Probing dS bubbles with holographic tools

Mainly based on Phys. Rev. D 108 (2023) 026006,
in collaboration with R. Auzzi, G. Nardelli, and G. Pedde Ungureanu [arXiv:2302.03584 [hep-th]]

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Extreme Universe
A New Paradigm for Spacetime and Matter from Quantum Information
Grant-in-Aid for Transformative Research Areas (A)

Nicolò Zenoni

Quantum Gravity and Information in Expanding Universe @YITP

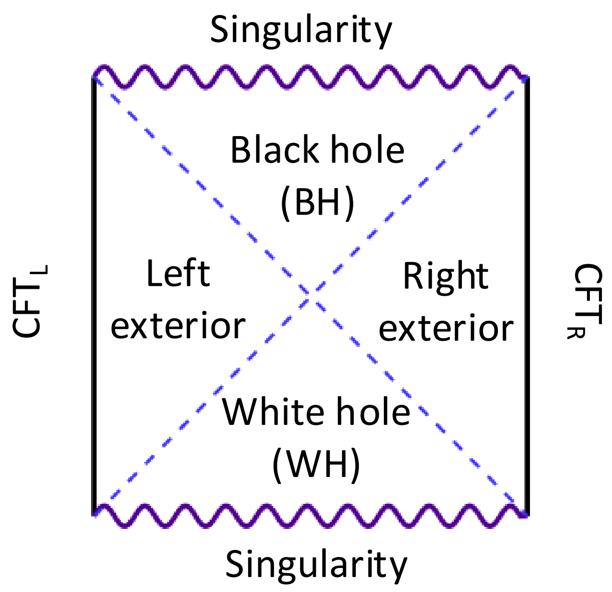
February 19^{th}, 2025

AdS/CFT

[J. M. Maldacena, 1997]

Theory of gravity
in anti-de Sitter (AdS_{d+1})

Conformal Field Theory (CFT_d)
on the AdS boundary



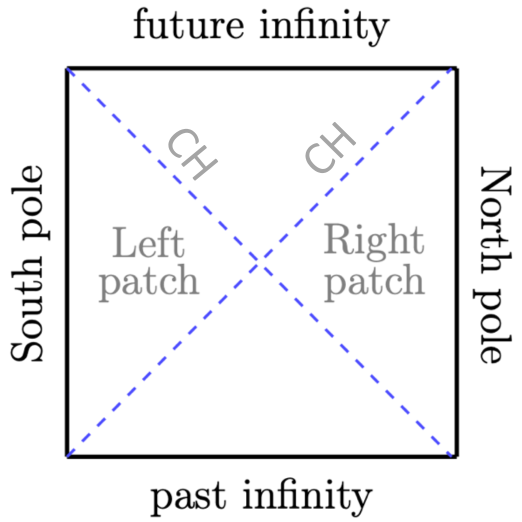
[J. M. Maldacena, 2003]

Eternal **two-sided black hole** in AdS_{d+1}

Thermofield double state
in two identical non-interacting and entangled CFT_d

An expanding Universe

Rather than AdS, our Universe resembles the expanding **de Sitter** (dS) spacetime

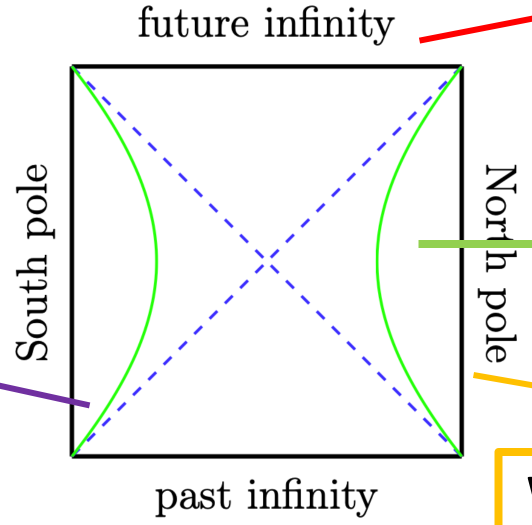


Due to the **inflationary expansion**, dS contains a **cosmological horizon** (CH)

- **No** singularity!
- **Observer** dependent!

Where is the boundary?

! dS has **no boundary**!



Euclidean CFT (**dS/CFT**) [Strominger, 2001]

Stretched horizon holography [L. Susskind, 2021]
Thermofield double-like state?

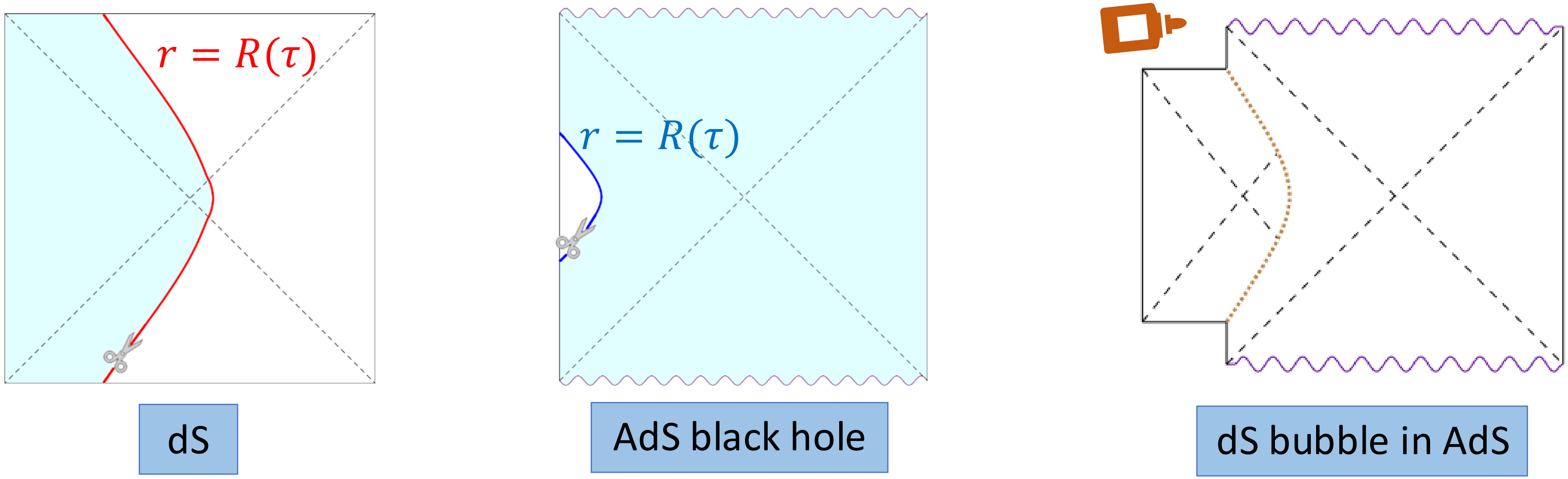
System with **finite** number of DOF [R. Bousso, 2000; T. Banks, 2000; L. Susskind, 2021]

Worldline holography [D. Anninos, S. A. Hartnoll, and D. M. Hofman, 2011]

A "Frankenstein" geometry

We can exploit the AdS **timelike boundary** by **embedding** dS_{d+1} into AdS_{d+1}

[B. Freivogel, V. E. Hubeny, A. Maloney, R. C. Myers, M. Rangamani, and S. Shenker, 2005]



The two spacetimes are glued along the (timelike) trajectory $r = R(\tau)$ of the bubble surface, according to **Israel junction conditions**:

- The **metric** is **continuous** across the bubble surface
- The jump in **extrinsic curvature** is $\left(K_\theta^\theta\right)_{dS} - \left(K_\theta^\theta\right)_{AdS} = \frac{\kappa}{d-1}$, with κ the **bubble tension**

The bubble dynamics

Let us specialize to the case $d = 2$ ($L_{AdS} = 1$):

$$ds^2 = -f_{i,o}(r)dt_{i,o}^2 + \frac{dr^2}{f_{i,o}(r)} + r^2 d\theta^2, \qquad f_{i,o}(r) = \begin{cases} 1 - \lambda\, r^2 \\ r^2 - \mu \end{cases}$$

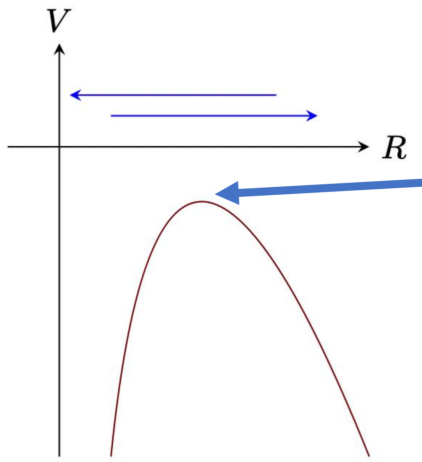
dS$_3$ spacetime

BTZ BH

Israel junction conditions dictate the **equation of motion** for the bubble:

$$\dot{R}^2 + V(R) = 0 \ , \qquad V(R) = f_o(R) - \frac{f_i(R) - f_o(R) - \kappa^2 R^2}{4\,\kappa^2 R^2} = -A\left(R^2 - \beta + \frac{\gamma}{R^2}\right)$$

We have
$A > 0, \gamma > 0$



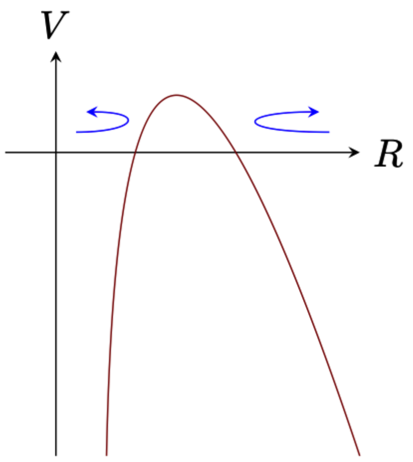
The maximum value of $V(R)$ is **negative** if $\beta^2 < 4\gamma$

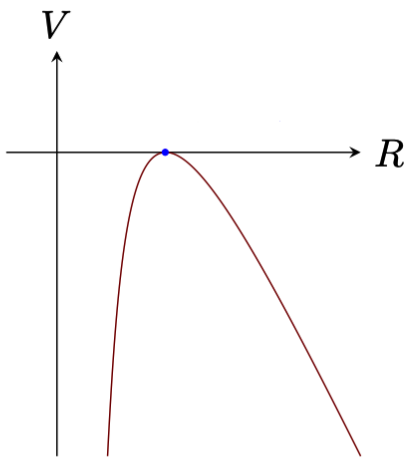
A time symmetric evolution

We consider **time symmetric** configurations, with $\beta^2 \geq 4\gamma$

$$R_{\mp}(\tau) = \sqrt{\frac{\beta}{2} \mp \frac{\sqrt{\beta^2 - 4\gamma}}{2} \cosh\left(2\sqrt{A}\,\tau\right)} \quad , \qquad \frac{dT_{i,o}}{dR} = \pm \frac{\sqrt{f_{i,o}(R) - V(R)}}{f_{i,o}(R)\sqrt{-V(R)}}$$

$\beta^2 > 4\gamma$





$\beta^2 = 4\gamma$

Collapsing bubble

$T_{i,o}(R_-(0)) = 0$

Expanding bubble

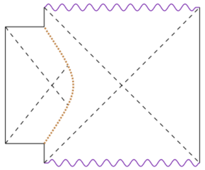
$T_{i,o}(R_+(0)) = 0$

Static bubble

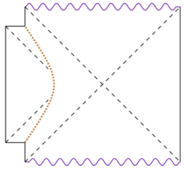
$R(\tau) = \sqrt{\beta/2}$

The bubble catalogue

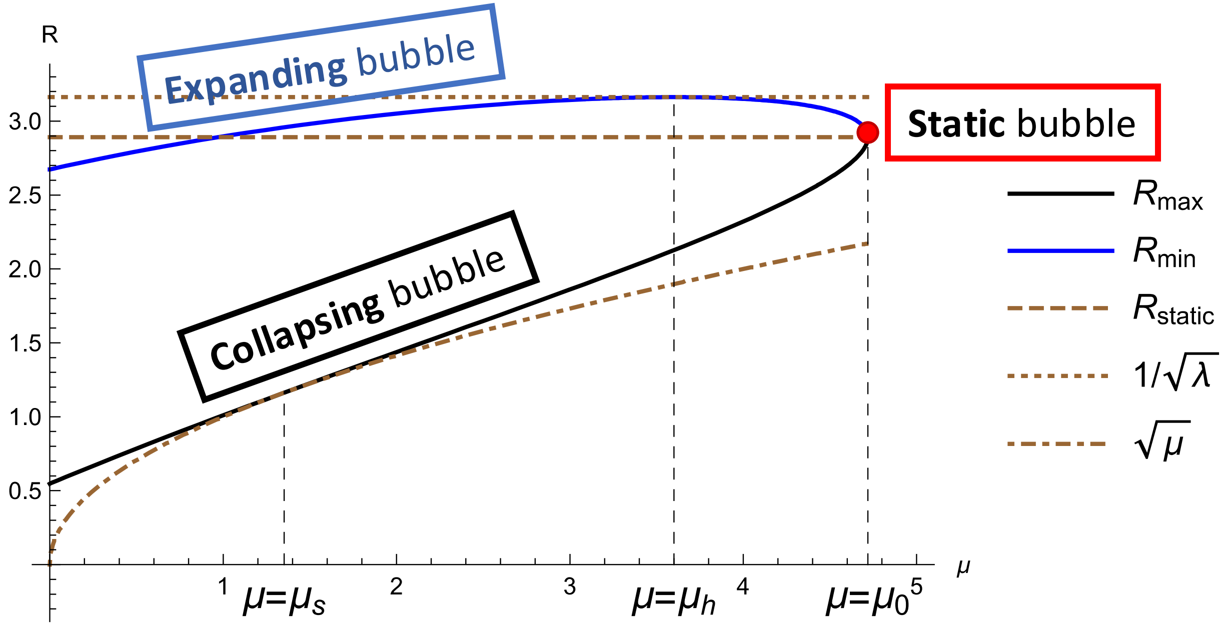
$0 < \mu < \mu_h$
(**Very large** bubble)

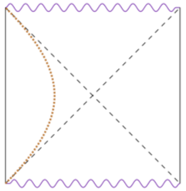


$$\mu_h = \frac{1-\kappa^2}{\lambda}$$



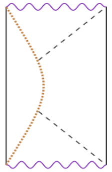
$\mu_h < \mu < \mu_0$
(**Not so large** bubble)



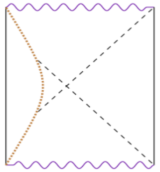


$$\mu_0 = \frac{\sqrt{F^2+4\lambda}-F}{2\lambda},$$
$$F = \kappa^2 + \lambda - 1$$

$0 < \mu < \mu_s$
(**Very small** bubble)



$$\mu_s = \frac{1}{\kappa^2+\lambda}$$

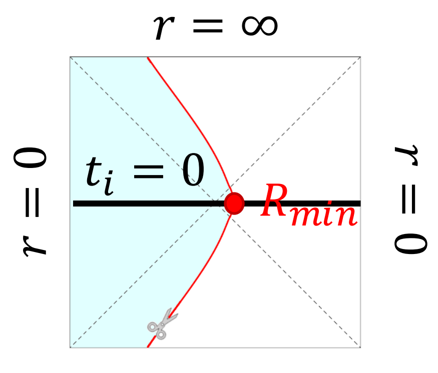


$\mu_s < \mu < \mu_0$
(**Not so small** bubble)

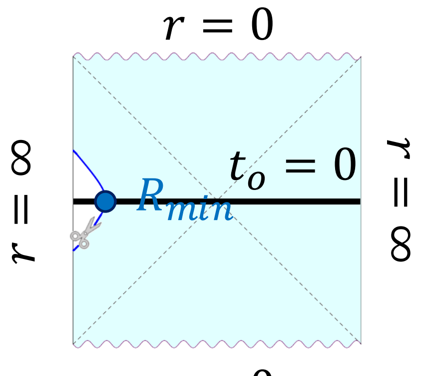
Bag of gold

1. **Inflating** regions are **behind** the event horizon (due to the **null energy condition**)!

2. **Entropy puzzle** for geometries containing inflating regions



$R_m \leq 1/\sqrt{\lambda}$



$\sqrt{\mu} \leq R_m$

$$S_{BH} = \frac{\pi\sqrt{\mu}}{2G} \leq \frac{\pi}{2G\sqrt{\lambda}} = S_{dS}$$

? How can a **pure state** which is built of **fewer** d.o.f. **contain** information on dS?

The inflating region must be described by a boundary **mixed** state
[B. Freivogel, V. E. Hubeny, A. Maloney, R. C. Myers, M. Rangamani, and S. Shenker, 2005]

Holographic entanglement entropy

A possible probe for the emergence of gravity from the boundary

$$S(A) = \frac{\mathcal{A}(\gamma_A)}{4G}$$

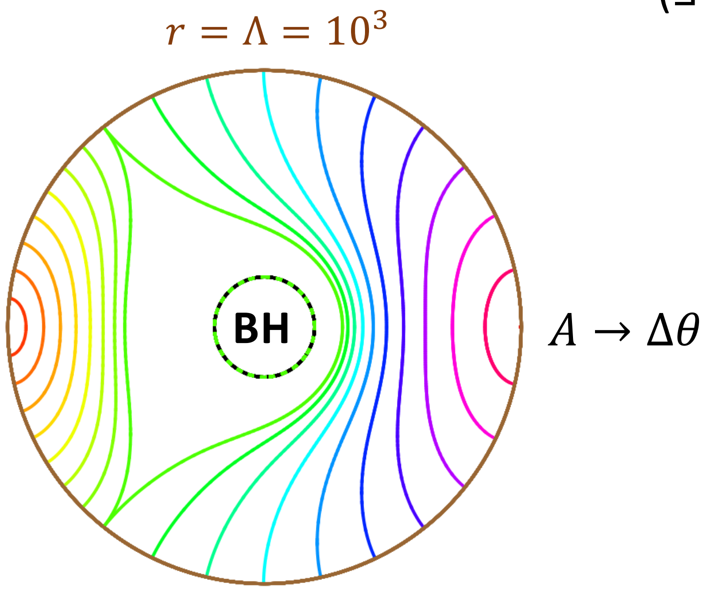
Holographic entanglement entropy

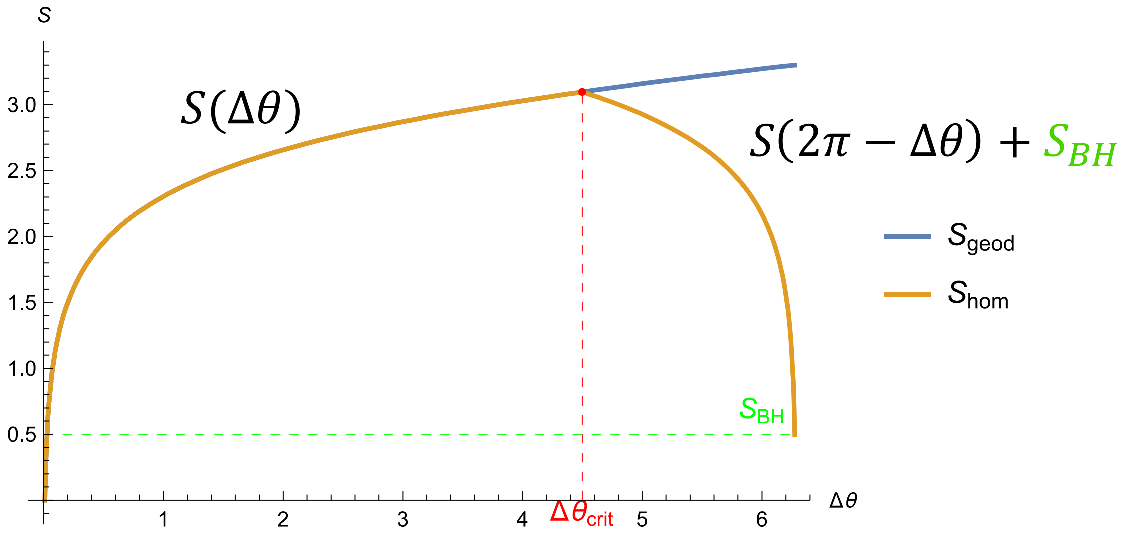
[S. Ryu and T. Takayanagi, 2006]

[V. E. Hubeny, M. Rangamani, and T. Takayanagi, 2007]

γ_A : **Minimal** (**extremal**) surface **homologous** to the boundary subregion A

($\exists$ **smooth** $\mathcal{R}$ $s.t.\, \partial\mathcal{R} = \gamma_A \cup A$)





Probing small bubbles

Very small bubbles are initially **outside** the horizon…

The "area" of a **1-dimensional surface** described by $r(l), \theta(l)$ at constant $t_{i,o} = 0$ is

$$\mathcal{A} = \int \mathcal{L}\, dl\,, \qquad \mathcal{L} = \frac{(r')^2}{f_{i,o}(r)} + r^2(\theta')^2$$

There is a **conserved** momentum $j \equiv \partial_{\theta'}\mathcal{L} = r^2\theta'$

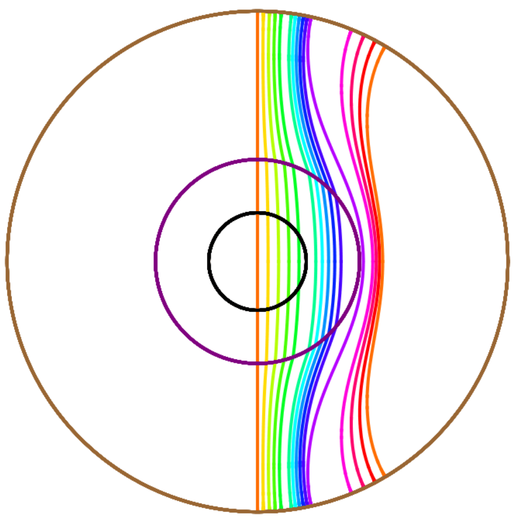
Plugging it into $\mathcal{L} = 1$ ⇒ $$\frac{d\theta}{dr} = \frac{\theta'}{r'} = \pm\frac{j}{r\sqrt{(r^2 - j^2)f_{i,o}(r)}}$$

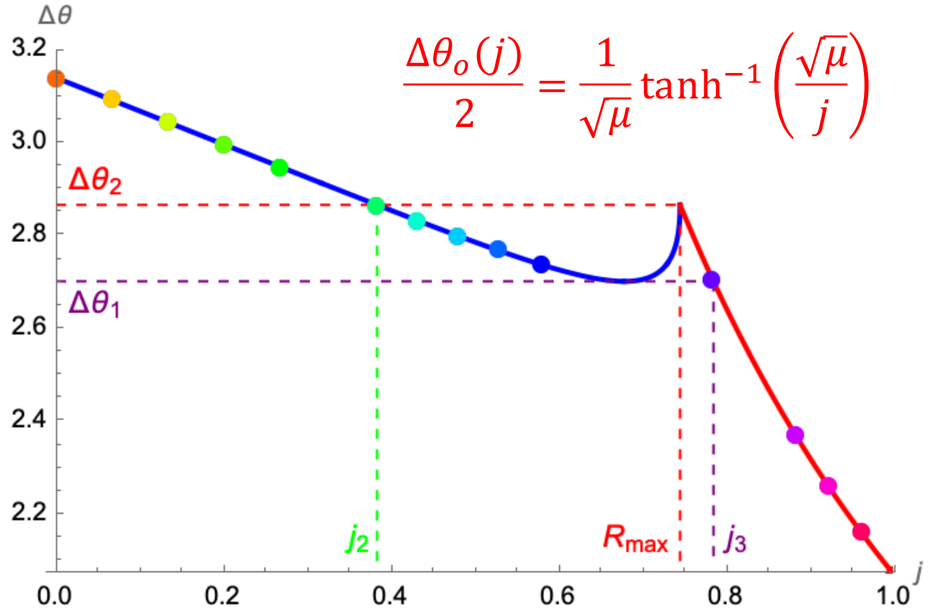
Minimal geodesics

The ‘’refraction’’ condition $\theta'_i = \theta'_o$ implies that j is the **same** inside and outside the bubble

$$\frac{\Delta\theta_{bub}(j)}{2} = \tan^{-1}\left(\frac{1}{j}\sqrt{\frac{R_{max}^2 - j^2}{1-\lambda R_{max}^2}}\right) + \frac{1}{\sqrt{\mu}}\tanh^{-1}\left(\frac{\sqrt{\mu}}{j}\right) - \frac{1}{\sqrt{\mu}}\tanh^{-1}\left(\frac{\sqrt{\mu}}{j}\sqrt{\frac{R_{max}^2 - j^2}{R_{max}^2-\mu}}\right)$$

$\lambda = 0.1, \mu = 0.1, \kappa = 0.4, \Lambda = 10^2$

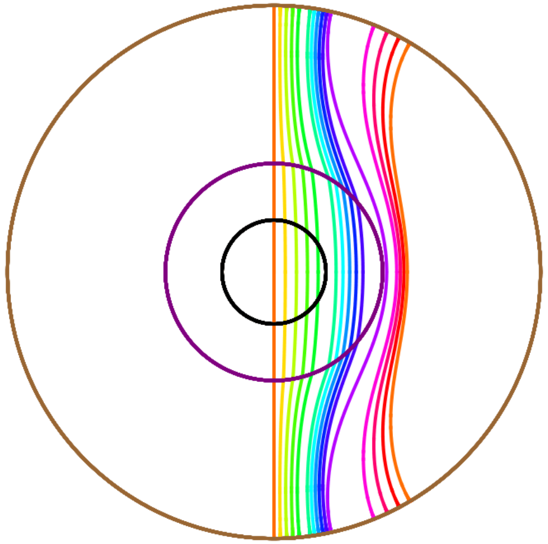




For $\Delta\theta_1 \leq \Delta\theta \leq \Delta\theta_2$ there are two geodesics inside and one outside the bubble

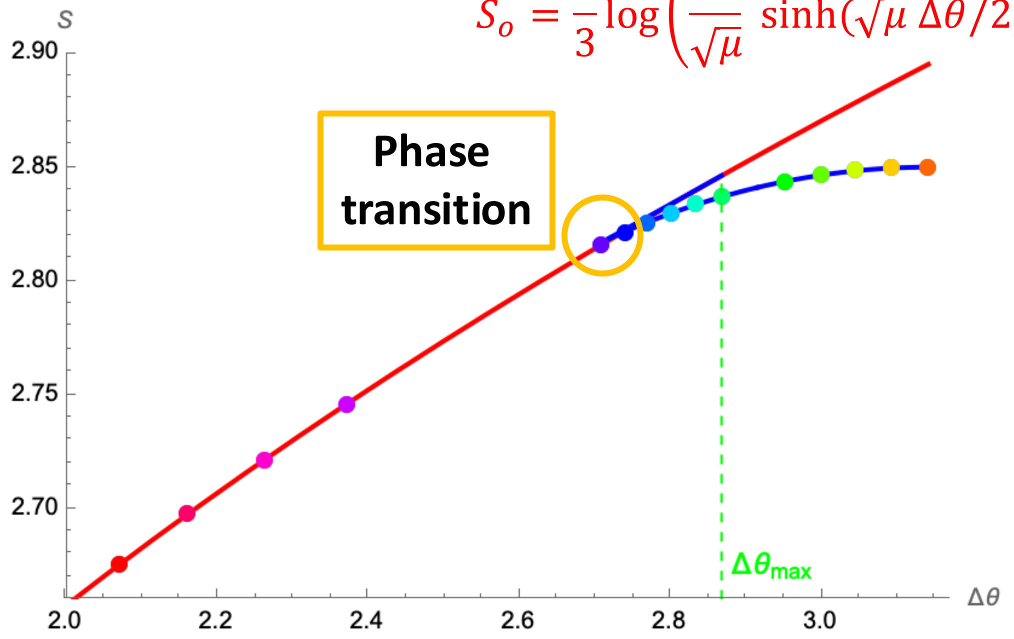
Entanglement entropy for small bubbles

$$S_{bub}(j) = \frac{c}{3}\left(\sin^{-1}\left(\sqrt{\frac{\lambda(R_{max}^2 - j^2)}{1-\lambda\, j^2}}\right) + \log\left(\frac{2\Lambda}{\sqrt{R_{max}^2 - j^2} - \sqrt{R_{max}^2 - \mu}}\right)\right)$$



$$S(\Delta\theta) = S(2\pi - \Delta\theta)!$$

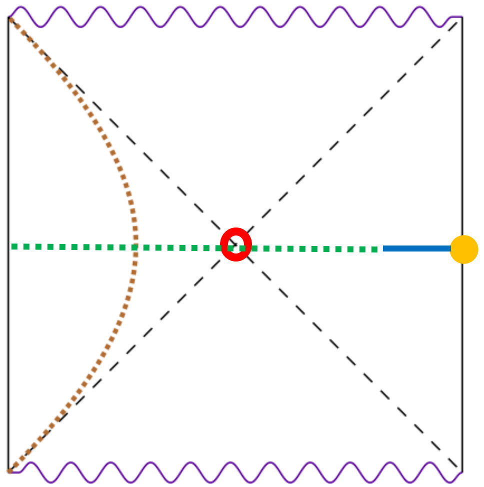
$$S_o = \frac{c}{3}\log\left(\frac{2\Lambda}{\sqrt{\mu}}\sinh(\sqrt{\mu}\,\Delta\theta/2)\right)$$



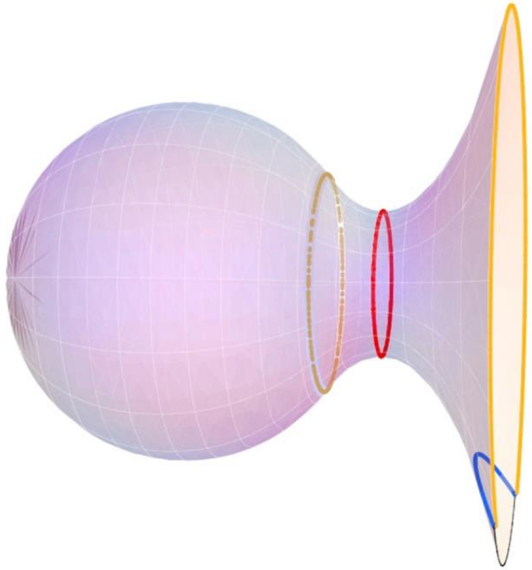
The entanglement entropy is **smaller** due to the bubble!
Similar for bubbles of vacuum AdS [R. Antonelli and I. Basile, 2018]

Entanglement for bags of gold

Holographic entanglement entropy can probe the bubble
[V. E. Hubeny, H. Maxfield, M. Rangamani, and E. Tonni, 2013]



Killing field ∂_t for **full** spacetime

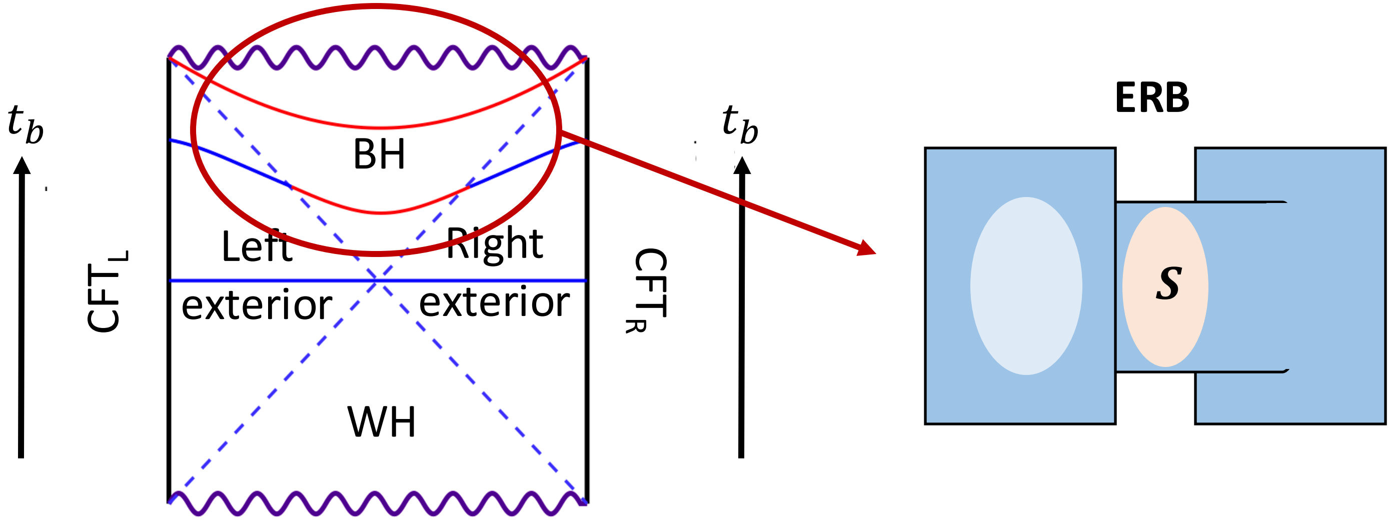


Taken from arXiv:1306.4004 [hep-th]

$$S(\Delta\theta) = S(2\pi - \Delta\theta)$$

The growth of the bridge

We can foliate spacetime with **maximal codimension-one** slices **anchored** at the boundaries



The **area** of the cross-section stops growing at the **thermalization** time
[T. Hartman and J. M. Maldacena, 2013]

[L. Susskind, 2014]

Entanglement entropy **is not enough**!

$$\mathcal{C}_V(B) = \max_{\partial\Sigma=B} \frac{V(\Sigma)}{G L_{AdS}}$$

Complexity = Volume
[D. Stanford and L. Susskind, 2014]

The ever growing bridge

The volume of a **2-dimensional surface** described by $t(l), r(l)$ is

$$V = 2\pi \int \mathcal{L}\, dl\,, \qquad \mathcal{L} = r\sqrt{-f_o(r)\,(t')^2 + \frac{(r')^2}{f_o(r)}}$$

$\mathcal{L}$ **does not** depend on t ⇒ $$P = \frac{\partial \mathcal{L}}{\partial t'} = -\frac{r^2\, f_o(r)\, t'}{\mathcal{L}}$$ **Conserved!**

We can exploit reparameterization invariance to fix

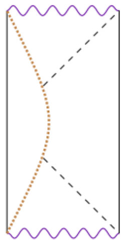
$\mathcal{L} = r^2$ ⇒ $$r'_{\pm} = \pm\sqrt{r^2\, f_o(r) + P^2}, \quad t' = -\frac{P}{f_o(r)}$$

The growth rate of the volume V at late times is
[D. Carmi, S. Chapman, H. Marrochio, R. C. Myers, and S. Sugishita, 2017]:

$$\lim_{t_b \to \infty} \frac{dV}{dt_b} = 16\pi G L_{AdS}^2\, M \qquad M = \frac{\mu}{8 G L_{AdS}^2}$$ BH mass

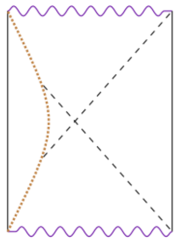
The bubble catalogue - reprise

Collapsing bubble



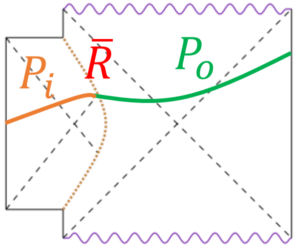
$0 < \mu < \mu_s$ (**Very small** bubble)

$$\mu_s = \frac{1}{\kappa^2 + \lambda}$$



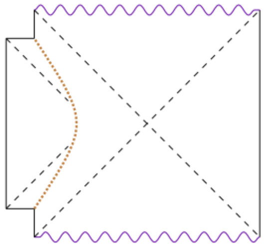
$\mu_s < \mu < \mu_0$ (**Not so small** bubble)

Expanding bubble



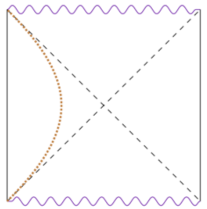
$0 < \mu < \mu_h$ (**Very large** bubble)

$$\mu_h = \frac{1 - \kappa^2}{\lambda}$$



$\mu_h < \mu < \mu_0$ (**Not so large** bubble)

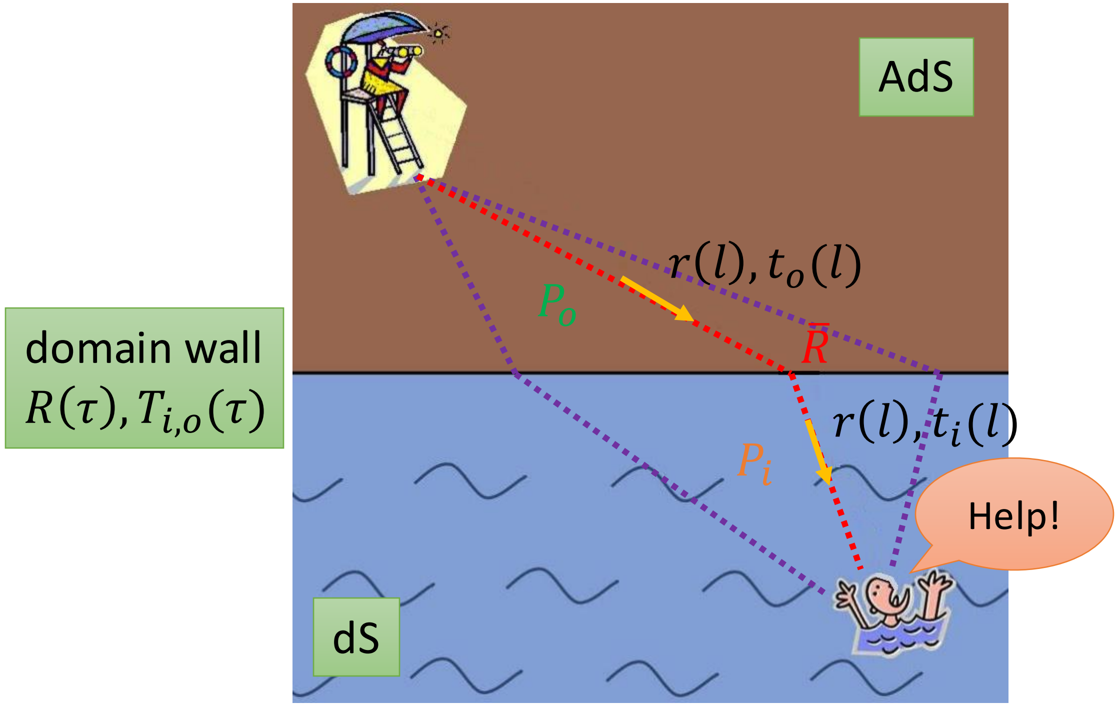
Static bubble



$$\mu = \mu_0 = \frac{\sqrt{(\kappa^2 + \lambda - 1)^2 + 4\lambda} - (\kappa^2 + \lambda - 1)}{2\lambda}$$

A "Frankenstein" extremal surface

The **full extremal surface** is obtained by **gluing** along the bubble surface
an **AdS part** attached to the boundary and a **dS part**



P_o, P_i, and $\bar{R}$ must be chosen so that the **full** surface has **maximal** volume!

$$P_i \frac{dT_i}{dR} + \frac{\xi_i(\bar{R})}{f_i(\bar{R})} = P_o \frac{dT_o}{dR} + \frac{\xi_o(\bar{R})}{f_o(\bar{R})}$$

Refraction-like law

$$\xi_{i,o}(\bar{R}) = \frac{dr}{dl}\Big|_{wall_{i,o}} = \pm\sqrt{f_{i,o}(\bar{R})\,\bar{R}^2 + P_{i,o}^2}$$

AdS holography prescription

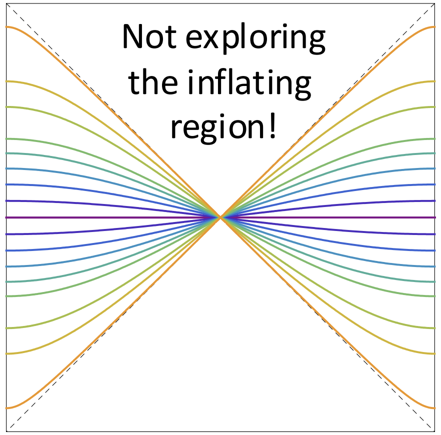
It is natural to anchor the maximal surface at the AdS boundary:

1. The **AdS boundary time** is fixed as $t_b = t_R$

2. To avoid a curvature singularity on the maximal surface at $r = 0$ in dS, we **impose** $P_i = 0$

$$t_i{}' = -\frac{P_i}{f_i(r)} = 0$$

The surfaces in the dS portion are at **constant time**!

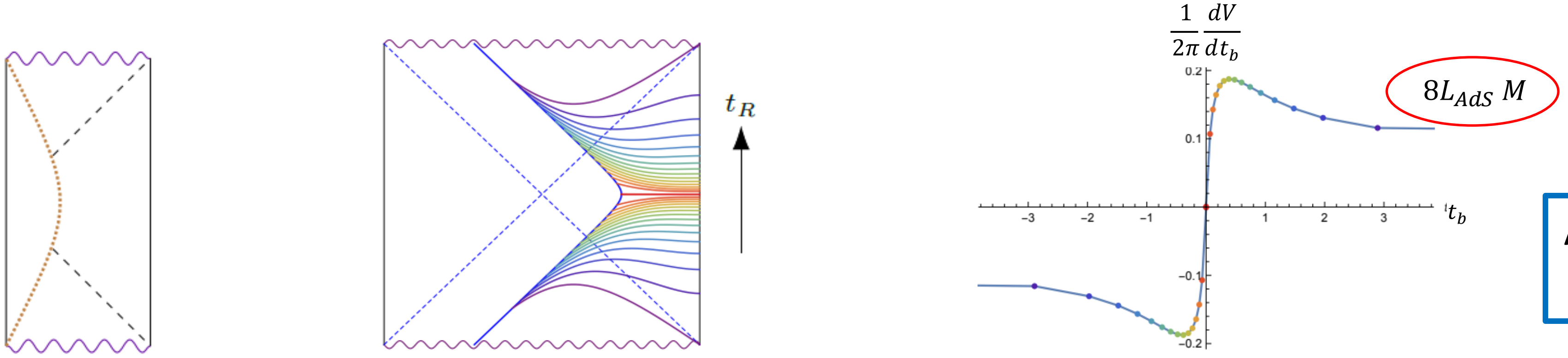


For **all kinds** of bubbles, the volume growth rate is:

$$\frac{1}{2\pi}\frac{dV}{dt_b} = P_o$$

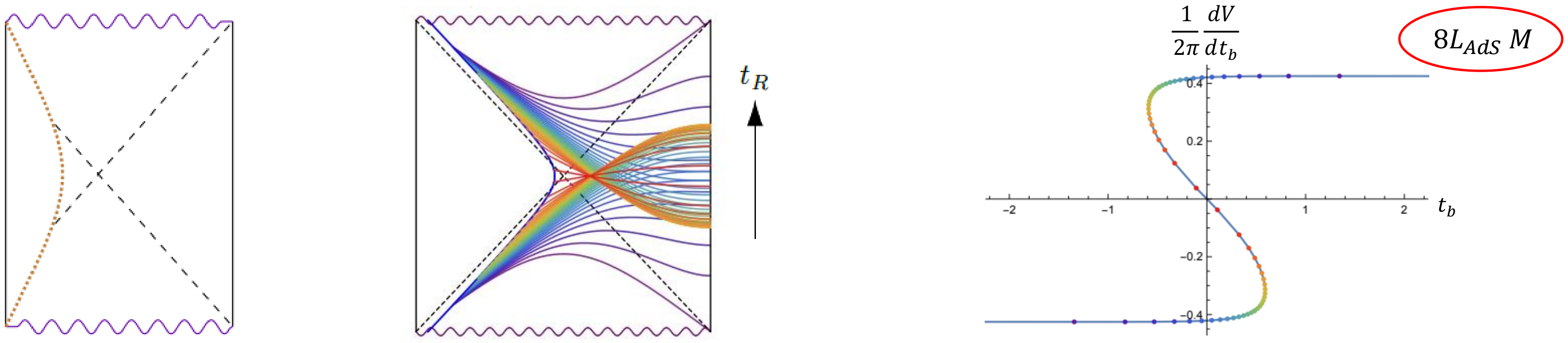
Complexity of collapsing bubbles

Very small bubbles $(0 < \mu < \mu_s)$



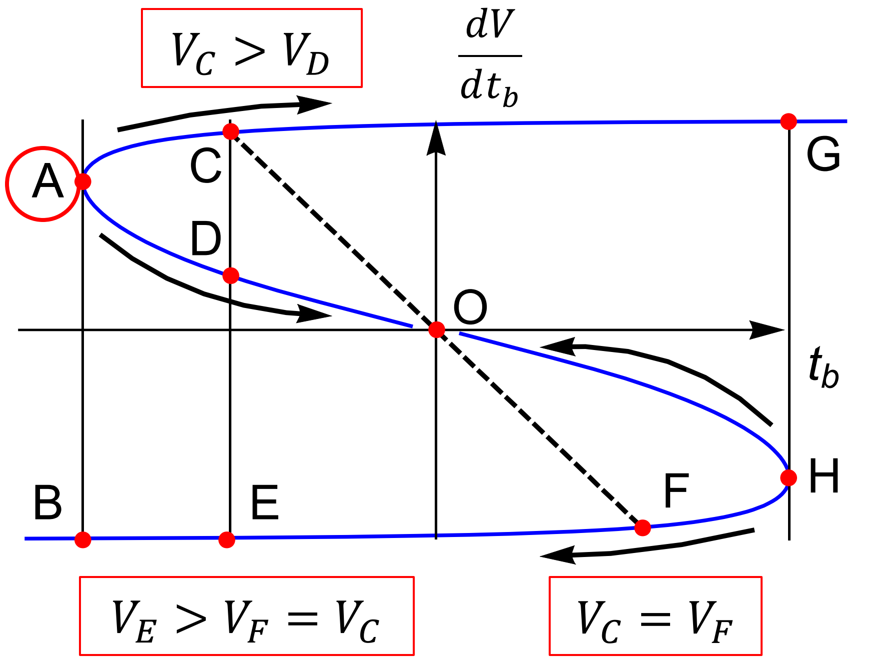
As in BTZ!

Not so small bubbles $(\mu_s < \mu < \mu_0)$

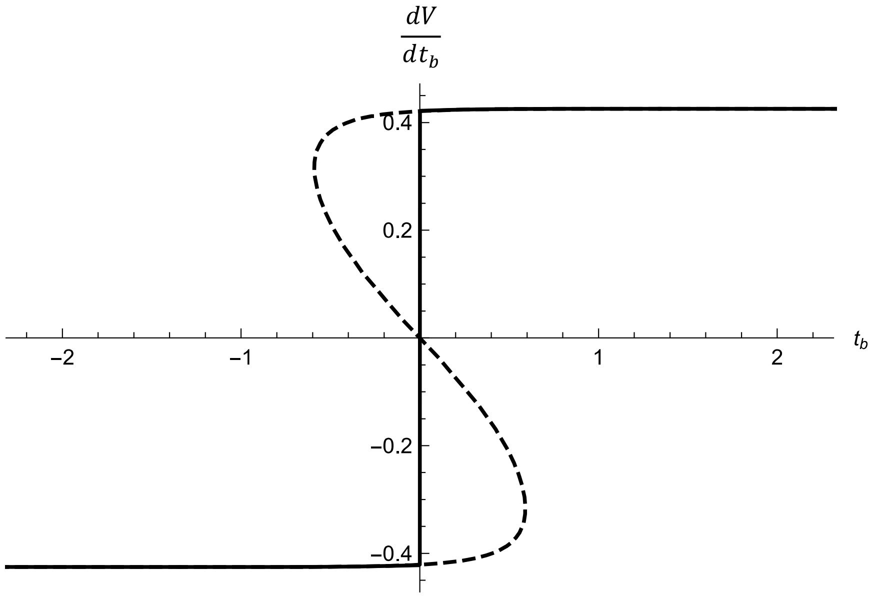


Maximizing the volume

For a given t_b, how to recognize surfaces with **maximal** volume?



Time-reversal symmetry



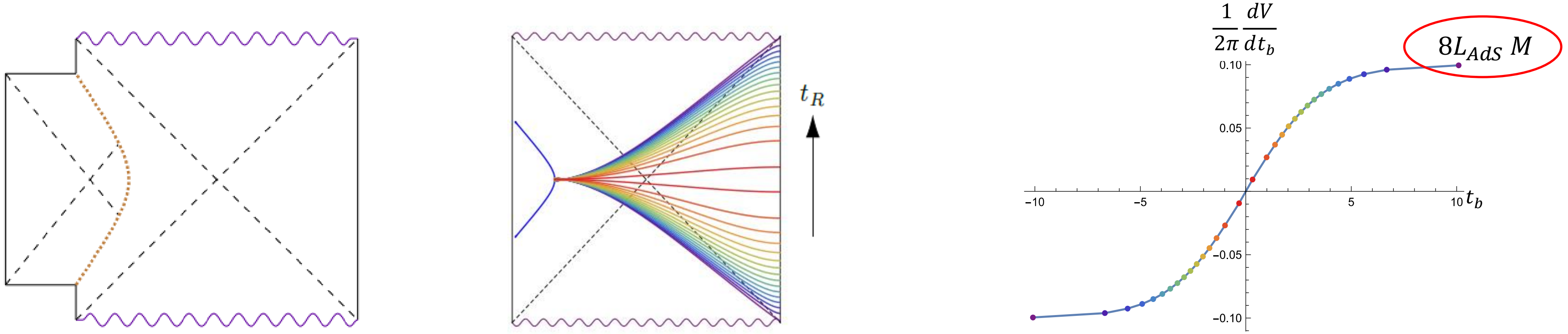
Maximization selects
the solid **step-like function**!

This situation **also** arises for bubbles with $0 < \mu < \mu_s$ when $\left.\frac{d^2V}{dt_b{}^2}\right|_{t_b=0} < 0,$

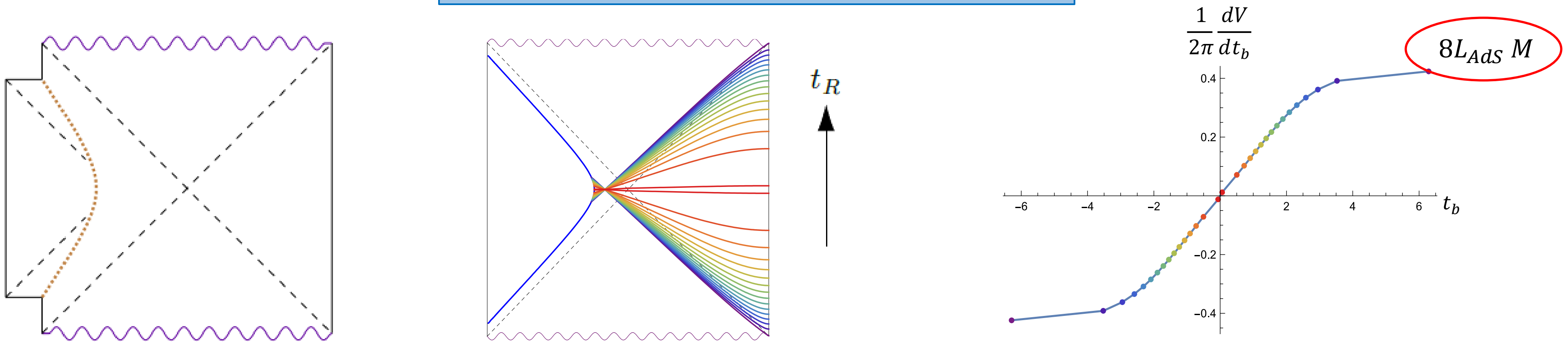
but **never** for expanding bubbles!

Complexity of expanding bubbles

Very large bubbles $(0 < \mu < \mu_h)$

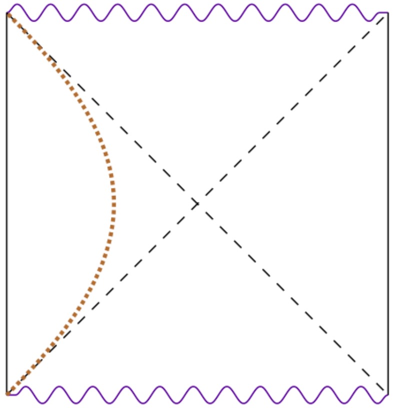


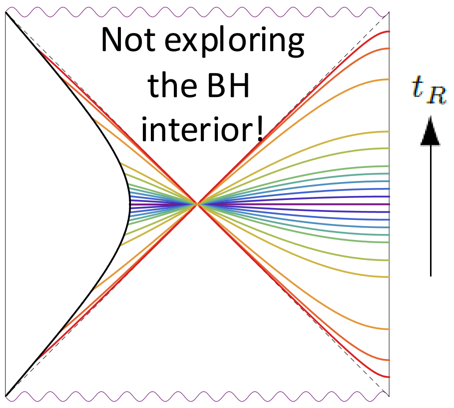
Not so large bubbles $(\mu_h < \mu < \mu_0)$



Complexity of static bubbles

$\mu = \mu_0$



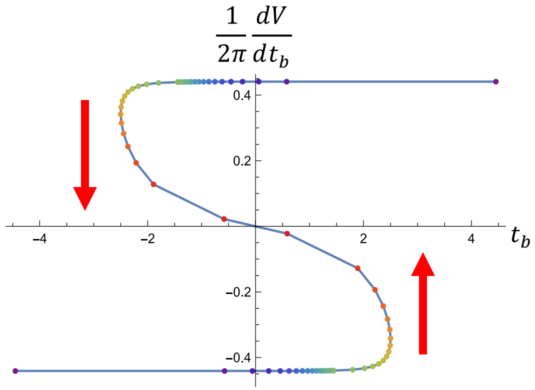


$$\frac{1}{2\pi}\frac{dV}{dt_b} = P_o = 0$$

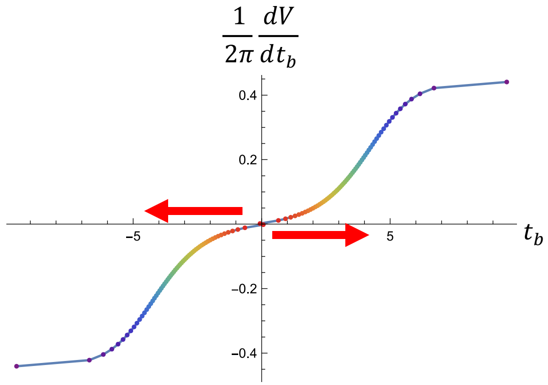
The Killing vector ∂_t is **preserved** by the bubble!

The static bubble can be realized as a **limit**:

Small bubble with $\mu \to \mu_0$

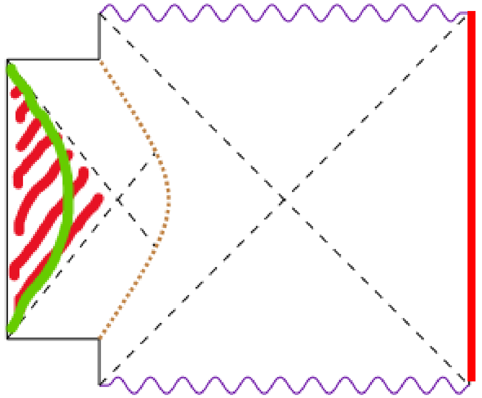


Large bubble with $\mu \to \mu_0$



Second prescription: hybrid holography?

A very large bubble $(0 < \mu < \mu_h)$ contains a **whole static patch**



We can introduce a **stretched horizon** $r_{st} = \frac{\rho}{\sqrt{\lambda}}$ ➡ The curvature singularity is **removed**, so we can **have** $P_i \neq 0$!

We fix the following **boundary** conditions:

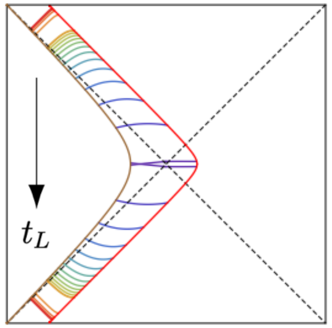
1. The **dS** stretched horizon time is fixed as $t_b = -t_L$

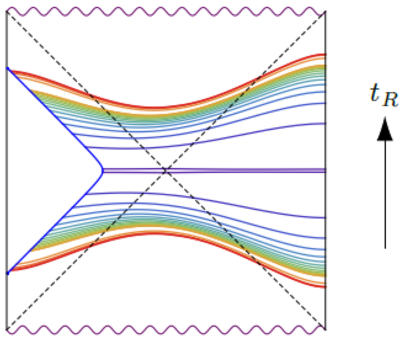
2. The **AdS** boundary time is fixed as $t_b = -\alpha_t \, t_R$

Hyperfast growth

$\alpha_t = -1$:

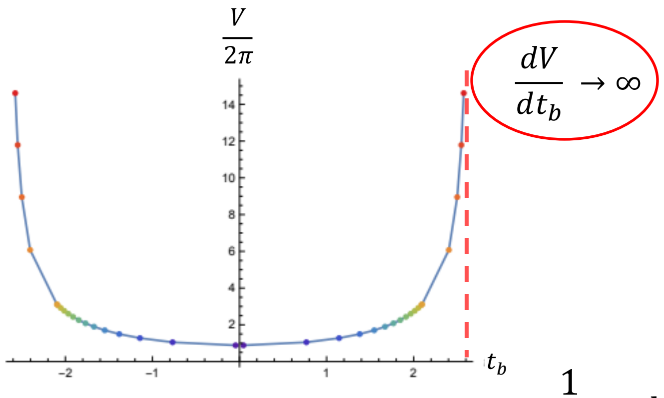
$t_L = -t_R$





dS

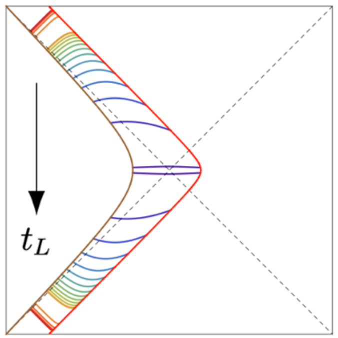
AdS BH

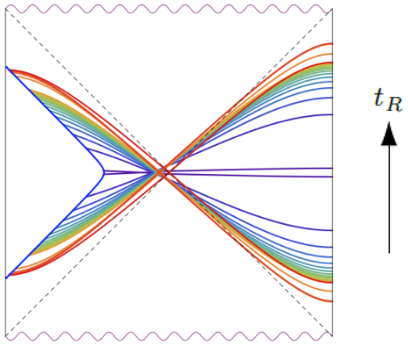


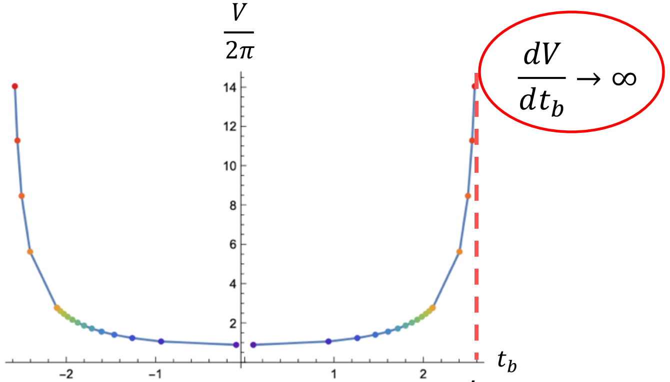
$$t_{cr} = \frac{1}{\sqrt{\lambda}} \tanh^{-1} \rho + const$$

$\alpha_t = 1$:

$t_L = t_R$







As in **pure dS**! [E. Jørstad, R. C. Myers, and S.-M. Ruan , 2022]

Conclusions and outlook

- ✓ We have considered **dS bubbles** embedded into BTZ spacetime [B. Freivogel, V. E. Hubeny, A. Maloney, R. C. Myers, M. Rangamani, and S. Shenker, 2005]
- ✓ **Holographic entanglement entropy** (initial time): for large subregions is smaller than the case of BTZ without bubble. What about later time?

? Where are the **extra** degrees of freedom of the **inflating** region?

The **two prescriptions** for holographic complexity suggest different interpretations:

- ➢ **AdS holography**: the volume is proportional to complexity of a **mixed** CFT state, obtained by **tracing** the thermofield double-like state over the dS degrees of freedom (no dS information)
- ➢ **Hybrid holography**: the volume is proportional to complexity of the **pure** thermofield double-like state (dS expansion is detected)

Thank you for your attention!