Bulk reconstruction of flat space from Carrollian CFT

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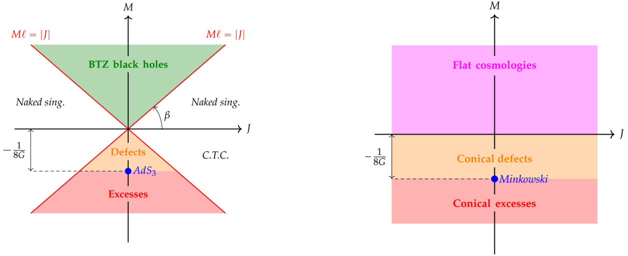
Contents

Asymptotically Flat Spacetime in 3d

[Barnich, et al. 06',10',12'; Compère, et al. 06', 18; $\cdots$]

Locally asymptotically flat spacetimes: metric tensor decays to $\eta_{\mu\nu}$ as some spacelike coordinate r reaches infinity.

- The flat solutions can be obtained by taking the large AdS radius limit $l \to \infty$, or zero cosmological constant limit $\Lambda \to 0$ equivalently. (Figure by Compère)



- This limit pushes the boundary cylinder to infinity as the length scale becomes infinite. Now the bulk looks like the previous canter of AdS.
- Asymptotic symmetry: two copies of Virasoro algebra becomes BMS_3 algebra via Wigner-Inonu contraction.

Flat/Carrollian CFT correspondence

Bottom-up approach: based on the asymptotic symmetry, is there an analog to AdS/CFT correspondence?

$$\boxed{\text{Flat/Carrollian CFT}}$$

[Susskind, Polchinski, de Boer, Dappiaggi, Bagchi, Grumiller, Duval, Barnich, Detournay, Donnay, $\cdots$]

Two roads towards flat holography:

- Flat/Carrollian CFT: the field theory lives on the asymptotic boundary which is one dimensional lower and contains the time direction.(evolution)

- Celestial holography: the CFTs live in two dimensional lower space without time direction.(scattering data)

Carrollian CFTs (BMS Field Theories) in 2d

[Arjun, et al. 09'; Barnich, et al. 06',09',12'; $\cdots$]

A 2d field theory on a plane (x, y) has the following symmetry

$$x \to f(x), \quad y \to f\,'(x)y + g(x). \tag{1}$$

- BMS_3 algebra (Carrollian conformal algebra in 2d):

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \frac{c_T}{12}n(n^2-1)\delta_{n,-m} \\ [L_n, M_m] &= (n-m)M_{n+m} + \frac{c_M}{12}n(n^2-1)\delta_{n,-m} \\ [M_n, M_m] &= 0 \end{aligned} \tag{2}$$

  Asymptotic symmetry for 3d flat spacetime in Einstein gravity: BMS_3 with $c_T = 0$.
- Ultra-relativistic ($c \to 0$)/ non-relativistic ($c \to \infty$) limit from CFT_2, via Wigner-Inonu contraction from two copies of Virasoro algebra.
- 2d non-Lorentzian quantum field theories. The time direction y is a null direction.

The Height Weight Representation

- Operators at the origin can be labelled by the eigenvalues (Δ, ξ) of (L_0, M_0)

$$[L_0, O] = \Delta O, \qquad [M_0, O] = \xi O. \tag{3}$$

By requiring L_0 bounded below, one has the highest weight conditions on the primary operators

$$[L_n, O] = 0, \quad [M_n, O] = 0, \quad n > 0. \tag{4}$$

- It is not unitary since multiplets appear naturally due to the algebra structure.

$$[M_0, L_{-2}O] = \xi L_{-2}O + 2M_{-2}O, \;\; [M_0, M_{-2}O] = \xi M_{-2}O. \tag{5}$$

on which M_0 has a non-diagonal action.

- It can be seen as a UR limit from flipped representation (highest weight $\times$ lowest weight) in CFT$_2$.

Induced Representation

- BMS algebra is the semi-direct sum of the Virasoro algebra and an Abelian ideal generated by M_n's.
- Induced Representation: representation induced from that ideal.

$$
\begin{aligned}
L_0|O\rangle &= \Delta|O\rangle, \qquad & M_0|O\rangle &= \xi|O\rangle, \\
M_n|O\rangle &= 0, & \forall n &\neq 0,\, n \in \mathbb{Z}.
\end{aligned} \tag{6}
$$

- It is ultra-local. The vacuum behaves as a direct product of states living at each point of the spatial slice

$$
\langle A(x_1, y_1) B(x_2, y_2) \rangle \sim \delta(x_1 - x_2) \tag{7}
$$

- It is unitary, and can be seen as the UR limit of the highest weight representation of CFT_2.

Bulk reconstruction in 3d flat holography

- Which representation in Carrollian CFT$_2$ will play a role in the holography?
- What is Hilbert space for the gravitational theory in the bulk? How to compere it with that on the boundary?
- How does a bulk locally excitation look like in the boundary?

We want to probe the bulk by considering reconstruction of the bulk locally excited state in terms of the states in Carrollian CFTs.

Global Minkowski spacetime

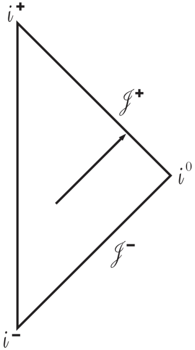
- In the Cartesian coordinate

$$ds^2 = -dt^2 + dx^2 + dy^2 \tag{8}$$

- It is useful to go to Bondi gauge with retarded (or advanced) time,

$$ds^2 = -du^2 - 2dudr + r^2 d\phi^2 \tag{9}$$

$$t = u + r, \quad x = r\cos\phi, \quad y = r\sin\phi \tag{10}$$



It is maximally symmetric with 6 isometries,

$$M_0 = (i, 0, 0),\ M_{\pm 1} = (\mp e^{\pm i\phi}, \pm e^{\pm i\phi}, \frac{i}{r} e^{\pm i\phi}) \tag{11}$$

$$L_0 = (0, 0, i),\ L_{\pm 1} = (-ie^{\pm i\phi} u, ie^{\pm i\phi}(u + r), \mp e^{\pm i\phi}(1 - \frac{u}{r})) \tag{12}$$

Bulk reconstruction in the induced representation

- Consider the scalar type excitation (with arbitrary mass m) at origin $(u=0, r=0, \phi)$, which rotationally invariant

$$L_{0,\pm1}|\Phi_{flat}(0,0,\phi)\rangle = 0 \tag{13}$$

- It is hard to solve. In practice, we consider the excitation at $(t, 0, \phi)$, satisfying

$$L_0|\Phi_{flat}(t,0,\phi)\rangle = 0 \tag{14}$$

$$(L_1 - itM_1)|\Phi_{flat}(t,0,\phi)\rangle = (L_{-1} + itM_{-1})|\Phi_{flat}(t,0,\phi)\rangle = 0 \tag{15}$$

There exists solution in the induced representation

$$|\Phi_{flat}(t,0,0)\rangle = \sum_{k=0}(-\frac{i}{t})^{k+1} c_k |k\rangle \tag{16}$$

$$|k\rangle = L_{-1}^k L_1^k |\xi\rangle, \quad c_k = \frac{1}{k!(2\xi)^k} \tag{17}$$

where $|\xi\rangle$ is a induced primary state with $\Delta = 0, \xi = \pm m$.

Bulk reconstruction in the induced representation

- For a generic bulk massive (or massless) scalar excitation at arbitrary point

$$|\Phi_{flat}(t,x,y)\rangle = e^{-iy/2(M_1+M_{-1})}e^{x/2(M_1-M_{-1})}|\Phi_{flat}(t,0,0)\rangle \tag{18}$$

- It reproduces the flat scalar two-point functions in the bulk

$$G_{flat}(t,x,y) = \langle\Phi_{flat}(0,0,0)|\Phi_{flat}(t,x,y)\rangle = -\frac{e^{-mD_{flat}}}{4\pi D_{flat}} \tag{19}$$

where D_{flat} is the geodesic distance in Flat$_3$.

- A technical point: the conjugation relations are as usual

$$(L_n)^\dagger = L_{-n}, \quad (M_n)^\dagger = M_{-n} \tag{20}$$

but for the states in induced representation, we should introduce the dual basis to calculate the inner product.

The highest weight representation: massless only

The solutions for the states in the highest weight representation only exist when considering the massless scalars,

$$|\Phi_{HWR}(0,0,0)\rangle = \sum_i c_{ij} L^i_{-1} M^i_{-1} |\Delta, \xi\rangle \tag{21}$$

$$c_i = \frac{\sqrt{-\Delta+1}\Delta(2\xi)^{\Delta+i}(-1)^i(\Delta+1)_{i-1}}{2(2)_{i-1}(3)_{i-1}} \tag{22}$$

- This can be seen from the Casimir operators (in the highest weight representation)

$$C_1|\Delta,\xi\rangle = \xi^2|\Delta,\xi\rangle, \quad C_2|\Delta,\xi\rangle = 2\xi(\Delta-1)|\Delta,\xi\rangle \tag{23}$$

- In the bulk

$$C_1|\Phi\rangle = m^2|\Phi\rangle, \quad C_2|\Phi\rangle = 0 \tag{24}$$

there are solutions with $\xi = m = 0$ case.

Conclusion

- There is a flat holography proposal with one-dimensional lower field theory in analog to (A)dS/CFT: Flat/Carrollian CFT. We provide more hints for it.
- We consider bulk reconstruction of scalar excitation with arbitrary mass in flat holography in terms of states in Carrollian CFTs, and find good match with the solution in induced representation, indicating that the bulk theory is unitary.

Outlook:

- More understandings on induced representation in Carrollian CFTs. (operator level)
- Generalization to higher dimensional cases. (In 4d, the picture changes totally and the conjugation relation is exotic.)
- Relation to the celestial holography.
- ...

Thanks for Your Attention!