Two-color QCD as a laboratory to explore diquark dynamics in medium

Daiki Suenaga (KMI/Nagoya U, Japan)

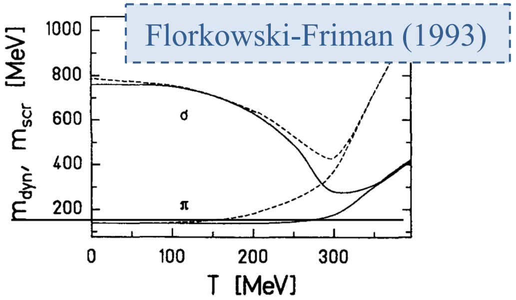
Introduction

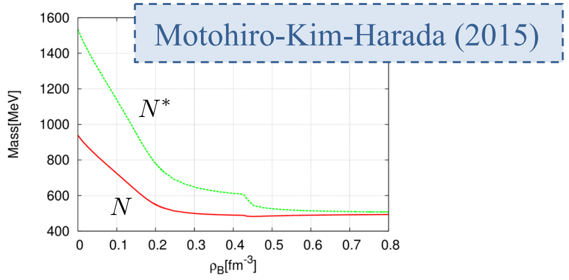
- **Chiral symmetry and hadron masses**
  - Mass difference of $\pm$-parity hadrons would be driven by chiral symmetry breaking

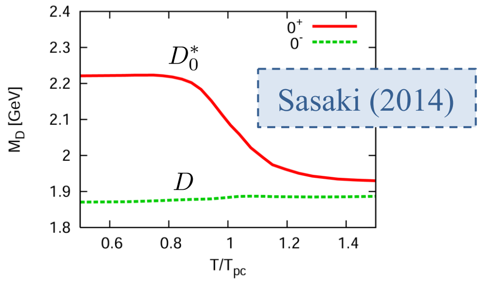
    = chiral partner structure (chiral doubling)

mass difference from chiral symmetry breaking

$f_0(500)$ (?)	$N^*(1535)$	D_0^*	(unobserved)	
π	N	D	Λ_c	$\cdots$







$\cdots$

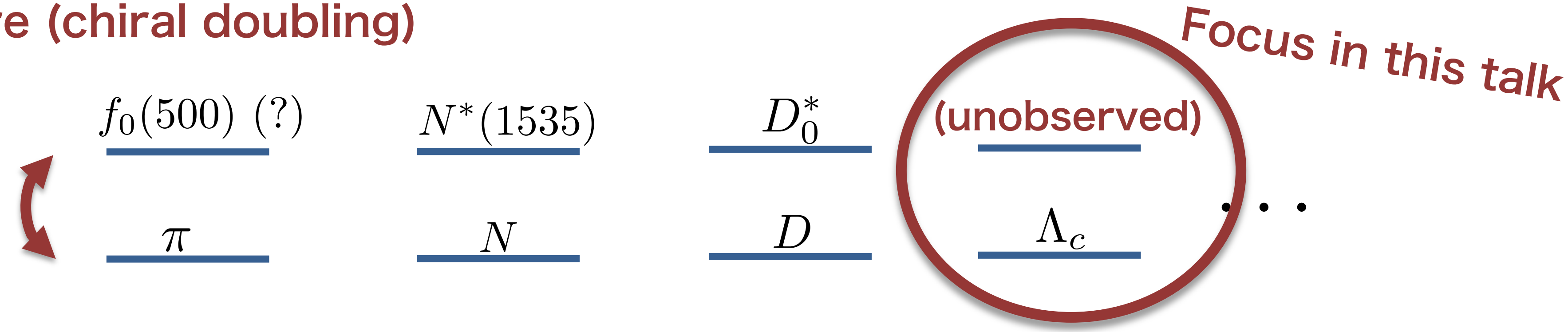
⇨ Unified picture of **hadron mass generation from chiral symmetry**

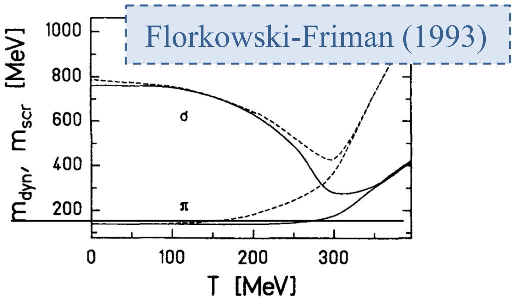
|| underlying QCD symmetry

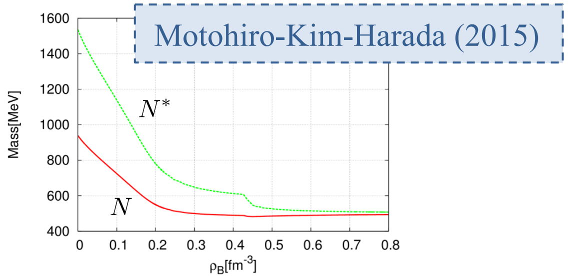
Introduction

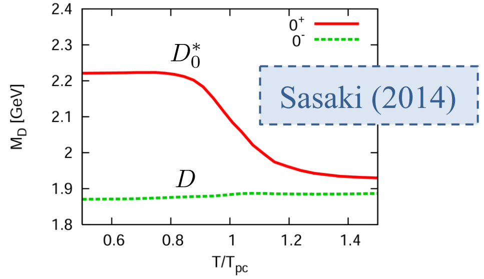
- **Chiral symmetry and hadron masses**
  - Mass difference of ± -parity hadrons would be driven by chiral symmetry breaking
    = chiral partner structure (chiral doubling)

mass difference from
chiral symmetry breaking









. . .

⇨ Unified picture of **hadron mass generation from <u>chiral symmetry</u>**

||

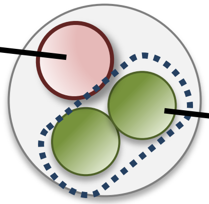
<u>underlying QCD symmetry</u>

Introduction

• Diquarks in singly heavy baryons (SHBs)

- Chiral dynamics of SHBs (Λ_c etc.) is derived by diquarks inside

heavy quark
= spectator



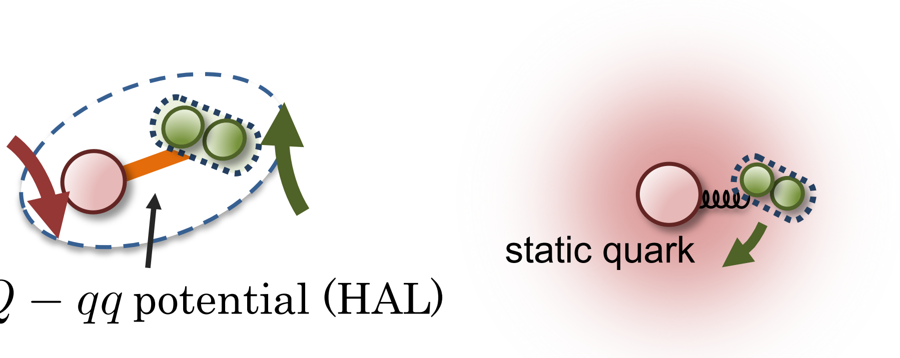
(light) diquark

controls interactions with "QCD vacuum", pions, etc.

Chiral model, QCD sum rule approaches

Hong-Song (2012), Harada et al (2020), Azizi-Turkan (2020) etc.

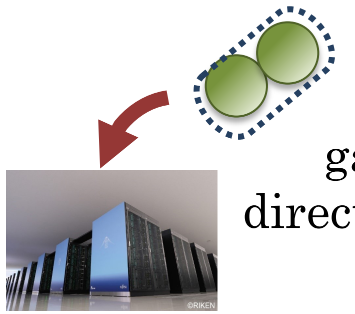
- Lattice studies of diquarks



$Q - qq$ potential (HAL)

static quark

by RCNP (Ishii-san) group



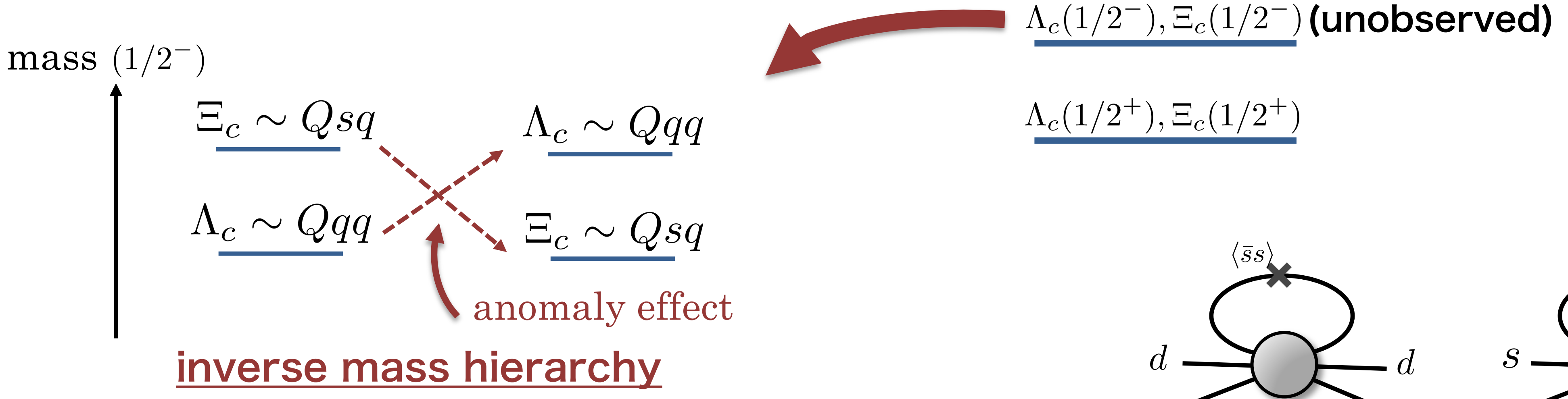
gauge-fixed
direct computation

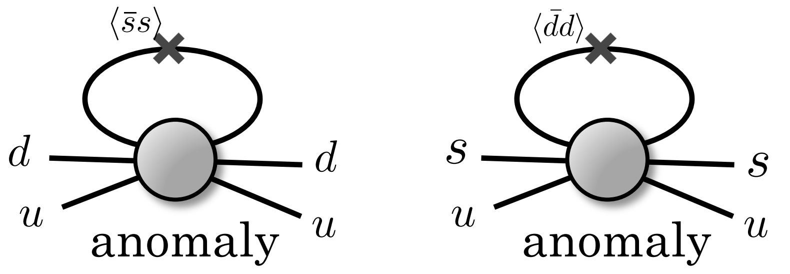
Bi-Cai-Chen-Gong-Liu-Qiao-Yang (2016), etc.

Introduction

- **U(1)$_A$ anomaly effects on HQS-singlet SHBs**
  - Mass hierarchy of HQS-singlet $\Lambda_c(1/2^-)$ and $\Xi_c(1/2^-)$ can be inverted [Harada-Liu-Oka- Suzuki, PRD(2020)]

    Theoretical suggestion from chiral models of diquark



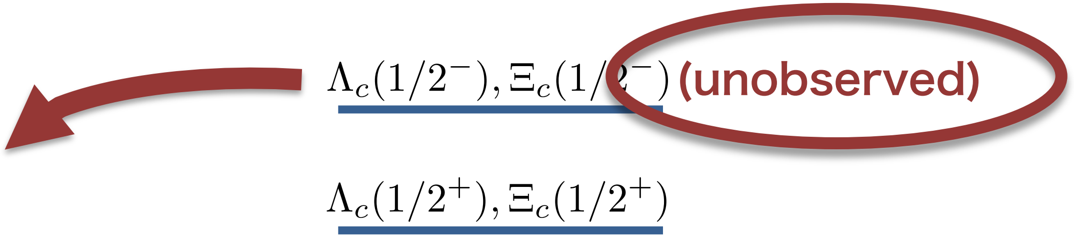


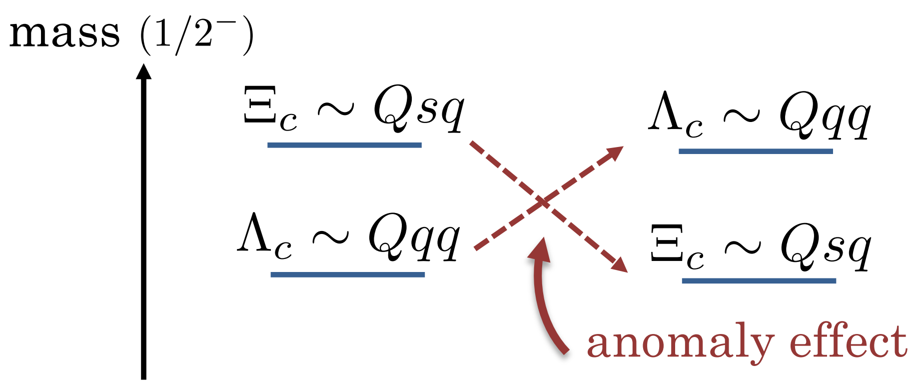
eg, $\langle \bar{s}s \rangle$ cont. to ud diquark
(flavor mixing structure)

• $U(1)_A$ anomaly effects on HQS-singlet SHBs

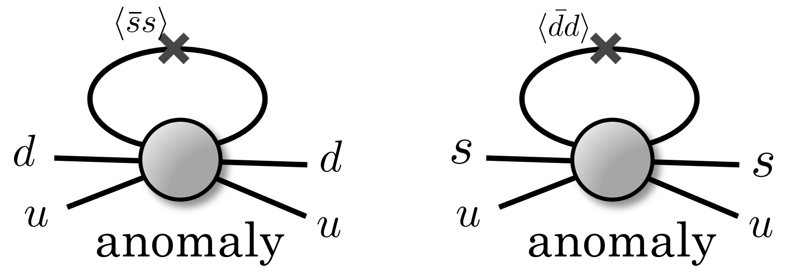
- Mass hierarchy of HQS-singlet $\Lambda_c(1/2^-)$ and $\Xi_c(1/2^-)$ can be inverted   Harada-Liu-Oka- Suzuki, PRD(2020)

Theoretical suggestion from chiral models of diquark





inverse mass hierarchy



eg, $\langle \bar{s}s \rangle$ cont. to ud diquark
(flavor mixing structure)

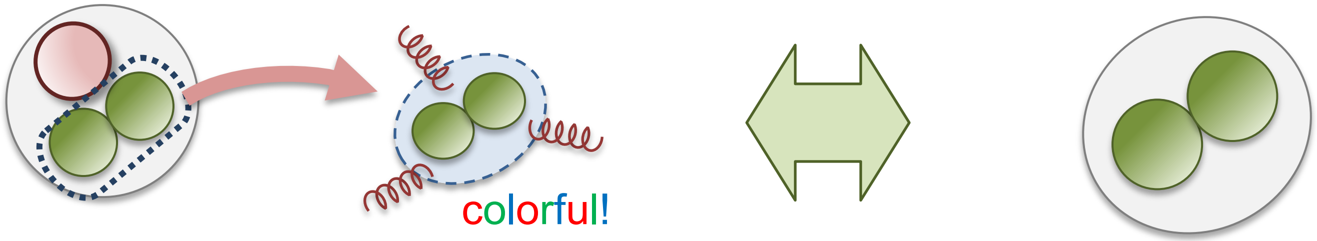
- Are there other testing ground to see those diquark properties?

Introduction

- **Two-color QCD (=QC_2D)**

Strong interaction with $N_c = 2$

- In QC_2D world diquarks become color-singlet



Color-singlet HADRON (baryon)!

well-defined!

for $N_c = 3$

for $N_c = 2$

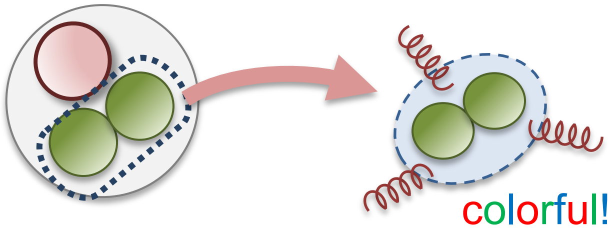
QC_2D world can be a useful testing ground to explore diquark dynamics with a solid argument

Introduction

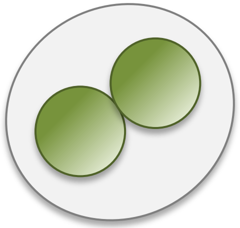
- **Two-color QCD (=QC$_2$D)**

Strong interaction with $N_c = 2$

- In QC$_2$D world diquarks become color-singlet



for $N_c = 3$



Color-singlet HADRON (baryon)!

well-defined!

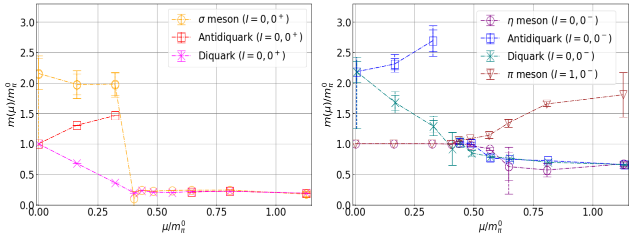
for $N_c = 2$

QC$_2$D world can be a useful testing ground to explore diquark dynamics with a solid argument

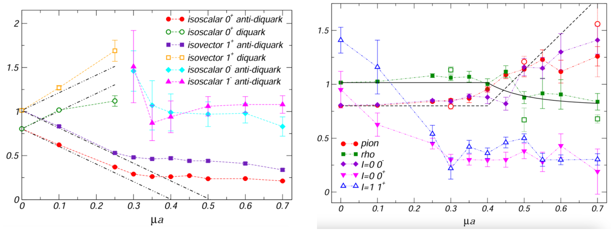
- Lattice computation on QC$_2$D is straightforward

Numerical experiments are being done!

eg, Hadron mass spectrum from lattice (at finite μ)



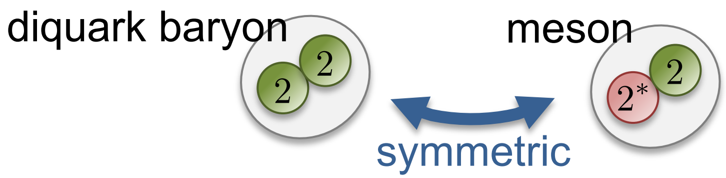
Murakami et al, PoS(2022)



Hands et al, PLB(2007)

• Chiral symmetry in QC_2D

- Diquark baryons and mesons are treated in a unified way

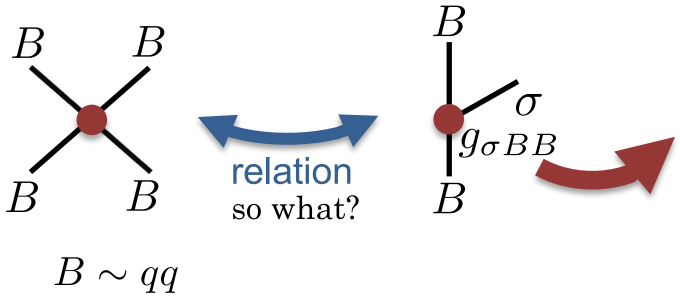


$2 \simeq 2^*$:pseudoreality of color SU(2)

→ Chiral symmetry (flavor structure) is extended from $SU(N_f)_L \times SU(N_f)_R$ to $SU(2N_f)$

- The extended $SU(2N_f)$ symmetry doesn't matter, since this symmetry just relates couplings among diquarks and mesons

eg



- From the viewpoint of mass generation, only $g_{\sigma BB}$ is important *regardless of the coupling relations*
- $U(1)_A$ anomaly *universally exists regardless of N_c*

Introduction

• History of QC_2D studies

- Lattice activities (with chemical potential)

  → Phase diagram, thermodynamics, conductivity, hadron mass, HAL method, etc.

numerical experiments

Nakamura, PLB 149, 391 (1984)
Hands-Kogut-Lombardo-Morrison, NPB 558, 327 (1999)
Muroya-Nakamura-Nonaka-Takaishi, PTP 110, 615 (2003)
:
Boz-Hajizadeh-Maas-Skullerud, PRD 99, 074514 (2019)
Buividovich-Smith-Smekal, PRD 102, 094510 (2020)
Astrakhantsev-Braguta-Ilgenfritz-Kotov-Nikolaev, PRD 102, 074507 (2020)
Iida-Itou-Murakami-Suenaga, JHEP 10, 022 (2024)
:

(currently)
Japanese group
Ireland/UK group
UK group
Russian group
:

- Model studies

  - ChPT / LSM / NJL approach
  - massive gluon model
  - pQCD approach
  - FRG / DS approach
  - :

Kogut-Stephanov-Toublan-Verbaarschot-Zhitnitsky, NPB 582, 477 (2000)
Ratti-Weise, PRD 70, 054013 (2004)
Sun-He-Zhuang, PRD 75, 096004 (2007)
:
Suenaga-Kojo, PRD 100, 076017 (2019)
Contant-Huber, PRD 101, 014016 (2020)
Suenaga-Murakami-Itou-Iida, PRD 107, 054001 (2023)
Suenaga, Symmetry 17, 124 (2025) [REVIEW PAPER]
:

Long history!
(broadly studied)

Model

- **Pauli-Gursey $SU(2N_f)$ symmetry in QC$_2$D**

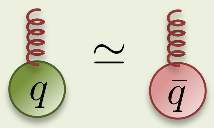
  - Pseudoreality of $SU(2)_c$ leads to rewriting QC$_2$D Lagrangian as

$$\mathcal{L}_{\mathrm{QC_2D}} = \bar{\psi} i \partial\!\!\!/ \psi - g_s \bar{\psi} A\!\!\!/^a T_c^a \psi = \Psi^\dagger i \partial_\mu \sigma^\mu \Psi - g_s \Psi^\dagger A_\mu^a T_c^a \sigma^\mu \Psi$$

pseudoreality: $\sigma^2 \sigma^a \sigma^2 = -(\sigma^a)^*$, $T_c^2 T_c^a T_c^2 = -(T_c^a)^*$

  - 2Nf-component vector: $\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \cdots, \tilde{u}_L, \tilde{d}_L, \cdots)$ with $\tilde{\psi}_L = \sigma^2 \tau_c^2 \psi_L^*$
  - Four-dimensional Pauli matrix: $\sigma^\mu = (1, \sigma^i)$

  - $\mathcal{L}_{\mathrm{QC_2D}}$ is obviously invariant under $\Psi \to g\Psi$ $[g \in SU(2N_f)]$



gluons are blind

$SU(N_f)_L \times SU(N_f)_R$ chiral symmetry → (enlarged) <u>Pauli-Gursey $SU(2N_f)$ symmetry</u> Pauli (1957), Gursey (1958)

Model

- **Nf=2+1 linear sigma model (LSM)**
  - (approximately) $SU(6)$-invariant $(N_f = 3)$ LSM Lagrangian is given by

$$\mathcal{L}_{\rm LSM} = {\rm tr}[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] + \underline{{\rm tr}[H^\dagger \Sigma + \Sigma^\dagger H]} - m_0^2 {\rm tr}[\Sigma^\dagger \Sigma] - \lambda_1 \left({\rm tr}[\Sigma^\dagger \Sigma]\right)^2 - \lambda_2 {\rm tr}\left[(\Sigma^\dagger \Sigma)^2\right] - c\, \epsilon_{ijklmn}\left(\Sigma_{ij}\Sigma_{kl}\Sigma_{mn} - \Sigma^\dagger_{ij}\Sigma^\dagger_{kl}\Sigma^\dagger_{mn}\right)$$

$U(1)_A$ anomaly

$$H = \bar{c} m_q E_q + \bar{c} m_s E_s \quad \text{with} \quad E_q = \begin{pmatrix} 0 & I_q \\ -I_q & 0 \end{pmatrix} \quad E_s = \begin{pmatrix} 0 & I_s \\ -I_s & 0 \end{pmatrix} \quad \text{and} \quad I_q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad I_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

current-quark mass effect

$$\Sigma = \frac{1}{2}\begin{pmatrix} B & M \\ -M^T & \bar{B} \end{pmatrix} \quad \text{with}$$

6×6 matrix

$$B = \begin{pmatrix} 0 & -B'_{[ud]} + iB_{[ud]} & -B'_{[su]} + iB_{[su]} \\ B'_{[ud]} - iB_{[ud]} & 0 & -B'_{[sd]} + iB_{[sd]} \\ B'_{[su]} - iB_{[su]} & B'_{[sd]} - iB_{[sd]} & 0 \end{pmatrix}$$

$$\bar{B} = \begin{pmatrix} 0 & -\bar{B}'_{[ud]} + i\bar{B}_{[ud]} & -\bar{B}'_{[su]} + i\bar{B}_{[su]} \\ \bar{B}'_{[ud]} - i\bar{B}_{[ud]} & 0 & -\bar{B}'_{[sd]} + i\bar{B}_{[sd]} \\ \bar{B}'_{[su]} - i\bar{B}_{[su]} & \bar{B}'_{[sd]} - i\bar{B}_{[sd]} & 0 \end{pmatrix}$$

$$M = \begin{pmatrix} \frac{\sigma_N - i\eta_N + a_0^0 - i\pi^0}{\sqrt{2}} & a_0^+ - i\pi^+ & \kappa^+ - iK^+ \\ a_0^- - i\pi^- & \frac{\sigma - i\eta_N - a_0^0 + i\pi^0}{\sqrt{2}} & \kappa^0 - iK^0 \\ \kappa^- - iK^- & \bar{\kappa}^0 - i\bar{K}^0 & \sigma_S - i\eta_S \end{pmatrix}$$

$\ni$

parity (chiral) partner

$\eta, \pi, K \leftrightarrow \sigma, a_0, \kappa$

0^- meson 0^+ meson

$B_{[ud]}, B_{[su]}, B_{[sd]} \leftrightarrow B'_{[ud]}, B'_{[su]}, B'_{[sd]}$

0^+ diquark 0^- diquark

Model

- **Some technical**

  - Mean fields $\langle\sigma_N\rangle, \langle\sigma_S\rangle$ determined by $\frac{\partial\langle\mathcal{L}_{\rm LSM}\rangle}{\partial\langle\sigma_N\rangle} = 0\,, \quad \frac{\partial\langle\mathcal{L}_{\rm LSM}\rangle}{\partial\langle\sigma_S\rangle} = 0$

  - Diquark masses $m^2_{B_{[qq]}(0^+)} = -\frac{\partial^2\mathcal{L}_{\rm LSM}}{\partial\bar{B}_{[ud]}\partial B_{[ud]}}\Big|_{\partial_\mu\to 0}\,, \quad \cdots$

  - Coupling for $B'_{[qq]}(0^-) \to B_{[qq]}(0^+) + \eta$ decay

$$g_{B'B\eta} = \frac{\partial\mathcal{L}_{\rm LSM}}{\partial\eta\partial\bar{B}_{[ud]}\partial B'_{[ud]}} = -\frac{\lambda_2}{2}\bar{\sigma}_N\cos\theta_\eta - 12c\sin\theta_\eta$$

with

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_\eta & -\sin\theta_\eta \\ \sin\theta_\eta & \cos\theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \\ \eta_S \end{pmatrix}$$

$$\tan 2\theta_\eta = \frac{2m^2_{\eta_N\eta_S}}{m^2_{\eta_S} - m^2_{\eta_N}} \quad (\eta - \eta' \text{ mixing})$$

$$\Sigma = \frac{1}{2}\begin{pmatrix} B & M \\ -M^T & \bar{B} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & -B'_{[ud]} + iB_{[ud]} & -B'_{[su]} + iB_{[su]} \\ B'_{[ud]} - iB_{[ud]} & 0 & -B'_{[sd]} + iB_{[sd]} \\ B'_{[su]} - iB_{[su]} & B'_{[sd]} - iB_{[sd]} & 0 \end{pmatrix}$$

$$\bar{B} = \begin{pmatrix} 0 & -\bar{B}'_{[ud]} + i\bar{B}_{[ud]} & -\bar{B}'_{[su]} + i\bar{B}_{[su]} \\ \bar{B}'_{[ud]} - i\bar{B}_{[ud]} & 0 & -\bar{B}'_{[sd]} + i\bar{B}_{[sd]} \\ \bar{B}'_{[su]} - i\bar{B}_{[su]} & \bar{B}'_{[sd]} - i\bar{B}_{[sd]} & 0 \end{pmatrix}$$

$$M = \begin{pmatrix} \frac{\sigma_N - i\eta_N + a_0^0 - i\pi^0}{\sqrt{2}} & a_0^+ - i\pi^+ & \kappa^+ - iK^+ \\ a_0^- - i\pi^- & \frac{\sigma - i\eta_N - a_0^0 + i\pi^0}{\sqrt{2}} & \kappa^0 - iK^0 \\ \kappa^- - iK^- & \bar{\kappa}^0 - i\bar{K}^0 & \sigma_S - i\eta_S \end{pmatrix}$$

- Let's focus on diquark mass hierarchy and stability of $B'_{[qq]}(0^-)$ ⇒
  - insight into inverse mass hierarchy
  - hint for unobserved $\Lambda_c(1/2^-)$
    - main mode: $\Lambda_c(1/2^-) \to \Lambda_c(1/2^+) + \eta$

Results

- **Mass and $B'_{[qq]}(0^-)$ width**
  - Parameters : $\bar{c}m_q, \bar{c}m_s, m_0^2, \lambda_1, \lambda_2, c$
  - Choose anomaly effect C as a variable

<u>input</u>

lattice: $f_\pi = 60\,\mathrm{MeV}$

Iida+ (2024)

physical: $f_K \approx 1.2 f_\pi$

large Nc: $\lambda_1 = 0$

+

Iida+ (2021)

$m_q = 100\,\mathrm{MeV}$

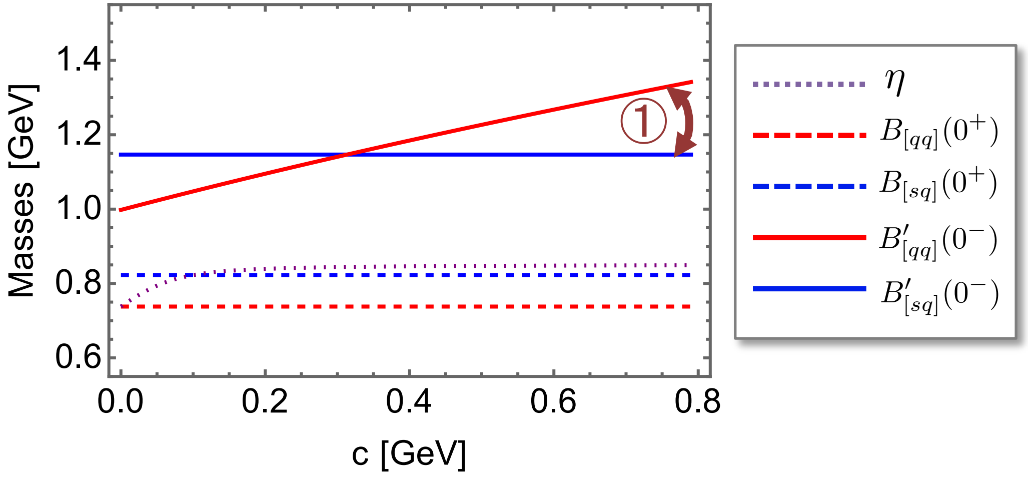
$m_s = 200\,\mathrm{MeV}$

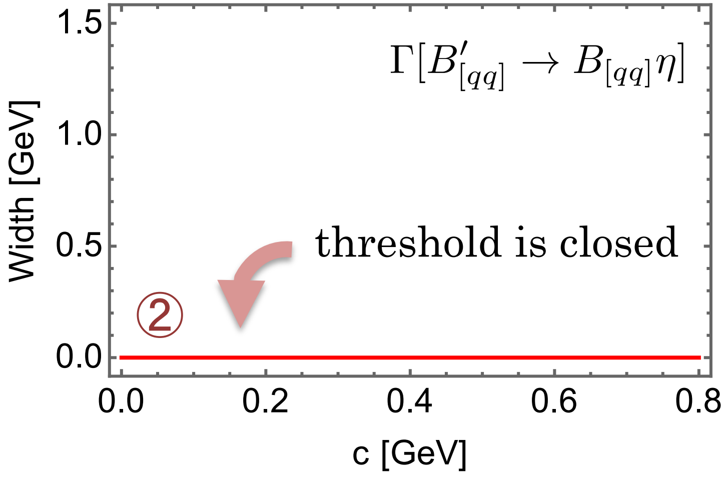
$(\bar{c} \approx 163\,\mathrm{MeV}^2)$

cf, $m_{qq(0+)} = 738\,\mathrm{MeV}$ from lattice

Murakami+ (2022)

mass spectrum





① The inverse mass hierarchy $M_{B'_{[qq]}} > M_{B'_{[sq]}}$ for larger C is derived ↔ consistent with Nc=3 SHB analysis

② This input makes $B'_{[qq]}$ stable for reasonable range of C ⇨ stably measurable

Results

- **Mass and $B'_{[qq]}(0^-)$ width**

  - Parameters : $\bar{c}m_q, \bar{c}m_s, m_0^2, \lambda_1, \lambda_2, c$

  - Choose anomaly effect C as a variable

input

lattice: $f_\pi = 60\,\mathrm{MeV}$

Iida+ (2024)

physical: $f_K \approx 1.2 f_\pi$

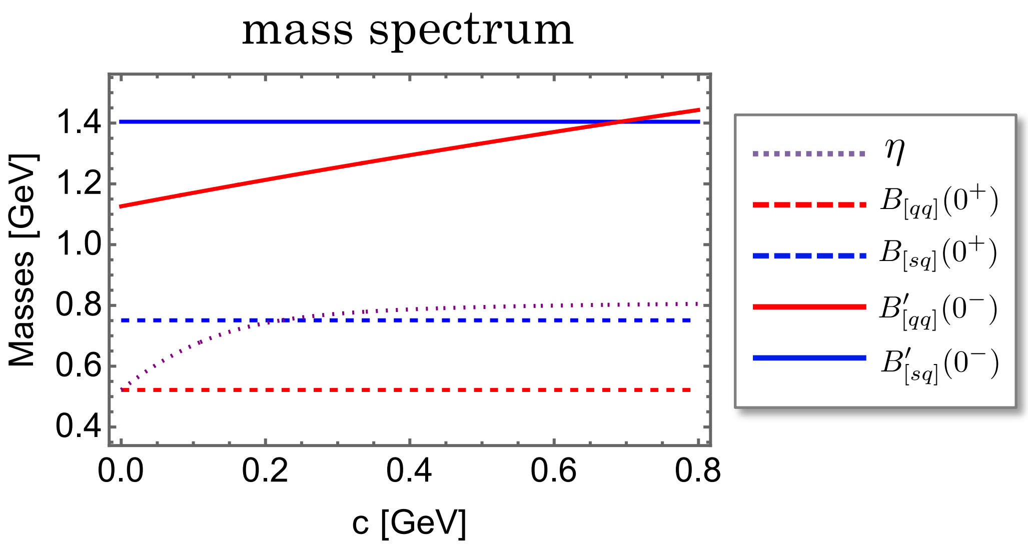
large Nc: $\lambda_1 = 0$

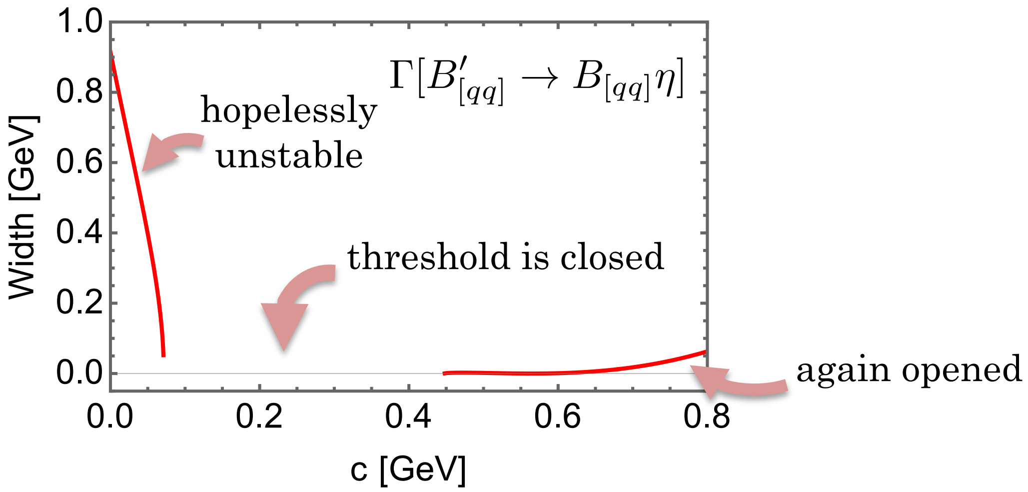
$+$

$m_q = 50\,\mathrm{MeV}$

$m_s = 200\,\mathrm{MeV}$

$(\bar{c} \approx 163\,\mathrm{MeV}^2)$





  - The inverse mass hierarchy $M_{B'_{[qq]}} > M_{B'_{[sq]}}$ for larger C

  - With this input $B'_{[qq]}$ is only stabilized for $0.07\,\mathrm{GeV} \lesssim c \lesssim 0.45\,\mathrm{GeV}$

Results

- **Mass and $B'_{[qq]}(0^-)$ width**
  - Parameters : $\bar{c}m_q, \bar{c}m_s, m_0^2, \lambda_1, \lambda_2, c$
  - Choose anomaly effect C as a variable

<u>input</u>

lattice: $f_\pi = 60\,\mathrm{MeV}$ Iida+ (2024)

physical: $f_K \approx 1.2 f_\pi$

large Nc: $\lambda_1 = 0$

\+

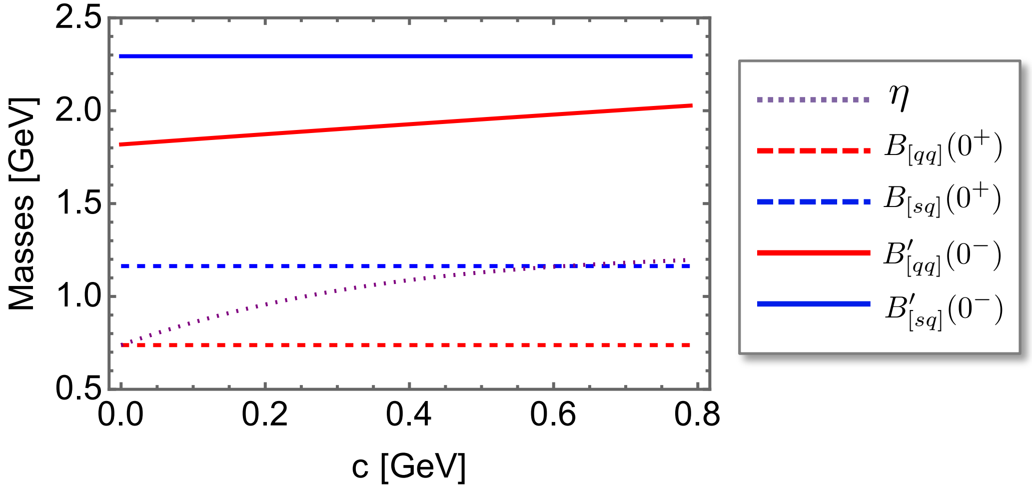
Iida+ (2021)

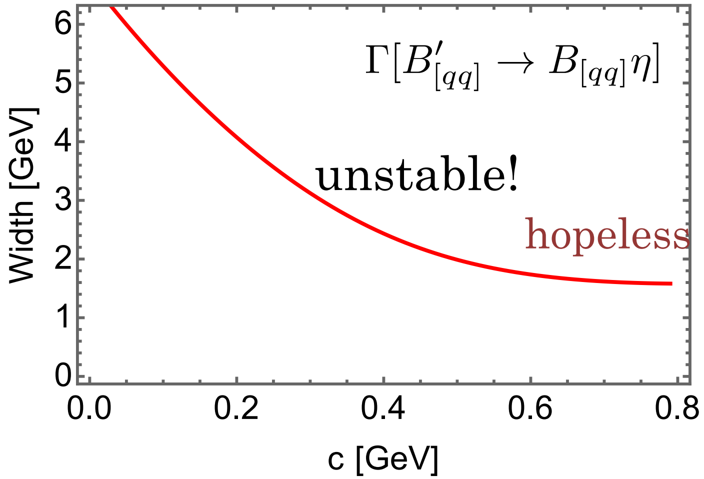
$m_q = 100\,\mathrm{MeV}$

$m_s = 500\,\mathrm{MeV}$

$(\bar{c} \approx 163\,\mathrm{MeV}^2)$

mass spectrum





hopeless to measure (observe)

- With this input $B'_{[qq]}$ is always unstable ⇨ quark mass and anomaly effect are key ingredients for $qq(0^-)$ stability ⇨ hints to the unobserved $\Lambda_c(1/2^-)$

Conclusions

- I constructed $N_f = 2+1$ linear sigma model in QC_2D to examine ud, su, sd diquark nature

- The inverse mass hierarchy of 0^- diquarks is derived similarly to Nc=3 SHBs

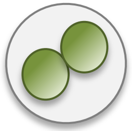
- Stability of $qq(0^-)$ largely depends on quark mass and anomaly effect

  ⇨ hints for unobserved $\Lambda_c(1/2^-)$ and more insights into SHB spectrum

- Looking forward to $N_f = 2+1$ QC_2D lattice simulation with various quark masses!

↔ NO NEED to access finite chemical potential!

- My message: QC_2D is helpful to pursue diquark dynamics as a SOLID argument



color-singlet!

- Work in progress: FRG analysis to check mass degeneracies at finite T

  w/ chiral symmetry restoration, $U(1)_A$ anomaly enhancement

Fejos-Suenaga, in preparation