Mean-field theory becomes exact under shear flow

a dynamical renormalization group study of the phi-4 model

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Harukuni Ikeda (YITP, Kyoto Univ.)
Hiroyoshi Nakano (ISSP, Tokyo Univ.)

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

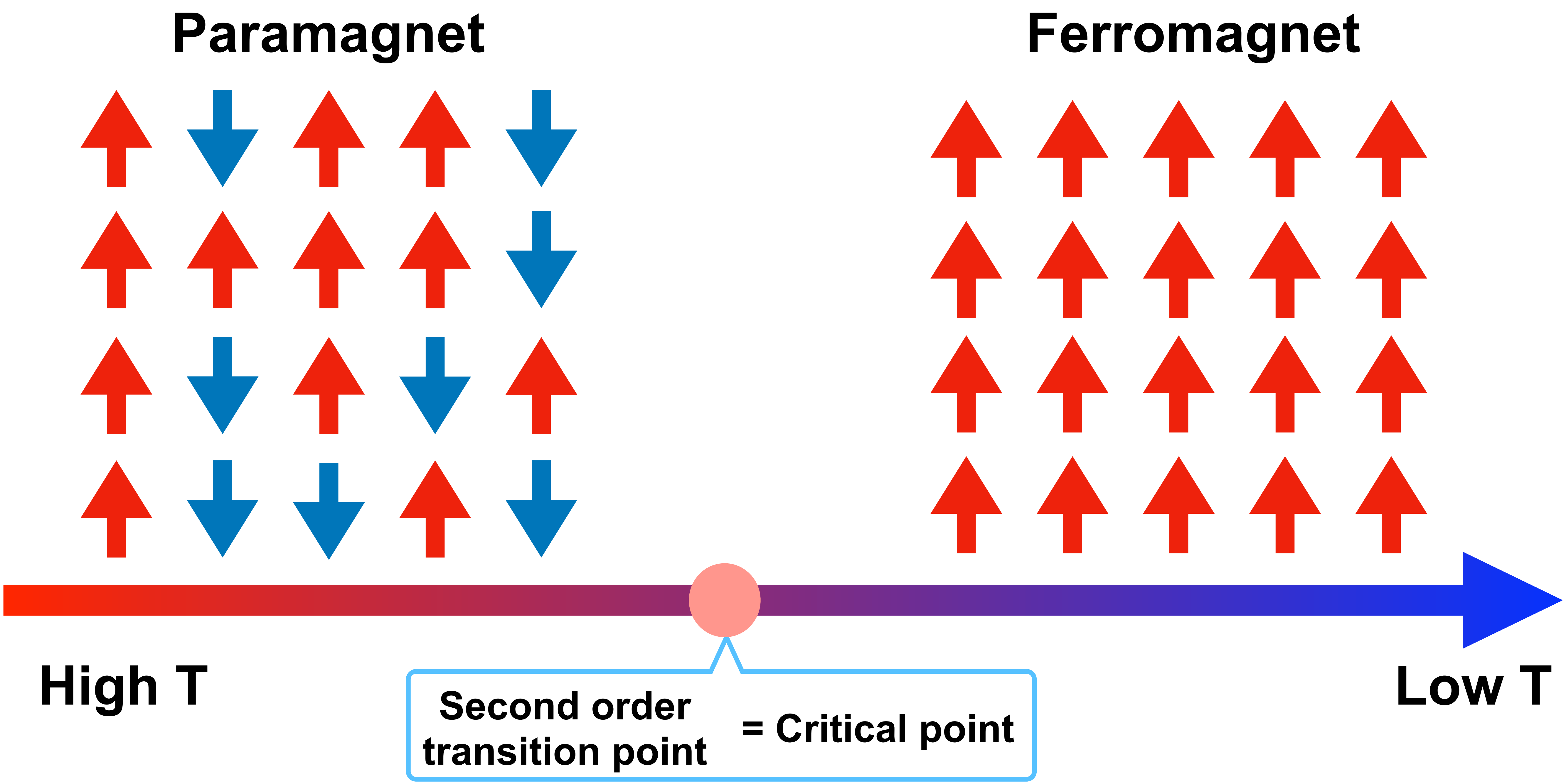
東京大学 物性研究所
THE INSTITUTE FOR SOLID STATE PHYSICS
THE UNIVERSITY OF TOKYO

Introduction

Phase transition

Ising model

$$H = -J\sum_{<ij>} S_i S_j,\quad S_i = \pm 1$$



Introduction

Mean-field theory (ϕ^4 theory)

Taylor expansion of free-energy by order parameter

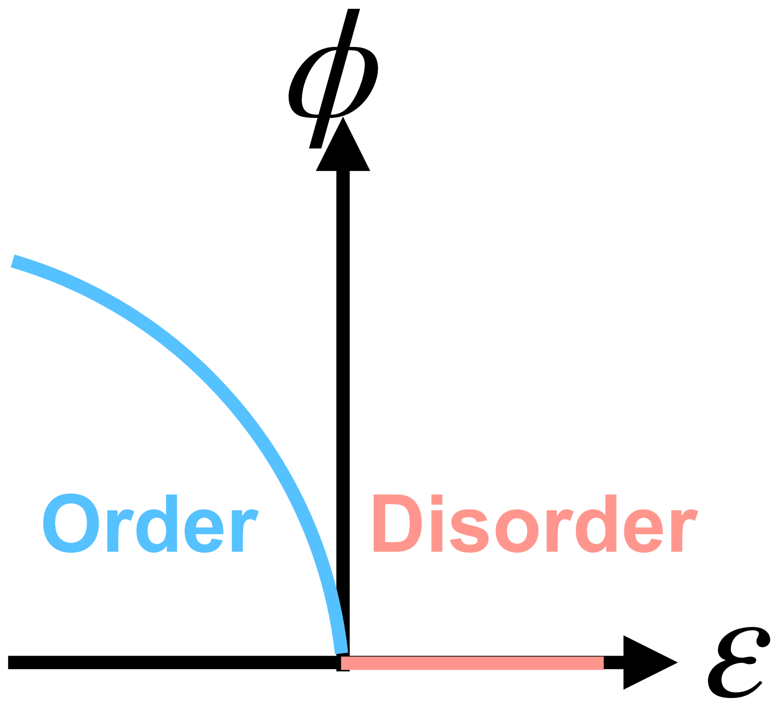
$$F(\phi) = \frac{\varepsilon}{2}\phi^2 + \frac{u}{4}\phi^4 + \cdots$$

Orderparameter

Z2 symmetry $F(\phi) = F(-\phi)$ prohibits odd terms.

Saddle point equation

$$\frac{dF(\phi)}{d\phi} = 0 \;\Rightarrow\; \phi = \begin{cases} 0 & \varepsilon > 0 \quad \text{Disorder} \\ \sqrt{-\varepsilon/u} & \varepsilon < 0 \quad \text{Order} \end{cases}$$



Scaling behavior

$$\phi = (-\varepsilon)^{\beta}$$

Mean-field critical exponent

$$\beta = 1/2$$

Introduction

Mean-field theory (ϕ^4 theory)

Mean field critical exponents

Order parameter $\phi = (-\varepsilon)^{\beta},\ \beta = 1/2$

Specific heat $C = (-\varepsilon)^{\alpha},\ \alpha = 0$

Correlation length $\xi = |\varepsilon|^{-\nu},\ \nu = 1/2$

Susceptibility $\chi = |\varepsilon|^{-\gamma},\ \gamma = 1$

Relaxation time $\tau \sim \xi^{z},\ z = 2$

Introduction

Mean-field theory (ϕ^4 theory)

Critical exponents of Ising model

	d=2	d=3	d>4
α (specific heat)	0	0.110	0
β (order parameter)	1/8	0.33	1/2
ν (correlation length)	1	0.63	1/2

Disagree with mean-field

Agree with mean-field

The mean-field theory fails to predict the critical exponents in d=2 and 3 in equilibrium.

Introduction

Purpose of this study

Motivation

- In equilibrium, the mean-field theory fails in d=2 and 3.
- What will happen far from equilibrium?
- We consider a model in shear flow as a prototypical example of a nonequilibrium system.

Question:
How does the steady shear flow affect the critical phenomena?

Our answer:
Mean-field theory becomes exact in d=2, and 3.

Overview

- **Introduction**
- Dynamical scaling in equilibrium (review)
- Dynamical scaling under shear
- Comparison with numerics
- Summary and discussions

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Dynamical scaling in equilibrium

Model

We first review the dynamical scaling in equilibrium.

Langevin equation (model-A: nonconservative order parameter)

$$\frac{\partial \phi(\boldsymbol{x},t)}{\partial t} = -\frac{\delta F[\phi(\boldsymbol{x},t)]}{\delta \phi(\boldsymbol{x},t)} + \sqrt{2T}\xi(\boldsymbol{x},t)$$

ϕ: order parameter

ξ: white noise

$\langle \xi(\boldsymbol{x},t)\xi(\boldsymbol{x}',t')\rangle = \delta(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t')$

Free energy functional (ϕ^4 free energy)

$$F[\phi] = \int d\boldsymbol{x} \left[\frac{k}{2}(\nabla\phi)^2 + \frac{\varepsilon}{2}\phi^2 + \frac{u}{4}\phi^4\right]$$

Steady state distribution

$$P_{\rm eq}[\phi] \propto e^{-\frac{F[\phi]}{T}}$$

Canonical distribution

Dynamical scaling in equilibrium

Linear analysis

To simplify the analysis, we linearize the equation

$$\frac{\partial \phi(\boldsymbol{x}, t)}{\partial t} = - \frac{\delta F[\phi(\boldsymbol{x}, t)]}{\delta \phi(\boldsymbol{x}, t)} + \sqrt{2T}\xi(\boldsymbol{x}, t)$$

$$= k\nabla^2\phi(x, t) - \varepsilon\phi(\boldsymbol{x}, t) - u\phi(\boldsymbol{x}, t)^3 + \sqrt{2T}\xi(\boldsymbol{x}, t)$$

Nonlinear term

Linearize

$$\frac{\partial \phi(\boldsymbol{x}, t)}{\partial t} = k\nabla^2\phi(x, t) - \varepsilon\phi(\boldsymbol{x}, t) + \sqrt{2T}\xi(\boldsymbol{x}, t)$$

Dynamical scaling in equilibrium

Critical exponents

We want to investigate the power-law scaling near the transition point

Scaling Ansatz

$$\boldsymbol{x} = b\boldsymbol{x}', \; t = b^z t', \; \phi(\boldsymbol{x}, t) = b^\chi \phi'(\boldsymbol{x}', t')$$

$$\frac{\partial \phi(\boldsymbol{x}, t)}{\partial t} = k\nabla^2\phi(x, t) - \varepsilon\phi(\boldsymbol{x}, t) + \sqrt{2T}\xi(\boldsymbol{x}, t)$$

$$b^{\chi - z}\frac{\partial \phi'}{\partial t'} = b^{\chi-2}k\nabla'^2\phi' - b^\chi\varepsilon\phi' + b^{-\frac{d+z}{2}}\sqrt{2T}\xi$$

$$\frac{\partial \phi'}{\partial t'} = k'\nabla'^2\phi' + -\varepsilon'\phi' + \sqrt{2T'}\xi$$

$k' = b^{z-2}k$

$\varepsilon' = b^z\varepsilon$

$T' = b^{\frac{z-2\chi-d}{2}}T$

Dynamical scaling in equilibrium

Critical exponents

$$\frac{\partial \phi'}{\partial t'} = k' \nabla'^2 \phi' + -\varepsilon' \phi' + \sqrt{2T'}\xi$$

$k' = b^{z-2}k$

$\varepsilon' = b^{z}\varepsilon$

$T' = b^{\frac{z-2\chi-d}{2}}T$

Scale invariance at transition point $\varepsilon = 0$

$$k' = k,\ T' = T$$

Scaling relations

$$z - 2 = 0,\ \frac{z - 2\chi - d}{2} = 0$$

Critical exponents of the linear model (mean-field)

$$z = 2,\ \chi = \frac{2-d}{2}$$

Dynamical scaling in equilibrium

Correlation functions

Two point correlation function

$$C(\boldsymbol{x}, t, \varepsilon) \equiv \langle \phi(\boldsymbol{x}, t)\phi(\boldsymbol{0},0)\rangle$$

Scaling Ansatz

$$\boldsymbol{x} = b\boldsymbol{x}', \ t = b^z t', \ \phi(\boldsymbol{x}, t) = b^\chi \phi'(\boldsymbol{x}', t')$$

$$C(\boldsymbol{x}, t, \varepsilon) = b^{2\chi} C(b^{-1}\boldsymbol{x}, b^{-z}t, b^z\varepsilon)$$

Set the scaling parameter $b = \varepsilon^{-1}$

$$C(x, t, \varepsilon) = \varepsilon^{-2\chi/z} C(x/\varepsilon^{-1/2}, t/\varepsilon^{-1}, 1)$$

Correlation length

$$\xi = \varepsilon^{-\nu}, \ \nu = 1/2$$

Relaxation time

$$\tau = \varepsilon^{-1}$$

Correlation length and relaxation time diverge with power-law.

Dynamical scaling in equilibrium

Static structure factor

Two point equal time correlation function

$$C(\boldsymbol{x}, t, \varepsilon) = b^{2\chi} C(b^{-1}\boldsymbol{x}, b^{-z}t, b^{z}\varepsilon)$$

Fourier transformation

$z = 2\chi + d$

$$\hat{C}(q, \varepsilon) = \int d\boldsymbol{x} e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} C(\boldsymbol{x}, \varepsilon) = b^{z}\hat{C}(b\boldsymbol{q}, b^{z}\varepsilon)$$

- $\hat{C}(q = 0, \varepsilon) = b^{z}\hat{C}(0, b^{z}\varepsilon) \xrightarrow{b=\varepsilon^{-1/z}} \varepsilon^{-1}\hat{C}(0,1) \sim \varepsilon^{-1}$
- $\hat{C}(q, \varepsilon = 0) = b^{z}\hat{C}(bq, 0) \xrightarrow{q=\varepsilon^{-1}} q^{-z}\hat{C}(1,0) \sim q^{-2}$

$$\hat{C}(q, \varepsilon) = \left(c_1\varepsilon + c_2 q^2 + \cdots\right)^{-1}$$

Ornstein-Zernike (OZ) equation

Dynamical scaling in equilibrium

Upper critical dimension

Scaling transform of the full equation of motion

$$\frac{\partial \phi(\boldsymbol{x},t)}{\partial t} = k\nabla^2\phi(x,t) - \varepsilon\phi(\boldsymbol{x},t) - u\phi(\boldsymbol{x},t)^3 + \sqrt{2T}\xi(\boldsymbol{x},t)$$

Nonlinear terms

Scaling transformation

$$b^{\chi-z}\frac{\partial \phi'}{\partial t'} = b^{\chi-2}k\nabla'^2\phi' - b^{\chi}\varepsilon\phi' - b^{3\chi}u\phi'^3 + b^{-\frac{d+z}{2}}\sqrt{2T}\xi$$

$$\frac{\partial \phi'}{\partial t'} = k'\nabla'^2\phi' + -\varepsilon'\phi' - u'\phi'^3 + \sqrt{2T'}\xi$$

$k' = b^{z-2}k$

$\varepsilon' = b^{z}\varepsilon$

$u' = b^{2\chi+z}u$

$T' = b^{\frac{z-2\chi-d}{2}}T$

Dynamical scaling in equilibrium

Upper critical dimensions

$$\frac{\partial \phi'}{\partial t'} = k' \nabla'^2 \phi' + -\varepsilon' \phi' - u' \phi'^3 + \sqrt{2T'}\xi$$

$k' = b^{z-2}k$

$\varepsilon' = b^{z}\varepsilon$

$u' = b^{2\chi+z}u$

$T' = b^{\frac{z-2\chi-d}{2}}T$

$$u' = b^{4-d}u \xrightarrow{b\to\infty} \begin{cases} \infty & d<4 \\ 0 & d>4 \end{cases}$$

relevant ($d<4$)

irrelevant ($d>4$)

For d<4, the non-linear term diverges in the large length scale b>>1

↓

The mean-field theory fails!!!

Upper critical dimension in equilibrium

$$d_{\mathrm{up}} = 4$$

What will happen far from equilibrium?

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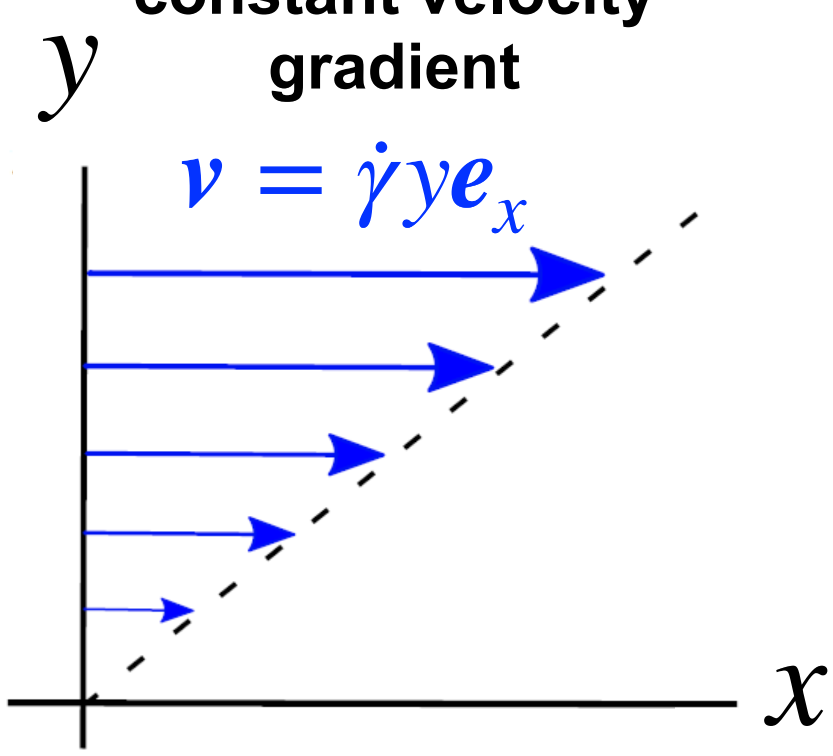
Model

Dynamical equation for non-conservative order parameter (model-A)

$$\frac{\partial \phi(\boldsymbol{x},t)}{\partial t} + \dot{\gamma} y \frac{\partial \phi(\boldsymbol{x},t)}{\partial x} = -\frac{\delta F[\phi(\boldsymbol{x},t)]}{\delta \phi(\boldsymbol{x},t)} + \sqrt{2T}\xi(\boldsymbol{x},t)$$

Advection by simple shear

Shear flow with constant velocity gradient



The simple shear is one of the simplest settings to realize nonequilibrium steady state.

Dynamical scaling under shear

Previous studies for critical phenomena in shear flow

- **1976, P.G. De Gennes: Structure factor in shear flow**
  At the transition point: $S(\boldsymbol{q}) \sim q_x^{-2/3}$.
  Much weaker than the standard OZ equation
  $S(\boldsymbol{q}) \sim q^{-2}$

- **2020 Nakano *et al.*: Simulation of O(2) model in shear**
  $\rightarrow$ Observed mean-field exponents even in d=2

Previous studies suggest the shear flow reduce upper critical dimension of the model-A. However the theoretical understanding is yet to be completed.

(but see Onuki&Kawasaki 1979 for RG analysis of model-H in shear)

Dynamical scaling under shear

Motivation & Main results

Motivation

- In equilibrium, the mean-field theory fails in d=2 and 3.
- Previous studies suggest that the shear flow suppress critical fluctuations.
- Here, we develop the scaling theory of model-A under shear.

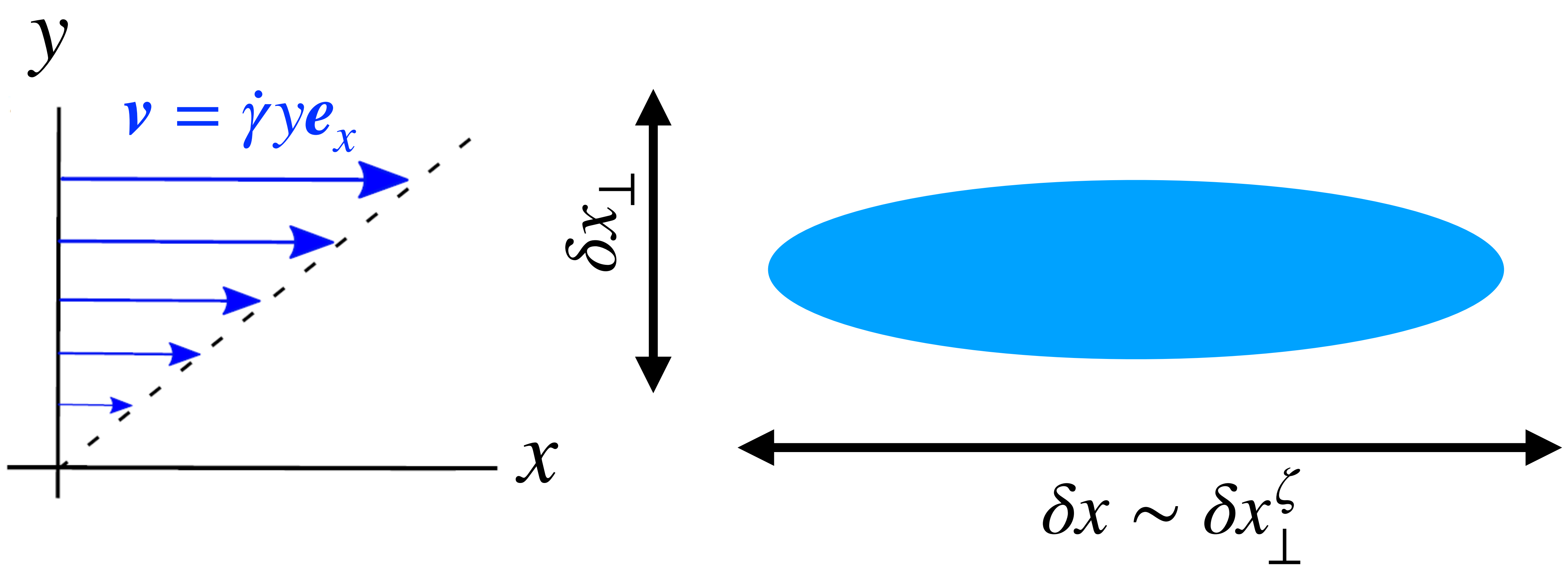
Main results

- A new Gaussian fixed point appears under simple shear.
- Upper critical dimensions of the new Gaussian fixed point is
  $d_{\mathrm{up}} = 2$
  → The mean-field becomes exact in d=2 and 3.

Dynamical scaling under shear

Scaling argument

Due to shear, critical fluctuations are anisotropic



ζ: anisotropy exponent

Anisotropic scaling Ansatz

$$x = b^\zeta x', \; \boldsymbol{x}_\perp = b\boldsymbol{x}'_\perp, \; t = b^z t', \; \phi(x, \boldsymbol{x}_\perp, t) = b^\chi \phi'(x', \boldsymbol{x}'_\perp, t')$$

Parallel to shear

Perpendicular to shear

Dynamical scaling under shear

Scaling argument

$$\frac{\partial \phi}{\partial t} + \dot{\gamma} y \frac{\partial \phi}{\partial x} = k\partial_x^2 \phi + k\nabla_\perp^2 \phi - \varepsilon\phi - u\phi^3 + \sqrt{2T}\xi$$

Anisotropic scaling Ansatz

$$x = b^\zeta x',\ \boldsymbol{x}_\perp = b\boldsymbol{x}'_\perp,\ t = b^z t',\ \phi(x, \boldsymbol{x}_\perp, t) = b^\chi \phi'(x', \boldsymbol{x}'_\perp, t')$$

$$b^{\chi-z}\frac{\partial \phi'}{\partial t'} + b^{1-\zeta+\chi}\dot{\gamma} y' \frac{\phi'}{x'} = b^{\chi-2\zeta} k\partial_{x'}^2 \phi' + b^{\chi-2} k\partial_{x'}^2 \phi' - b^\chi \varepsilon\phi' - b^{3\chi} u\phi'^3 + b^{-\frac{\zeta+d-1+z}{2}}\sqrt{2T}\xi'$$

$$\frac{\partial \phi'}{\partial t'} + \dot{\gamma}' y' \frac{\partial \phi'}{\partial x'} = k'_\parallel \partial_{x'}^2 \phi' + k'_\perp \nabla_\perp'^2 \phi' - \varepsilon'\phi' - u'\phi'^3 + \sqrt{2T'}\xi'$$

$T' = b^{\frac{z-2\chi-(d-1)-\zeta}{2}} T$

$\dot{\gamma}' = b^{z-\zeta+1}\dot{\gamma}$

$k'_\parallel = b^{z-2\zeta} k$

$k'_\perp = b^{z-2} k$

$\varepsilon' = b^z \varepsilon$

$u' = b^{2\chi+z} u$

Nonlinear term

Dynamical scaling under shear

Stability of equilibrium fixed point

Critical exponents in equilibrium

$$z = 2 \qquad \chi = \frac{2-d}{2} \qquad \zeta = 1$$

The system is isotropic

$\dot{\gamma}' = b^{z-\zeta+1}\dot{\gamma}$ $\quad k'_{\parallel} = b^{z-2\zeta}k$ $\quad k'_{\perp} = b^{z-2}k$ $\quad T' = b^{\frac{z-2\chi-(d-1)-\zeta}{2}}T$

$$\frac{\partial \phi'}{\partial t'} + \dot{\gamma}' y' \frac{\partial \phi'}{\partial x'} = k'_{\parallel}\partial_{x'}^2\phi' + k'_{\perp}\nabla_{\perp}'^{2}\phi' - \varepsilon'\phi' + \sqrt{2T'}\xi'$$

$k_{\parallel}$, $k_{\perp}$, T **are scale invariance**

However, the coefficient of the shear diverges.

$$\dot{\gamma}' = b^{z-\zeta+1}\dot{\gamma} = b^2\dot{\gamma} \xrightarrow{b\to\infty} \infty$$

The equilibrium fixed point is unstable (even in d>4).

Dynamical scaling under shear

New fixed point

Advection **Diffusion**

$$\dot{\gamma} y \frac{\partial \phi}{\partial x} \gg k_{\parallel} \partial_x^2 \phi$$

$k_{\parallel}$ is irrelevant!

We require the scale invariance of $\dot{\gamma}, \ k_{\perp}, \ T$

$$\dot{\gamma}' = b^{z-\zeta+1}\dot{\gamma}, \ k_{\perp} = b^{z-2}k, \ T' = b^{\frac{z-2\chi-(d-1)-\zeta}{2}}T$$

Scaling relations

$$z - \zeta + 1 = 0, \ z - 2 = 0, \ \frac{z - 2\chi - (d-1) - \zeta}{2} = 0$$

New critical exponents

Anisotropic

$$z = 2, \chi = -\frac{d}{2}, \zeta = 3$$

Self-consistency check

$$k_{\parallel}' = b^{z-2\zeta}k = b^{-4}k \xrightarrow{b\to\infty} 0$$

Dynamical scaling under shear

Upper critical dimension

Now, we shall discuss the effects of the nonlinear terms

$$T' = b^{\frac{z-2\chi-(d-1)-\zeta}{2}} T$$

$$\frac{\partial \phi'}{\partial t'} + \dot{\gamma}' y' \frac{\partial \phi'}{\partial x'} = k'_{\parallel} \partial_{x'}^2 \phi' + k'_{\perp} \nabla'^{2}_{\perp} \phi' - \varepsilon' \phi' - u' \phi'^3 + \sqrt{2T'} \xi'$$

$\dot{\gamma}' = b^{z-\zeta+1} \dot{\gamma}$

$k'_{\parallel} = b^{z-2\zeta} k$

$k'_{\perp} = b^{z-2} k$

$\varepsilon' = b^z \varepsilon$

$u' = b^{2\chi+z} u$

Nonlinear term

Coefficient of the nonlinear term

$$u' = b^{2\chi+z} u = b^{2-d} u \xrightarrow{b\to\infty} \begin{cases} 0 & d > 2 \\ \infty & d < 2 \end{cases}.$$

Irrelevant ($d > 2$)

Relevant ($d < 2$)

But, the shear is ill-defined for d<2.

The upper critical dimension

$$d_{\mathrm{up}} = 2$$

Dynamical scaling under shear

Static structure factor

Two point correlation function

$$C(x_{\|}, \boldsymbol{x}_{\perp}, \varepsilon) \equiv \langle \phi(\boldsymbol{x}, t)\phi(\boldsymbol{0},0)\rangle$$

Fourier transformation

$$\hat{C}(q_{\|}, q_{\perp}, \varepsilon) = b^z \hat{C}(b^{\zeta} q_{\|}, b q_{\perp}, b^z \varepsilon)$$

- $\hat{C}(0,0,\varepsilon) = b^z \hat{C}(0,0,b^z\varepsilon) \xrightarrow{b=\varepsilon^{-1/z}} \varepsilon^{-1}\hat{C}(0,0,1) \sim \varepsilon^{-1}$
- $\hat{C}(q_{\|},0,0) = b^z \hat{C}(b^{\zeta} q_{\|},0,0) \xrightarrow{b=q_{\|}^{-1/\zeta}} q_{\|}^{-z/\zeta}\hat{C}(1,0,0) \sim q_{\|}^{-2/3}$
- $\hat{C}(0,q_{\perp},0) = b^z \hat{C}(0,b q_{\perp},0) \xrightarrow{b=q_{\perp}^{-1}} q_{\perp}^{-z}\hat{C}(0,1,0) \sim q_{\perp}^{-2}$

$$\hat{C}(q_{\|}, q_{\perp}, \varepsilon) = \left(c_1\varepsilon + c_2 q_{\|}^{2/3} + c_3 q_{\perp}^2 + \cdots\right)^{-1}$$

Anisotropic correlation function

DeGennes 1976

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Comparison with numerics

Finite size scaling

$$\phi(\varepsilon, L_x, L_y) = b^{\chi}\phi(b^{z}\varepsilon, b^{-\zeta}L_x, b^{-1}L_y) = L_x^{\chi/\zeta}\phi(L_x^{z/\zeta}\varepsilon, 1, L_x^{-1/\zeta}L_y)$$

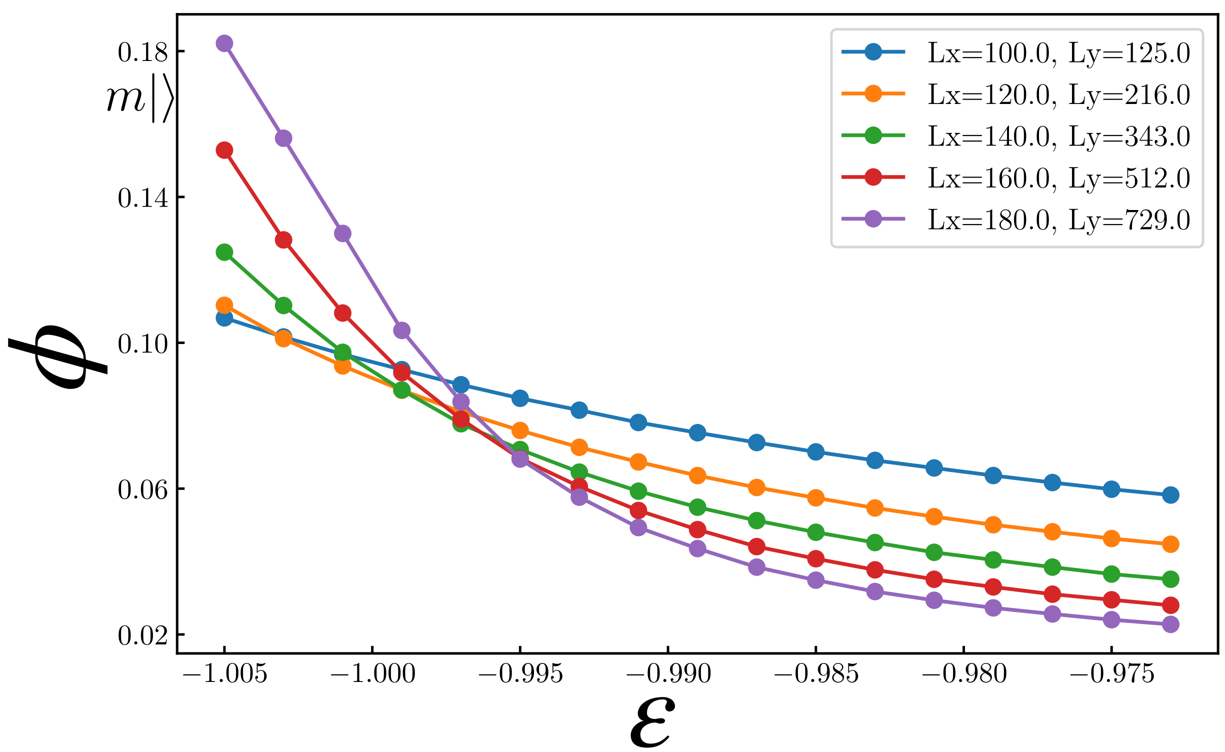
Scaling exponents

$$z = 2,\ \zeta = 3,\ \chi = -1$$

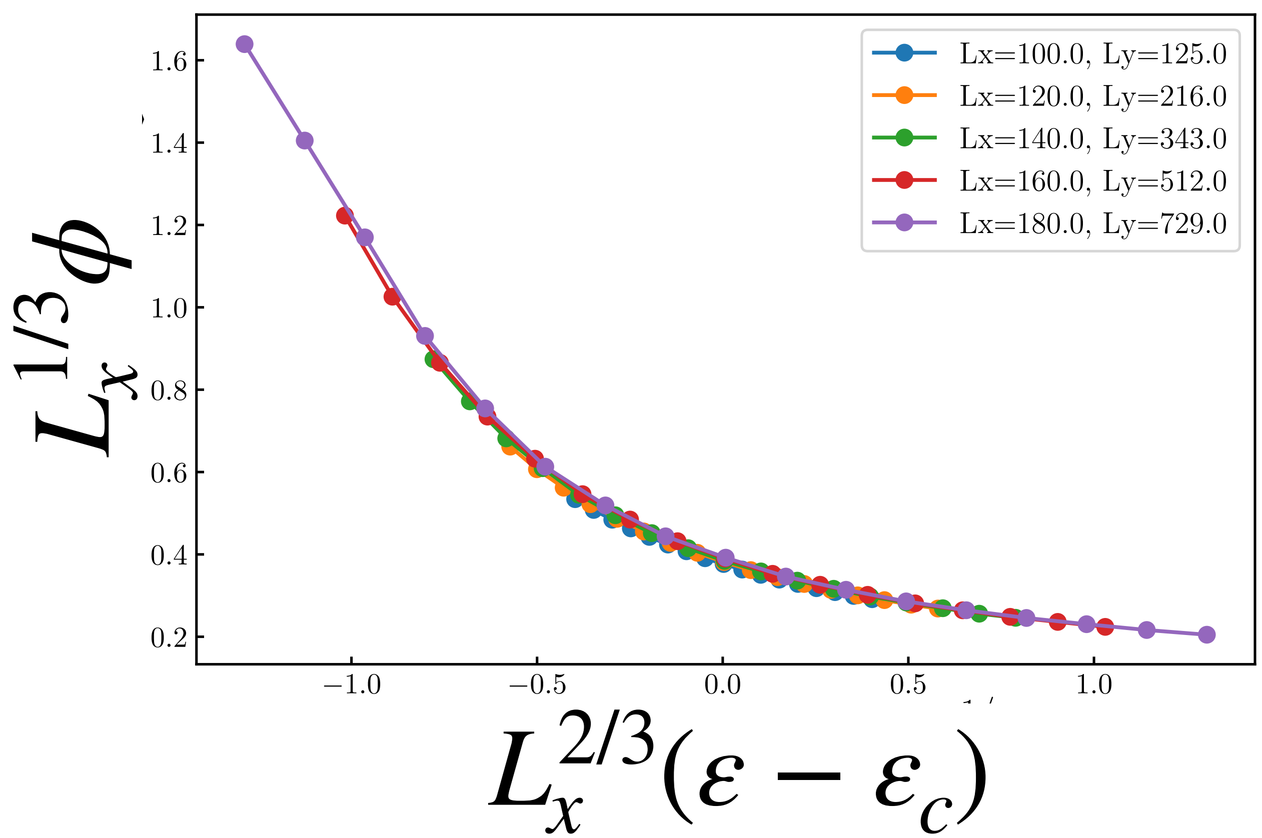
Scaling function

$$\phi(\varepsilon, L_x, L_y) = L_x^{-1/3}\Phi(L_x^{2/3}\varepsilon, L_x^{-1/3}L_y)$$

Order parameter for $\dot{\gamma} = 5.0$



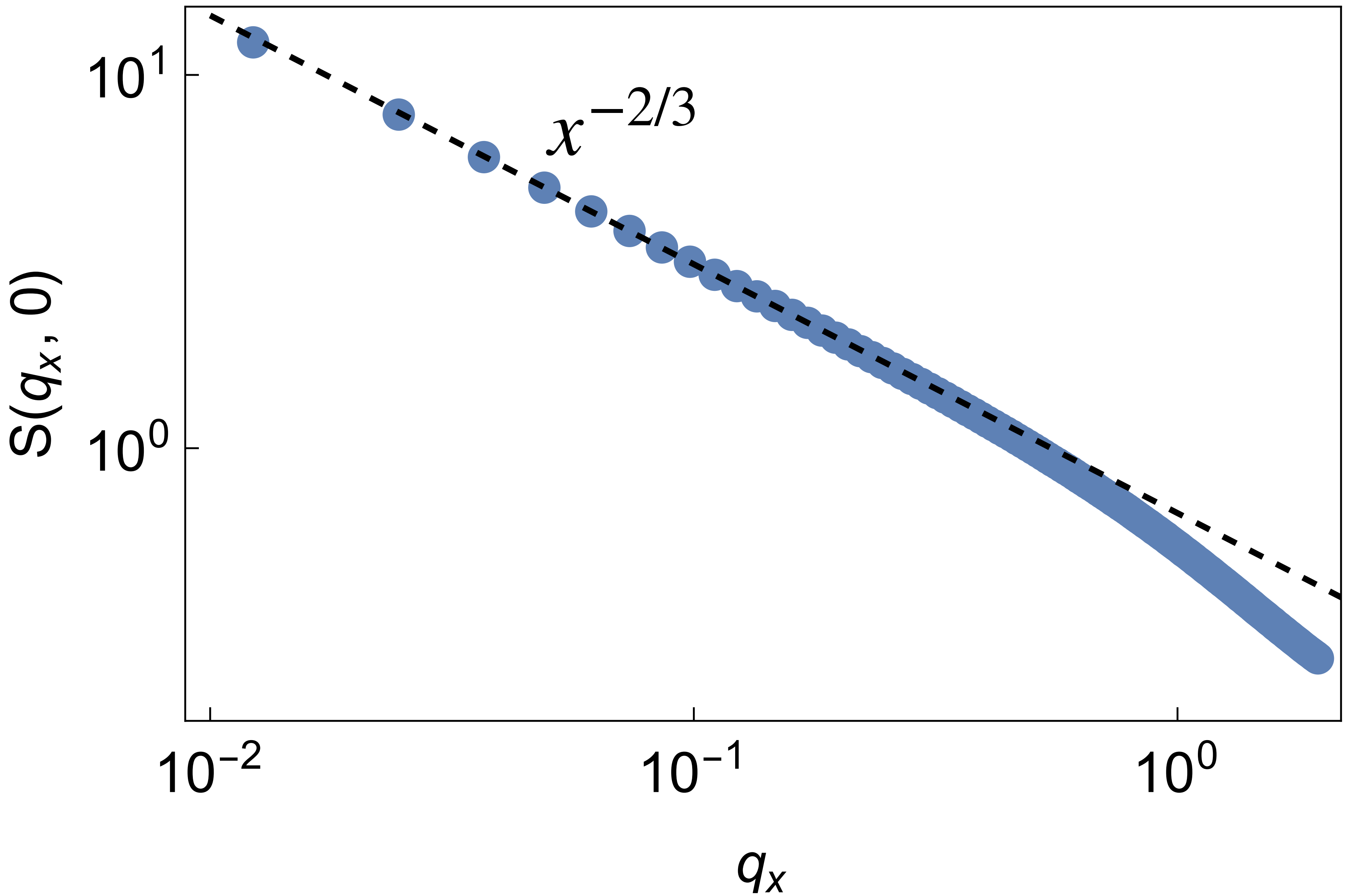
Scaling plot for $\dot{\gamma} = 5.0$



Comparison with numerics

Structure factor at critical point

Static structure factor at critical point for $\dot{\gamma} = 5.0$



$S(q_x, 0) \sim q_x^{-2/3}$ **is indeed observed in numerics.**

Comparison with numerics

Crossover phenomenon

Scaling behavior for crossover phenomenon

$$S(q_x,0) = \dot{\gamma}^{-a}\mathcal{S}(\dot{\gamma}^{-b}q_x)$$

$\dot{\gamma} \to 0$ → **Equilibrium Ising model (2D)**

$$S(q_x,0) \sim q_x^{-2+\eta},\ \eta = 1/4$$

$q_x \ll \dot{\gamma}^b$ → **Anisotropic scaling**

$$S(q_x,0) \sim \dot{\gamma}^{-2/3}q_x^{-2/3}$$

Scaling exponents

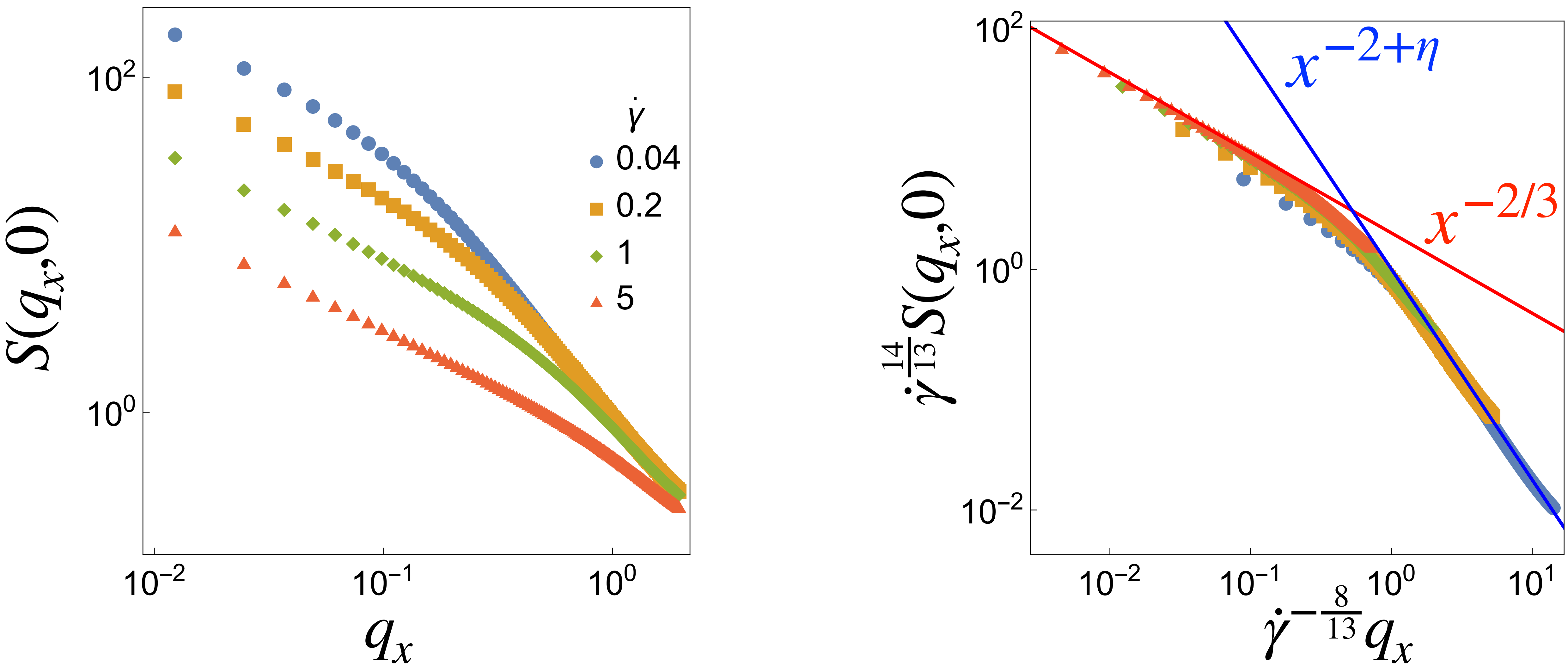
$$a = \frac{14}{13},\quad b = \frac{8}{13}$$

Comparison with numerics

Crossover phenomenon

Scaling function

$$S(q_x,0) = \dot{\gamma}^{-\frac{14}{13}} \mathcal{S}(\dot{\gamma}^{-\frac{8}{13}} q_x)$$



The mean-field scaling is always observed for $q_x \ll \dot{\gamma}^{\frac{8}{13}}$

→ **Even infinitesimal shear rate changes the universality class!**

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Summary and discussions

Generalization for other critical phenomena

$$\frac{\partial \phi}{\partial t} + \dot{\gamma} y \frac{\partial \phi}{\partial x} = \cdots \quad \xrightarrow{\text{Scale transformation}} \quad b^{\chi - z}\frac{\partial \phi}{\partial t} + b^{\chi+1-\zeta}\dot{\gamma} y \frac{\partial \phi}{\partial x} = \cdots$$

Advection by simple shear

General relation

$$\chi - z = \chi + 1 - \zeta \;\rightarrow\; \zeta = z + 1$$

Volume integral

$$\int d\boldsymbol{x} = b^{d-1+\zeta}\int dx_{\parallel} d\boldsymbol{x}_{\perp} \sim b^{d_{\mathrm{eff}}},\; d_{\mathrm{eff}} = d + z$$

Since the dimensional dependence appears only through the volume integral, **the scaling behavior of the shared system can be identified with the equilibrium model in** $d_{\mathrm{eff}} = d + z$

Simple shear reduces the upper critical dimension as

$$d_{\mathrm{up}} \rightarrow \max[d_{\mathrm{up}} - z,\; 2]$$

Shear is ill-defined for d<2

Summary and discussions

Generalization for other critical phenomena

	z	d_{up} in equilibrium	d_{up} in shear
Model-A	2	4	2
Model-B	4	4	2
A+A→0	2	2	2
A+B→0	2	4	2
Directed percolation	2	4	2

Mean-field behaviors are expected in d=2 for various critical phenomena.

Summary

- We investigated the phi-4 model with simple shear by using the scaling analysis.
- We found a new Gaussian fixed point.
- The upper critical dimension of the new fixed point is $d_{\rm up} = 2$, meaning that the mean-field theory becomes exact in d=2 and 3.
- In general, the simple shear reduces the upper critical dimension as $d_{\rm up} \to d_{\rm up}^{\rm eq} - z$