Quantum Gravity and Information
in Expanding Universe @YITP, Kyoto U. Feb.19, 2025

Bulk reconstruction of de Sitter space from CFT

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Based on JHEP02(2025)093 with
Kazuki Doi, Naoki Ogawa, Kotaro Shinmyo, Yu-ki Suzuki

Bulk reconstruction of flat space from Carrollian CFT → Peng-Xiang Hao's talk

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Extreme Universe

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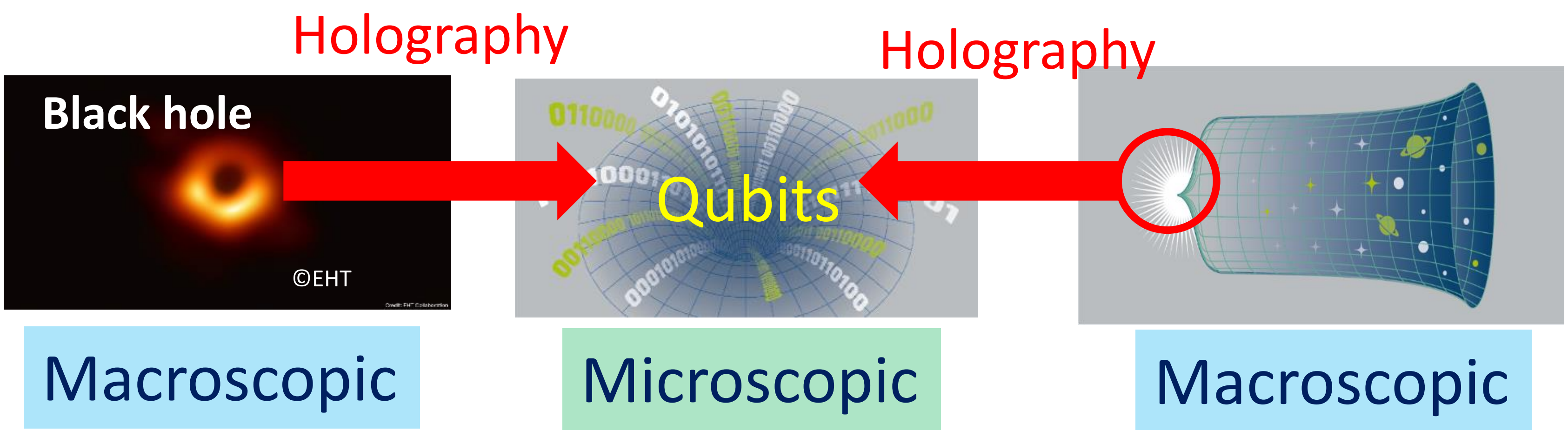
① Introduction

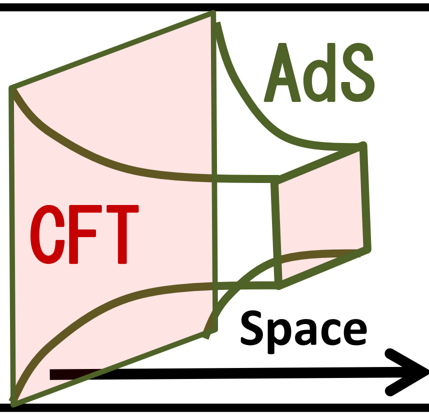
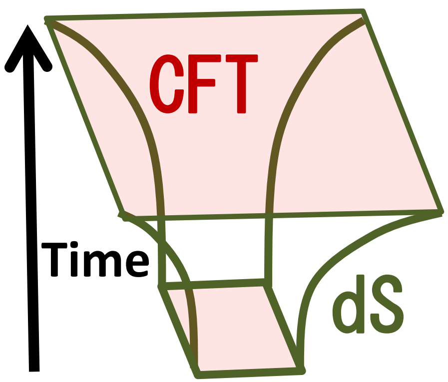
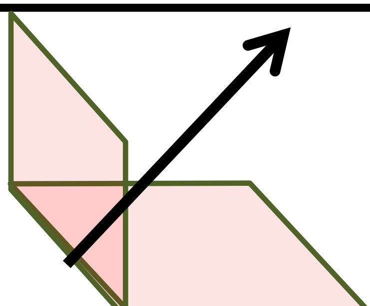
Holography is a very promising approach to quantum gravity.

This is because holography is much like ***a microscope*** in thought experiments of quantum gravity.



However, the application of holography to cosmological spacetimes is very challenging !

Classification of Max. Symmetric Spaces and Holography

Type	Geometry	Holography	Central charge
AdS $\Lambda<0$	 	**AdS/CFT** [Maldacena 1997] Gravity in d+1 dim. AdS = d dim. CFT on $R^{1,d-1}$ ▶Emergent Space	For d=2, $C=\frac{3R_{AdS}}{2G_N}$
dS $\Lambda>0$	 	**dS/CFT** [Strominger 2001] Gravity in d+1 dim. dS $\overset{?}{=}$ d dim. Euclid CFT on S^d ▶Emergent Time ?	For d=2, $C=i\frac{3R_{dS}}{2G_N}$
Flat $\Lambda=0$		String theory can describe quantum gravity. Also celestial holography is proposed.	$C=i\infty$?

Thermodynamics

AdS3 BH $S_{AdS} = 2\pi\sqrt{\frac{cE}{3}}$

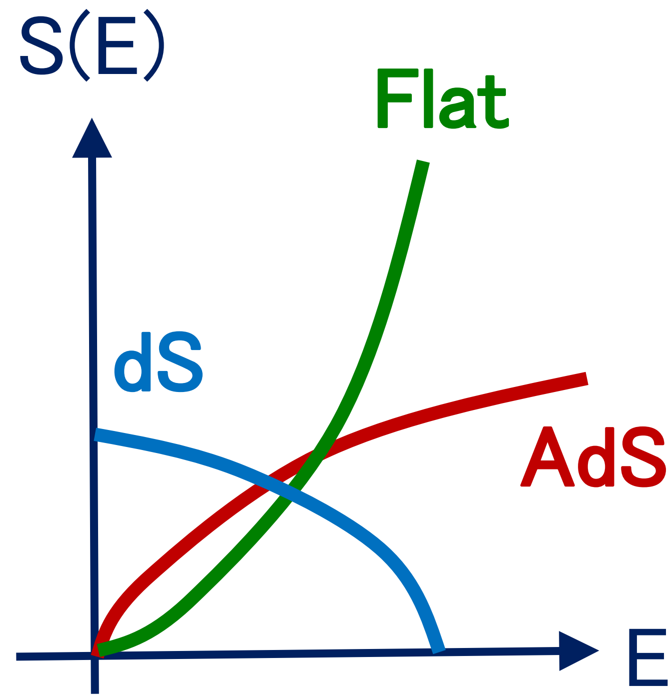
→It has a positive specific heat and is thermodynamically stable.

dS3 BH $S_{dS} = \frac{\pi R_{dS}}{2G_N}\sqrt{1-8G_N E}$

→The vacuum E= 0, the state is maximally entangled !

4D flat BH $S_{Flat} = 4\pi G_N E^2$

→ This leads to a negative specific heat. It is thermodynamically, unstable.



This is one of the main reasons why holography in dS/flat space is very difficult !

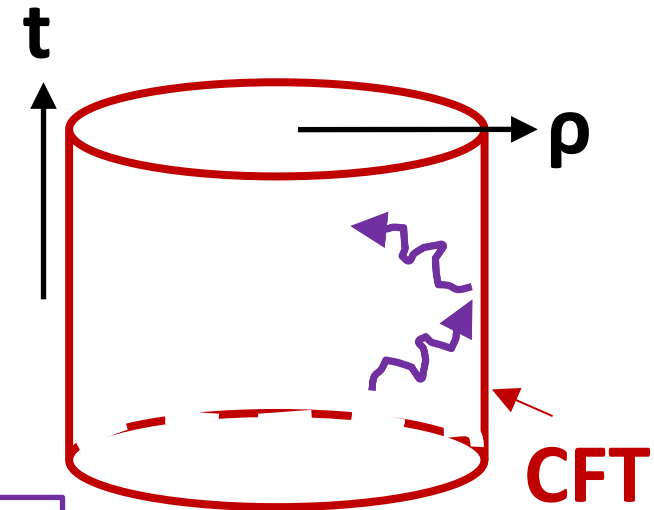
Geometries

AdS

$$ds^2 = R_{AdS}^2(-\mathrm{Cosh}^2\rho dt^2 + d\rho^2 + \mathrm{Sinh}^2\rho d\varphi^2)$$

Gravity in a box → closed quantum system (unitary)

$$\Delta = 1 + \sqrt{1 + M^2 R_{AdS}^2} \approx M R_{AdS}$$



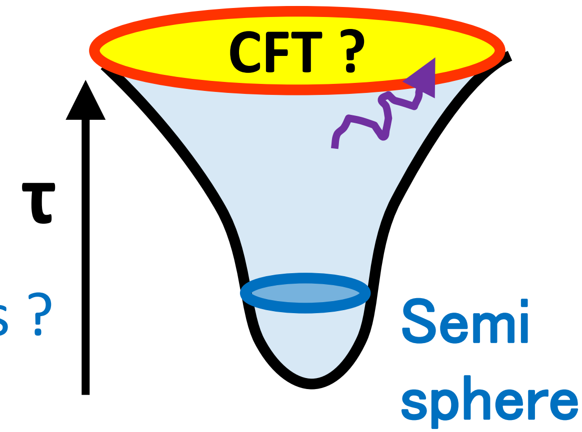
dS

$$ds^2 = R_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Closed universe, ∃cosmological horizon

→ open quantum system ? Non-Unitary CFTs ?

$$\Delta = 1 + \sqrt{1 - M^2 R_{dS}^2} \approx i M R_{dS}$$

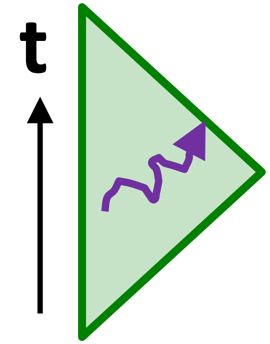


Flat

$$ds^2 = -dt^2 + dx^2$$

The boundary is situated at null Infinity

→Open system ? Non-unitary ? Codim=2 holography ?



A sketch of dS/CFT [Strominger 2001, Maldacena 2002,⋯.]

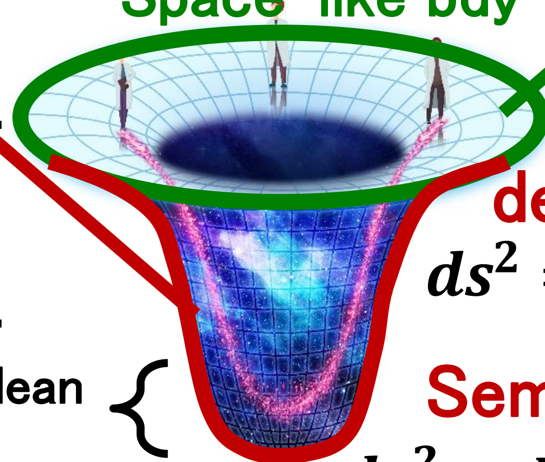
Gravity in d+1 dim.
de−Sitter spacetime

Dual

Euclidean d. dim
non−unitary CFT on S^d

Space−like bdy

Time



$t=\infty$

Lorentzian time

de Sitter

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Euclidean time

Semi sphere

$$ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta\, d\Omega^2)$$

Time emerges from Euclidean CFT !

Ψ [dS gravity]=Z [CFT]

Hartle-Hawking wave function

Why dS/CFT is much more difficult than AdS/CFT ?

[1] Dual Euclidean CFTs should be exotic and non-unitary !

A "standard" Euclidean CFTs is dual to gravity on hyperbolic space.

e.g. dS3/CFT2 → ***Imaginary valued*** central charge $c \approx i\frac{3L_{dS}}{2G_N}$!

[2] Time should emerge from Euclidean CFT !

From a usual Euclidean CFT, a space-like direction will emerge as RG scale.

How does a ***time-like direction emerge*** from a Euclidean CFT ?

[3] Entanglement entropy becomes complex valued !

Extremal surfaces in dS which end on its boundary are ***time-like*** !

Non-unitary CFT dual of 3 dim. dS

[Hikida–Nishioka–Taki–TT, 2021, 2022]


Physics ABOUT BROWSE PRESS COLLECTIONS
VIEWPOINT
Steps toward Quantum Gravity in a Realistic Cosmos
Jordan Cotler
Society of Fellows, Harvard University, Cambridge, MA, USA
July 18, 2022 • Physics 15, 107
Theorists have modeled an expanding spacetime—akin to our Universe—by taking inspiration from a string theory framework in which spacetime is emergent.

Large c limit of SU(2)k WZW model (a 2dim. CFT)
= Einstein Gravity on 3 dim. de Sitter (radius L_{ds})

Level

$$k \approx -2 + \frac{4iG_N}{L_{dS}}$$

→ **Central charge**

$$c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$$

$$Z[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E}}$$

CFT partition function — De Sitter Entropy

This non-unitary CFT is equivalent to the Liouville CFT

at $b^{-2} \approx \pm\frac{i}{4G_N}.$

$$I_{CFT}[\phi] = \int d^2x \left[\frac{1}{4\pi}(\partial_a\varphi\partial_a\varphi) + \mu e^{2b\varphi}\right].$$

complex !

[→Reproduced by Verlinde–Zhang 2024 via the Double Scaled SYK]

[cf. Complex saddle approach: Heng–Yu Cheng's talk and Yusuke Taki's poster]

Holographic Entanglement Entropy in dS3/CFT2 ?

[Doi–Harper–Mollabashi–Taki–TT 2022, 2023]

In dS3/CFT2, the geodesic Γ_A becomes time-like and we find:

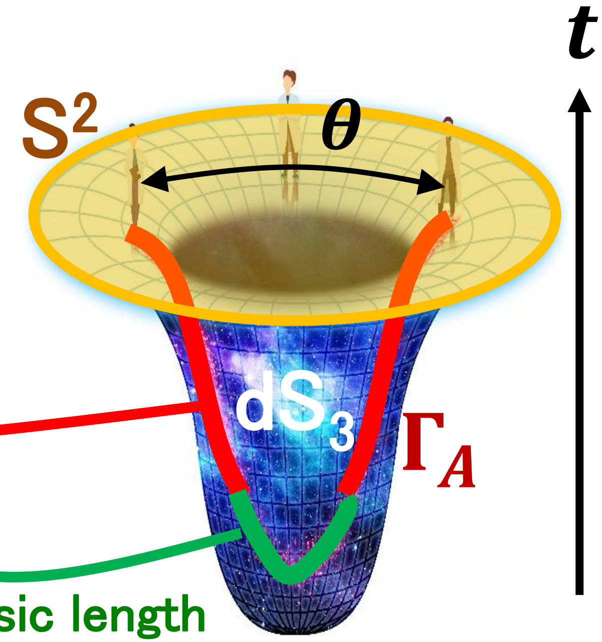
$$S_A = \frac{L(\Gamma_A)}{4G_N} = i\frac{c_{dS}}{3}\log\left(\frac{2}{\epsilon}\sin\frac{\theta}{2}\right) + \frac{c_{dS}}{6}\pi.$$

$S_{dS}/2$

Time-like geodesics length
→imaginary part !

Space-like geodesic length
→Real part

Agree !



CFT calculation

$$S_A = \frac{c_{CFT}}{6}\log\left[\frac{\sin^2\frac{\theta}{2}}{\tilde{\varepsilon}^2}\right],$$ by setting

$$c_{CFT} = ic_{dS} \text{ and } \tilde{\varepsilon} = i\varepsilon = ie^{-t_\infty}.$$

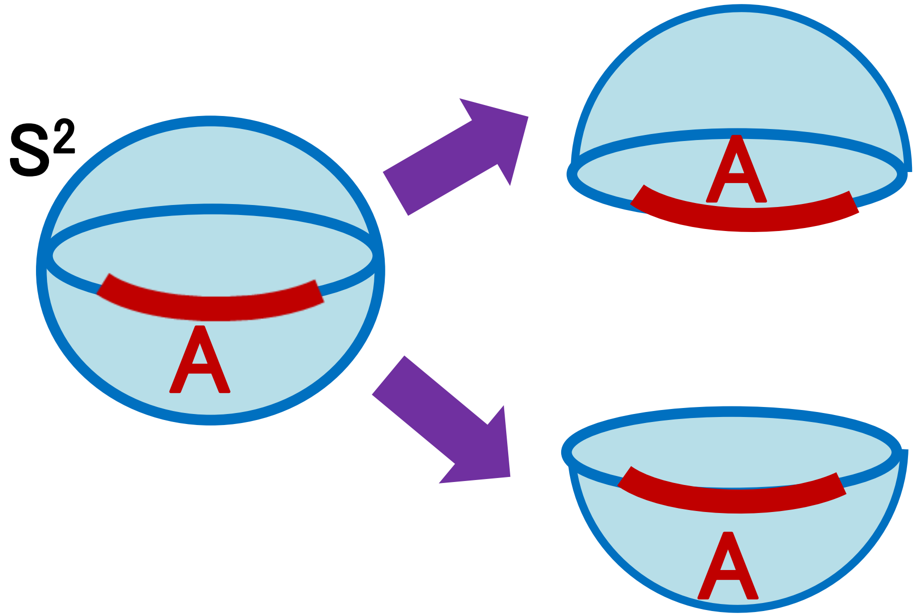
Complex valued entropy !
(should not be EE !)

We argue it is more properly considered as pseudo entropy (PE).

Even though we still have $S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A$ (=pseudo entropy),

the reduced density matrix ρ_A **is not Hermitian !**

$\rho_A =$



$\langle \varphi | \sim \int_{S^2_+} D\varphi\, e^{-I_{CFT}[\varphi]}$

Different states !

$| \psi \rangle \sim \int_{S^2_-} D\varphi\, e^{-I_{CFT}[\varphi]}$

→ $\rho_A \neq \rho_A^\dagger$

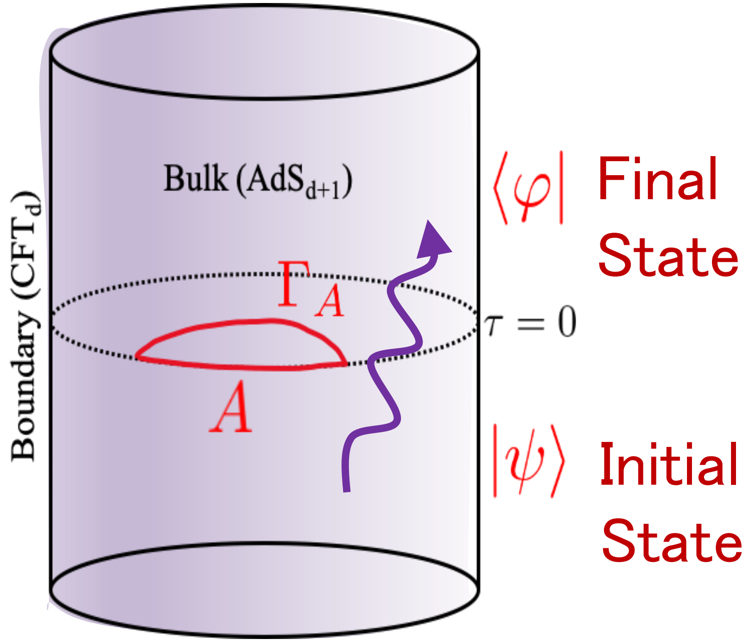
Note: the emergent time coordinate = imaginary part of PE.

Comment: Pseudo entropy in AdS/CFT

Q. What is HEE in a *Euclidean time-dependent AdS* geometry ?

Holographic Pseudo Entropy [Nakata–Taki–Tamaoka–Wei–TT, 2020]

$$S(\mathcal{T}_A^{\psi|\varphi}) = \min_{\Gamma_A} \frac{\text{Area}(\Gamma_A)}{4G_N}$$



Transition matrix

$$H_{tot} = H_A \otimes H_B \quad . \qquad \tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

$$\tau_A^{\psi|\varphi} = \text{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

Pseudo entropy: $$S\left(\tau_A^{\psi|\varphi}\right) = -\text{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].$$

In general, complex valued.

③ Bulk reconstruction in AdS3/CFT2

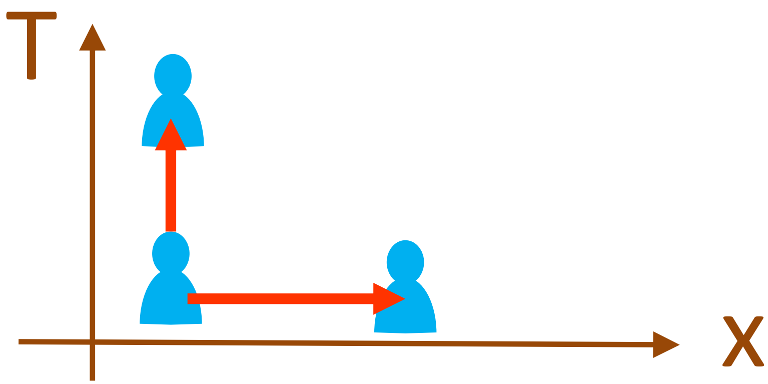
[Miyaji–Numasawa–Shiba–Watanabe–TT 2015, equivalent to HKLL formulation]

Consider an observer in 2d CFT.

➡How does the observer feel that he or she lives in AdS3 ?

To probe the spacetime of AdS3, we introduce an local excitation.

"How many directions can the observer move ?"



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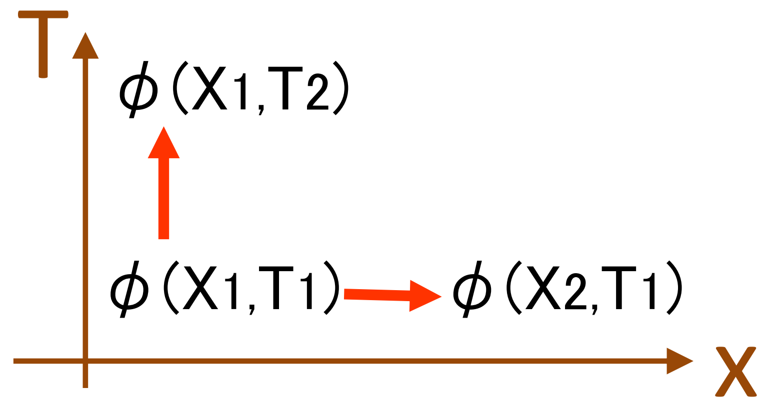
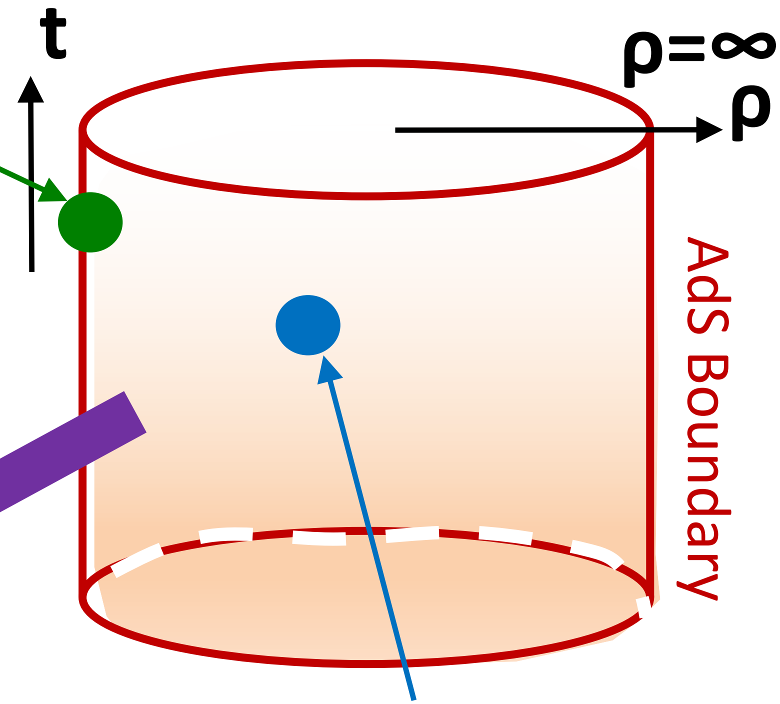


In flat spacetime, $\left|\Psi_{\text{Flat}}(t,x)\right\rangle = e^{-iHt} e^{iPx} \hat{\phi}(0)\left|0\right\rangle$

AdS metric $ds^2 = R_{AdS}^2(-\mathrm{Cosh}^2\rho dt^2 + d\rho^2 + \mathrm{Sinh}^2\rho d\varphi^2)$

$O^{(\Delta)}(t,\varphi)|0\rangle$



Primary operator in CFT $O^{(\Delta)}(t,\varphi)$

$\Delta_{\pm} = 1 \pm \sqrt{1 + M^2 R_{AdS}^2}$

Geometric Symmetry of AdS3
= SO(2,2)=SL(2,R)$_L$ × SL(2,R)$_R$

$L_0, L_{-1}, L_1 \quad \tilde{L}_0, \tilde{L}_{-1}, \tilde{L}_1$

$|\Psi_{AdS}^{(\Delta)}(\rho,t,\varphi)\rangle$

Bulk locally excited state

$$L_0 = \frac{i}{2}(\partial_\tau + \partial_\phi), \quad \tilde{L}_0 = \frac{i}{2}(\partial_\tau - \partial_\phi),$$

$$L_{\pm 1} = \frac{i}{2}e^{\pm i(\tau+\phi)}\left[\frac{\sinh\rho}{\cosh\rho}\partial_\tau + \frac{\cosh\rho}{\sinh\rho}\partial_\phi \mp i\partial_\rho\right],$$

$$\tilde{L}_{\pm 1} = \frac{i}{2}e^{\pm i(\tau-\phi)}\left[\frac{\sinh\rho}{\cosh\rho}\partial_\tau - \frac{\cosh\rho}{\sinh\rho}\partial_\phi \mp i\partial_\rho\right],$$

$$[L_n, L_m] = (n-m)L_{n+m}$$

Among 6 generators, 3 linear combinations act trivially on $\left|\Psi_{AdS}^{(\Delta)}(\rho,t,\varphi)\right\rangle$:

For $\rho=\tau=0$, $\left|\Psi_{AdS}^{(\Delta)}(\rho=0,t=0,\varphi)\right\rangle$ satisfies

$$(L_0-\tilde{L}_0)\,|\Psi_{\mathrm{AdS}}^{(\Delta)}(0,0)\rangle=(L_1+\tilde{L}_{-1})\,|\Psi_{\mathrm{AdS}}^{(\Delta)}(0,0)\rangle=(L_{-1}+\tilde{L}_1)\,|\Psi_{\mathrm{AdS}}^{(\Delta)}(0,0)\rangle=0,$$

In terms of SL(2,R) Ishibashi state

$$(L_n-\tilde{L}_{-n})\,|I_\Delta\rangle=0,\quad n=0,\pm1.$$

$$|I_\Delta\rangle=\sum_{k=0}^{\infty}|k\rangle_{\mathrm{L}}\,|k\rangle_{\mathrm{R}}\,.\qquad |k\rangle=\prod_{j=1}^{k}\sqrt{\frac{1}{j^2+(\Delta-1)j}}(L_{-1})^k\,|\Delta\rangle$$

the bulk locally excited state is solved as follows:

$$|\Psi_{\mathrm{AdS}}^{(\Delta)}(\tau=0,\rho=0)\rangle=e^{\frac{i\pi}{2}(L_0+\tilde{L}_0-\Delta)}\,|I_\Delta\rangle\,,$$

Finally we find quantum states in the CFT which describe localized excitations in AdS (cf. HKLL):

$$\left|\Psi^{(\Delta)}_{AdS}(\rho,t,\varphi)\right\rangle = e^{-i(L_0+\tilde{L}_0)t} e^{i\varphi(L_0-\tilde{L}_0)} e^{-\frac{\rho}{2}\left(L_{-1}+\tilde{L}_{-1}-L_1-\tilde{L}_1\right)} e^{\frac{\pi i}{2}(L_0+\tilde{L}_0)} \left|I^{(\alpha)}\right\rangle.$$

CFT state dual to a localized excitation in the bulk AdS

SL(2,R) Crosscap State

SL(2,R) Ishibashi State

$$(L_n - \tilde{L}_{-n})\left|I^{(\alpha)}\right\rangle = 0, \qquad n = 0, \pm 1.$$

$$\left|I^{(\alpha)}\right\rangle = \sum_{k=0}^{\infty} c_k \left(L_{-1}\tilde{L}_{-1}\right)^k \left|\Delta\right\rangle$$

Note: {Ln} (|n|>1) are not relevant as they change the CFT vacuum.

2 pt function **This leads to the correct 2pt function in AdS3:**

$$\left\langle \Psi^{(\Delta)}_{AdS}(X) \middle| \Psi^{(\Delta)}_{AdS}(X') \right\rangle = G_{AdS}(X,X') = \frac{e^{-(\Delta-1)\cdot D_{AdS}(X,X')}}{2\sinh D_{AdS}(X,X')}$$

Green function

Geodesic distance in AdS3

④ Bulk reconstruction in dS3/CFT2

[Doi-Ogawa-Shinmyo-Suzuki-TT 2024]

First we note the "formal" relation between AdS and dS:

AdS metric

$$ds^2 = R_{AdS}^2(-\mathrm{Cosh}^2\rho dt^2 + d\rho^2 + \mathrm{Sinh}^2\rho d\varphi^2)$$

$$R_{AdS}^2 = -R_{dS}^2$$

$$t = i\tau$$

$$\rho = i\theta$$

$$(L_0)^\dagger = L_0, \quad (L_{\pm 1})^\dagger = L_{\mp 1}$$

dS metric

$$ds^2 = R_{dS}^2(-\mathrm{Cos}^2\theta d\tau^2 + d\theta^2 + \mathrm{Sin}^2\theta d\varphi^2)$$

Unusual conjugation

$$(L_0)^\dagger = -\tilde{L}_0, \quad (L_{\pm 1})^\dagger = \tilde{L}_{\pm 1}$$

$$H^\dagger = -H$$

Hamiltonian is anti-Hermitian ! →Non-unitary CFT!

Global Virasoro algebra of dS3 :

$$L_0 = \frac{1}{2}(\partial_t + i\partial_\phi), \quad \tilde{L}_0 = \frac{1}{2}(\partial_t - i\partial_\phi),$$

$$L_{\pm 1} = \frac{i}{2} e^{\pm(-t+i\phi)} \left[\frac{\sin\theta}{\cos\theta}\partial_t - i\frac{\cos\theta}{\sin\theta}\partial_\phi \mp i\partial_\rho \right],$$

$$\tilde{L}_{\pm 1} = \frac{i}{2} e^{\pm(-t-i\phi)} \left[\frac{\sin\theta}{\cos\theta}\partial_t + i\frac{\cos\theta}{\sin\theta}\partial_\phi \mp i\partial_\rho \right],$$

$$[L_m, L_n] = (m-n)L_{m+n}, \quad [\tilde{L}_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n}.$$

Among 6 generators, 3 linear combinations act trivially on $\left|\widetilde{\Psi}_{dS}^{(\Delta+)}(\rho,t,\varphi)\right\rangle$:

For $\theta = \tau = 0,$ $\left|\widetilde{\Psi}_{dS}^{(\Delta+)}(\theta=0,t=0,\varphi)\right\rangle$ satisfies

$$(L_0 - \tilde{L}_0)|\Psi_\Delta\rangle = 0,$$

$$(L_1 + \tilde{L}_{-1})|\Psi_\Delta\rangle = 0,$$

$$(L_{-1} + \tilde{L}_1)|\Psi_\Delta\rangle = 0.$$

The bulk locally excited state is solved as follows:

$$|\Psi_\Delta\rangle = e^{i\frac{\pi}{2}(L_0+\tilde{L}_0-\Delta)}|I_\Delta\rangle,$$

A naïve analytical continuation from AdS leads to the following quantum state for a localized excitation in dS:

$$\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle = e^{(L_0+\tilde{L}_0)\tau} e^{i\varphi(L_0-\tilde{L}_0)} e^{i\frac{\theta}{2}\left(L_1+\tilde{L}_1-L_{-1}-\tilde{L}_{-1}\right)} e^{\frac{\pi i}{2}(L_0+\tilde{L}_0)} \left|I^{(\alpha)}\right\rangle.$$

Non-unitary evolution

→emergent time

SL(2,R) Crosscap State

Primary operator in CFT

$$O^{(\Delta_+)}(t,\varphi)$$

$$\Delta_\pm = 1 \pm i\sqrt{M^2 R_{dS}^2 - 1}$$

SL(2,R) Ishibashi State

$$(L_n - \tilde{L}_{-n})\left|I^{(\alpha)}\right\rangle = 0$$

However, this state ***leads to the confusing result*** (due to the *unusual conjugation*):

$$\left\langle \tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi) \middle| \tilde{\Psi}_{dS}^{(\Delta_+)}(\tau',\theta',\varphi') \right\rangle = 0.$$

Resolution

The correct answer is found by requiring the CPT invariant state:

$$\left|\Psi_E^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle = \frac{1}{\sqrt{i}}\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle + CPT\cdot\left(\frac{1}{\sqrt{i}}\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle\right)$$

$$= \frac{1}{\sqrt{i}}\left|\tilde{\Psi}_{dS}^{(\Delta_+)}(\tau,\theta,\varphi)\right\rangle + \sqrt{i}\left|\tilde{\Psi}_{dS}^{(\Delta_-)}(\underline{\tau,\pi-\theta,\varphi+\pi})\right\rangle.$$

Antipodal map

[Gauging CPT in QG: Harlow–Numasawa 2023]

Indeed, this reproduces the correct dS Green function at Euclidean vacuum

$$\left\langle \Psi_E^{(\Delta_+)}(x)\middle|\Psi_E^{(\Delta_+)}(x')\right\rangle = G_{dS}^E(x,x') = \frac{\sinh\mu(\pi - D_{dS}(x,x'))}{4\pi\sinh\pi\mu\cdot\sin D_{dS}(x,x')}.$$

Summary

- In dS/CFT, the Hamiltonian is anti Hermitian.

  →Emergence of time

- In dS/CFT, we need to gauge the CPT symmetry.

⑤ Conclusions

- Holography for Λ>0 (dS) spacetimes is still in the development stage due to many exotic properties of dual CFTs.
- Complex valued entanglement entropy in dS/CFT can be interpreted as pseudo entropy.
- An analysis of CFT dual of bulk local excitation again reveal the non-Hermitian nature of the dual CFT.
- The CPT symmetry on a de-Sitter space should be gauged.

Future problems

- What does the reconstruction actually mean in dS/CFT ?
- Holography for more general cosmological spacetimes ?
- CFT Models of creation of the Universe ?

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