Dbar-N interaction from Lattice QCD

Hadron in Nucleus 2025 (HIN25)

April 2, 2025 @ YITP

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RIKEN iTHEMS

For HAL QCD collaboration

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RIKEN interdisciplinary Theoretical & Mathematical Sciences

Content

This Talk

Study the $\bar{D}N$ interaction with Lattice QCD using configurations generated at "physical point" (pion mass $m_\pi = 137$ MeV) by the HAL QCD collaboration

- $\bar{D}N$ potential (LO derivative expansion)
- Scattering quantities (e.g. s-wave phase shift, scattering length, effective range)

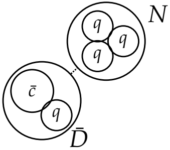
Collaborated work by the HAL QCD group.
Especially, Yan Lyu, Kotaro Murakami and Takumi Doi.

- Introduction
- Method: HAL QCD, Lattice Setup
- Results

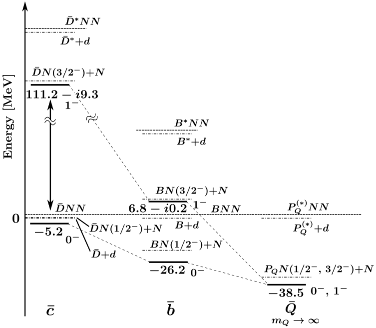
$\bar{D}N$ system

- "$\bar{c}$-quark version" of KN system

- No $q\bar{q}$ annihilation:
  Bound state $\rightarrow$ Pentaquark



A. Hosaka et al. / Progress in Particle and Nuclear Physics 96 (2017) 88–153

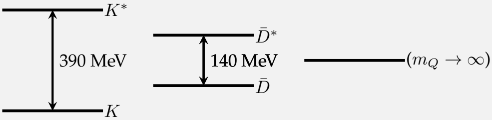


- Charmed nuclei, $\bar{D}NN$...
  Approximate degenarate states
  from HQSS ($j_l \geq 1$)
- in-medium effects to the $\bar{D}$-meson
  e.g. mass modification, impurity effects

Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP 96, 88 (2017)

Introduction

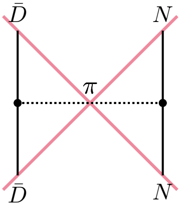
Coupling to the $\bar{D}^*N$ channel (is thought to) enhance attraction ($\leftrightarrow$ KN system)

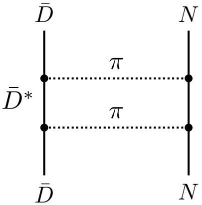


- Contact SU(4)$_\mathrm{F}$ “Tomozawa-Weinberg”

$$C^{(I=0)}_{\bar{D}N\bar{D}N} = 0 \quad \to \quad C^{(I=0)}_{ij} = \begin{bmatrix} 0 & \sqrt{12} \\ \sqrt{12} & -4 \end{bmatrix} \simeq \begin{bmatrix} -6 & 0 \\ 0 & 2 \end{bmatrix}$$

- π-exchange

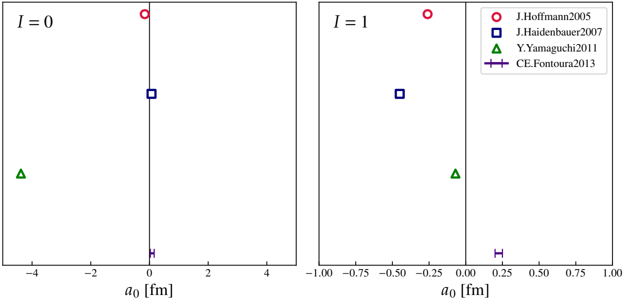




Current Status: Models

- No consistency of existance/absence of bound state, attractive/repulsive phase shift behavior between models

Model	Bound State ($I=0$) [MeV]	Bound State ($I=1$)[MeV]
Hoffmann 2005	×	×
Haidenbauer 2007	×	×
Gamermann 2010	2805	×
Yamaguchi 2011	2804	×
Yamaguchi 2022	2804	2800



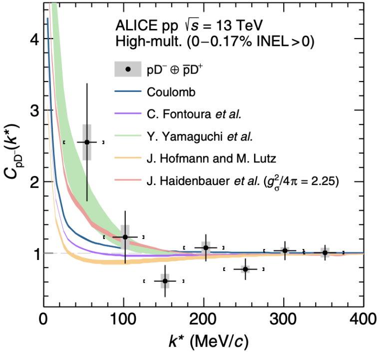
J.Hofmann, M.F.M.Lutz, Nucl.Phys. A763 (2005) 90-139
Y. Yamaguchi, S. Ohkoda, S. Yasui, A. Hosaka, Phys.Rev. D 84 (2011) 014032
D.Gamermann, C.Garcia-Recio, J.Nieves, L.L.Salcedo, L.Tolos, Phys.Rev. D81 (2010) 094016
J.Haidenbauer, G.Krein, U.-G.Meissner, A.Sibirtsev, Eur.Phys.J. A33 (2007) 107-117
C.E. Fontoura, G. Krein, V.E. Vizcarra, Phys.Rev. C 87 (2) (2013) 025206
Y. Yamaguchi, S. Ohkoda, S. Yasui, A. Hosaka, Phys.Rev. D 106, 094001 (2022)

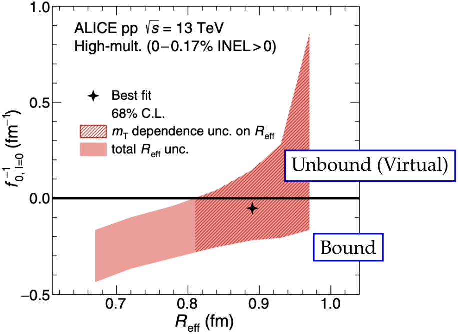
Current Status: Experiment

$D^{-}p$ correlation from pp collision

S. Acharya et al. ALICE collab. Phys.Rev.D 106 (2022) 5, 052010

- First experimental study of the two-body scattering of $\bar{D}N$
- Assuming negligible interaction in the $I = 1$ channel





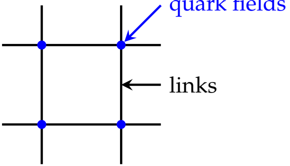
- inverse scattering length a_0^{-1}: -0.4 ~ +0.9 fm → Bound state or virtual state

Numerically evaluating the path integral of QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-S_{QCD}} \mathcal{O}$$

↓ Decretize spacetime

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod dU d\bar{\psi} d\psi e^{-S_{LQCD}} \mathcal{O}$$



- Evaluate the decritized path integral with Monte-Carlo methods $\rightarrow \langle \mathcal{O} \rangle$

$\bar{D}N$ system is suited for lattice simulations:

- No open channels ($\leftrightarrow DN$ system has lower open channels e.g. $\pi\Lambda_c$, $\pi\Sigma_c^{(*)}$)
- No $q\bar{q}$ annihilation suited for Lattice simulations ($q\bar{q}$ annihilation $\rightarrow$ large computational cost)

Scattering on the Lattice

(2-body) scattering on the Lattice

$$\mathcal{C}(t,\vec{r}) = \sum_{\vec{x}} \langle \mathcal{O}_1(t,\vec{x}+\vec{r})\mathcal{O}_2(t,\vec{x})\bar{\mathcal{J}}(0)\rangle$$
$$= \sum_{n} A_n \psi(\vec{r};E_n)e^{-E_n t}$$

Obtain scattering info (Phase shift) from Euclidean-time correlation function:

$$\mathcal{C}(t,\vec{r}) \longrightarrow \text{Phase shift } \delta$$

- Lüscher's finite volume method

  M. Lüscher, Nucl.Phys.B 354 531-578 (1991)

  Energy spectra in finite volume $\{E_n\}$ $\rightarrow$ quatization condition (e.g. Lüscher's formula) $\rightarrow$ Phase shift

- HALQCD method

  N. Ishii, S. Aoki, and T. Hatsuda PRL 99, 022001 (2007)

  Temporal + spacial info. $\psi(\vec{r};E_n)$ $\rightarrow$ Potential $\rightarrow$ Phase shift

HALQCD

$$\mathcal{C}(t,\vec{r}) = \sum_{\vec{x}} \langle \mathcal{O}_1(t,\vec{x}+\vec{r})\mathcal{O}_2(t,\vec{x})\bar{\mathcal{J}}(0)\rangle = \sum_n A_n \psi(\vec{r};E_n)e^{-E_n t}$$

$$\psi(\vec{r}) \to \frac{\sin(kr - l\pi/2 + \delta(k))}{kr} e^{i\delta(k)} \quad (r \gg R)$$

$$(\nabla^2 + k^2)\psi(\vec{r};E_n) = \int d\vec{r}' V(\vec{r},\vec{r}')\psi(\vec{r}';E_n)$$

- Energy-independent interaction kernel $V(\vec{r},\vec{r}')$ produces the correct phase shift of the QCD S-matrix in the elastic region

Time-dependent method

Ishii et al. (HAL QCD), PLB712, 437(2012)

$$R(t,\vec{r}) = \frac{\mathcal{C}(t,\vec{r})}{\langle \mathcal{O}_1(t)\rangle \langle \mathcal{O}_2(t)\rangle}$$

$$\left[\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right]R(t,\vec{r}) = \int d\vec{r}' V(\vec{r},\vec{r}')R(t,\vec{r}')$$

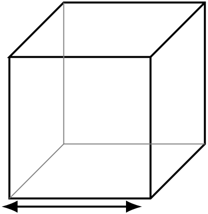
- Excited-state contamination surpressed

Setup

Configuration (F-conf)

Phys. Rev. D 110, 094502 (HAL QCD collab.)

- Volume: 96×96^3
- Iwasaki gauge action (β = 1.82)
- 2+1 flavor, $\mathcal{O}$(a)-improved Wilson quark action
- Relativistic heavy-quark action for c quark
  slightly heavy / light c quark (set1 / set2)
- Statistics: 360 configurations $\times$ 96 source $\times$ 4

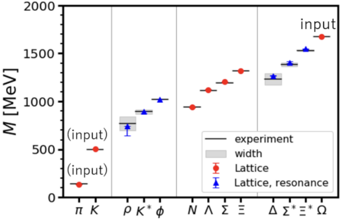


$L \simeq 8$ fm
$m_\pi \simeq 137$ MeV

$a \simeq 0.084$ fm

	Lattice [MeV]	PDG [MeV]
m_π	137.1	138
m_N	939.7	938

	set-1 [MeV]	set-2 [MeV]	PDG [MeV
m_D	1880.1	1854.3	1865
m_{D^*}	2017.8	1994.1	2007



Phys. Rev. D 110, 094502 (HALQCD)

Setup

In the following:

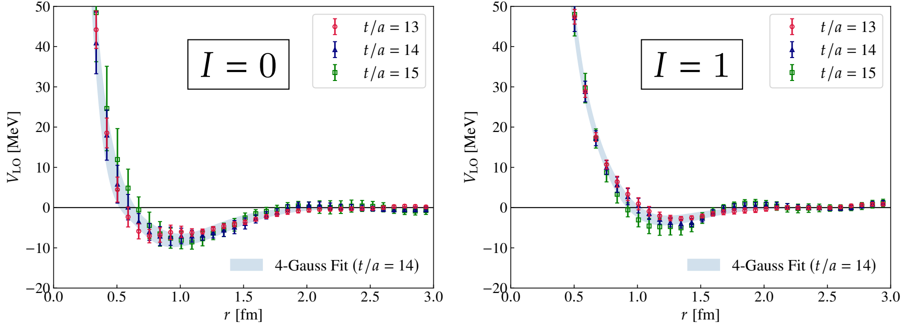
- Only consider $\bar{D}N$ channel (valid upto $\bar{D}^*N$ threshold)
- Jackknife sampling methods for statistical uncertainties from MC
  sbin = 30 (binsize dependence negligible)
- A_1^+ projection + Misner method (T. Miyamoto et.al. Phys. Rev. D 101, 074514 (2020))
  Suppress $l = 4,\ 6,\ \cdots$ contributions
- Linear extrapolation of set-1 and set-2 correlators to physical D^0 mass (PDG)

$$R^{\text{phys}}(t,\vec{r}) \simeq 0.411 \times R^{(1)}(t,\vec{r}) + 0.589 \times R^{(2)}(t,\vec{r})$$

s-wave $\bar{D}N$ potential

LO of the derivative expansion of $V(r, r')$

$$V(\vec{r}, \vec{r}') = V_{\text{LO}}(r)\,\delta(\vec{r} - \vec{r}') + \sum V_k \nabla^k \delta(\vec{r} - \vec{r}')$$



- Repulsive core + Attractive pocket
- $I = 0$ potential is more attractive than $I = 1$ potential in all regions

4 Gaussian Fit

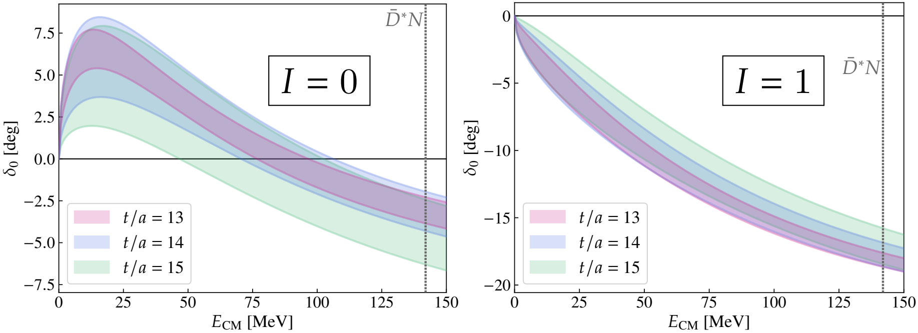
$$V_{LO}^{\text{fit}}(r) = \sum_{k=1}^{4} a_i \, \exp[-r^2/b_i^2]$$

	χ^2/dof $(I = 0)$	χ^2/dof $(I = 1)$
$t/a = 13$	0.31	1.36
$t/a = 14$	0.46	0.82
$t/a = 15$	0.24	0.66

s-wave Phase Shifts

Solve schrödinger equation using fitted potential

$$\left[H_0 + V_{LO}^{\text{fit}}\right]\psi(r) = E\psi(r)$$

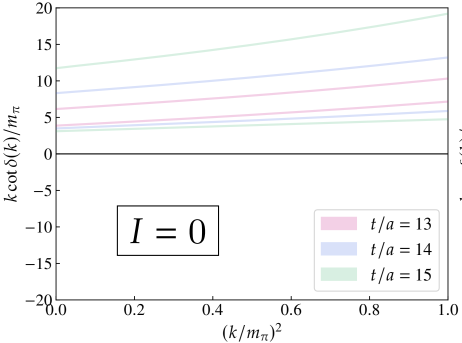


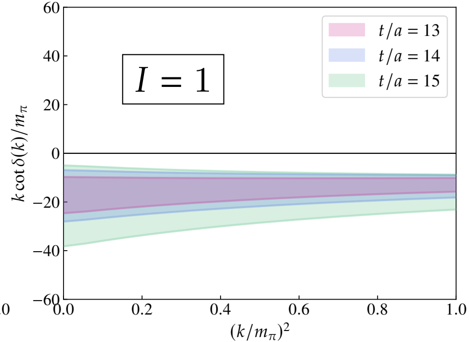
- I=0: Phase shift shows weak attractive behavior in the low-energy region
- I=1: Phase shift shows repulsive behavior in all energy regions
- <u>No bound state in I=0 and I=1 channel</u>

Scattering Length, Effective Range

Effective-Range Expansion

$$k \cot \delta(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$





	a_0 [fm]	r_0 [fm]
$I = 0$ ($t/a = 14$)	0.247 ± 0.105	8.92 ± 2.97
$I = 1$ ($t/a = 14$)	-0.084 ± 0.049	15.21 ± 26.49

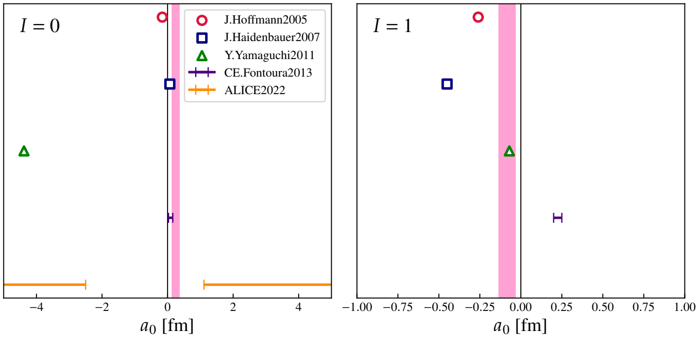
- Scattering length: systematic uncertainties from choosing different time slices ~20% (I=0), ~12% (I=1)

Comparison: Models, Femtoscopy

- No bound states in I=0 and I=1

Study	Bound State ($I = 0$) [MeV]	Bound State ($I = 1$)[MeV]
Hoffmann 2005	×	×
Haidenbauer 2007	×	×
Gamermann 2010	2805	×
Yamaguchi 2022	2804	2800
ALICE 2022	△	—
HAL 2025	×	×

- Scattering Length a_0 (Pink band: Our Result)



Summary

Summary

- $\bar{D}N$ system

  Possibility of Pentaquark, Charmed nuclei...
- Limited experimental data input for theoretical studies

  $\rightarrow$ Lattice simulations (expect good signal because no lower open channels, no $q\bar{q}$ annihilation)
- Our Results:
  - (Small) Attractive behavior in the low energy region of I=0 channel
  - Repulsive behavior in the I=1 channel
  - No bound states for both I=0 and I=1
  - Results differ from models (& ALICE results)

Outlook

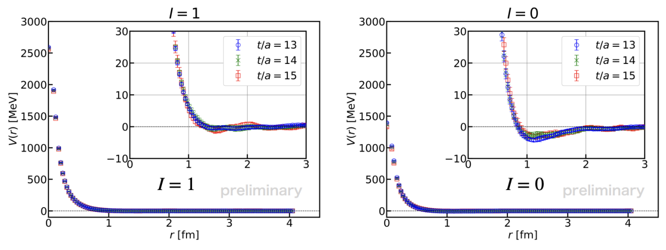
- Coupled-channel analysis of $\bar{D}N$-$\bar{D}^*N$

  Coupling of $\bar{D}N$-$\bar{D}^*N$ is important to explain the attraction
- Multi-body systems (e.g. $\bar{D}NN$...) or Femtoscopy analysis using the $\bar{D}N$ HALQCD potential

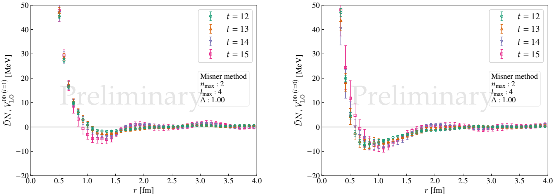
Supplementary Materials

$\bar{D}N$ has stronger attraction than KN

KN potential by Kotaro Murakami (HADRON 2025)



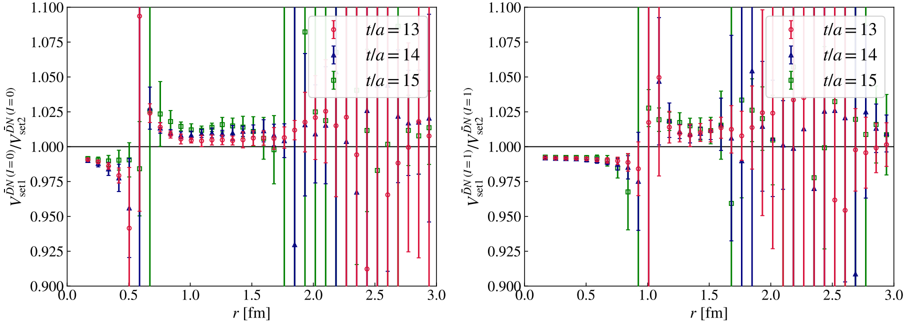
$\bar{D}N$



LO potential: m_c dependence

Ratio between set-1 LO potential and set-2 LO potential

$$\frac{V^{I}_{LO}}{V^{II}_{LO}}$$



- Heavier c quark $\rightarrow$ stronger attraction ($\sim$ 1-2 %)
  c.f. mass difference of $\bar{D}$ meson between set-1 & set-2 $\approx$ 1.5%

4-gauss Fit and tail behavior of the LO potential

4-gauss Fit of the $\bar{D}N$ LO potential

$$V_{LO}^{\text{fit}} = a_1\, e^{-(b_1 r)^2} + a_2\, e^{-(b_2 r)^2} + a_3\, e^{-(b_3 r)^2} + a_4\, e^{-(b_4 r)^2}$$

	$I = 0\ (t/a = 14)$	$I = 1\ (t/a = 14)$
a_1 [latt unit]	0.12496607 ± 0.10321802	0.16384792 ± 0.05043241
b_1 [latt unit]	0.11414286 ± 0.04742497	0.11571616 ± 0.00822781
a_2 [latt unit]	0.10975638 ± 0.01943871	0.16256342 ± 0.01280945
b_2 [latt unit]	0.3704377 ± 0.03930028	0.31399445 ± 0.0126255
a_3 [latt unit]	0.15532092 ± 0.02018262	0.30985708 ± 0.01617404
b_3 [latt unit]	0.75811416 ± 0.04901691	0.6783387 ± 0.01245769
a_4 [latt unit]	−0.11483858 ± 0.11301206	−0.13140795 ± 0.05119623
b_4 [latt unit]	0.10176603 ± 0.01632742	0.10814738 ± 0.00685318

Range of attractive tail $\sim\ b_4$

$$b_4^{I=0} = 238 \pm 38 \text{ MeV}$$
$$b_4^{I=1} = 253 \pm 16 \text{ MeV}$$

c.f. $2m_\pi \approx 274$ MeV

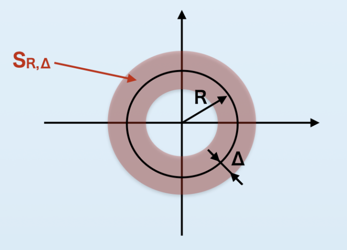
$\bar{D}N$ potential: Misner Method

Higher partial-wave contamination in the A_1^+ R-correlator

- Misner method

  C. W. Misner, Class. Quantum Grav. 21 (2004) S243-S247

  T. Miyamoto et. al. Phys. Rev. D 101, 074514 (2020)



From T.Miyamoto slides 2019

$$R(\vec{r}) \approx \sum_{nlm} c_{nlm} G_n^{R,\Delta}(R) Y_{lm}(\hat{r})$$

$$G_n^{R,\Delta}(R) = \frac{1}{r}\sqrt{\frac{2n+1}{2\Delta}} P_n\left(\frac{r-R}{\Delta}\right)$$

$$c_{nlm} = \langle \tilde{\mathcal{Y}}_{nlm} | \mathcal{Y}_{nlm} \rangle$$

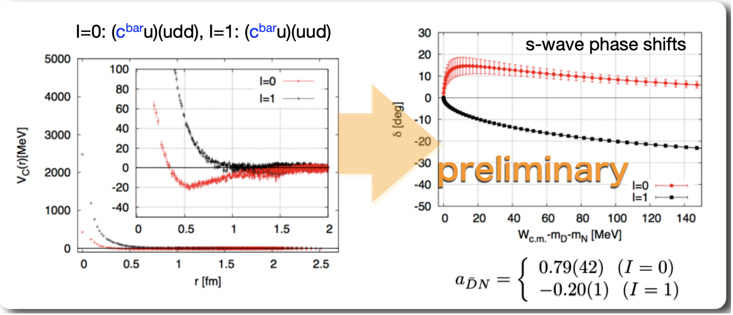
In the following the expansion of the R-correlator is trancated at

- nmax=2, lmax=4
- Δ=1.00 lattice unit

m_π dependence

Comparison to results with heavier pion mass $m_\pi = 410$ MeV (PACS-CS config.)

Y. Ikeda slides 10th APCTP-BLTP/JINR-RCNP-RIKEN Joint Workshop Aug.2016



$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I=0) \\ -0.20(1) & (I=1) \end{cases}$$

- $I = 0$: smaller pion mass, shallower attractive pocket
  ($m_\pi \simeq 137$ MeV case) has smaller scattering length but is still positive
- $I = 1$: ($m_\pi \simeq 137$ MeV case) has slightly smaller repulsion
  scattering length closer to zero