Comments on Time Dependent Backgrounds in 2d String Theory

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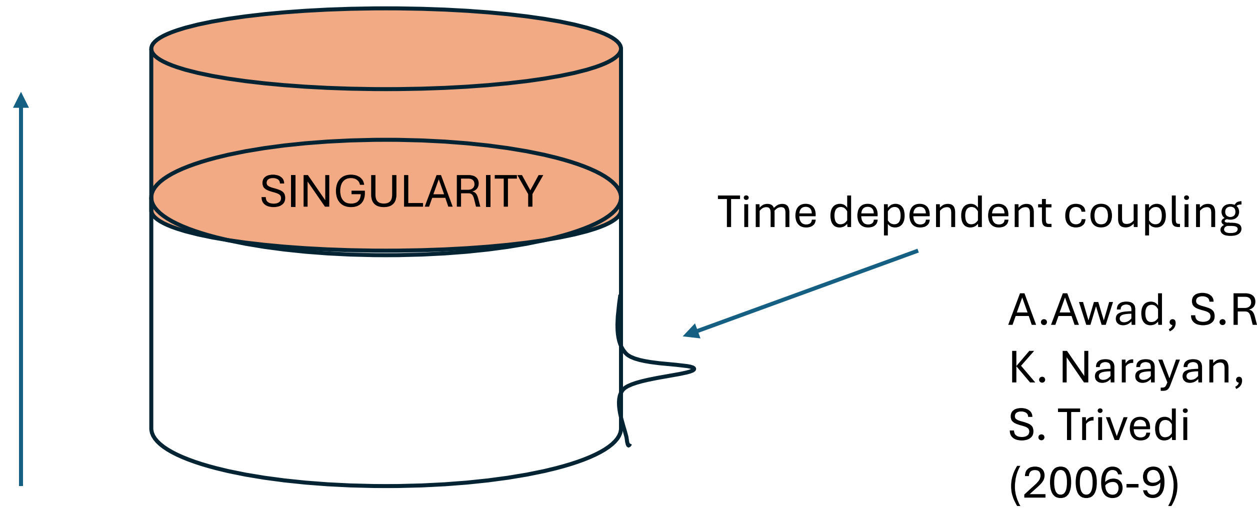
S.R.D., Sinong Liu & S. Hampton – JHEP (04) 2021 and work in progress

Cosmological Singularities in Holography

- Cosmological backgrounds pose several challenges – particularly when they involve space-like singularities, as in Big Bang- Big Crunch singularities, or in the interior of Schwarzschild black holes.
- From the low energy effective theory point of view these backgrounds imply an "end of time", or "beginning of time" – these are puzzling from the point of view of our usual notion of time evolution in physics.
- Over the past several decades there have been many attempts to understand this puzzle by considering situations where the gravitational description is emergent – from a non-gravitational holographic theory, e.g. Matrix Theory, AdS/CFT correspondence, or the holographic description of 2d String Theory in terms of the c=1 Matrix Model.

- One hope in these investigations is that while time evolution in the low energy gravitational description is problematic, there is a smooth time evolution in the more fundamental holographic description.
- For example, in the AdS/CFT correspondence such cosmological background results from a quantum quench in the boundary field theory – e.g. a time dependent coupling constant in the N=4 SYM.
- Normally such a quantum quench starting from the ground state results in black hole formation. However, in some situations one gets a space-like singularity which extends to the boundary



A.Awad, S.R.D, J. Michelson, K. Narayan, S. Nampuri, S. Trivedi (2006-9)

- This setup has been useful in understanding aspects of the *approach to the singularity* – as long as we do not go all the way to it

➢ Holographic Correlators encode some signatures of the singularity (*Engelhardt, Hertog, Horowitz (2005-2015)*.

➢ Complexity = Area *decreases as we approach the singularity*. (*Barbon & Rabinovici (2015)*

➢ *Path Integral complexity* monotonically decreases in a universal fashion, regardless of the Kasner exponents (*P. Caputa, D. Das & S.R.D. (2021*))

- However, the main question of a continuation of the holographic theory through what appears as a space-like singularity has not been resolved.
- There have been some suggestions of such a continuation (*Craps, Hertog, Turok*) – but things are not quite clear.
- If we have NULL singularity, a continuation is indeed possible both in AdS/CFT (*Chu & Ho*, *S.R.D., J. Michelson & S. Trivedi*) and in similar models in Matrix String Theory (*Craps, Sethi & Verlinde*)

- The difficulty in resolving this issue is two-fold

  1) Typically, the holographic theory is not solvable.

  2) String Theory effects become important – and not much is calculable in the bulk String Theory.

- Two-dimensional String Theory provides an example where we can make substantial progress with these two problems.
- The holographic description is well known – this is the quantum mechanics of a single Hermitian Matrix. The duality of this theory with the two-dimensional string was developed in the early 1990's and recent work of the past 3-4 years has solidified this duality to a remarkable extent.
- In fact, in the beginning of this century there were several works on developing time dependent backgrounds in the c=1 Matrix Model (*Alexandrov, Kazakov & Kostov (2002*); *Karczmarek & Strominger (2004*); *S.R.D., F. Davis, F. Larsen & P. Mukhopadhyay (2004*)).
- In particular, there were solutions which have space-like boundaries (*S.R.D. & J. Karczmarek).*

- More recently, there has been improved understanding of the String Theory, both at the level of world-sheet and at the level of String Field Theory (*Balthazar, Rodriguez & X. Yin (2017-2022)*; *A. Sen (2000-2024).*
- This improved understanding has been applied to a class of time dependent backgrounds – leading to world-sheet calculations of the S-Matrix and particle production (*Rodriguez (2023)*; *Balthazar, Chu & Kutasov (2023)*).
- It turns out that while for some of these backgrounds the world-sheet theory provides reasonable results for scattering amplitudes and for particle production, for a perhaps more interesting class, the problem cannot be posed on the world-sheet.
- In this talk I will resort to the Matrix Model description to understand what is going on in these problematic cases.

Time dependent background in 2d String Theory

(see review A. Jevicki, hep-th/9309115)

- The worldsheet action for 2d String Theory around the standard background is given by

$$S_{WS} = \frac{1}{8\pi} \int d^2 z \sqrt{h} \left(-\partial X \bar{\partial} X + \partial\varphi \bar{\partial}\varphi - 2\sqrt{2}\varphi R^{(2)}(h) + \mu \varphi e^{-\sqrt{2}\varphi} \right)$$

- The worldsheet field $X(z, \bar{z})$ is identified as time in the target space, while the Liouville field $\varphi(z, \bar{z})$ is treated as a space coordinate.
- The physical vertex operators for propagating fields are given by

$$T_p^+ = \frac{\Gamma(|p|)}{\Gamma(1-|p|)} e^{ip(X+\varphi)} e^{-\sqrt{2}\varphi} \qquad T_p^- = \frac{\Gamma(|p|)}{\Gamma(1-|p|)} e^{-ip(X-\varphi)} e^{-\sqrt{2}\varphi} \qquad p > 0$$

- The standard target space background has a flat metric, a linear dilaton and a "massless tachyon" field represented by these vertex operators.

- The linear dilaton gives rise to a position dependent string coupling

$$S = \frac{1}{2}\int dt d\varphi \left[(\partial_t T)^2 - (\partial_\varphi T)^2 - \frac{1}{3} e^{-\sqrt{2}\varphi} T^3 + \cdots \right]$$

- Correlation functions of the vertex operators provide a "wall" S Matrix of modes coming in from the asymptotic region $\varphi \to \infty$ and getting reflected from the Liouville wall.
- In addition to these propagating modes, there are a set of discrete modes – these have imaginary values of the energy and Liouville momenta

$$W^{+}_{jm} = e^{\sqrt{2}mX} e^{-\sqrt{2}(1-j)\varphi} \qquad j = 0, \frac{1}{2}, 1 \cdots \qquad m = -j, -j+1, \cdots + j$$

- There are also modes with the *opposite* dressing, which are thought of as providing non-trivial backgrounds

$$W^{-}_{jm} = e^{\sqrt{2}mX} e^{-\sqrt{2}(1+j)\varphi} \qquad j = 0, \frac{1}{2}, 1 \cdots \qquad m = -j, -j+1, \cdots + j$$

- The time dependent background is described by the following addition to the world-sheet action (*Balthazar, Chu & Kutasov, 2023*)

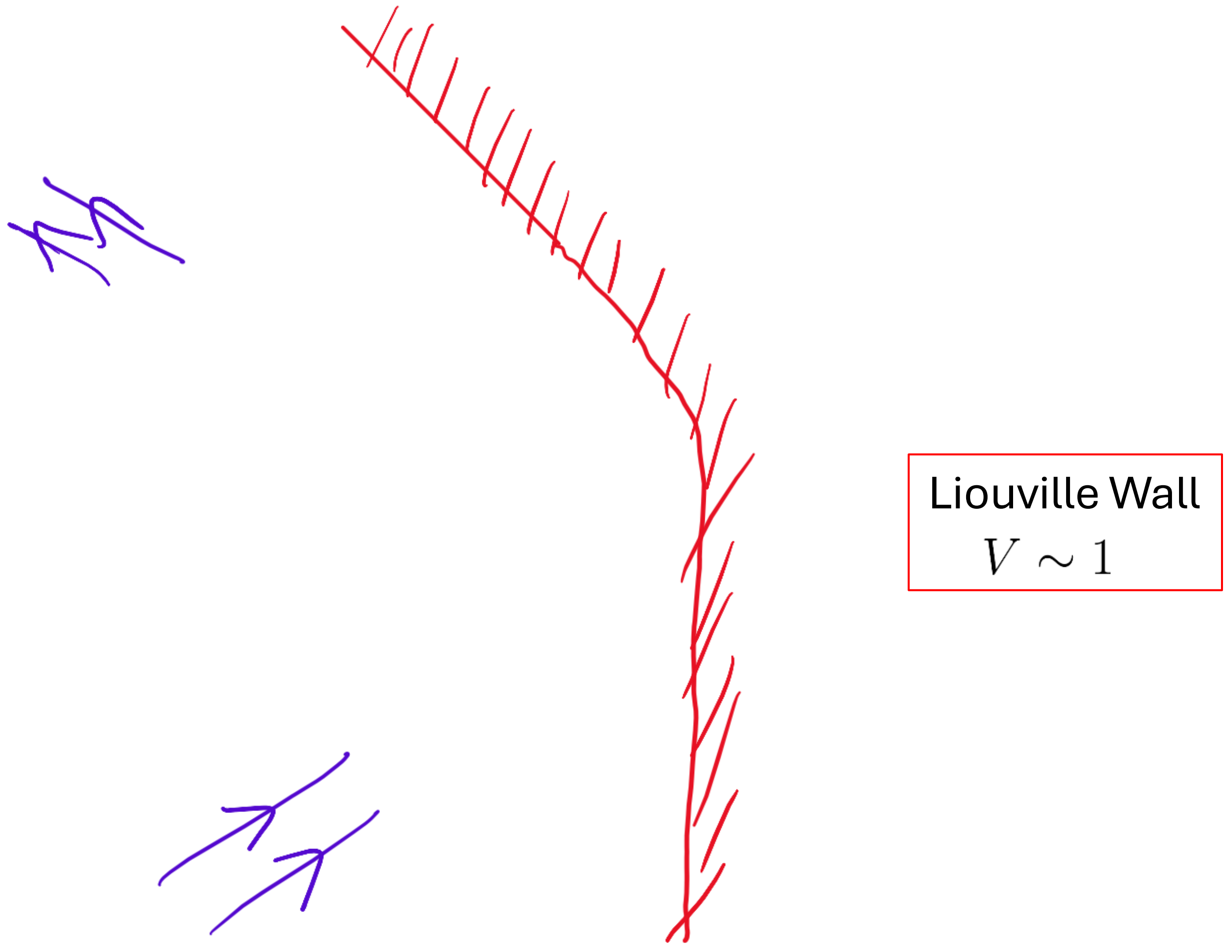
$$\delta S_{WS} = \lambda \frac{\Gamma(r)}{\Gamma(1-r)} \int d^2 z e^{-\sqrt{2}(1-\frac{r}{2})\varphi} e^{\frac{r}{\sqrt{2}} X}$$

- Where the Euclidean momentum r is taken to be real, arbitrary.
- This modifies the Liouville wall – the worldsheet potential is now

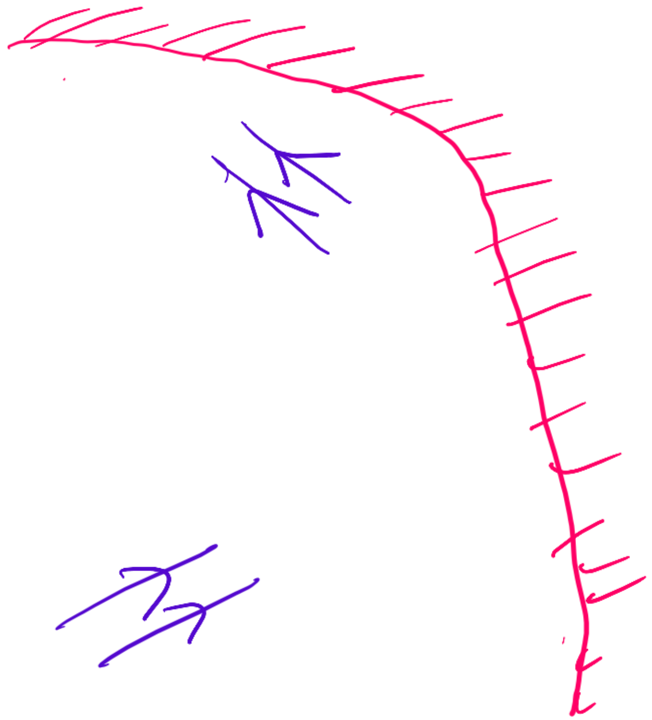
$$V = -\mu e^{-\sqrt{2}\varphi} + \lambda e^{-\sqrt{2}(1-r/2)\varphi + \frac{r}{\sqrt{2}} X}$$

- This corresponds to an accelerating Liouville wall – when r > 1 the wall in fact moves at superluminal speeds at late time.
- The calculations of Balthazar are in practice involved bringing down the deformation in a power series in the parameter

- For r < 1 there are asymptotic past and future null infinities.
- Various processes can now be calculated – scattering off the wall as well as particle production from the wall



- When r > 1, however, there is no asymptotic future infinity.
- This is because the Liouville wall accelerates to super-luminal speeds.
- The meaning of world-sheet calculations become unclear.
- In the following we will resort to the holographic description in terms of Matrix Quantum Mechanics to get a physical picture of what is happening.



Liouville Wall
$V \sim 1$

The c=1 Matrix Model

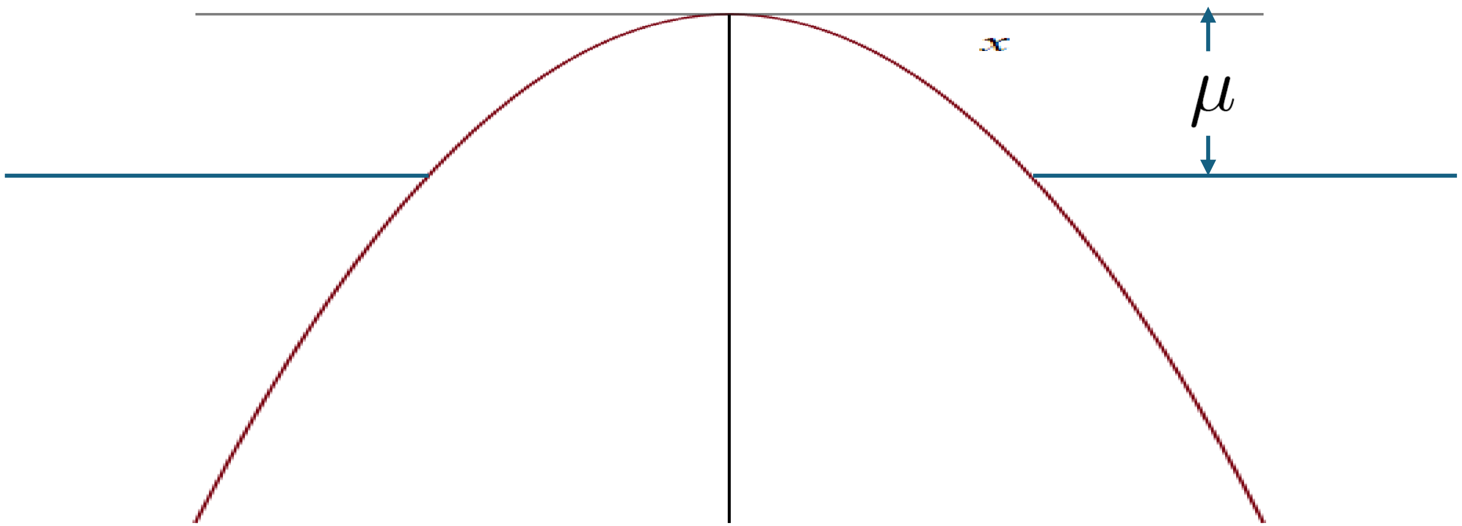
- The holographic description of this String Theory is given by the gauged quantum mechanics of a single $N \times N$ matrix $M_{ij}(t)$ in the double scaling limit,

$$S = \frac{\beta}{2} \int dt \, \mathrm{Tr} \left[(D_t M)^2 + M^2 \right] \qquad \beta \sim N$$

- One can pick a gauge in which the matrix is diagonal. Because of a non-trivial measure in the change of variables from the matrices to the eigenvalues

$$\frac{[dM]}{\mathrm{Vol}[U(N)]} = \prod_i d\lambda_i [\prod_{i<j} (\lambda_i - \lambda_j)]^2$$

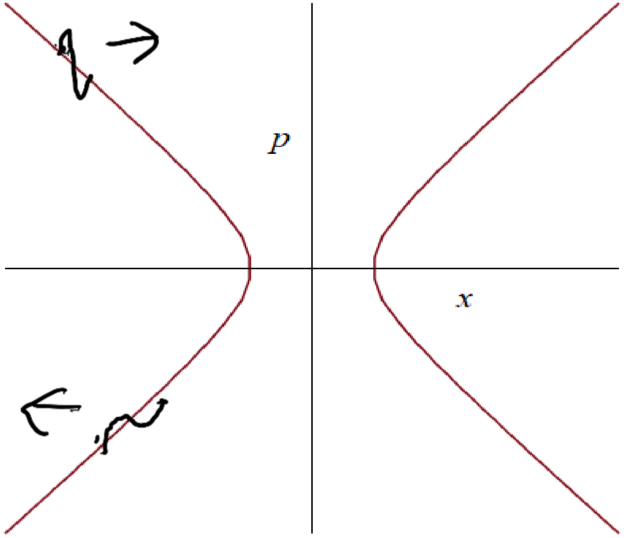
- The eigenvalues behave as free fermions moving in one dimension in the presence of an inverted oscillator potential. These are D particles in the String Theory.
- The ground state is the filled Fermi sea.



- The action of the second quantized fermion field $\psi(x,t)$ is

$$S_F = \int dx dt\ \psi^\dagger(x,t) \left[i\partial_t + \frac{1}{2}\{g_s \partial_x^2 + \frac{1}{g_s}x^2 - \frac{1}{g_s}\} \right] \psi(x,t) \qquad g_s = \frac{1}{\mu}$$

- Excitations are particle-hole pairs which come from the asymptotic region and scatter off the potential.
- In the following we will rescale things to set $2\mu = 1$



- In a phase space these are deformations of the Fermi surface, which is given by the equation

$$x^2 - p^2 = 1$$

- Even though each fermion is free – the decomposition in terms of bosonic degrees of freedom is non-trivial, because of the non-relativistic dispersion relation - resulting in an interacting bosonic theory.

- The bosonic description is Collective Field Theory of the density of eigenvalues

$$\partial_x \zeta(x,t) = \psi^\dagger(x,t)\psi(x,t)$$

- To leading order in large N

$$S_{coll} = \frac{1}{g_s^2}\left[\frac{1}{2}\frac{(\partial_t\zeta)^2}{\partial_x\zeta} - \frac{\pi^2}{6}(\partial_x\zeta)^3 + \frac{1}{2}[x^2-1](\partial_x\zeta)\right]$$

- The filled fermi sea is then described by a static classical solution

$$\partial_x\zeta_0 = \frac{1}{\pi}\sqrt{x^2-1}\;\theta(|x|-1)$$

- Expanding around this

$$\zeta(x,t) = \zeta_0(x,t) + \frac{g_s}{\sqrt{\pi}}\eta(x,t)$$

- One gets a non-polynomial action for the fluctuations. However, the Hamiltonian expressed in terms of the field and its canonically conjugate momentum is cubic to leading order.

- The quadratic part of the fluctuation action is

$$S^{(2)} = \frac{1}{2\pi}\int dx dt \left[\frac{(\partial_t \eta)^2}{\partial_x \zeta_0} - \pi^2 (\partial_x \zeta_0)(\partial_x \eta)^2\right]$$

- The cubic interaction is

$$S^{(3)} = -\frac{1}{\pi^{3/2}}\int dx dt \left[\frac{1}{2}\frac{1}{(\partial_x \zeta_0)^2}(\partial_t \eta)^2(\partial_x \eta) + \frac{\pi^2}{6}(\partial_x \eta)^3\right]$$

- The quadratic part clearly shows that the fluctuation field is a relativistic massless scalar in 1+1 dimensions.
- The space of eigenvalues, x , is an emergent direction.
- It is useful to transform to coordinates in which the underlying metric is conformal to Minkowski metric. This is achieved by the transformation

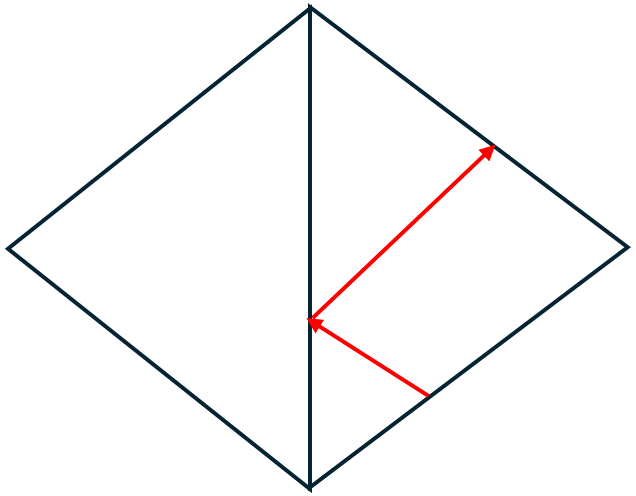
$$x \to q \qquad x = \cosh(q)$$

- The quadratic and cubic terms then become

$$S^{(2)} = \frac{1}{2}\int dqdt \left[(\partial_t\eta)^2 - (\partial_q\eta)^2\right]$$

$$S^{(3)} = \frac{g_s}{2}\int dqdt \frac{1}{\sinh^2 q}\left[(\partial_t\eta)^2(\partial_q\eta) + \frac{\pi^2}{6}(\partial_q\eta)^3\right]$$

- The asymptotic regions are $q = \pm\infty$, which correspond to the two sides of the potential. The coupling becomes strong at $q = 0$ - which is like a wall.
- The Penrose diagram of the emergent space-time is very simple



- The effective coupling in the asymptotic region is

$$g_{eff} \sim e^{-2|q|}$$

- Which motivates the identification of q with the zero mode of the worldsheet Liouville field, φ and the time of the matrix model with X (*S.R.D. & A. Jevicki*)

$$q \sim \frac{1}{\sqrt{2}}\varphi \qquad\qquad t \to \frac{1}{\sqrt{2}}X$$

- However, it turns out that a more precise identification is with the macroscopic loop operator

$$W(e^{-\sqrt{2}\varphi}) = \int dx\ e^{-e^{-\sqrt{2}\varphi}x}\partial_x\eta$$

- In the asymptotic region this agrees with the so called "leg pole" Kernel which relates e.g. right moving Tachyon field to the collective field

$$S(t-\varphi) = e^{-\sqrt{2}\varphi}\int dv\ \left(\frac{d}{dv}J_0(v)\right)\eta(t-v-\varphi)$$

- These give different notions of emergent space which differ from each other at the string scale.
- We are interested in the global nature of the space-time. The Penrose diagrams we draw should be "fuzzed over" the string scale.
- The leg pole transforms are, however, necessary to establish the equivalence of S-Matrix of the Matrix Model with the world-sheet String Theory (*Demeterfi, Jevicki & Rodrigues*; *Gross & Klebanov*; *Sengupta & Wadia*; *Moore, Plesser & Raamgoolam*).

Time dependent backgrounds in the Matrix Model

- There is a natural identification of the deformation of the matrix model Hamiltonian which corresponds to the addition of a time dependent term in the world-sheet action of the type we considered

$$\delta S_{WS} = \lambda \frac{\Gamma(r)}{\Gamma(1-r)} \int d^2 z e^{-\sqrt{2}(1-\frac{r}{2})\varphi} e^{\frac{r}{\sqrt{2}}X}$$

- In the Matrix Model this is represented by a deformed Fermi sea described by the equation

$$x^2 - p^2 + \lambda e^{rt}(p-x)^r = 1$$

- Which follows from the identification of the usual vertex operators with (*Jevicki*)

$$T_+^E = e^{-iEt}\,\mathrm{Tr}\,(M+P)^{iE}, \qquad T_-^E = e^{-iEt}\,\mathrm{Tr}\,(M-P)^{-iE}.$$

- When r is a positive or negative integer, these are deformations by a spectrum generating $W_{1+\infty}$ symmetry algebra of the theory.
- The identification of the deformation vertex operator with the deformation of the Matrix Model Hamiltonian follows from the realization of this symmetry in the Matrix Model (*Avan & Jevicki*) and on the worldsheet (*Witten*).

A digression: Quantum Quench

- To see what is going on, we will now use some earlier results in a slightly different problem to explore the global structure of the emergent space-time.
- This is the problem of quantum quench in the Matrix Model (*S.R.D., S. Hampton and S. Liu, 2020*) – where the coupling of the theory is made time-dependent

$$S = \frac{\beta_0 f(\tau)}{2} \int d\tau \ \mathrm{Tr}\left[(D_\tau M)^2 - U(M)\right] \qquad \beta \sim N$$

- By changing the definition of time $\quad dt = \dfrac{d\tau}{f(\tau)}$
- One can perform a double scaling limit

$$S = \frac{\beta_0}{2} \int dt \ \mathrm{Tr}\left[(D_t M)^2 - f(t) M^2\right] \qquad \beta \sim N$$

- The corresponding collective field theory is

$$S_{coll} = \frac{1}{g_s^2} \int dt dx \left[\frac{1}{2} \frac{(\partial_t \zeta)^2}{\partial_x \zeta} - \frac{\pi^2}{6} (\partial_x \zeta)^3 + \frac{1}{2} [f(t)^2 x^2 - 1](\partial_x \zeta) \right]$$

- It turns out that one can go to a moving frame and remove the time dependence

$$y = \frac{x}{\sqrt{|\rho(t)|}} \qquad T = \int^t \frac{dt'}{\rho(t)}$$

- If the function $p(t)$ obeys the non-linear equation – related to a modified Ermakov-Pinney equation

$$\partial_t \rho(t) - \frac{1}{2\rho(t)} (\partial_t \rho(t))^2 - 2f(t)^2 \rho(t) = -\frac{2}{\rho(t)}$$

- The action becomes

$$S_{coll} = \frac{1}{g_s^2} \int dy dT \left[\frac{1}{2} \frac{(\partial_T \zeta)^2}{\partial_y \zeta} - \frac{\pi^2}{6} (\partial_y \zeta)^3 + \frac{1}{2} [y^2 - 1](\partial_y \zeta) \right]$$

- Therefore, starting from a static solution we can generate time dependent solutions by solving the modified Ermakov-Pinney equation.
- For a quench function which interpolates between constant values of the coupling at early and late times, we need to solve the modified EP equation with adiabatic boundary conditions at early times.
- Once that is done, the response to the quench is given by the time dependent solution of collective field theory,

$$\partial_x \zeta_0(x,t) = \frac{1}{\rho(t)} \left[x^2 - \rho(t)\right] \; \theta(|x| > \sqrt{\rho(t)})$$

- The nature of the emergent space-time can be gleaned from the action for fluctuations around this solution

$$\zeta(x,t) = \zeta_0(x,t) + \frac{g_s}{\sqrt{\pi}}\eta(x,t)$$

- The quadratic action is

$$S^{(2)} = \frac{1}{2\pi}\int dxdt \left[\frac{(\partial_t\eta)^2}{\partial_x\zeta_0} - 2\frac{(\partial_t\zeta_0)}{(\partial_x\zeta_0)^2}(\partial_t\eta)(\partial_x\eta) + \left(\frac{(\partial_t\zeta_0)^2}{(\partial_x\zeta_0)^3} - \pi^2\partial_x\zeta_0\right)(\partial_x\eta)^2\right]$$

- This is the action of a massless scalar which is in a metric which is conformal to

$$ds^2 = -dt^2 + \frac{1}{(\pi\partial_x\zeta_0)^2}\left(dx + \frac{\partial_t\zeta_0}{\partial_x\zeta_0}dt\right)^2$$

- For the static geometry, the space of eigenvalues, x, eventually becomes the emergent space. Now we see that x = constant lines need not be always time-like. They are time-like only when

$$(\partial_t\zeta_0)^2 \leq \pi^2(\partial_x\zeta_0)^4$$

- The cubic interaction of the fluctuations is given by

$$S^{(3)} = -\frac{1}{\pi^{3/2}} \int dx dt \left[\frac{1}{2} \frac{1}{(\partial_x \zeta_0)^2} (\partial_t \eta)^2 (\partial_x \eta) - \frac{\partial_t \zeta_0}{(\partial_x \zeta_0)^3} (\partial_t \eta)(\partial_x \eta)^2 + \left(\frac{(\partial_t \zeta_0)^2}{(\partial_x \zeta_0)^4} + \frac{\pi^2}{6} \right) (\partial_x \eta)^3 \right]$$

Back to our problem

- Presently, we are not interested in quantum quench – rather we are looking at solutions of the collective field theory with the standard constant inverted oscillator potential which represent excited states and hence time dependent geometries.
- However, the method we outlined works for constant potentials as well – we just need to solve the equation for $p(t)$ by setting $f(t) = 1$.
- Furthermore, we can think of a rapid quantum quench as a way to prepare an excited state – in fact the late time behavior would be given by one of the solutions with constant potential.

- Coming back to the world-sheet deformation

$$\delta S_{WS} = \lambda \frac{\Gamma(r)}{\Gamma(1-r)} \int d^2 z e^{-\sqrt{2}(1-\frac{r}{2})\varphi} e^{\frac{r}{\sqrt{2}}X}$$

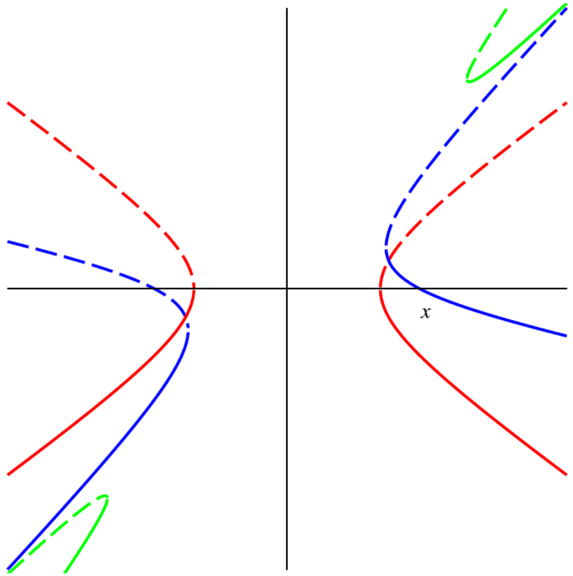
- The world-sheet theory became confusing for r > 1.
- We will now realize the r =2 deformation as a time dependent solution of the type we just discussed, and use this to figure out what is really happening.
- The solution corresponding to r=2 is given by the function

$$p(t) = 1 + \lambda e^{2t}$$

- This corresponds to a Fermi surface given by

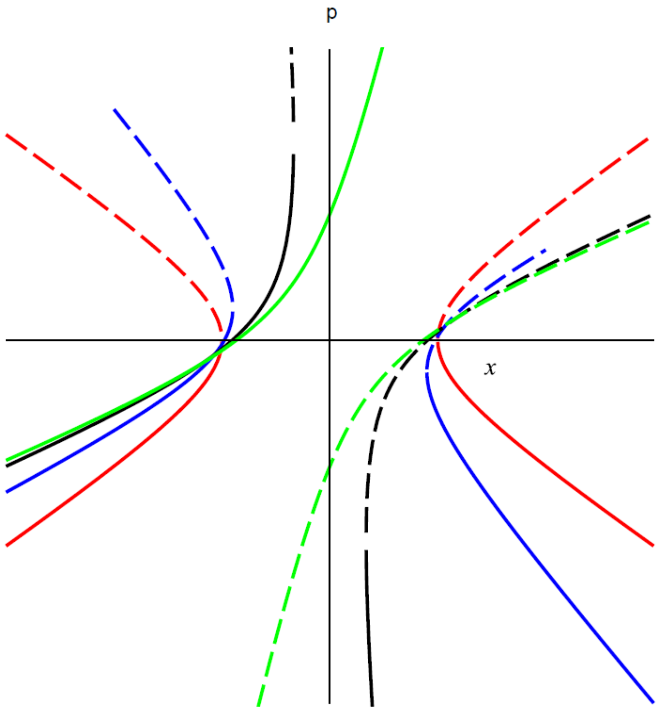
$$x^2 - p^2 - \lambda e^{2t}(x-p)^2 = 1$$

- When $\lambda > 0$ the moving Fermi surface is of the form



- Fermions on one side of the potential remain on that side.
- As time elapses, they turn back at locations further and further away from the maximum of the potential.
- The edge of the surface moves to the asymptotic infinity at long times.

- When $\lambda < 0$ fermions spill over to the other side, and the wall disappears at a finite Matrix Model time



- Time evolution continues smoothly beyond this point – at late times the fermi level asymptotes to the $p = x$ line.

The Emergent Space-time

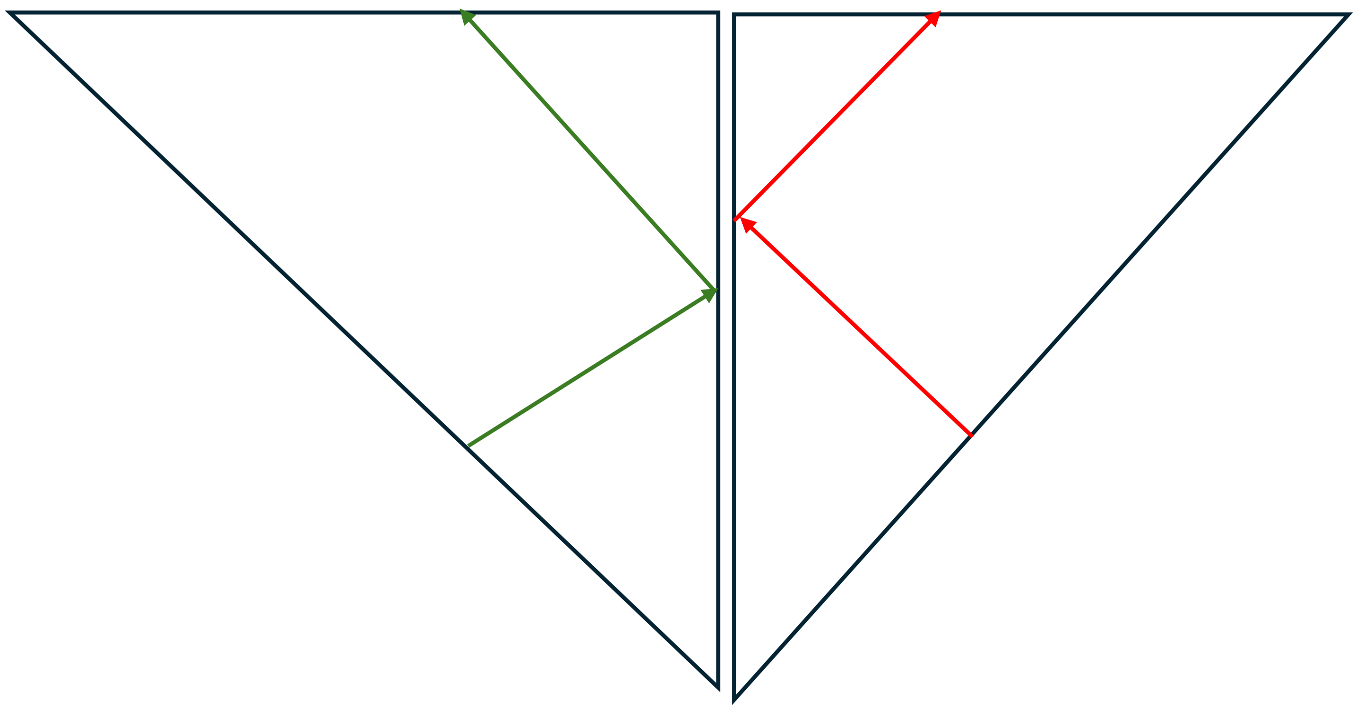
- To find the global nature of the emergent space-time it is convenient to go to coordinates in which the metric is conformal to Minkowski space. This is provided by the following transformations

$$x = \pm\sqrt{|1+\lambda e^{2t}|}\cosh(q) \qquad T = t - \frac{1}{2}\log(1+\lambda e^{2t})$$

- In these coordinates the metric is conformal to

$$ds^2 = -dT^2 + dq^2$$

- For $\lambda > 0$ we see that as $t \to \infty$ the time T reaches $T = 0$
- Normally we would have continued the space-time beyond this time. However, this is the end of time evolution in the Matrix Model.
- The Matrix Model is telling us to stop time evolution at this time.
- The boundary of the emergent space-time is space-like !



- To probe a little more what is happening at this time, consider the cubic couplings

$$S^{(3)} = -\frac{1}{\pi^{3/2}} \int dx dt \left[\frac{1}{2} \frac{1}{(\partial_x \zeta_0)^2} (\partial_t \eta)^2 (\partial_x \eta) - \frac{\partial_t \zeta_0}{(\partial_x \zeta_0)^3} (\partial_t \eta)(\partial_x \eta)^2 + \left(\frac{(\partial_t \zeta_0)^2}{(\partial_x \zeta_0)^4} + \frac{\pi^2}{6} \right) (\partial_x \eta)^3 \right]$$

- The three different terms have coefficients which all diverge in the limit of $t \to \infty$ or $T = 0$

$$\frac{\partial_t \zeta_0}{(\partial_x \zeta_0)^3} = -\frac{1}{2} \frac{x(\partial_t p(t))}{[x^2 - (1 + \lambda e^{2t})]} \sim e^{2t}$$

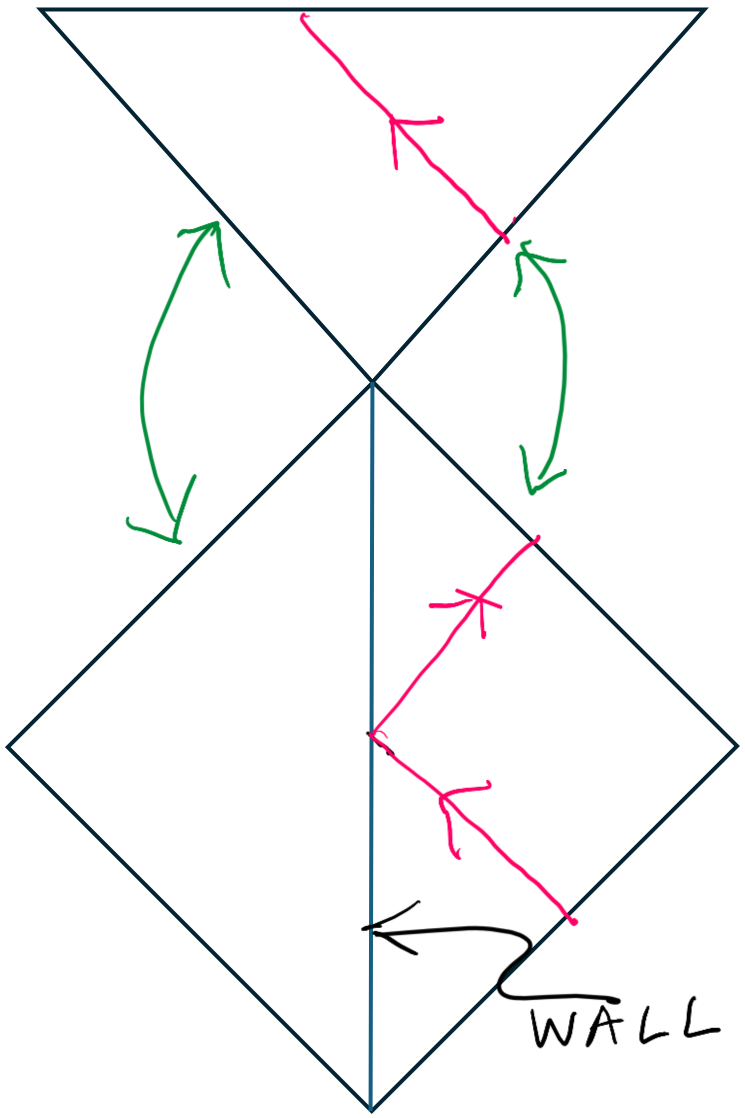
$$\frac{(\partial_t \zeta_0)^2}{(\partial_x \zeta_0)^4} = \frac{(\partial_t p)^2 x^2}{p^{3/2}[x^2 - (1 + \lambda e^t)]} \sim e^t$$

$$\frac{1}{(\partial_t \zeta_0)^2} = \frac{p(t)}{[x^2 - (1 + \lambda e^{2t})]} \sim e^{2t}$$

- This is a space-like singularity – a BIG CRUNCH space-time.

- Perturbative collective field theory – which is related to perturbative string theory is not useful here. However more exact methods should remain useful.
- However, the holographic description in terms of fermions (D branes) remains well defined – since this happens when the matrix model time is infinite.

- When $\lambda < 0$ something very different happens.
- Now the function $p(t)$ hits a zero at finite Matrix Model time, $t = t_0 = -\frac{1}{2}\log(-\lambda)$ which however corresponds to $T = \infty$
- From the point of view of emergent space-time, it is natural to stop time evolution here. In fact, at this time we have reached a null infinity.
- This would have been fine. However, the fundamental description is the Matrix Model – and clearly the system continues to evolve after this time, when the function $p(t)$ becomes negative.
- We interpret this to mean that we need to attach another piece of space-time, in which there is no wall which separates the two sides of the potential.


WALL

- There is a good semi-classical description in terms of a different collective field which is obtained by subtracting the upper edge of the right side fermi surface from the lower edge of the left side Fermi surface

$$\partial_x \zeta_0 = -\frac{1}{e^{2t}-1}\sqrt{x^2 + (e^{2t}-1)}$$

- Finally, we reach the end of Matrix Model time evolution, $t \to \infty$. This is a space-like boundary at a finite time in the emergent space-time.
- As in the previous case, the coupling diverges there – one has a BIG CRUNCH.
- The excitations, represented by fluctuations of the collective field, all fall into this singularity.
- From the point of view of the 2d world-sheet theory, this manifests itself as the absence of a future asymptotic region.

Outlook

- The time dependent deformation in the world-sheet theory was of the form

$$\delta S_{WS} = \lambda \frac{\Gamma(r)}{\Gamma(1-r)} \int d^2 z e^{-\sqrt{2}(1-\frac{r}{2})\varphi} e^{\frac{r}{\sqrt{2}}X}$$

- This is for any real, positive r and there was a problem in interpreting this theory for r > 1
- The Matrix model description can be treated analytically for integer values of r – we explored this in detail for r =2 and found that the difficulties in thinking about the world-sheet theory are related to the development of space-like singularities in a semi-classical description.

- For non-integer r < 2 involves fermi surfaces which are described by equations which have non-integer powers of the light cone variables in phase space

$$z_{\pm} = x \pm p$$

- In these cases, it is possible to study the problem numerically (Balthazar et.al.)
- It is also possible to develop the *semi-classical* collective field theory description using the usual map between Fermi surface variables and the collective field.
- We are currently studying this – we expect results which are quite similar to what we obtained for r =2
- For higher integer powers of r the Fermi surfaces are not quadratic profiles – leading to “folds” on the fermi surface. These are highly “quantum” states.
- There is still a *quantum* collective field description – though an effective semi-classical description requires introduction of more classical collective variables.

- The physics at times close to the space-like “singularity” is clearly non-perturbative in the string coupling, which is why it is invisible in perturbative String theory.
- Exact results in collective field theory are known (*Avan & Jevicki*) in terms of exact creation and annihilation operators

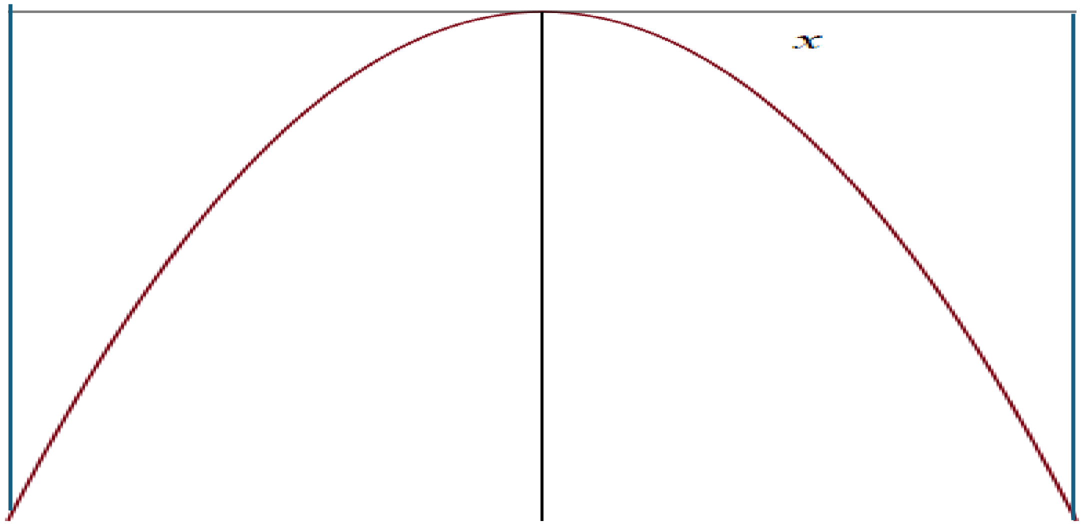
$$B_n^{\pm} = \mathrm{Tr}\left(P \pm M\right)^n , \qquad n = 0, 1, 2, \ldots$$

$$\left[\, H, B_n^{\pm} \,\right] = \mp i n B_n^{\pm}$$

- Analytic continuation to imaginary values of n have been useful in understanding properties of scattering amplitudes: maybe continuation to arbitrary real values could be useful as well.

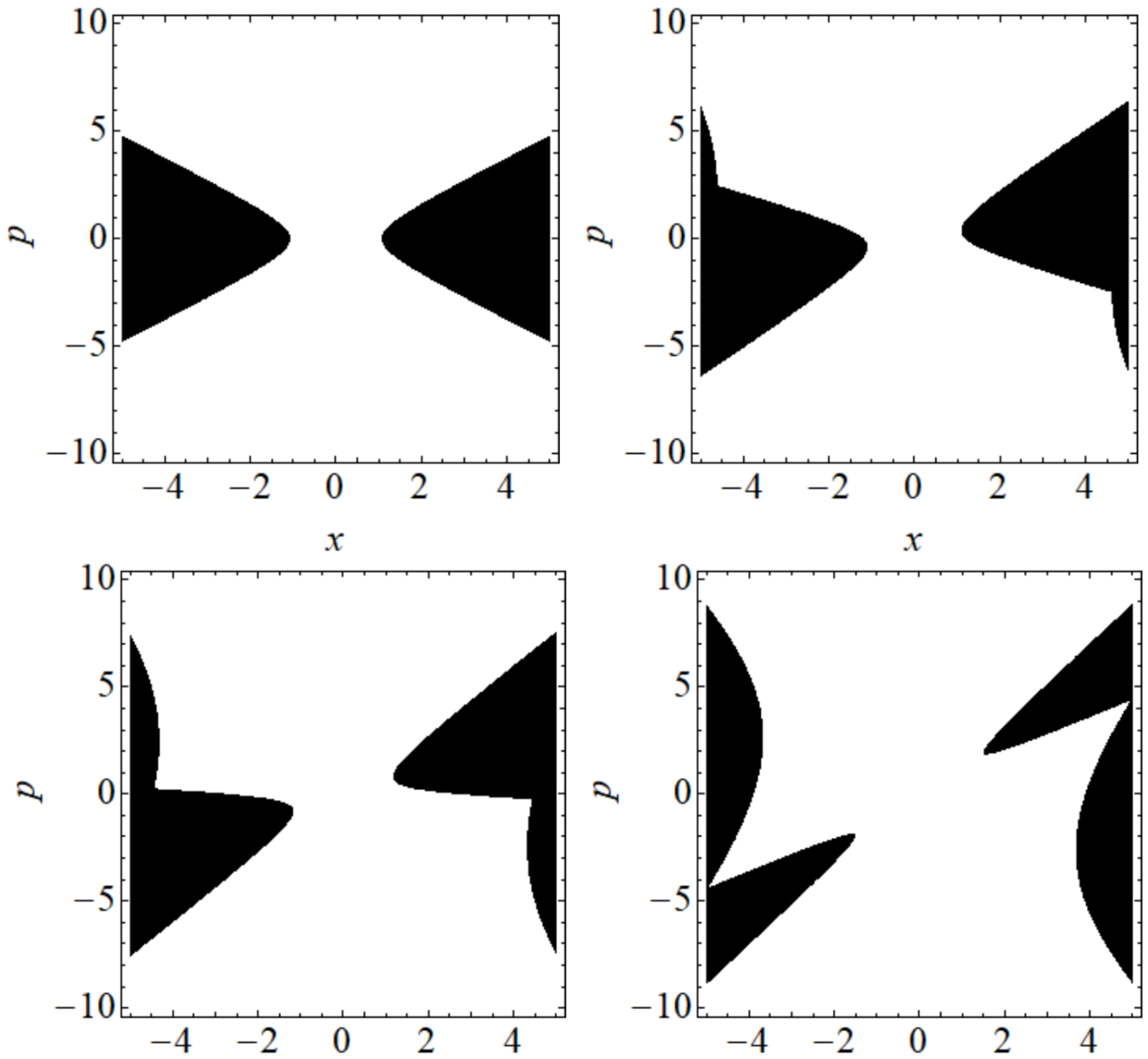
- Even though the fermionic description is still free, there is an important caveat which makes probing this region non-trivial.
- This has to do with finite N effects.
- Our entire discussion has been in the double scaling limit, in which $N = \infty$ and the string coupling is held fixed.
- The filled Fermi surfaces shrank or expanded – this of course cannot happen for any fixed N , however large.

- In fact, the inverted oscillator potential has to be regulated for finite N by e.g. hard walls which are $O(\sqrt{N})$ away. These come from the non-linear terms in the Matrix Model potential which get scaled away in the string double scaling limit.



- Fermions reaching this wall will get reflected back.

- As a result, the time dependent Fermi surfaces look like



- At early enough times, this effect does not modify the bulk physics.
- However, at times close to the singularity this effect could become important.
- From the point of view of the parent Matrix Model this is the usual problem of trace relations – the collective field which we used

$$\partial_x \zeta(x,t) = \frac{1}{N} \mathrm{Tr}\ \delta(x - M(t)) = \int dk\ e^{ikx} \mathrm{Tr}\ e^{-ikM}$$

  is overcomplete.
- What we are finding is that at late times one needs a more accurate description which incorporates these trace relations (*Jevicki*; *Corley, Jevicki & Ramgolam*).
- It is interesting that we may need to this effect – called the "Stringy Exclusion Principle" in the context of AdS/CFT is necessary to fully understand what is going on.

- The main lesson in this two-dimensional theory is that it is possible to have situations where a semi-classical description indicates a space-like singularity, but an underlying holographic description is *in principle* well defined.
- Two dimensional models are of course too simple – however two-dimensional models of gravity have been useful in teaching us about the black hole evaporation problem.
- Maybe, with some more thought one can make progress in understanding space-like singularities.

THANK YOU