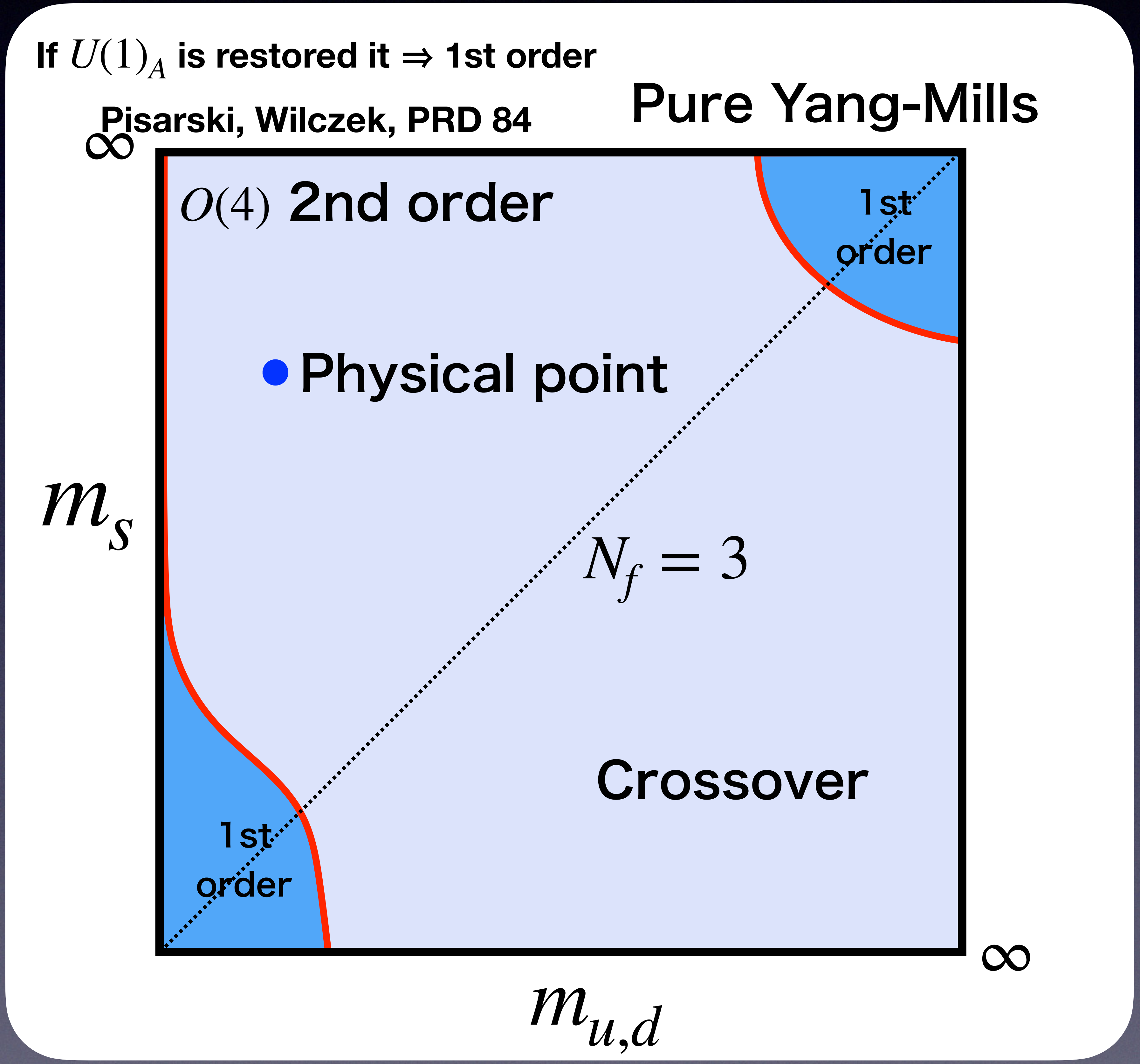
Screening mass

HotQCD, PRD 100, 094510 (2019)



Recent results in finite temperature QCD

Columbia Plot



Continuum limit

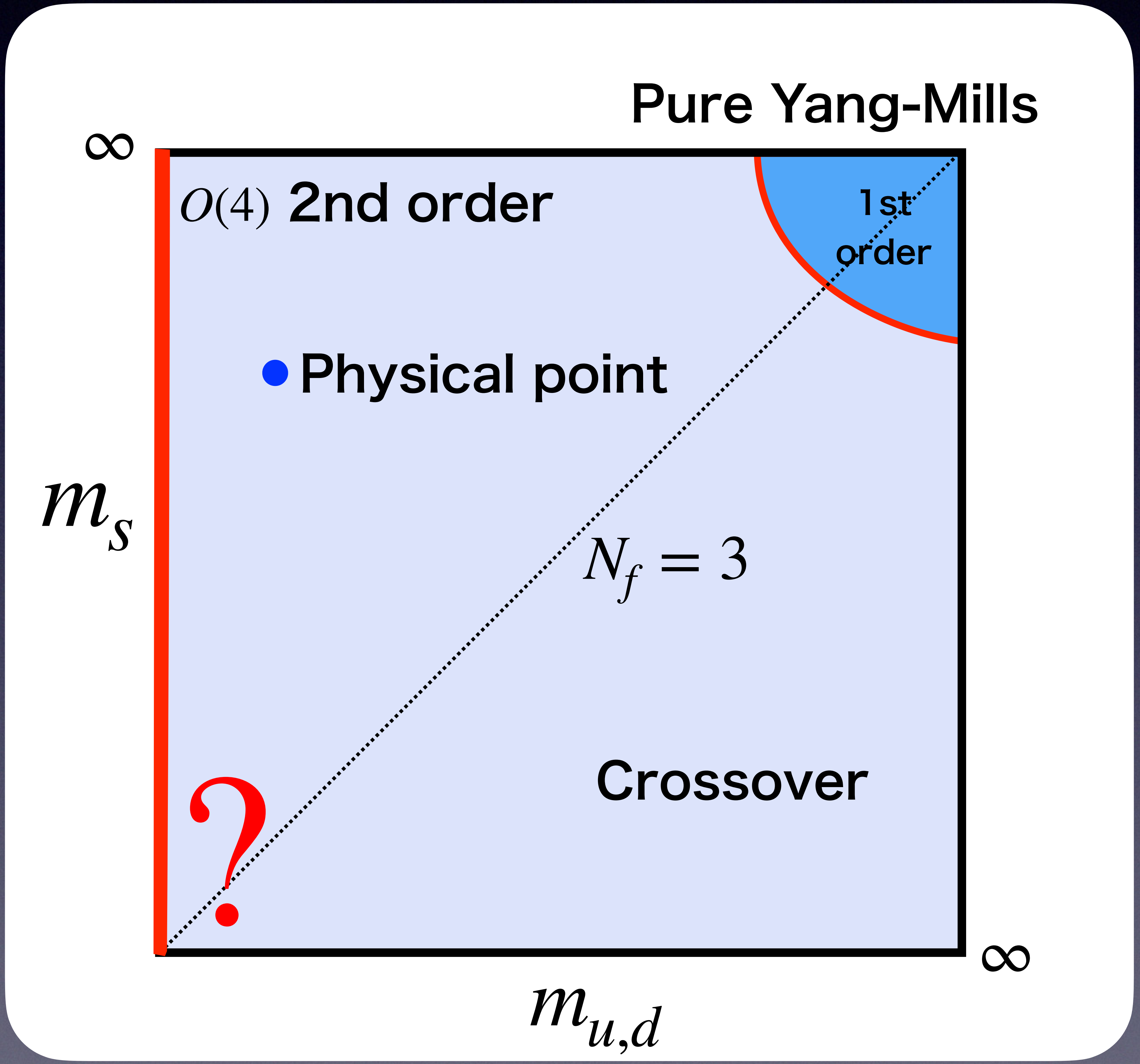
$a \to 0$

Kuramashi et al ('20)
1st order region
shrinks as $a \to 0$

Cuteri et al ('21)
no 1st order region
for Staggered Fermion

Dini et al (22')
Consistent with no 1st order
for improved Staggered
Fermion

Cuteri, Phillipsen, Sciarra JHEP 11, 141(2021)



c.f. non lattice study: Fejos ('22), Kousvos, Stegio ('23), Pisarski, Rennecke ('23), Fejos, Hatsuda ('24)

QCD at finite density

What happens at finite density ?

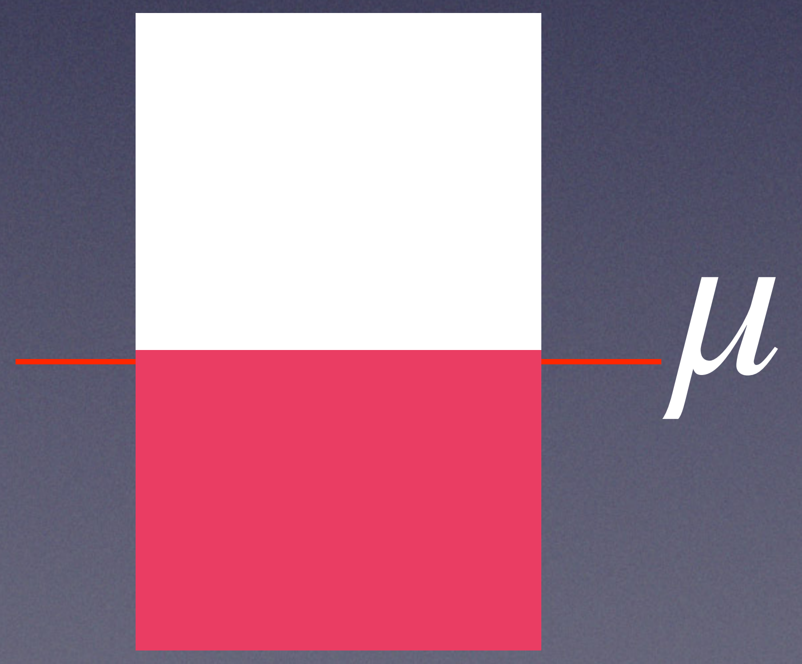
No positivity of Fermion determinant → QCD inequality does not work.

Anomaly matching condition still works. → The ground state is nontrivial.

Unlike at finite T, chiral restoration at finite density is nontrivial.

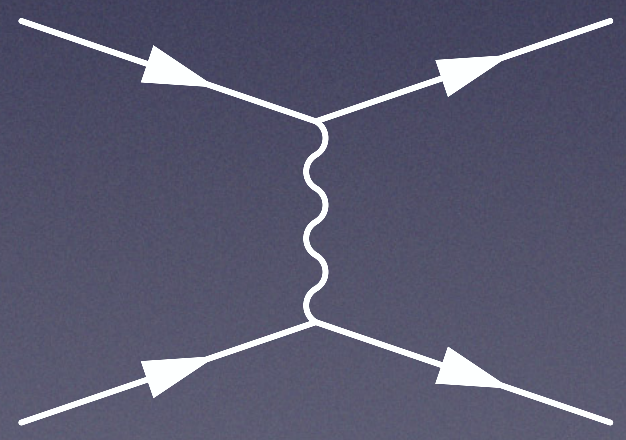
Consider very dense QCD system

Fermi surface



+

Attractive interaction



$3 \otimes 3 = 6 \otimes \bar{3}$

Repulsive Attractive

→

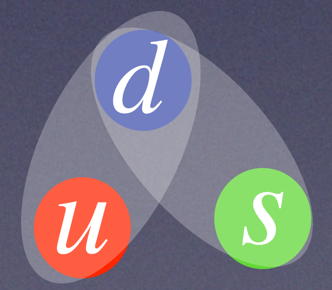
Color superconductivity

BCS pairing of diquarks

2SC

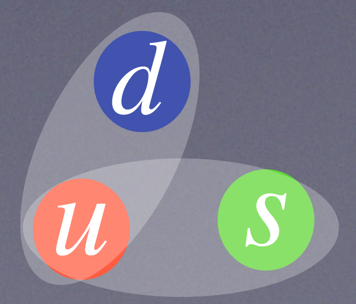
Bailin. Love ('84)

dSC

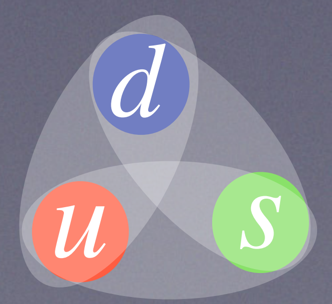


Iida, Matsuura, Tachibana, Hatsuda ('04)

uSC



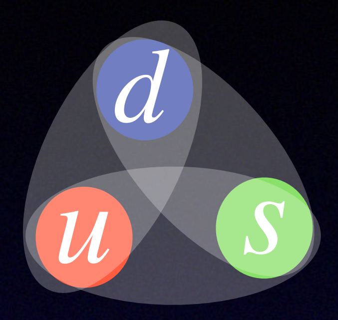
CFL



Alford, Rajagopal, Wilcheck ('99)

Color superconductivity

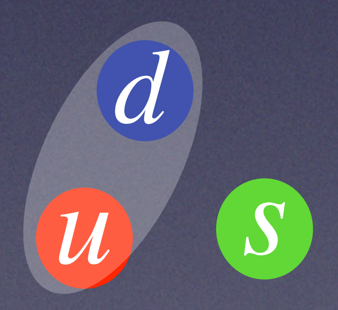
CFL phase



- Chiral symmetry is spontaneously broken
- $U(1)_B$ symmetry is spontaneously broken
- gapped Baryons ~ quarks
- gapped vector mesons~ gluons
- Kaon are lighter than pions

Anomaly is matched by NG bosons

2SC phase



- Chiral symmetry is unbroken
- There are massless and massive baryons ~quarks

Anomaly is matched by massless baryons

➡ **Chiral symmetry can be restored or spontaneously broken**

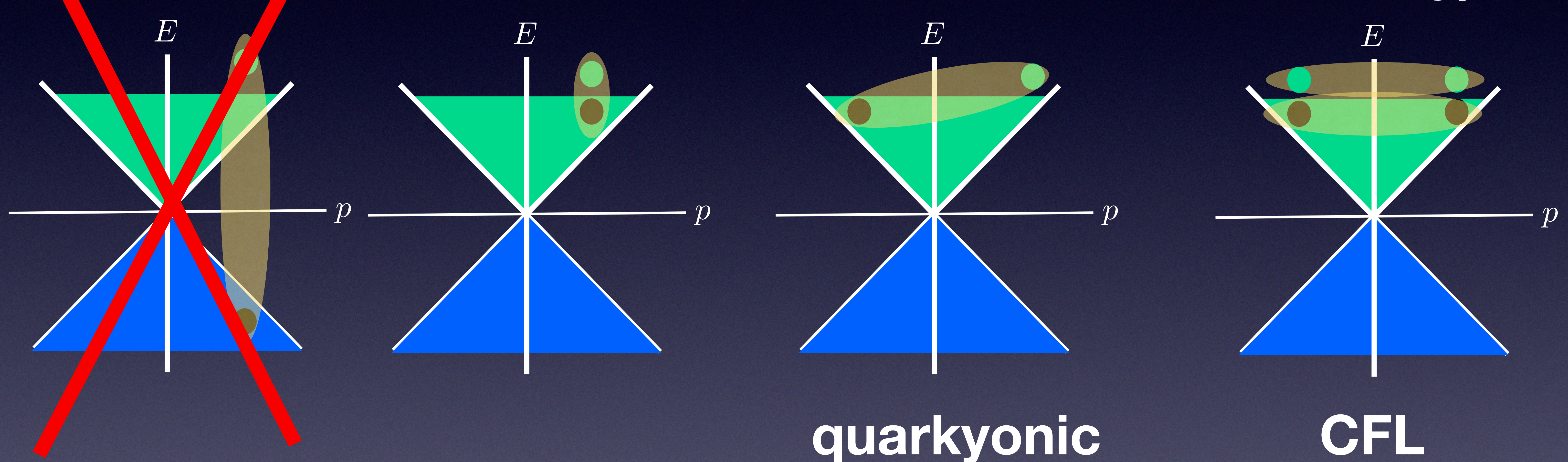
Chiral condensate at finite density

Low density

Drukarev, Levin ('90)

$$\langle \bar{q}q\rangle_\rho / \langle \bar{q}q\rangle_0 \approx 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2}\rho$$

Dirac Type | **Exciton Type** | **Density Wave Type** | **BCS Type**



Dirac Type: disfavored

$$\langle \bar{q}q\rangle \approx 0$$

Density Wave Type: quarkyonic senario

$$\langle \bar{q}q\rangle \sim \cos pz$$

BCS Type: CFL senario

$$\langle \bar{q}q\rangle \approx 0$$

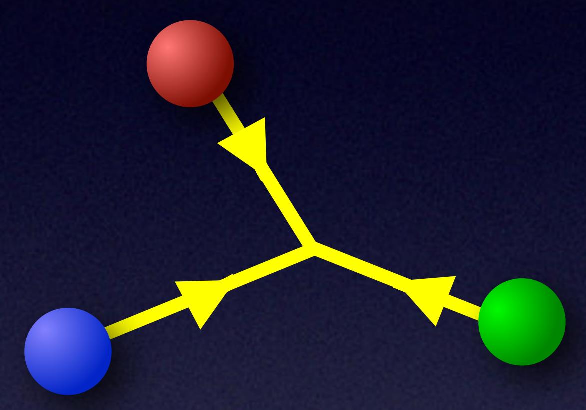
$$\langle (\bar{q}q)^2\rangle \neq 0$$

Quark hadron continuity

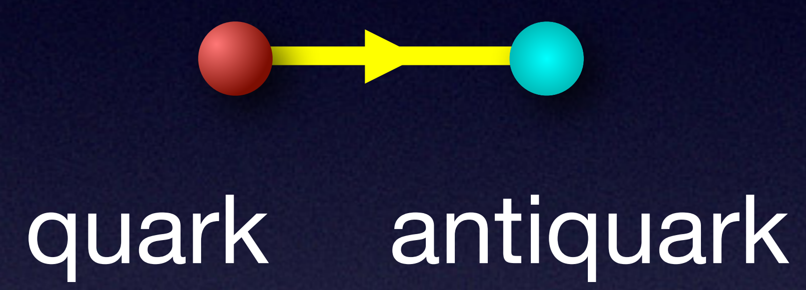
Schafer and Wilczek, PRL 82 , 3956(1999)
Hatsuda, Tachibana, Yamamoto, Baym, PRL 97 122001 (2006)

Vacuum

Baryon

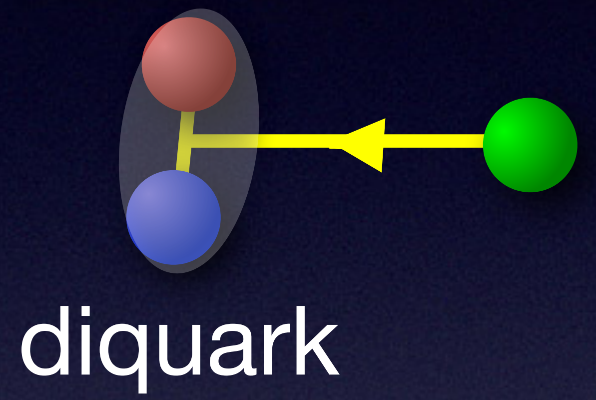


Vector meson

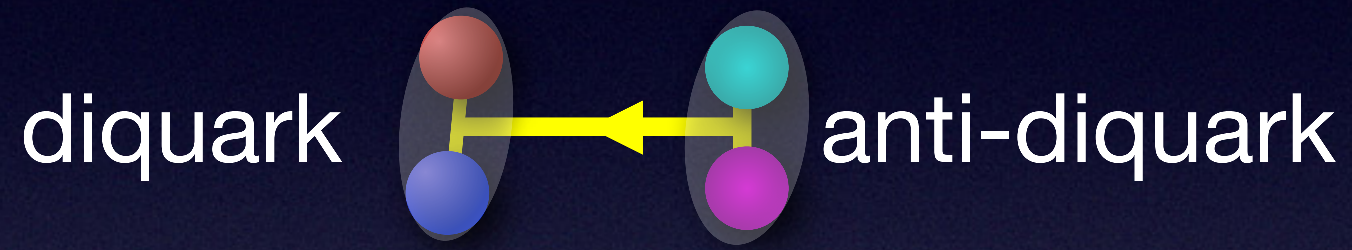


CFL

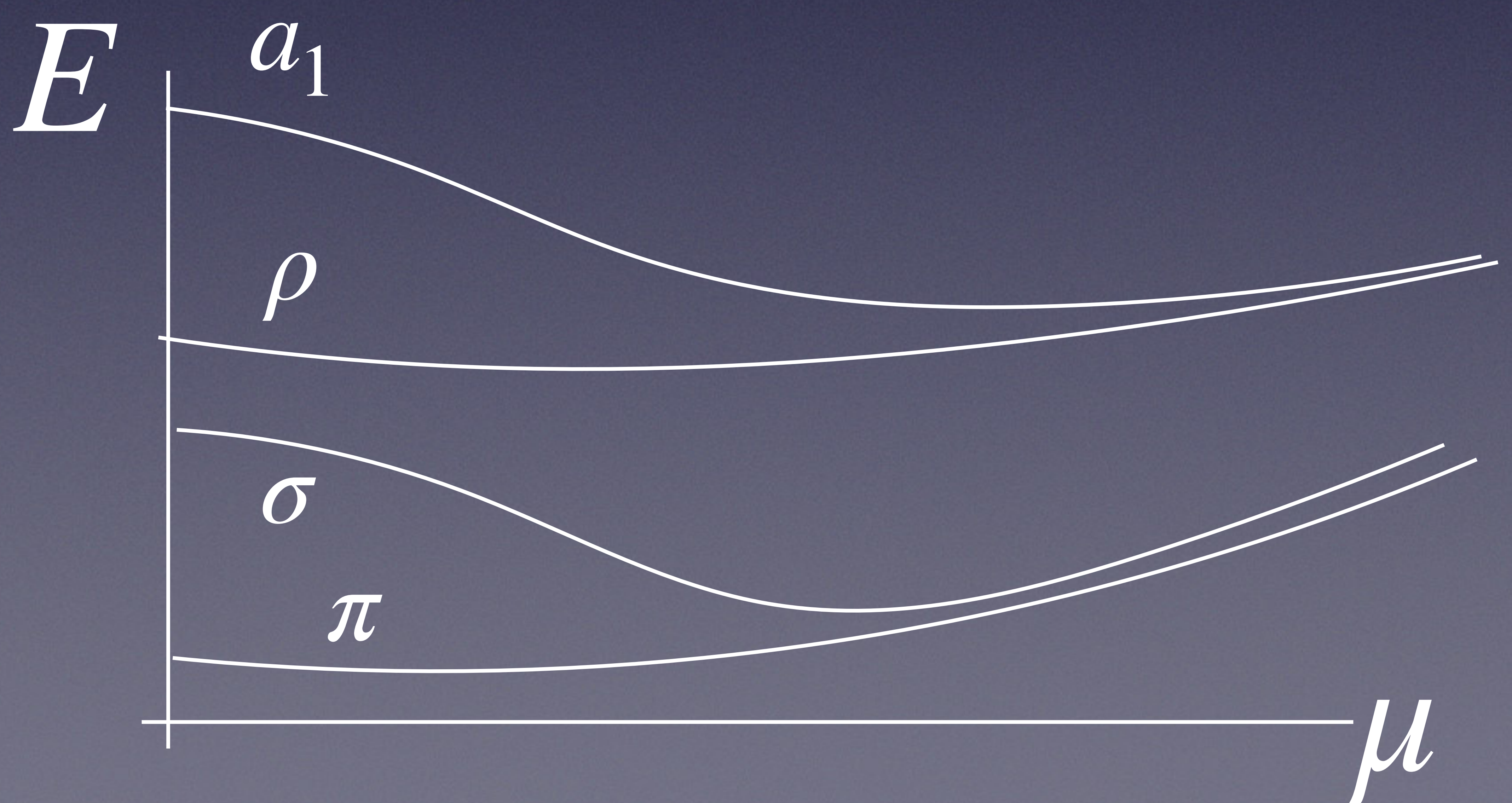
Quark like



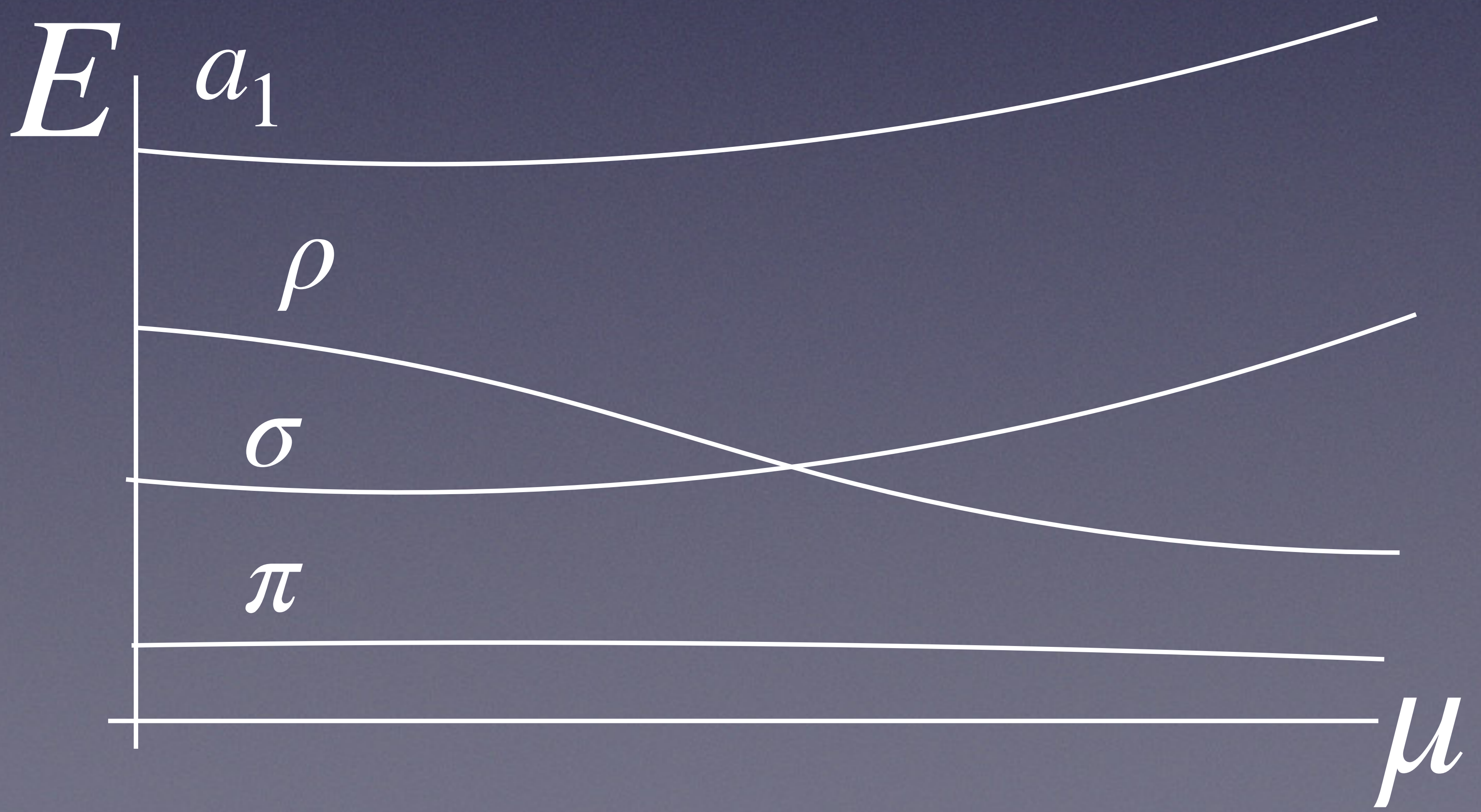
Gluons like



Chiral restoration senario



CFL senario



Recent Development
in numerical calculations

First-principle calculation

Complex fermion determinant

➡ Sign problem Obstruction to Monte Carlo methods

Toward a breakthrough:

- Complex Langevin method
- Lefschetz thimble method
- Tensor Network
- Quantum simulation

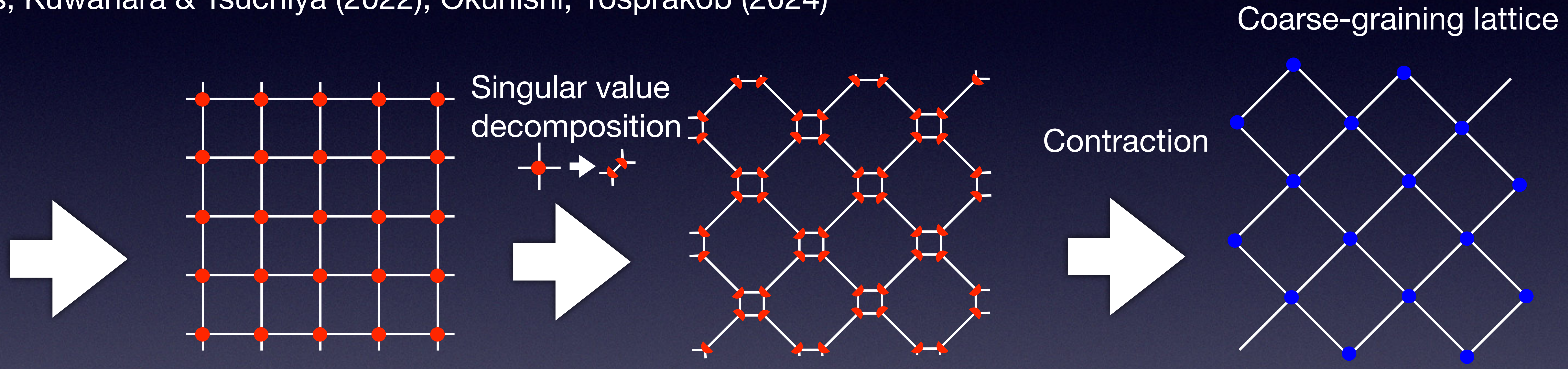
Tensor Network approach

Lagrangian approach

Tensor network renormalization group

Application to Nonabelian gauge theory:
2D pure Yang-Mills, Fukuma, Kadoh, Matsumoto (2021), Hirasawa, Matsumoto, Nishimura, Yosprakob (2021)
3D pure Yang-Mills, Kuwahara & Tsuchiya (2022), Okunishi, Yosprakob (2024)

$$Z = \int DAe^{-S[A]}$$



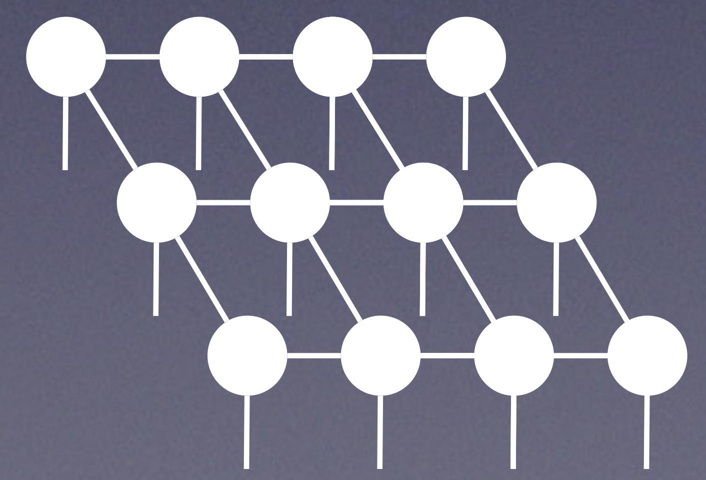
Hamiltonian approach

$$|\psi\rangle = \sum_{\{n_i\}} C_{n_1 n_2 \cdots n_N} |n_1\rangle \cdots |n_N\rangle$$

d^N **d.o.f**

Coefficients are approximated by tensor network

$$C_{n_1 n_2 \cdots n_N} \simeq$$



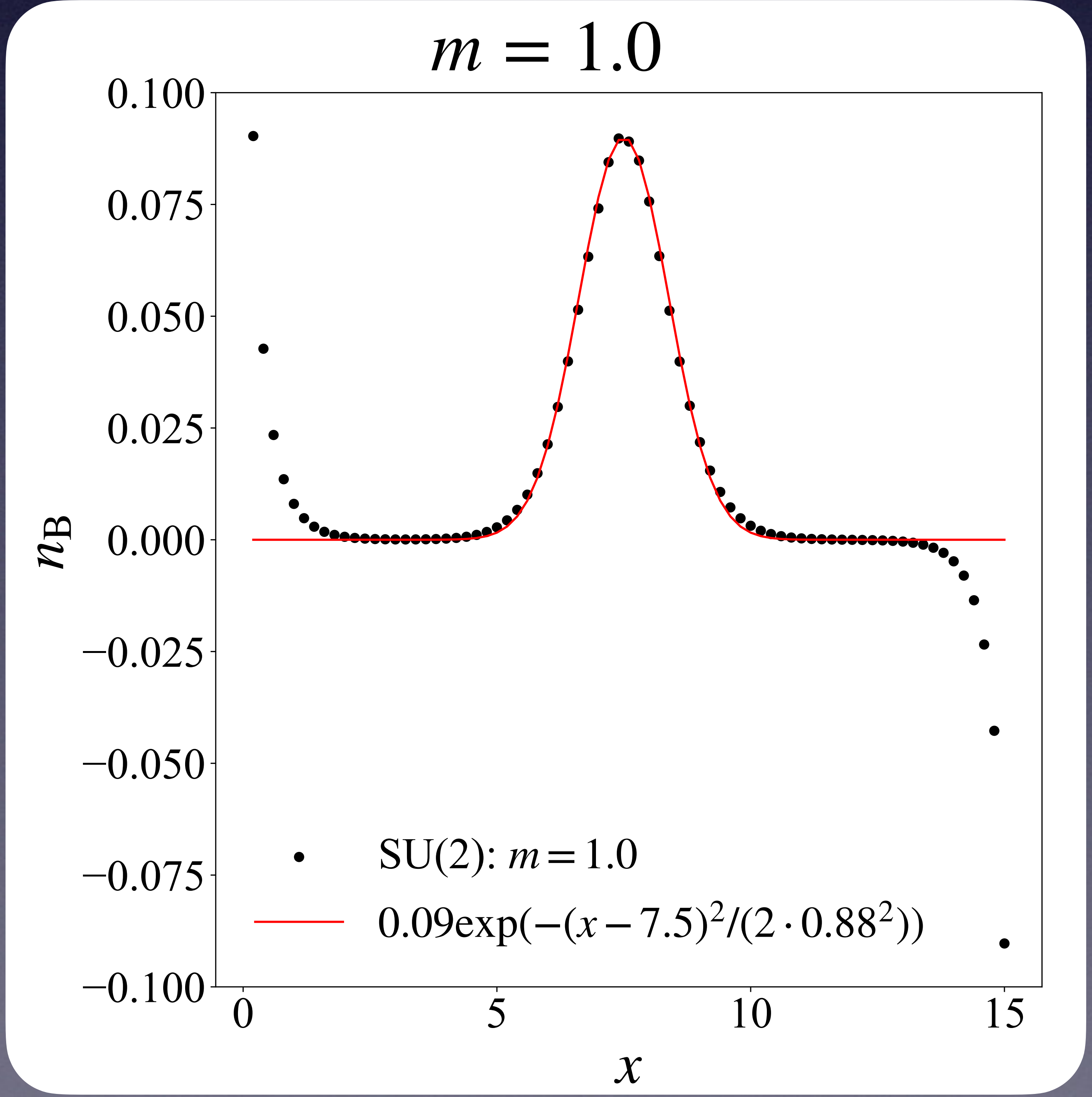
C is optimized s.t. $E = \min_{C} \langle \psi | H | \psi \rangle$

QCD_2 Hamiltonian Lattice simulation
with density matrix renormalization technique

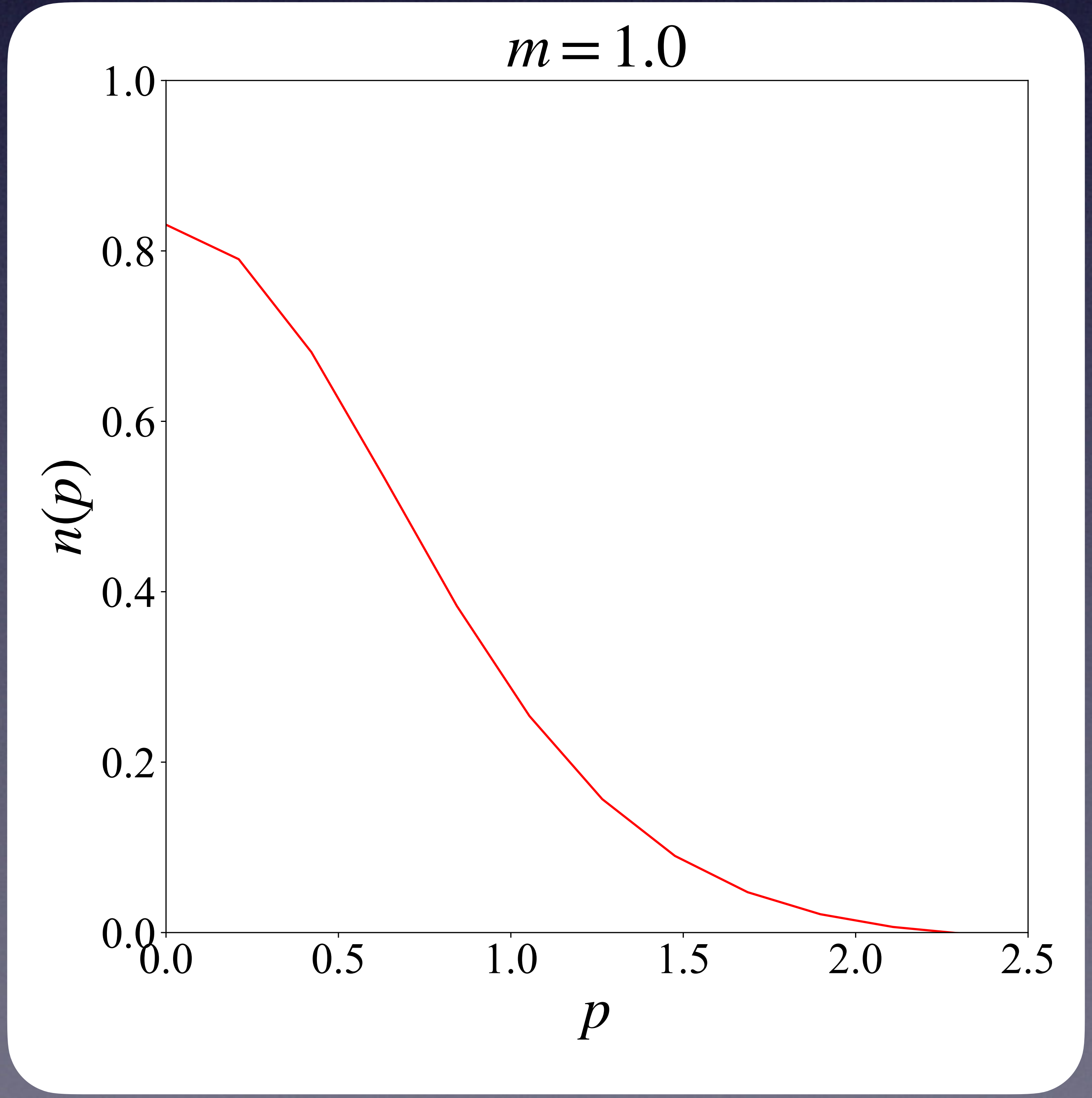
Tomoya Hayata, YH, Nishimura, 2311.11643

Two color QCD $N_f = 1$

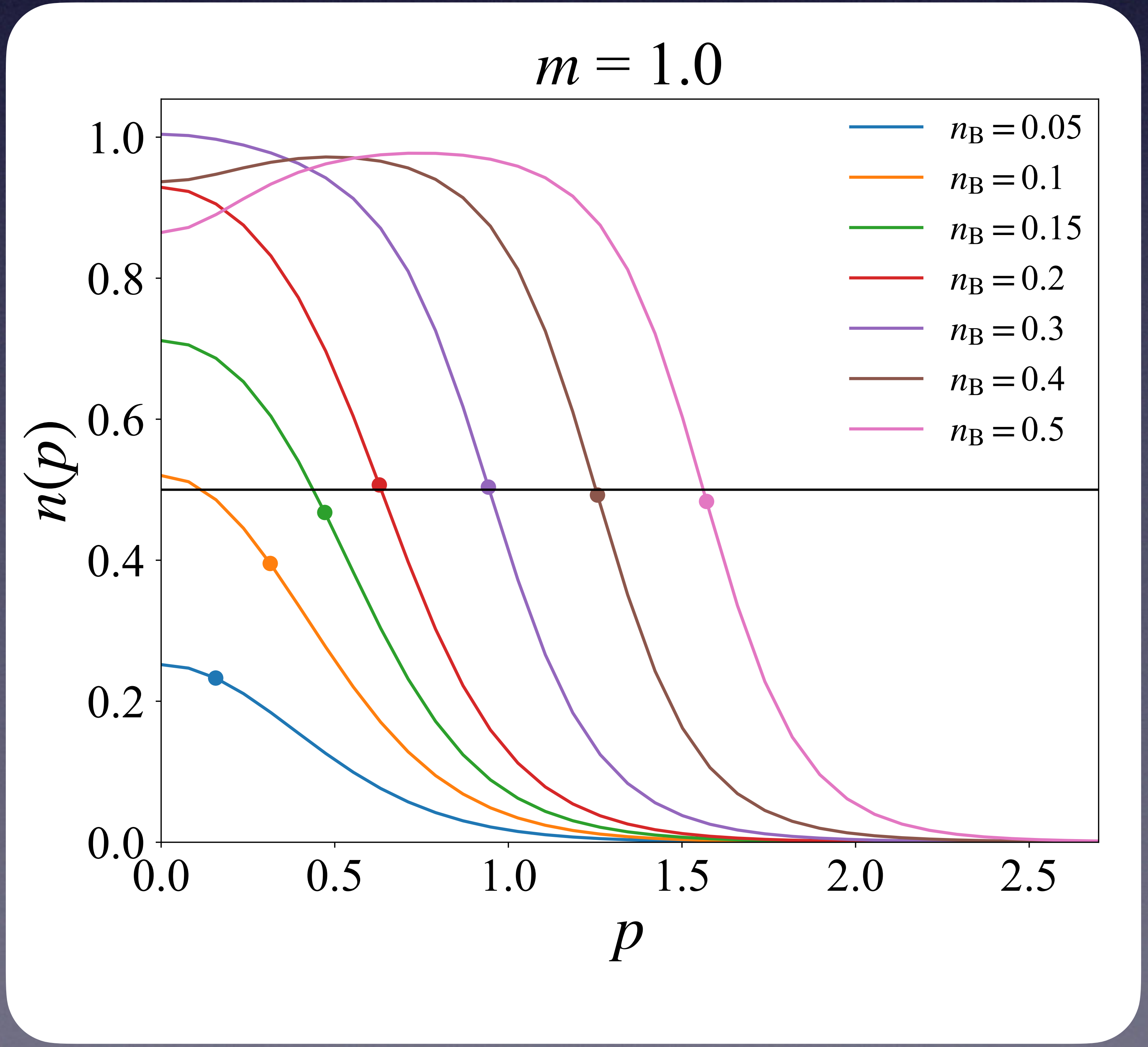
Single baryon state
$\dim\mathscr{H} = 2^{300}$



Quark distribution function
$\dim\mathscr{H} = 2^{300}$



Finite density
$\dim\mathscr{H} = 2^{480}$

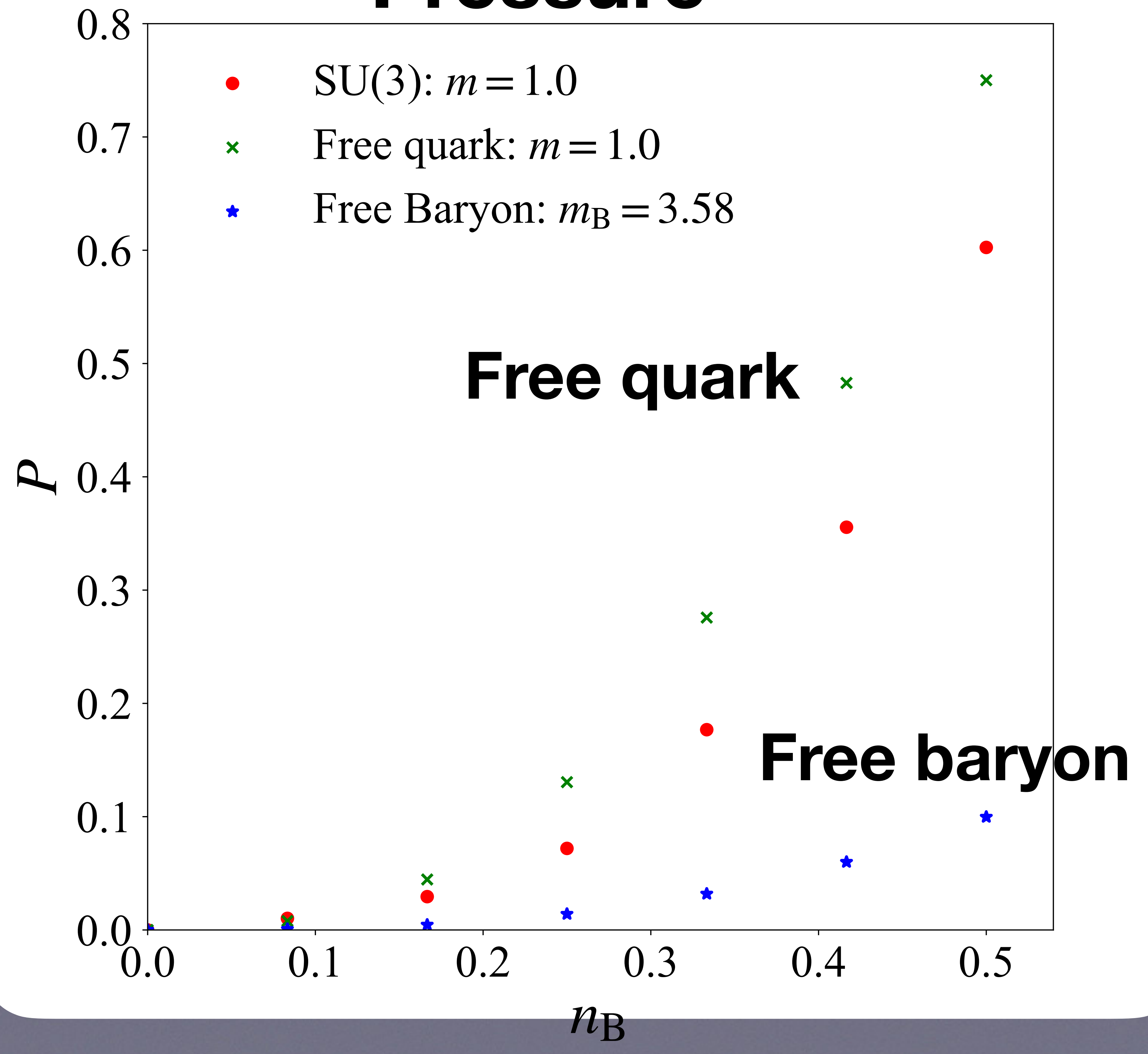


QCD_2 Hamiltonian Lattice simulation
with density matrix renormalization technique

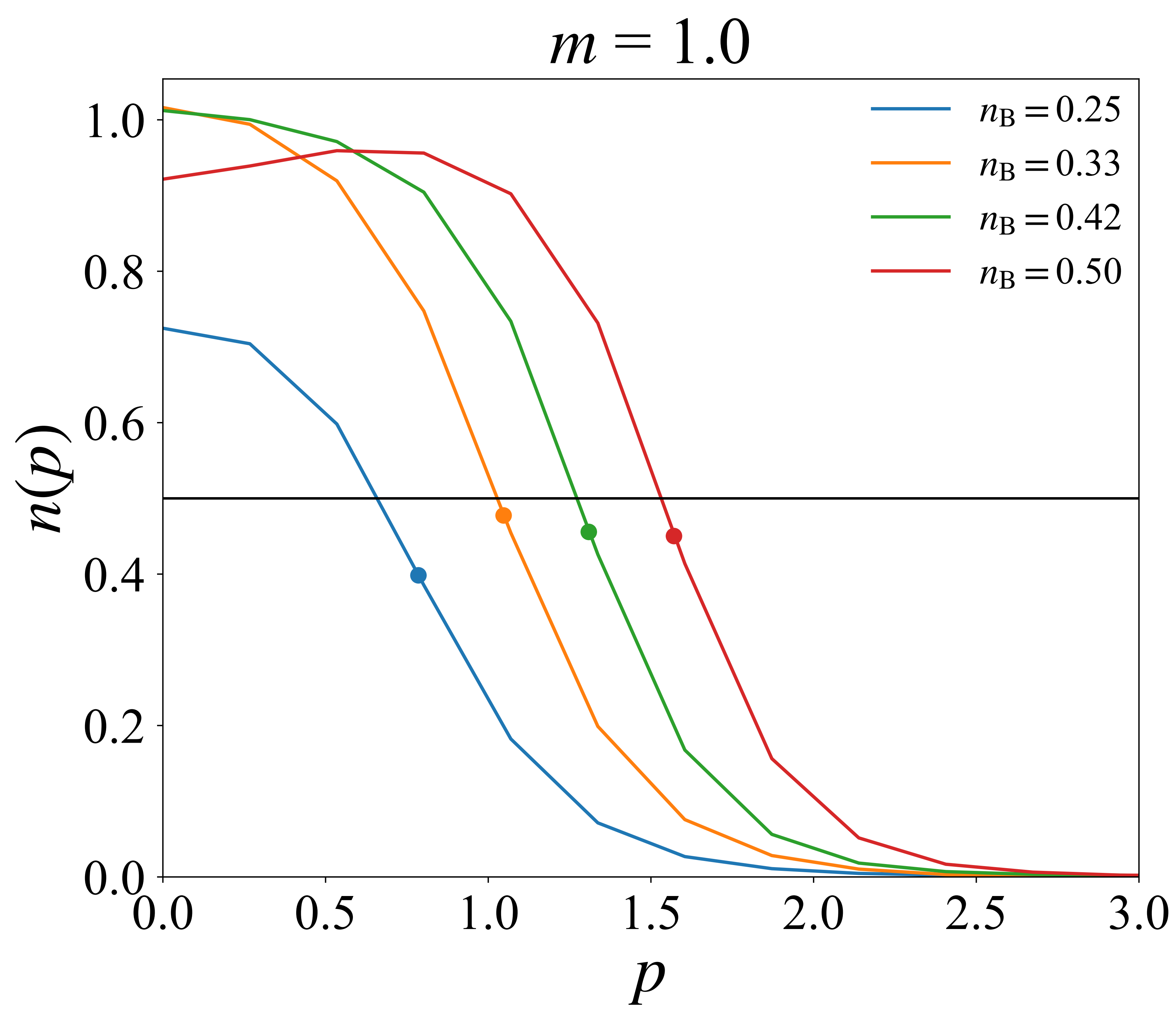
Tomoya Hayata, YH, Nishimura, 2311.11643

Three color QCD, $N_f = 1$ $\dim\mathscr{H} = 2^{144}$

Pressure



Quark distribution

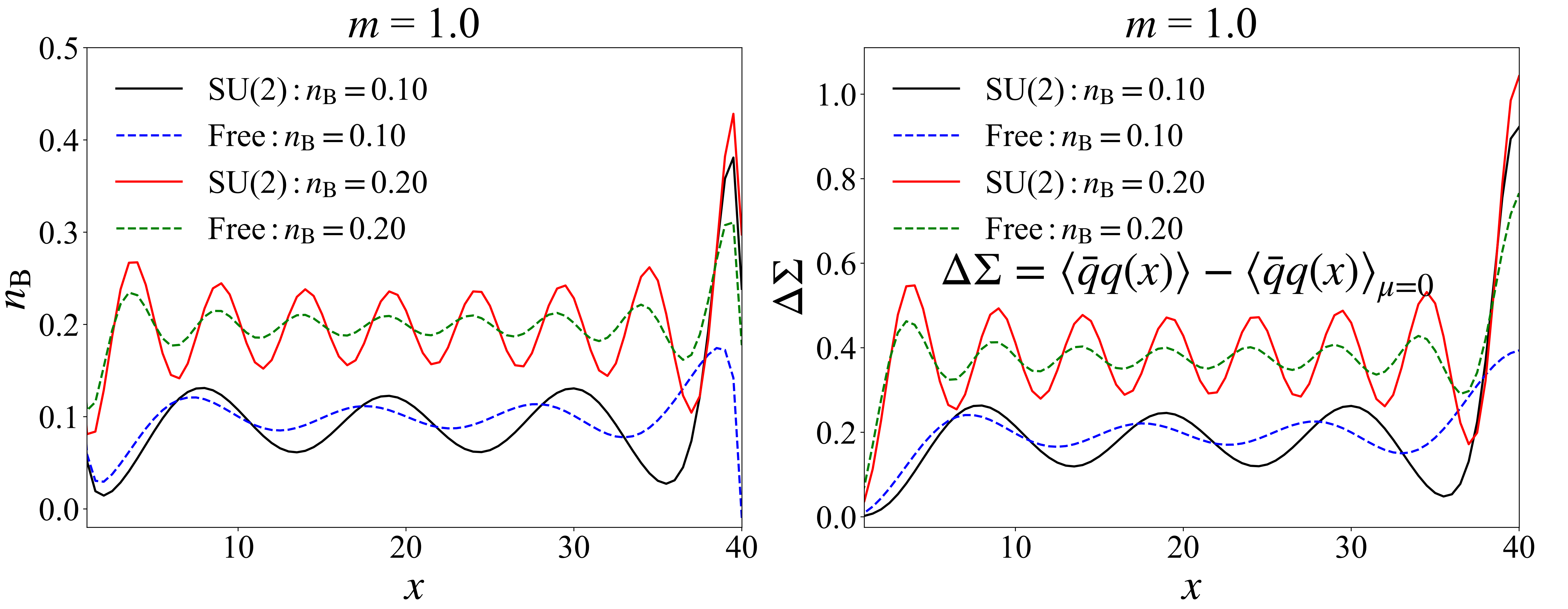


Inhomogeneous phase in QCD_2

corresponding to ‘quarkyonic chiral spirals’ Kojo, Hidaka, McLerran, Pisarski (2010)

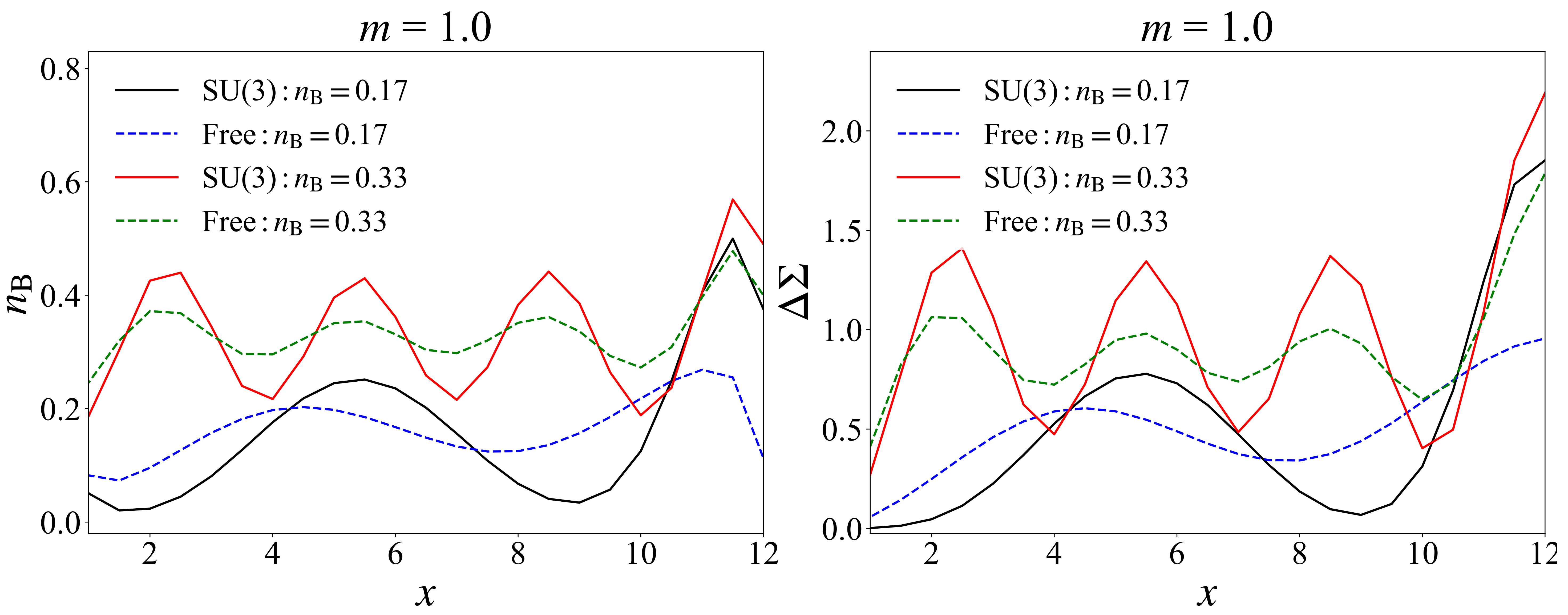
SU(2)

$\dim\mathscr{H} = 2^{320}$

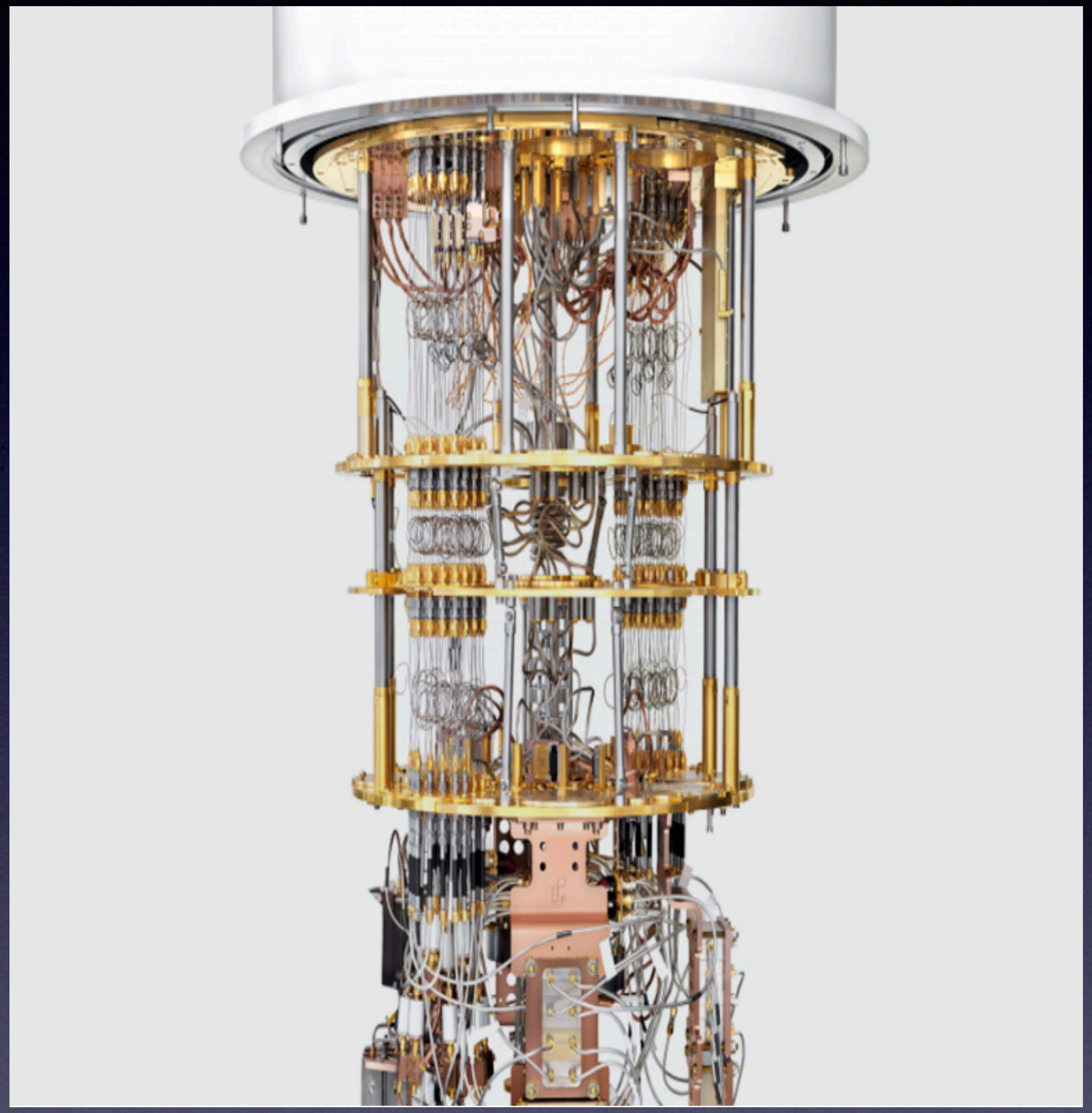


SU(3)

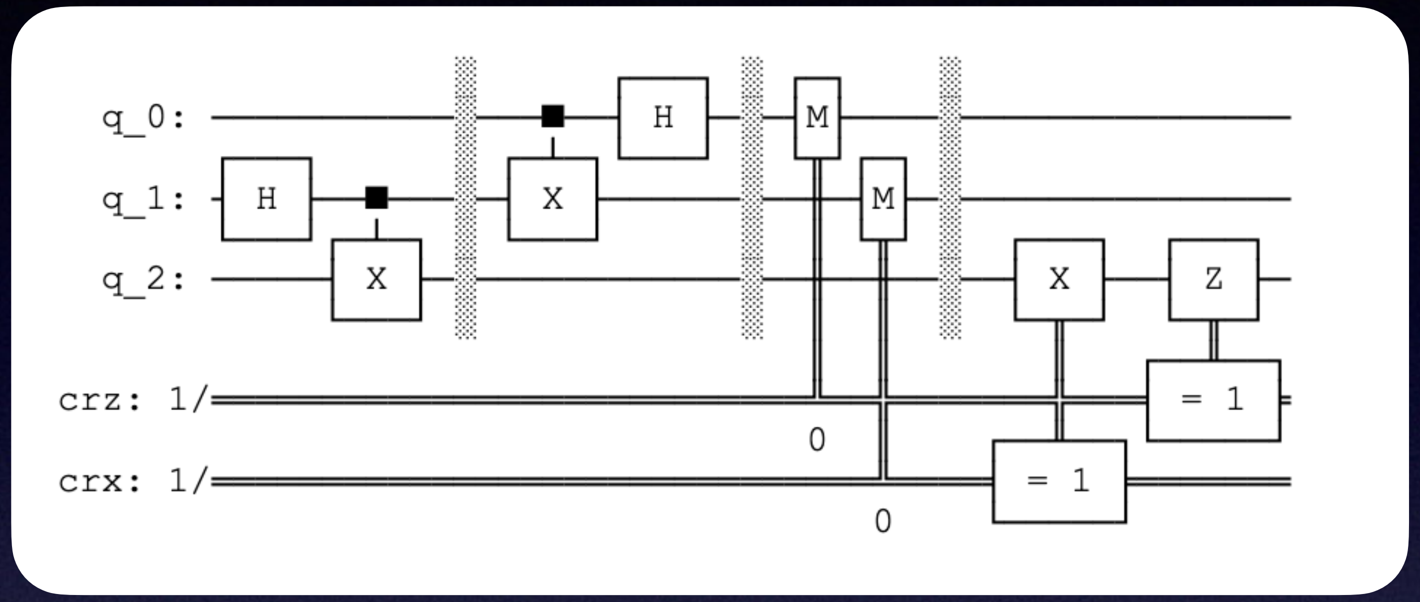
$\dim\mathscr{H} = 2^{320}$



Quantum computing



$$i\partial_t|\psi\rangle = H|\psi\rangle$$



- Natural method to solve quantum systems

So far, noise is large,
the number of qubits are small.

Selected Recent Results

String breaking dynamics

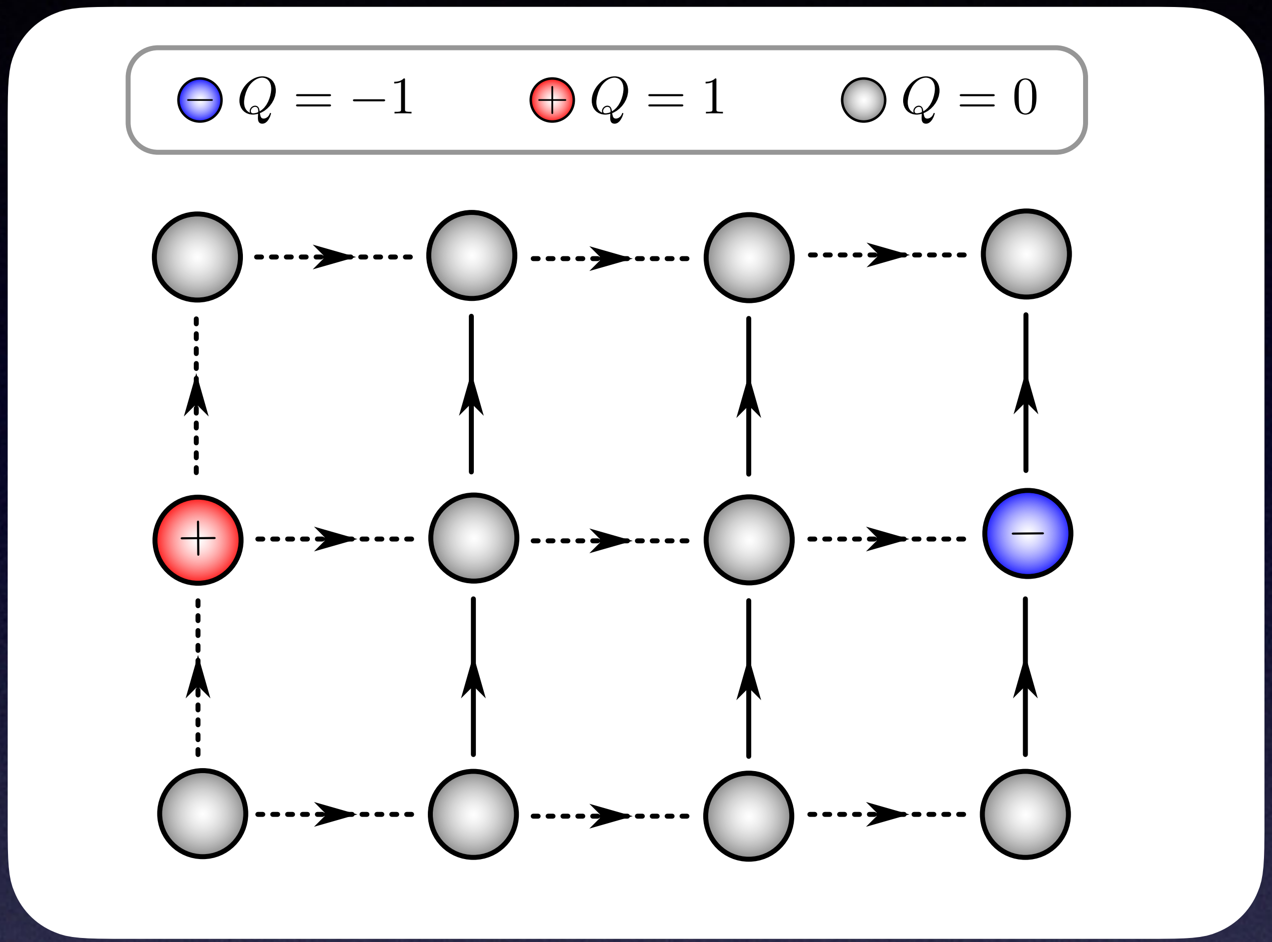
Crippa, Jansen, Rinaldi, 2411.05628

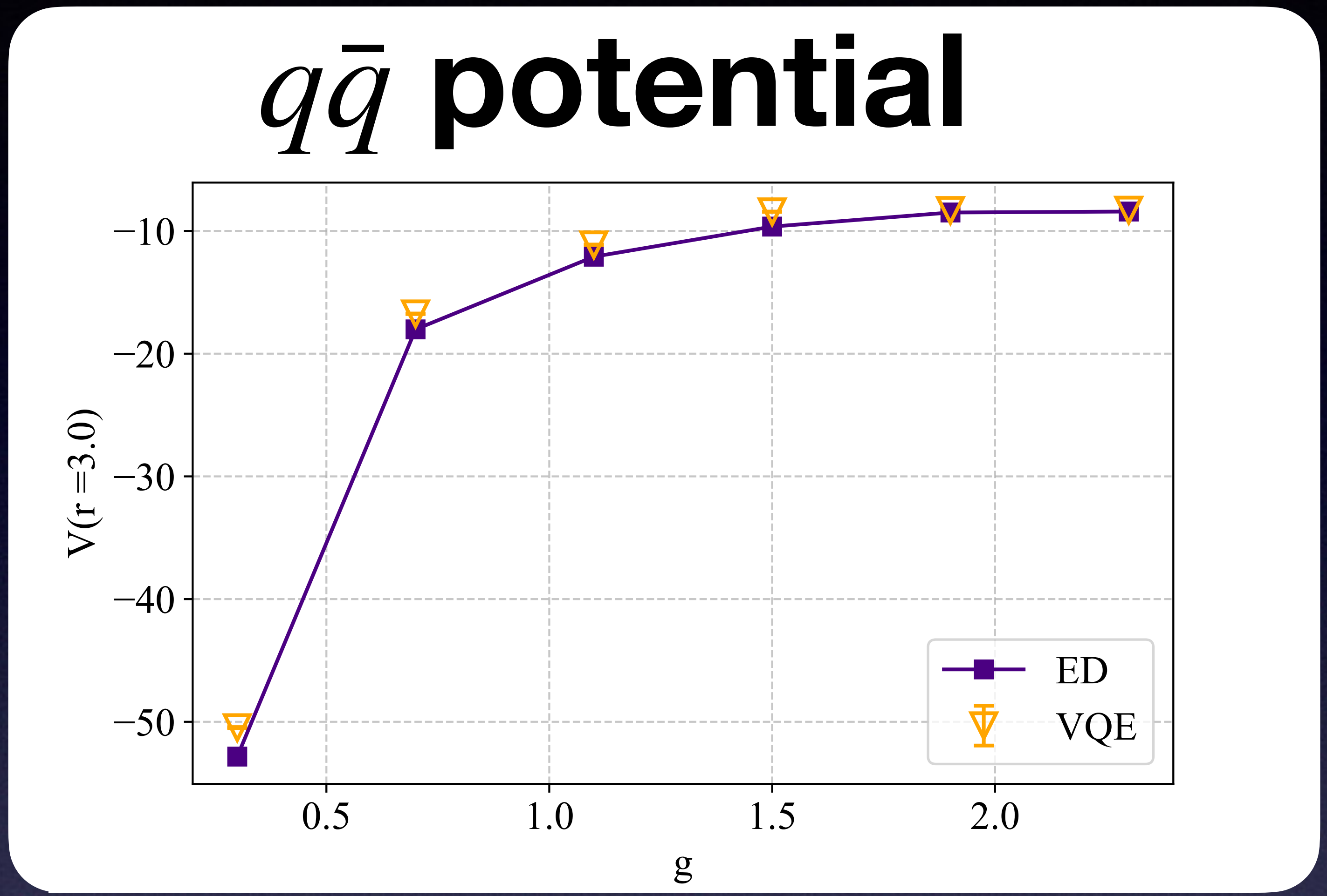
QED (QZ_3D)

(4×3) system

Ion traps

(20 qubits)





$\mathbb{Z}_3$ lattice gauge theory at nonzero baryon density

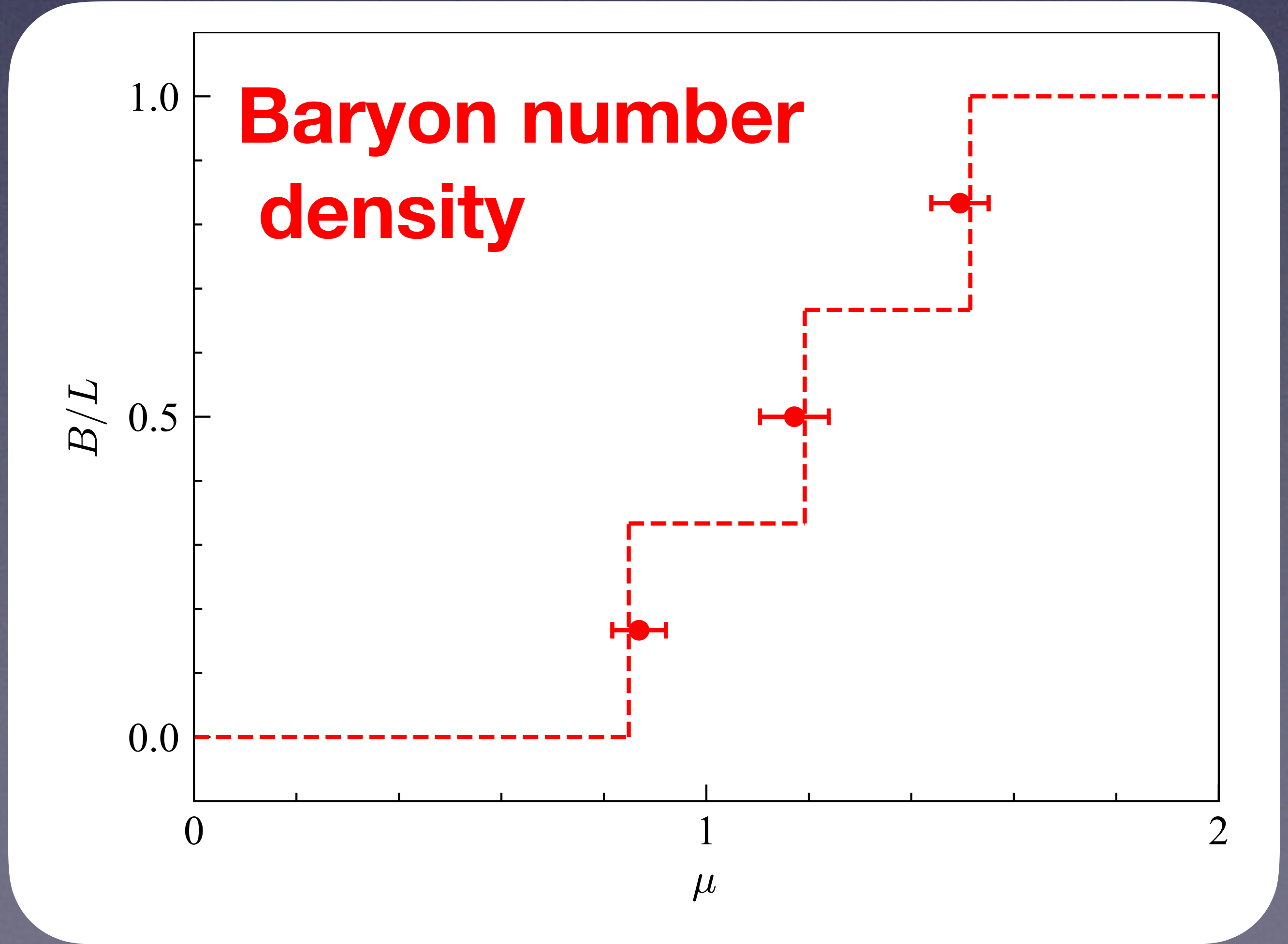
Hidaka, Yamamoto 2409.17349

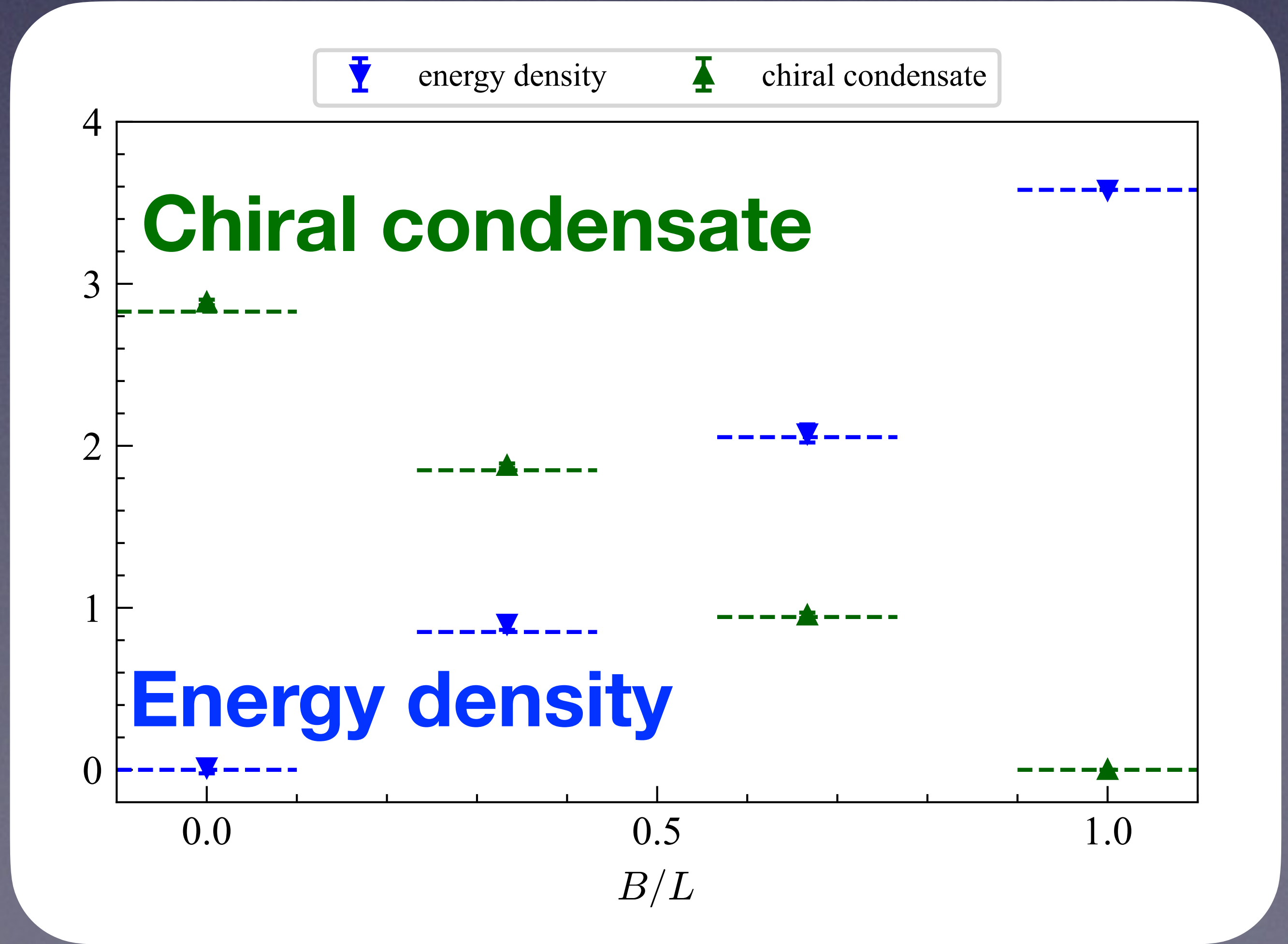
Three flavor QZ_3D

$(1+1)d$, 3-sites

Noiseless simulator

Quantum adiabatic algorithm





Summary

QCD is a fundamental theory of strong interactions

- Describes quarks and gluons with SU(3) gauge symmetry.
- Key features: confinement, chiral symmetry breaking.

Understanding QCD in medium is a frontier of hadron physics

- High T: chiral restoration (crossover).
- High μ: color superconductivity.
- Neutron star observations constrain EOS.

New approaches are needed for intermediate density

- Sign problem limits lattice QCD.
- Tensor networks and quantum computing are new promising tools.