Decays of Exotic Double-Heavy Hadrons into Pairs of Heavy Hadrons

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EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

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Outline

The Dilemma of Exotic Double-Heavy Hadrons

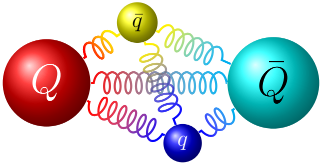
- For a long time, it was believed that every hadron is either:
  - a quark-antiquark meson;
  - a 3-quark baryon.

- Dozens of exotic hadrons with additional constituents have been discovered in the last 20 years.

- Many of them contain heavy quarks (Q) or antiquarks ($\bar{Q}$):
  - dozens of exotic $Q\bar{Q}$ hadrons
  - one exotic QQ hadron

- In addition to being interesting in themselves, they have surprising properties in a dense hadronic medium (cf. Laura's talk on Wednesday).

Born-Oppenheimer Approximation for QCD

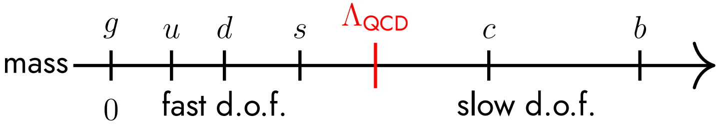


Two-step procedure:

1. calculate potentials in lattice QCD
2. solve a Schrödinger equation

EFT expansion in $\Lambda_{\mathrm{QCD}}/m_Q$

1. LO: $m_Q \to \infty$ (static quarks)
2. NLO: $\mathcal{O}(1/m_Q)$ (kinetic energy, spin splittings,...)



Coupled Channels

Adiabatic Born-Oppenheimer Approximation

- $-\frac{1}{m}\left(\vec{\nabla} + \vec{\boldsymbol{\Pi}}(\vec{r})\right)^2 \boldsymbol{\Psi}(\vec{r}) + \mathbf{V}_{\text{diag}}(r)\boldsymbol{\Psi}(\vec{r}) = E\boldsymbol{\Psi}(\vec{r})$
- channels coupled by nonadiabatic couplings

unitary transformation

Diabatic Born-Oppenheimer Approximation

- $-\frac{1}{m}\nabla^2 \boldsymbol{\Psi}(\vec{r}) + \mathbf{V}(\vec{r})\boldsymbol{\Psi}(\vec{r}) = E\boldsymbol{\Psi}(\vec{r})$
- channels coupled by mixing potentials

Diabatic Schrödinger Equation

RB [2303.17533]

$$-\frac{1}{m_Q}\nabla^2\mathbf{\Psi}(\vec{r}) + \boxed{\mathbf{V}(\vec{r})}\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$$

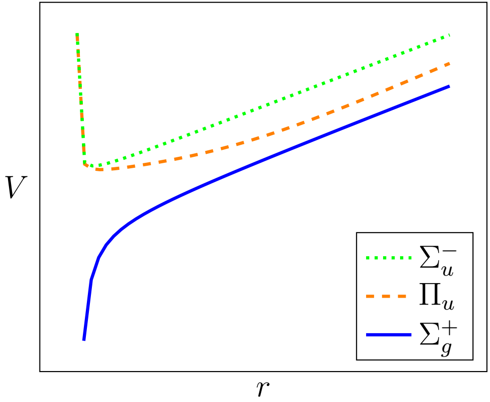
$\mathbf{V}(\vec{r})$: multichannel potential matrix

- diagonal elements $\longrightarrow$ potentials $\longrightarrow$ spectrum
- off-diagonal elements $\longrightarrow$ transitions $\longrightarrow$ mixing effects and decays
- completely determined by:
  - lattice QCD with static color sources
  - angular momentum algebra

Outline

Confining $Q\bar{Q}$ Potentials

Juge, Kuti & Morningstar [hep-lat/0207004]
Capitani, Philipsen, Reisinger, Riehl & Wagner [1811.11046]; Schlosser & Wagner [2111.00741]
Bicudo, Cardoso & Sharifian [2105.12159]; Sharifian, Cardoso & Bicudo [2303.15152]



Π_u, Σ_u^-: quarkonium hybrid

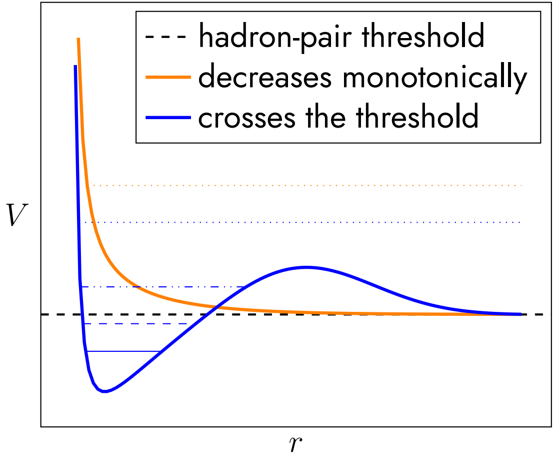
- $r \to 0$: 1^{+-} gluelump
- $r \to \infty$: $N = 1, 3$ string

Σ_g^+: quarkonium

- $r \to 0$: 0^{++} vacuum
- $r \to \infty$: $N = 0$ string

Hadron-Pair $Q\bar{Q}$ Potentials

E. Braaten & RB [2409.08002]



Decreases monotonically

- scattering states (hadron pairs)

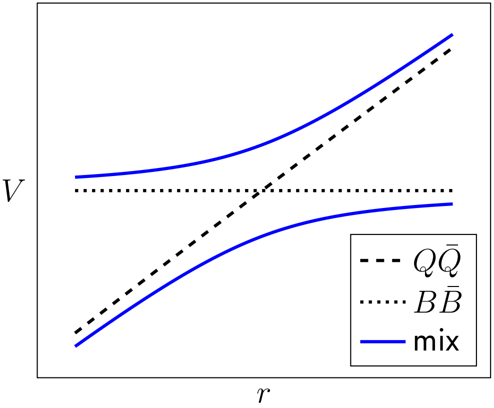
Crosses the threshold

- deeply bound states (compact multiquarks)
- shallow bound states (hadron molecules)
- scattering states (hadron pairs, resonances)

Transition Potentials

G. Bali, H. Neff, T. Düssel, T. Lippert & K. Schilling [hep-lat/0505012]
J. Bulava, F. Knechtli, V. Koch, C. Morningstar & M. Peardon [2403.00754]

$Q\bar{Q} \leftrightarrow B\bar{B}$ potential mixing



Transition effects

- bound $\leftrightarrow$ bound:
  - configuration mixing
- bound $\leftrightarrow$ scattering:
  - mixing (below threshold)
  - decays (above threshold)
- scattering $\leftrightarrow$ scattering:
  - mixing/decays (resonances)
  - inelasticity (hadron pairs)

Decays from Transition Potentials

E. Braaten & RB [2403.12868]

Decay widths are calculated by solving a coupled-channel Schrödinger equation:

$g_{\lambda,\eta}$ transition potentials from lattice QCD

$V_{L,\eta}$ transition potentials inside the Schrödinger equation

$$V_{L,\eta}(j, L_{Q\bar{Q}} \to j', L'_{Q\bar{Q}}) = (-1)^{j-j'} \sum_{\lambda} \left\langle \begin{matrix} j & L \\ \lambda & -\lambda \end{matrix} \middle| \begin{matrix} L_{Q\bar{Q}} \\ 0 \end{matrix} \right\rangle \left\langle \begin{matrix} j' & L \\ \lambda & -\lambda \end{matrix} \middle| \begin{matrix} L'_{Q\bar{Q}} \\ 0 \end{matrix} \right\rangle g_{\lambda,\eta}(j \to j')$$

Model-independent selection rules

- conservation of Q and $\bar{Q}$ spins
- conservation of Born-Oppenheimer quantum numbers λ and η
- conservation of the angular-momentum vector $\vec{L} = \vec{J}_{\text{light}} + \vec{L}_{Q\bar{Q}}$

Hadron Pairs and Heavy-Quark Spin Symmetry

E. Braaten & RB [2403.12868]

- Natural angular momenta for a hadron pair:
  - orbital angular momentum: $\vec{L}_{Q\bar{Q}}$
  - hadron spins: $\vec{J}_1 = \vec{S}_Q + \vec{j}_1$, $\vec{J}_2 = \vec{S}_{\bar{Q}} + \vec{j}_2$
- Most convenient angular momenta in B-O:
  - $\vec{L} = \vec{j}_1 + \vec{j}_2 + \vec{L}_{Q\bar{Q}}$
  - $\vec{S}_{Q\bar{Q}} = \vec{S}_Q + \vec{S}_{\bar{Q}}$
- Conversion from hadron pair to B-O angular momenta:
  1. Sum hadron spins: $\vec{S} = \vec{J}_1 + \vec{J}_2$.
  2. Add orbital angular momentum: $\vec{J} = \vec{S} + \vec{L}_{Q\bar{Q}}$.
  3. Express $\vec{S}$ as $\vec{S}_{Q\bar{Q}} + \vec{j} \implies$ Wigner 9-j symbol.
  4. Express $\vec{J}$ as $\vec{S}_{Q\bar{Q}} + \vec{L} \implies$ Wigner 6-j symbol.
- Some relative partial decay rates are model-independent rational numbers!

Quarkonium ($Q\bar{Q}$) vs. Quarkonium-Hybrid ($Q\bar{Q}g$) Decays

RB [2306.17120]; E. Braaten & RB [2403.12868]

$J^{PC} = 1^{--}$ Quarkonia into $B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$

- allowed
- $B\bar{B} : B^*\bar{B} : B\bar{B}^* : B^*\bar{B}^* = 1 : 2 : 2 : 7$
- in agreement with the prediction from constituent models

$J^{PC} = 1^{--}$ Quarkonium hybrids into $B\bar{B}$, $B^*\bar{B}$, $B\bar{B}^*$, $B^*\bar{B}^*$

- forbidden for bound states in the Π_u potential
- allowed for bound states in the Σ_u^- potential and in coupled Π_u/Σ_u^- potentials
- $B\bar{B} : B^*\bar{B} : B\bar{B}^* : B^*\bar{B}^* = 1 : 0 : 0 : 3$
- in disagreement with the prediction from constituent models ($0 : 0 : 0 : 0$)

Decays of $Q\bar{Q}q\bar{q}$ Tetraquarks

E. Braaten & RB [2409.08002]

- $Q\bar{Q}q\bar{q}$ tetraquarks can decay into lower-energy heavy-meson pairs.
- The spin splittings between heavy mesons have to be taken into account.
- The spins of the Q and $\bar{Q}$ are approximately conserved in the decay.

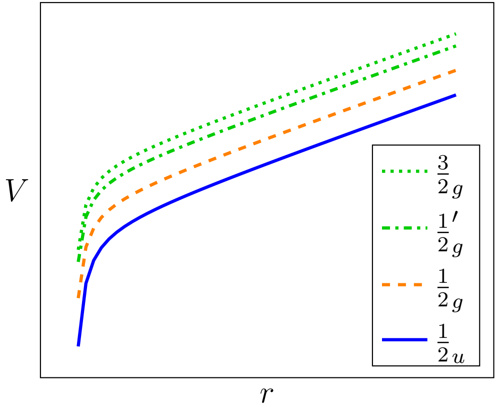
The remarkable decays of $Z_b(10650)$ (cf. Voloshin [1601.02540])

- The mass of $Z_b(10650)$ is near the $B^*\bar{B}^*$ threshold.
- The decays of $Z_b(10650)$ into $B^*\bar{B}$ and $B\bar{B}^*$ are not observed even though they are kinematically favored.
- This suppression can be explained by $Z_b(10650)$ being an equal-amplitude superposition of heavy-quark spin 0 and 1.

Outline

Confining QQ Potentials

J. Najjar & G. Bali [0910.2824]

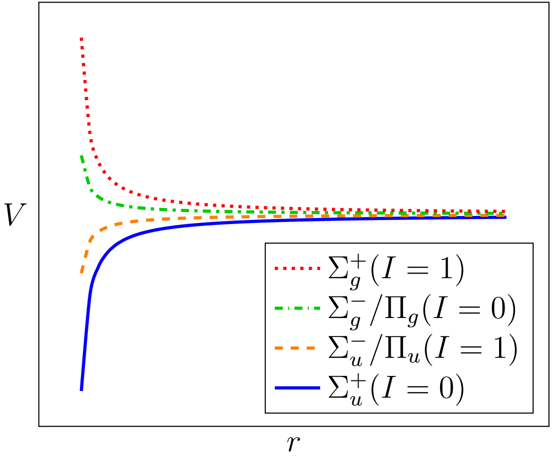


QQq baryons

- $r \to 0$: $\mathbf{3}^* q$ potential
- $r \to \infty$: string-like potential

Hadron-Pair QQ Potentials

P. Bicudo, K. Cichy, A. Peters, & M. Wagner [1510.03441]



$Q\bar{q}$-$Q\bar{q}$ meson pairs

- $r \to 0$:
  - $\mathbf{3}^*\bar{q}\bar{q}$ potential (attractive) or
  - $\mathbf{6}\bar{q}\bar{q}$ potential (repulsive)
- $r \to \infty$: threshold

Decays of QQ Hadrons

RB [2408.05150]

Born-Oppenheimer exclusion principle

- The parity P, Born-Oppenheimer quantum number η, and total heavy-quark spin S_{QQ} of a QQ hadron must satisfy the constraint $P = \eta(-1)^{S_{QQ}}$.
- Consistent with exclusion principles from identical heavy quarks in quark models.

Transition potentials

The equations for QQ transition potentials look the same as in the $Q\bar{Q}$ case. Decay widths are calculated from a Schrödinger equation. There are model-independent:

- selection rules
- relative partial decay rates

Summary

- The Born-Oppenheimer approximation gives model-independent results for:
  - selection rules for decays of double-heavy hadrons into heavy-hadron pairs;
  - relative partial decay rates of double-heavy hadrons into heavy-hadron pairs.
- These results agree with constituent models for quarkonium (in simple cases).
- These results contradict the conventional wisdom of the last 40 years that quarkonium hybrids cannot decay into the lowest pairs of heavy mesons.
- The decay widths can be calculated by solving a Schrödinger equation with:
  - transition potentials
  - spin splittings of heavy hadrons