Holographic entanglement entropy in the FLRW universe

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Based on a paper in preparation

w/Fumiya Sano (Science Tokyo & IBS) and Yuki Suzuki (YITP)

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Why Quantum Gravity & Information in Expanding Universe?

(motivation from a cosmologist point of view)

1. How **time** emerges in quantum gravity?

2. Can we derive **consistency conditions on cosmological scenarios** in the same spirit as bootstrap, landscape/swampland etc?

As an introduction part of this talk,

let me spend some time to share the second motivation with you all.

Cosmology

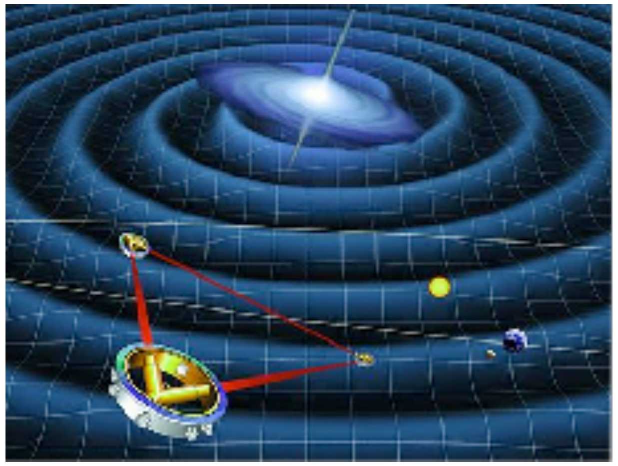
Progress in observational cosmology:

- We obtained a new eye, gravitational waves, of the universe!
- There are many ongoing/future observations & experiments!

(GW, CMB pol., Large Scale Structure, 21cm, tabletop experiments for DM, …)

DESI

SKA



LISA

Cosmology

Progress in observational cosmology:

- We obtained a new eye, gravitational waves, of the universe!
- There are many ongoing/future observations & experiments!

(GW, CMB pol., Large Scale Structure, 21cm, tabletop experiments for DM, …)

Status of the theory side:

- SUSY and WIMP were told to be promising (in my undergrad age).
- But now there is a vast space of cosmological models (ex. DM).

→ Intuitions/lore of QFT (on flat space) do not work anymore?

→ We need new guiding principles in theoretical cosmology!

Recent attempts to **bootstrap approaches for cosmology**

S-matrix/conformal bootstrap

S-matrix bootstrap

- Unitarity has been an important principle for UV completion of EFTs.
- Together w/analyticity, an infinite set of bounds on scattering amplitudes.

※ Positivity bounds on 4pt forward amplitudes [..., Adams et al '06, ...]

※ Non-forward amplitudes w/crossing symmetry
[Bellazzini et al '20, Caron-Huot et al '20, Tolley et al '20, Sinha et al '20, Arkani-Hamed et al '20, ...]

※ Gravitational EFT [Tokuda et al '20, Caron-Huot et al '22, ...]

- Application to particle pheno (ex. SMEFT) and cosmology (ex. inflation, DE)

Conformal bootstrap + AdS/CFT

Similar bounds on AdS scattering [Hartman et al '15, ..., Caron-Huot et al '21, ...]

※ This motivated cosmologists to explore **bootstrap in expanding universe!**

Towards bootstrap in de Sitter spacetime

Unitarity has been important also in phenomenology of inflation.

- Positivity bounds on 4pt functions in multi-field inflation [Suyama-Yamaguchi '07]
- Non-analyticity of inflationary correlators as a probe of new particles
  [Chen-Wang '09, Baumann-Green '11, **TN**-Yamaguchi-Yokoyama '12, Arkani Hamed-Maldacena '15, …]
- Further studies on symmetries and non-analyticity of inflationary correlators
  with catch copy "cosmological bootstrap" [Arkani Hamed-Baumann-Lee-Pimentel '18, …]

The studies so far are mostly on symmetry and non-analyticity,

but the community is aiming at implementing notion of UV completion.

※ dS/CFT gives some inspiration, but bulk unitarity ≠ boundary unitarity

→ **dS bootstrap = conformal bootstrap w/modified notion of unitarity**

Sounds good so far,

but current studies rely on full Lorentz/AdS/dS symmetries.

Actually, phenomenologically interesting situations in cosmology

often come with **spontaneous breaking of boost symmetry**.

Boost symmetry breaking in cosmology

ex. EFT of inflation (EFT of quantum fluctuations during inflation)

$$\mathscr{L} = -M_{\rm Pl}^2 \dot{H}(\partial_\mu \pi)^2 + \alpha \left(-2\dot{\pi} + (\partial_\mu \pi)^2\right)^2 + \beta \left(-2\dot{\pi} + (\partial_\mu \pi)^2\right)^3 + \dots$$

※ $\pi \simeq \frac{\delta\phi}{\dot{\phi}}$ is NG boson for time diffs that nonlinearly realize boosts.

※ Necessary for non-Gaussianities within the scope of near-future observations.

There are some attempts to S-matrix w/o boost symmetry,

but complication of analyticity is an obstruction [Hui et al '23, Creminelli et al '23].

Necessary to explore alternative approaches for less symmetric setups!

Bootstrap w/o boost symmetry?

\# density matrix & equal-time correlators

- Perturbative unitarity from purity bounds [Pueyo-Goodhew-McCulloch-Pajer '24]
- Analytic structure of equal-time correlators in spatial momentum space [Hui-Nicolis-Podo-Zhou '25]

\# Thermodynamic & quantum information perspective of gravity?

- Causality constraints on BH thermo & similarity w/weak gravity [Hamada-**TN**-Shiu '18, …, Abe-**TN**-Medevielle-Yoshimura to appear]
- Toward QG constraints from holography [ex. Harlow-Ooguri '18]

※ consistency of gravity/geometry ⇄ consistency of thermo & info

Is holography for expanding universe useful in this context too?

It is important to explore what we can learn about cosmology *if* we succeed in establishing holography for the universe.

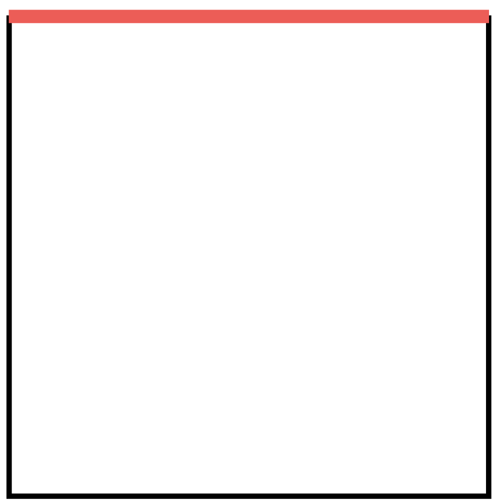
For this purpose,

it is mandatory to extend dS holography to more general FLRW universe,

even at a bottom-up approach level (e.g., assuming the RT formula).

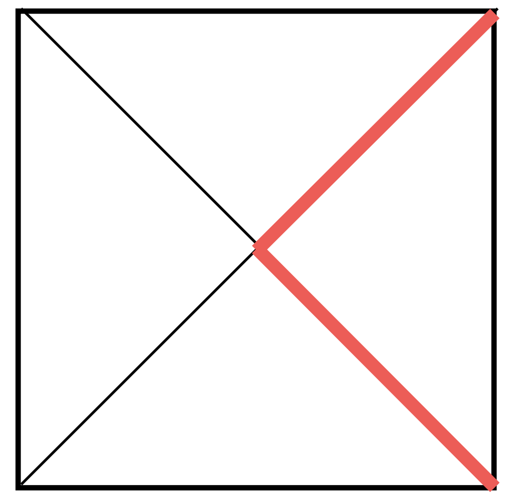
cf. ER = EPR in closed FRW universe [Franken-Partouche-Rondeau-Toumbas ’23]

de Sitter holography



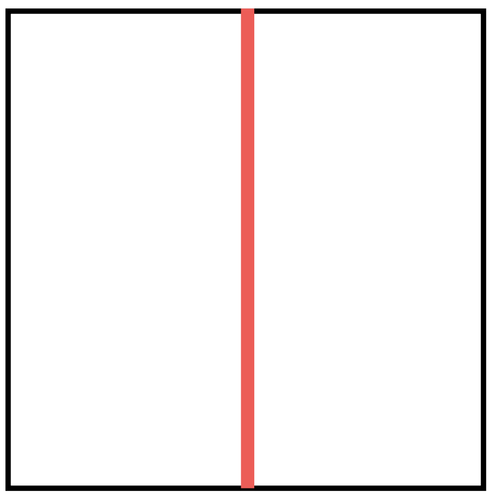
dS/CFT

[Strominger '01]



static patch holography

[Susskind '21]



half holography

[Kawamoto et al '23]

CFT is non-unitary

Bulk and boundary share time & notion of unitarity

In this talk

I consider **analogue of static patch & half holography in 3D flat universes**

and discuss when **subadditivity of holographic entanglement entropy** is satisfied.

Contents

1. Motivation

2. dS holography

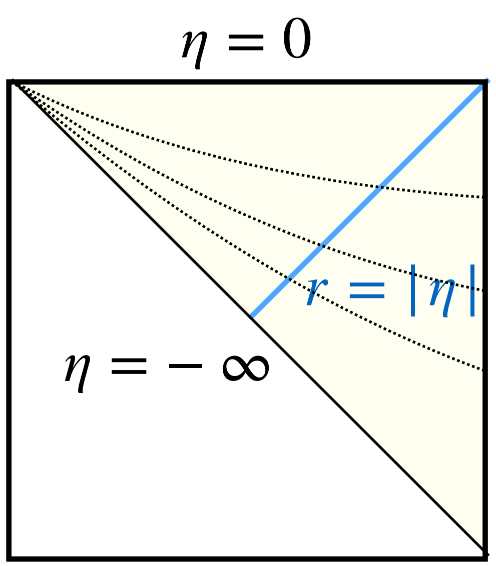
3. FLRW holography

4. Summary and discussion

Setup

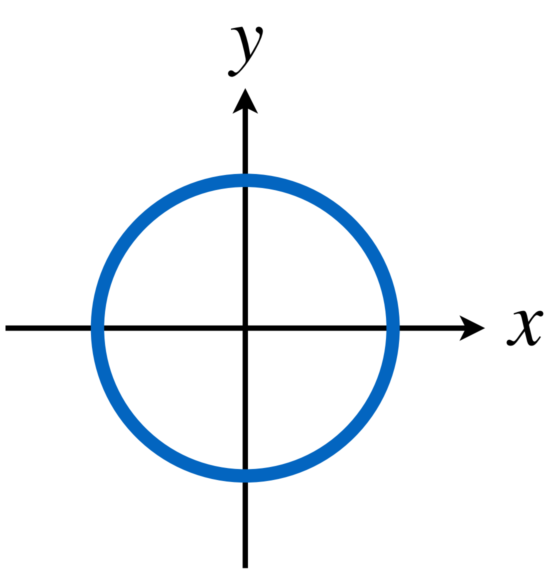
Flat de Sitter universe

Consider flat de Sitter universe in 3 dimensions.



$$ds^2 = \frac{-d\eta^2 + dr^2 + r^2 d\theta^2}{\eta^2} \quad (-\infty < \eta < 0)$$

※ constant-η spatial slice

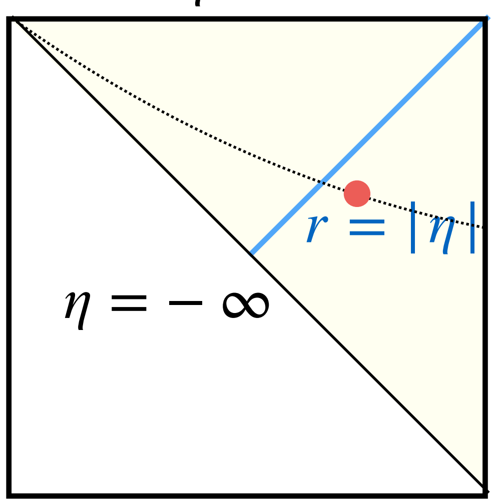


The event horizon of an observer at $r = 0$ is $r = |\eta|$.

We consider two types of symmetric embedding of a 1+1 dim holographic screen.

Horizon type scenario

Consider flat de Sitter universe in 3 dimensions.



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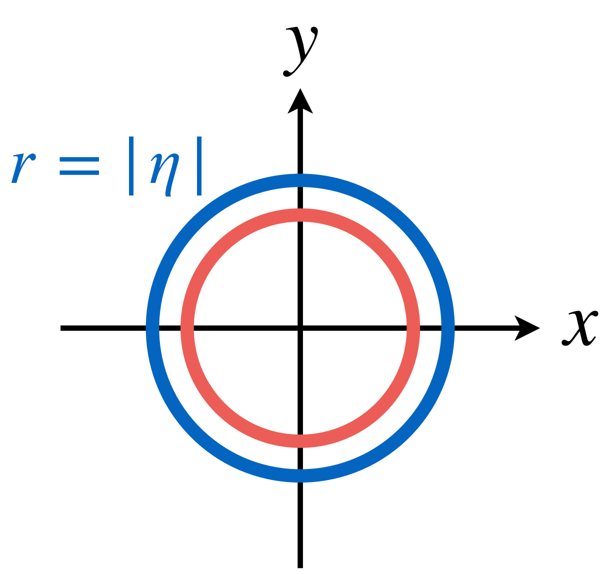


<u>Horizon type scenario</u>

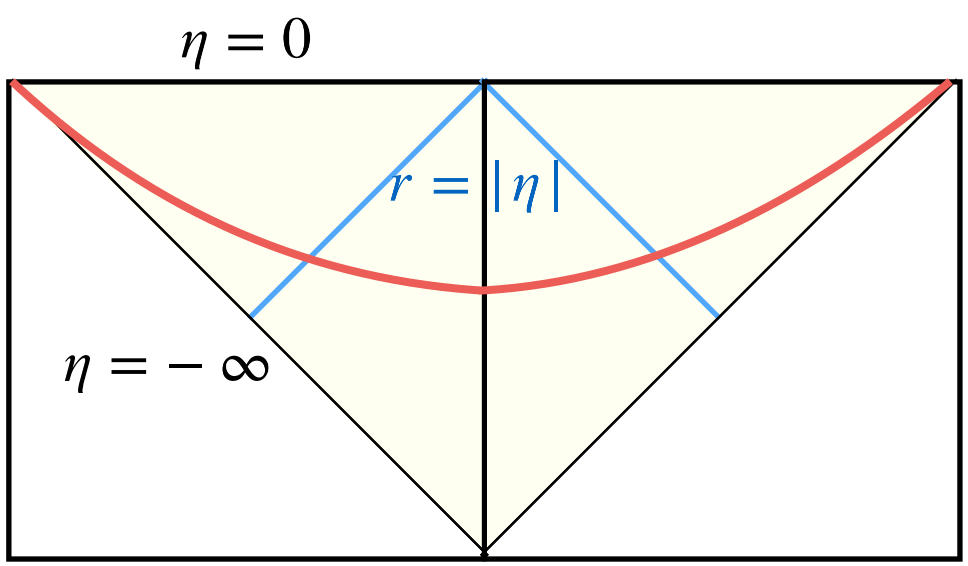
Holographic screen is located at $r = \lambda|\eta|$.

※ λ: screen size relative to horizon size

※ shift symmetry of the spatial coordinate ϕ

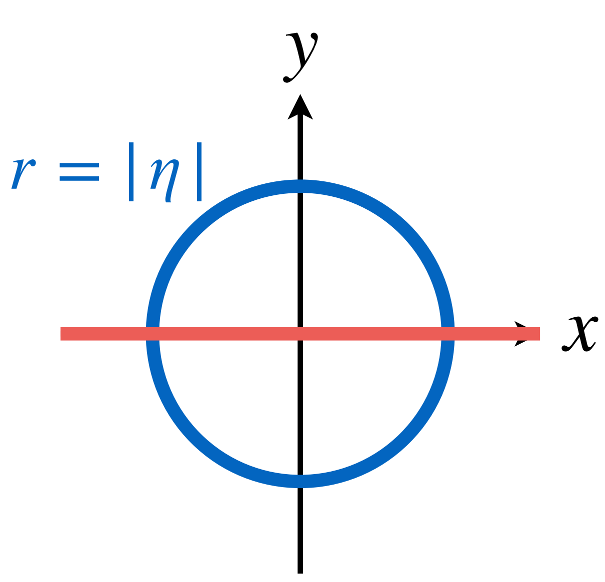
Half holography type scenario

Consider flat de Sitter universe in 3 dimensions.



$$ds^2 = \frac{-d\eta^2 + dr^2 + r^2 d\theta^2}{\eta^2} \quad (-\infty < \eta < 0)$$

※ constant-η spatial slice



<u>Half holography type scenario</u>

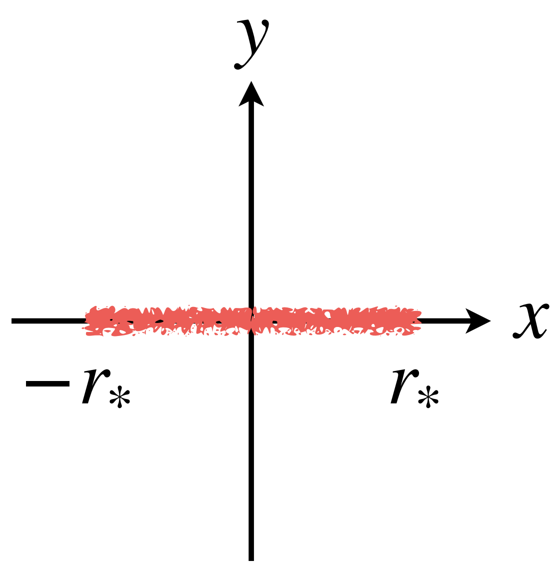
Holographic screen cuts the flat universe in half.

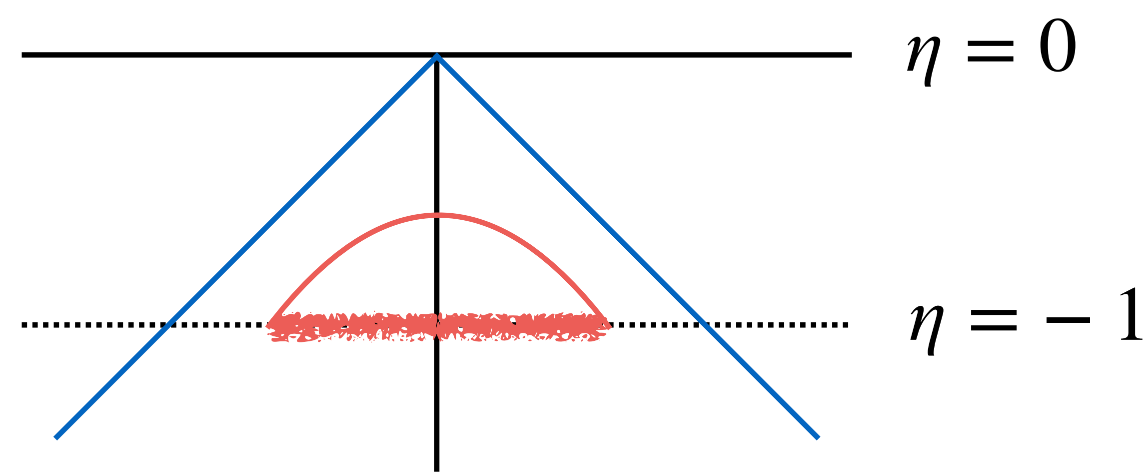
※ shift symmetry of the spatial coordinate x

Holographic entanglement entropy in half holography type scenario

Half holography type scenario (1/2)

Consider a subregion $-r_* \leq x \leq r_*$ at a time $\eta = -1$.





Assuming the RT formula,

holographic EE is determined by geodesic length of the two boundary points.

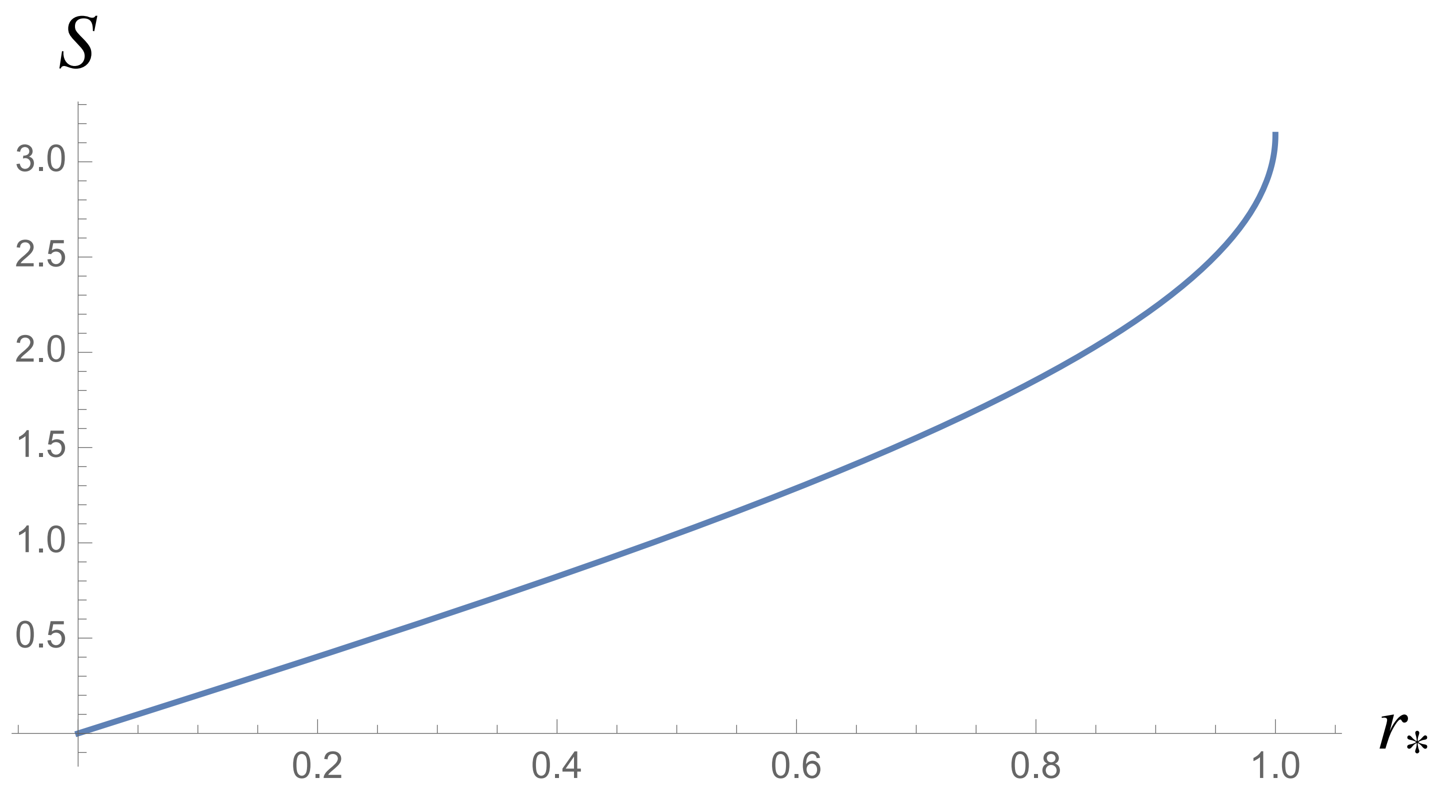
※ Boundary points are connected by a spatial geodesic only when $r_* < 1$,

for which we have $S = \frac{1}{4G} \arccos\left[1 - 2r_*^2\right]$.

※ Otherwise, the RT curve is a union of time-like & spatial curves.

[Kawamoto-Ruan-Suzuki-Takayanagi '23]

Half holography type scenario (2/2)



The holographic EE is convex, so that subadditivity is violated.

More explicitly, we find $\frac{\partial^2 S}{\partial r_*^2} = \frac{1}{2G}\frac{r_*}{(1-r_*^2)^{\frac{3}{2}}} > 0$.

$\rightarrow$ no standard holographic dual in this scenario.

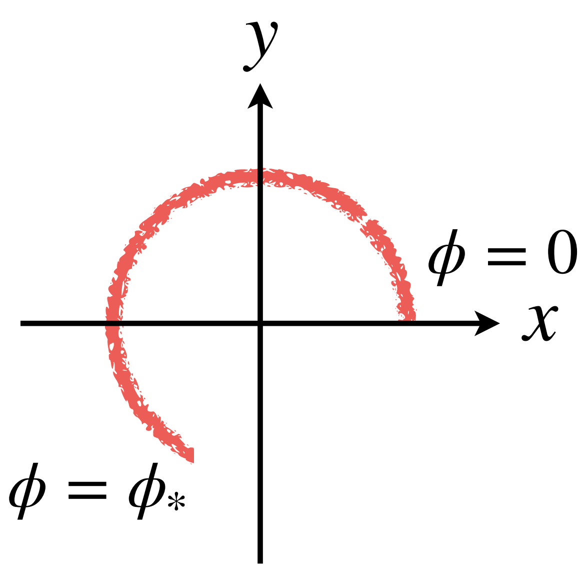
※ For symmetric embedding, $S_{AB} \leq S_A + S_B \leftrightarrow \frac{\partial^2 S(\ell)}{\partial \ell^2} \leq 0$.

※ $4G = 1$ in the plot

Holographic entanglement entropy in horizon type scenario

Horizon type scenario (1/2)

Consider a subregion $0 \leq \phi \leq \phi_*$, $r = \lambda$ at a time $\eta = -1$.



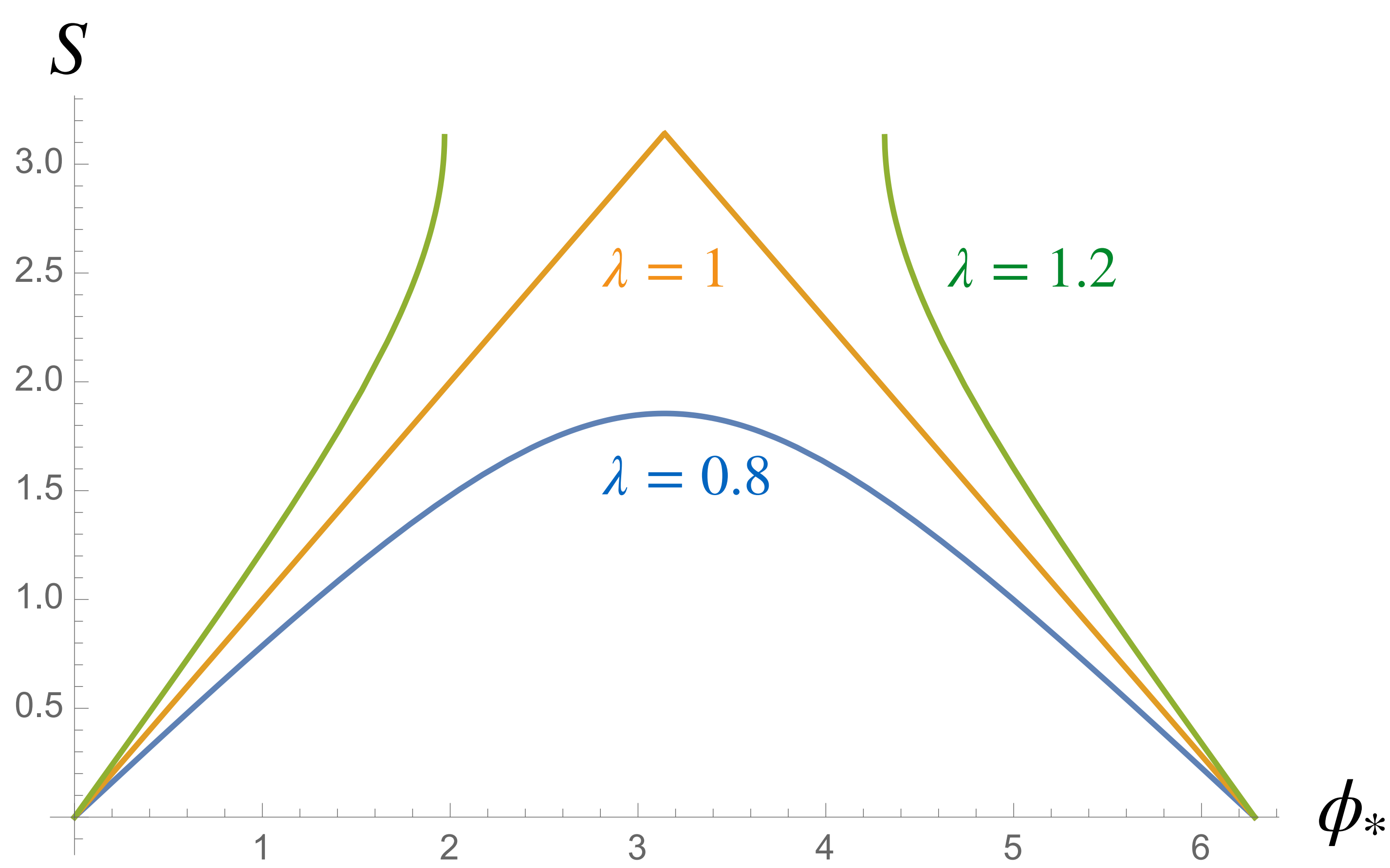
Geodesic length depends only on boundary points, so that we can recycle the previous analysis simply by the replacement $r_* \to \lambda \sin \frac{\phi_*}{2}$.

Holographic EE and its second derivative in the subsystem size read

$$S = \frac{1}{4G} \operatorname{arccos}\left[1 - 2\lambda^2 \sin^2 \frac{\phi_*}{2}\right], \quad \frac{\partial^2 S}{\partial \phi_*^2} = -\frac{(1-\lambda^2)}{8G} \frac{\sqrt{\lambda^2 \sin^2 \frac{\phi_*}{2}}}{(1 - \lambda^2 \sin^2 \frac{\phi_*}{2})^{\frac{3}{2}}}$$

※ Concave when the screen is on or inside the horizon $\lambda \leq 1$.

Horizon type scenario (2/2)



- Subadditivity is satisfied in the horizon type scenario when the holographic screen is on or inside the horizon.
- It is saturated when the screen is on the horizon.

※ $4G = 1$ in the plot

summary

Subadditivity is satisfied only in the horizon type scenario with $\lambda \leq 1$.

※ The same conclusion holds for closed/open de Sitter universe too.

Q. What about more general FLRW universes,

for which apparent horizon $\neq$ event horizon?

Contents

1. Motivation

2. dS holography

3. FLRW holography

4. Summary and discussion

Flat FLRW universe

Consider a flat universe in 3 dimensions: $ds^2 = a(\eta)^2\left[-d\eta^2 + dr^2 + r^2 d\theta^2\right]$.

※ The apparent horizon has a speed of light,

so that its location $r = r_H$ is determined by the Hubble law: $\dot{a}\, r_H = \dfrac{a'}{a}\, r_H = 1$.

For concreteness, we assume the universe is filled with a perfect fluid

of constant equation of state $w = \dfrac{p}{\rho}$ ($\rho > 0$: energy density, p : pressure).

※ Scale factor enjoys a simple power law $a \propto |\eta|^{1/w}$, so that $r_H = w\eta$.

※ Accelerating expansion for $w < 0$: $ds^2 = (-\eta)^{2/w}\left[-d\eta^2 + dr^2 + r^2 d\theta^2\right]$ $(-\infty < \eta < 0)$.

※ Null energy condition is satisfied when $w \geq -1$ (dS: $w = -1$).

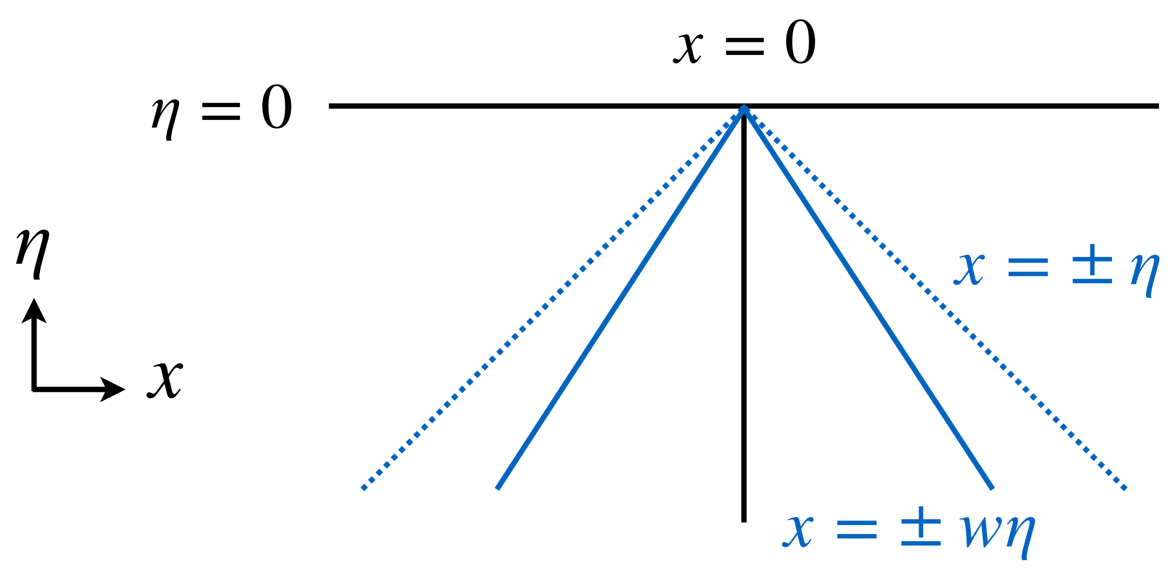
In the following I focus on accelerating universes ($w < 0$)

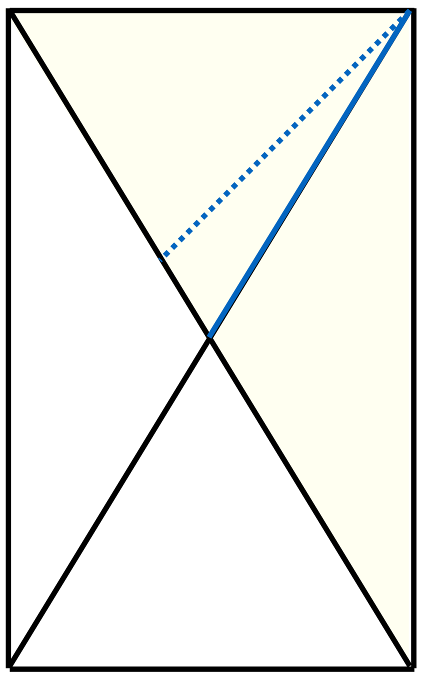
to see the difference of NEC satisfying and violating universe.

※ See our coming paper for decelerating universe and closed/open universe.

(1) Accelerating universe satisfying NEC $-1 \leq w < 0$

Apparent horizons vs event horizon





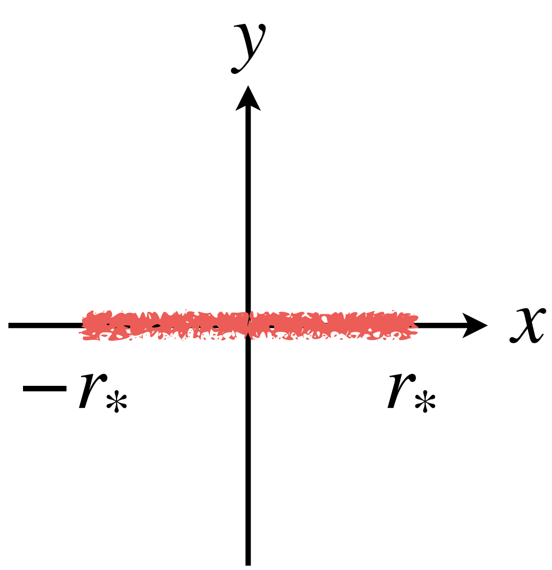
When the null energy condition is satisfied ($-1 < w < 0$),

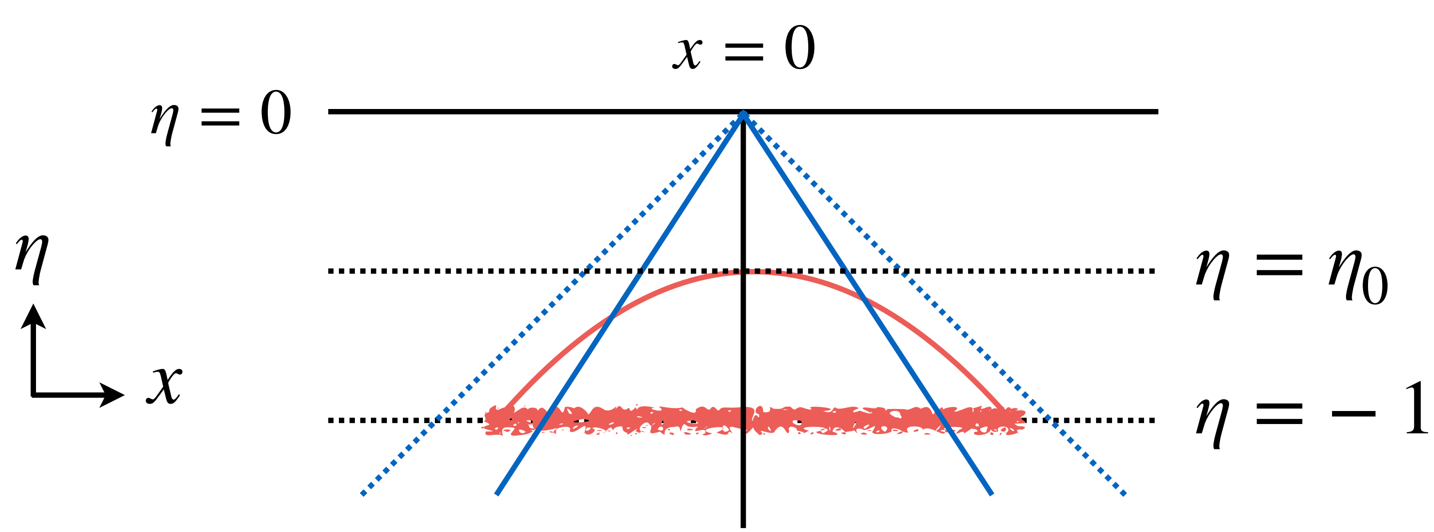
the apparent horizon is inside the event horizon of an observer at the origin.

Half holography type scenario

Half holography type scenario (1/2)

Assume that the holographic screen cuts the flat universe in half and consider a subregion $-r_* \leq x \leq r_*$ at a time $\eta = -1$.





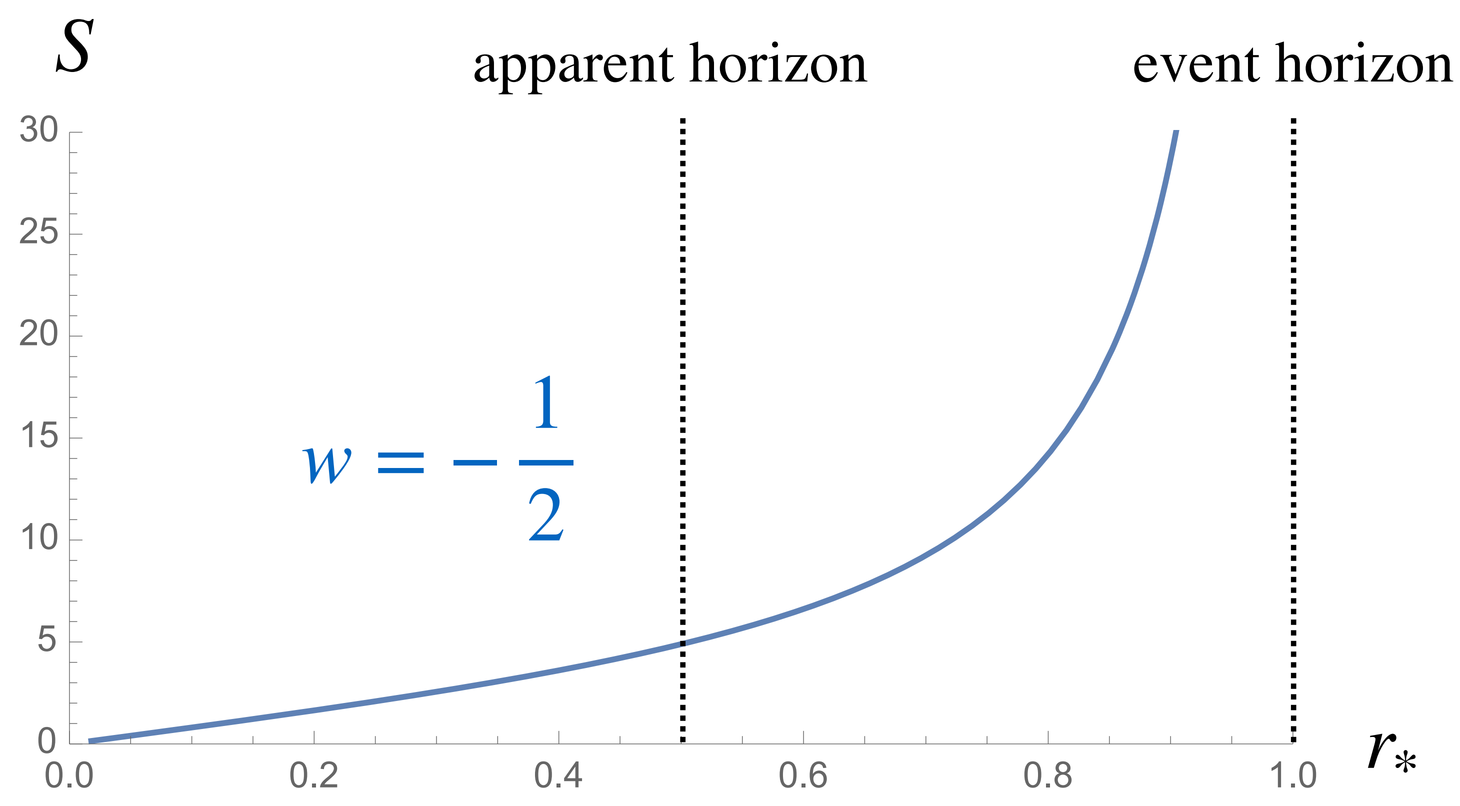
- Boundary points are connected by a spatial geodesic as long as $r_* \leq 1$.

- Subsystem size r_* and holographic EE S as a function of turning time η_0:

$$r_* = \frac{\sqrt{\pi}\Gamma\left(\frac{2+w}{2}\right)}{\Gamma\left(\frac{1+w}{2}\right)}\eta_0 + {}_2F_1\left[\frac{1}{2}, \frac{w}{2}; \frac{2+w}{2}; (-\eta_0)^{-2/w}\right]$$

$$S = \frac{1}{4G}(w\eta_0)^{1+\frac{1}{w}}\left[\frac{\sqrt{\pi}\Gamma\left(\frac{2+w}{2}\right)}{\Gamma\left(\frac{3+w}{2}\right)} - \frac{(-\eta_0)^{-1-\frac{2}{w}}}{1+\frac{w}{2}}{}_2F_1\left[\frac{1}{2}, \frac{2+w}{2}; \frac{4+w}{2}; (-\eta_0)^{-2/w}\right]\right]$$

Half holography type scenario (2/2)



Holographic EE is convex, so that subadditivity is violated.

→ no standard holographic dual in this scenario.

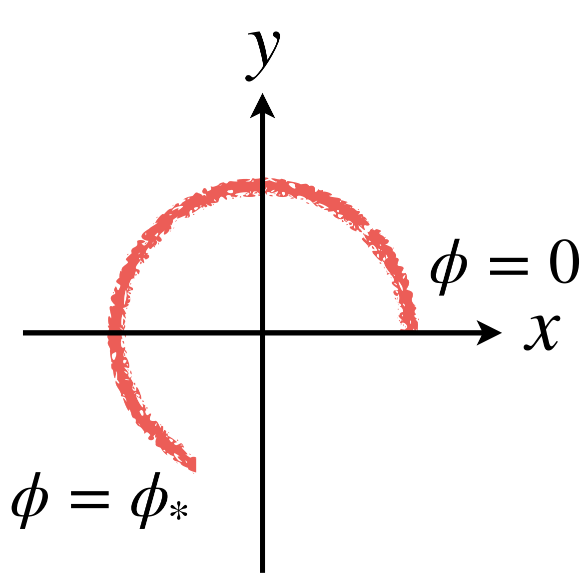
Horizon type scenario

Horizon type scenario (1/2)

Assume that the holographic screen is at $r = \lambda |w|$

and consider a subregion $0 \leq \phi \leq \phi_*$ at a time $\eta = -1$.

※ λ is screen size relative to apparent horizon size.

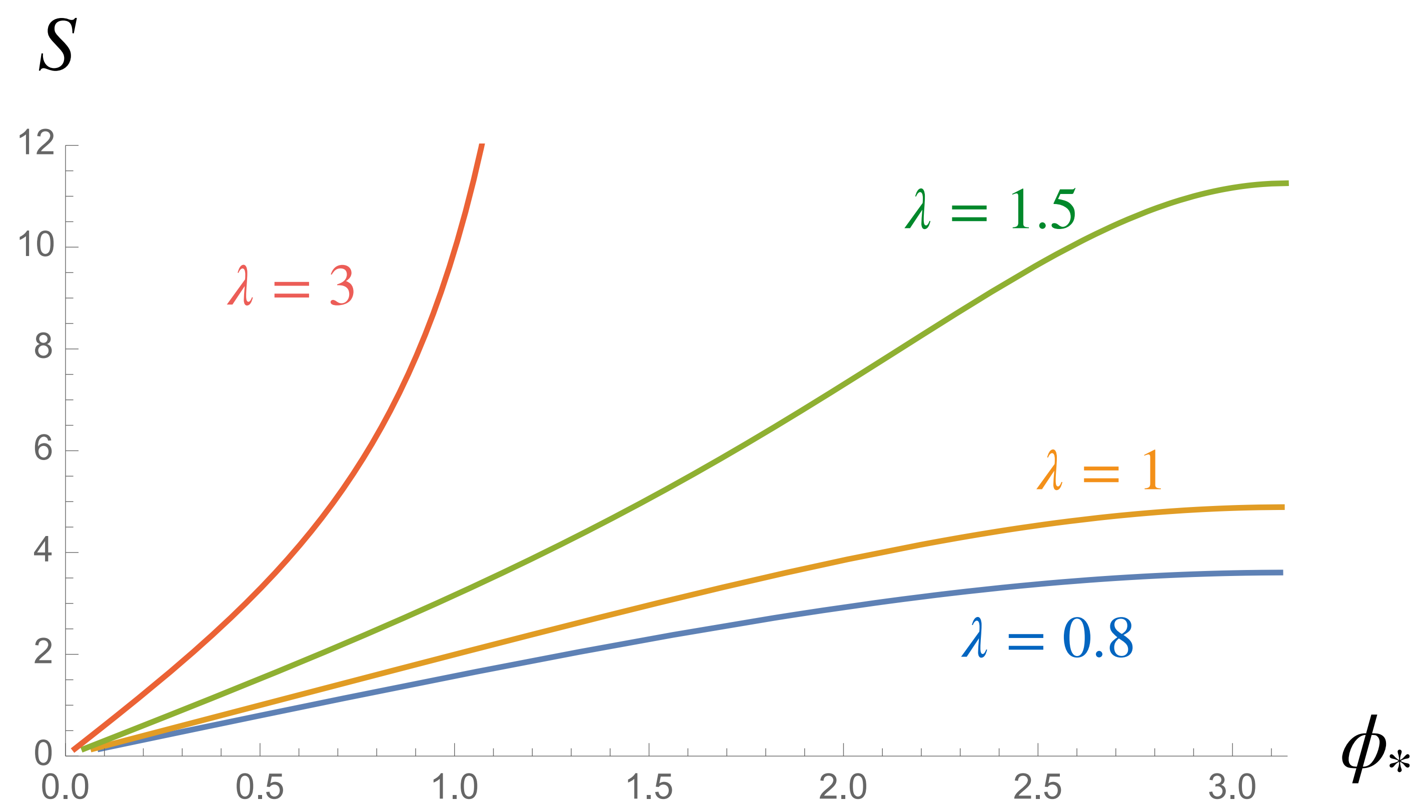


Geodesic length depends only on boundary points,

so that we can recycle the previous analysis

simply by the replacement $r_* \to \lambda |w| \sin \frac{\phi_*}{2}$.

Horizon type scenario (2/2)



$$w = -\frac{1}{2}$$

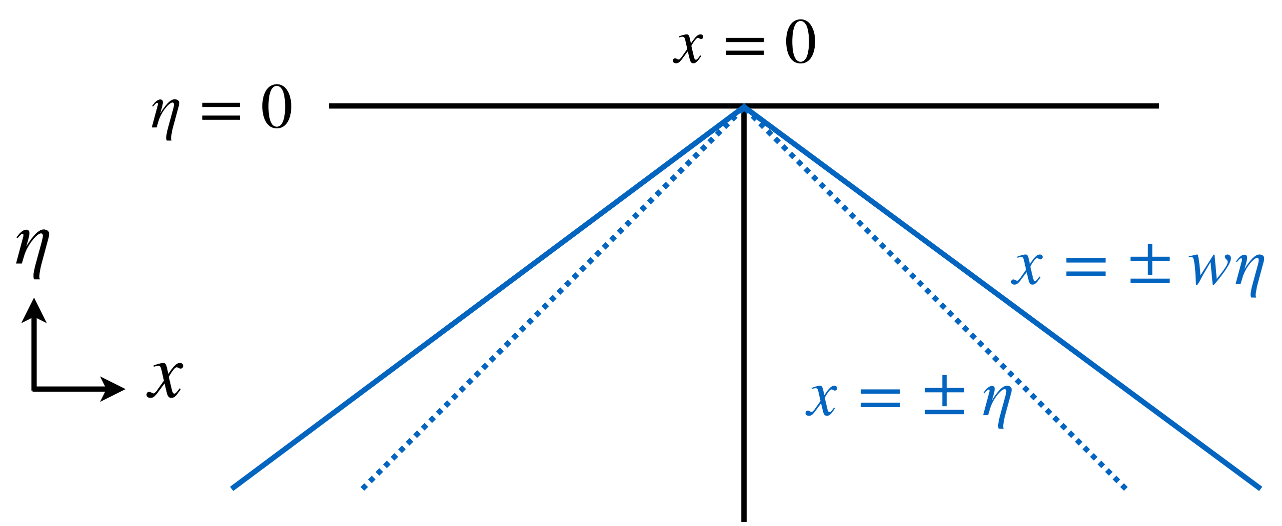
Subadditivity is satisfied in the horizon type scenario when holographic screen is on or inside apparent horizon.

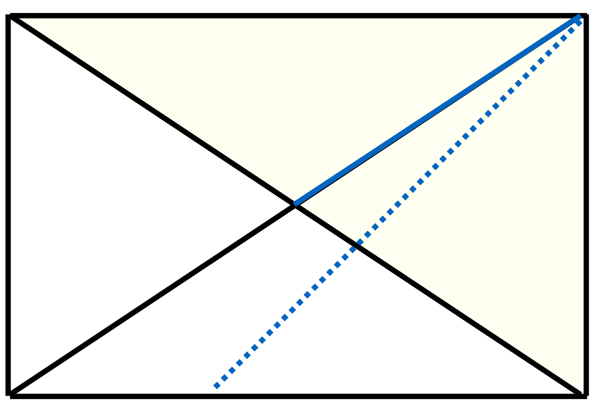
※ This can be checked analytically for general $w \geq -1$, e.g., by performing the small ϕ_* expansion.

※ $4G = 1$ in the plot

(2) Accelerating universe violating NEC $w < -1$

Apparent horizons vs event horizon





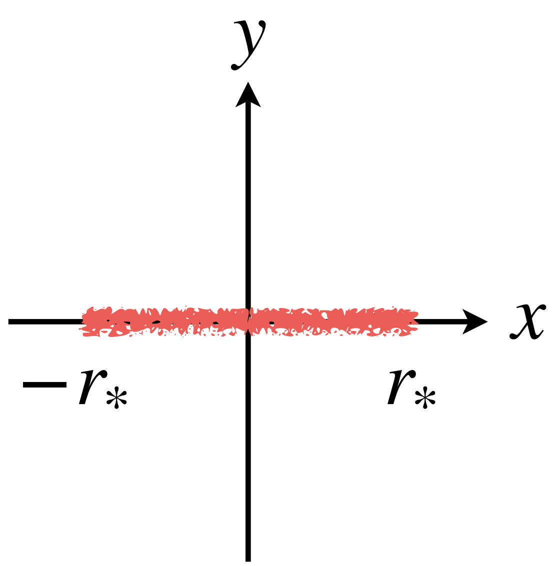
When the null energy condition is violated ($w < -1$),

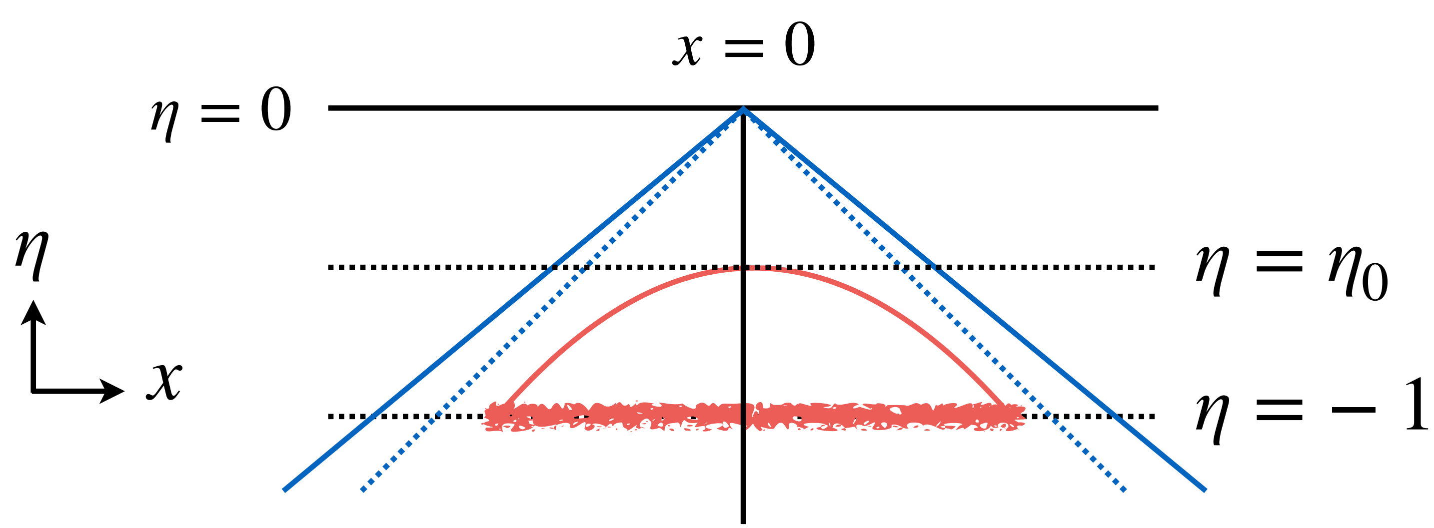
the apparent horizon is outside the event horizon of an observer at the origin.

Half holography type scenario

Half holography type scenario (1/3)

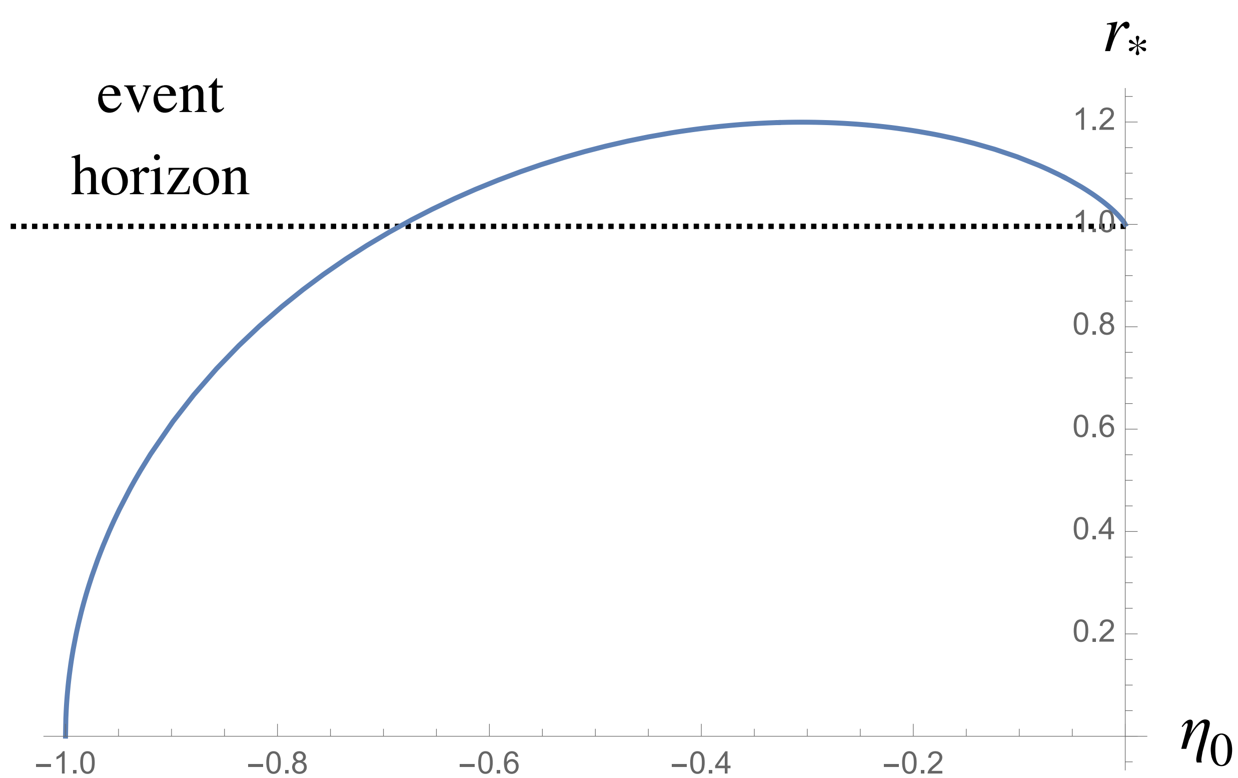
Assume that the holographic screen cuts the flat universe in half and consider a subregion $-r_* \leq x \leq r_*$ at a time $\eta = -1$.



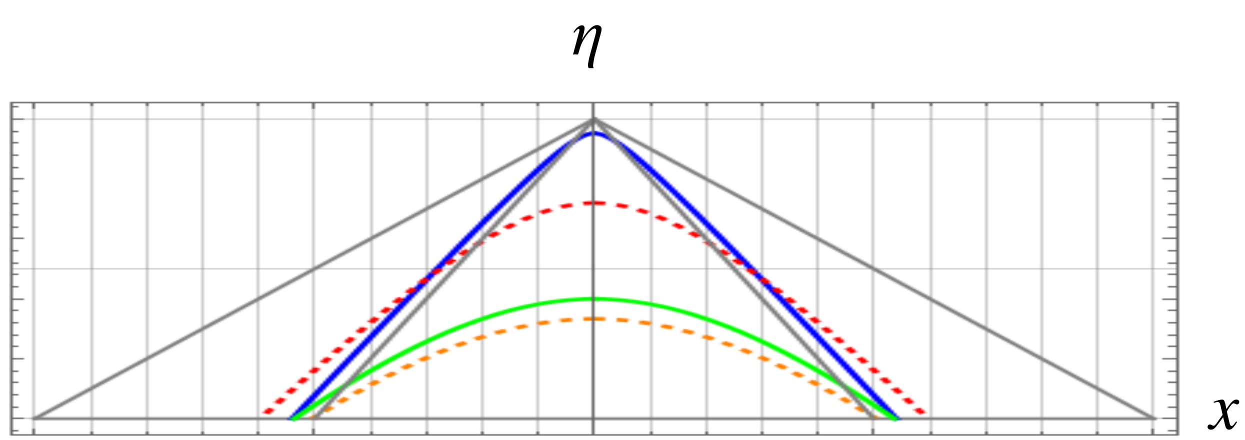


- r_* and holographic EE S as a function of turning time η_0 enjoy the same analytic expression as the $-1 \leq w < 0$ case.
- The geodesic is not unique in some range with $r_* \geq 1$ (next page).

Half holography type scenario (2/3)



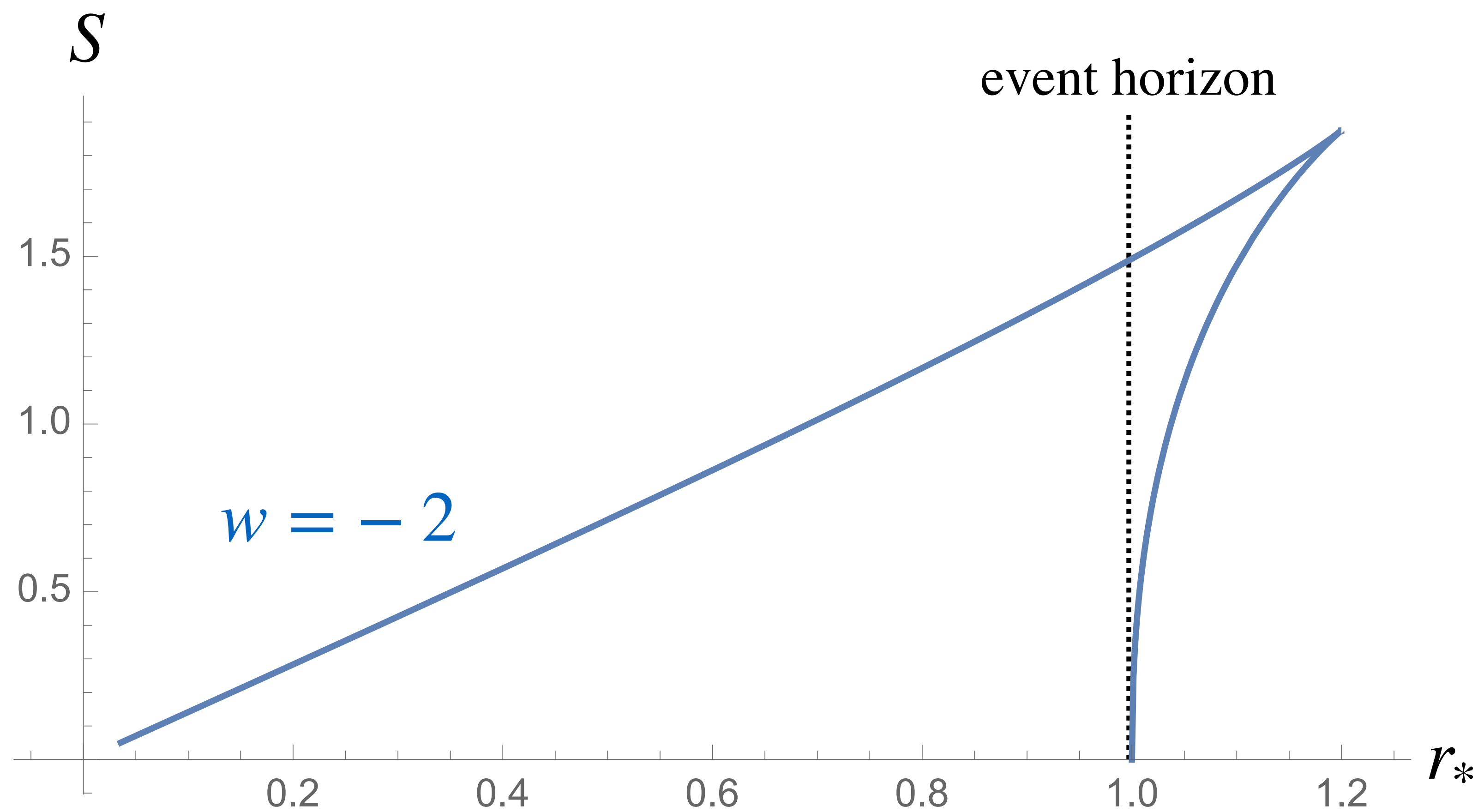
subsystem size vs turning time



profile of geodesics

※ plot is for $w = -2$

Half holography type scenario (3/3)



Subadditivity is violated due to the transition at the event horizon scale $r_* = 1$.

→ no standard holographic dual in this scenario.

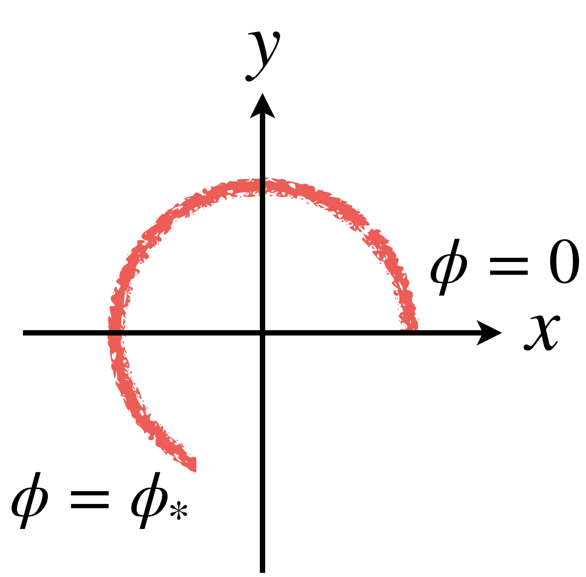
Horizon type scenario

Horizon type scenario (1/2)

Assume that the holographic screen is at $r = \lambda |w|$

and consider a subregion $0 \leq \phi \leq \phi_*$ at a time $\eta = -1$.

※ λ is screen size relative to apparent horizon size.

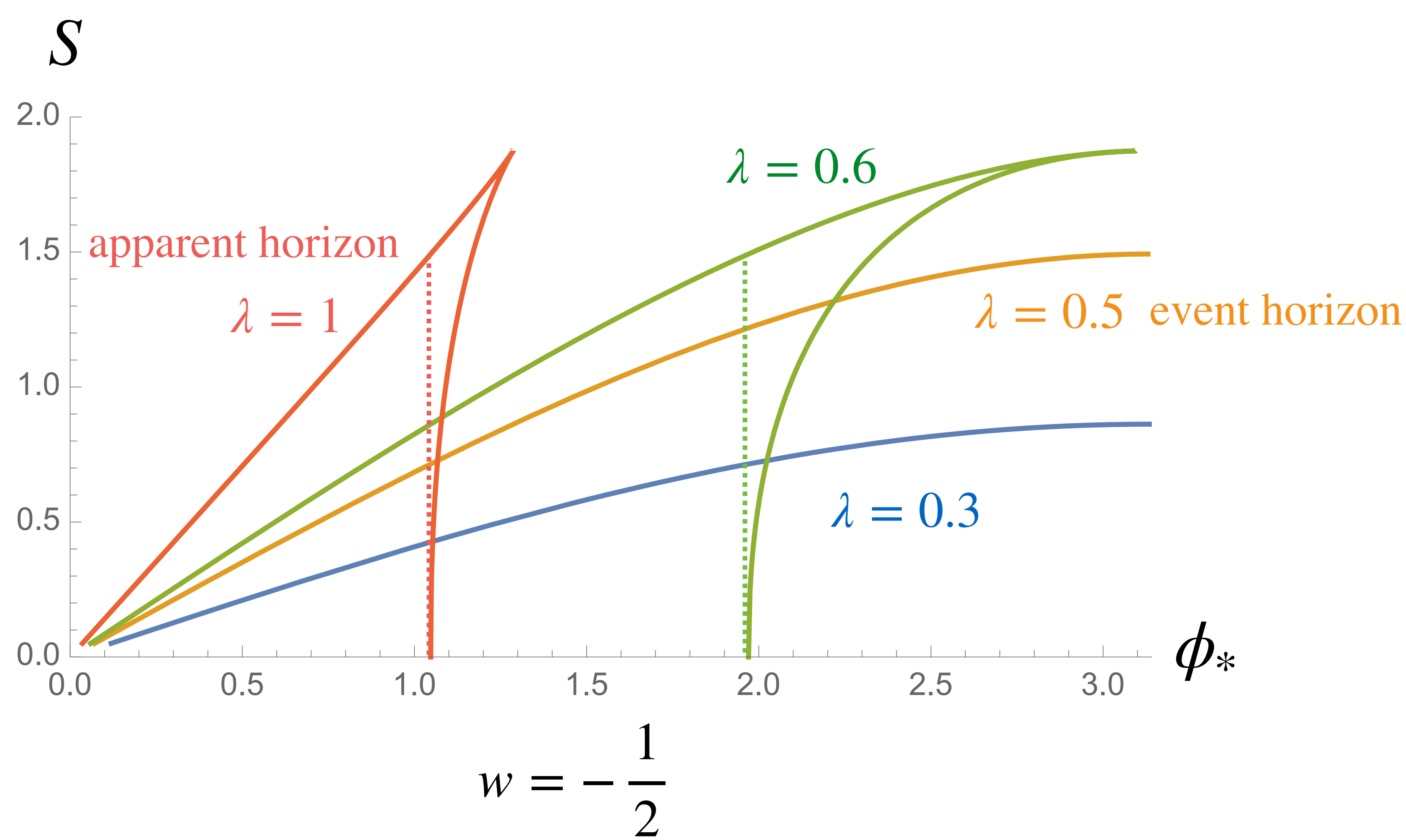


Geodesic length depends only on boundary points,

so that we can recycle the previous analysis

simply by the replacement $r_* \to \lambda |w| \sin \frac{\phi_*}{2}$.

Horizon type scenario (2/2)



Subadditivity is satisfied in the horizon type scenario when holographic screen is on or inside **event horizon**.

※ $4G = 1$ in the plot

Contents

1. Motivation

2. dS holography

3. FLRW holography

4. Summary and discussion

Bootstrap for less symmetric cosmological scenarios is important.

→ Can holography & quantum information offer a new approach?

※ consistency of info ⇄ consistency of geometry (numerically calculable).

※ Discussion: is brane world holography useful to study landscape/swampland?

We studied analogue of static patch & half holography in 3D flat universes.

Holographic EE satisfies subadditivity in horizon like scenario

if the screen is on/inside apparent horizon (NEC satisfying case $w \geq -1$)

event horizon (NEC violating case $w < -1$)

※ Transition at the horizon scale was relevant for $w < -1$.

※ Consistent with [Franken et al'23] based on Bousso bound perspective

Future directions

- Can we derive NEC by studying consistency of other quantities?
- In cosmology, dominant components of energy density are not necessarily dominant components of (typically coarse-grained) entropy density.

  How such "hidden" matter entropy affects the story?

Thank you!