Quantum Gravity and Information in Expanding Universe
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Gravity with the Gauss-Bonnet term
- Black Holes and Cosmology -

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Contents

I. Motivation

Modified Gravity beyond Einstein - Is it needed?

Dilaton-Einstein Gauss-Bonnet(dEGB) theory

II. Black Holes

Black Hole solutions in Gravity theories

- Schwarzschild BH, AdS BH, RN AdS BH, RN Gauss-Bonnet AdS BH
- dEGB BH

III. Dilaton-Einstein-Gauss-Bonnet (dEGB) Cosmology

- WIMP indirect detection
  constraints from the GW signals from BH-BH and BH-NS merger events

- New Phase at High T and GWs

IV. Summary

I. Motivation : Modified Gravity beyond Einstein - Is it needed?

1. Theoretical Aspect: Gravity beyond Einstein?

- GR is an **effective theory** valid below UV cut-off, $M_{Pl} \sim 10^{19} GeV$

Ex) String theory $\xrightarrow[\text{Low Energy}]{(\alpha'\text{-expansion})}$ Einstein Gravity ($\sim R$) + higher curvatures ($\sim R^n, n \geq 2$; (R^2_{GB}?))

Einstein Gravity is the simplest theory of the gravity, linear in the curvature scalar:

- **Holography**: Needs the dual geometry (beyond Einstein)

5dim. classical gravity ⇔ 4dim. strongly interacting theory
Ex) QCD & CMT

* Black Holes are thermal system.

2. Observational Aspect: Alternatives to Λ CDM?

- **Some challenging observations**

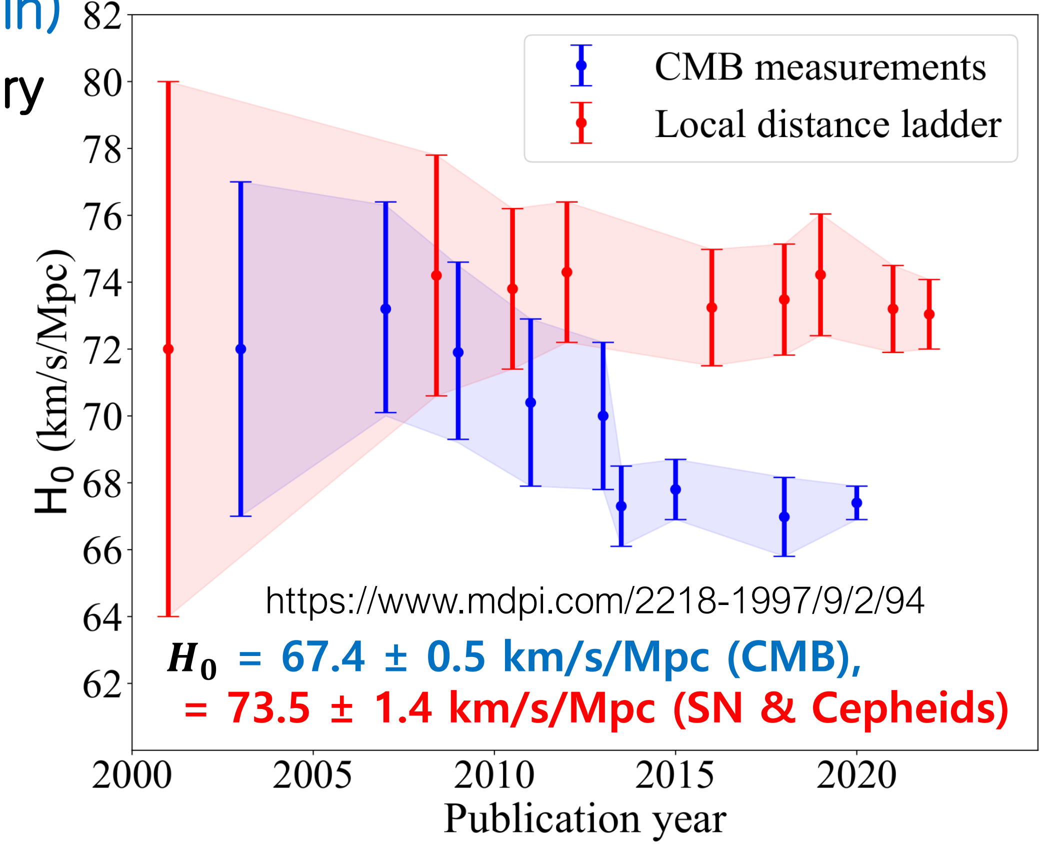
1) H_0 tension (~5.8σ)

2) Cosmic Birefringence(~3σ)

3) σ_8(S_8)etc.

J. Kochappan, L. Yin, B-HL, T. Ghosh e-Print: 2408.09521

B-HL, W. Lee, M.M. Sheikh-Jabbari, S. Thakur, JCAP 04 (2022)



Note: Lovelock theory (dim. $D = 2t+1$ or $2t$)

Lagrangian with only **1) metric 2)2nd order e.o.m** (for no ghosts and instabilities) will be

$$\mathcal{L}_D = \sqrt{-g}\sum_{n=0}^{t}\alpha_n L^n$$

Ex) D-dim

$\mathcal{L}_2 = L^1 = \sqrt{-g}\ R$ topological

$\mathcal{L}_3 = L^1 = \sqrt{-g}\ R$

$\mathcal{L}_4 = L^1 + L^2 = \sqrt{-g}(R + R^2_{GB}) \approx \sqrt{-g}\ R$

$\mathcal{L}_5 = L^1 + L^2 = \sqrt{-g}(R + R^2_{GB})$

$\mathcal{L}_6 = L^1 + L^2 + L^3 = \sqrt{-g}(R + R^2_{GB} + R^3_{E.C}) \approx \sqrt{-g}(R + R^2_{GB})$

$\mathcal{L}_7 = L^1 + L^2 + L^3 = \sqrt{-g}(R + R^2_{GB} + R^3_{E.C})$

L^n : Lovelock term, topological in 2n D

Ex) $L^1 = R$ Einstein-Hilbert term topol in 2 D

$L^2 = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$

$= R^2_{GB}$ Gauss-Bonnet term. topol in 4 D

$$L^m = \frac{1}{2^m}\delta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_m\nu_m}_{a_1b_1a_2b_2\cdots a_mb_m} R_{a_1b_1}{}^{\mu_1\nu_1} R_{a_2b_2}{}^{\mu_2\nu_2}\cdots R_{a_mb_m}{}^{\mu_m\nu_m}$$

Euler characteristic of dim 2m topol in $2m$ D

$$\delta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_m\nu_m}_{a_1b_1a_2b_2\cdots a_mb_m} = (2m)!\,\delta^{\mu_1}_{[a_1}\delta^{\nu_1}_{b_1}\cdots\delta^{\mu_m}_{a_m}\delta^{\nu_m}_{b_m]}$$

Note :

- Higher ($n(\geq 2)$-th order) curvature terms lead to the higher ($2n$-th) order e.o.m.
- Among the 3 possible quadratic terms ($R^2,\ R_{\mu\nu}R^{\mu\nu},\ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$) , the Gauss-Bonnet term ($R^2_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$) is the only combination with the 2nd order eom in general dim.
- In d=4, the G-B term is topological (boundary term) hence nondynamical.

Lovelock's theorem (in dim =4 (& 3))

The Einstein eqns (w/ c.c.) are the only possible 2nd-order eqns derived in 4 dim. solely from the metric.

'Theory beyond Einstein' needs to relax the assumptions of Lovelock's theorem.

→ Adding a **new degree of freedom**

1) **Fields (scalars, etc.)** other than the metric

2) **Higher order curvatures** (G-B terms, etc. in $d \geq 5$)

Note :

- We want to keep the eom in 2nd order to avoid problems related to the unphysical d.o.f.s
- Horndeski theory (the most general scalar-metric tensor theory w/ 2nd-order field eqn in 4dim.) is classified by 4 arbitrary functions $\{G_i(\phi, X),\ i = 2,3,4,5\}$.
- We extend the theory by adding the Gauss-Bonnet term.
- In d=4, G-B with a scalar field coupling function belongs to the Horndeski theory .

Einstein Gauss-Bonnet Gravity

The general theory with quadratic curvature terms Λ in $d > 4$

$$S_{quad} = \int d^d x \sqrt{-g} \left[\frac{1}{2\kappa}\left(R - 2\Lambda + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\varrho\sigma}\right)\right]$$

Gauss-Bonnet term

$$R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

The e.o.m. doesn't include more than 2 derivatives only if $b = -4a$ & $c = a$, i.e., Gauss-Bonnet term .

1) the Einstein-Gauss-Bonnet (EGB) w/ or w/o Λ in $d \geq 5$

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g} \left[\frac{1}{2\kappa} (R - 2\Lambda + \alpha R_{GB}^2) + \mathcal{L}_m^{matt} \right]$$

Note
Λ: Cosmological constant
$\kappa = 8\pi G, \quad g = \det g_{\mu\nu}$
$[\alpha] =$ [length]^2

2) the Dilaton-Einstein-Gauss-Bonnet (DEGB) Gravity in $d = 4$

$f(\phi) = \alpha e^{\gamma\phi}$ polynomial etc.

$$S_{dEGB} = \int d^4 x \sqrt{-g} \left[\frac{1}{2\kappa} \left(R - 2\Lambda e^{\lambda\phi(r)} + f(\phi) R_{GB}^2\right) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathcal{L}_m^{matt} \right]$$

Note : belong to the Horndeski theory

Goal : To understand the physics of the role of the Gauss-Bonnet term
In the Simple Modified Gravity beyond Einstein.

Question: Is the modified theory better? ⇒ We investigate through

1) **Black Hole** properties &
2) the implication to the **cosmology**.

II. Black Holes

- Schwarzschild BH, AdS BH, RN AdS BH,
- RN Gauss-Bonnet AdS BH
- dEGB BH

Black Holes (in d-dim) (Review)

1. Einsten theory – Schwarzschild BH

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Sigma_k^{d-2}$$

$$f(r) = k - \frac{\mu}{r^{d-3}} \xrightarrow{d=4;k=1} 1 - \frac{\mu}{r} \quad (\mu > 0),$$

Note: Dimension (c=1)

$[S] = ML\,;\,[G] = \frac{L^{d-3}}{M}\,;$

$[\mu] = L^{d-3};$

Σ_k^{d-2}: Einstein mfld ($R_{ij} \propto h_{ij}$), codim.2 curvature $= k$

$$d\Sigma_k^{d-2} = h_{ij}(x)\,dx^i dx^j$$

Ex) $\Sigma_1^2 = S^2$; $\Sigma_0^2 = T^2$; $\Sigma_1^{d-2} = \int d^{d-2}x\sqrt{|h_{ij}|}$

$\Sigma_{-1}^2 = H^2$

$$d\Sigma_k^2 = \begin{cases} d\Omega_{d-2}^2 & \text{for } k = +1 \\ \Sigma\, dx_i^2 & \text{for } k = 0 \\ dH_{d-2}^2 & \text{for } k = -1 \end{cases}$$

ex) $(k = 1)$

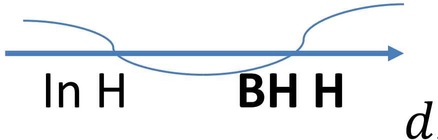
$$M = \frac{(d-2)\Sigma_1^{d-2}}{16\pi G}\mu = \frac{(d-2)\pi^{\frac{d-3}{2}}}{8G\Gamma[\frac{d-1}{2}]}\mu;$$

$$\Sigma_1^{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma[\frac{d-1}{2}]}$$

Ex) $d = 4$; $M = \frac{1}{2G}\mu$

$\Sigma_1^1 = 2\pi$; $\Sigma_1^2 = 4\pi$; $\Sigma_1^3 = 2\pi^2$

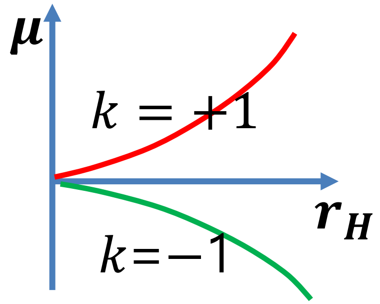
Horizon $(f(r_H) = 0)$& **(μ–r_H) relation**



$$\mu = k r_H^{d-3} \xrightarrow{d=4;k=1} r_H = 2GM$$

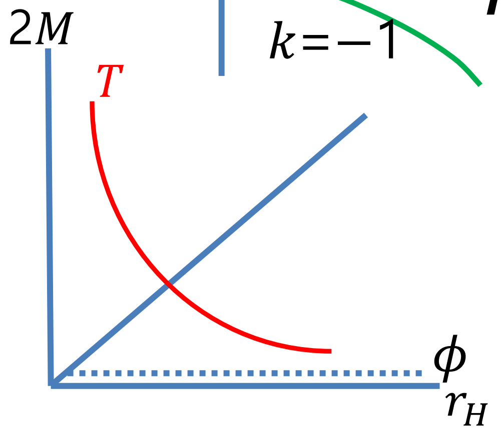
Note: BH exists only for $(k = +1)$

No minimum mass for BH



Hawking Temperature

$$T_H = \frac{\hbar\kappa_{SG}}{2\pi} = \frac{\hbar}{4\pi}f'(r_H) = \frac{\hbar}{4\pi}\frac{(d-3)\mu}{r_H{}^{d-2}} = \frac{\hbar}{4\pi}\frac{(d-3)k}{r_H} \xrightarrow{d=4;k=1} \frac{\hbar}{8\pi GM}$$



A **BH** in **asymp flat sp** is **thermally unstable**

$C_V = \frac{dM}{dT} = -\frac{1}{8\pi G}\frac{1}{T^2}$ **<0 : Unstable**

To make the BH thermodynamically stable,

1) Place the BH inside a finite cavity (a heat bath around the cavity.)

Or 2) Put the BH in AdS space (acting as a reflecting box.)

2. Schwarz **AdS Black Holes**

Black Hole solution

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Sigma_k^{d-2}$$

$$f(r) = k - \frac{\mu}{r^{d-3}} + \frac{r^2}{l^2}$$

Horizon $f(r_H) = 0$ & (μ–r_H) relation

$$\mu = r_H^{d-3}\left(k + \frac{r_H^2}{\ell^2}\right)$$

Note: BH exists for all k $(k = +1, 0, -1)$

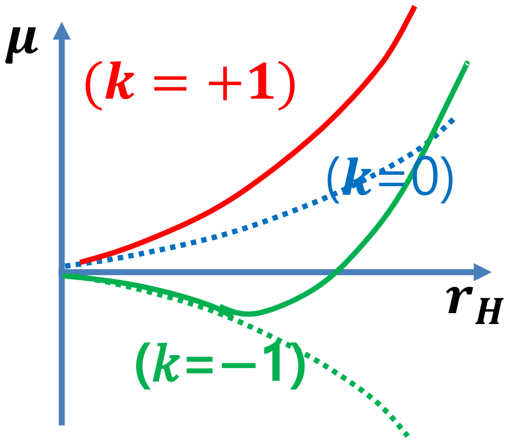
No minimum mass for BH

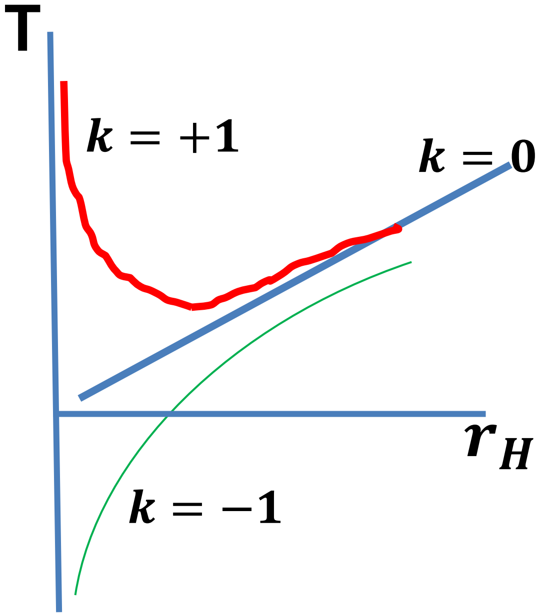
Hawking Temperature

$$T_H = \frac{1}{4\pi} f'(r_H) = \frac{1}{4\pi}\left((d-3)\frac{\mu}{r_H^{d-2}} + 2\frac{r_H}{\ell^2}\right)$$

$$= \frac{1}{4\pi}\left(\frac{k(d-3)}{r_H} + (d-1)\frac{r_H}{\ell^2}\right)$$

Note: Natural reference scale : ℓ (in addition to μ).





Note: Dimension (c=1)

$[S] = ML\,;\, [G] = \frac{L^{d-3}}{M}\,;$

$[\mu] = L^{d-3};\, [\ell^2] = L^2$

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$$

Note :

1) Two branches:
   Small BH ($r_H \ll \ell$): unstable
   Large BH ($r_H \gg \ell$) : stable.

2) Horizon geometry can be
   sphere $(k = +1)$,
   plane $(k = 01)$,
   or hyperbolic $(k = -1)$.

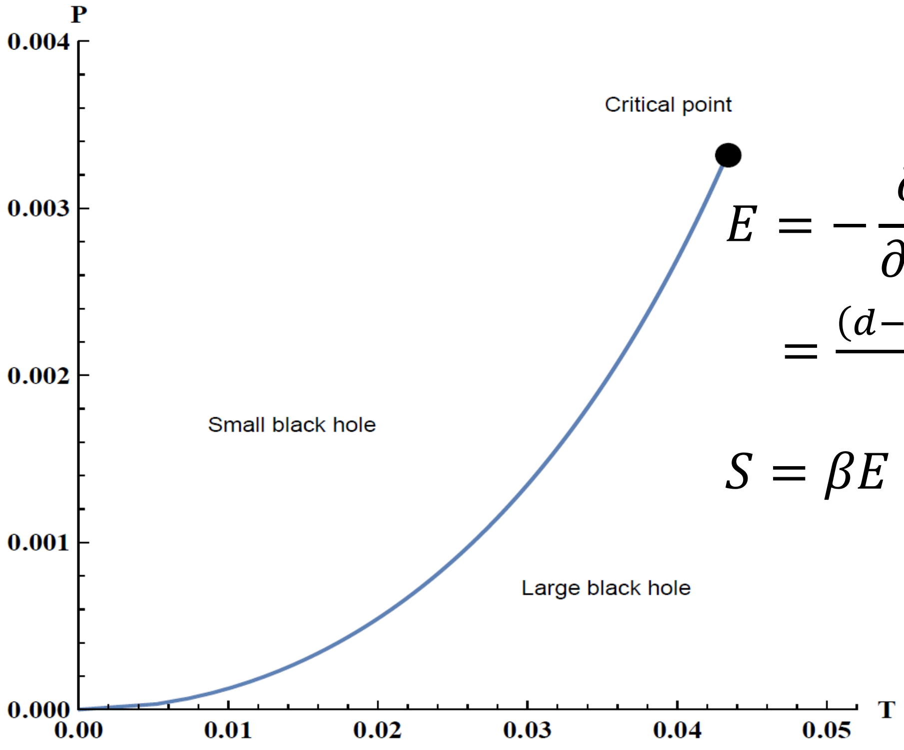
3) For $k = +1$, (Schw. AdS BH)
   - $T \geq T_0 = \frac{\sqrt{2}}{\pi\ell}$,
   - Hawking-Page Tr.

$$-\ln Z = I_{Euc} = \beta F$$

$$I_{Euc} = \frac{1}{16\pi G}\int d^d x\,\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{\ell^2}\right]$$

$$= \frac{\Sigma^1_{d-2}}{4G}\,\frac{\ell^2 r_H^{d-2} - r_H^d}{(d-1)r_H^2 + (d-3)\ell^2}$$

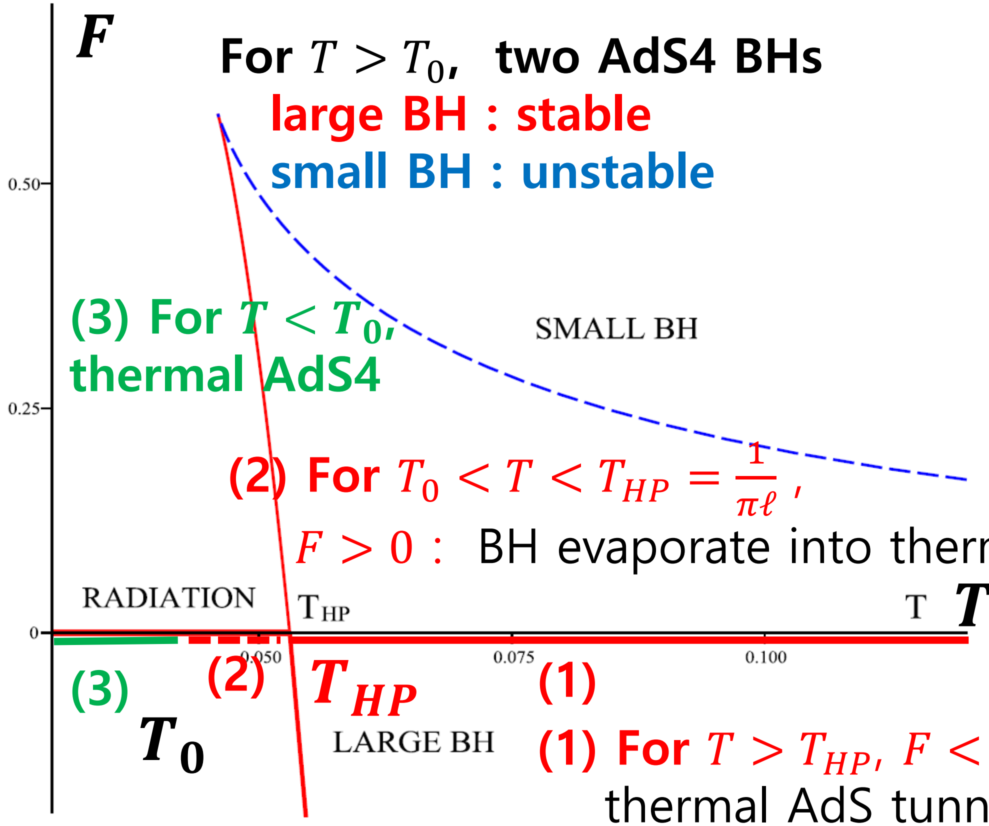
$$Z[\beta] = \int [dg][d\Phi_{matter}]\, e^{-I_{Euc}} = e^{-\beta F}$$



$$E = -\frac{\partial}{\partial\beta}\ln Z = \frac{\partial}{\partial\beta}\beta F = \frac{\partial I_{Euc}}{\partial\beta}$$

$$= \frac{(d-2)\Sigma^1_{d-2}(\ell^{-2} r_H^{d-1} + r_H^{d-3})}{16\pi G} = M$$

$$S = \beta E - I = \frac{\Sigma^1_{d-2} r_H^{d-2}}{4G} = \frac{A}{4G}$$



For $T > T_0$, two AdS4 BHs

large BH : stable

small BH : unstable

(3) For $T < T_0$, thermal AdS4

(2) For $T_0 < T < T_{HP} = \frac{1}{\pi\ell}$,

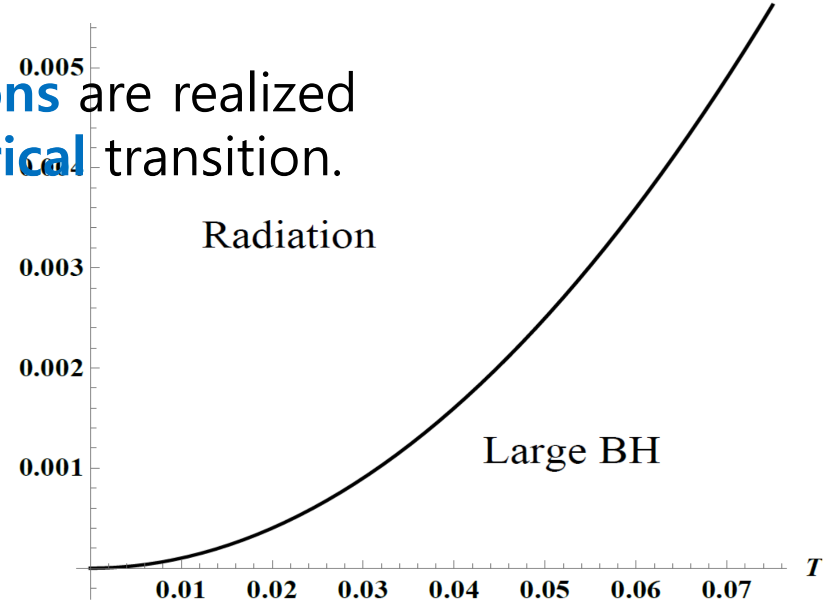
$F > 0$: BH evaporate into thermal AdS

(3) (2) T_{HP} (1)

T_0

(1) For $T > T_{HP}$, $F < 0$ (large BH): thermal AdS tunnels into BH

Phase Transitions are realized as the **geometrical** transition.



3. RNAdS Black Holes

$$2\Lambda = -\frac{(d-1)(d-2)}{\ell^2}$$

$$2\Lambda \xrightarrow{d=4} -\frac{6}{\ell^2} \xrightarrow{d=5} -\frac{12}{\ell^2}$$

Note: Dimension (c=1)

$$[S] = ML\,;\,[G] = \frac{L^{d-3}}{M};$$

$$[\mu] = L^{d-3};\,[\ell^2] = L^2;$$

Black Hole solution

$$f(r) = k - \frac{\mu}{r^{d-3}} + \frac{d-3}{2}\frac{q^2}{r^{2(d-3)}} + \frac{r^2}{l^2} \qquad ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2 d\Sigma_k^{d-2}$$

$$A = \left(-\frac{1}{c}\frac{q}{r^{d-3}} + \Phi\right)dt \qquad \text{(gauge choice) } A(r_H) = 0$$

$$c = \sqrt{\frac{2(d-3)}{d-2}} \quad \text{and } \Phi = \frac{1}{c}\frac{q}{r_H^{d-3}} \qquad \text{(Electric field)} \quad E(r) = \frac{Q}{r^{(d-2)}}$$

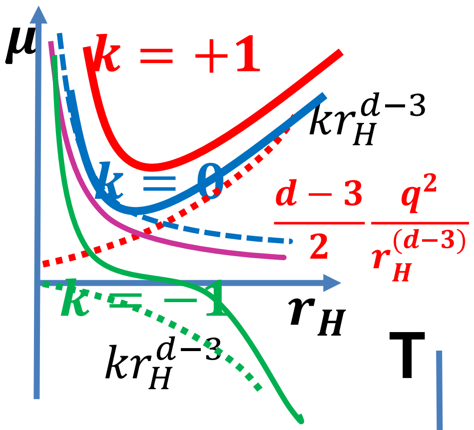
$$[q^2] = L^{2(d-3)}$$

$$\frac{[Q^2]}{[g^2]} = ML^{d-3};$$

$$\frac{[GQ^2]}{[g^2]} = [q^2]$$

Horizon $f(r_H) = 0$ Horizon-Mass (μ–r_H) relation

$$\mu = r_H^{d-3}\left(k + \frac{d-3}{2}\frac{q^2}{r_H^{2(d-3)}} + \frac{r_H^2}{\ell^2}\right) = k r_H^{d-3} + \frac{d-3}{2}\frac{q^2}{r_H^{(d-3)}} + \frac{r_H^{d-1}}{\ell^2}$$

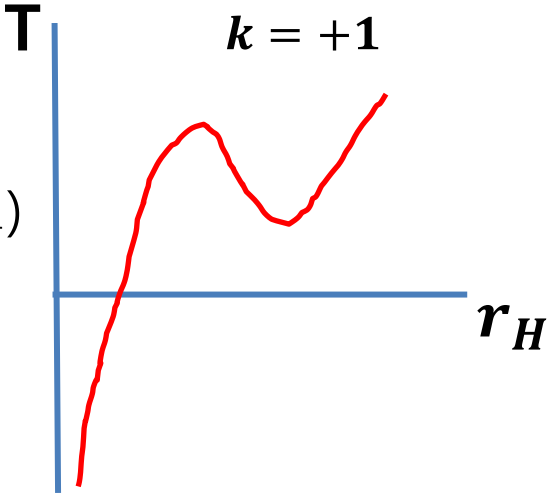


Extremal solution $f(r_{ex}) = \left.\frac{df}{dr}\right|_{r=r_{ex}} = 0$

$$\mu_{ex} = 2r_{ex}^{d-3}\left(k + \frac{(d-2)r_{ex}^2}{(d-3)\ell^2}\right) \quad q_{ex}^2 = \frac{2r_{ex}^{2(d-3)}}{(d-3)^2}\left(\frac{(d-1)r_{ex}^2}{\ell^2} + (d-3)k\right)$$

Note

$\mu \geq \mu_{ex}$ $(k = +1)$

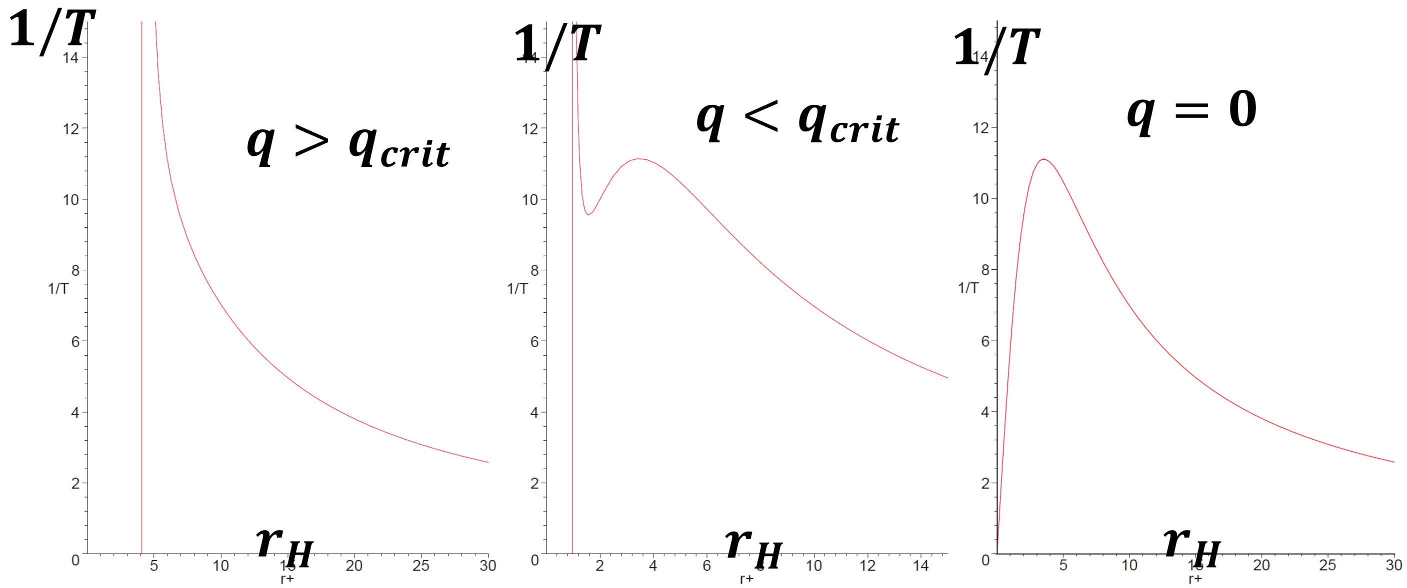


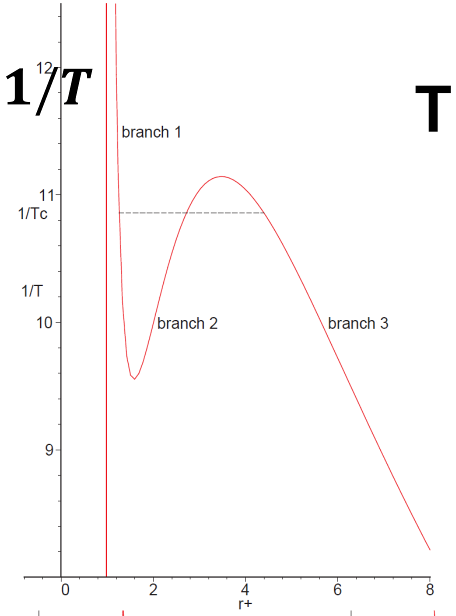
Hawking Temperature

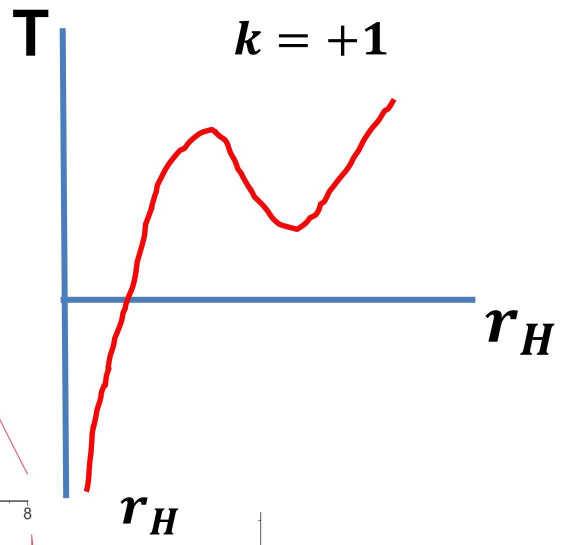
$$T_H = \frac{1}{4\pi}f'(r_H) = \frac{1}{4\pi}\left((d-3)\frac{\mu}{r_H^{d-2}} - (d-3)^2\frac{q^2}{r_H^{2d-5}} + 2\frac{r_H}{\ell^2}\right)$$

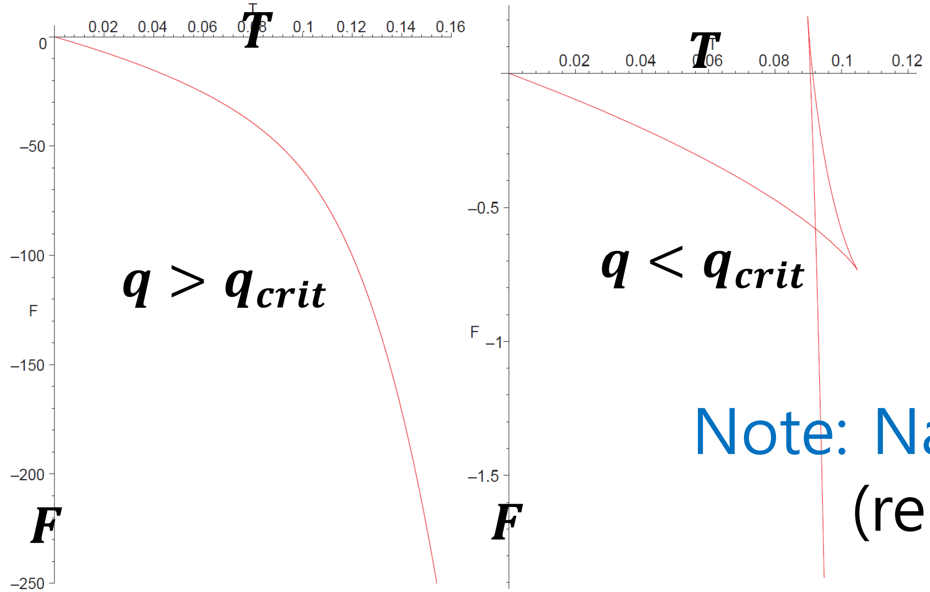
$$= \frac{1}{4\pi}\left(\frac{(d-3)k}{r_H} - \frac{(d-3)^2}{2}\frac{q^2}{r_H^{2d-5}} + (d-1)\frac{r_H}{\ell^2}\right) = \frac{k(d-3)\ell^2 r_H^{2d-6} - \frac{(d-3)^2}{2}q^2\ell^2 + (d-1)r_H^{2d-4}}{4\pi\ell^2 r_H^{2d-5}}$$

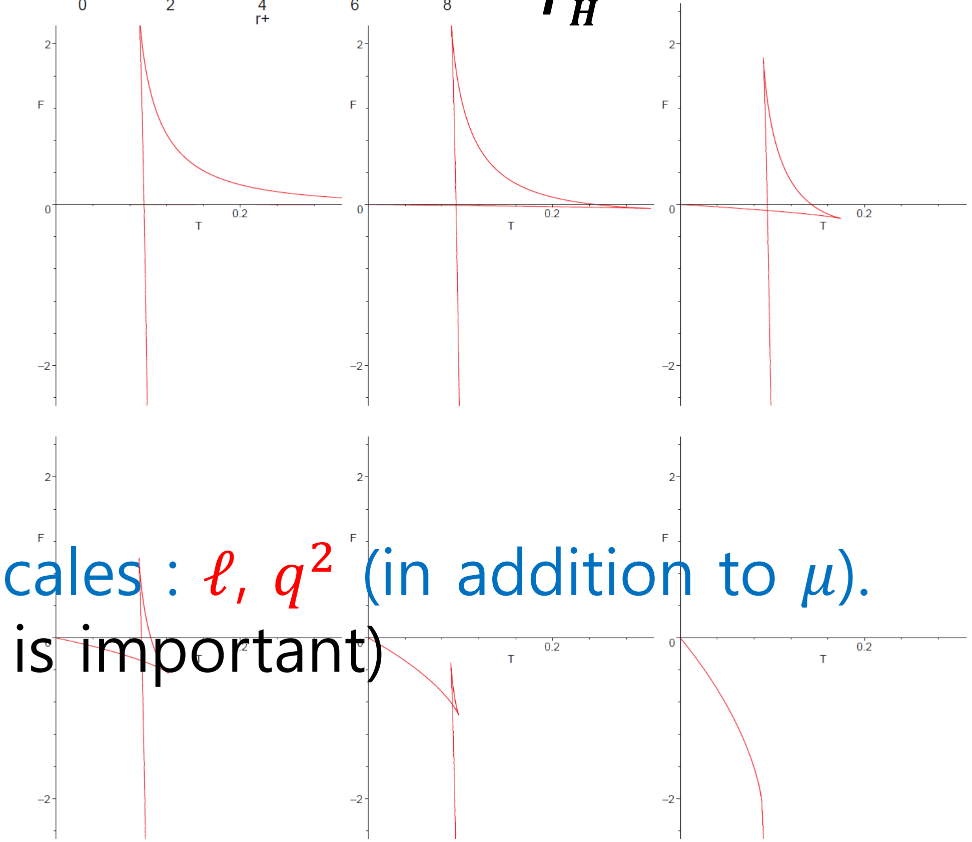
RN**AdS** : **Thermodynamics** $dM = TdS + \Phi dQ$











Note: Natural reference scales : ℓ, q^2 (in addition to μ).

(relative magnitude is important)

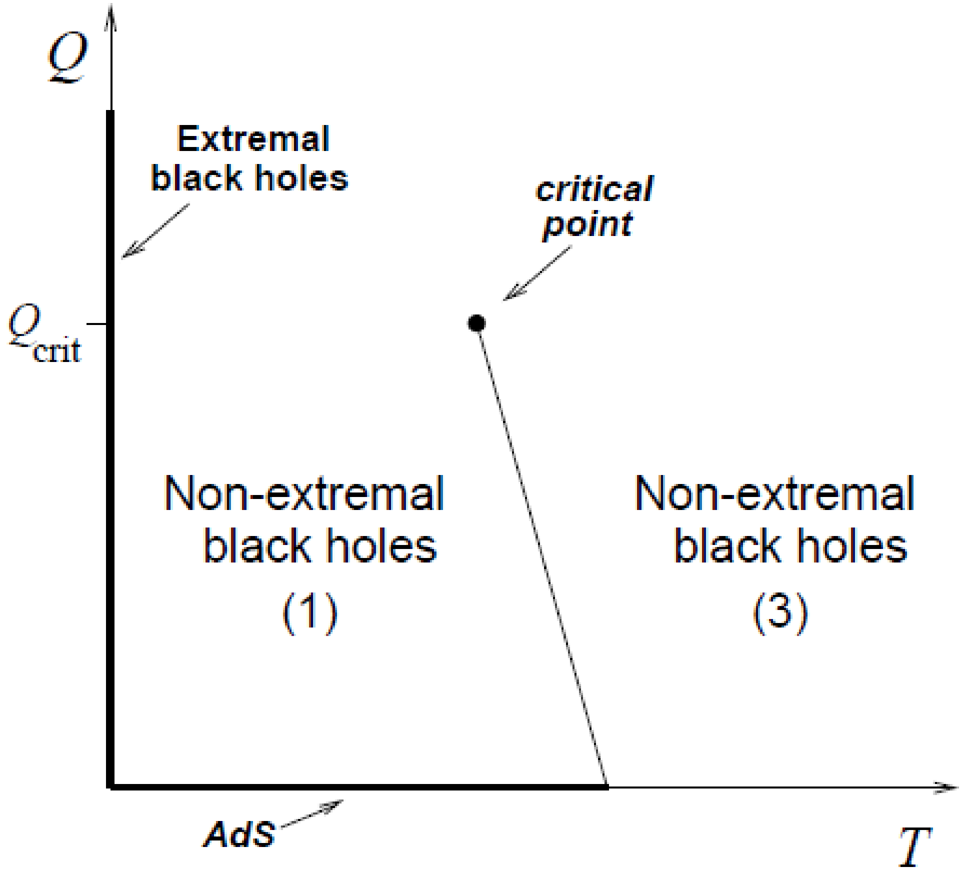
Chamblin Emparan Johnson Myers 1999 PRD

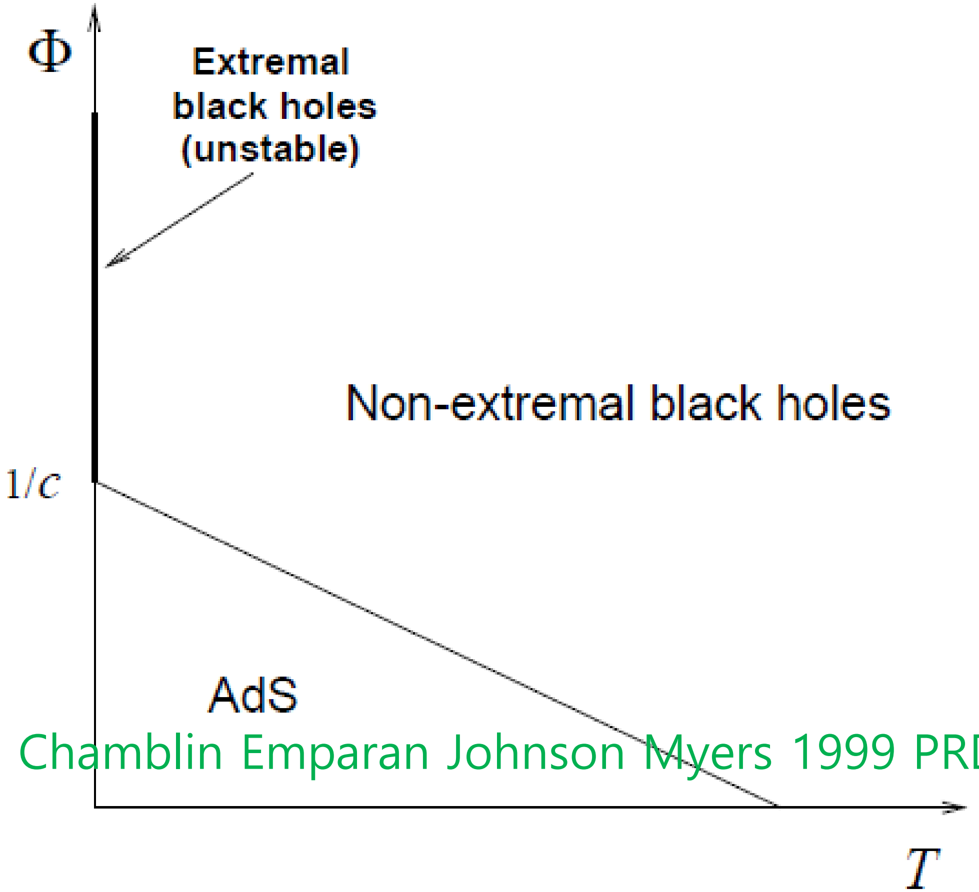
Thermodynamics RN AdS BH

Euclidean Action

$$I_E = \frac{1}{16\pi G}\int_{\mathcal{M}} d^d x\,\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{\ell^2}\right] + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^{d-1}x\,\sqrt{-h}\mathcal{K} - I_{\text{subtr}}$$

Evaluate other thermodynamic quantities, such as energy, entropy, etc.





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RN GB-AdS BH

D. RNAdS in Einstein-Gauss-Bonnet

R. -G. Cai, PRD (2002)   Wei, Liu, PRD (2013)   T.Torii, H.Maeda (2005)

Action

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R + \frac{(d-1)(d-2)}{l^2} + \alpha_{GB} R_{GB}^2\right)\right] + S_{matter}$$

$$S_{matter} = -\frac{1}{4\pi g^2}\int d^d x \sqrt{-g}\, F_{\mu\nu}F^{\mu\nu}$$

$\kappa = 8\pi G,$

$g = \det g_{\mu\nu};$

Note: Dimension (c=1)

$[S] = ML\,; [G] = \frac{L^{d-3}}{M};\ [\ell^2] = L^2 = [\alpha_{GB}];$

$[\mu] = L^{d-3};\ [q^2] = L^{2(d-3)}$

Note

AdS$_d$ limit $\tilde{\alpha} \to 0$: well defined at the action level.

Ansatz

$$ds^2 = -f(r)\, dt^2 + f^{-1}(r)\, dr^2 + r^2 d\Sigma_k^2$$

Σ_k^{d-2}: Einstein mfld (codim.2)

$(R_{ij} = (d-3)k h_{ij}$, curvature $= k$

$$d\Sigma_k^{d-2} = h_{ij}(x)\, dx^i dx^j$$

$$\Sigma_k^{d-2} = \int d^{d-2}x \sqrt{|h_{ij}|}$$

$$d\Sigma_k^2 = \begin{cases} d\Omega_{d-2}^2 & \text{for } k = +1 \ sphere \\ \Sigma\, dx_i^2 & \text{for } k = 0 \ plane \\ dH_{d-2}^2 & \text{for } k = -1 \ hyperspace \end{cases}$$

$$\Sigma_1^{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma[\frac{d-1}{2}]}$$

Ex)Σ_k^2: $(d = 4)$

$\Sigma_1^2 = S^2;\ \Sigma_0^2 = T^2; \Sigma_{-1}^2 = H^2$

$\Sigma_1^1 = 2\pi;\ \Sigma_1^2 = 4\pi;\ \Sigma_1^3 = 2\pi^2$

Eqns of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda\, g_{\mu\nu} = \kappa T_{\mu\nu} - \alpha_{GB}H_{\mu\nu} \equiv T_{\mu\nu}^{tot} \quad \text{(Einstein Eq.)}$$

$$H_{\mu\nu} = 2\left(RR_{\mu\nu} - 2R_{\mu\alpha}R^{\alpha}{}_{\nu} - 2R^{\alpha\beta}R_{\mu\nu\alpha\beta} + R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\right) - \frac{1}{2}g_{\mu\nu}R_{GB}^2$$

$$\nabla_\alpha F^{\alpha\mu} = 0 \quad \text{(Maxwell Eq.)}$$

RN GB AdS BH solution

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}}\left(1 \mp \sqrt{1 - 4\tilde{\alpha}\left[-\frac{\mu}{r^{d-1}} + \frac{(d-3)}{2}\frac{q^2}{r^{2(d-2)}} + \frac{1}{\ell^2}\right]}\right)$$

$$A(r) = \left(-\frac{1}{c}\frac{q}{r^{d-3}} + \Phi\right)dt \qquad c = \sqrt{\frac{2(d-3)}{d-2}} \quad \text{and } \Phi = \frac{1}{c}\frac{q}{r_H^{d-3}}$$

$$\tilde{\alpha} = (d-3)(d-4)\alpha_{GB}$$

Note: mass M & charge Q

$$\mu = \frac{16\pi G}{(d-2)\Sigma_k^{d-2}} M$$

$$q^2 = \frac{8\pi G}{2(d-2)\pi g^2} Q^2 \ or$$

$$Q^2 = \frac{\pi(d-2)(d-3)\left(1-\frac{4\alpha}{\ell^2}\right)}{2G\alpha} q^2$$

Ex) $d = 4 \Longrightarrow \mu = 2GM, q = GQ$

$d = 5 \Longrightarrow \mu = \frac{8G}{3\pi} M, q = \frac{2G}{\sqrt{3}\pi} Q$

Note

(Upper sign branch)

$$f(r) \xrightarrow{\tilde{\alpha}\to 0} k - r^2\left[\frac{\mu}{r^{d-1}} - \frac{(d-3)}{2}\frac{q^2}{r^{2(d-2)}} - \frac{1}{\ell^2}\right]$$

$$\xrightarrow{\tilde{\alpha}\to 0} k - \frac{\mu}{r^{d-3}} + \frac{(d-3)}{2}\frac{q^2}{r^{2(d-3)}} + \frac{r^2}{\ell^2}$$

$= f(r)$ in RN AdS BH

(Lower sign branch)

$$\xrightarrow{\tilde{\alpha}\to 0} k + \frac{\mu}{r^{d-3}} - \frac{(d-3)}{2}\frac{q^2}{r^{2(d-3)}} - \frac{r^2}{\ell^2} + \frac{r^2}{\tilde{\alpha}}$$

$$\xrightarrow{\tilde{\alpha}\to 0} \infty$$

Branch singularity $\sqrt{} = 0$; $(\mu\text{–}r_b)$ relation

$$\mu = -\left(1 - \frac{4\tilde{\alpha}}{\ell^2}\right)\frac{r_b^{d-1}}{4\tilde{\alpha}} + \frac{(d-3)}{2}\frac{q^2}{r_b^{d-3}}$$

$r_b \searrow$ as $\mu \nearrow$;

$r_b \to 0$ as $\mu \to \infty$

∃ the branch singularity

- only for $\mu < 0$ if $Q = 0$,
- always if $Q \neq 0$.

$$f(r) \approx \left(k + \frac{r_b^2}{2\tilde{\alpha}}\right) \mp \frac{r_b^2}{2\tilde{\alpha}}\sqrt{\frac{d-1}{r_b}\left(1 - \frac{4\tilde{\alpha}}{\ell^2}\right) + \frac{2(d-3)^2\tilde{\alpha}q^2}{r_b^{2d-3}}}(r - r_b)^{1/2}$$

Kretschmann invariant

$$I \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \mathcal{O}\left((r - r_b)^{-3}\right)$$

Horizon $f(r_H)=0$

R. -G. Cai, PRD (2002).

$$\pm\left(1+\frac{2\tilde{\alpha}k}{r_H^2}\right)=\sqrt{1+\frac{4\tilde{\alpha}}{r_H^2}\left(\frac{\mu}{r_H^{d-3}}-\frac{(d-3)q^2}{2r_H^{2(d-3)}}-\frac{r_H^2}{\ell^2}\right)}$$

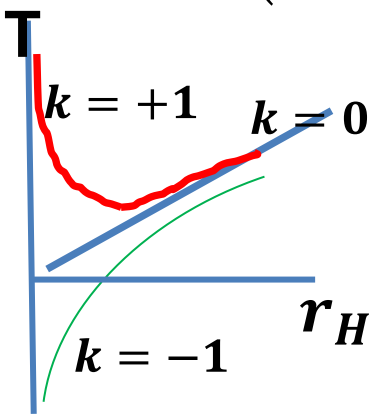
$$\mu=r_H^{d-3}\left\{k+\frac{\tilde{\alpha}k^2}{r_H^2}+\frac{(d-3)}{2}\frac{q^2}{r_H^{2(d-3)}}+\frac{r_H^2}{\ell^2}\right\}$$

$$f(r)=k+\frac{r^2}{2\tilde{\alpha}}\left(1\mp\sqrt{1+\frac{4\tilde{\alpha}}{r^2}\left(\frac{\mu}{r^{d-3}}-\frac{(d-3)q^2}{2\,r^{2(d-3)}}-\frac{r^2}{\ell^2}\right)}\right)$$

(Upper sign + branch $r_H^2-2\tilde{\alpha}k<0$,

(Lower sign − branch) $r_H^2-2\tilde{\alpha}k>0$,

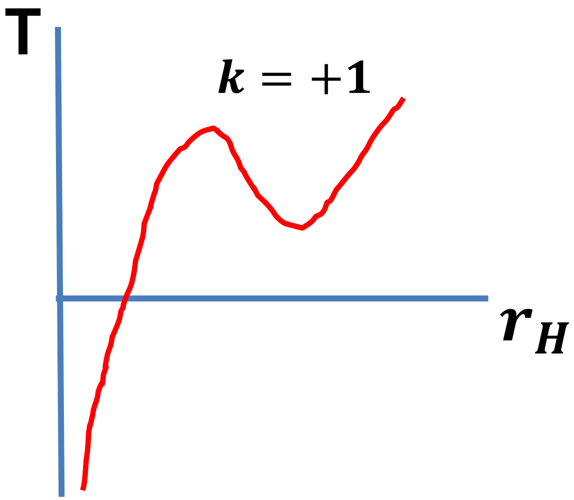
$$f'(r)=\frac{r}{\tilde{\alpha}}\left(1\mp\sqrt{1+\frac{4\tilde{\alpha}}{r^2}\left(\frac{\mu}{r^{d-3}}-\frac{(d-3)q^2}{2\,r^{2(d-3)}}-\frac{r^2}{\ell^2}\right)}\right)\mp\frac{\left[-\frac{\mu(d-1)}{r^{d-3}}+(d-3)(d-2)\frac{q^2}{r^{2(d-3)}}\right]}{r\sqrt{1+\frac{4\tilde{\alpha}}{r^2}\left(\frac{\mu}{r^{d-3}}-\frac{(d-3)q^2}{2\,r^{2(d-3)}}-\frac{r^2}{\ell^2}\right)}}$$



Hawking Temperature

$$T_H=\frac{1}{4\pi}f'(r_H)$$

$$=\frac{1}{4\pi}\frac{1}{r_H\left(1+\frac{2\tilde{\alpha}k}{r_H^2}\right)}\left\{-(d-3)^2\frac{q^2}{r_H^{2(d-3)}}+(d-5)\frac{\tilde{\alpha}k^2}{r_H^2}+(d-3)k+(d-1)\frac{r_H^2}{\ell^2}\right\}$$



AdS$_d$ limit $\tilde{\alpha}\,\&\,q\to 0$

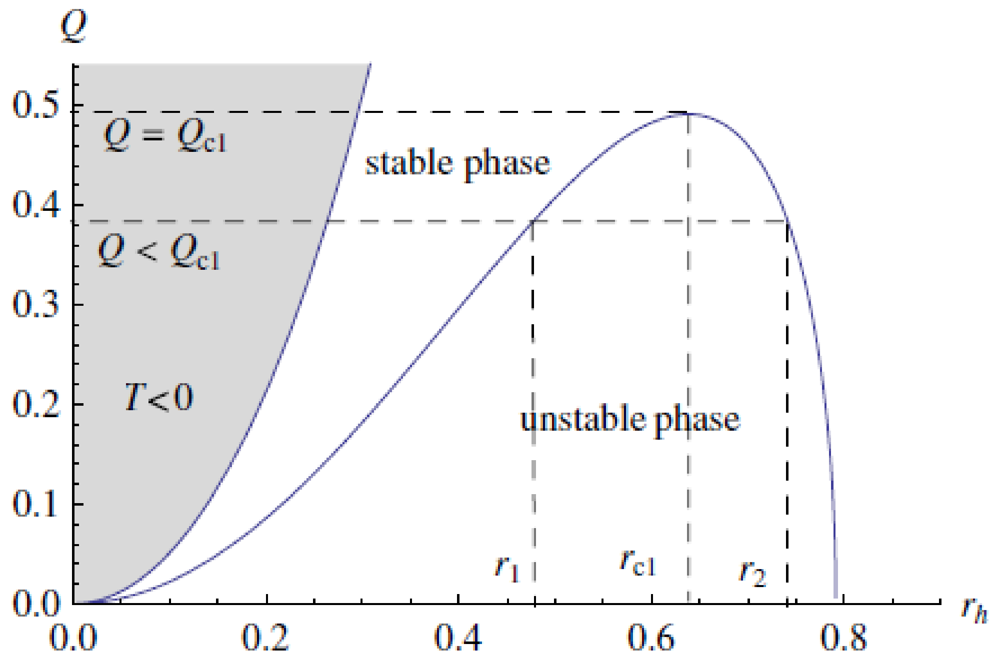
$$T_H=\frac{1}{4\pi r_H}\left((d-3)k+(d-1)\frac{r_H^2}{\ell^2}\right)$$

RN AdS BH $\tilde{\alpha}\to 0$

$$T_H=\frac{1}{4\pi r_H}\left\{-(d-3)^2\frac{q^2}{r_H^{2(d-3)}}+(d-3)k+(d-1)\frac{r_H^2}{\ell^2}\right\}$$

RNAdS in Einstein-Gauss-Bonnet : Phases

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}}\left(1 \mp \sqrt{1-\frac{4\alpha}{\ell^2}}\sqrt{1+\frac{\mu}{r^{d-1}}-\frac{q^2}{r^{2(d-2)}}}\right)$$



Note : $Q^2 = \frac{\pi(d-2)(d-3)\left(1-\frac{4\alpha}{\ell^2}\right)}{2G\alpha}q^2$

Wei & Liu, PRD (2013) mass

$$M = \frac{(d-1)Q^2 r_H^8 + 2\pi r_H^{2d}(d-3)\left((d^2-3d+2)(kr_H^2+k^2\alpha)-2\Lambda r_H^4\right)}{8\pi^2(d^2-4d+3)r_H^{d+5}}\Sigma_{\mathrm{d}-2}^k$$

Hawking Temperature

$$T_H = \frac{1}{4\pi}f'(r_H) = \frac{-Q^2 r_H^8 + 2\pi r_H^{2d}\left((d-2)k\left((d-3)r_H^2+(d-5)k\alpha\right)-2\Lambda r_H^4\right)}{32\pi^2(d-2)r_H^{2d+1}(2k\alpha+r_H^2)}$$

Near Extremal behavior etc.
I. Jeon, BHL, W. Lee, M. Mishra,
To appear in PRD

4. dEGB theory - Black Holes

Guo,Ohta & Torii, Prog.Theor.Phys. (2008); (2009); (2010); Maeda,Ohta Sasagawa, PRD(2009);(2011) Ohta Torii, PRD (2013).

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda e^{\lambda\phi(r)} + f(\phi)R^2_{GB}\right) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}^{matt}_m\right]$$

BHL, W. Lee, D. Rho, PRD (2019)

Equations of motion

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} = \kappa\left(T^{\phi}_{\mu\nu} + T^{GB}_{\mu\nu}\right)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}\, g^{\mu\nu}\partial_\nu\Phi] = \alpha\gamma e^{-\gamma\phi(r)}R^2_{GB}$$

$$T^{GB}_{\mu\nu} = 4\left(\nabla_\mu\nabla_\nu f(\Phi)\right)R - 4g_{\mu\nu}(\nabla^2 f(\Phi))R$$
$$-8\left(\nabla_\rho\nabla_\mu f(\Phi)\right)R^\rho_\nu - 8\left(\nabla_\rho\nabla_\nu f(\Phi)\right)R^\rho_\mu + 8(\nabla^2 f(\Phi))R_{\mu\nu}$$
$$+8g_{\mu\nu}\left(\nabla_\rho\nabla_\sigma f(\Phi)\right)R^{\rho\sigma} - 8(\nabla^\rho\nabla^\sigma f(\Phi))R_{\mu\nu\rho\sigma}$$

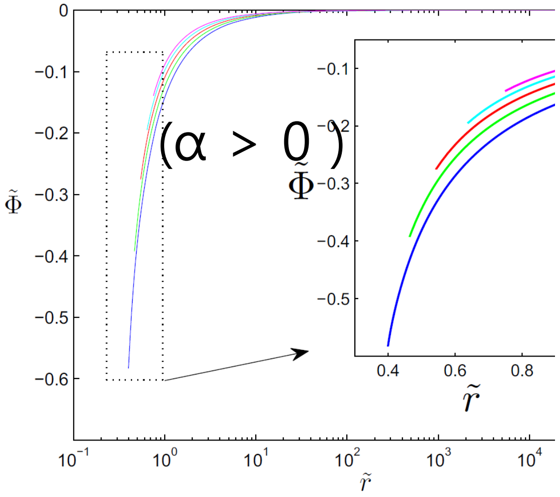
Note :

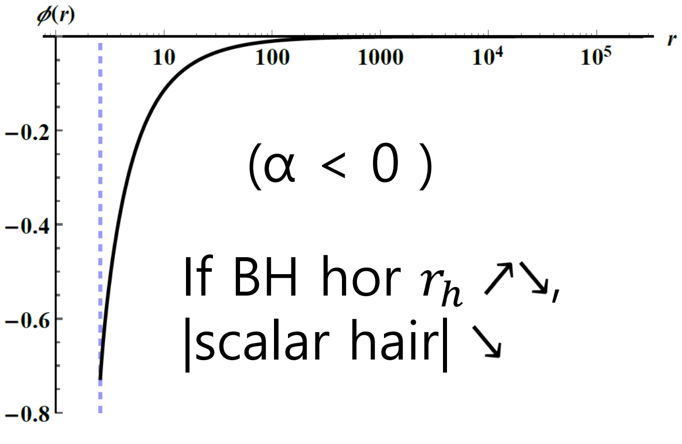
1) For $\alpha = 0$ (or $\gamma = 0$), DEGB theory becomes the Einstein theory.
2) α scaling :The α could be absorbed by the r → r/√α. Sign of α is important
3) We may treat the Gauss-Bonnet term as "matter" which is a source of the metric.
4) The effect of the G-B term is expected stronger for smaller r region, and negligible as $r \to \infty$

New Properties of the Black Holes

1) Scalar Hair - BH hair ↘ as M ↗

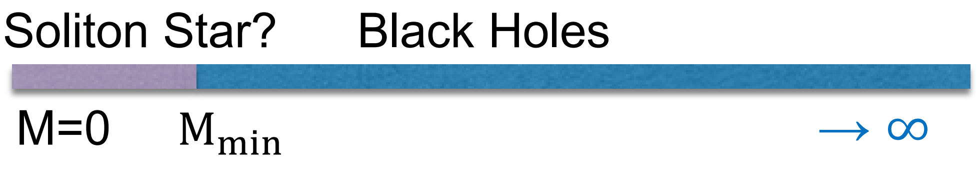
- All DEGB BHs have hairs.
- If Φ = 0, e.o.m. impose $R^2_{GB} = 0$.
- (consistent with the no hair theorem).
- Hair Charge is dependent : 2ndary charge.



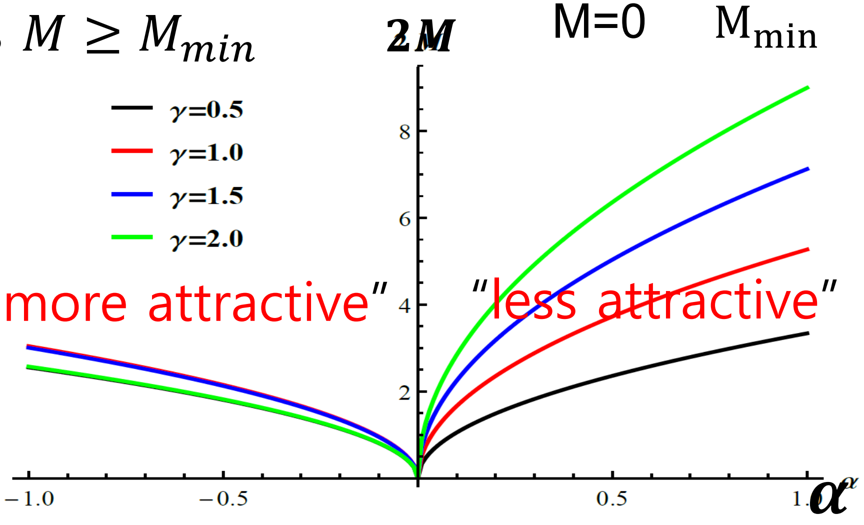


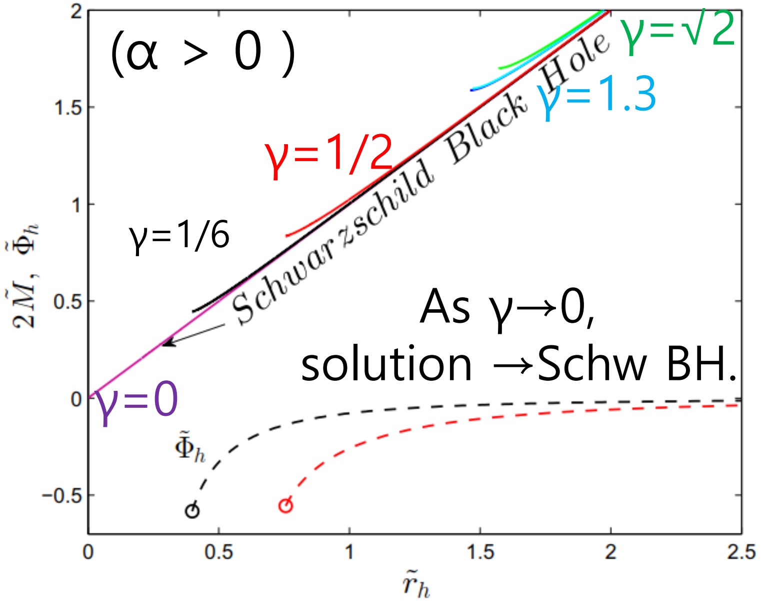
(b) $\phi(r)$ vs. r

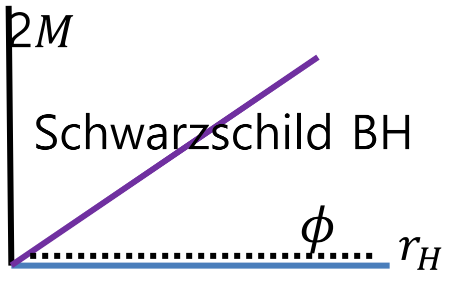
2) Minimum Mass



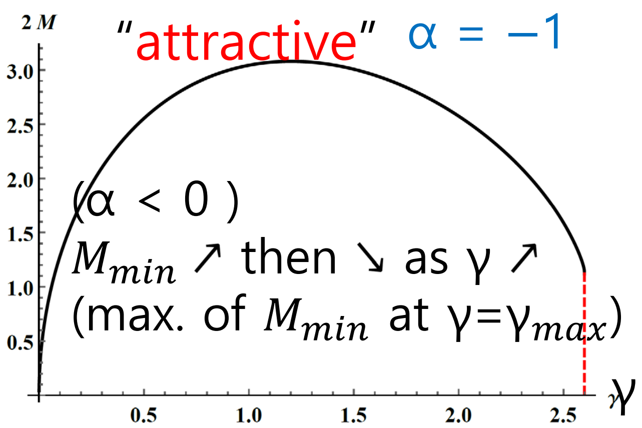
BH mass $M \geq M_{min}$

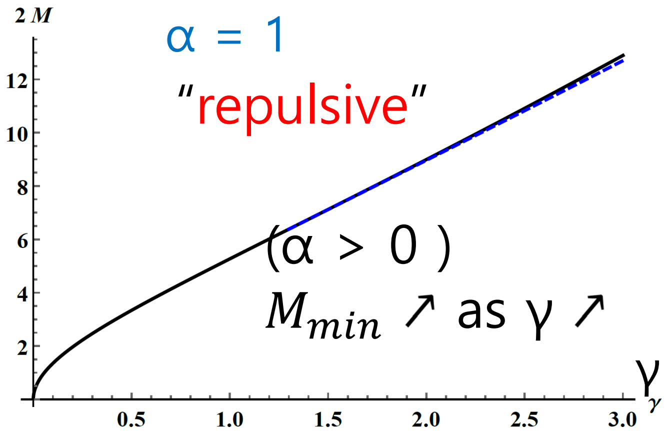


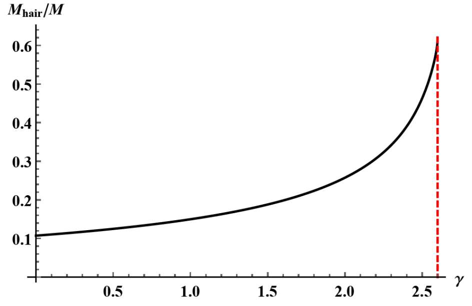




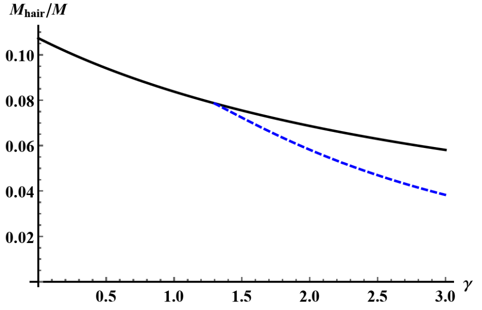
M_{min} vs. γ







(a) The hairy mass ratio vs. γ with α = −1.



(b) The hairy mass ratio vs. γ with α = 1.

GB term → makes gravity "less attractive" (for α >0) (making the black hole "smaller") !!!

The BH properties strongly depends on the sign of α.

Q: minimum mass → New Phase?

With Cosmological Constant :

BHL, H. Lee, W. Lee, in prepation

S. KHIMPHUN, BHL, W. LEE PRD(2016)

$$S = \int dx^4 \sqrt{-g}\left(\frac{R + 2\Lambda e^{\lambda\Phi(r)}}{2\kappa} - \frac{1}{2}\nabla^{\mu}\Phi\nabla_{\mu}\Phi + f(\Phi)R_{GB}^2\right)$$

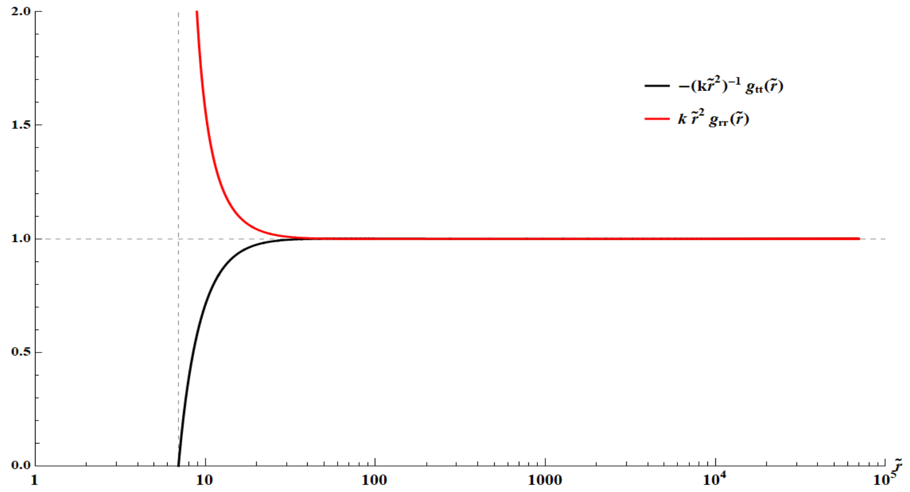
$$\kappa = 1$$
$$\alpha = 1$$
$$r_h = 1$$

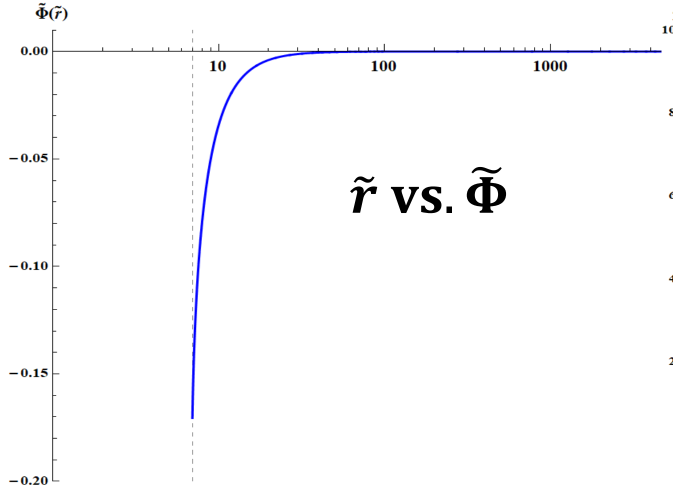
Negative cosmo const w/ G-B

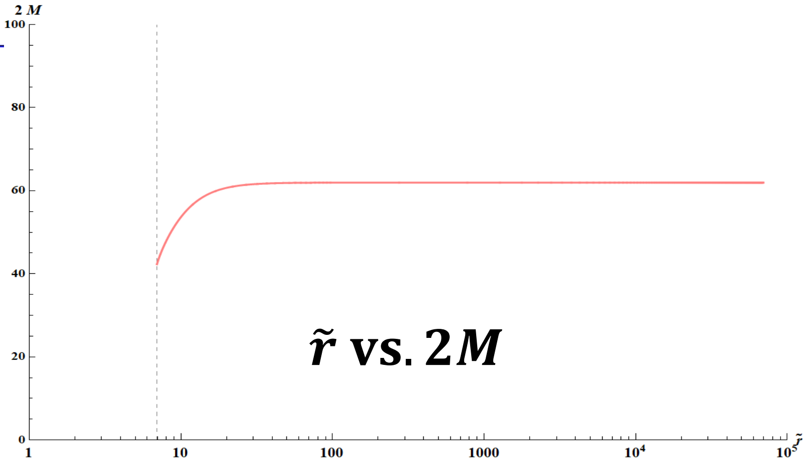
$$f(\Phi) = \alpha e^{\gamma\Phi(r)}$$

Remind: $\gamma + \lambda = 0, \quad \Lambda = \frac{3\lambda}{8\kappa\alpha\gamma} e^{-(\gamma+\lambda)\Phi_{\infty}} \Longrightarrow \Phi(r) = \Phi_{\infty}$ **(Constant)**

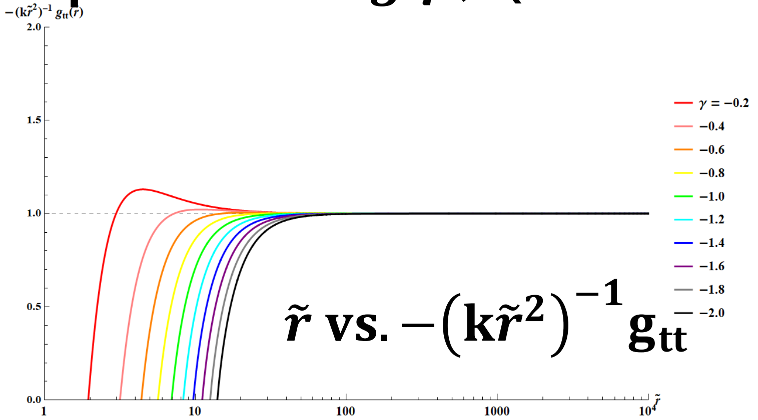
Example 1: $\gamma = -1$ **(with γ+λ = 0, Λ = -3/8κα= - 0.375)**

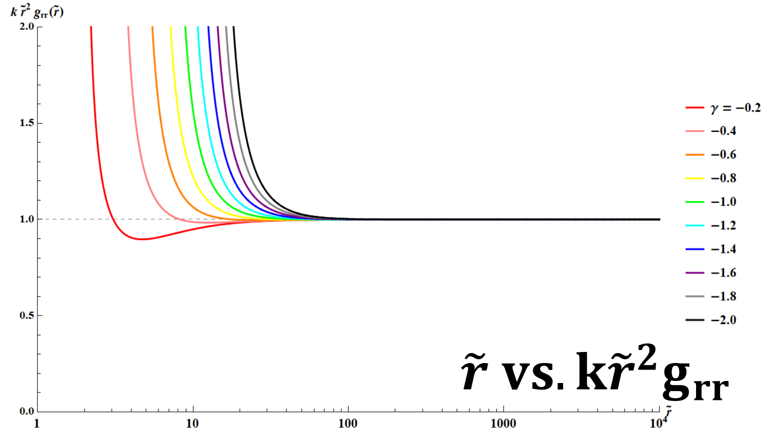


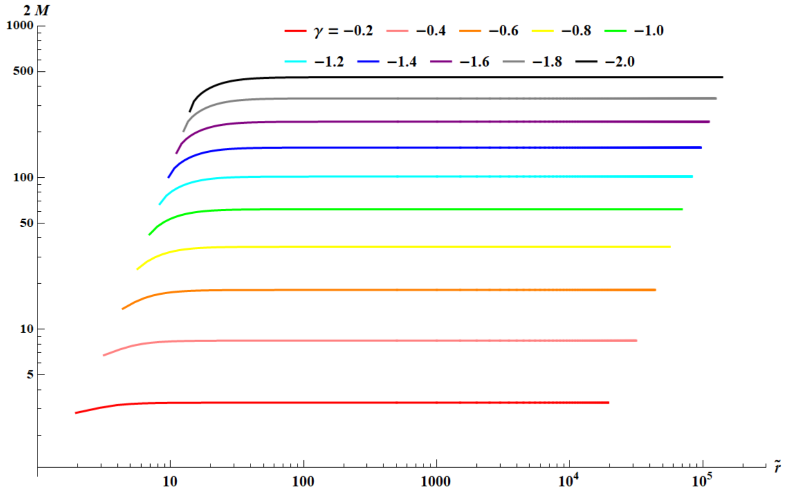


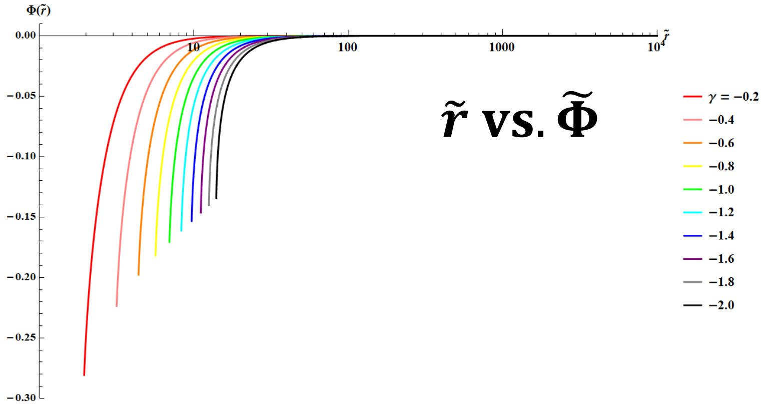


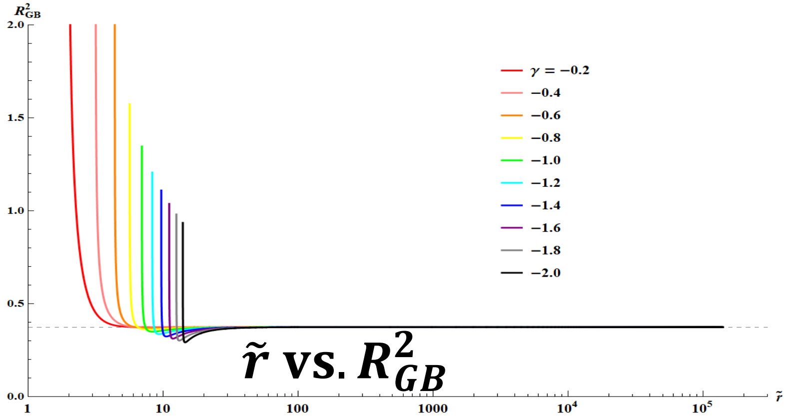
Example 2: Varing γ , (while keeping γ+λ = 0, Λ = -3/8κα)

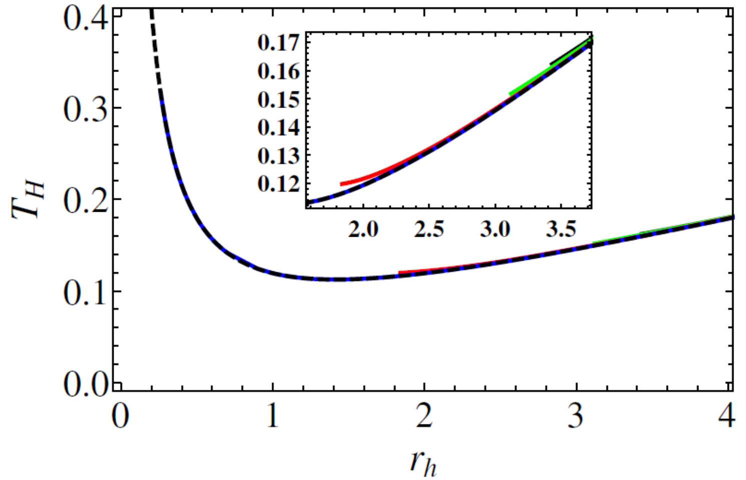


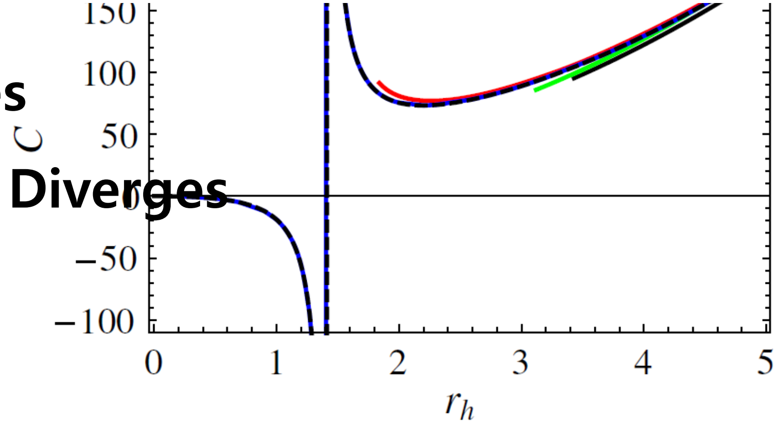












Example 3: γ+λ ≠ 0 $\left(\text{with } \Lambda = \frac{3\lambda}{8\kappa\alpha\gamma}\right)$ γ+λ ≠ 0 Φ Diverges

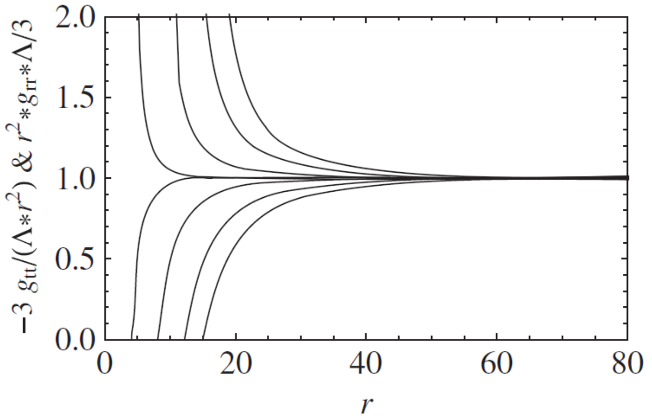
Example 4: $\gamma = -1$, γ+λ = 0, varying Λ ≠ -3/8κα ➡ Φ Diverges

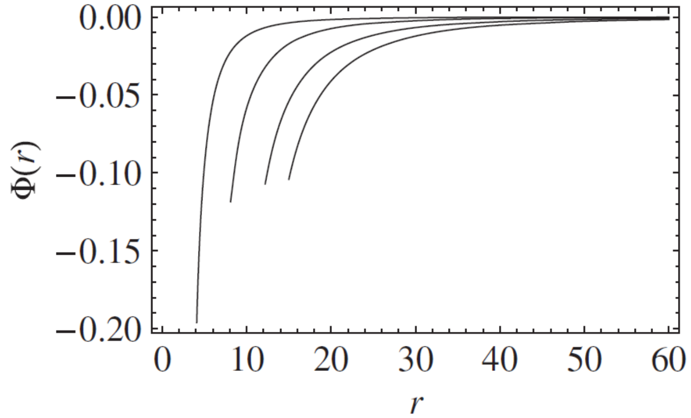
Note: BH sol w/ finte Φ_∞ for +tive c.c. is har to get.

Negative cosmological constant with Gauss-Bonnet term : Phases

S. KHIMPHUN, BHL, W. LEE PRD(2016)

α = 1.0 γ=1/2, Λ=-1/2, κ=1



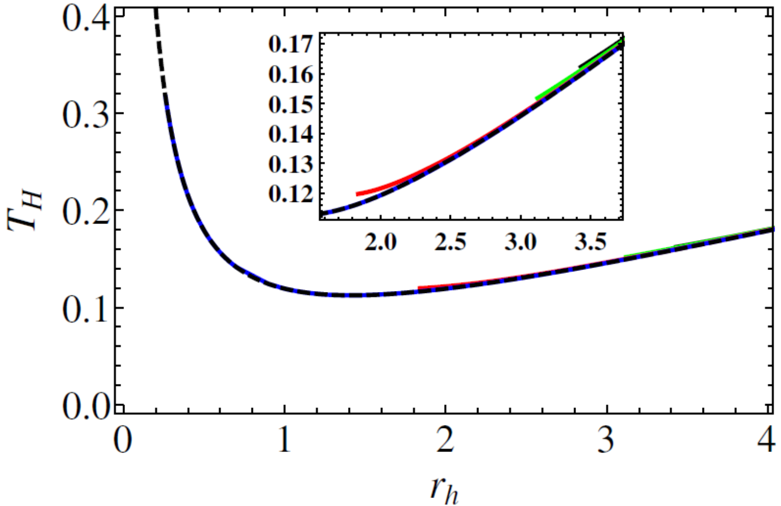


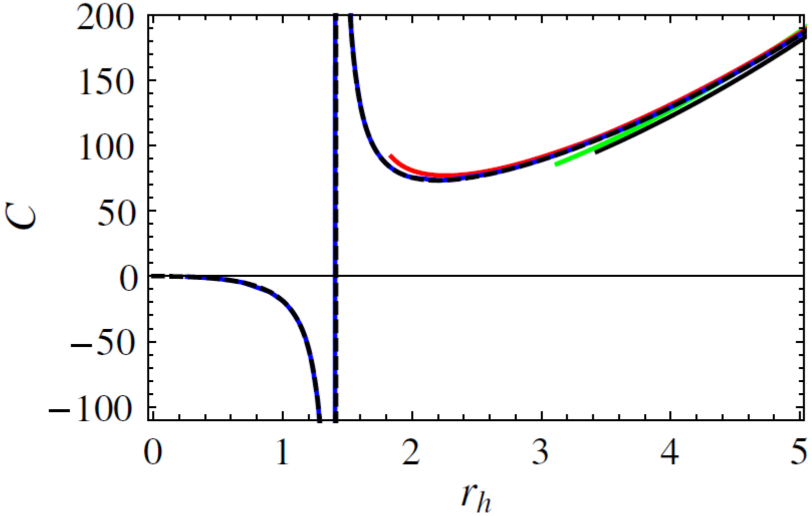
r_h = 4.09, 8.1, 12.2, and 15, respectively

There exists the minimum mass of a black hole.

If the black hole horizon r_h becomes larger, the magnitude of the scalar hair becomes smaller.

γ=1/2, -Λ=1/2, κ=1



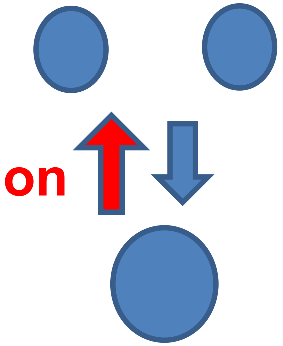


α = 0 for the dashed line
α = 0.005 for the blue line,
α = 0.4 for the red line,
α = 0.8 for the green line,
α = 1.0 for the black line

(In)stability of the DEGB Blackholes under fragmentation

B. Gwak & BHL, PRD (2015).
B.Gwak, BHL, D. Rho, PL.B (2016)

B) fragmentation

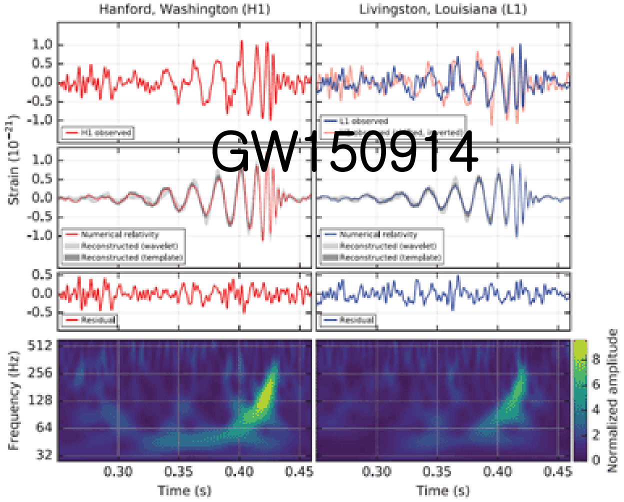


?

A) Merging

+ Gravitational Wave

Observed!



Fragmentation instability is based on the entropy preference between the solutions.

Emparan and Myers, JHEP 0309, 025 (2003).

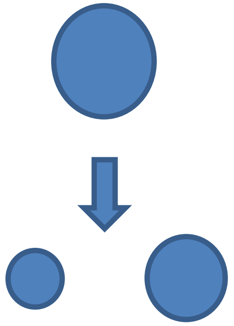
entropy of 1 BH < entropy of 2 fragmented BHs ⟶ (transition to) instability

B) Fragmentation Process : one BHs → two BH ?

Schwarzschild BHs are always stable under the fragmentation, is marginally stable under the fragmentation of shooting off the infinitesimal mass BH .

$$\frac{S_f}{S_i} = \frac{M_1^2+M_2^2}{(M_1+M_2)^2} = \frac{(\delta r_h)^2+((1-\delta)r_h)^2}{r_h^2} = \delta^2 + (1-\delta)^2 \leq 1$$

(equality only if $\delta \rightarrow 0$))



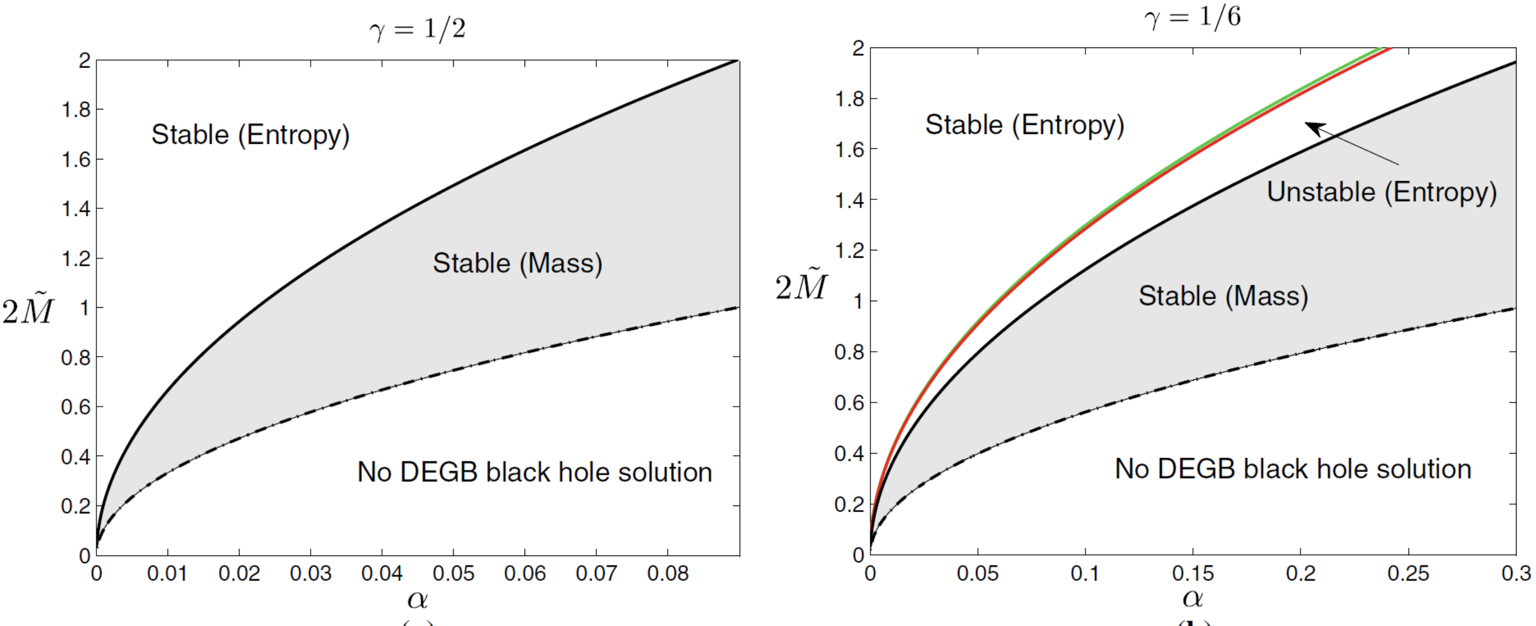
δ 1-δ

These phenomena could happen in the theory with the higher order of curvature term with appropriate parameters.

Fragmentation Instability for DEGB Black Holes

DEGB black hole with mass M decaying into two daughter BHs with mass fraction (1-δ, δ).

(1/2 ,1/2)
($\bar{\delta}$,1 − $\bar{\delta}$)



(a) (b)

Note :

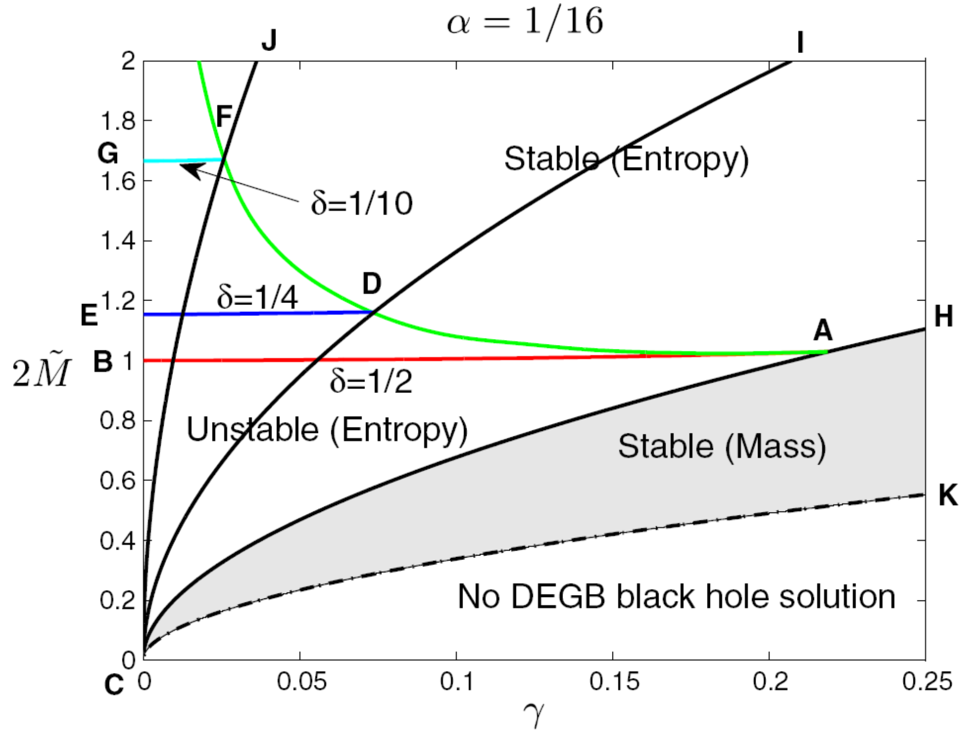
1) It cannot decay into black holes with mass smaller than the minimum mass M_{min}. Hence, $\delta_m \le \delta \le 1/2$, $\delta_m = M_{min}/M$.

2) The BHs with $M < 2M_{min}$ are absolutely stable.

The black hole can be fragmented only when its mass exceeds twice of minimum mass.

The phase diagrams in γ & $\tilde{M}$

(1/2 ,1/2)
(1/4 , 3/4)
(1/10 , 9/10)
($\bar{\delta}$,1 − $\bar{\delta}$)



For δ=1/4, the regions are as follows

the stable (mass) : region ICK

the unstable (mass) : region ECD

the stable (entropy) : above the line EDI .

III. dEGB Cosmology

III. Dilaton-Einstein-Gauss-Bonnet (dEGB) Cosmology

A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, Velasco-Sevilla, L. Yin **JCAP08 (2023) 023**

Biswas, Kar, **BHL**,H.Lee, W.Lee, **Scopel**, Velasco-Sevilla, L. Yin **JCAP (2024)**

S. Koh, BHL, Tumurtushaa **PRD98 (2018) 10, 103511**

S. Koh, BHL, W. Lee, Tumurtushaa **PRD90 (2014) no.6, 063527**

S. Koh, BHL, Tumurtushaa **PRD 95 (2017)**

Action

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right]$$

$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ Gauss-Bonnet term

$\mathcal{L}_m = \mathcal{L}_{SM} + \mathcal{L}_{CDM} - \frac{1}{\kappa}\Lambda \rightarrow \mathcal{L}_{rad}$ $\quad \kappa \equiv 8\pi G,\ [\kappa] = \sqrt{\frac{[L]}{[M]}}$

Note:

1) If $f(\phi) = $ const and $\phi = $ const, the theory is reduced to **ΛCDM**.

$\tilde{\alpha}\gamma = 0,\ \rho_\phi(T_{BBN}) = 0\ (\epsilon = 0)$

$$S = \int d^4x \sqrt{-g}\,[\frac{1}{2\kappa}R + \mathcal{L}_m - \frac{1}{\kappa}\Lambda] \quad \mathcal{L}_m = \mathcal{L}_{rad} + \mathcal{L}_{matt} + \mathcal{L}_{CDM}$$

(dEGB ⟶ **ΛCDM**) — No GB($\tilde{\alpha}\gamma=0$), No Dilaton ($\dot{\phi}(t)=0$)

2) If $f(\phi) = $ const, the theory is reduced to a **quintessence model.**

$\tilde{\alpha}\gamma = 0,\ \rho_\phi(T_{BBN}) \neq 0\ (\epsilon \neq 0)$

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right]$$

(dEGB ⟶ **Quintessence**) — No GB($\tilde{\alpha}\gamma=0$), Dilaton ($\phi(t)$)

Note:

dEGB Model (Generic) $\quad \tilde{\alpha}\gamma \neq 0,\ \rho_\phi(T_{BBN}) \neq 0\ (\epsilon \neq 0)$

The spatially flat Friedmann-Lemaˆıtre-Robertson-Walker (FLRW) metric,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

Application:
1) Inflation in DEGB
2) Reconstruction of Infl $V(\phi)$
3) Primor GWs & RH param
4) WIMPs
5) New Phase & GWs, etc.

The Einstein and scalar Eqs.

$$H^2 = \frac{\kappa}{3}\left(\rho_{\{\phi+GB\}} + \rho_m\right)$$
$$= \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 - 24\dot{f}H^3 + \rho_m\right) = \frac{\kappa}{3}\rho_{tot}$$
$$\dot{H} = -\frac{\kappa}{2}\left[\left(\rho_{\{\phi+GB\}} + p_{\{\phi+GB\}}\right) + (\rho_m + p_m)\right]$$
$$= -\frac{\kappa}{2}\left[\dot{\phi}^2 + 8\frac{d(\dot{f}H^2)}{dt} - 8\dot{f}H^3 + (\rho_m + p_m)\right]$$
$$\equiv -\frac{\kappa}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa}{2}\rho_{tot}(1 + w_{tot})$$
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + V'_{GB} = 0$$

where: $V'_{GB} \equiv -f'R^2_{GB}$
$$= -24f'H^2(\dot{H} + H^2) = 24\alpha\gamma e^{\gamma\phi}qH^4$$

$$\rho_{rad} = 3\,p_{rad} = \frac{\pi^2}{30}\,g_* T^4$$
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 = p_\phi \; (V(\phi) = 0)$$
$$\rho_{GB} = -24\dot{f}H^3 = -24f'H^3 = -24\alpha\gamma e^{\gamma\phi}\dot{\phi}H^3$$
$$p_{GB} = 8\left(f''\dot{\phi}^2 + f'\ddot{\phi}\right)H^2 + 16f'\dot{\phi}H\left(\dot{H} + H^2\right)$$
$$= 8\frac{d(\dot{f}H^2)}{dt} + 16\dot{f}H^3 = 8\frac{d(\dot{f}H^2)}{dt} - \frac{2}{3}\rho_{GB}$$

the continuity equation

$$\dot{\rho}_I + 3H(\rho_I + p_I) = \dot{\rho}_I + 3H(1 + w_I)\rho_I = 0$$

$H = \frac{\dot{a}}{a}$ Hubble Parameter

$w_I = \frac{p_I}{\rho_I}$ (Eq. of state parameter)

acceleration ⟶ ⟵ deceleration

w_I: -1 $-1/3$ 0 $+\frac{1}{3}$ $+1$

$\rho_I \sim a^{-3(1+w_I)}$ a^{0-} a^{-1} a^{-2} a^{-3} a^{-4} a^{-5} a^{-6}

$a(t) \sim t^{\frac{2}{3(1+w)}}$ $t^{\infty} \sim e^{Ht}$ $t^{2/1}$ $t^{2/2}$ $t^{2/3}$ $t^{2/4}$ $t^{2/5}$ $t^{2/6}$

$H \sim a^{-3(1+w)/2}$ a^{0-} $a^{-1/2}$ $a^{-2/2}$ $a^{-3/2}$ $a^{-4/2}$ $a^{-5/2}$ $a^{-6/2}$

Acceleration (deceleration) of a

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{\kappa}{6}\left[\left(\rho_{\{\phi+GB\}} + 3p_{\{\phi+GB\}}\right) + (\rho_m + 3p_m)\right]$$
$$= -\frac{\kappa}{6}\rho_{tot}(1 + 3w_{tot}) = -\frac{1}{2}H^2(1 + 3w_{tot}) \equiv -H^2 q$$
$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2}(1 + 3w_{tot})$$ Deceleration parameter

Bdry Conditions (constraints) at BBN for $\phi, \dot{\phi}$, a, $\dot{a}$

$\phi_{BBN} = 0$ (shift of $\phi_{BBN} \Leftrightarrow \alpha$-scaling) ($T_{BBN} \simeq 1\,MeV$)

$\eta \leq 3 \times 10^{-2}$ from $N_{eff} \leq 2.99 \pm 0.17$, $\eta \equiv \frac{\rho_\phi(T_{BBN})}{\rho_{tot}(T_{BBN})}$

(choose $\dot{\phi}_{BBN} \geq 0$: (sign change sym of both $\dot{\phi}_{BBN}$ & γ)

H_{BBN}: from $8\sqrt{6\kappa\eta}f'(0)H^4_{BBN} + (1-\eta)H^2_{BBN} + \frac{\kappa}{3}\rho_{rad}(T_{BBN}) = 0$

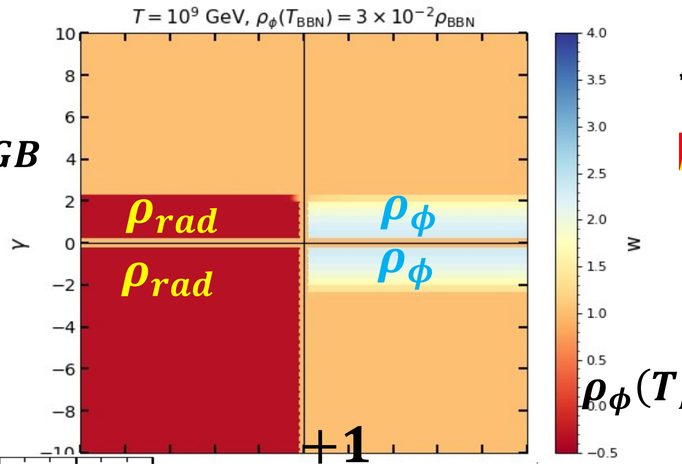
Note:

1) ρ_{GB} p_{GB} w_ϕ $\rho_{\{\phi+GB\}}$ & $p_{\{\phi+GB\}}$: NOT necessarily positive.
2) We treat the Gauss-Bonnet term (as well as a scalar) some “matters”.
3) The effect of the G-B term is expected to be stronger for earlier universe.

Goal : Constrain the **dEGB gravity**

Solutions

Mitigating role of ρ_{GB} by cancelling the leading energy component.



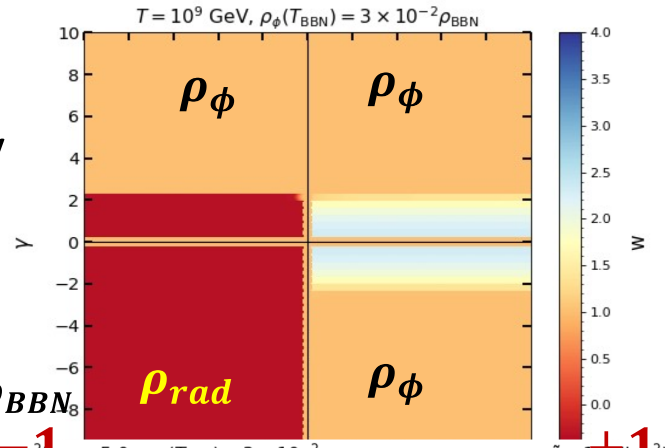
ρ_{GB} Mitigation

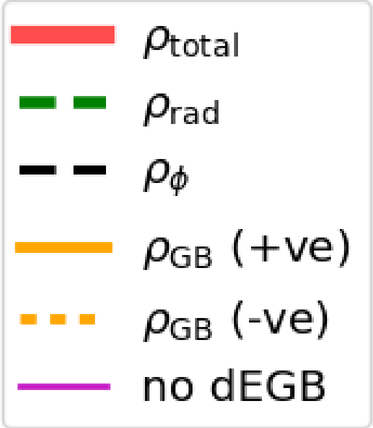
ρ_{rad} effectively

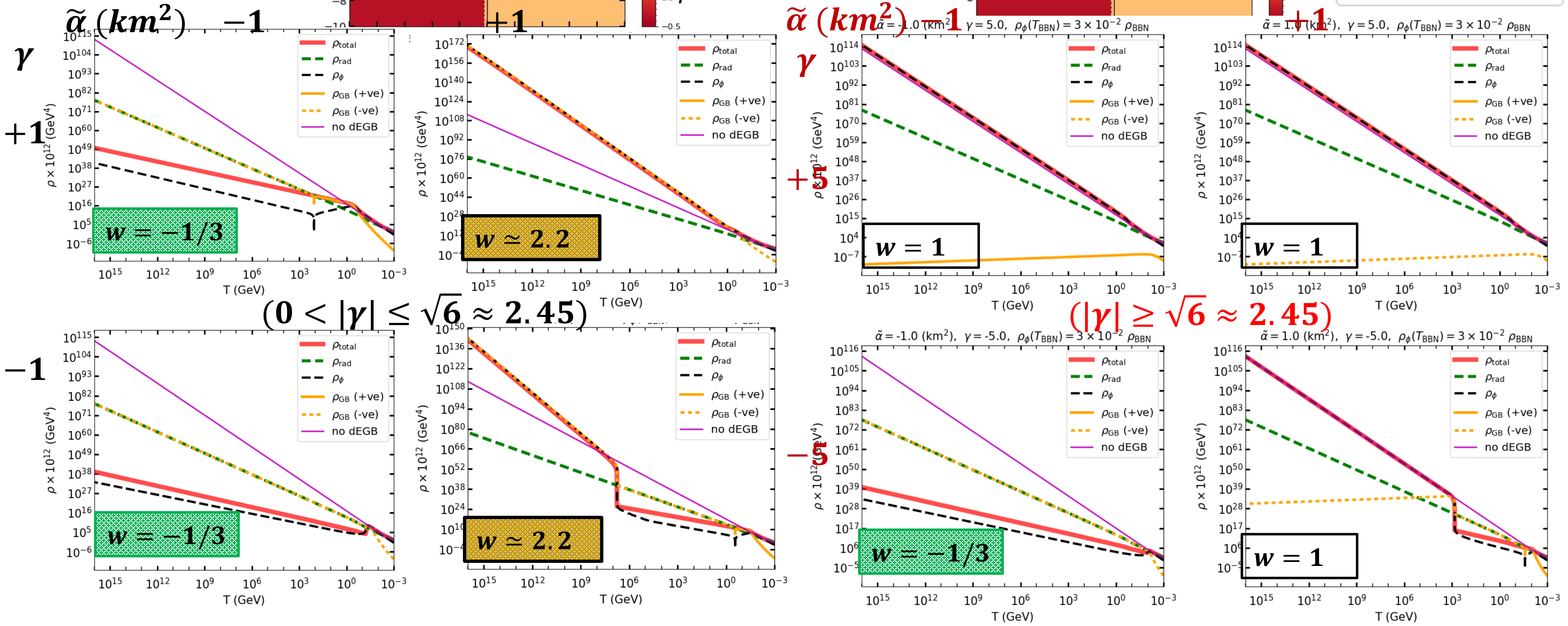
ρ_ϕ partially

ρ_ϕ no effect

$\rho_\phi(T_{BBN}) = 3\times10^{-2}\rho_{BBN}$



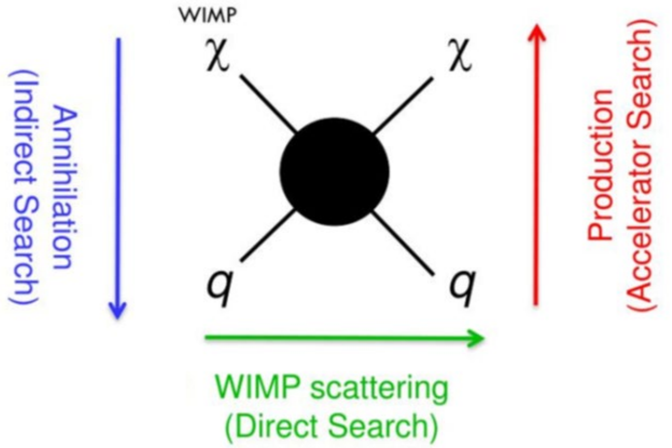


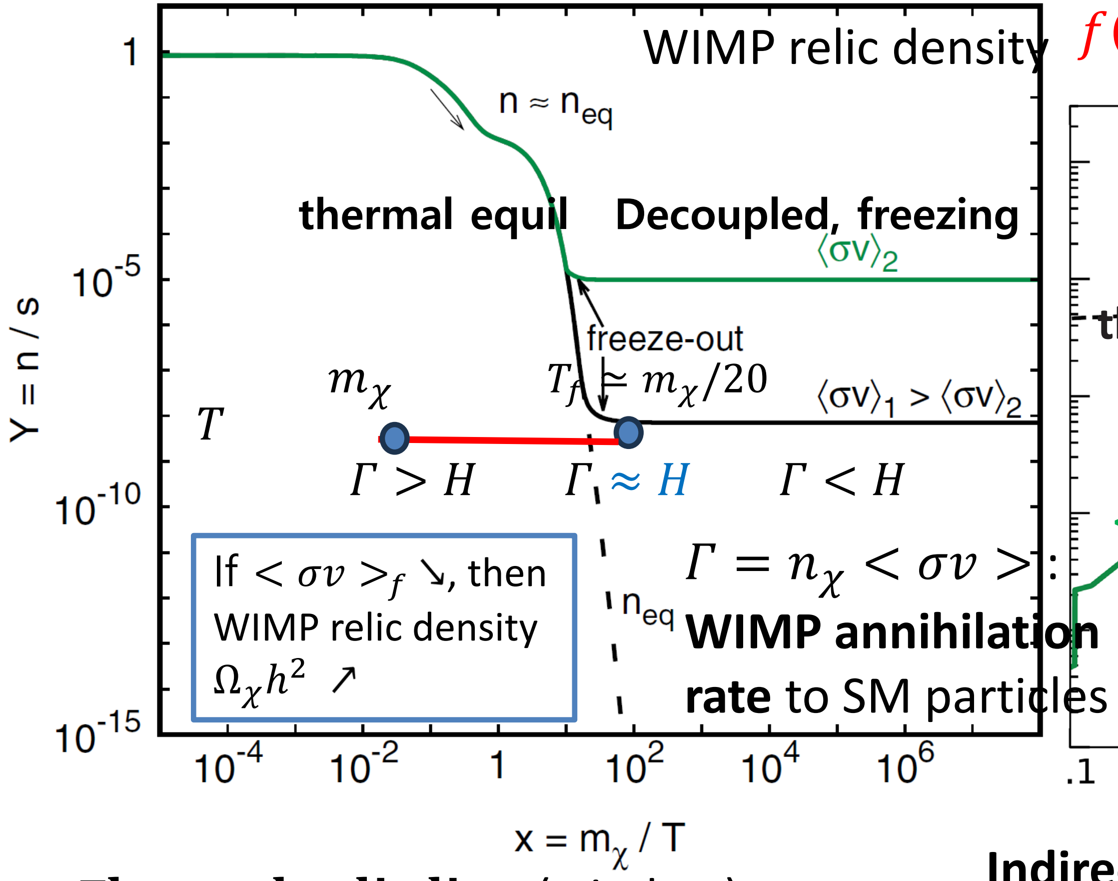


WIMPs in DEGB cosmology

A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee, **S. Scopel**, L. Velasco-Sevilla, L. Yin **JCAP08 (2023) 023**

$$S_{DEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R^2_{GB} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m^{rad} + \mathcal{L}_{DM}^{WIMP}\right]$$





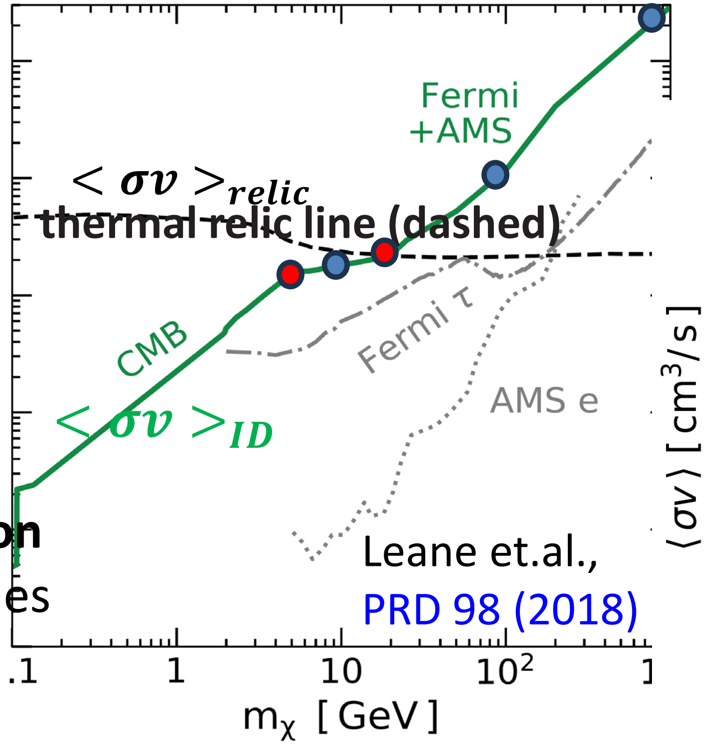
Thermal relic line (window)

$< \sigma v >_{relic} = < \sigma v >_f$

for $\Omega_{\chi(WIMP)} = \Omega_{DM} \simeq 0.12/h^2$

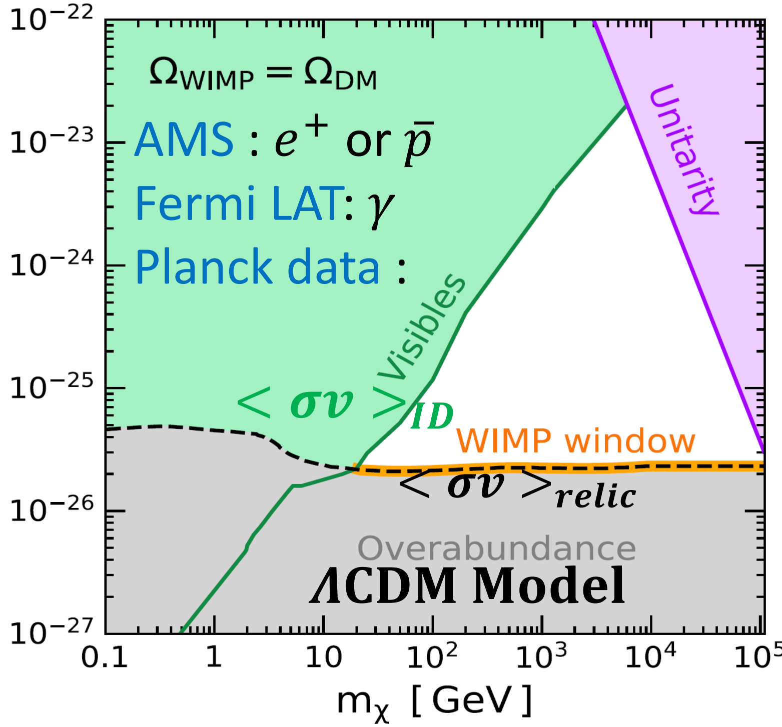
as a fn of m_χ

$f(\phi) = \alpha e^{\gamma\phi(r)}$ $\quad V(\phi) = 0$



Indirect detection bounds

Nonobservation of the WIMP annihilation in the Galaxy today ⇝ an upper bound $< \sigma v >_{ID}$ on $< \sigma v >_{gal}$



The favoured parameters of dEGB cosmology are those satisfying

$< \sigma v >_{gal} / < \sigma v >_{ID} \lesssim 1$

the constraints from the GW signals from BH-BH and BH-NS merger events

Yagi, (2012)
Nair, Perkins, Silva, Yunes, (2019)
Perkins,Nair,Silva,Yunes(2021),
Lyu, Jiang, Yagi, PRD (2022)

- ϕ freezes at $T_L \ll T_{BBN}$ to a bgr value $\phi(T_L)$, while near a BH or a NS, distorted ϕ can modify the GW signal in a merger event.
- The LIGO-Virgo data for constraints $\alpha_{GB}^{1/2} \leq \mathcal{O}(2\text{ km})$ or $\alpha_{GB}^{1/2} \leq 1.18$ km

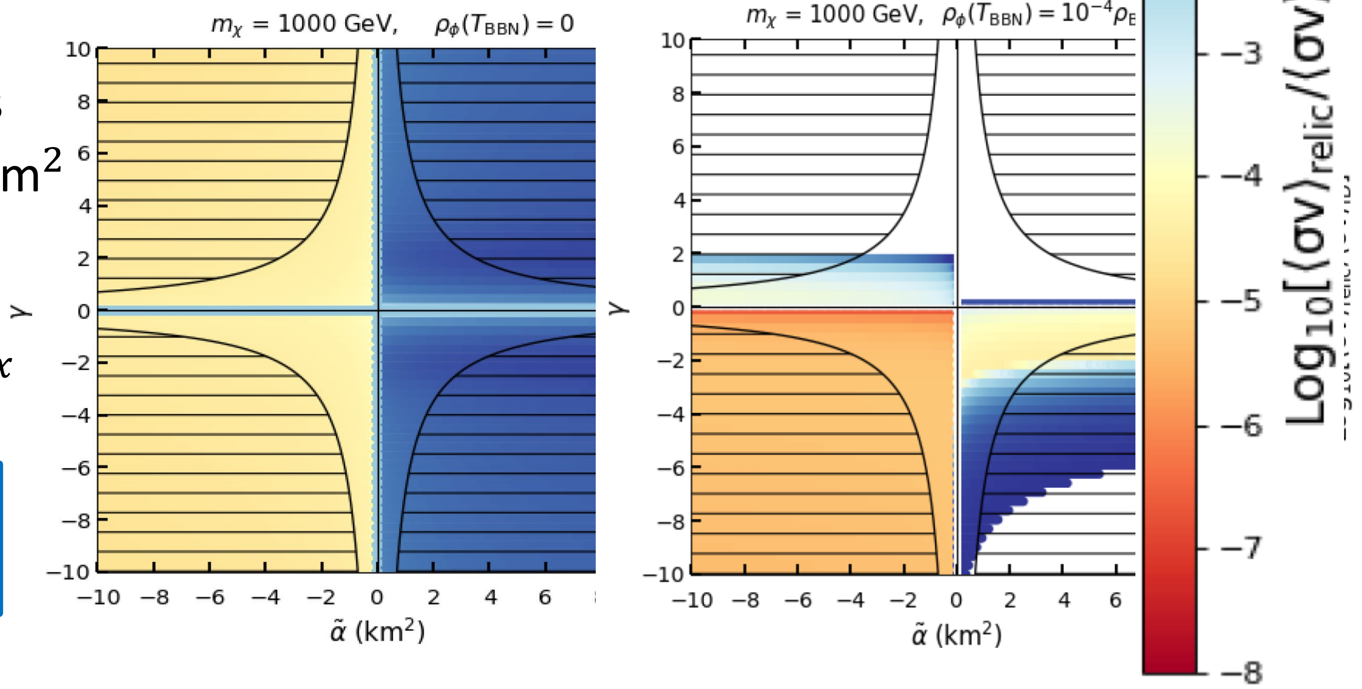
	LMXB	GW (BBH)		GW (NSBH)	
		O1–O2	O1–O3	GW200115	combined
$\alpha_{GB}^{1/2}$ [km]	1.9	5.6	1.7,	1.33	1.18

- **Other Bounds** do not provide competitive constraints

- **The favoured parameters** of dEGB cosmology by WIMP indirect detection are those satisfying
$$<\sigma v>_{gal} / <\sigma v>_{ID} \lesssim 1$$
- **White regions** ($\frac{<\sigma v>_{relic}}{<\sigma v>_{ID}} > 1$) are **disfavoured** by WIMP indirect detection.

The dEGB constraints from compact binary mergers
$$|f'(\phi(T_L))| \leq \sqrt{8\pi}\alpha_{GB}^{max} \text{ with } \alpha_{GB}^{max} = (1.18)^2\text{km}^2$$

- If $\dot{\phi}\ (T_{BBN}) = 0$, then $|\tilde{\alpha}\gamma| \leq \sqrt{8\pi}\alpha_{GB}^{max}$
- If $\dot{\phi}\ (T_{BBN}) \neq 0$, then $|\tilde{\alpha}\gamma e^{\gamma\frac{\dot{\phi}_{BBN}}{H_{BBN}}}| \leq \sqrt{8\pi}\alpha_{GB}^{max}$



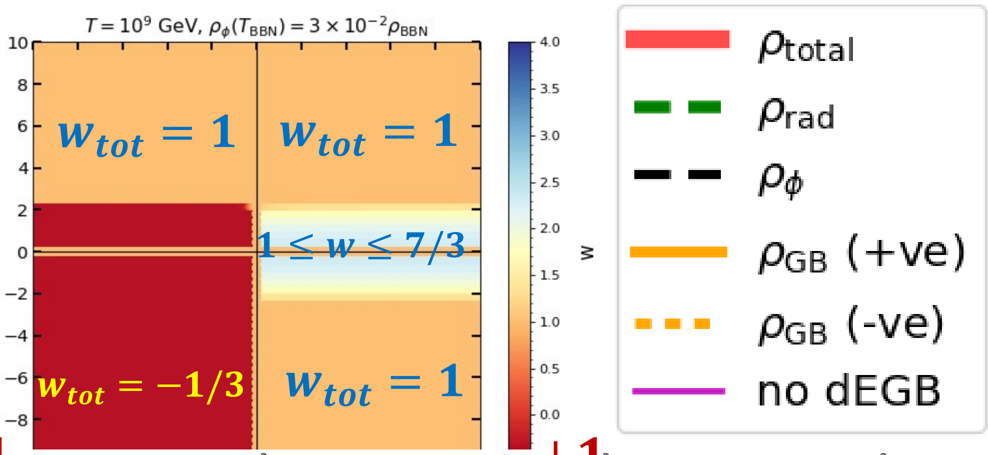
Hatched areas of the $\tilde{\alpha}$-γ plane are disallowed by the constraints of GWs in BBH merge

New Phases in High T of dEGB cosmology

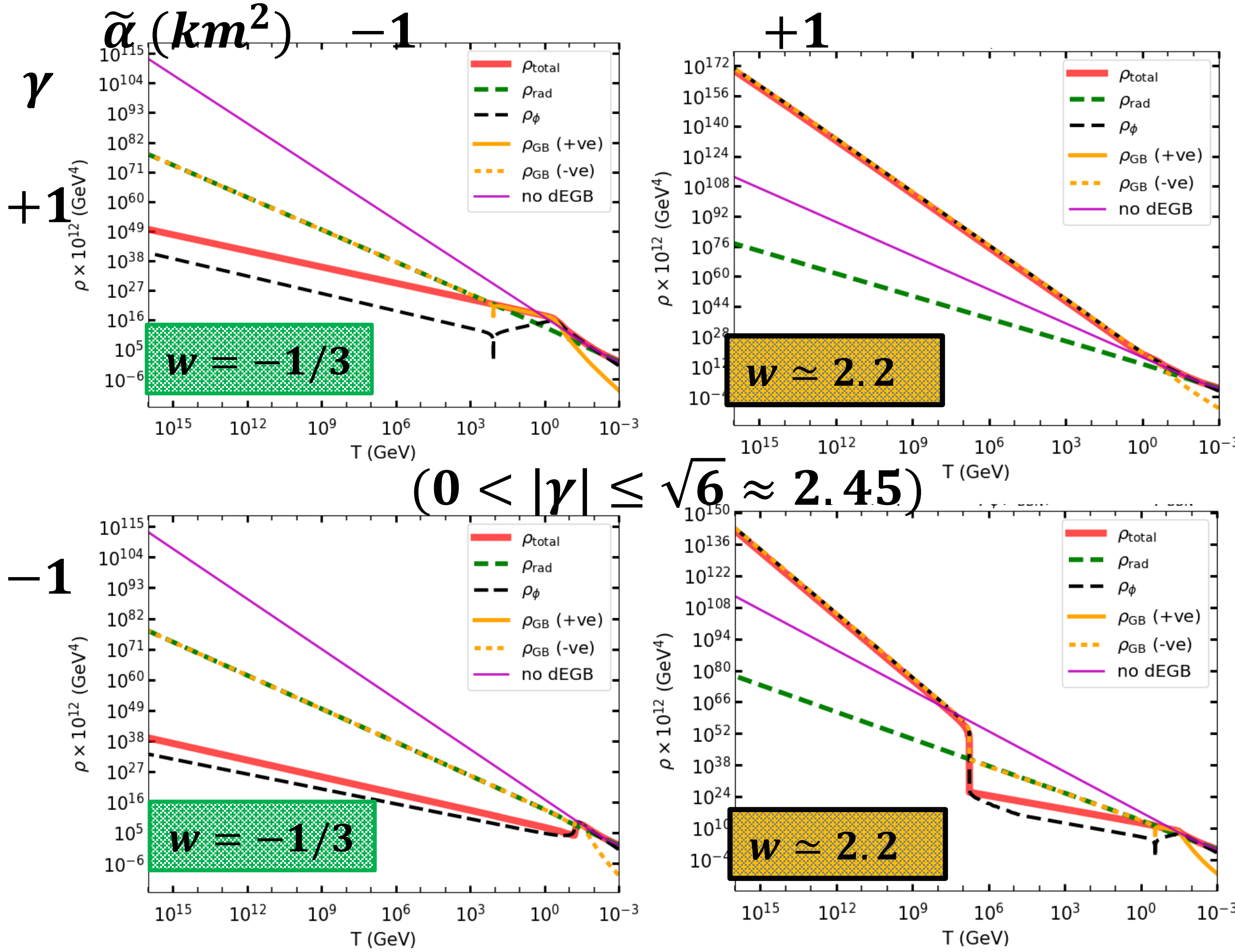
ρ_{tot} reaches an asymptotics at large T

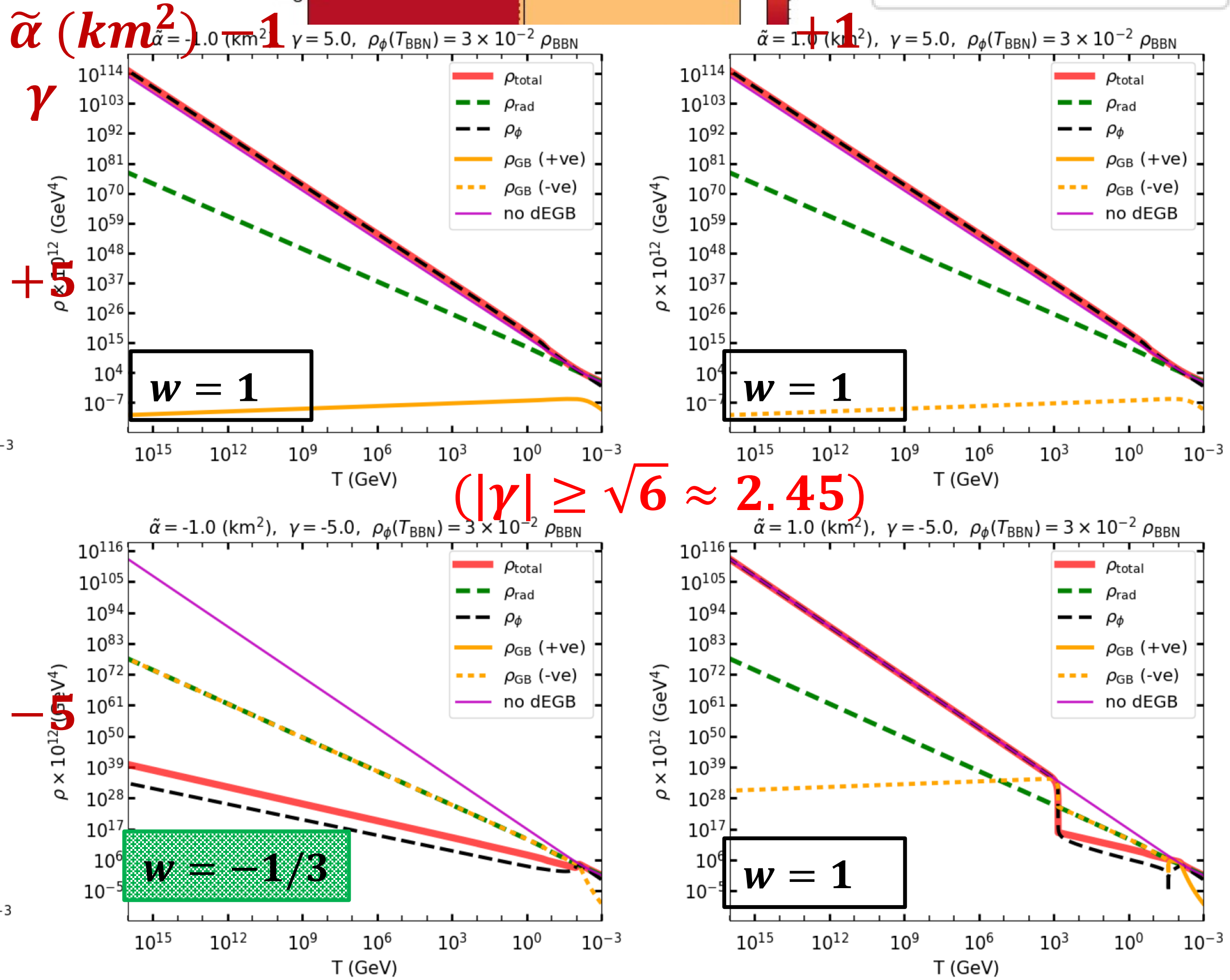
- $1 < w_{tot} < 7/3 \approx 2.3$ fast rolling phase
- $w_{tot} = 1$ Kination Phase
- $w_{tot} \approx -1/3$ Slow rolling phase

$$w_{tot} = \frac{p_{tot}}{\rho_{tot}} = \frac{p_\phi + p_{rad} + p_{GB}}{\rho_\phi + \rho_{rad} + \rho_{GB}}$$



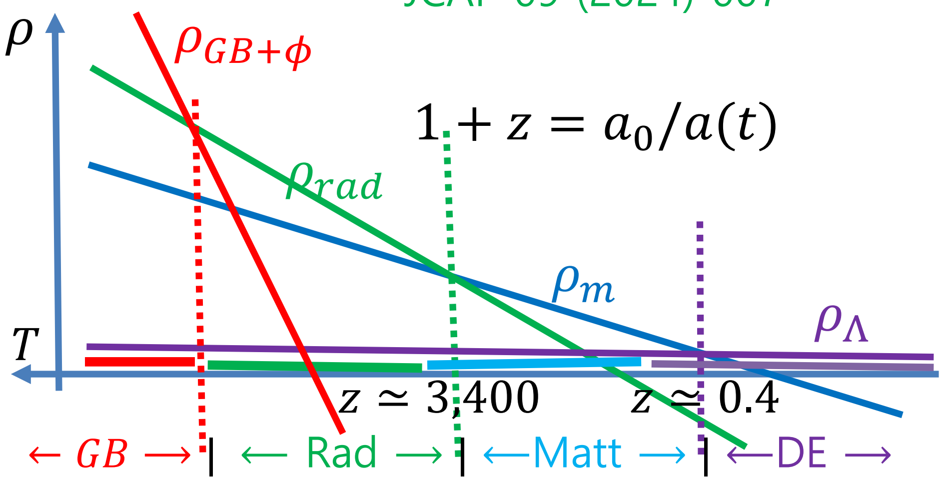
$\rho_\phi(T_{BBN}) = 3\times10^{-2}\rho_{BBN}$





New Phases

Biswas, Kar, **BHL**, H.Lee, W.Lee, S.**Scopel**, L.Velasco-Sevilla, L.Yin
JCAP 09 (2024) 007



acceleration⟶ ⟵ deceleration

w_I: -1 $-1/3$ 0 $+\frac{1}{3}$ $+1$ $7/3$

$\rho_I \sim a^{-3(1+w_I)}$ a^0– a^{-1} a^{-2} a^{-3} a^{-4} a^{-5} a^{-6} a^{-10}

$a(t) \sim t^{\frac{2}{3(1+w)}}$ $t^\infty \sim e^{Ht}$ $t^{2/1}$ $t^{2/2}$ $t^{2/3}$ $t^{2/4}$ $t^{2/5}$ $t^{2/6}$ $t^{2/10}$

$H \sim a^{-3(1+w)/2}$ a^0– $a^{-1/2}$ $a^{-2/2}$ $a^{-3/2}$ $a^{-4/2}$ $a^{-5/2}$ $a^{-6/2}$ $a^{-10/2}$

NEW PHASE⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶

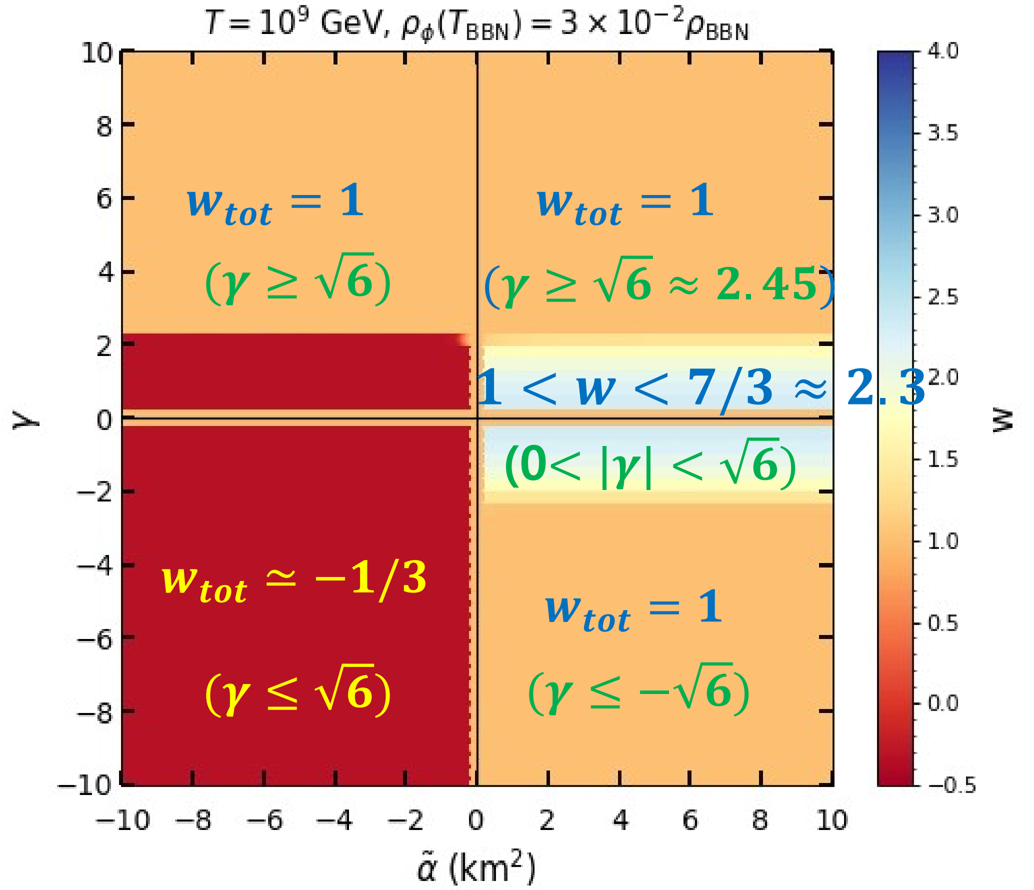
1) New Phases appear
   Ex) Super Kination phase ($w > 1$)
   Kination Phase ($w = 1$)
   Slow rolling phase ($w \approx -1/3$)
2) These are attractor/fixed points
3) May affect observation -Ex) GWs

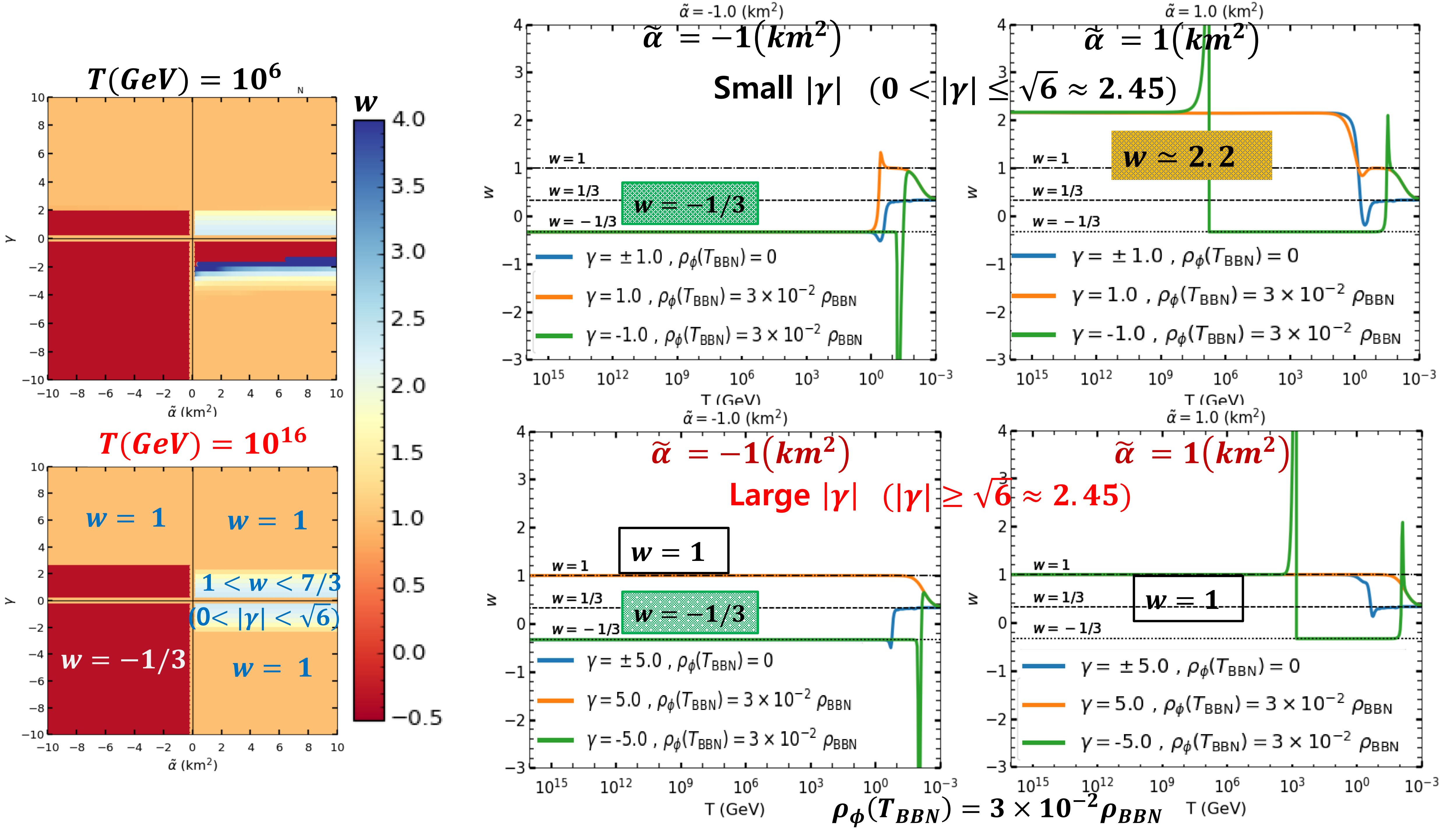
$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot\phi^2 - V(\phi)}{\frac{1}{2}\dot\phi^2 + V(\phi)}$$

$$-1 \le w_\phi \le +1$$

How to get big numbers of w_I?

$$w_I = \frac{p_I}{\rho_I} = \frac{100 - (-99)}{100 + (-99)}$$





T(GeV) = 10^6
w
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
γ
α̃ (km²)
T(GeV) = 10^16
w = 1
w = 1
1 < w < 7/3
(0< |γ| < √6)
w = −1/3
w = 1
α̃ = -1.0 (km²)
α̃ = −1(km²)
α̃ = 1.0 (km²)
α̃ = 1(km²)
Small |γ| (0 < |γ| ≤ √6 ≈ 2.45)
w = 1
w = 1/3
w = −1/3
w = −1/3
w ≃ 2.2
γ = ±1.0 , ρ_ϕ(T_BBN) = 0
γ = 1.0 , ρ_ϕ(T_BBN) = 3 × 10^−2 ρ_BBN
γ = -1.0 , ρ_ϕ(T_BBN) = 3 × 10^−2 ρ_BBN
T (GeV)
Large |γ| (|γ| ≥ √6 ≈ 2.45)
w = 1
γ = ±5.0 , ρ_ϕ(T_BBN) = 0
γ = 5.0 , ρ_ϕ(T_BBN) = 3 × 10^−2 ρ_BBN
γ = -5.0 , ρ_ϕ(T_BBN) = 3 × 10^−2 ρ_BBN
ρ_ϕ(T_BBN) = 3 × 10^−2 ρ_BBN

Equivalent system of autonomous eqns

Define the following variables

$$x \equiv \frac{\rho_\phi}{\rho} = \frac{\kappa}{6}\left(\frac{\dot\phi}{H}\right)^2$$

$$y \equiv \frac{\rho_{rad}}{\rho} = \frac{\kappa g_* \pi^2}{90}\frac{T^4}{H^2} \qquad x>0 \quad y>0 \quad -\infty < z < \infty$$

$$z \equiv \frac{\rho_{GB}}{\rho} = -8\kappa \dot f H = -8\kappa \frac{\partial f}{\partial \phi}\dot\phi H$$

Note:
2 indep d.o.f. $(x+y+z=1)$

the e.o.ms can then be reexpressed: $\left(' = \frac{d}{d\ln a} = \frac{1}{H}\frac{d}{dt} \right)$

$$x' = 2x\left(\frac{\beta}{z} + \epsilon - \mu\sqrt{x}\right) = 2(\epsilon-3)x + (\epsilon-1)z$$

$$y' = 2(\epsilon-2)y$$

$$z' = \beta - \epsilon z = 6x + 4y + (1+\epsilon)z - 2\epsilon$$

where

$$\beta \equiv -8\kappa \ddot f; \; \epsilon \equiv -\frac{\dot H}{H^2} = 1+q; \;\; \mu \equiv \sqrt{\frac{\kappa}{6}\frac{\partial^2 f}{\partial\phi^2}} \Big/ \frac{\partial f}{\partial \phi} = \sqrt{6}\gamma$$

Writing in terms of 2 indep. variables x and z,

$$x' = 2[\epsilon(x,z)-3]x + [\epsilon(x,z)-1]z \equiv F(x,z)$$

$$z' = 2x + [\epsilon(x,z)-3]z + 2[2-\epsilon(x,z)] \equiv G(x,z)$$

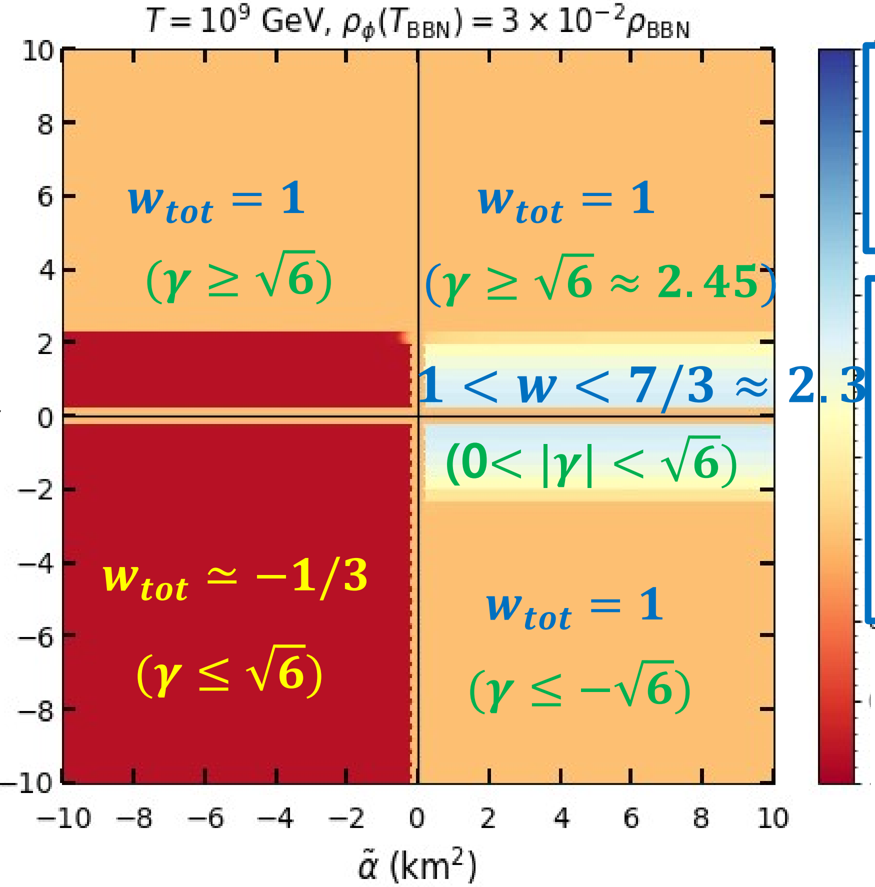
where $\epsilon(x,z)$ is explicitly given by

$$\epsilon(x,z) = \frac{4x^2+8x+z^2+2\sqrt{6}\,\mathrm{sign}(\alpha)|\gamma||z|x^{3/2}}{4x-4xz+z^2}$$

acceleration ⟶ ⟵ deceleration

w_I:	-1		$-1/3$	0	$+\frac{1}{3}$		$+1$	$7/3$
$\rho_I \sim a^{-3(1+w_I)}$	a^0–	a^{-1}	a^{-2}	a^{-3}	a^{-4}	a^{-5}	a^{-6}	a^{-10}
$a(t) \sim t^{\frac{2}{3(1+w)}}$	$t^\infty \sim e^{Ht}$	$t^{2/1}$	$t^{2/2}$	$t^{2/3}$	$t^{2/4}$	$t^{2/5}$	$t^{2/6}$	$t^{2/10}$
$H \sim a^{-3(1+w)/2}$	a^0–	$a^{-1/2}$	$a^{-2/2}$	$a^{-3/2}$	$a^{-4/2}$	$a^{-5/2}$	$a^{-6/2}$	$a^{-10/2}$

NEW PHASEs⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶



Note:
$w = \frac{2}{3}\epsilon - 1$

Note: Eqns & w_{tot} depend on $\mathrm{sgn}(\tilde\alpha)$ & γ (indep. of $|\tilde\alpha|$).

Biswas, Kar, **BHL**, Lee, Lee, **Scopel**, Yin **arXiv 2405.15998**

Constraints from Gravitational Waves

J. Ghiglieri and M. Laine, JCAP (2015), [1504.02569].
Ghiglieri, Jackson, Laine, Zhu, JHEP (2020), [2004.11392]

- Any plasma of relativistic particles in thermal equilibrium emits a stochastic GW background (SGWB)
- SGWB : potential probe of Cosmological models before BBN. Ex) the Standard Model : peak around 80 GHz
  (Present detectors are only sensitive to few Hertz, some proposals exist to extend to the GHz range.

Energy liberated into GW radiation,

$$\Omega_{GW}(f,T_0)h^2 \equiv \frac{1}{\rho_{crit}(T_0)}\frac{d\rho_{GW}(T_0)}{d\ln f}h^2$$

$$= \Omega_{\gamma 0}h^2 \frac{\lambda}{M_{PL}}\int_{T_{EWCO}}^{T_{max}} dT \left(\frac{g_{*0}}{g_*(T)}\right)^{\frac{4}{3}} T^2 \hat{k}(f,T)^3 \frac{\eta(\hat{k},T)}{\sqrt{\rho(T)}}\beta(T)$$

$$\hat{k}(f,T) = \left[\frac{g_{*s}(T)}{g_{*s}(T_0)}\right]^{\frac{1}{3}}\frac{2\pi f}{T_0} \qquad f = \frac{1}{2\pi}\left[\frac{g_{*s}(T_0)}{g_{*s}(T_{EWCO})}\right]^{\frac{1}{3}}\left(\frac{T_0}{T_{EWCO}}\right)k_{EWCO}$$

The BBN bound:

$$\Omega(f,T_0)h^2 < 1.3\times 10^{-6}$$

$\eta(k,T)$:the shear viscosity of the plasma

T_{EWCO} = 160 GeV $\qquad \hat{k} = k/T$

$h = H_0/(100km/s/Mpc) \qquad T_0 = 2.7K$

f: freq. measured today; $\lambda = 30\sqrt{3}/\pi^4$,

$\Omega_{\gamma,0} = 2.47\times 10^{-5}$ photon density (now)

$g_{*S}(T_0) = 3.91,\ g_{*S}(T_{EWCO}) = 106.75$

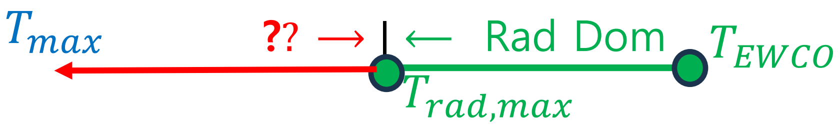
$g_{*0} = 2, \quad \beta = \left(1+\frac{1}{3}\frac{d\ln g_{*s}}{d\ln T}\right)$

Note - $\eta(\hat{k},T)$ peak at $\hat{k}_{peak} \simeq 3.92$ (at production) or $f_{peak} \simeq 74$GHz (today)

- $\frac{d\Omega_{GW}}{d\ln f} \propto \frac{T^3}{\sqrt{\rho}}$ ($\propto T, \Lambda$CDM) a sizeable GWs produced when rad is the dominant comp of ρ_{tot}
- UV-dominated, by GWs emitted at high T.

Ex) ΛCDM : rad. Dom. for all $T_{max}> T_{EWCO}$,
- Ω_{GW} is a monotonically growing fn of T_{max}.,
- Ωh^2 at $T_{max} = T_{RH}(10^{16}GeV)$ < the BBN bound

Ex) non–standard cosmol: If no rad dom at $T > T_{rad,max}$,
- SGWB is dominated by the GWs produced at $T_{rad,max}$,
- increasing T_{max} beyond $T_{rad,max}$, has no effects, detection are worse than in standard Cosmology.



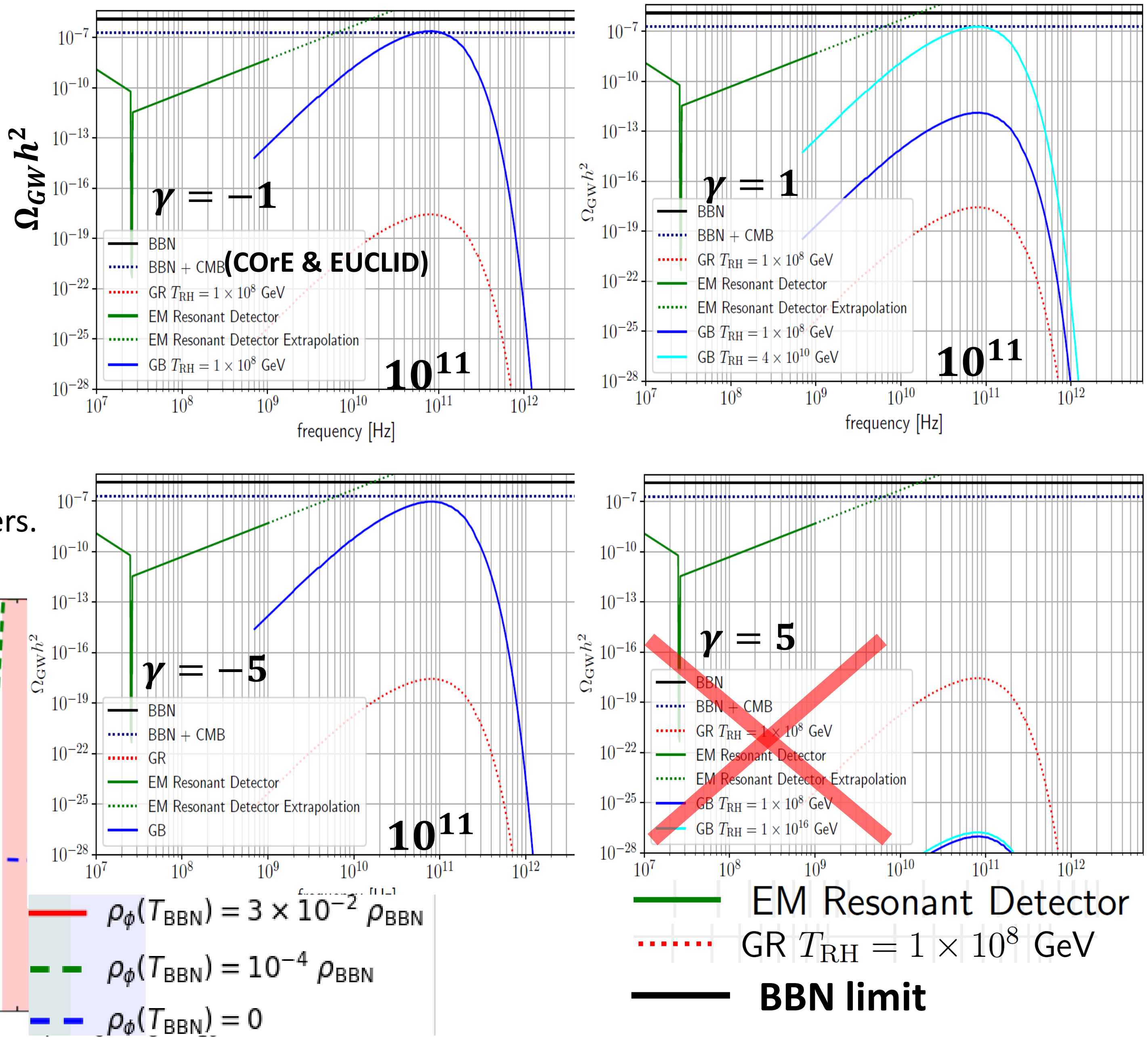
the dEGB scenario-GW bounds

For **slow roll (w = −1/3)**, strong enhancement ρ_{rad} dominate, and $\rho_{tot} \propto T^2$
Slower dilution rate than ΛCDM. Hence, $d\Omega_{GW}/d\ln a \propto T^3/\sqrt{\rho} \propto T^2$ during GW prod.

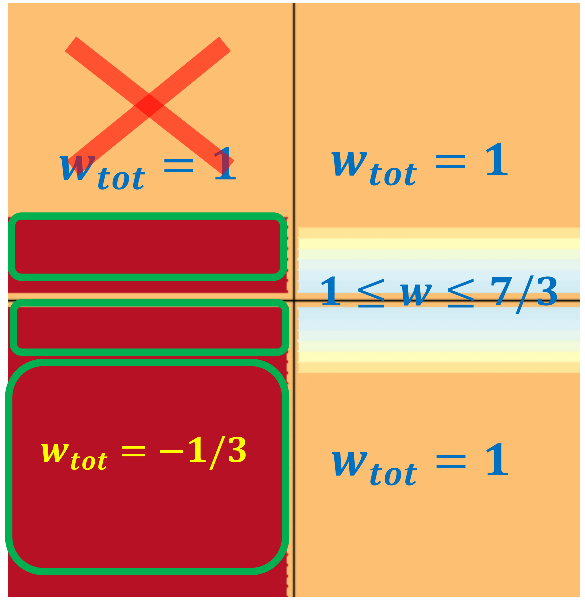
This strongly enhances the GW expected signal compared to the standard case
$\boldsymbol{\Omega_{GW}(f_{peak})h^2} \gg (\Lambda CDM)$.

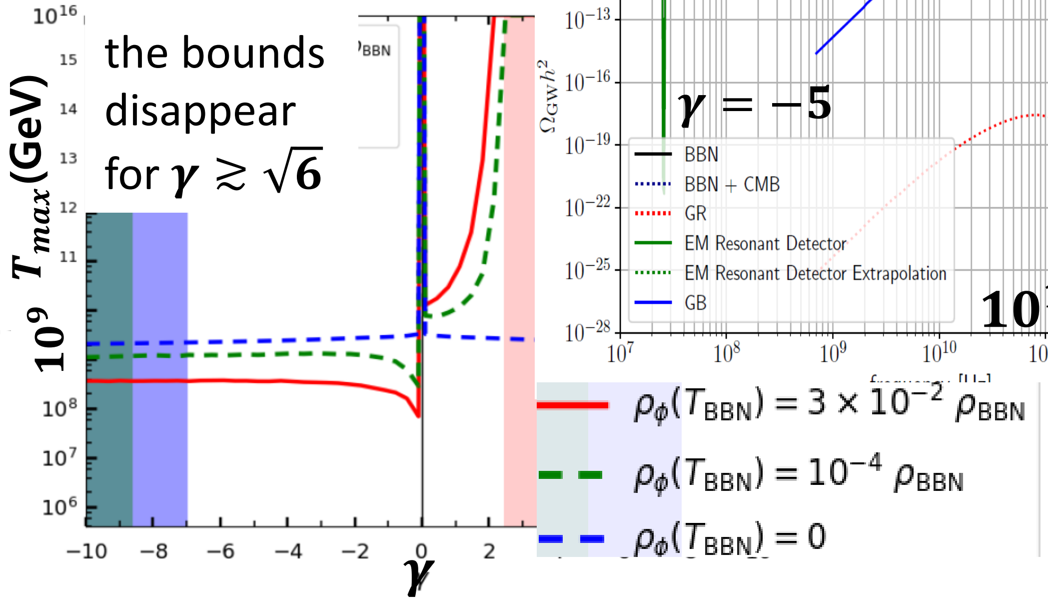
Put bounds on $T_{RH} \simeq 10^9$ GeV $\ll 10^{16}$ GeV
(by imposing the GW peak not exceeding the BBN upper limit)
Shaded areas excluded by the observed GW from binary mergers.

$\tilde{\alpha} = -1\mathrm{km}^2 \quad T_{RH} = T_{max} = 1 \times 10^8 \mathrm{GeV}$



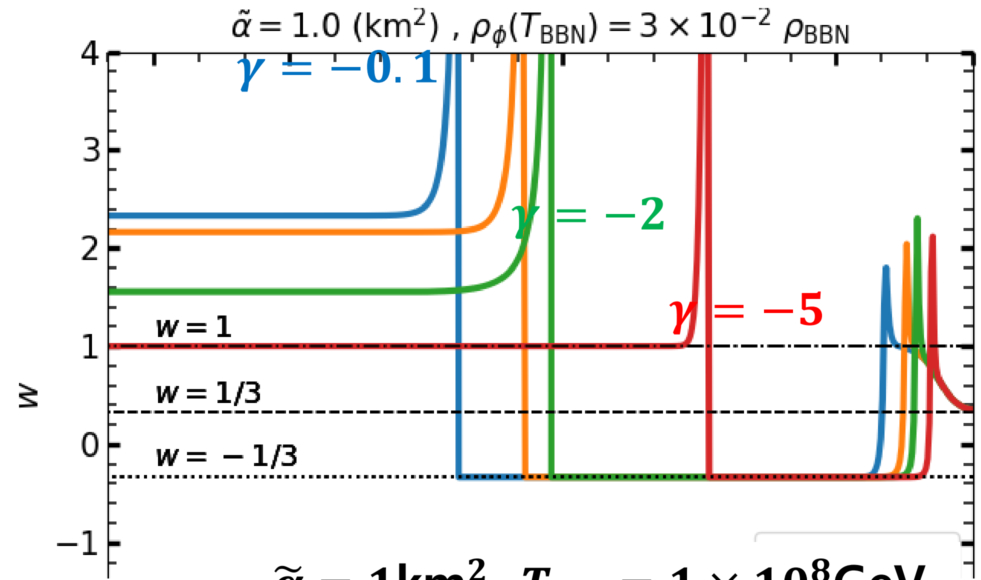
$\tilde{\alpha} = -1\mathrm{km}^2$



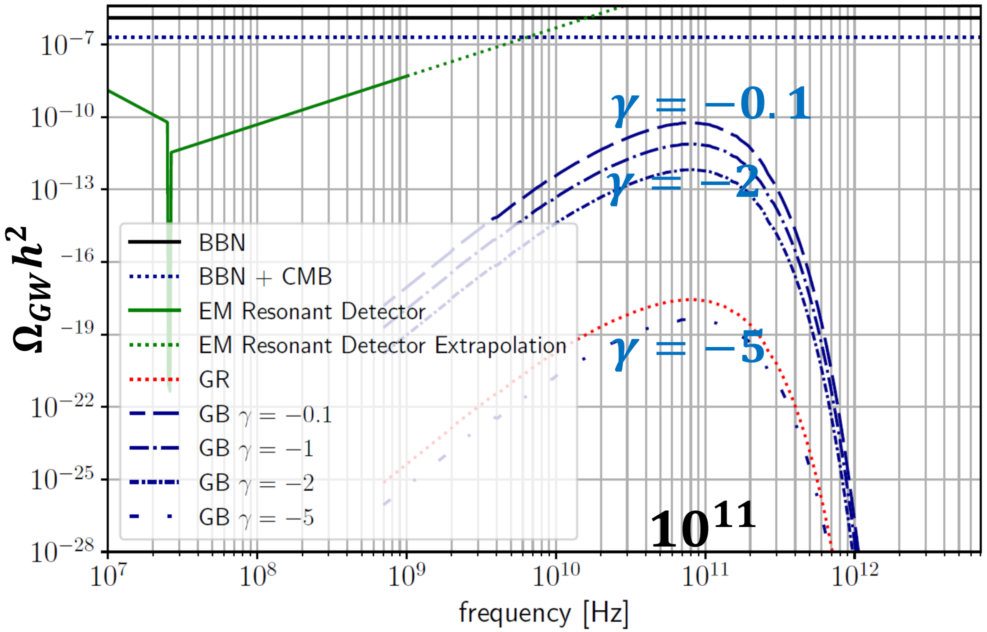


- a metastable slow-roll regime ($\tilde{\alpha} > 0, \gamma < 0$)

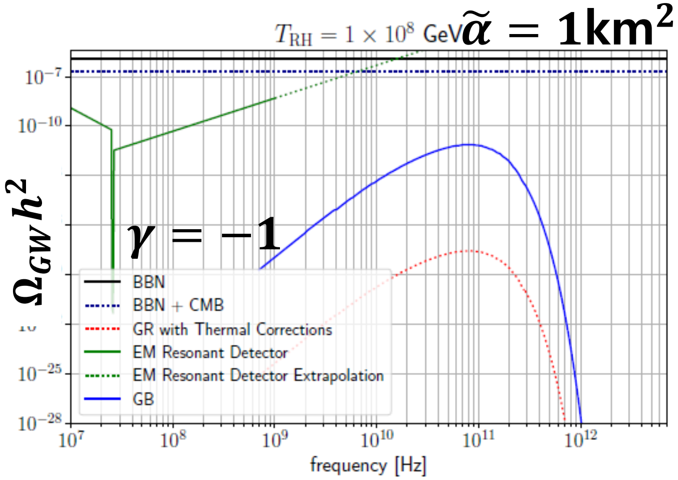
GW enhancement as $\gamma \to 0$.

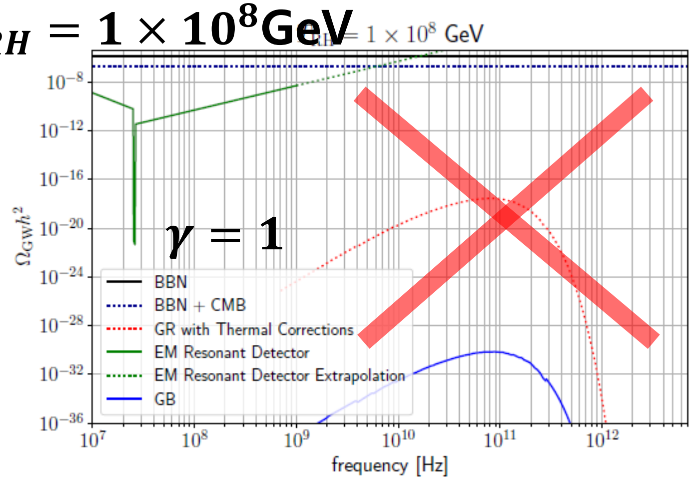


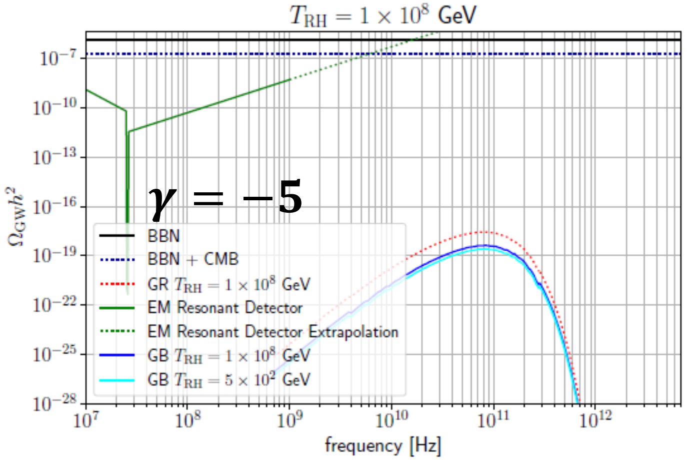
$\tilde{\alpha} = 1\mathrm{km}^2 \quad T_{RH} = 1 \times 10^8 \mathrm{GeV}$

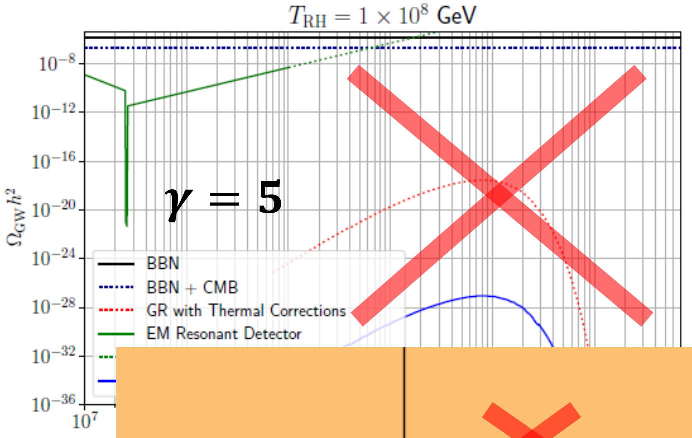


$\tilde{\alpha} = 1\mathrm{km}^2 \quad T_{RH} = 1 \times 10^8 \mathrm{GeV}$







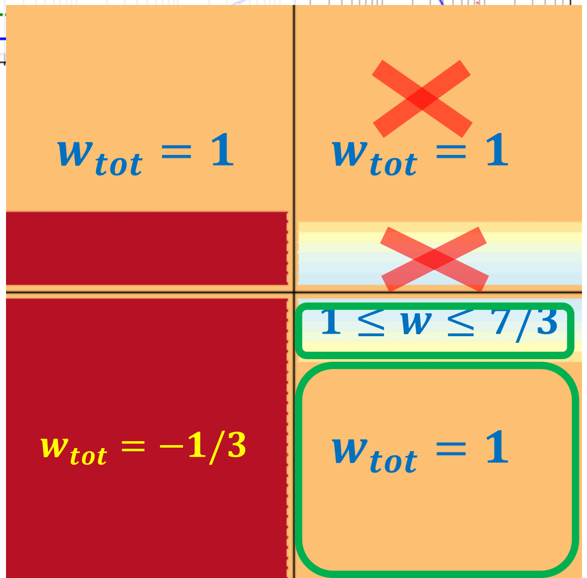


The system follows the z axis with w = −1/3 before jumping to the attractor,

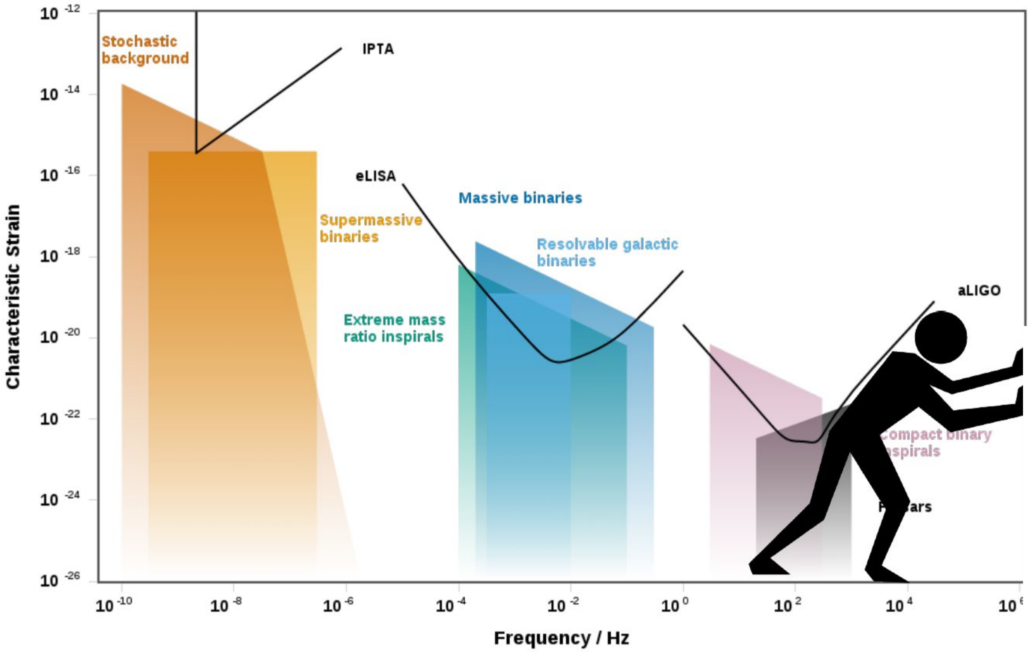
-an enhancement of the GW signal,

As |γ| →0 detaches "later" from the z<0 axis,and the metastable slow-roll lasts longer.

GW signal is way below the BBN bounds.

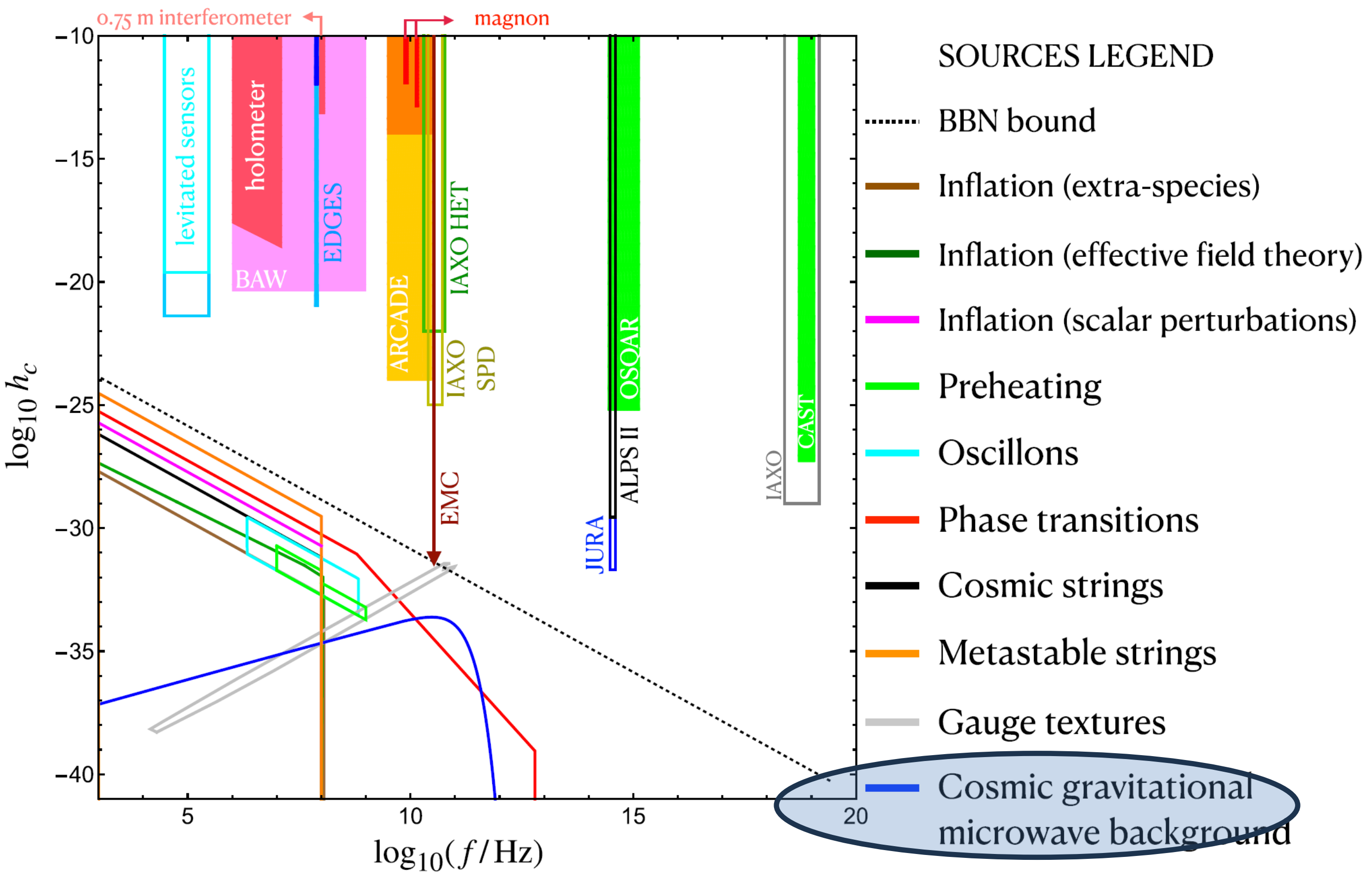


GW with high frequency - Future Observation



https://www.ctc.cam.ac.uk/activities/UHF-GV

Agarwal et.al, LRR (2021)



IV.Summary

Modified Gravity beyond Einstein needed?

Theoretical Aspect

- an **effective theory** below UV cut-off, $M_{Pl}\sim 10^{19}GeV$
- **Holography**

Observational Aspect - H_0 tension, Cosmological Birefringence etc.

Modification of GR - needs to introduce additional d.o.f.

Want understand the modified gravity with the G-B term.

In dim>4, consider the AdS Gravity with Gauss-Bonnet term **(allows 2nd order e.o.m.)**

$$S_{EGB-\Lambda} = \int d^d x \sqrt{-g}\left[\frac{1}{2\kappa}\left(R - 2\Lambda + \alpha R_{GB}^2\right) + \mathcal{L}_m^{matt}\right]$$

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$$

$$\kappa = 8\pi G, \quad g = \mathrm{det} g_{\mu\nu}$$

Briefly introduced the black hole thermodynamics, and phases:

- Schwarzschild BH - AdS Schwarzschild BH, - RN AdS BH, - AdS GB Black Holes
- charged GB AdS BH, etc.

IV.Summary (continued)

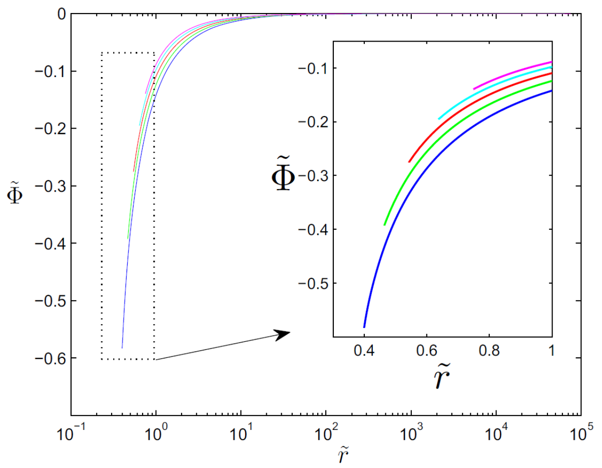
In dim=4 the Dilaton-Einstein-Gauss-Bonnet (dEGB) Gravity (belongs to Horndeski theory)

$$S_{dEGB} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}R + f(\phi)R_{GB}^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_m\right] \qquad f(\phi) = \alpha e^{\gamma\phi}$$

dEGB Black Holes

Scalar Hair

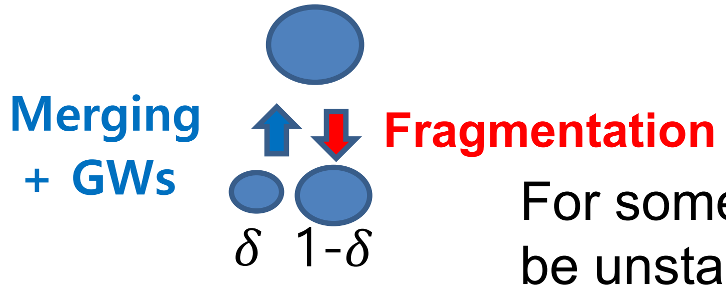
- All DEGB BHs have hairs.
- Hair Charge is 2[nd]ary charge.



Minimum Mass

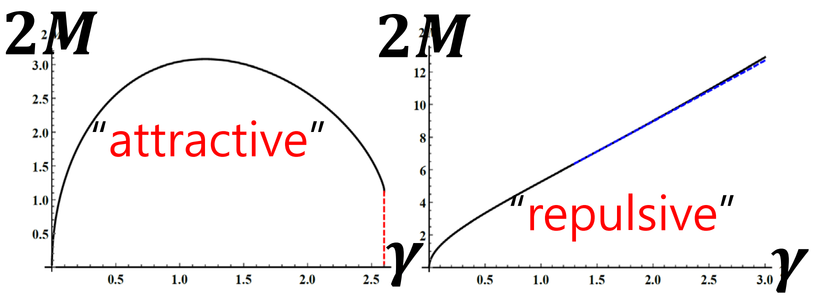
BH mass $M \geq M_{min}$

Fragmentation instability of BHs:



Observed! **?**

For some parameter range, the dEGB BH can be unstable under fragmentation, even if these phases are stable under perturbation.



(a) The black hole mass vs. γ with $\alpha = -1$.

(b) The black hole mass vs. γ with $\alpha = 1$.

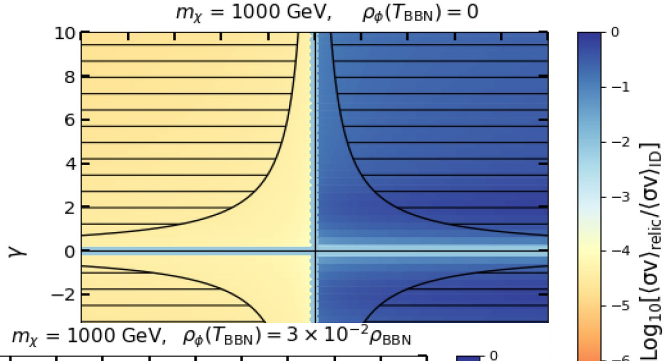
IV. Summary (continued)

dEGB Cosmology

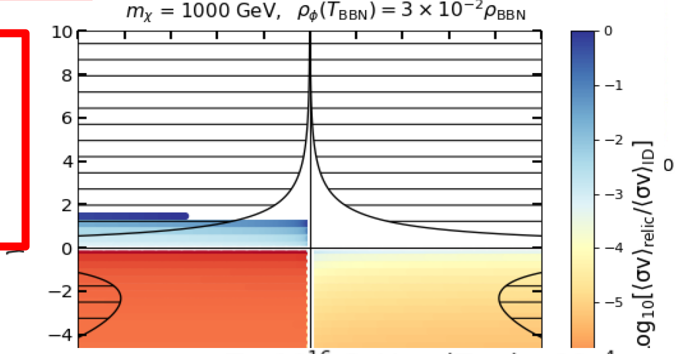
- **WIMPs indirect detection** put some constraints
- **Bounds from GWs** of BH-BH & BH-NS mergers

The WIMP indirect detection bounds are complementary to late-time BBH merger constraints.

White regions in the figures are disfavored by WIMP indirect detection



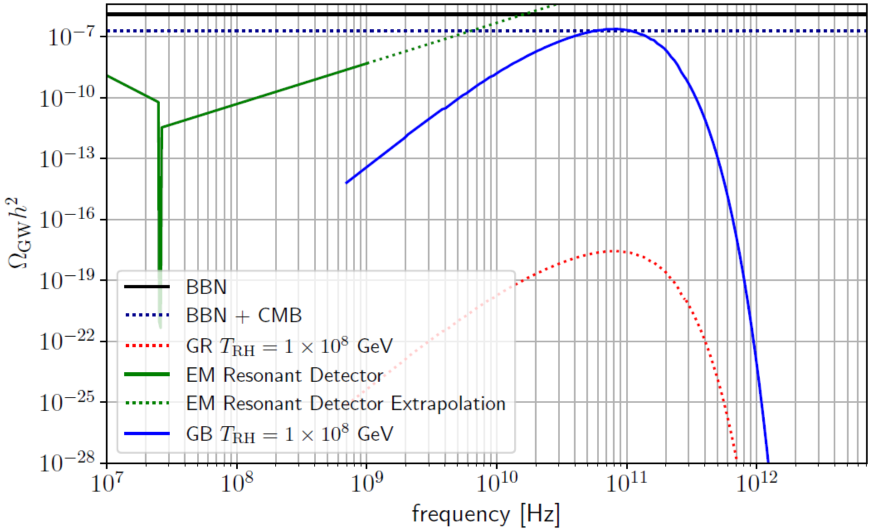
Hatched areas are disallowed by the BBHs

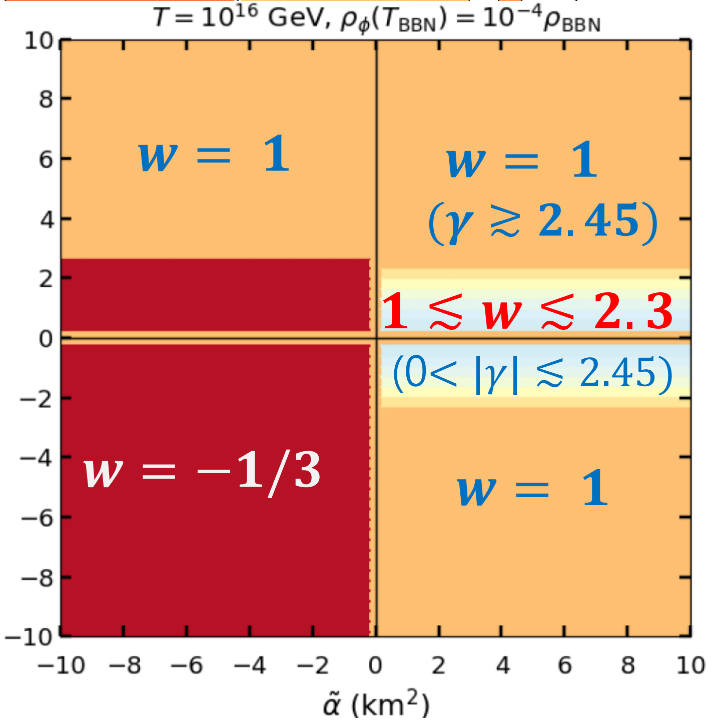


- **New Phases** exists at high enough temperature

NEW PHASEs⟶ | ⟵Rad Dom⟶ | ⟵ Matt ⟶ | ⟵ Λ(DE) ⟶

- the regions w = −1/3 imply a strong enhancement of the expected GWSG produced by the primordial plasma of relativistic particles.
- This allows to put bounds on $T_{RH} \simeq 10^8 - 10^9$ GeV $\ll 10^{16}$ GeV.





Thank you!