Hadron in Nucleus 2025 (HIN25)

Yukawa Institute for Theoretical Physics, Kyoto University

Femtoscopy correlation functions and mass distributions from production experiments [PRD 110 (2024) 114052]

M. Albaladejo, A. Feijoo, E.Oset, I. Vidaña (U. Catania, Italy)
J. Nieves

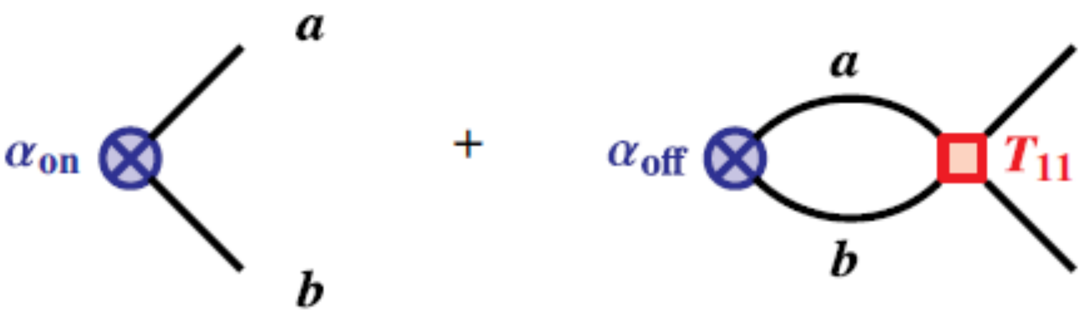
IFIC INSTITUT DE FÍSICA CORPUSCULAR
CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
VNIVERSITAT DE VALÈNCIA
EXCELENCIA SEVERO OCHOA

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
AGENCIA ESTATAL DE INVESTIGACIÓN
PID2020-112777GBI00

Outline

- Invariant mass distributions *versus* Koonin-Pratt CFs
- Production CFs. Results for $T_{cc}(3875)^+$
- Limitations of the on-shell Lednicky-Lyuboshits approximation for CFs
  - ✓ proton-proton CFs
  - ✓ $\pi^{\pm}K_S$ CFs and $\kappa/K_0^*(700)$
- Conclusions

Invariant mass distribution from a production experiment



half off-shell T –matrix

$$\frac{d\sigma}{dm_{ab}^2} \propto \left| 1 + \int d^3p \, \frac{T^{QM}(k \leftarrow p)}{E - \frac{\vec{p}^2}{2\mu_{ab}} + i\varepsilon} \, F(k,p) \right|^2$$

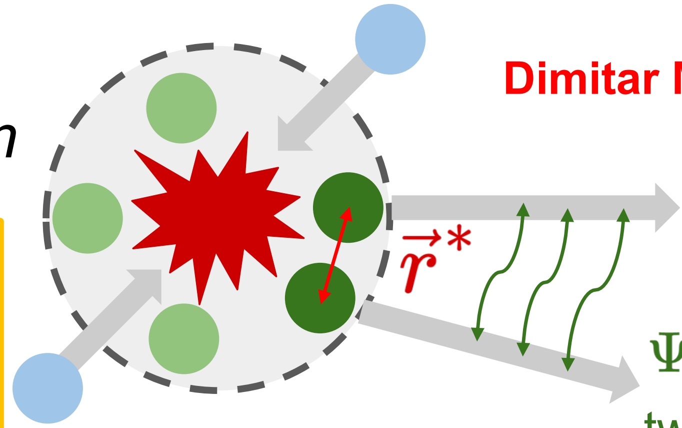
$$\frac{\alpha_{on}}{\alpha_{off}}$$

form-factor associated to the production of the virtual pair

$k = \sqrt{2\mu_{ab} \, E}$ (on-shell momentum)

Femtoscopy
Koonin-Pratt equation

Dimitar Mihaylov @ HIN 2025



$\Psi(\vec{k}^*, \vec{r}^*)$

two-particle wave function

quotient of the number of pairs of particles with the same relative momentum produced in the same collision event over the reference distribution of pairs originated from mixed events (pairs originated in different collisions)

$$C(k^*) = \frac{N_{\mathrm{SE}}(k^*)}{N_{\mathrm{ME}}(k^*)} = \int S(r^*) \left| \Psi(\vec{k}^*, \vec{r}^*) \right|^2 d^3 r^* \xrightarrow{k^* \to \infty} 1$$

Lisa et al. *Ann.Rev.Nucl.Part.Sci.55:357-402, 2005*

Relative distance and ½ relative momentum evaluated in the pair rest frame

- Measure C(k*), fix S(r*), study the interaction.
- Detailed studies of the source in pp:
  ALICE Coll. *Phys.Lett.B 811 (2020) 135849*
  Mihaylov and Gonzalez Gonzalez, *EPJC* 83 (2023) 7, 590
- CATS framework to evaluate the above integral Mihaylov et al. *EPJC* 78 (2018) 5, 394

source $S(r)$: probability density of finding the two hadrons of the emitted pair at a given relative distance *r*

basic object: wave-function

half off-shell T −matrix

$$\psi^*(\vec{r};\vec{k}) = e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\,\frac{e^{-i\vec{p}\cdot\vec{r}}}{E-\frac{\vec{p}^2}{2\mu_{ab}}+i\epsilon}\langle\vec{k}|\hat{T}^{\rm QM}(E+i\epsilon)|\vec{p}\rangle.$$

$$= e^{-i\vec{k}\cdot\vec{r}} + 4\pi\int dp\,p^2\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)$$

only S-wave interaction

$$|\psi(\vec{r};\vec{k})|^2 = 1 + 2{\rm Re}\left(e^{i\vec{k}\cdot\vec{r}}\int d^3\vec{p}\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)\right) + \left|\int d^3\vec{p}\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)\right|^2$$

spherical symmetric source

$$C(\vec{k}) = \int d^3\vec{r}\,S(\vec{r})\,|\psi(\vec{r};\vec{k})|^2 = 1 + 4\pi\int dr\,r^2 S(r)\left\{\left|j_0(kr) + \int d^3\vec{p}\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu_{ab}}+i\epsilon}T^{\rm QM}(k\leftarrow p;E)\right|^2 - j_0^2(kr)\right\}$$

We follow a different approach and first perform the d^3r integration

$$S(r) = \frac{1}{(4\pi R^2)^{3/2}} \exp\left(-\frac{r^2}{4R^2}\right)$$

Gaussian source of radius R

$$C(k) = 1 + 4\pi \int dr r^2 S(r) \left\{ \left| j_0(kr) + \int d^3\vec{p}\, \frac{j_0(pr)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} T^{\rm QM}(k \leftarrow p; E) \right|^2 - j_0^2(kr) \right\}$$

$$\int d^3r \int d^3p \Rightarrow \int d^3p \int d^3r$$

$$F_R(q, q') = \int d^3\vec{r}\, S(r) j_0(qr) j_0(q'r) = \frac{e^{-(q^2+q'^2)R^2} \sinh(2qq'R^2)}{2qq'R^2} \simeq 1 + O(q^2R^2,\, q'^2R^2)$$

$$C(k) = 1 + 2\mathrm{Re}\left(\int d^3\vec{p}\, \frac{T^{\rm QM}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} F_R(k, p) \right) + \int d^3\vec{p}\, d^3\vec{p}\,'\, \frac{T^{\rm QM}(k \leftarrow p; E) [T^{\rm QM}(k \leftarrow p'; E)]^*}{\left(E - \frac{p^2}{2\mu_{ab}} + i\epsilon\right)\left(E - \frac{p'^2}{2\mu_{ab}} - i\epsilon\right)} F_R(p, p')$$

$$= C^{\rm prod}(k) + \delta C(k),$$

$$C^{\rm prod}(k) = \left| 1 + \int d^3\vec{p}\, \frac{T^{\rm QM}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \widetilde{F}_R(k, p) \right|^2,$$

$$\tilde{F}_R(k, p) = \frac{F_R(k, p)}{F_R(k, k)}$$

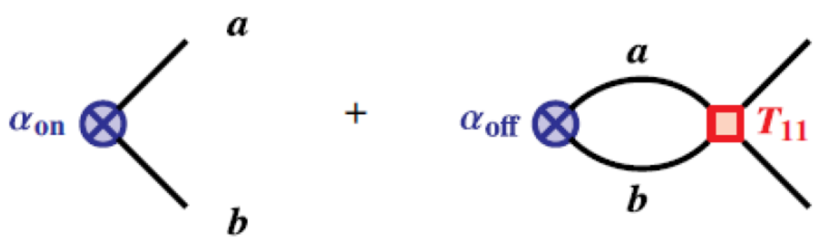
$$\delta C(k) = 2\mathrm{Re}\left(\int d^3\vec{p}\, \frac{T^{\rm QM}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \left[F_R(k, p) - \widetilde{F}_R(k, p) \right] \right)$$

$$+ \int d^3\vec{p}\, d^3\vec{p}\,'\, \frac{T^{\rm QM}(k \leftarrow p; E) [T^{\rm QM}(k \leftarrow p'; E)]^*}{\left(E - \frac{p^2}{2\mu_{ab}} + i\epsilon\right)\left(E - \frac{p'^2}{2\mu_{ab}} - i\epsilon\right)} \left[F_R(p, p') - \widetilde{F}_R(k, p) \widetilde{F}_R(k, p') \right]$$

$$C^{\text{prod}}(k) = \left| 1 + \int d^3\vec{p}\, \frac{T^{\text{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \tilde{F}_R(k, p) \right|^2,$$

$$\tilde{F}_R(k,p) = \frac{F_R(k,p)}{F_R(k,k)} = \frac{\int d^3r\, S(r) j_0(kr) j_0'(pr)}{\int d^3r\, S(r) j_0(kr) j_0'(kr)}$$

Invariant mass distribution from a production experiment



half off-shell T −matrix

$$\frac{d\sigma}{dm_{ab}^2} \propto \left| 1 + \int d^3p\, \frac{T^{QM}(k \longleftarrow p)}{E - \frac{\vec{p}^2}{2\mu_{ab}} + i\varepsilon} F(k,p) \right|^2$$

form-factor associated to the production of the virtual pair

$k = \sqrt{2\mu_{ab}\, E}$ (on-shell momentum)

- $C(k) = \boldsymbol{C^{\text{prod}}(k)} + \delta C(k)$
- $\frac{d\sigma}{dm_{ab}^2} \propto \boldsymbol{C^{\text{prod}}(k)}$, with

  $F(k,p) = \frac{\alpha_{\text{on}}}{\alpha_{\text{off}}} = \tilde{F}_R(k,p)$
- $\tilde{F}_R(k,p)$: form-factor that accounts for the off-shell production of a and b in the first step, with a regulator determined by the size of the source R

Assuming that only the S-wave part of the wave function is modified by the hadronic interaction, $C^{\text{prod}}(k)$ can also be written as

$$C^{\text{prod}}(k) = \left| \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r)\, j_0(kr)\, \psi^*(\vec{r};\vec{k}\,) \right|^2$$

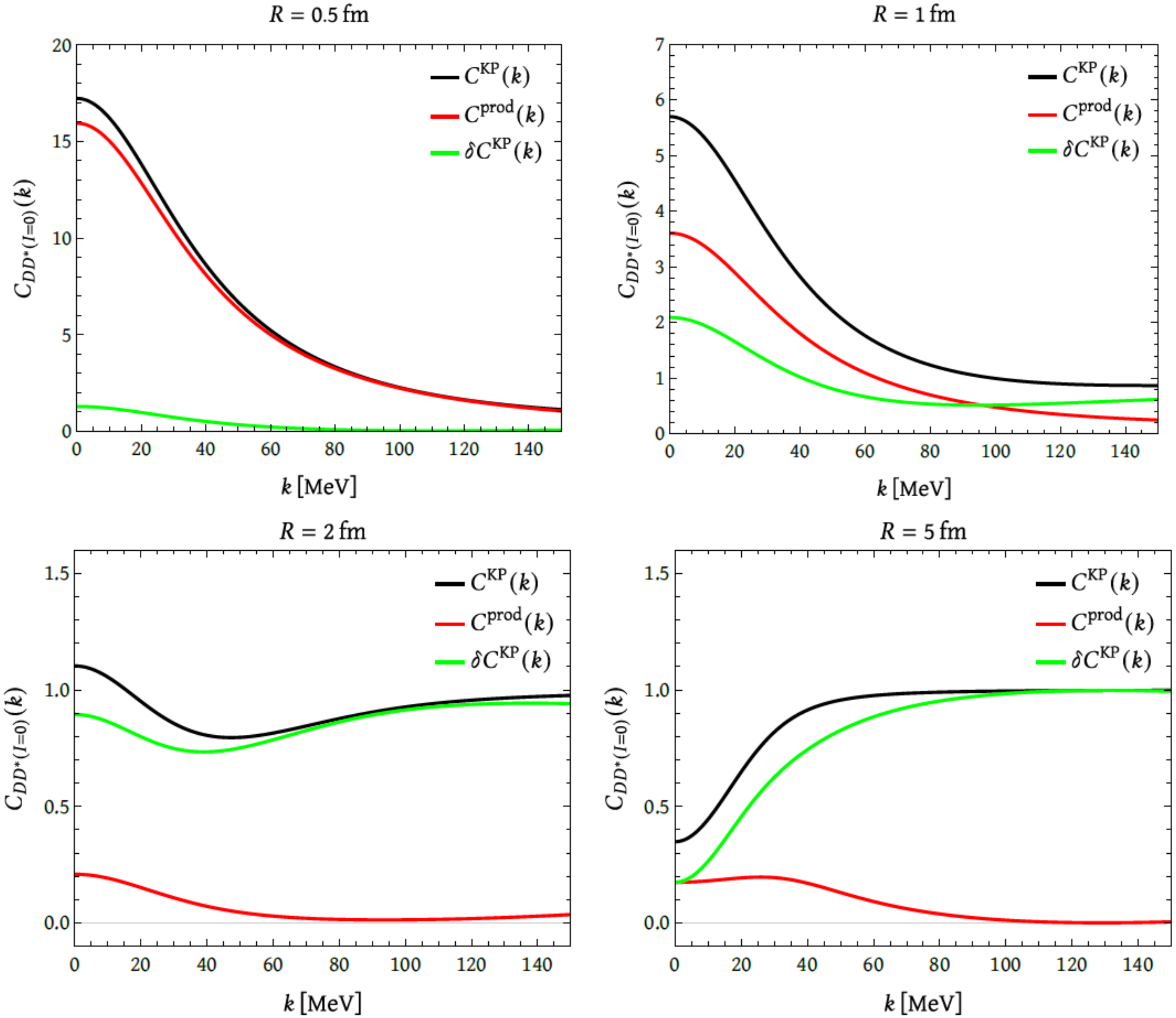
The Koonin–Pratt femtoscopic correlation function and invariant mass distributions from production experiments are different objects which are identical only for a zero source-size.

$$\begin{aligned} 1 + \int d^3\vec{p}\, \frac{T^{\text{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \widetilde{F}_R(k,p) &= \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r) j_0(kr) \left[j_0(kr) + \int d^3\vec{p}\, j_0(pr) \frac{T^{\text{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \right] \\ &= \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r) j_0(kr) \left[e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\, e^{-i\vec{p}\cdot\vec{r}} \frac{T^{\text{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \right] \\ &= \frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r)\, j_0(kr)\, \psi^*(\vec{r};\vec{k}\,) \end{aligned}$$

For point-like sources $\lim_{R\to 0} S(r) = \delta^3(\vec{r})$

$$C(\vec{k}\,) = \int d^3\vec{r}\, S(\vec{r}\,)\, |\psi(\vec{r};\vec{k}\,)|^2$$

$$\lim_{R\to 0} C(k) = |\psi(\vec{r} = \vec{0}; \vec{k}\,)|^2 = \left| 1 + \int d^3\vec{p}\, \frac{T^{\text{QM}}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} \right|^2 = \lim_{R\to 0} C^{\text{prod}}(k)$$

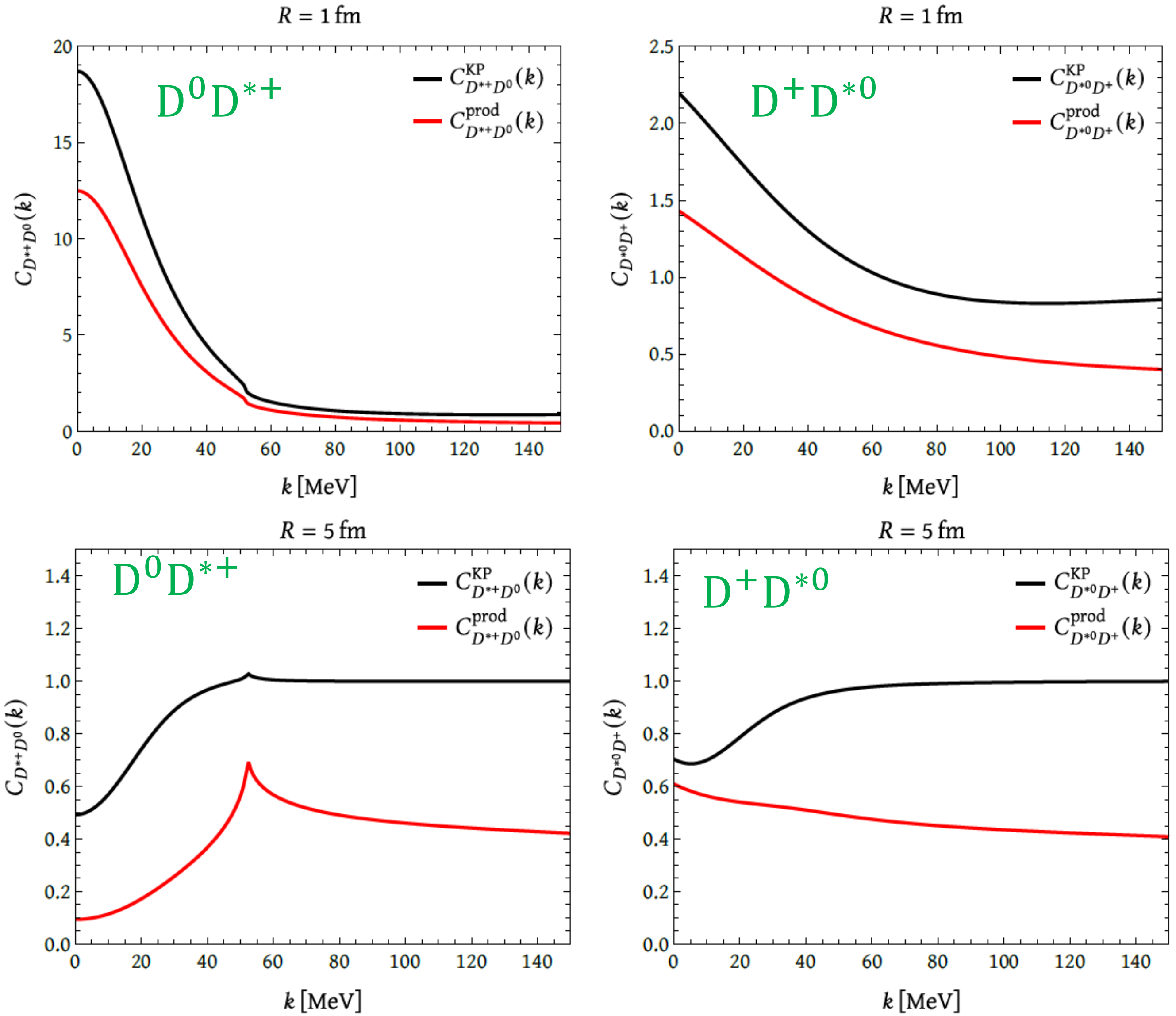


$$C(k) = \mathbf{C^{prod}(k)} + \delta C(k)$$

- isoscalar DD^* contact interaction regularized with hard-cutoff of around 400 MeV, which gives rise to a $T_{cc}(3875)^+$ bound by 860 keV

- The differences arise mostly from the T–matrix quadratic terms and increase with the source size

DD^* SINGLE channel



$$C(k) = \boldsymbol{C^{\mathrm{prod}}(k)} + \delta C(k)$$

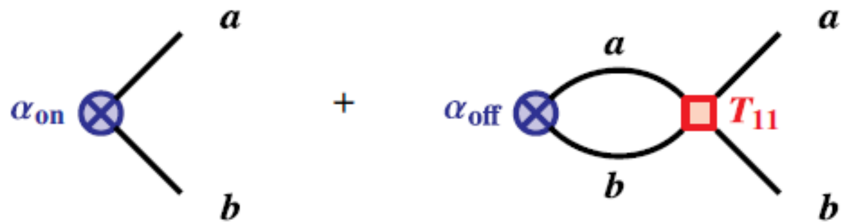
- D^0D^{*+} and D^+D^{*0} correlation functions of interest to unravel the dynamics of the exotic $T_{cc}(3875)^+$.

Interaction inspired from the realistic model of A. Feijoo, W. H. Liang and E. Oset, Phys. Rev. D104, 114015 (2021). It gives rise to a $T_{cc}(3875)^+$ bound by 354 keV

$D^0D^{*+} - D^+D^{*0}$ COUPLED channels

$$C(\vec{k}) = \int d^3\vec{r}\, S(\vec{r})\, |\psi(\vec{r};\vec{k})|^2 \qquad C^{\rm prod}(k) = \left|\frac{1}{F_R(k,k)} \int d^3\vec{r}\, S(r)\, j_0(kr)\, \psi^*(\vec{r};\vec{k})\right|^2$$

$C^{\rm prod}(k)$: coherent sum of amplitudes. This is correct for exclusive processes where only the observed particles a and b are produced



This is obviously not neccesarily the case in high-multiplicity event experiments. For inclusive reactions, for example initiated by pp collisions, the coherence is still preserved when the real and virtual production vertices, $\alpha_{\rm on}$ and $\alpha_{\rm off}$ stand for the $pp \to X + ab$ (real) and $pp \to X + ab$ (virtual) processes, respectively, with the rest of the particles (X) in the final state being the same for both reactions. In that case, the two Feynman amplitudes must be coherently added since both of them contribute to the same quantum amplitude of the reaction $pp \to X + ab$ (real).

In high-multiplicity events of pp, pA and AA collisions, where the hadron production yields are described by statistical models which lead to the extended sources $S(r)$, it is unlikely that the coherent sum of amplitudes, which is assumed in $C^{\rm prod}(k)$, is still appropriate.

The Koonin–Pratt scheme implies some kind of summation of probabilities, and not of amplitudes, since $C(k)$ is obtained from the spatial superposition between the extended source and the squared modulus of the wave-function pair $|\psi(\vec{r};\vec{k})|^2$

Source function in pp collisions at the LHC

- Emitting source function anchored to p-p correlation function

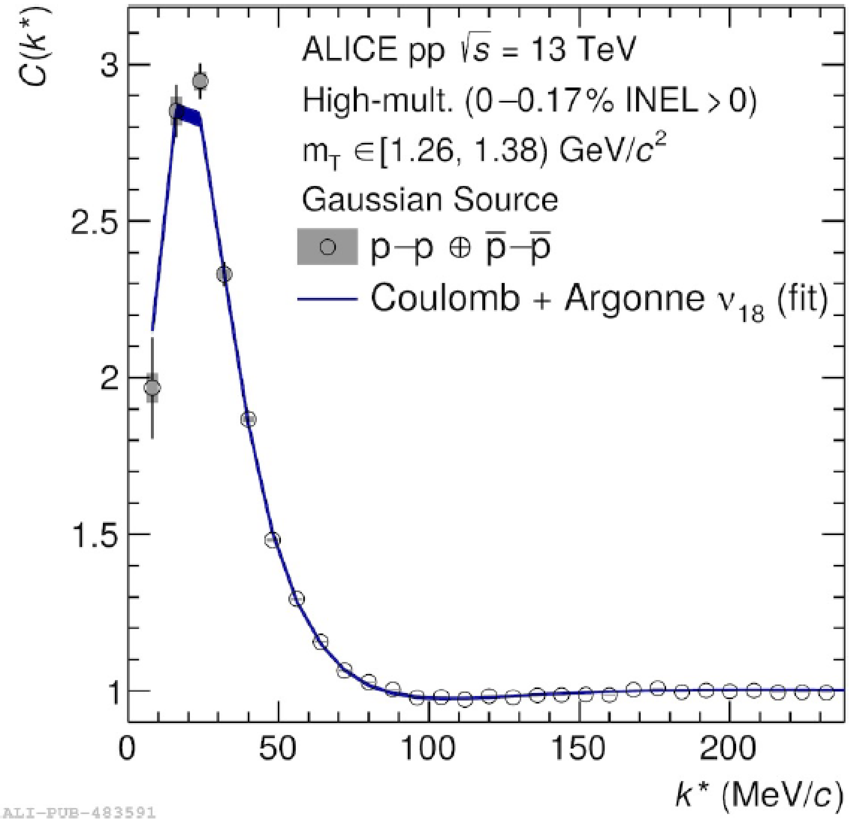
$$C(k^*) = \int S(\vec{r}\,)\left|\psi(\vec{k}^*,\vec{r}\,)\right|^2 d^3\vec{r}$$

measured — known interaction

- Gaussian parametrization

$$S(r) = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4 r_{core}^2}\right) \times$$ Effect of short lived resonances (cτ ~ 1 fm)

ALICE Coll., PLB, 811 (2020), 135849



ALICE Coll., PLB, 811 (2020)

moreover Koonin-Pratt $C(\vec{k})$ describes very well accurate CF data…. It seems more realistic a formalism based on the Koonin–Pratt equation.

Laura Fabbietti@HADRON25

on-shell factorization schemes [R. Lednicky, V.L. Lyuboshits Yad. Fiz. 35, 1316 (1981)]

$$C(\vec{k}) = \int d^3\vec{r}\, S(\vec{r})\, |\psi(\vec{r};\vec{k})|^2$$

$$\psi^*(\vec{r};\vec{k}) = e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\, \frac{e^{-i\vec{p}\cdot\vec{r}}}{E - \frac{\vec{p}^2}{2\mu_{ab}} + i\epsilon} \langle \vec{k} | \hat{T}^{\rm QM}(E+i\epsilon) | \vec{p} \rangle.$$

$$= e^{-i\vec{k}\cdot\vec{r}} + 4\pi \int dp\, p^2 \frac{j_0(pr)}{E - \frac{p^2}{2\mu_{ab}} + i\epsilon} T^{\rm QM}(k \leftarrow p; E)$$

half off-shell T −matrix

$\approx T^{QM}(k \leftarrow k)$ on-shell T −matrix

only S-wave interaction

If the half off-shell $T^{QM}(k \leftarrow p)$ is **approximated by the on-shell amplitude** $T^{QM}(k \leftarrow k)$ the integrations can be done analytically….

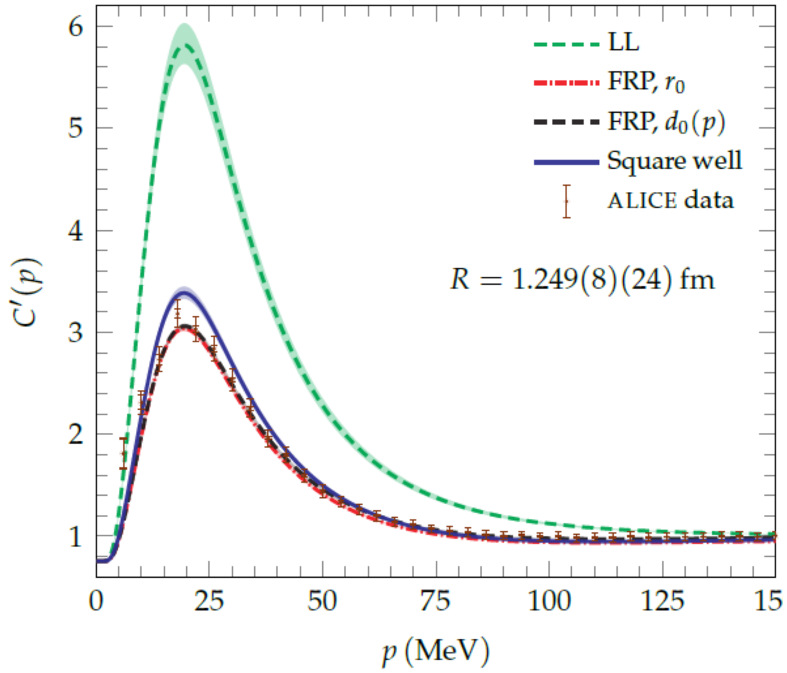
asymptotic wave function

$$\psi^*_{LL}(\vec{r};\vec{k}) = e^{-i\vec{k}\cdot\vec{r}} + f_0(k)\frac{e^{ikr}}{r} \qquad f_0(k) = \frac{1}{k\cot\delta_0(k) - ik}$$

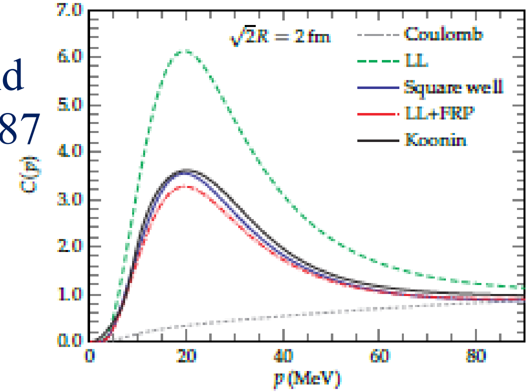
$x = 2kR$, $F_{1,2}$ analytical known functions

$$C_{LL}(k) = 1 + \frac{2{\rm Re}f_0(k)}{\sqrt{\pi}R}F_1(x) - \frac{{\rm Im}f_0(k)}{R}F_2(x) + \frac{|f_0(k)|^2}{2R^2} = 1 + \frac{2{\rm Re}f_0(k)}{\sqrt{\pi}R}F_1(x) + \frac{|f_0(k)|^2}{2R^2}e^{-x^2}$$

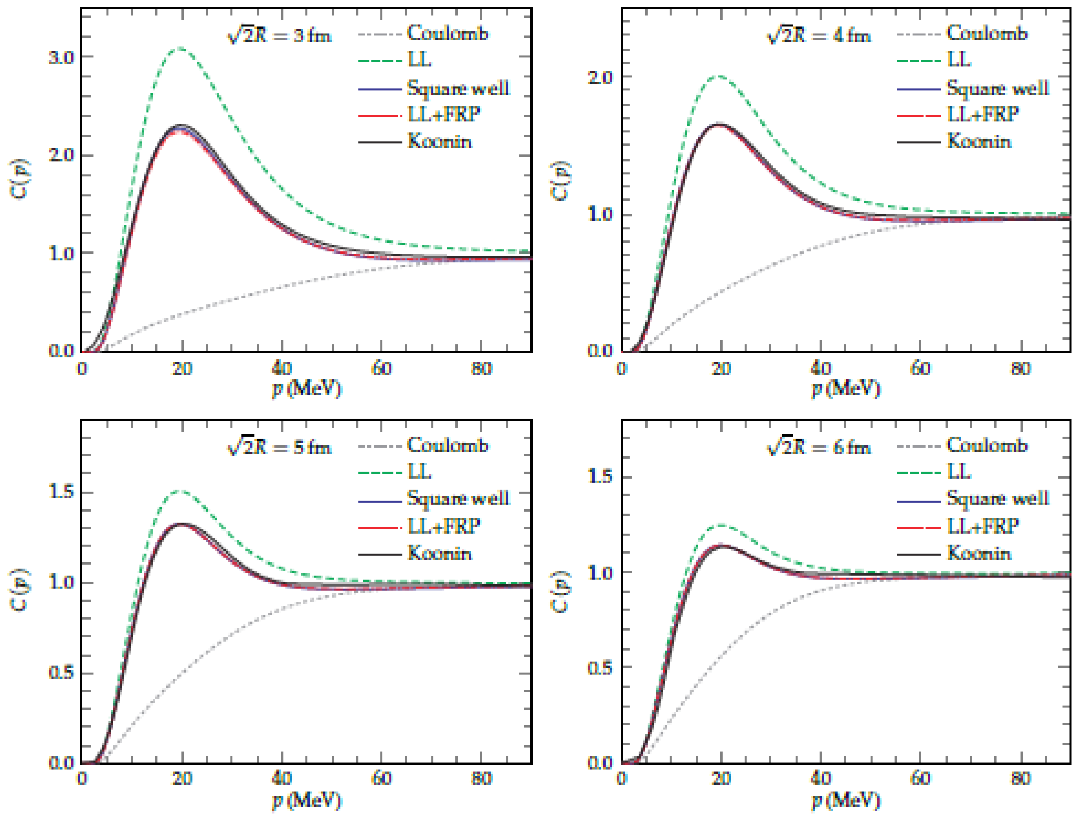
ALICE data: PLB 805 (2020) 135419



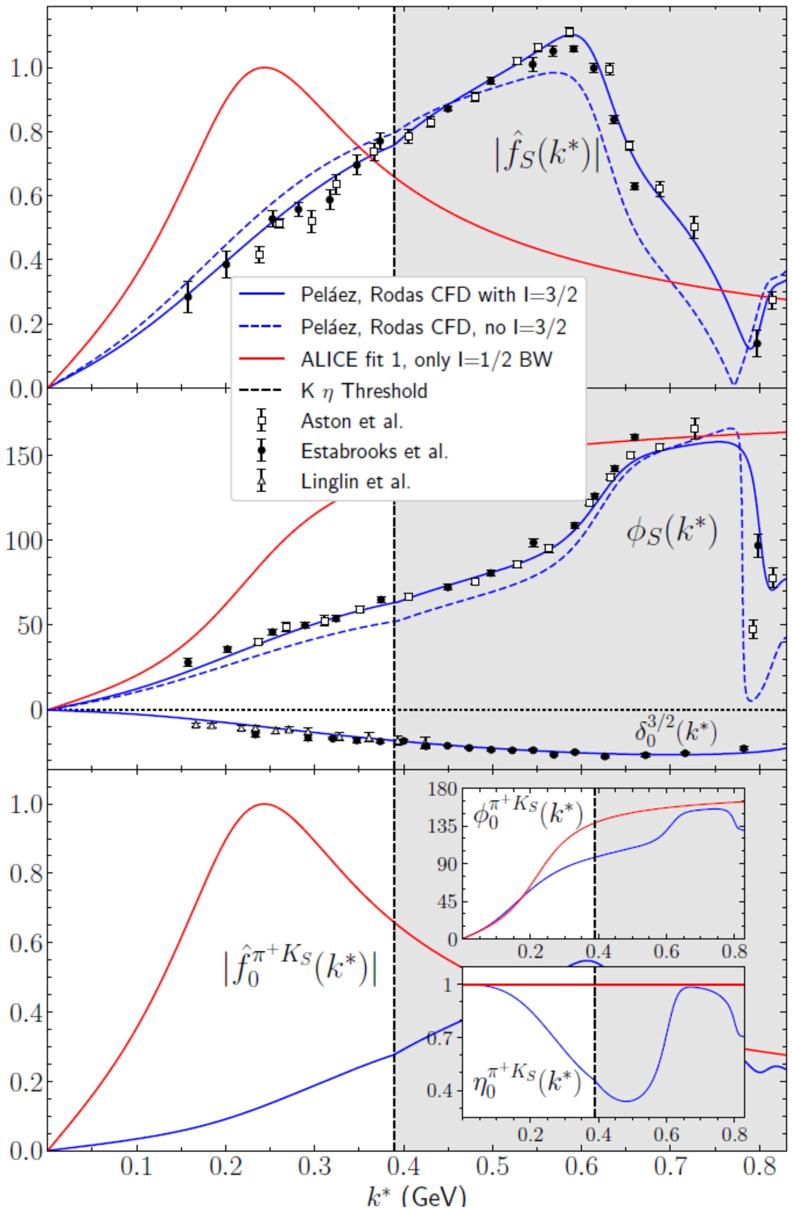
M. Albaladejo, A. García-Lorenzo and J.N., arXiv:2503.187



black curves:
S. E. Koonin, PLB 70 (1977) 43
(NN Reid Soft Core potential)



proton-proton CFs: deficiencies of on-shell **Lednicky & Lyuboshits model (green curves)** for small source-radius!



M. Albaladejo, A. Canoa, J. N., J.R. Peláez, E. Ruiz Arriola, and J. Ruiz de Elvira, 22503.19746

ALICE data $\pi^{\pm}K_S$ CFs [PLB 856 (2024) 138915]

$\kappa/K_0^*(700)$

Within **the on-shell scheme**:

i) improve on the ALICE πK BW interaction and use realistic $\pi^{\pm}K_S, \pi^{\pm}K_L, \pi^0 K^{\pm}$ interactions obtained from a dispersive analysis of scattering data [J. R. Peláez and A. Rodas, Phys. Rept. **969** (2022) 1]

“QCD spontaneous chiral symmetry breaking threshold suppression”

ii) relativistic corrections

We get smaller-than-usual source radius which should raise concerns about its applicability

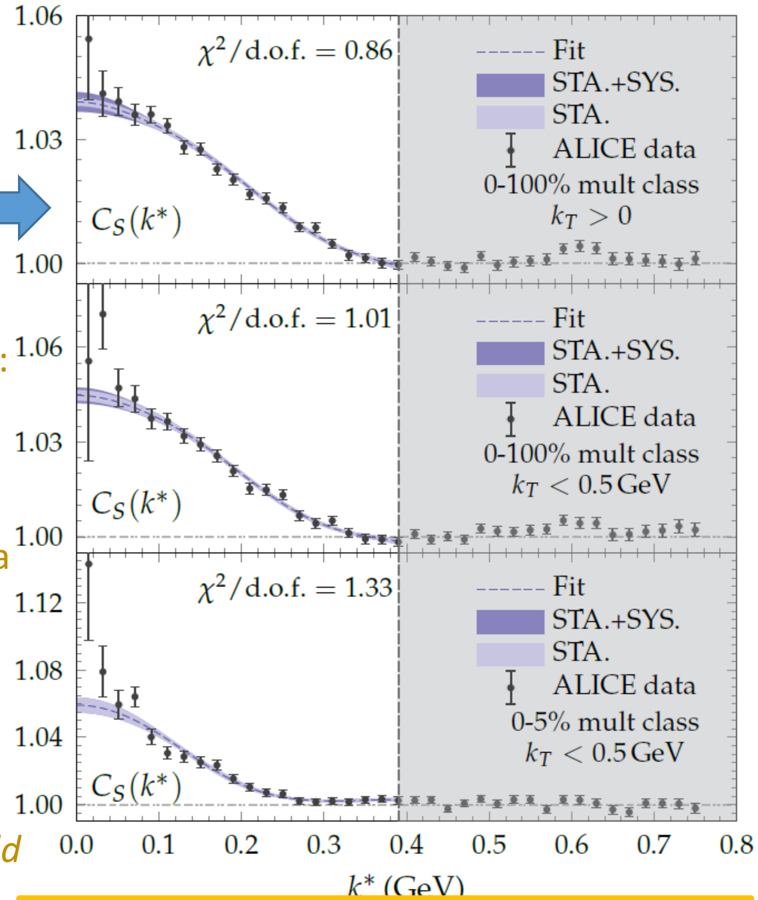


TABLE I. Parameters and χ^2/d.o.f. of the fits in Fig. 1.

	0-100% m. cl. $k_T > 0$	0-100% m. cl. $k_T < 0.5$ GeV	0-5% m. cl. $k_T < 0.5$ GeV
χ^2/d.o.f.	0.86	1.01	1.33
~~R (fm)~~	~~0.36(3)(3)~~	~~0.41(3)(3)~~	~~0.68(5)(3)~~
λ	0.19(2)(4)	0.29(4)(5)	0.80(14)(13)
$(N-1)\times10^2$	0.80(8)(7)	0.85(8)(6)	0.97(6)(4)

Conclusions

- We have discussed the relation between the Koonin–Pratt femtoscopic correlation function and invariant mass distributions from production experiments, and shown that the equivalence is total for a zero source-size.

- We have also shown that a Gaussian finite-size source provides a form-factor for the virtual production of the particles.

- We have seen quite significant differences between production and Koonin–Pratt CFs. One might think that from a QFT perspective, the production CF is more theoretically sound than the Koonin–Pratt one, however the presumably lack of coherence in high-multiplicity-event reactions and in the creation of the fire-ball source that emits the hadrons certainly make much more realistic a formalism based on the Koonin–Pratt equation.

- The off-shell effects neglected within the Lednicky & Lyuboshits approximation, turn out be essential to obtain realistic predictions of CFs.

- High-statistics femtoscopic correlations, such as those recently observed by ALICE for $\pi^{\pm}K_S$ pairs, may provide useful information on hadron-hadron interactions, particularly when data are scarce or inexistent. However, to calibrate these methods, it is important to understand first the cases when interactions (phase-shifts and inelasticities) are known, and learn on the sensitivity of CF data to short distance (off-shell effects) physics.