Internal structure of $X(3872)$ by compositeness with coupled channel potential

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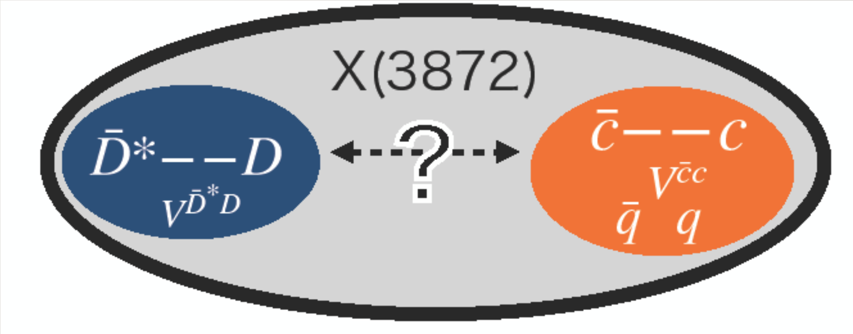
[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

Exotic hadron $X(3872)$

- There is no restriction by QCD which prohibits the mixing with each d.o.f
  - States with same quantum numbers mix by definition
- Structure of $X(3872)$ [A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP **2016** (2016)]
  - **Mixing with quark and hadron** degrees of freedom
  - Not enough experimental data and lattice QCD results

> - How about a channel coupling between **quark** and **hadron** degrees of freedom like $\mathbf{X(3872)}$?
> - Revealing the internal structure of exotic hadrons by compositeness



1 : Molecule ⟺ Compositeness ⟺ 0 : Elementary

Channel coupling

✓ Formulation according to Feshbach method [H. Feshbach, Ann. Phys. 5, 357 (1958); ibid., 19, 287 (1962)]

- Hamiltonian H with channel between quark potential V^q and hadron V^h

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

T^q, T^h:Kinetic energy

Δ:Threshold energy

V^t:Transition potential

- Schrödinger equation with wave functions of quark and hadron channels $|q\rangle, |h\rangle$

$$H \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix} = E \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix}$$

$$\langle \boldsymbol{r}'_h \mid V^h_{\text{eff}}(E) \mid \boldsymbol{r}_h \rangle = \langle \boldsymbol{r}'_h \mid V^h \mid \boldsymbol{r}_h \rangle + \sum_n \frac{\langle \boldsymbol{r}'_h \mid V^t \mid \phi_n \rangle \langle \phi_n \mid V^t \mid \boldsymbol{r}_h \rangle}{E - E_n}$$

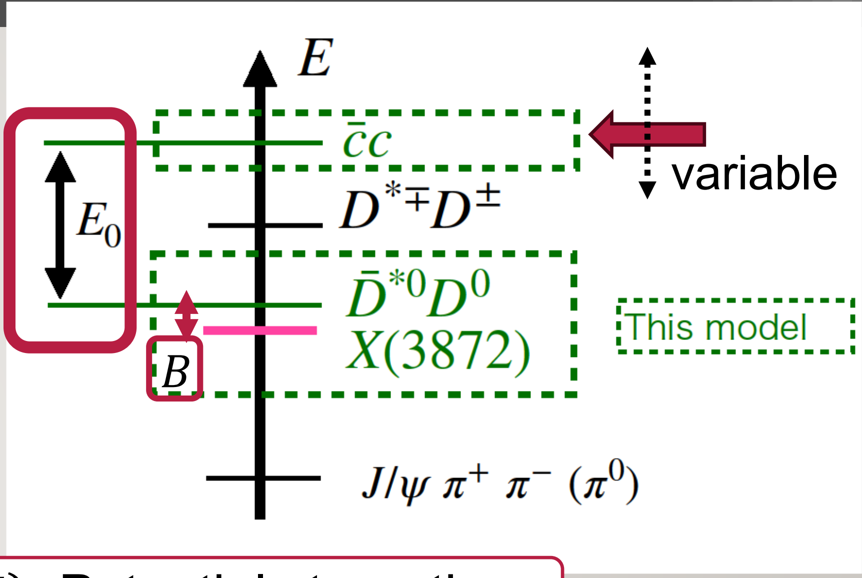
➢ Quark channel contribution. Sum of discrete eigenstates E_n

Formulation of $X(3872)$

◆Quark channel : $\bar{c}c$

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

◆Hadron channel : $D^0\,\bar{D}^{*0}$

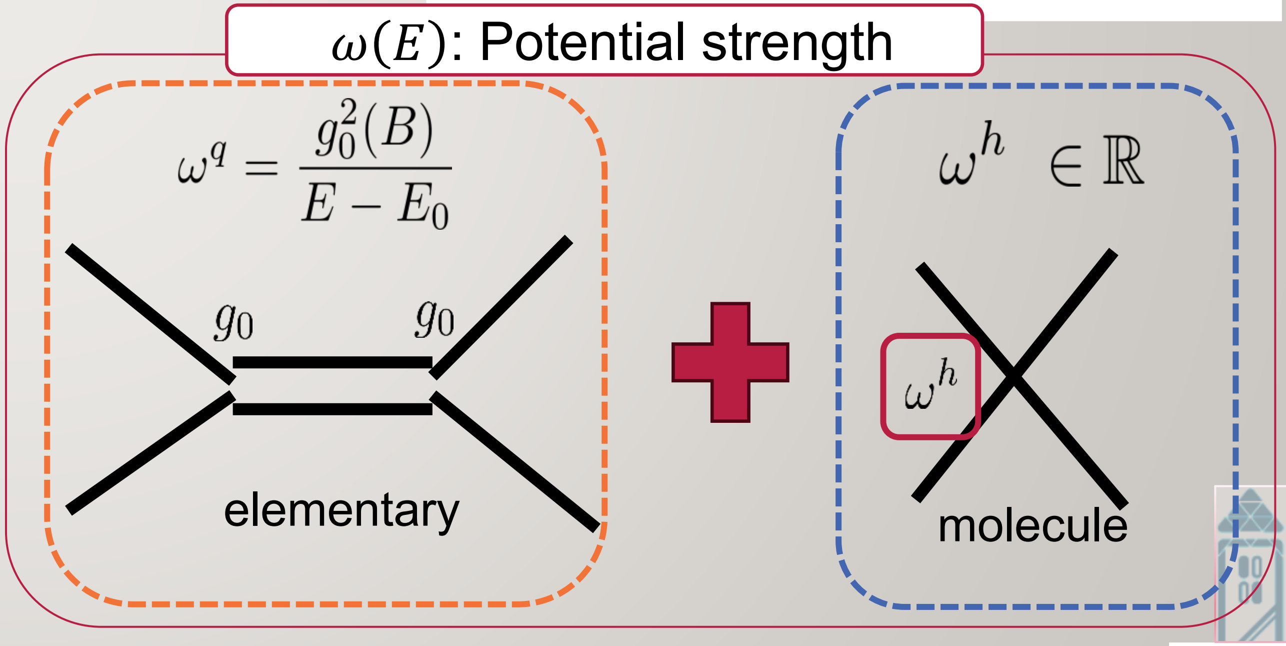


$\langle \boldsymbol{r}'_h \mid V^t \mid \boldsymbol{r}_h \rangle$ ✓ Separable
✓ Yukawa

$$V_{\rm eff}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h] V(\boldsymbol{r}) V(\boldsymbol{r}')$$

$$= \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

μ: cut-off



$g_0(B)$: coupling constant

➢ Determine to reproduce mass of $X(3872)$

$$g_0^2(B) = (B + E_0) \cdot (-1/G(E = -B) + \omega^h)$$

$G(E)$ is a loop function

Formulation: Compositeness

- Bound state wave function is normalized as:

[Kenta Miyahara and Tetsuo Hyodo. Phys. Rev. C, 93(1):015201, 2016.]

$$1 = \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')(\delta(\boldsymbol{r}-\boldsymbol{r}') - \frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E))\Psi_E(\boldsymbol{r})$$

- Definition of compositeness $1 = X_1 + Z_1 = X_2 + Z_2$

$$X_1 = \int \mathrm{d}\boldsymbol{r}|\Psi_{E=-B}(\boldsymbol{r})|^2 \quad Z_1 = -\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')\frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E)\Psi_E(\boldsymbol{r})$$

$$X_1 = [1 + \frac{g_0^2\kappa\mu(\kappa+\mu)^3}{8\pi m^2(g_0^2 + (E-E_0)\omega^h)^2}]^{-1} = X_2 = [1 + 2\pi\frac{g_0^2}{(B+E_0)^2}\frac{\kappa}{\mu(\mu+\kappa)}]^{-1}$$

X_2 from L-S e.q.

- Scattering wave function ψ^s

$$\psi_k^s(r) = \frac{\sin[kr+\delta(k)] - \sin\delta(k)e^{-\mu r}}{kr}$$

$$k\cot\delta(k) = -\frac{\mu[4\pi m\omega(E) + \mu^3]}{8\pi m\omega(E)} + \frac{1}{2\mu}\left[1 - \frac{2\mu^3}{4\pi m\omega(E)}\right]k^2 - \frac{1}{8\pi m\omega(E)}k^4$$

- Bound state wave function ψ^b

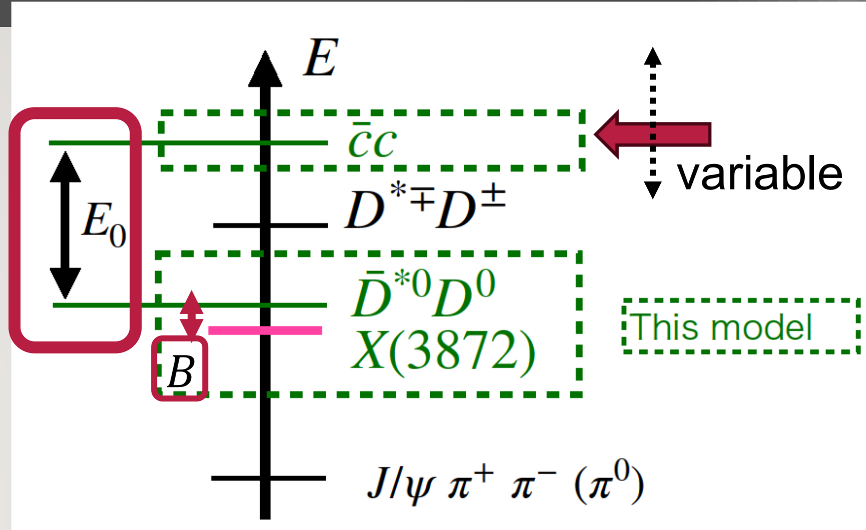
$$\Psi = \sqrt{X}\frac{\mu\kappa(\mu+\kappa)}{2\pi(\mu-\kappa)^2}(\frac{e^{-\mu r}}{r} - \frac{e^{-\kappa r}}{r})$$

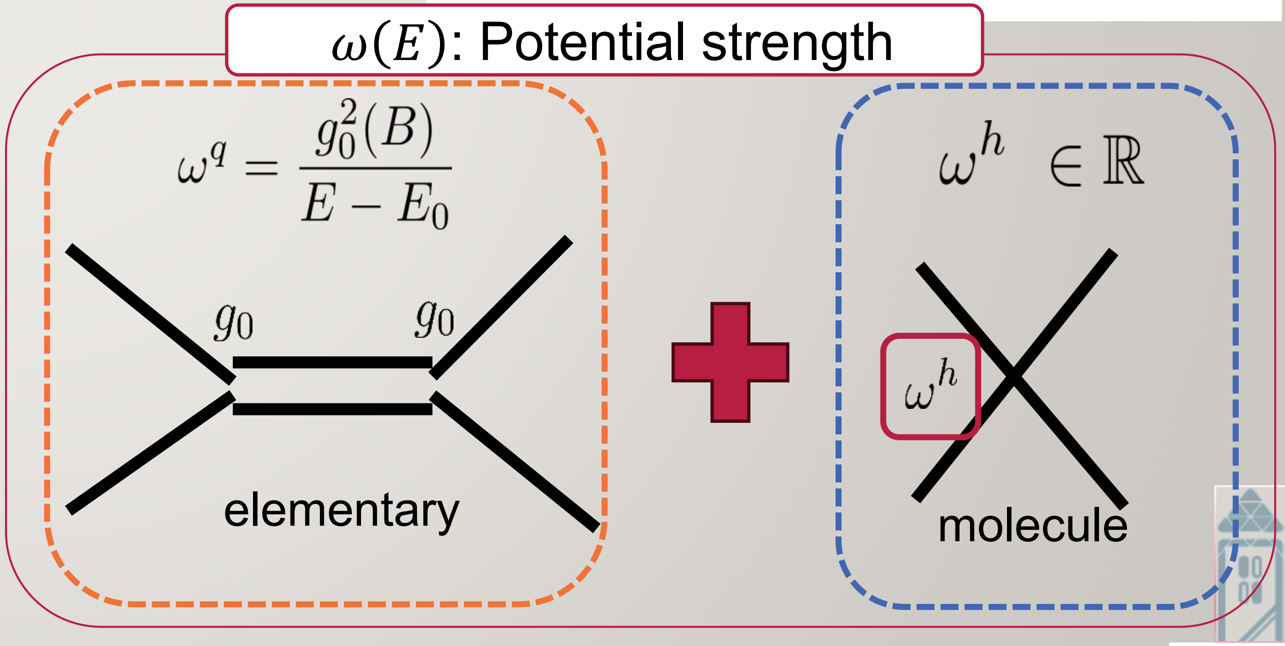
Parameters

- Parameters in this model

Physical observable	Typical value
$\boldsymbol{E_0}$	0.0078 [GeV] ($\chi_{c1}(2P)$)
B	4×10^{-5} [GeV]
μ	0.14 [GeV]
ω^h	0 [dim'less]

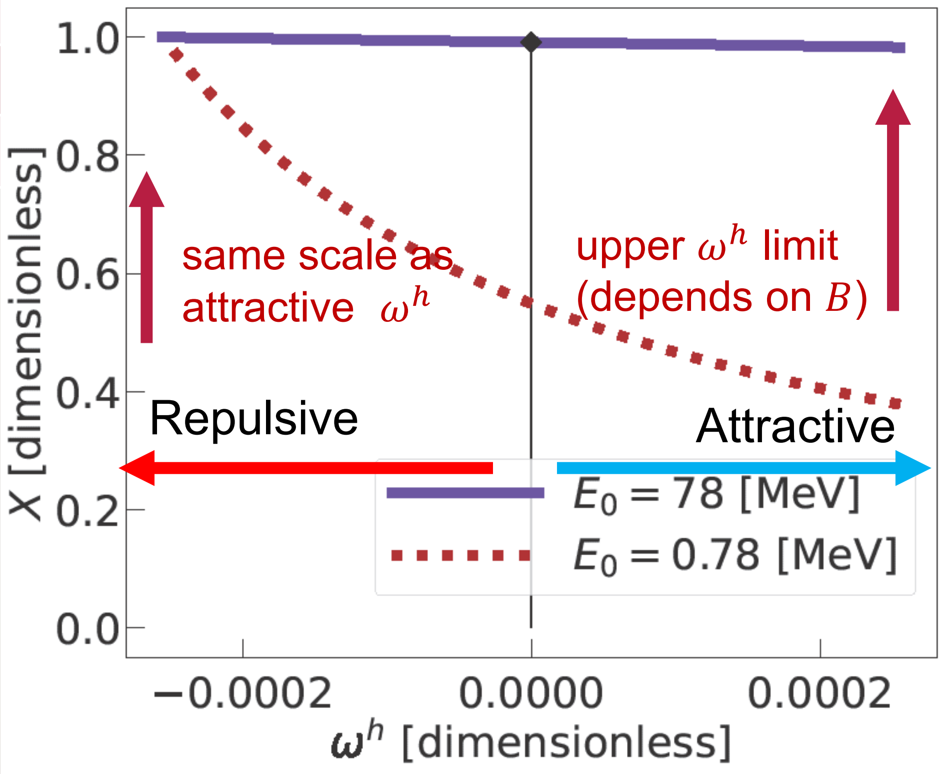
$$V_{\text{eff}}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r'}, E) = \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$





Result : ω^h dependence

Physical observable	Fixed quantity	Typical value
E_0	0.0078 [GeV] ($\chi_{c1}(2P)$)	0.0078 [GeV] ($\chi_{c1}(2P)$)
B	4×10^{-5} [GeV]	4×10^{-5} [GeV]
μ	0.14 [GeV]	0.14 [GeV]
$\boldsymbol{\omega^h}$	-	0 [dim'less]

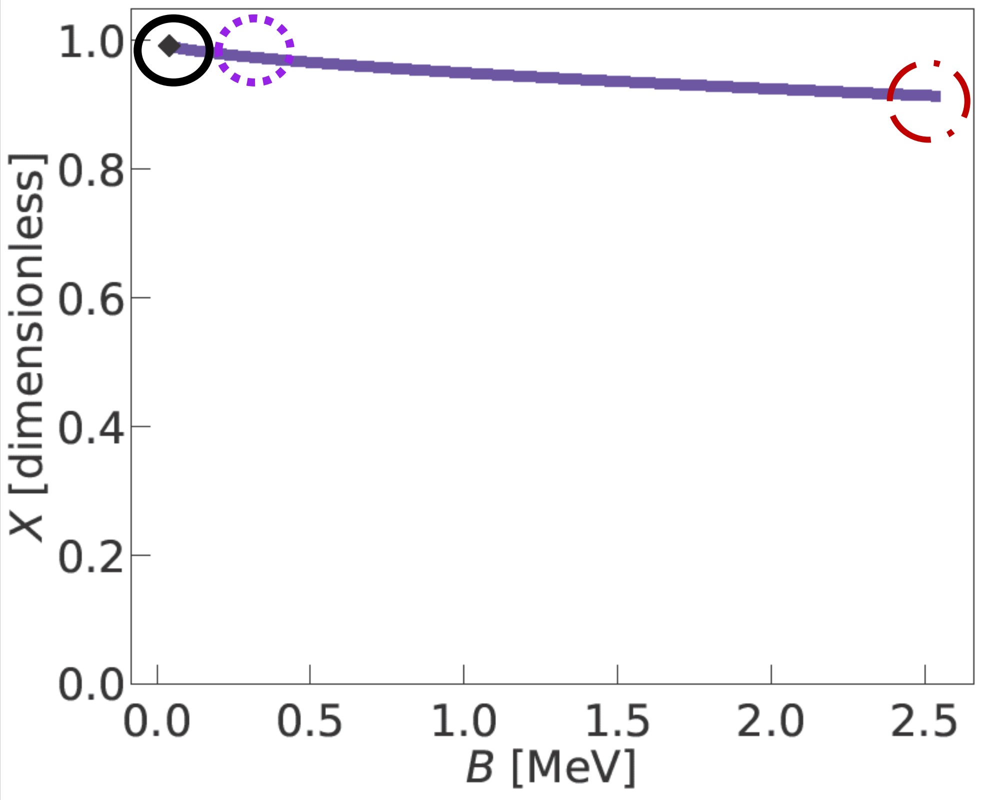


- At typical values of E_0 reproducing $X(3872)$, the compositeness changes little and the internal structure holds molecular
- When E_0 is small (close to the $D\overline{D}^*$ threshold), a change in compositeness is observed.

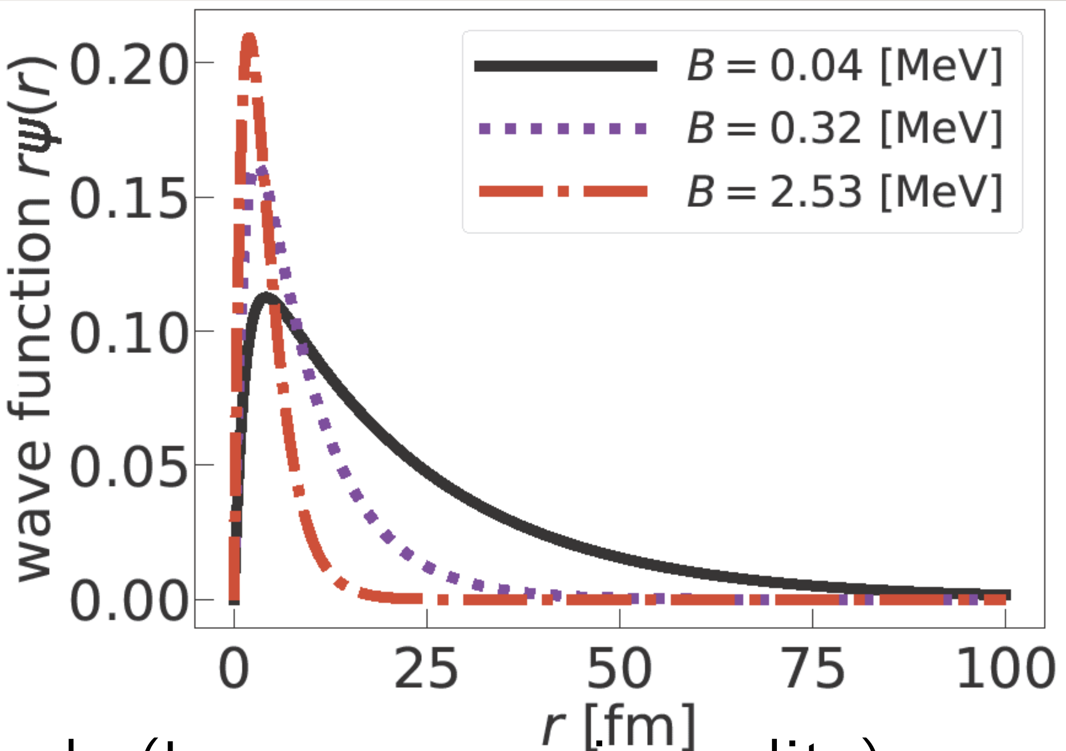
Result : B dependence

Case: X not so changes

- Comparison of changes in compositeness X with changes in wavefunction



- ➢ Binding Energy ↔ upper B limit by Yukawa potential(depends on μ)

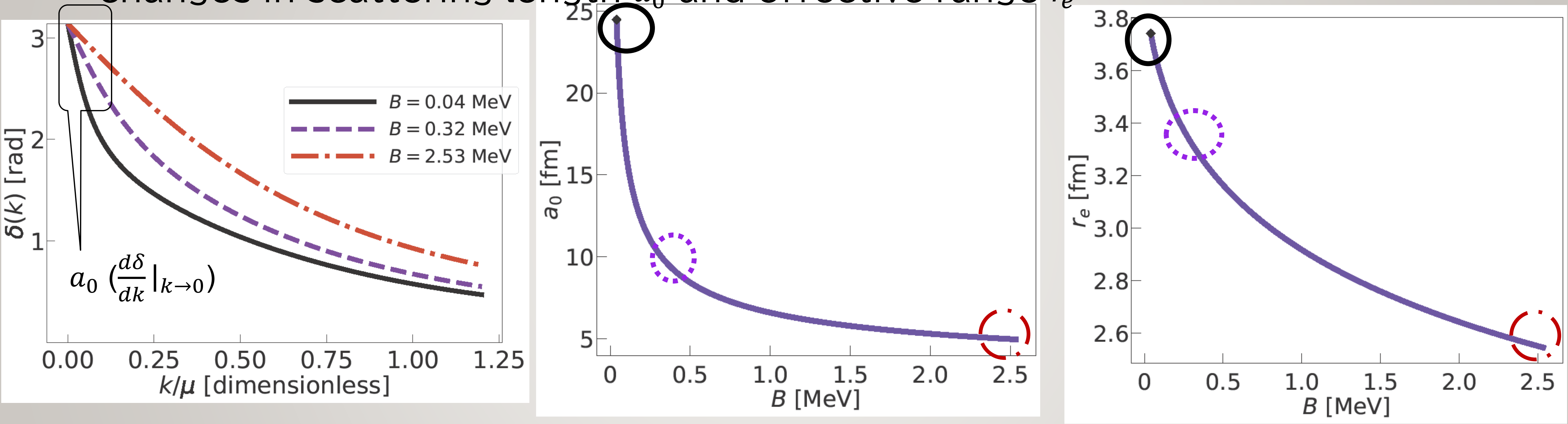


- Compositeness does not change much. (Low energy universality)。
- Wave functions are transformed to fit B

Result : B dependence

B [MeV]	X [dim'less]	a_0 [fm]	r_e [fm]
0.04	0.99	24	3.7
0.32	0.98	10	3.3
2.53	0.91	5	2.5

- Comparison of changes in phase shift $\delta(k)$ with changes in scattering length a_0 and effective range r_e



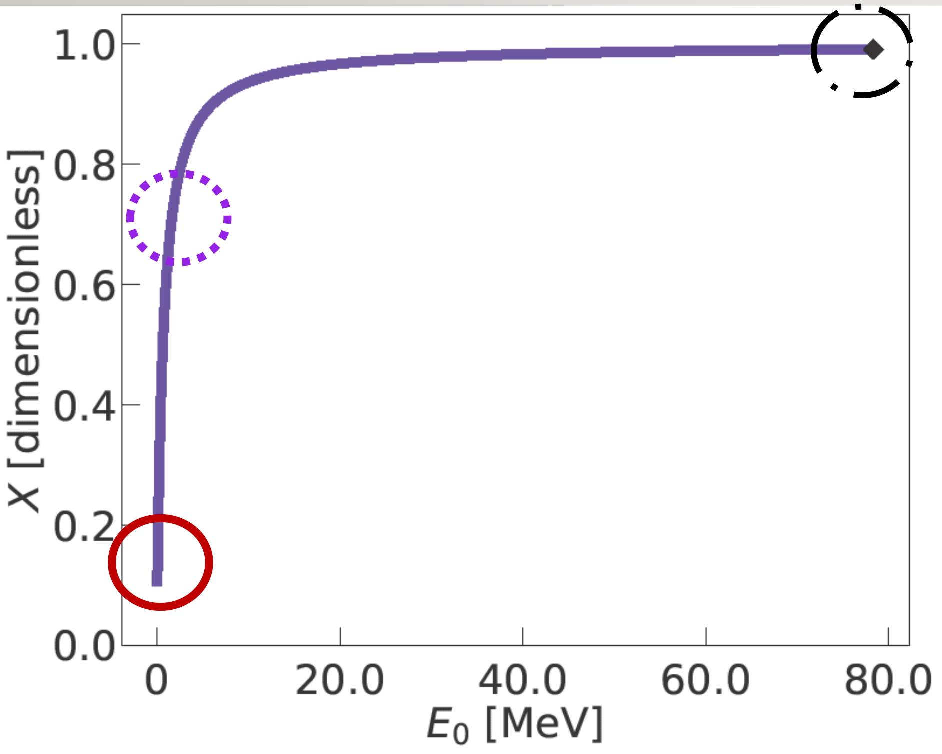
$$r_e = \frac{1}{\mu} - \frac{\mu^2}{2\pi m(\omega^h - g_0^2/E_0)} - \frac{\mu^4 g_0^2}{8\pi m^2 E_0^2(\omega^h - g_0^2/E_0)^2}$$

- $\delta(k)$, a_0: monotonic change
- r_e: Very small monotonic change

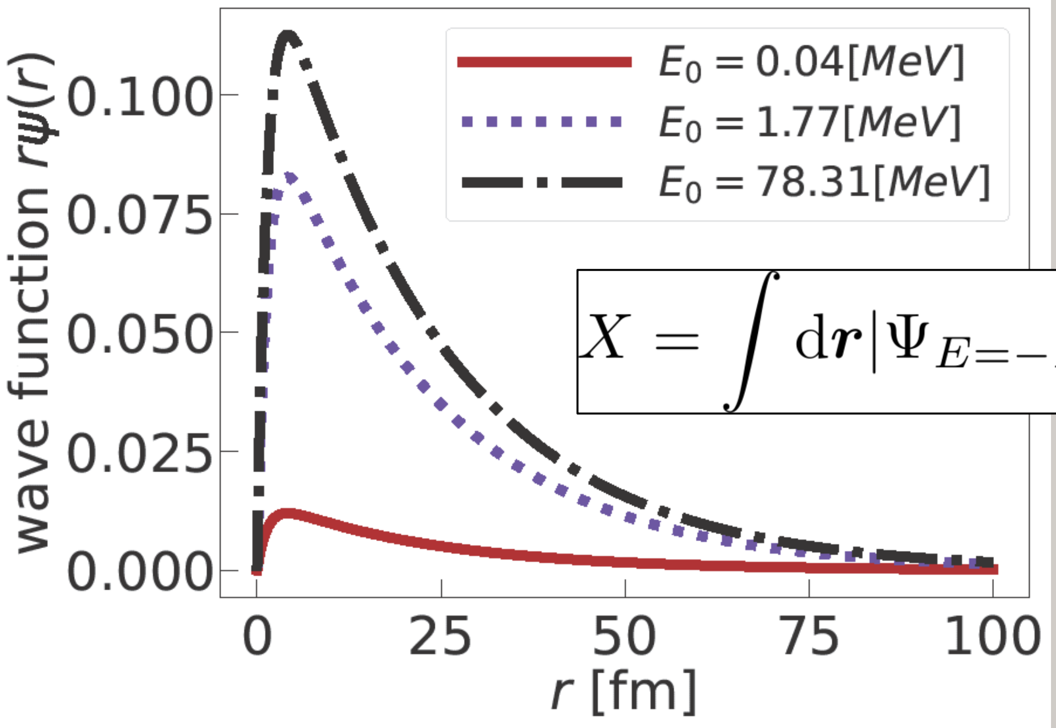
Result : E_0 dependence

Case: X changes

- Comparison of change in compositeness X and change in wave function



- Binding energy ↔ Quark model ($\chi_{c1}(2P)$)



$$X = \int \mathrm{d}\boldsymbol{r} |\Psi_{E=-B}(\boldsymbol{r})|^2$$

✓ When E_0 is extremely small (enough to break the low-energy universality), the compositeness changes significantly

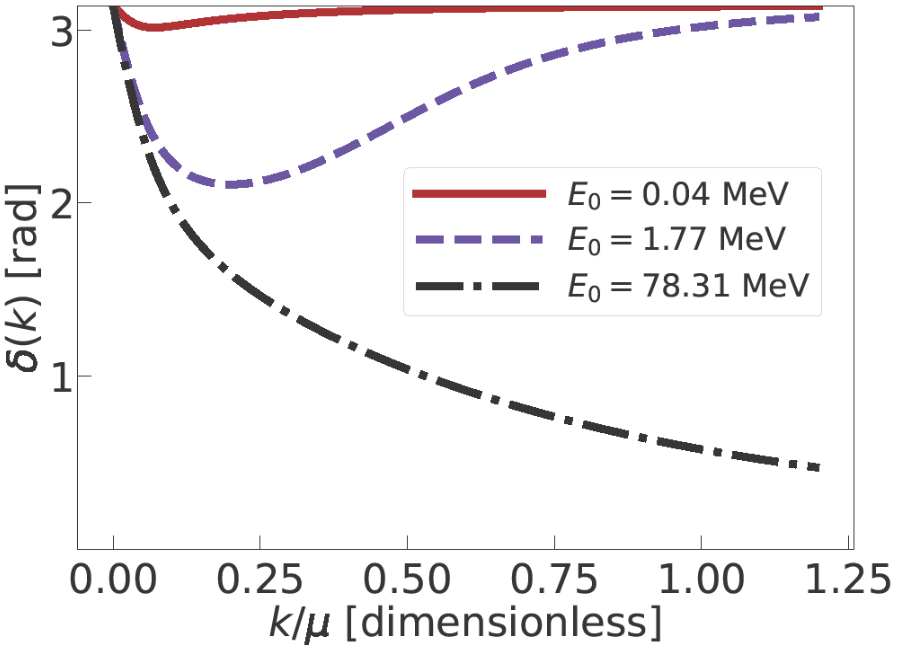
✓ The shape of the wave function changes with compositeness X from

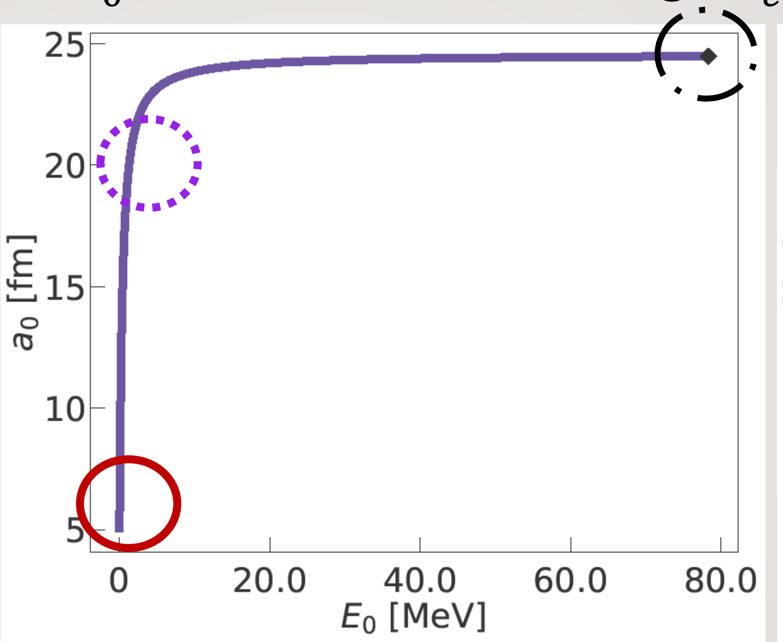
$$\Psi = \sqrt{X} \frac{\mu\kappa(\mu+\kappa)}{2\pi(\mu-\kappa)^2} \left(\frac{e^{-\mu r}}{r} - \frac{e^{-\kappa r}}{r} \right)$$

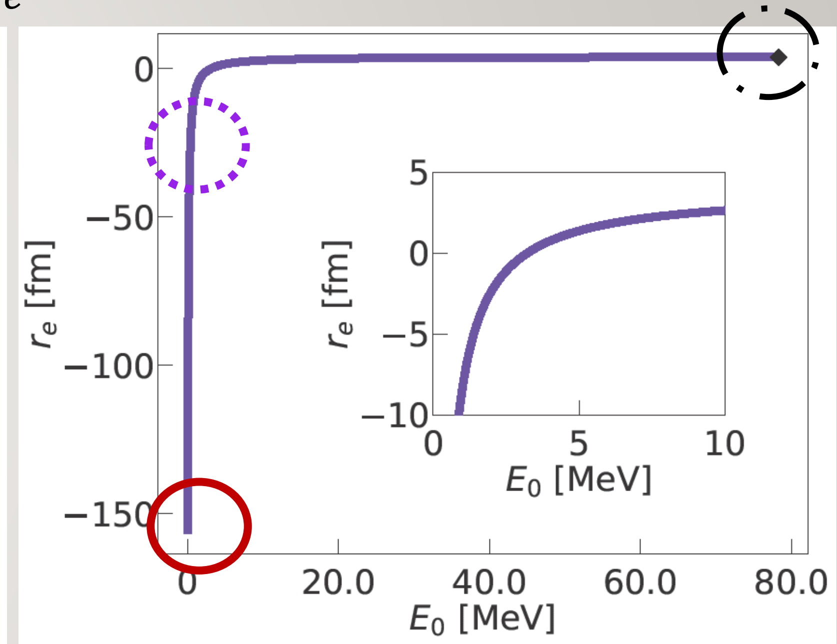
Result : E_0 dependence

E_0 [MeV]	X [dim'less]	a_0 [fm]	r_e [fm]
0.04	0.1	5	-155
1.77	0.7	21	-3
78.31	1	24	3

- Comparison of changes in phase shift $\delta(k)$ with changes in scattering length a_0 and effective range r_e





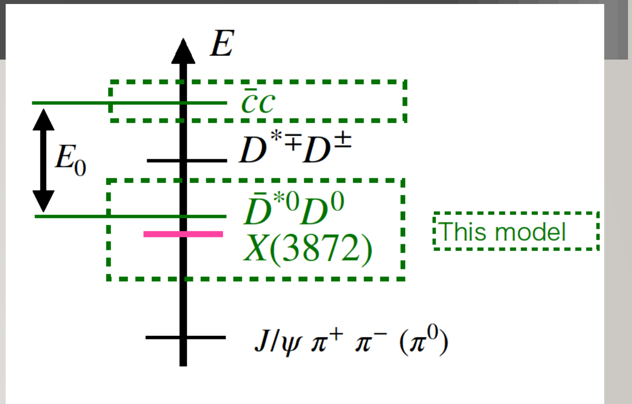


✓When E_0 is extremely small → neither the scatter length (slope of k=0) nor the effective range matches

✓Scatter length is always positive but effective range changes from negative to positive

Summary

[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

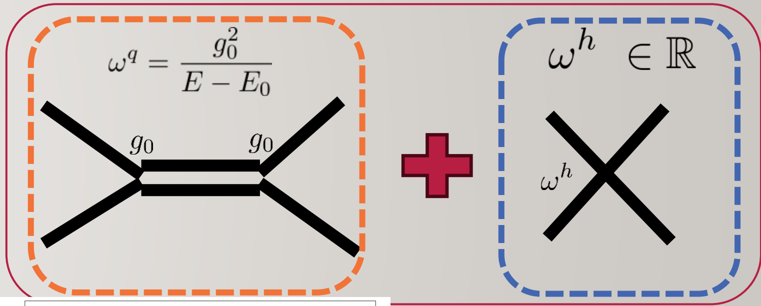


- Channel coupling between $c\bar{c}$ and $D\bar{D}^*$ in $X(3872)$

$$H = \begin{pmatrix} T^{c\bar{c}} & 0 \\ 0 & T^{\bar{D}^*D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{c\bar{c}} & V^t \\ V^t & V^{\bar{D}^*D} \end{pmatrix}$$

- Effective potential with explicit V^q and V^h

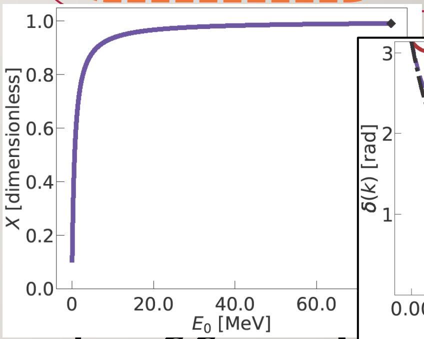
$$V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h(E)]V(\boldsymbol{r})V(\boldsymbol{r}')$$

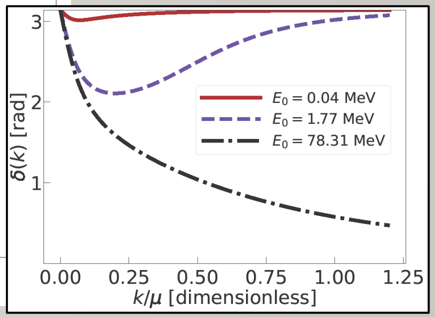


- Compositeness X in analytical form

$$X = [1 + \frac{g_0^2\kappa\mu(\kappa+\mu)^3}{8\pi m^2(g_0^2 + (E - E_0)\omega^h)^2}]^{-1} = [1 + 2\pi\frac{g_0^2}{(B+E_0)^2}\frac{\kappa}{\mu(\mu+\kappa)}]^{-1}$$

- Parameter dependences for compositeness
  - Especially for E_0, When fine-tuned enough to break the low-energy universality, the compositeness changes from 0 (elementary) to 1 (molecule).





- Wave function, phase shift, scattering length, and effective range change as compositeness change