Efimov effect at the Kardar-Parisi-Zhang roughening transition

Yu Nakayama (YITP)

with Yusuke Nishida (Institute of Science Tokyo)
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KPZ roughening Phase Transition

- KPZ equation

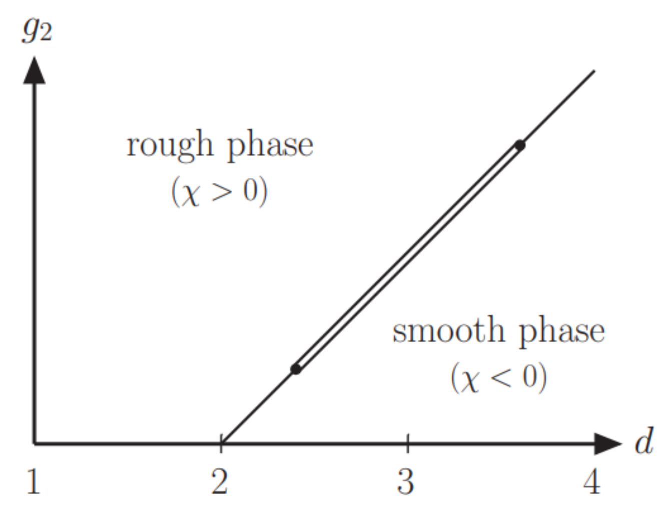
$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \sqrt{D}\,\eta,$$

$$\langle \eta(t, \boldsymbol{r}) \eta(t', \boldsymbol{r}') \rangle = \delta(t - t')\delta(\boldsymbol{r} - \boldsymbol{r}').$$

- Edwards-Wilkinson smooth phase: $g_2 = D\lambda^2/\nu^3 \to 0$
- Rough phase (KPZ phase): $g_2 = D\lambda^2/\nu^3 \to \infty$
- Depending on spatial dimension d, phase transition exists

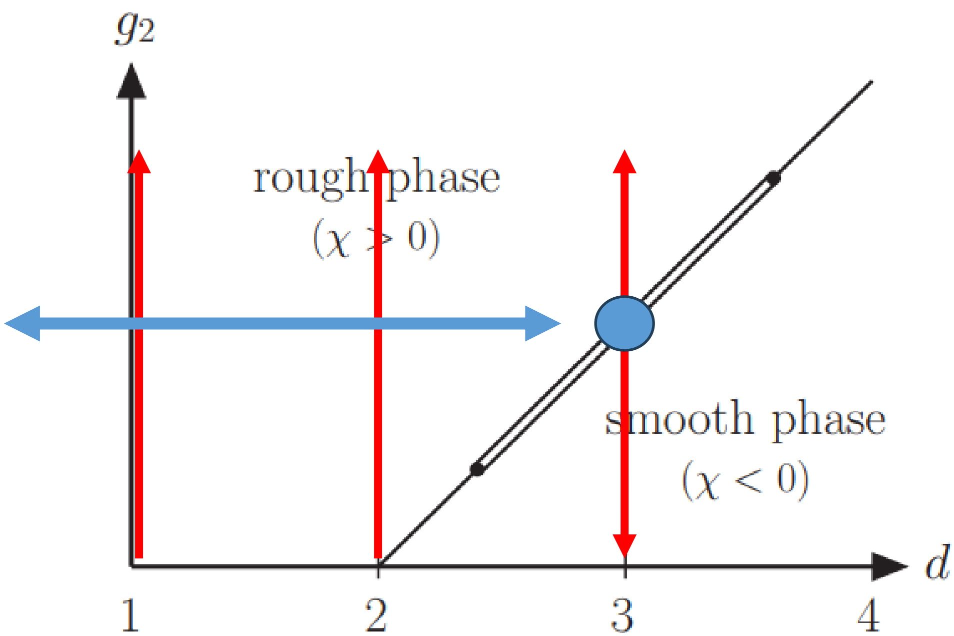
$$\langle [(h(t, \boldsymbol{r}) - h(0, \boldsymbol{0})]^2 \rangle \sim r^{2\chi} F\left(\frac{t}{r^z}\right),$$

- I'm interested in the (unstable) phase transition point in d=3



Phase Transition

- I love critical phenomena such as CFT, 2nd order phase transition, renormalization group etc, so I got very much wondered why people are not talking about this roughening phase transition point
- We found it is related to Efimov effect in atomic physics, which shows extremely interesting behavior



Why atomic physics and KPZ?

- KPZ equation $$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \sqrt{D}\,\eta,$$

$$\langle \eta(t, \boldsymbol{r})\eta(t', \boldsymbol{r}')\rangle = \delta(t-t')\delta(\boldsymbol{r}-\boldsymbol{r}').$$

Can be mapped to non-relativistic interacting particles

- Stochastic path integral approach (Martin, Siggia, Rose, Janssen, and De Dominicis)

$$S_\Lambda[h, \bar{h}] = \int dt d\boldsymbol{r}\, i\bar{h}\left[\frac{\partial h}{\partial t} - \nu \nabla^2 h - \frac{\lambda}{2}(\nabla h)^2\right]$$
$$- \frac{D}{2}\int dt d\boldsymbol{r} d\boldsymbol{r}' i\bar{h}(t, \boldsymbol{r}) V_\Lambda(\boldsymbol{r}-\boldsymbol{r}') i\bar{h}(t, \boldsymbol{r}').$$

- Do Cole-Hopf transformation $h(t, \boldsymbol{r}) = \frac{2\nu}{\lambda}\ln\phi(t, \boldsymbol{r}), \quad i\bar{h}(t, \boldsymbol{r}) = \frac{\lambda}{2\nu}\bar{\phi}(t, \boldsymbol{r})\phi(t, \boldsymbol{r})$

$$S_\Lambda[\phi, \bar{\phi}] = \int d\tau d\boldsymbol{r}\, \bar{\phi}(\tau, \boldsymbol{r})\left(\frac{\partial}{\partial \tau} - \frac{\nabla^2}{2}\right)\phi(\tau, \boldsymbol{r})$$
$$- \frac{g_2}{4}\int d\tau d\boldsymbol{r} d\boldsymbol{r}' \bar{\phi}(\tau, \boldsymbol{r})\phi(\tau, \boldsymbol{r}) V_\Lambda(\boldsymbol{r}-\boldsymbol{r}')\bar{\phi}(\tau, \boldsymbol{r}')\phi(\tau, \boldsymbol{r}').$$

- → Interacting non-relativistic bosons

Suspicious?

- My first impression was "suspicious"
- I'm a theoretical physicist, so "suspicious" means, if true, an opportunity (e.g. string duality, AdS/CFT whatsoever)

$$S_\Lambda[\phi,\bar{\phi}] = \int d\tau d\boldsymbol{r}\,\bar{\phi}(\tau,\boldsymbol{r})\left(\frac{\partial}{\partial\tau} - \frac{\nabla^2}{2}\right)\phi(\tau,\boldsymbol{r})$$
$$-\frac{g_2}{4}\int d\tau d\boldsymbol{r} d\boldsymbol{r}'\bar{\phi}(\tau,\boldsymbol{r})\phi(\tau,\boldsymbol{r})V_\Lambda(\boldsymbol{r}-\boldsymbol{r}')\bar{\phi}(\tau,\boldsymbol{r}')\phi(\tau,\boldsymbol{r}').$$

- Reality of fields? $h(t,\boldsymbol{r}) = \frac{2\nu}{\lambda}\ln\phi(t,\boldsymbol{r}),\quad i\bar{h}(t,\boldsymbol{r}) = \frac{\lambda}{2\nu}\bar{\phi}(t,\boldsymbol{r})\phi(t,\boldsymbol{r})$
- Replica trick to compute $\langle h\rangle \sim \langle\log\phi\rangle$ from $\langle\phi^n\rangle$
- Strong evidence in d=1: interacting bosons in d=1 (Lieb-Liniger model) is exactly solvable
- Takeuchi-san is famously known as the only experimental physicist on earth who can use Bethe-Ansatz and solve it; he demonstrated the "exact solution" to the problem in agreement with (his) experiments

In d>1?

- Two-body problems can be solved in any dimensions

$$S_\Lambda[\phi,\bar{\phi}] = \int d\tau d\boldsymbol{r}\, \bar{\phi}(\tau,\boldsymbol{r}) \left(\frac{\partial}{\partial\tau} - \frac{\nabla^2}{2}\right) \phi(\tau,\boldsymbol{r})$$
$$- \frac{g_2}{4} \int d\tau d\boldsymbol{r} d\boldsymbol{r}' \bar{\phi}(\tau,\boldsymbol{r})\phi(\tau,\boldsymbol{r}) V_\Lambda(\boldsymbol{r}-\boldsymbol{r}')\bar{\phi}(\tau,\boldsymbol{r}')\phi(\tau,\boldsymbol{r}').$$

- d=1,any small g_2 is relevant (QM 101: existence of bound states in delta potential)
- d=2, needs renormalization, but relevant (Advanced QM: important in BCS superconductor)
- d>2, small g_2 perturbation is irrelevant; the beta function is

$$\Lambda \frac{\partial \hat{g}_2}{\partial \Lambda} = (d-2)\hat{g}_2 - \frac{\hat{g}_2^2}{(4\pi)^{d/2}\Gamma(d/2)},$$

- Exist an (exact) UV fixed point at $g_2 > 0$
- Should describe the roughening phase transition with non-relativistic conformal invariance!!
- The same equation can be derived in dynamical RG

Now it’s getting interesting

I’m talking about something not found in KPZ literature

In d=3

- Since we are talking about UV fixed point, beyond perturbation theory, finding g_2 fixed point may not be sufficient (say in d=3)

- I consulted an expert (and a friend), Yusuke Nishida, who has been studying this non-relativistic interacting bosons system (near criticality) in d=3

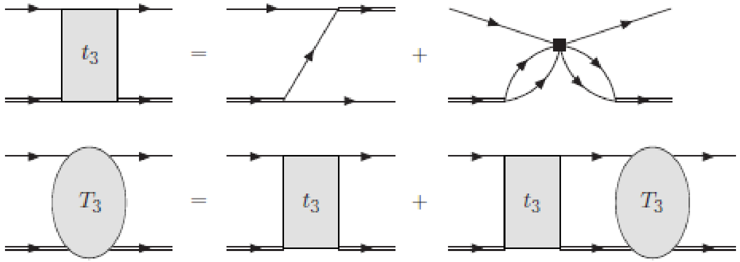
- He said, "Oh, if you consider the other couplings, there are NO fixed point in this system. It shows only discrete scale invariance"
- This is the Efimov effect!

matrioshka

Further RG in three-body coupling

- Higher order (contact) interaction IS generated under the RG flow

$$S_\Lambda[\phi,\bar\phi] \to \int d\tau\, dr \left[\bar\phi\left(\frac{\partial}{\partial\tau} - \frac{\nabla^2}{2}\right)\phi - \frac{g_2}{4}(\bar\phi\phi)^2 - \frac{g_3}{36}(\bar\phi\phi)^3 + \cdots\right].$$



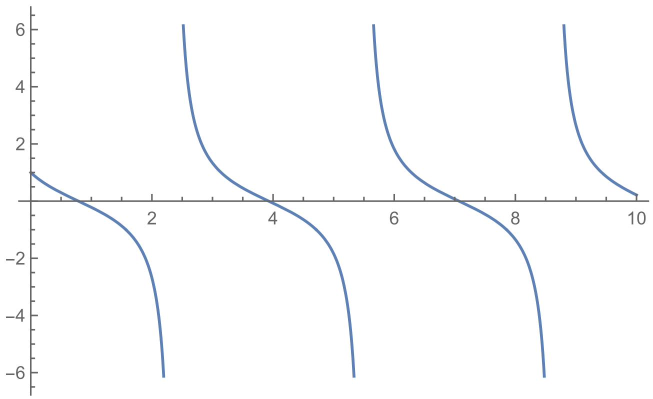
- In non-relativistic bosons (think about QM!) higher order term cannot affect the lower order term
- The g_2 fixed point cannot be corrected
- The RG of g_3 has been studied extensively in the atomic physics: Omitting the details, at g_2 fixed point, they have

$$\Lambda\frac{\partial \hat g_3}{\partial\Lambda} = -\frac{1+s_0^2}{2}\left(\frac{\hat g_3^2}{c} + c\right) + \frac{1-s_0^2}{2}\hat g_3,$$

- It has NO fixed point

Further RG

- Try to integrate the RG equation

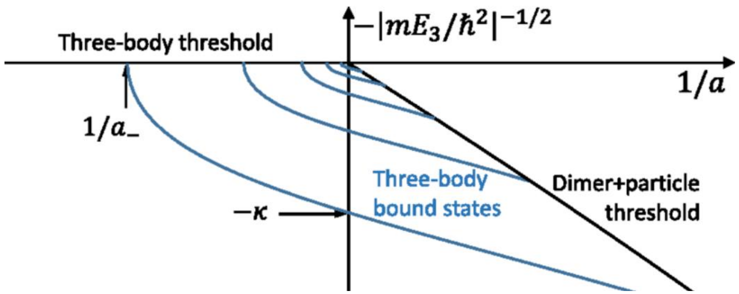


$$\Lambda\frac{\partial \hat{g}_3}{\partial \Lambda} = -\frac{1+s_0^2}{2}\left(\frac{\hat{g}_3^2}{c}+c\right)+\frac{1-s_0^2}{2}\hat{g}_3,$$

- Can be integrated:

$$\hat{g}_3 = c\,\frac{1-s_0\tan(s_0\ln\Lambda/\Lambda_*)}{1+s_0\tan(s_0\ln\Lambda/\Lambda_*)},$$

- It is suspicious if we can trust beta functions when it becomes infinity (e.g. Landau pole), but if we do so, it has no fixed point but it has only discrete scale invariance
- Surprisingly, solving three-body Schrödinger equation (numerically) shows discrete scale invariant energy spectrum (Efimov states, Efimov effect)



What we have learned

- This means that in the Cole-Hopf picture
  - $\langle \phi^2 \rangle$ is scale invariant
  - $\langle \phi^3 \rangle$ is only discrete scale invariant
- Unfortunately, in non-relativistic particle physics, it is NOT known if $\langle \phi^n \rangle$ has the same discrete scale invariance or is more chaotic
- It is not clear what the replica trick should imply

$$\langle \log \phi \rangle = \langle \frac{\partial \phi^n}{\partial n} \rangle |_{n \to 0}$$

- But our best bet is either of the two
  - The roughening transition is second order and has discrete scale invariance

$$\langle [(h(t,r) - h(0,0)]^2 \rangle \sim F_{s0} \left(\ln r\Lambda_*, \frac{t}{r^2} \right) ,$$

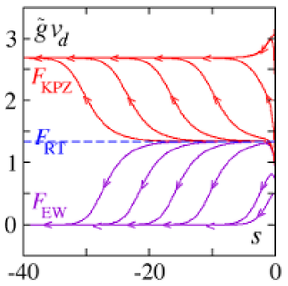
  - The chaotic RG flow make the second order impossible → first order

A comment on functional RG

- RG and the fixed point properties should not depend on the formulation (in principle)

- But with a full of suspicious things, it is interesting to look at different approach

- There is a functional dynamical RG approach based on the original variables h rather than Cole-Hopf
- Perturbatively equivalent for g_2 with fixed point in d>2
- No findings in discrete scale invariance in a study by Delamotte et al (!)
- But this is because their functional ansatz did not introduce g_3



- "Exact RG" has never been exact…

$$\Gamma_\kappa[\psi,\tilde\psi]=\int_{t,\mathbf{x}}\Big\{\tilde\psi f_\kappa^\lambda(D_t,\nabla)\Big[\partial_t\psi-\frac{\lambda}{2}(\nabla\psi)^2\Big] -\tilde\psi f_\kappa^\nu(D_t,\nabla)\,\nabla^2\psi - f_\kappa^D(D_t,\nabla)\tilde\psi^2\Big\}$$

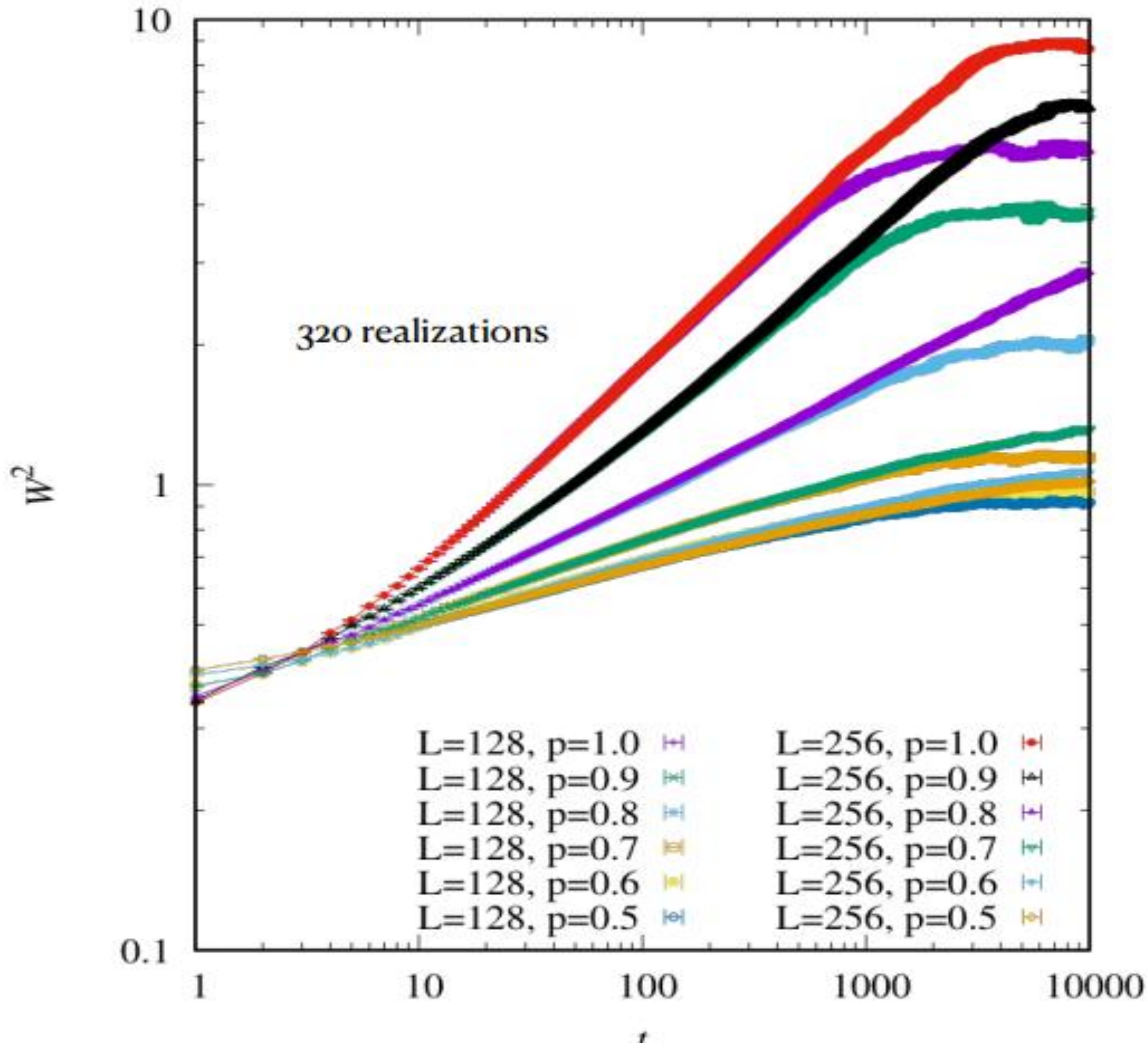
Numerical evidence?

Unpublished

Thanks to Kato-san (now in Fukui) for the simulation

RSOS model in d=2+1

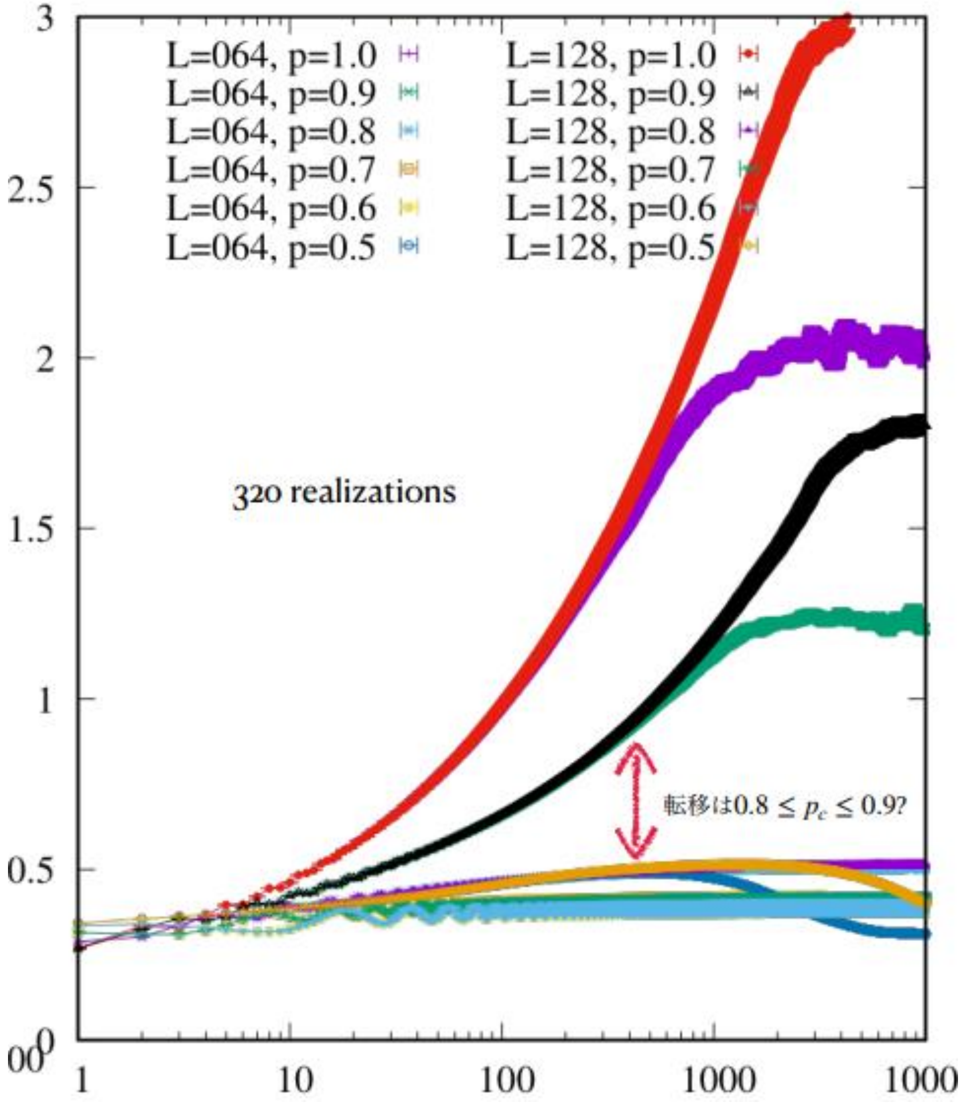
$$W^2(t,L) = \frac{1}{N}\sum_r \left[h(r) - \frac{1}{N}\left[\sum_{r'} h(r')\right]\right]^2$$



Any p leads to KPZ phase?

RSOS model in d=3+1

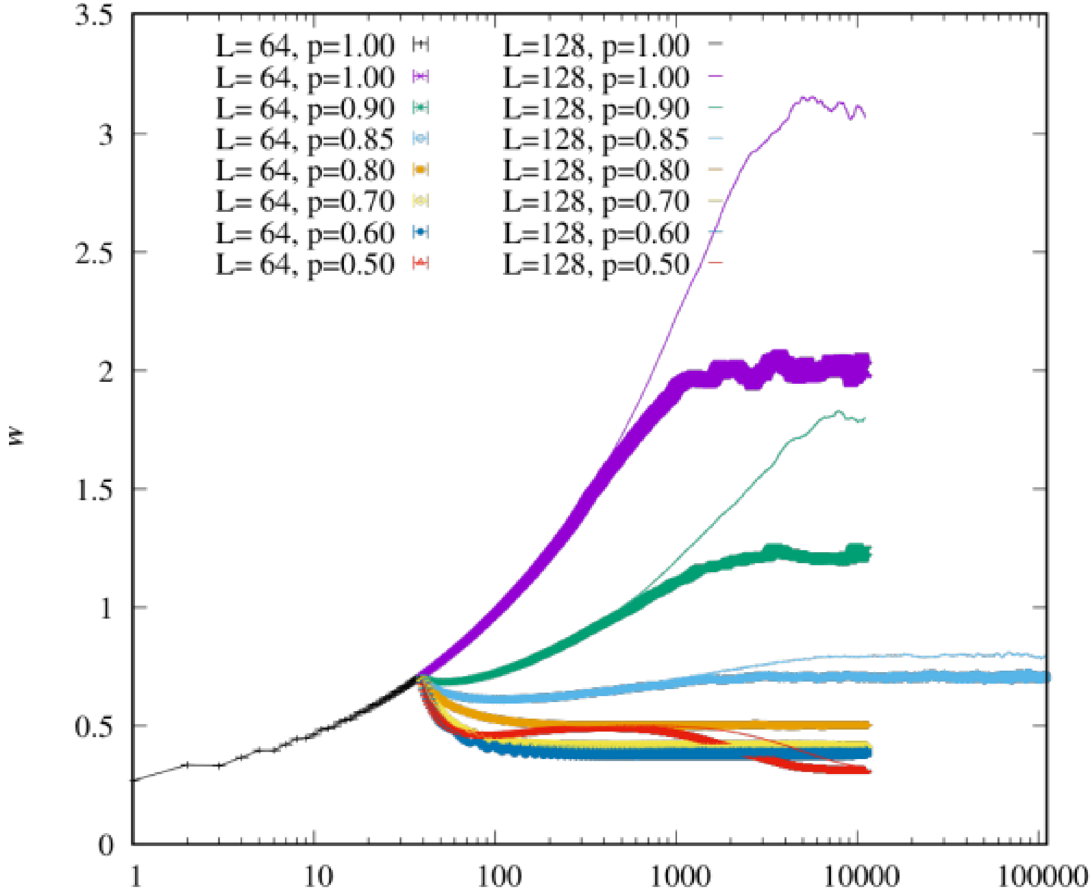
$$W^2(t,L) = \frac{1}{N}\sum_r \left[h(r) - \frac{1}{N}\left[\sum_{r'} h(r')\right]\right]^2$$



Phase transition near p = 0.85?

RSOS model in d=3+1

$$W^2(t,L) = \frac{1}{N}\sum_r \left[h(r) - \frac{1}{N}\left[\sum_{r'} h(r')\right]\right]^2$$



Smooth vs Rough

$$t \sim 500t?$$

No numerical conclusion

- I haven’t seen any clear evidence for or against the Efimov effect in KPZ in numerical simulations

- Any thought will be welcome

- Discretization and Cole-Hopf transform does not commute unless we take the continuum limit

- Efimov effect in Polymer in random media may be compromise?