Vacuum Solutions to Einstein Double Field Equation:

Traversable Wormhole and Alternative to de Sitter

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Quantum Gravity and Information in Expanding Universe

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- In electrodynamics, the electric field is denoted by E for obvious reason.

  But, the magnetic field is denoted by B or H instead of M. Why?

Physics: the History of Unification

- Originally (1861), Maxwell wrote his equations with neighboring nine alphabets,

$$B,\ C,\ D\,,\ E,\ F,\ G\,,\ H,\ I,\ J$$

  lacking vector notation.

- It was Heaviside (1864), or **SO**(3), who reformulated them into modern four equations,

$$\nabla\cdot\mathbf{E}=\rho\,,\qquad \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\,,\qquad \nabla\cdot\mathbf{B}=0\,,\qquad \nabla\times\mathbf{B}=\mathbf{J}+\frac{\partial\mathbf{E}}{\partial t}$$

- Minkowski (1908), or **SO**$(1,3)$, then made further simplification,

$$\partial_\lambda F^{\lambda\mu}=J^\mu\,,\qquad \partial_{[\lambda}F_{\mu\nu]}=0$$

- Nonetheless, these simplifications are all rewriting of the same 8 equations in component.

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Physics: the History of Unification

– Similar simplification has been made for the gravitational sector in string theory.

The vanishings of the three β-functions on string worldsheet,

$$R_{\mu\nu} + 2\nabla_{\mu}(\partial_{\nu}\phi) - \tfrac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma} = 0$$

$$\tfrac{1}{2}e^{2\phi}\nabla^{\rho}\left(e^{-2\phi}H_{\rho\mu\nu}\right) = 0$$

$$R + 4\Box\phi - 4\partial_{\mu}\phi\partial^{\mu}\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} = 0$$

have been unified, thanks to $\mathbf{O}(D,D)$, into a single formula, w/ S. Rey, W. Rim, Y. Sakatani 2015

$$G_{AB} = 0\,.$$

which is the vacuum case of more general, **Einstein Double Field Equation,**

$$G_{AB} = T_{AB}$$

where A, B are $\mathbf{O}(D,D)$ vector indices. w/ S. Angus and K. Cho 2018

In contrast to electrodynamics, this simplification turns out to be more than just rewriting,

which I will explain.

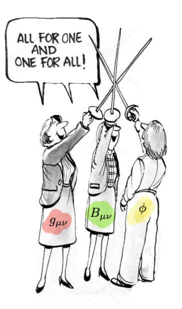
What is the gravitational theory that string theory predicts?

– A conventional answer is General Relativity (GR), in view of $g_{\mu\nu}$ appearing in the quantization of closed string. Needless to say, ever since the formulation of GR, Riemannian geometry has been the mathematical paradigm for theoretical physics where $g_{\mu\nu}$ is privileged to be the only fundamental variable that defines the concept of 'spacetime'.

– However, $g_{\mu\nu}$ is only one segment of the closed string massless sector that includes two additional fields: a two-form potential $B_{\mu\nu}$ and a scalar dilaton ϕ . A better answer is

$$S_{\text{SUGRA}} = \int \mathrm{d}^D x \, \underbrace{\sqrt{-g}e^{-2\phi}\left(R + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right)}_{\text{closed string massless sector as } \textit{gravity}} + \underbrace{\mathcal{L}_{\text{matter}}}_{\text{other sectors as } \textit{matter}}$$

This action secretly keeps $\mathbf{O}(D,D)$ symmetry which transforms the trio $\{g, B, \phi\}$ to one another, and may suggest to regard the whole sector as gravitational and also geometric.



– This idea has come true through the developments in Double Field Theory (DFT) Siegel 1993; Hull-Zwiebach 2009 (*c.f.* Generalised Geometry *à la* Hitchin-Gualtieri) which reformulated the above action in an $\mathbf{O}(D,D)$ manifest way and further evolved into an autonomous gravitational theory.

Our answer: **DFT = $\mathbf{O}(D,D)$ completion of GR .**

Plan of the Talk

I. Review of the $\mathbf{O}(D, D)$-symmetric differential geometry underlying the central formula,

$$G_{AB} = T_{AB} \quad : \quad \textbf{EDFE}$$

where A, B are $\mathbf{O}(D, D)$ vector indices running from 1 to $D+D$.

II. Two vacuum solutions to EDFE, $T_{AB} = 0$:

- Traversable Wormhole for String but not for Particle 2412.04128 w/ H. Jang, H. Lee, & M. Kim
- Accelerating Open Universe as a realistic alternative to de Sitter 2308.07149 w/ H. Lee, L. Velasco-Sevilla, & L. Yin

► Essentially, the negative kinetic term of ϕ realises these solutions in the string frame.

DFT as Gravity of String Theory

– Its Autonomous Structure –

DFT = $\mathbf{O}(D,D)$ completion of GR

– GR is characterised by

$$\mathcal{L}_{\xi}\,, \quad g_{\mu\nu}\,, \quad \nabla_{\lambda}g_{\mu\nu}=0 \quad\Rightarrow\quad \gamma^{\lambda}_{\mu\nu}=\tfrac{1}{2}g^{\lambda\rho}(\partial_{\mu}g_{\rho\nu}+\partial_{\nu}g_{\mu\rho}-\partial_{\rho}g_{\mu\nu})\,, \quad G_{\mu\nu}=\kappa T_{\mu\nu}$$

– Dictated by $\mathbf{O}(D,D)$ Symmetry Principle, DFT has its own version of each item above.

– *A priori*, DFT should be formulated in terms of $\mathbf{O}(D,D)$ covariant fields, rather than $\{g,B,\phi\}$. DFT describes then not only Riemannian but also **non-Riemannian geometries** ($\nexists\, g_{\mu\nu}$).

- DFT becomes a universal framework for (Riemannian) SUGRA as well as (exotic) non-relativistic Newton–Cartan, ultra-relativistic Carroll gravities and fracton physics which are all non-Riemannian.

- DFT enlarges the concept of spacetime geometries, redefining the notion of spacetime singularity, and provides novel string vacua.

Notation

Index	Representation	Metric (raising/lowering indices)
$A, B, \cdots, M, N, \cdots$	$\mathbf{O}(D, D)$ vector	$\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$p, q, \cdots$	$\mathbf{Spin}(1, D{-}1)$ vector	$\eta_{pq} = \text{diag}(-++\cdots+)$
$\alpha, \beta, \cdots$	$\mathbf{Spin}(1, D{-}1)$ spinor	$C_{\alpha\beta}, \quad (\gamma^p)^T = C\gamma^p C^{-1}$
$\bar{p}, \bar{q}, \cdots$	$\mathbf{Spin}(D{-}1, 1)$ vector	$\bar{\eta}_{\bar{p}\bar{q}} = \text{diag}(+--\cdots-)$
$\bar{\alpha}, \bar{\beta}, \cdots$	$\mathbf{Spin}(D{-}1, 1)$ spinor	$\bar{C}_{\bar{\alpha}\bar{\beta}}, \quad (\bar{\gamma}^{\bar{p}})^T = \bar{C}\bar{\gamma}^{\bar{p}}\bar{C}^{-1}$

– DFT employs ‘doubled’ coordinates which the $\mathbf{O}(D, D)$ metric $\mathcal{J}_{AB}$ splits into two parts,

$$x^A = (\tilde{x}_\mu, x^\nu), \qquad \partial_A = (\tilde{\partial}^\mu, \partial_\nu), \qquad \partial^A = \mathcal{J}^{AB}\partial_B = (\partial_\mu, \tilde{\partial}^\nu).$$

– The existence of two separate local Lorentz symmetries, $\mathbf{Spin}(1, D{-}1) \times \mathbf{Spin}(D{-}1, 1)$, indicates the twofold locally inertial frames of the closed-string left and right movers. Duff 1986

Doubled-yet-Gauged Coordinates & Generalised Lie Derivative

- All the functions in DFT $\{\Phi_i, \Phi_j, \cdots\}$ are required to satisfy **section condition**, $\partial_A\partial^A = 0$:

$$\partial_A\partial^A\Phi_i = 0 \quad \& \quad \partial_A\partial^A\left(\Phi_i\Phi_j\right) = 0 \quad \Longrightarrow \quad \partial_A\Phi_i\partial^A\Phi_j = 0\,,$$

which can be generically solved by setting $\tilde{\partial}^\mu = 0$ up to $\mathbf{O}(D, D)$ rotations $\Rightarrow$ choice of section.

- DFT-diffeomorphisms are then given by generalised Lie derivative: Siegel 1993

$$\hat{\mathcal{L}}_\xi T_{M_1\cdots M_n} = \underbrace{\xi^N\partial_N T_{M_1\cdots M_n}}_{\text{transport}} + \underbrace{\omega_T\,\partial_N\xi^N T_{M_1\cdots M_n}}_{\text{weight}} + \sum_{i=1}^n \underbrace{(\partial_{M_i}\xi_N - \partial_N\xi_{M_i})}_{\mathbf{so}(D,D)\ \text{rotation}} T_{M_1\cdots M_{i-1}}{}^N{}_{M_{i+1}\cdots M_n}\,,$$

whose commutators are only closed under the section condition.

With $\xi^M = (\lambda_\mu, \zeta^\nu)$, it unifies B-field gauge symmetry $\delta B = \mathrm{d}\lambda$ and ordinary Lie derivative $\mathcal{L}_\zeta$.

- The section condition is mathematically equivalent to a certain translational invariance:

$$\Phi_i(x) = \Phi_i(x + \Delta)\,, \qquad \Delta^M = \Phi_j\partial^M\Phi_k\,,$$

where Δ^M is said to be 'derivative-index-valued'. JHP 2013

- ► Physics should be invariant under such a shift of the doubled coordinates, suggesting

The doubled coordinates are gauged by derivative-index-valued shifts, satisfying $\Delta^M\partial_M = 0$,

$$x^M \;\sim\; x^M + \Delta^M(x) \quad : \quad \text{Coordinate Gauge Symmetry}$$

Each equivalence class or gauge orbit in $\mathbb{R}^{D+D}$ corresponds to a single physical point.

Fundamental Fields: $\mathcal{H}_{MN}$, d

– DFT has its own dynamical metric $\mathcal{H}_{MN}$ ("generalised metric") satisfying two defining properties,

$$\mathcal{H}_{MN}=\mathcal{H}_{NM}\,,\qquad\qquad \mathcal{H}_{M}{}^{K}\mathcal{H}_{N}{}^{L}\mathcal{J}_{KL}=\mathcal{J}_{MN}$$

Combined with $\mathcal{J}_{MN}=\begin{pmatrix}0&1\\1&0\end{pmatrix}$, it generates a pair of projectors (orthogonal and complete),

$$P_{MN}=\tfrac{1}{2}(\mathcal{J}_{MN}+\mathcal{H}_{MN})\,,\qquad \bar{P}_{MN}=\tfrac{1}{2}(\mathcal{J}_{MN}-\mathcal{H}_{MN})\,;\qquad \begin{array}{ll} P_{L}{}^{M}P_{M}{}^{N}=P_{L}{}^{N}\,, & \bar{P}_{L}{}^{M}\bar{P}_{M}{}^{N}=\bar{P}_{L}{}^{N} \\ P_{L}{}^{M}\bar{P}_{M}{}^{N}=0\,, & P_{M}{}^{N}+\bar{P}_{M}{}^{N}=\delta_{M}{}^{N}\end{array}$$

– Further, taking the 'square root' of each projector,

$$P_{MN}=V_{M}{}^{p}V_{N}{}^{q}\eta_{pq}\,,\qquad \bar{P}_{MN}=\bar{V}_{M}{}^{\bar{p}}\bar{V}_{N}{}^{\bar{q}}\bar{\eta}_{\bar{p}\bar{q}}\,,$$

we obtain a pair of DFT-vielbeins for $\mathbf{Spin}(1,D{-}1)\times\mathbf{Spin}(D{-}1,1)$,

$$V_{Mp}V^{M}{}_{q}=\eta_{pq}\,,\qquad \bar{V}_{M\bar{p}}\bar{V}^{M}{}_{\bar{q}}=\bar{\eta}_{\bar{p}\bar{q}}\,,\qquad V_{Mp}\bar{V}^{M}{}_{\bar{q}}=0\,.$$

Namely, $\mathcal{J}_{MN}$ and $\mathcal{H}_{MN}$ are simultaneously diagonalisable as $\mathbf{diag}(\eta,\bar{\eta})$ and $\mathbf{diag}(\eta,-\bar{\eta})$.

– The $\mathbf{O}(D,D)$ singlet dilaton d sets the DFT-integral measure e^{-2d} (unit diffeomorphic weight).

We shall see $\exists$ various ways of parametrising the fundamental fields: Riemannian *vs.* non-Riemannian.

Christoffel & Spin Connections

w/ Imtak Jeon & Kanghoon Lee 2010, 2011

– In GR, the Christoffel symbol is the unique metric-compatible connection, $\nabla_\lambda g_{\mu\nu} = 0$, which satisfies either a torsionless condition, or an alternative condition that the metric is the only ingredient to form the connection.

– Similarly, the DFT-Christoffel connection can be uniquely fixed,

$$\Gamma_{LMN} = 2\left(P\partial_L P\bar{P}\right)_{[MN]} + 2\left(\bar{P}_{[M}{}^{J}\bar{P}_{N]}{}^{K} - P_{[M}{}^{J}P_{N]}{}^{K}\right)\partial_J P_{KL} - \tfrac{4}{D-1}\left(\bar{P}_{L[M}\bar{P}_{N]}{}^{K} + P_{L[M}P_{N]}{}^{K}\right)\left(\partial_K d + (P\partial^J P\bar{P})_{[JK]}\right)$$

satisfying, in particular, the compatibility

$$\nabla_L \mathcal{J}_{MN} = 0\,, \qquad \nabla_L \mathcal{H}_{MN} = 0\,, \qquad \nabla_L d = -\tfrac{1}{2}e^{2d}\nabla_L\left(e^{-2d}\right) = 0$$

where $\nabla_L = \partial_L + \Gamma_L$ is defined by

$$\nabla_L T_{M_1\cdots M_n} := \partial_L T_{M_1\cdots M_n} - \omega_T \Gamma^K{}_{KL} T_{M_1\cdots M_n} + \sum_{i=1}^{n} \Gamma_{LM_i}{}^{N} T_{M_1\cdots M_{i-1}NM_{i+1}\cdots M_n}\,.$$

– One can further obtain the twofold spin connections,

$$\Phi_{Mpq} = V^N{}_p \nabla_M V_{Nq}\,, \qquad \bar{\Phi}_{M\bar{p}\bar{q}} = \bar{V}^N{}_{\bar{p}} \nabla_M \bar{V}_{N\bar{q}}$$

from the requirement that the 'master' covariant derivative

$$\mathcal{D}_M = \partial_M + \Gamma_M + \Phi_M + \bar{\Phi}_M = \nabla_M + \Phi_M + \bar{\Phi}_M$$

should be compatible with the DFT-vielbeins,

$$\mathcal{D}_M V_{Np} = \nabla_M V_{Np} + \Phi_{Mp}{}^{q} V_{Nq} = 0\,, \qquad \mathcal{D}_M \bar{V}_{N\bar{p}} = \nabla_M \bar{V}_{N\bar{p}} + \bar{\Phi}_{M\bar{p}}{}^{\bar{q}} \bar{V}_{N\bar{q}} = 0\,.$$

Curvature & (Semi-)covariance

w/ Imtak Jeon & Kanghoon Lee 2010, 2011

– Semi-covariant Riemann curvature :

$$S_{KLMN} = S_{[KL][MN]} = S_{MNKL} := \tfrac{1}{2}\left(R_{KLMN} + R_{MNKL} - \Gamma^{J}{}_{KL}\Gamma_{JMN}\right) , \qquad S_{[KLM]N} = 0 ,$$

where R_{ABCD} denotes the ordinary "field strength",

$$R_{CDAB} = \partial_A\Gamma_{BCD} - \partial_B\Gamma_{ACD} + \Gamma_{AC}{}^{E}\Gamma_{BED} - \Gamma_{BC}{}^{E}\Gamma_{AED} .$$

By construction, like in GR, it varies as 'total derivative':

$$\delta S_{ABCD} = \nabla_{[A}\delta\Gamma_{B]CD} + \nabla_{[C}\delta\Gamma_{D]AB} \qquad \Longrightarrow \qquad \text{hence good for variational principle.}$$

– Our formalism is 'semi-covariant', meaning

$$\delta_\xi\left(\nabla_L T_{M_1\cdots M_n}\right) = \hat{\mathcal{L}}_\xi\left(\nabla_L T_{M_1\cdots M_n}\right) + \textstyle\sum_{i=1}^{n} 2(\mathcal{P}+\bar{\mathcal{P}})_{LM_i}{}^{NEFG}\partial_E\partial_F\xi_G\, T_{M_1\cdots M_{i-1}NM_{i+1}\cdots M_n}$$

$$\delta_\xi S_{KLMN} = \hat{\mathcal{L}}_\xi S_{KLMN} + 2\nabla_{[K}\left[(\mathcal{P}+\bar{\mathcal{P}})_{L][MN]}{}^{EFG}\partial_E\partial_F\xi_G\right] + 2\nabla_{[M}\left[(\mathcal{P}+\bar{\mathcal{P}})_{N][KL]}{}^{EFG}\partial_E\partial_F\xi_G\right]$$

$$\delta_\xi\Gamma_{CAB} = \hat{\mathcal{L}}_\xi\Gamma_{CAB} + 2\left[(\mathcal{P}+\bar{\mathcal{P}})_{CAB}{}^{FDE} - \delta_C^F\delta_A^D\delta_B^E\right]\partial_F\partial_{[D}\xi_{E]}$$

where $\mathcal{P}_{LMN}{}^{EFG} = P_L{}^E P_{[M}{}^{[F}P_{N]}{}^{G]} + \frac{2}{P_K{}^K-1}P_{L[M}P_{N]}{}^{[F}P^{G]E}$ and similarly $\bar{\mathcal{P}}_{LMN}{}^{EFG}$ is set with $\bar{P}_M{}^N$.

► The red-colored anomalies can be easily projected out to give fully covariant objects, *e.g.*

$$\mathcal{D}_p T_{\bar{q}} = \nabla_L T_M V^L{}_p \bar{V}^M{}_{\bar{q}} , \qquad S_{p\bar{q}} = S^L{}_{MLN}V^M{}_p\bar{V}^N{}_{\bar{q}} \ \ (\text{Ricci}) , \qquad S_{(0)} = S_{pq}{}^{pq} - S_{\bar{p}\bar{q}}{}^{\bar{p}\bar{q}} \ \ (\text{scalar})$$

$$\gamma^p\mathcal{D}_p\rho ,\ \mathcal{D}_{\bar{p}}\rho\ (\text{Dirac}) , \quad \mathcal{D}_\pm\mathcal{C} = \gamma^p\mathcal{D}_p\mathcal{C} \pm \gamma^{(D+1)}\mathcal{D}_{\bar{p}}\mathcal{C}\bar{\gamma}^{\bar{p}} , \quad (\mathcal{D}_\pm)^2 = 0 \ \Rightarrow\ \mathcal{F} = \mathcal{D}_+\mathcal{C} \ \ (\text{bispinorial RR})$$

$\mathbf{O}(D,D)$ symmetric 'minimal' coupling

The pure DFT action is then given by $e^{-2d}S_{(0)}$ and can further 'minimally' couple to 'matter':

– $D=10$, Type II SDFT (full order 32 SUSY, pseudo action) w/ I. Jeon, K. Lee & Y. Suh 2012

$$\mathcal{L}_{\text{type II}} = e^{-2d}\Big[\,\tfrac{1}{8}S_{(0)} + \tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}}) + i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q} + i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}_{p}\rho - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'$$
$$-i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}_{q}\psi_{\bar{p}} + i\bar{\psi}'^{p}\mathcal{D}_{p}\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_{p}\,\Big]$$

which unifies IIA and IIB SUGRAs as different solution sectors.

The full order SUSY, *i.e.* quartic order in fermions, has been recently verified by D. Butter 2022.

– $D=4$ DFT minimally coupled to the Standard Model w/ K. Choi 2015 PRL

$$\mathcal{L}_{\text{SM}} = e^{-2d}\Big[\tfrac{1}{16\pi G_N}S_{(0)} + \textstyle\sum_{A}\mathrm{Tr}(F_{p\bar{q}}F^{p\bar{q}}) - \mathcal{H}^{MN}(\mathcal{D}_{M}\phi)^{\dagger}\mathcal{D}_{N}\phi - V(\phi)\Big]$$
$$+\textstyle\sum_{\psi}\bar{\psi}\gamma^{p}\mathcal{D}_{p}\psi + \sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\psi' + y_{d}\,\bar{q}\cdot\phi\,d + y_{u}\,\bar{q}\cdot\tilde{\phi}\,u + y_{e}\,\bar{l}'\cdot\phi\,e'$$

Conjecture: quarks and leptons are distinct kinds of spinors, one for **Spin**$(1,3)$ and the other for **Spin**$(3,1)$.

- Every single term in the above Lagrangians is completely covariant, w.r.t. $\mathbf{O}(D,D)$ rotations, DFT-diffeomorphisms, and twofold local Lorentz symmetries.

Einstein Double Field Equation

w/ S. Angus and K. Cho 2018

– Now we consider a general DFT action coupled to generic matter, say Υ's,

$$\text{Action} = \int_{\Sigma} e^{-2d}\Big[\,\tfrac{1}{2\kappa}S_{(0)} + L_{\text{matter}}\big(\Upsilon, \mathcal{D}_M\Upsilon\big)\Big]\,.$$

The variational principle,

$$\delta\text{Action} = \int_{\Sigma} e^{-2d}\left[2\bar{V}^{M\bar{q}}\delta V_M{}^{p}\big(\tfrac{1}{\kappa}S_{p\bar{q}} - K_{p\bar{q}}\big) - \delta d\big(\tfrac{1}{\kappa}S_{(0)} - T_{(0)}\big) + \delta\Upsilon\,\frac{\delta L_{\text{matter}}}{\delta\Upsilon}\right]$$

leads us to define out of L_{matter},

$$K_{p\bar{q}} := \tfrac{1}{2}\left(V_{Mp}\frac{\delta L_{\text{matter}}}{\delta\bar{V}_M{}^{\bar{q}}} - \bar{V}_{M\bar{q}}\frac{\delta L_{\text{matter}}}{\delta V_M{}^{p}}\right) = -2V_{Mp}\bar{V}_{N\bar{q}}\frac{\delta L_{\text{matter}}}{\delta\mathcal{H}_{MN}}\,, \qquad T_{(0)} := e^{2d}\times\frac{\delta\big(e^{-2d}L_{\text{matter}}\big)}{\delta d}$$

– Subsequently, the 'General Covariance',

$$0 = \int_{\Sigma} e^{-2d}\left[\tfrac{1}{\kappa}\xi^N\mathcal{D}^M\Big\{4V_{[M}{}^{p}\bar{V}_{N]}{}^{\bar{q}}\big(S_{p\bar{q}} - \kappa K_{p\bar{q}}\big) - \tfrac{1}{2}\mathcal{J}_{MN}\big(S_{(0)} - \kappa T_{(0)}\big)\Big\} + \hat{\mathcal{L}}_{\xi}\Upsilon\,\frac{\delta L_{\text{matter}}}{\delta\Upsilon}\right]$$

guides us to identify the Einstein curvature, w/ S. Rey, W. Rim, Y. Sakatani 2015

$$G_{MN} := 4V_{[M}{}^{p}\bar{V}_{N]}{}^{\bar{q}}S_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{MN}S_{(0)}\,, \qquad \nabla_M G^{MN} = 0 \qquad \text{(off-shell)}$$

and the Energy-Momentum tensor,

$$T_{MN} := 4V_{[M}{}^{p}\bar{V}_{N]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{MN}T_{(0)}\,, \qquad \nabla_M T^{MN} = 0 \qquad \text{(on-shell)}$$

► **Equating them, we obtain the Einstein equation of DFT, or EDFEs:** $G_{MN} = \kappa T_{MN}$

Question: Is DFT a mere reformulation of SUGRA in an $\mathbf{O}(D,D)$ manifest manner?

The answer would be (and had been) yes, if we assume

$$\mathcal{H}_{MN}=\begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g-Bg^{-1}B \end{pmatrix}\,, \qquad e^{-2d}=\sqrt{|g|}e^{-2\phi}$$

Giveon, Rabinovici, Veneziano '89, Duff '90

Upon this parametrisation, EDFE, $G_{MN}=T_{MN}$, reduces to

$$\begin{array}{rcccc} R_{\mu\nu}+2\bigtriangledown_{\mu}(\partial_{\nu}\phi)-\frac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma} & = & K_{(\mu\nu)} & \Leftarrow & \delta g_{\mu\nu} \\ \frac{1}{2}e^{2\phi}\bigtriangledown^{\rho}\left(e^{-2\phi}H_{\rho\mu\nu}\right) & = & K_{[\mu\nu]} & \Leftarrow & \delta B_{\mu\nu} \\ R+4\Box\phi-4\partial_{\mu}\phi\partial^{\mu}\phi-\frac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} & = & T_{(0)} & \Leftarrow & \delta d \end{array}$$

And the 'pure' DFT action reduces

$$\int \mathrm{d}^{D}x\ e^{-2d}S_{(0)} \ = \int \mathrm{d}^{D}x\ \sqrt{-g}e^{-2\phi}\left(R+4\partial_{\mu}\phi\partial^{\mu}\phi-\tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right).$$

– However, DFT works perfectly fine, with any generalised metric that satisfies the defining properties: $\mathcal{H}_{MN}=\mathcal{H}_{NM},\ \ \mathcal{H}_{M}{}^{K}\mathcal{H}_{N}{}^{L}\mathcal{J}_{KL}=\mathcal{J}_{MN}$. And the above parametrisation is not the most general solution to them. Hence the answer to the question is **No.**

– In fact, the most or perfectly symmetric vacua of DFT are given by

$$\mathcal{H}_{MN}=\pm\mathcal{J}_{MN}=\begin{pmatrix} \mathbf{0} & \pm\mathbf{1} \\ \pm\mathbf{1} & \mathbf{0} \end{pmatrix}$$ which do not admit any Riemannian interpretation.

Non-Riemannian Geometry

- **First example** w/ Kanghoon Lee 2013
- **Non-Relativistic String** w/ Sung Moon Ko, Charles Melby-Thompson, Rene Meyer 2015
- **Classification** w/ Kevin Morand 2017
- **Moduli-free Kaluza–Klein reduction** w/ Kyoungho Cho and Kevin Morand 2018
- **-Dynamics through EDFE** w/ Kyoungho Cho 2019
- **Quantum Consistency on Worldsheet** w/ Shigeki Sugimoto 2020 PRL
- ∞**-dimensional Isometries** w/ Chris Blair and Gerben Oling 2020
- **Some Riemannian Singularities** $=$ **Non-Riemannian Regularity** w/ Kevin Morand and Miok Park 2021 PRL
- **Fracton Physics** w/ Stephen Angus and Minkyoo Kim 2021

Classification of DFT geometries

w/ K. Morand 2017

The most general parametrisations of the DFT-metric, $\mathcal{H}_{MN}=\mathcal{H}_{NM}$, $\mathcal{H}_{M}{}^{K}\mathcal{H}_{N}{}^{L}\mathcal{J}_{KL}=\mathcal{J}_{MN}$, can be classified by two non-negative integers, $(n,\bar{n})$, $0\leq n+\bar{n}\leq D$:

$$\mathcal{H}_{MN}=\begin{pmatrix} H^{\mu\nu} & -H^{\mu\sigma}B_{\sigma\lambda}+Y_{i}^{\mu}X_{\lambda}^{i}-\bar{Y}_{\bar{\imath}}^{\mu}\bar{X}_{\lambda}^{\bar{\imath}} \\ B_{\kappa\rho}H^{\rho\nu}+X_{\kappa}^{i}Y_{i}^{\nu}-\bar{X}_{\kappa}^{\bar{\imath}}\bar{Y}_{\bar{\imath}}^{\nu} & K_{\kappa\lambda}-B_{\kappa\rho}H^{\rho\sigma}B_{\sigma\lambda}+2X_{(\kappa}^{i}B_{\lambda)\rho}Y_{i}^{\rho}-2\bar{X}_{(\kappa}^{\bar{\imath}}B_{\lambda)\rho}\bar{Y}_{\bar{\imath}}^{\rho}\end{pmatrix}$$

$$=\begin{pmatrix}1&0\\B&1\end{pmatrix}\begin{pmatrix}H & Y_{i}(X^{i})^{T}-\bar{Y}_{\bar{\imath}}(\bar{X}^{\bar{\imath}})^{T}\\ X^{i}(Y_{i})^{T}-\bar{X}^{\bar{\imath}}(\bar{Y}_{\bar{\imath}})^{T} & K\end{pmatrix}\begin{pmatrix}1&-B\\0&1\end{pmatrix}$$

where

$$H^{\mu\nu}=H^{\nu\mu}\,,\qquad K_{\mu\nu}=K_{\nu\mu}\,,\qquad B_{\mu\nu}=-B_{\nu\mu}$$

$$H^{\mu\nu}X_{\nu}^{i}=0=H^{\mu\nu}\bar{X}_{\nu}^{\bar{\imath}}\,,\qquad K_{\mu\nu}Y_{j}^{\nu}=0=K_{\mu\nu}\bar{Y}_{\bar{\jmath}}^{\nu}\quad:\quad i,j=1,2,\cdots,n;\quad \bar{\imath},\bar{\jmath}=1,2,\cdots,\bar{n}$$

$$H^{\mu\rho}K_{\rho\nu}+Y_{i}^{\mu}X_{\nu}^{i}+\bar{Y}_{\bar{\imath}}^{\mu}\bar{X}_{\nu}^{\bar{\imath}}=\delta^{\mu}{}_{\nu}\quad:\quad \text{completeness relation}$$

- It follows that $Y_{i}^{\mu}X_{\mu}^{j}=\delta_{i}{}^{j}$, $\bar{Y}_{\bar{\imath}}^{\mu}\bar{X}_{\mu}^{\bar{\jmath}}=\delta_{\bar{\imath}}{}^{\bar{\jmath}}$, $Y_{i}^{\mu}\bar{X}_{\mu}^{\bar{\jmath}}=0=\bar{Y}_{\bar{\imath}}^{\mu}X_{\mu}^{j}$, *etc.*
- Obviously, only $(0,0)$ is Riemannian but all others are non-Riemannian.

Examples of Non-Riemannian Geometries, $(n,\bar{n})\neq(0,0)$

i) $(1,0)$ Newton–Cartan gravity, $\mathrm{d}s^2=-c^2\mathrm{d}t^2+\mathrm{d}\mathbf{x}^2$, $\lim_{c\to\infty} g^{-1}$ is finite & degenerate

ii) $(D-1,0)$ ultra-relativistic Carroll gravity, $\mathrm{d}\tau^2=\mathrm{d}t^2-c^{-2}\mathrm{d}\mathbf{x}^2$, $\lim_{c\to 0} g^{-1}$ is finite & degenerate

iii) $(1,1)$ Stringy/torsional Newton–Cartan gravity, Gomis–Ooguri non-relativistic string theory

w/ Ko, Melby-Thompson and Meyer 2015; Blair 2019

iv) $(D,0)$ and $(0,D)$ are the two perfectly symmetric vacua, $\mathcal{H}=\pm\mathcal{J}$ with the trivial coset $\frac{\mathbf{O}(D,D)}{\mathbf{O}(D,D)}$.

Taken as an internal space, K-K reductions on them are moduli-free. w/ Cho and Morand 2018

"*Riemannian spacetime emerges after SSB of* $\mathbf{O}(D,D)$*, identifying* $\{g,B\}$ *as Nambu–Goldstone boson moduli.*" *Berman, Blair and Otsuki 2019*

- EDFEs, $G_{MN}=T_{MN}$, govern all the dynamics of various non-Riemannian geometries. One needs to insert the $(n,\bar{n})$ parametrisations and organise the expressions. w/ K. Cho 2019

- Besides, a class of singular geometries known in GR/SUGRA can be identified as regular $(1,1)$ non-Riemannian geometries of DFT. w/ K. Morand and M. Park 2021 PRL

Properties of Non-Riemannian Geometries

– The trace is given by $\mathcal{H}_M{}^M = 2(n - \bar{n})$ which $\mathbf{O}(D, D)$ rotations cannot alter.

– One can identify the underlying coset $\frac{\mathbf{O}(D,D)}{\mathbf{O}(t+n,s+n)\times\mathbf{O}(s+\bar{n},t+\bar{n})}$ with dimensions $D^2 - (n - \bar{n})^2$.

– Analysing DFT Killing eqns, $\hat{\mathcal{L}}_\xi \mathcal{H}_{MN} = 8\bar{P}_{(M}{}^{[K} P_{N)}{}^{L]} \nabla_K \xi_L = 0$, one can address the notion of non-Riemannian isometries. Constant non-Riemannian backgrounds turn out to admit 'super-translational', (*i.e.* infinitely many) isometries. Further, within SDFT, they imply infinitely many Killing spinors or 'super-supersymmetries'. w/ C. Blair and G. Oling 2020

– In fact, strings become chiral and anti-chiral over the n and $\bar{n}$ non-Riemannian directions:

$$X^i_\mu\, \partial_+ x^\mu(\tau,\sigma) = 0\,, \qquad\qquad \bar{X}^{\bar{\imath}}_\mu\, \partial_- x^\mu(\tau,\sigma) = 0$$

such that the central charges read

$$\mathbf{c}_{\mathbf{L}/\mathbf{R}} = D \pm (n - \bar{n}) - 26 \quad \text{(bosonic string)}\,; \qquad \mathbf{c}_{\mathbf{L}/\mathbf{R}} = D \pm (n - \bar{n}) - 10 \quad \text{(superstring)}$$

Thus, necessarily we require $n = \bar{n}$ and $D = 26$ or 10. w/ Shigeki Sugimoto 2020 PRL

– On the other hand, particles 'freeze' over the $n + \bar{n}$ non-Riemannian directions:

$$X^i_\mu \frac{\mathrm{d}x^\mu(\tau)}{\mathrm{d}\tau} = 0 = \bar{X}^{\bar{\imath}}_\mu \frac{\mathrm{d}x^\mu(\tau)}{\mathrm{d}\tau}\,.$$

Spherical Vacuum Solution to EDFE

Traversable wormhole for string, but not for particle

2412.04128 w/ Hun Jang, Hocheol Lee, and Minkyoo Kim

– The wormhole geometry we propose is a two-parameter family of solutions and is traceable to the work (1994) by Burgess, Myers, and Quevedo who obtained more general three-parameter family of solutions by performing **SL**$(2,\mathbb{R})$ S-duality rotations of a dilaton–metric solution in Einstein frame.

– The three-parameter solutions were later re-derived as the most general spherically symmetric vacuum solutions to EDFE, by analogy with Schwarzschild geometry of GR.

w/ S. Ko and M. Suh 2016.

– The two-parameter family of solutions were further singled out as an example of Riemann-wise singular but DFT-wise regular non-Riemannian geometry.

w/ K. Morand and M. Park 2021 PRL.

– Without further ado, let me spell the solution in a convenient coordinate system.

NS-NS Wormhole

$$ds^2 = \frac{-dt^2 + dy^2}{\mathcal{F}(y)} + \mathcal{R}(y)^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right) ,$$

$$H_{(3)} = h \sin\vartheta \, dt \wedge d\vartheta \wedge d\varphi ,$$

$$e^{2\phi(y)} = \frac{1}{|\mathcal{F}(y)|}$$

where

$$\mathcal{F}(y) = \frac{(y - b_-)(y - b_+)}{y^2 + \frac{1}{4}h^2} , \qquad \mathcal{R}(y) = \sqrt{y^2 + \frac{1}{4}h^2} \geq \frac{1}{2}|h| .$$

- The geometry has two free parameters, $b \neq 0$ and electric H-flux h, in terms of which we set

$$\gamma_\pm = \frac{1 \pm \sqrt{1 - h^2/b^2}}{2} , \qquad b_+ = -b\gamma_+ , \qquad b_- = b\gamma_- .$$

- For the solution to be real, we require $h^2 \leq b^2$.
- While $\mathcal{R}(y) = \mathcal{R}(-y)$, $\mathcal{F}(y)$ is not parity symmetric, except the case of saturation, $h^2 = b^2$.

  Therefore, in general one cannot identify y with $-y$ to perform a Z_2-orbifolding.

  We should then set the range of the y-coordinate to be all real numbers, $y \in \mathbb{R}$.

Wormhole Metric

$$ds^2 = \frac{-dt^2 + dy^2}{\mathcal{F}(y)} + \mathcal{R}(y)^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right) ,$$

$$\begin{cases} \mathcal{F}(y) = \dfrac{(y - b_-)(y - b_+)}{y^2 + \frac{1}{4}h^2} \\ \mathcal{R}(y) = \sqrt{y^2 + \frac{1}{4}h^2} \ \geq \ \frac{1}{2}\,|h| \ . \end{cases}$$

– The geometry consists of two separate, asymptotically flat spacetime letting $\mathcal{F}(y) \to 1$, one by $y \to \infty$ and the other by $y \to -\infty$, which are to be connected by a wormhole.

– The minimum of the areal radius is at $y = 0$ which we identify as the throat of the wormhole.

– With nontrivial H-flux, $h \neq 0$, a flare-out condition is satisfied in terms of the y-coordinate,

$$\left.\frac{d\mathcal{R}}{dy}\right|_{y=0} = 0 \,, \qquad \left.\frac{d^2\mathcal{R}}{dy^2}\right|_{y=0} = \frac{2}{|h|} \ > \ 0 \,.$$

Riemann-wise Singular but DFT-wise Regular Non-Riemannian

w/ Kevin Morand and Miok Park 2021 PRL

– The curvatures defined in Riemannian geometry are singular at the points of $y = b_{\pm}$, such as

$$R = -\frac{2b^2(y^2 + \frac{1}{4}h^2)^2 + 3h^2(y - b_+)^2(y - b_-)^2}{2(y - b_+)(y - b_-)(y^2 + \frac{1}{4}h^2)^3} \,.$$

– However, the geometry sets the G_{AB} hence DFT-curvatures, $S_{(0)}$ & $(PS\bar{P})_{AB}$, all trivial.

– In fact, by choosing the B-field appropriately to include a pure-gauge term,

$$B_{(2)} = h\cos\vartheta\,\mathrm{d}t \wedge \mathrm{d}\varphi + \frac{\mathrm{d}t \wedge \mathrm{d}y}{\mathcal{F}(y)}\,, \qquad \mathrm{d}B_{(2)} = H_{(3)}\,,$$

both $\mathcal{H}_{AB}$ and e^{-2d} can be made everywhere non-singular, such as $e^{-2d} = \mathcal{R}(y)^2 \sin\vartheta$.

As the B-field gauge transformation is a part of doubled diffeomorphisms,
the curvature singularity characterised within Riemannian geometry is to be identified
as a coordinate singularity within DFT.

– From the perspective of DFT, the geometry is regular everywhere: it is Riemannian away from $y = b_{\pm}$ and non-Riemannian at the points.

– It is of the same $(1, 1)$ type of non-Riemannian geometry as to the non-relativistic string.

Null Convergence Condition (NCC)

- In terms of (ordinary) Ricci curvature, the NCC stipulates $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ for $\forall$ null vector k^{μ}.
- Any vacuum solution to EDFE decomposes the Ricci curvature into dilaton and H-flux terms,

$$R_{\mu\nu} = -2\bigtriangledown_{\mu}(\partial_{\nu}\phi) + \tfrac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma}\,.$$

- Our wormhole geometry gives, with a null radial vector in a simple form $k^{\mu} = (1,1,0,0)$,

$$\begin{aligned}
R_{\mu\nu}\,k^{\mu}k^{\nu} &= -\frac{4(b_{+}+b_{-})y(y^2-\frac{1}{2}h^2)+5h^2(y^2-\frac{1}{20}h^2)}{2\mathcal{R}(y)^4(y-b_{+})(y-b_{-})}\,,\\
-2\bigtriangledown_{\mu}(\partial_{\nu}\phi)\,k^{\mu}k^{\nu} &= -\frac{2(b_{+}+b_{-})y(y^2-\frac{3}{4}h^2)+3h^2(y^2-\frac{1}{12}h^2)}{\mathcal{R}(y)^4(y-b_{+})(y-b_{-})}\,,\\
\tfrac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma}\,k^{\mu}k^{\nu} &= \frac{h^2}{2\mathcal{R}(y)^4} > 0\,.
\end{aligned}$$

  This shows that the H-flux always respects the NCC but the dilaton exhibiting the negative kinetic term in the string framed action does not. The NCC can be broken.

- Nonetheless, from the DFT perspective, $\{g, B, \phi\}$ are all gravitational fields which constitute the LHS of the EDFE, *i.e.* G_{AB}.

  The matter part is on the RHS, *i.e.* T_{AB}, which has its own energy conditions.

  There is no need to care about the energy condition for the vacuum in DFT, including the present wormhole solution.

Asymmetric 'Wine-Glass' Wormhole embedded in Ambient space

– Embedding of the wormhole into an ambient spacetime:

$$\mathrm{d}\hat{s}^2 = \frac{-\mathrm{d}t^2}{\mathcal{F}} \pm \mathrm{d}z^2 + \mathrm{d}\mathcal{R}^2 + \mathcal{R}^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right) ,$$

by $\mathcal{R}(y) = \sqrt{y^2 + \frac{1}{4}h^2}$ and $z(y)$ satisfying

$$\frac{\mathrm{d}z}{\mathrm{d}y} = \sqrt{\pm\left[\frac{1}{\mathcal{F}} - \left(\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}y}\right)^2\right]} = \sqrt{\pm\left[\frac{-b\sqrt{1-h^2/b^2}\,y^3 + \frac{3}{4}h^2y^2 + \frac{1}{16}h^4}{(y-b_+)(y-b_-)(y^2+\frac{1}{4}h^2)}\right]}$$

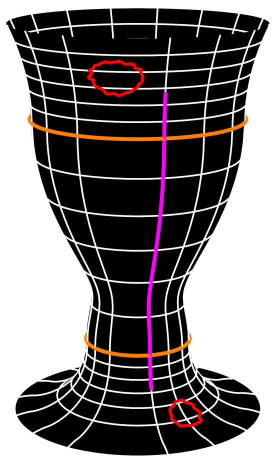
where the sign must be chosen to ensure the realness of the square roots: the embedding is inevitably piecewise.

i) For $b^2 > h^2 > 0$ and large $\mathcal{R}$ as $y \to \pm\infty$,

$$z \sim \pm 2(b^2 - h^2)^{1/4}\sqrt{\mathcal{R}}$$

This supplements the throat region depicted on the RHS.

ii) When $b^2 = h^2 > 0$, we get instead $z \sim \pm\frac{\sqrt{3}}{2}\,|b|\ln\mathcal{R}$.



Strings either traversing or non-traversing are colored in pink or red respectively. The Riemann-wise singular but DFT-wise regular non-Riemannian points at $y = b_\pm$ are colored in orange.

Traversable by String, but Not by Particle

– The massless particles' null geodesics reduce, with conserved energy $E \neq 0$ and angular momentum L_φ to $\dot{t} = E\mathcal{F}(y)$, $\dot{\varphi} = L_\varphi \mathcal{R}(y)^{-2}$, and pivotally for the y-coordinate,

$$0 = \dot{y}^2 + V(y)\,, \qquad V(y) = \left[-E^2\mathcal{F}(y) + L_\varphi^2/\mathcal{R}(y)^2\right]\mathcal{F}(y)$$

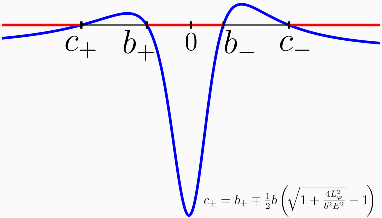
w/ K. Morand and M. Park 2021 PRL.

i) When $L_\varphi \neq 0$, the effective potential $V(y)$ features two positive peak, such that a massless particle cannot traverse $y = b_+$ nor $y = b_-$, as depicted on the RHS.

ii) If $L_\varphi = 0$, it takes infinite amount of affine parameter, λ, to reach $y = b_\pm$, as

$$\int \mathrm{d}\lambda = \int \frac{\mathrm{d}y}{E\mathcal{F}(y)}$$

is logarithmically divergent.



Effective potential $V(y)$ for $b > 0$ and $L_\varphi \neq 0$. Geodesics are confined in each of the three regions (red colored) divided by the points of $y = b_\pm$.

⇒ **Each of the three regions divided by $y = b_+$ and $y = b_-$ is geodesically complete and the wormhole is non-traversable by particles.**

Traversable by String, but Not by Particle

- We now turn to strings,

$$\frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma \ -\tfrac{1}{2}\sqrt{-h}h^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu g_{\mu\nu} + \tfrac{1}{2}\epsilon^{ab}\partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}\,.$$

  With $\sigma^\pm = \sigma \pm \tau$ and conformal gauge, the propagation of a string is dictated by

$$\partial_+\partial_- x^\mu + \left(\Gamma^\mu_{\rho\sigma} + \tfrac{1}{2}H^\mu{}_{\rho\sigma}\right)\partial_+ x^\rho \partial_- x^\sigma = 0\,,$$

  subject to Virasoro constraints,

$$\partial_+ x^\mu \partial_+ x^\nu g_{\mu\nu} = 0\,, \qquad\qquad \partial_- x^\mu \partial_- x^\nu g_{\mu\nu} = 0\,.$$

- As mentioned earlier, if the string ever approaches the non-Riemannian points of $y = b_\pm$, the string must be chiral or anti-chiral there.
- Rather than pursuing general solutions, we focus on the radial propagation of the string, by letting the two angular variables, ϑ, φ constant, and present solutions: *i)* non-traversing and *ii)* traversing.
- With constant ϑ and φ, the Virasoro constraints imply either *i)* $\partial_+ y \partial_- y = \partial_+ t \partial_- t$ (non-traversing) or *ii)* $\partial_+ y \partial_- y = -\partial_+ t \partial_- t$ (traversing).

Traversable by String, but Not by Particle

i) When $\partial_+ y \partial_- y = \partial_+ t \partial_- t$, our non-traversing solution assumes either $y = t$ (forward-moving) or $y = -t$ (backward-moving). All the equations of the string dynamics boil down to

$$\partial_+ \partial_- \mathcal{G}(y) = 0$$

where

$$\mathcal{G}(y) = \int \frac{\mathrm{d}y}{\mathcal{F}(y)} = y + \left(\frac{b_-^2 + h^2/4}{b}\right) \ln|y - b_-| - \left(\frac{b_+^2 + h^2/4}{b}\right) \ln|y - b_+|$$

Naturally, $\mathcal{G}(y)$ decomposes into left- and right-movers,

$$\mathcal{G}(y) = y_0 + 2\alpha' p \tau + f_+(\sigma^+) + f_-(\sigma^-)$$

where for a closed string, $f_\pm(\sigma^\pm)$ are arbitrary periodic functions, leading to vibrational mode expansions, while an open string needs to meet Neumann or Dirichlet boundary conditions.

– In any case, $\mathcal{G}(y)$ determines $y = \pm t$ completely, at least locally.

– In particular, far away from $y = b_\pm$, we have $\mathcal{G}(y) \simeq y$ and thus, not surprisingly, the string propagates like a free string on a flat background.

– However, such a string cannot approach nor cross the points of $y = b_\pm$ with finite amount of τ (logarithmic divergence). Only in the limit, $\tau \to \pm\infty$, the string may arrive at $y = b_\pm$.

– **These are all consistent with the non-traversing particle geodesics.**

– In fact, from the target spacetime perspective, the string setting $y = \pm t$ appears as if a point particle, without any spatial extension.

Traversable by String, but Not by Particle

ii) When $\partial_+ y \partial_- y = -\partial_+ t \partial_- t$, our traversing closed-string solution is given by

$$y = f_+(\sigma^+) + f_-(\sigma^-)\,, \qquad\qquad t = f_+(\sigma^+) - f_-(\sigma^-)\,,$$

such that $y + t$ is chiral and $y - t$ is anti-chiral, like the Gomis–Ooguri non-relativistic string.

- If the amplitudes of $f_\pm$, are large enough, this chiral string traverses the wormhole.
- One such example forms an ellipsoid in the target spacetime encompassing the wormhole,

$$\begin{pmatrix} f_+(\sigma^+) = b\sin\sigma^+ \\ \\ f_-(\sigma^-) = b\sin\sigma^- \end{pmatrix}, \qquad \begin{pmatrix} y = 2b\cos\tau\sin\sigma \\ \\ t = 2b\sin\tau\cos\sigma \end{pmatrix}, \qquad \left(\frac{t}{\cos\sigma}\right)^2 + \left(\frac{y}{\sin\sigma}\right)^2 = 4b^2\,.$$

- Although this traversing solution seems ignorant about the details of the wormhole geometry, it is H-flux that enables the chiral string to traverse: $\partial_+\partial_- x^\mu + \left(\Gamma^\mu_{\rho\sigma} + \frac{1}{2} H^\mu{}_{\rho\sigma}\right)\partial_+ x^\rho \partial_- x^\sigma = 0$.
- The periodic B.C. lets the traversing closed string localised not only in space but also in time.
- The $\mathbf{O}(D, D)$-symmetric volume of the middle throat region is independent of the H-flux:

$$\int_{\Sigma_t} e^{-2d} = \int_{b_+}^{b_-} \mathrm{d}y \int_0^\pi \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi\, \mathcal{R}(y)^2 \sin\vartheta = \frac{4\pi}{3} b^3\,.$$

Given that the throat has the minimal area $4\pi\mathcal{R}(0)^2 = \pi h^2$, the (averaged) height of the throat region is, roughly speaking, inversely proportional to the H-flux squared, $\propto b^3/3h^2$. It remains to be seen what would be the holographic interpretation, if any. Raamsdonk 2010.

Cosmological Vacuum Solution to EDFE

Late-Time Cosmology without Dark Sector
but with Closed String Massless Sector

2308.07149 w/ Hocheol Lee, Liliana Velasco-Sevilla, and Lu Yin

Cosmological Exact Vacuum

– In GR, de Sitter is the simplest cosmological solution: $\Omega_\Lambda = 0.73$ for ΛCDM.
Yet, the Hubble tension is getting worse by James Webb telescope: 67 vs. 73 km/s/Mpc.
Besides, there is swampland no-go argument for the existence of de Sitter. Vafa *et al.*

– What would be the cosmological vacuum solution to EDFE?
The answer is traceable to the work (1994) by Copeland, Lahiri, and Wands.

Here we elaborate their solution further to feature three free parameters,
$\{H_0,\, \mathfrak{h},\, l \equiv 1/\sqrt{-k}\,\}$ as for an open Universe which turns out to fit observational data.
Dilaton ϕ which does not run away because $k < 0$,

$$e^{2\phi(\eta)} = \frac{1-\sqrt{1-\frac{1}{12}(\mathfrak{h}l\sinh\zeta)^2}}{2}\left(\frac{\tanh\left(\frac{\eta}{l}+\frac{\zeta}{2}\right)}{\tanh\frac{\zeta}{2}}\right)^{\sqrt{3}} + \frac{1+\sqrt{1-\frac{1}{12}(\mathfrak{h}l\sinh\zeta)^2}}{2}\left(\frac{\tanh\left(\frac{\eta}{l}+\frac{\zeta}{2}\right)}{\tanh\frac{\zeta}{2}}\right)^{-\sqrt{3}}$$

Magnetic *H*-flux and FLRW metric (homogeneous & isotropic),

$$H_{(3)} = \frac{\mathfrak{h}\,r^2\sin\vartheta}{\sqrt{1+r^2/l^2}}\,\mathrm{d}r\wedge\mathrm{d}\vartheta\wedge\mathrm{d}\varphi\,, \qquad \mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2+\frac{\mathrm{d}r^2}{1+r^2/l^2}+r^2\left(\mathrm{d}\vartheta^2+\sin^2\vartheta\mathrm{d}\varphi^2\right)\right]$$

with the scale factor and the Hubble constant,

$$a^2(\eta) = e^{2\phi(\eta)}\,\frac{\sinh(2\eta/l+\zeta)}{\sinh\zeta}\,, \qquad H_0 = \frac{1}{2l\sinh\zeta}\left[2\cosh\zeta+\sigma\sqrt{12-(\mathfrak{h}l\sinh\zeta)^2}\,\right].$$

Bayesian Inference of Observational Data

– **Type Ia Supernovae by Pantheon+:** Distance Modulus $\mu(z)$ & Luminosity Distance $d_L(z)$,

$$\mu(z) = 5\,\mathrm{Log}_{10}\left[\frac{d_L(z)}{10\,\mathrm{pc}}\right], \qquad d_L(z) = \frac{1+z}{\sqrt{-k}}\,\sinh\left[\sqrt{-k}\int_0^z \frac{dz'}{H(z')}\right]$$

$\Rightarrow$ 1583 data points over $0.01 \leq z \leq 2.26$ Riess *et al.* 2021

– **Quasar Absorption Spectrum:** Temporal Variation of the Fine Structure Constant,

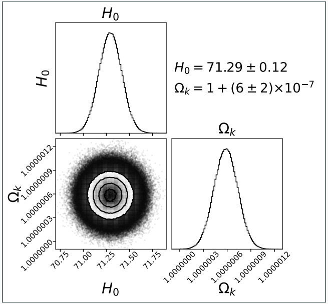
$$\frac{e^{-2\phi(t)}}{\alpha} F_{\mu\nu}F^{\mu\nu} = \frac{1}{\alpha_{eff.}(t)} F_{\mu\nu}F^{\mu\nu}$$

$\Rightarrow$ 199 data points over $0.22 \leq z \leq 7.06$

King *et al.* 2012; Wilczynska *et al.* 2015 & 2020; Martins *et al.* 2017

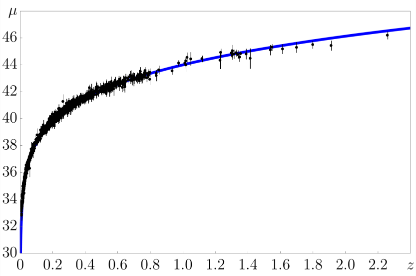
– We perform analyses of **Bayesian Inference (BI)** against these observational data.
We use Markov Chain Monte Carlo (MCMC) ensemble sampler called ‘emcee’.
With 100 walkers, we run the samplers on a supercomputer (KiSTi) for 10^6 steps.

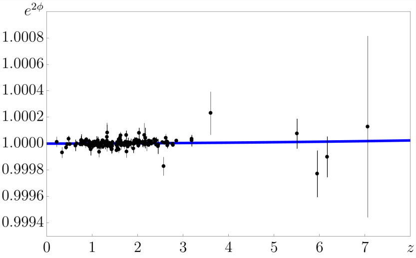
Two Parameter Fitting by the Exact Vacuum (trivial H-flux)



- BI: very well converged, $\Omega_k = 1/(lH_0)^2$
- Distance Modulus μ : Complete agreement with the type Ia supernova data.
- Suppressed time-evolution of $e^{2\phi}$ or the fine-structure constant: Consistency with the quasar data.

* Admirable agreement, without DE or DM.





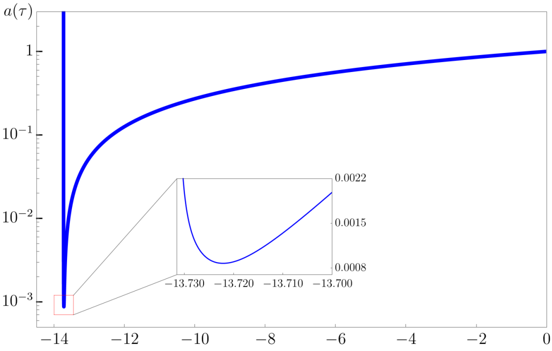
Extrapolations to Future and Past

- The exact vacuum solution predicts that, at future infinity the dilaton converges to constant, and the Universe expands forever as $a(\eta) \propto e^{\eta/l}$, with vanishing H such that

$$\lim_{\eta\to\infty} \Omega_k = 1$$

  which agrees with our BI fitting. Thus, there is No Coincidence Problem in our scenario.

- Extrapolated to the past, the Universe bounces without big bang about 13.72 gigayears ago which is intriguingly close to the estimated "age" of the flat Universe in ΛCDM.



Conclusion

- We have proposed a Lorentzian wormhole that is traversable by strings but not by particles.
- This wormhole is a vacuum solution to EDFE: $G_{AB} = 0$.
- In the string frame, ϕ exhibits a negative kinetic term, enabling the existence of the wormhole.
- Point-particle geodesics are complete within each region but non-traversable across regions.
- Strings perceive the geometry differently, allowing a chiral string to traverse freely.

* ⋆ The cosmological vacuum agrees admirably well with the supernova and quasar data.
* ⋆ The only requirements are an open Universe ($k = -1/l^2 < 0$) and a string frame.
* ⋆ We estimate the Hubble constant as $H_0 \simeq 71.2 \pm 0.2$ km/s/Mpc, and the spatial curvature length scale as $l = 1/\sqrt{-k} \simeq 1/H_0 \simeq 4.2$ Gpc.
* ⋆ It remains to be seen whether the early Universe, or CMB, is also consistent with DFT or not.

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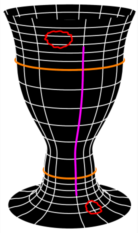


Cheers!

APPENDIX

Fracton Physics on non-Riemannian backgrounds

w/ Minkyoo Kim and Angus 2022

– 'Fracton Physics' is naturally realised on non-Riemannian backgrounds:

- Particle freezes with vanishing velocities $\rightarrow$ Fracton's immobility
- Infinitely many isometries and hence Noether charges $\rightarrow$ Fracton's large degeneracy

– Easy to produce fracton models, e.g. doubled YM on non-Riemannian backgrounds:

$$\mathrm{Tr}\Big(P^{AC}\bar{P}^{BD}F_{AB}F_{CD}\Big)\Big|_{(n,\bar{n})} = \mathrm{Tr}\left[\begin{array}{l} -\frac{1}{4}(f_{ab}+i[\varphi_a,\varphi_b])(f^{ab}+i[\varphi^a,\varphi^b]) \\ -\frac{1}{4}u_{ab}u^{ab} - f^{-}_{ai}D^{-a}\varphi^{i} + f^{+}_{a\bar{\imath}}D^{+a}\varphi^{\bar{\imath}} \\ -2D_i\varphi^{\bar{\imath}}D_{\bar{\imath}}\varphi^{i} - 2if_{i\bar{\imath}}[\varphi^{i},\varphi^{\bar{\imath}}] \end{array}\right]$$

which contains a symmetric strain tensor

$$u_{ab} = D_a\varphi_b + D_b\varphi_a$$

and features infinitely many multi-pole conservations. Note the decomposition, $\mu = a, i, \bar{\imath}$.

Riemannian Singularity? or Non-Riemannian Regularity!

w/ Kevin Morand and Miok Park 2021 PRL

– A class of known "singular" geometries in SUGRA assumes an ansatz: with $x^{\mu}=(t,y,z^{i})$,

$$\begin{aligned} ds^2 &= \tfrac{1}{F(x)}\left(-dt^2+dy^2\right)+G_{ij}(x)dz^i dz^j \\ B_{(2)} &= \tfrac{1}{F(x)}\,dt\wedge dy+\tfrac{1}{2}\,\beta_{\mu\nu}(x)dx^{\mu}\wedge dx^{\nu} \\ e^{-2\phi} &= F(x)\Psi(x) \end{aligned}$$

where $G_{ij},\beta_{\mu\nu}$ and Ψ are all regular.

– They solve the EOMs of

$$\int d^D x\,\sqrt{-g}e^{-2\phi}\left(R+4\partial_{\mu}\phi\partial^{\mu}\phi-\tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}-2\Lambda\right)$$

or equivalently EDFEs,

$$G_{MN}=-\Lambda\mathcal{J}_{MN}\,.$$

– Examples include

- $D=10$ black 5-brane *á la* Horowitz-Strominger ($\Lambda=0$);
- $D=4$ spherical solution *á la* Burges-Meyers-Quevedo ($\Lambda=0$);
- $D=2$ black hole *á la* Witten ($\Lambda\neq 0$).

– When $F=0$, the ansatz features coordinate singularity and further curvature singularity,

$$R\rightarrow\infty\quad,\quad R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\rightarrow\infty\quad\text{as}\quad F\rightarrow 0.$$

– However, DFT-curvatures should be all finite (if not vanishing for $\Lambda=0$).

– Substitution into the DFT-dilaton and DFT-metric removes the coordinate singularity:

$$e^{-2d} = \sqrt{-g}e^{-2\phi} = \Psi\sqrt{G}\,, \qquad \mathcal{H}_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \beta & \mathbf{1} \end{pmatrix} \mathring{\mathcal{H}} \begin{pmatrix} \mathbf{1} & -\beta \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

where no negative power of F appears:

$$\mathring{\mathcal{H}}_{AB} = \begin{pmatrix} -F\sigma_3 & 0 & \sigma_1 & 0 \\ 0 & G^{-1} & 0 & 0 \\ \sigma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & G \end{pmatrix}$$

– In fact, it corresponds to $(1,1)$ non-Riemannian geometry when $F \to 0$.

– In the case of $D = 2$, the B-field is a pure gauge hence removable by DFT-diffeomorphisms.

  ▶ The curvature singularity in GR becomes, at worst, a coordinate singularity in DFT.

– Depending on the choice of the framework, *i.e.* Riemannian GR *vs.* non-Riemannian DFT, the backgrounds appear either singular or regular.

– To determine if the singularity is physical or not, we have examined the geodesic motions.

- In all the examples, the geodesics turn out to be **complete**.
- Further, the tidal force of geodesic deviation is all **finite**:

$$\frac{D^2\xi^\mu}{D\lambda^2} = R^\mu{}_{\nu\rho\sigma}\dot{x}^\nu\dot{x}^\rho\dot{\xi}^\sigma\,, \qquad\qquad g_{\mu\nu}\frac{D^2\xi^\mu}{D\lambda^2}\frac{D^2\xi^\nu}{D\lambda^2} << \infty\,.$$

– In fact, approaching the "singular" points of $F=0$, particle freezes:

$$\dot{t}\rightarrow 0 \qquad \& \qquad \dot{y}\rightarrow 0 \qquad\qquad \text{as} \qquad F\rightarrow 0$$

and string becomes chiral/anti-chiral: with $y^\pm = y\pm t$,

$$\partial_- y^+ \rightarrow 0 \qquad \& \qquad \partial_+ y^- \rightarrow 0 \qquad \text{as} \qquad F\rightarrow 0\,,$$

which are expected features from the doubled-yet-gauged particle and string actions on generic non-Riemannian backgrounds.

Solar System Test: $D = 4$

Post-Newtonian Feasibility of the Closed String Massless Sector

2202.07413 w/ Kang-Sin Choi PRL

$$G_{AB} = T_{AB}$$

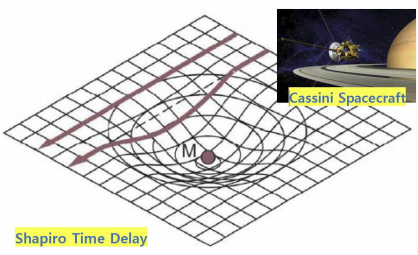
$$\Downarrow \quad \text{Riemannian Reduction}$$

$$\begin{aligned}
R_{\mu\nu} + 2\bigtriangledown_{\mu}(\partial_{\nu}\phi) - \tfrac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma} &= K_{(\mu\nu)} \\
\tfrac{1}{2}e^{2\phi}\bigtriangledown^{\rho}\left(e^{-2\phi}H_{\rho\mu\nu}\right) &= K_{[\mu\nu]} \\
R + 4\Box\phi - 4\partial_{\mu}\phi\partial^{\mu}\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} &= T_{(0)}
\end{aligned}$$

Parametrised Post Newtonian (PPN) formalism

– Two dimensionless PPN parameters $\beta_{PPN}, \gamma_{PPN}$ *à la* Eddington-Robertson-Schiff are defined in an asymptotically flat isotropic coordinate system: with $r = \sqrt{x^i x^j \delta_{ij}}$,

$$ds^2 = -\left(1 - \frac{2MG_N}{r} + \frac{2\beta_{PPN}(MG_N)^2}{r^2} + \cdots\right)dt^2 + \left(1 + \frac{2\gamma_{PPN}MG_N}{r} + \cdots\right)dx^i dx^j \delta_{ij}$$



- **Observational values** Will 2014
  – Shapiro Time Delay:

$$\gamma_{PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

  – Perihelion shifts of Mercury:

$$\beta_{PPN} - 1 = (-4.1 \pm 7.8) \times 10^{-5}$$

  – Earth Gravity:

$$4\beta_{PPN} - \gamma_{PPN} - 3 = (4.44 \pm 4.5) \times 10^{-4}$$

  – Galactic size scale: $\gamma_{PPN} = 0.98 \pm 0.07$

GR predicts $\beta_{PPN} = \gamma_{PPN} = 1$

– In GR, the geometry of a spherical object, or "star", is in general

$$ds^2 = -e^{-2\Delta(r)}\left(1 - \frac{2G_N M(r)}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2G_N M(r)}{r}} + r^2 d\Omega^2 ,$$

where r denotes areal radius and

$$M(r) := -\int_0^r dr'\, 4\pi r'^2\, T_t{}^t(r') , \qquad \Delta(r) := 4\pi G_N \int_r^\infty dr'\, \frac{\{T_r{}^r(r') - T_t{}^t(r')\} r'}{1 - \frac{2G_N M(r')}{r'}} .$$

– Outside the star $r > r_\star$ (star radius), $T_{\mu\nu} = 0$ hence $\Delta(r) = 0$. The outer geometry is given by Schwarzschild metric having the only one parameter $M = M(r_\star)$: Birkhoff's theorem

– Mapped to the isotropic coordinate system, one gets rather exactly $\beta_{PPN} = \gamma_{PPN} = 1$.

This has been viewed as the "success" of GR.

Stringy Spherical Vacuum

Burgess-Myers-Quevedo 1994

– The spherical vacuum solution to $G_{AB} = 0$ in DFT has three "free" parameters $\{a, b, h\}$,

$$e^{2\phi} = \gamma_+ \left(\frac{4r-\sqrt{a^2+b^2}}{4r+\sqrt{a^2+b^2}}\right)^{\frac{2b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{4r+\sqrt{a^2+b^2}}{4r-\sqrt{a^2+b^2}}\right)^{\frac{2b}{\sqrt{a^2+b^2}}} ,$$

$$H_{(3)} = h\,\mathrm{d}t \wedge \mathrm{d}\varphi \wedge \mathrm{d}\cos\vartheta\,, \qquad \mathrm{d}s^2 = g_{tt}(r)\,\mathrm{d}t^2 + g_{rr}(r)\left[\mathrm{d}r^2 + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)\right] ,$$

where $\gamma_\pm = \frac{1}{2}(1 \pm \sqrt{1-h^2/b^2})$, $g_{tt}(r) = -e^{2\phi(r)}\left(\frac{4r-\sqrt{a^2+b^2}}{4r+\sqrt{a^2+b^2}}\right)^{\frac{2a}{\sqrt{a^2+b^2}}}$ and

$$g_{rr}(r) = e^{2\phi(r)}\left(\frac{4r+\sqrt{a^2+b^2}}{4r-\sqrt{a^2+b^2}}\right)^{\frac{2a}{\sqrt{a^2+b^2}}}\left(1-\frac{a^2+b^2}{16r^2}\right)^2 .$$

– One can read off the mass and the two PPN parameters,

$$MG_N = \frac{1}{2}(a + b\sqrt{1-h^2/b^2})\,, \quad (\beta_{PPN}-1)(MG_N)^2 = \frac{h^2}{4}\,, \quad (\gamma_{PPN}-1)MG_N = -b\sqrt{1-\frac{h^2}{b^2}}\,,$$

and further take $\{MG_N, \beta_{PPN}, \gamma_{PPN}\}$ as alternative three parameters, such that

$$\phi \simeq \frac{(\gamma_{PPN}-1)MG_N}{2r} + \frac{(\beta_{PPN}-1)(MG_N)^2}{r^2}\,, \qquad H_{(3)} = \pm 2\sqrt{\beta_{PPN}-1}\,MG_N\,\mathrm{d}t \wedge \mathrm{d}\varphi \wedge \mathrm{d}\cos\vartheta$$

Namely, the deviations $\gamma_{PPN}-1$ and $\sqrt{\beta_{PPN}-1}$ correspond to the dilaton and H-flux *charges*.

Stringy Star has $\beta_{PPN} = 1$ due to weak energy condition:

– In a similar fashion to GR, the vacuum solution in the previous page can be identified as the outer geometry of a stringy star (non-singular), while it becomes possible to relate the three parameters to the stress-energy tensor of the star: [Angus-Cho-JHP 2018]

$$MG_N = \frac{1}{4\pi}\int \mathrm{d}^3x\, e^{-2d}\left(-K_t{}^t - H_{t\vartheta\varphi}H^{t\vartheta\varphi}\right), \qquad \sqrt{\beta_{PPN}-1}\,MG_N = \left|\frac{K_{(tr)}g^{rr}(e^{-2d}/\sin\vartheta)}{2\int_r^{r_\star}\mathrm{d}r'(e^{-2d}K^{[\vartheta\varphi]})}\right|,$$

$$(\gamma_{PPN}-1)MG_N = \frac{1}{4\pi}\int \mathrm{d}^3x\, e^{-2d}\left(K_\mu{}^\mu - T_{(0)} + \tfrac{1}{6}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right)$$

– Inside the star, while the magnetic flux can be nontrivial, the electric H-flux is persistently of the same rigid form as outside:

$$H^{r\vartheta\varphi} = -2e^{2d}\int_r^{r_\star}\mathrm{d}r'\, e^{-2d}K^{[\vartheta\varphi]}, \qquad H_{t\vartheta\varphi} = h\sin\vartheta\,.$$

– If $h \neq 0$, the electric H-flux contribution to the mass MG_N diverges at $r = 0$,

$$\frac{1}{4\pi}\int \mathrm{d}^3x\, e^{-2d}\left(-H_{t\vartheta\varphi}H^{t\vartheta\varphi}\right) = h^2\int_0^\infty \frac{\mathrm{d}r\, e^{-2\phi}}{\sqrt{-g_{tt}g_{rr}}\, r^2} \sim h^2\int_0^\infty \frac{\mathrm{d}r}{r^2} \to \infty$$

In order to have the mass finite, the energy density $-K_t{}^t$ should be negative. This violates weak energy condition. Thus, we conclude $h = 0 = H_{t\vartheta\varphi}$ and hence $\beta_{PPN} = 1$.

PPN parameter γ_{PPN} is an equation-of-state parameter:

– With the vanishing electric H-flux ($h=0$), the volume integrals of the mass and γ_{PPN} are now all restricted to the star's interior,

$$MG_N = \frac{1}{4\pi}\int_{star} d^3x\; e^{-2d}\left(-K_t{}^t\right)\,, \qquad \gamma_{PPN} = 1 + \frac{\int_{star} d^3x\; e^{-2d}\left(K_\mu{}^\mu - T_{(0)} + H_{r\vartheta\varphi}H^{r\vartheta\varphi}\right)}{\int_{star} d^3x\; e^{-2d}(-K_t{}^t)}$$

where the magnetic H-flux is set by $K^{[\vartheta\varphi]}$.

– The PPN parameter γ_{PPN} is then a sum of (volume-averaged) equation-of-state parameters,

$$\gamma_{PPN} = 3w_K - w_T + \delta_{H-flux}$$

where we let

$$w_K = \frac{\int_{star} d^3x\; e^{-2d}\,\frac{1}{3}K_i{}^i}{\int_{star} d^3x\; e^{-2d}\,(-K_t{}^t)}\,, \qquad w_T = \frac{\int_{star} d^3x\; e^{-2d}\,T_{(0)}}{\int_{star} d^3x\; e^{-2d}\,(-K_t{}^t)}\,, \qquad \delta_{H-flux} = \frac{16\pi\int_0^{r_\star} \mathrm{d}r\; r^2 e^{2\phi}\sqrt{-g_{rr}^3/g_{tt}}\left(\int_r^{r_\star} \mathrm{d}r'\; e^{-2d}K^{[\vartheta\varphi]}\right)^2}{\int_{star} d^3x\; e^{-2d}\,(-K_t{}^t)}$$

– As δ_{H-flux} is suppressed by G_N, the experimental bound implies

$$|\gamma_{PPN} - 1| \simeq |3w_K - w_T - 1| = \left|\frac{\int_{\mathrm{SUN}} d^3x\;\; e^{-2d}\left(K_\mu{}^\mu - T_{(0)}\right)}{\int_{\mathrm{SUN}} d^3x\;\; e^{-2d}\,(-K_t{}^t)}\right| \lesssim\; 10^{-5}$$

Failure or NOT? $\Rightarrow$ the choice of right degrees-of-freedom Weinberg

- If a star were modeled as an ideal gas of particles, we have $w_T = \delta_{H-flux} = 0$ and simply

$$\gamma_{PPN} = 3\langle p/\rho\rangle \simeq \langle v^2\rangle \,.$$

  To be consistent with the observation, the average speed v should be close to $1{=}\,c$, meaning that the constituting particles should be ultrarelativistic rather than being “pressureless dusts”.

- The pressure outside an atom may be negligible, but this is also true for the energy density.

  Both ρ and p should be confined inside baryons.

  Recent experiment reveals $\rho \sim p$ inside proton. Burkert-Elouadrhiri-Girod 2018 Nature

- Instead, chiral effective theory of nuclear physics,

$$S_{eff.} = -\int \mathrm{d}^4x\, e^{-2d} g^{\mu\nu}\partial_\mu\Phi^I\partial_\nu\Phi^J\mathcal{G}_{IJ}(\Phi)$$

  sets $K_\mu{}^\mu = T_{(0)},\ K^{[\vartheta\varphi]} = 0$, and thus rather precisely $\gamma_{PPN} = 1$.

- Applied to QCD, $\gamma_{PPN} - 1$ essentially amounts to the gluon and quark condensates,

$$\gamma_{PPN} - 1 \simeq \frac{1}{4\pi MG_N}\int_{star}\mathrm{d}^3x\,\left[\tfrac{1}{4}e^{-2d}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \tfrac{1}{2}m\bar{\psi}\psi\right] = \frac{\int_{star}\mathrm{d}^3x\,\left[e^{-2d}\mathrm{Tr}(B^2-E^2) - m\bar{\psi}\psi\right]}{\int_{star}\mathrm{d}^3x\,\left[e^{-2d}\mathrm{Tr}(E^2) + i\bar{\psi}\gamma^t D_t\psi\right]}$$

  which may vanish, as suggested by some empirical measurements Barate *et al.* 1998 Phys. Rept. and theoretical ‘pseudo-conformal’ scenarios Del Debbio-Zwicky, Hyun Kyu Lee, Mannque Rho 2022.

  The electric and magnetic fields may cancel each other, while the quarks get negligible.

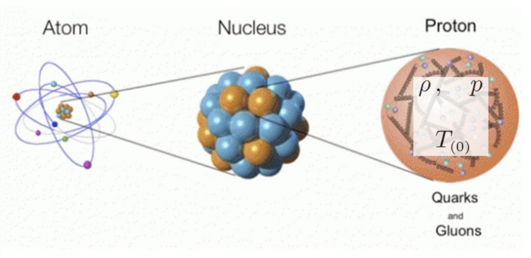
Solar System Test: Gravitational Probe into the Inside of Hadrons

– In summary, DFT sets $\beta_{PPN} = 1$ and lets γ_{PPN} be the equation-of-state parameters.

– Rather than ruling out the theory, applied to baryons' interior where the energy and pressure are both confined, the apparently universal observations $\gamma_{PPN} \simeq 1$ including the Sun and the Earth may signify pseud-conformal equation of state inside baryons,

$$E = mc^2 + \tfrac{1}{2}mv^2 + \cdots \quad \Longrightarrow \quad \rho = \rho_{intrinsic} + \rho_{thermal}$$

$$\rho \simeq 3p \quad \Longrightarrow \quad p = p_{intrinsic} + p_{thermal} \quad \textbf{where} \quad p_{intrinsic} \simeq \tfrac{1}{3}\rho_{intrinsic} \neq 0\,.$$



An open problem in Nuclear Physics.