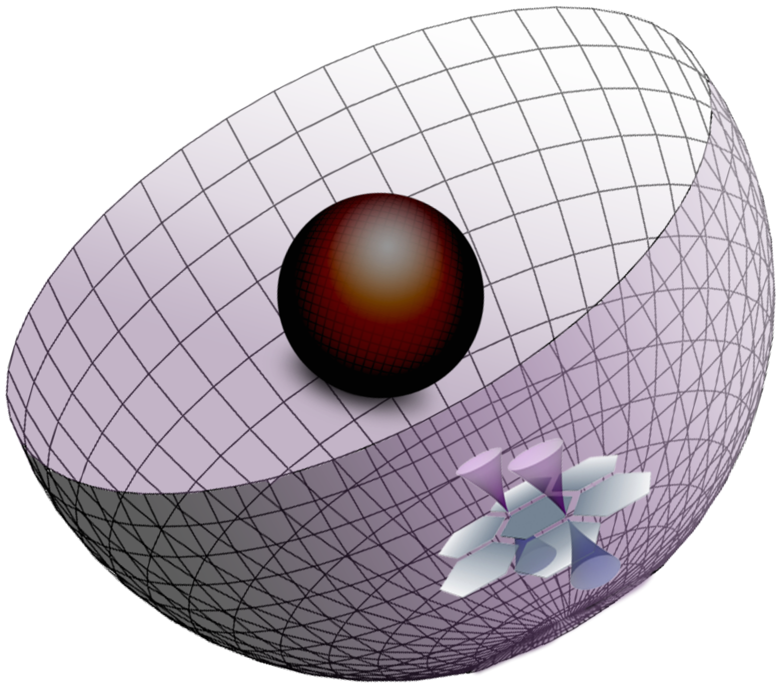
Holographic Mean field theory & How to use it for Condensed matter?



Sang-Jin Sin (Hanyang U.)
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I. Introduction

II. Holographic mean field theory

III. Examples

- 1. Kondo condensation and Kondo lattice.
- 2. Mott gap classes.
- 3. Topology in interacting system
- 4. Encoding lattice (UV) in the holography.

Mean field theory and holographic Kondo lattice, 2407.01978

Mean field theory for strongly coupled systems: Holographic approach *JHEP* 06 (2024) 100

I. Introduction: what happen if interaction is not weak?

- 1. Particle nature is lost.
- 2. Entire system is strongly entangled.

$$H_{tot} = H(x_1) + H(x_2)$$
$$\Rightarrow \psi_{tot} = \psi_i(x_1)\psi_j(x_2) \Rightarrow No\ entanglement$$

- Introduce the interaction:

$$H_{tot} = H(x_1) + H(x_2) + H_{int}(x_1, x_2)$$
$$\Rightarrow \psi_{tot} = \sum_{ij} c_{ij}\psi_i(x_1)\psi_j(x_2) \Rightarrow entanglement$$

What if interaction is not weak?

- Weak coupling:

  in $\psi_{tot} = \sum_{ij} c_{ij}\psi_i(x_1)\psi_j(x_2)$, one term dominance remained.

- For strong coupling,

  c_{ij} in $\psi_{tot} = \sum_{ij} c_{ij}\psi_i(x_1)\psi_j(x_2)$ are more evenly distributed

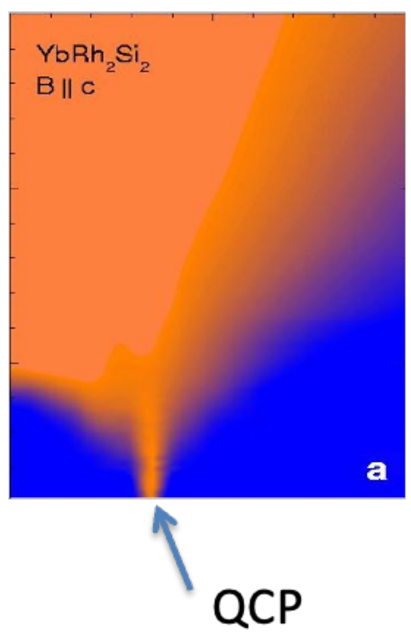
  => No. of the important terms increases.

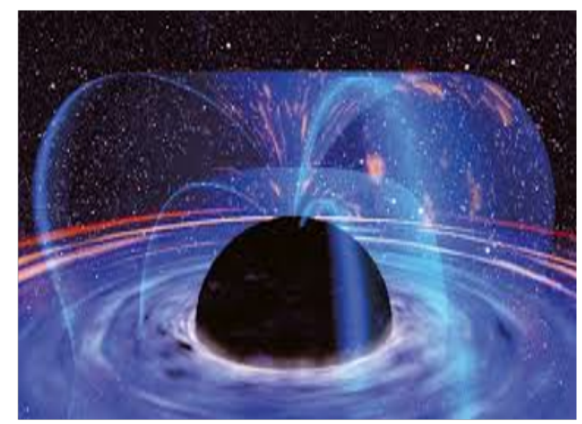
  => every scale is equally important=> QCP

  => Entire system becomes one object.

Recipe?

The whole system ~ one object => Emergence
If the object is like a black hole => Simplicity restored!





Apply AdS/CFT to CMT

The only known calculational scheme for the Emergence!

Any Quantitative evidence?

Well known η/s + Transport anomaly in Graphene

PRL **118**, 036601 (2017) PHYSICAL REVIEW LETTERS week ending 20 JANUARY 2017

Holography of the Dirac Fluid in Graphene with Two Currents

Yunseok Seo,[1] Geunho Song,[1] Philip Kim,[2,3] Subir Sachdev,[2,4] and Sang-Jin Sin[1]
[1]*Department of Physics, Hanyang University, Seoul 133-791, Korea*
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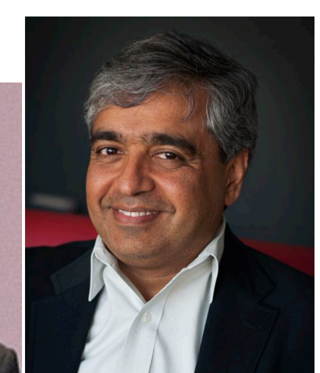
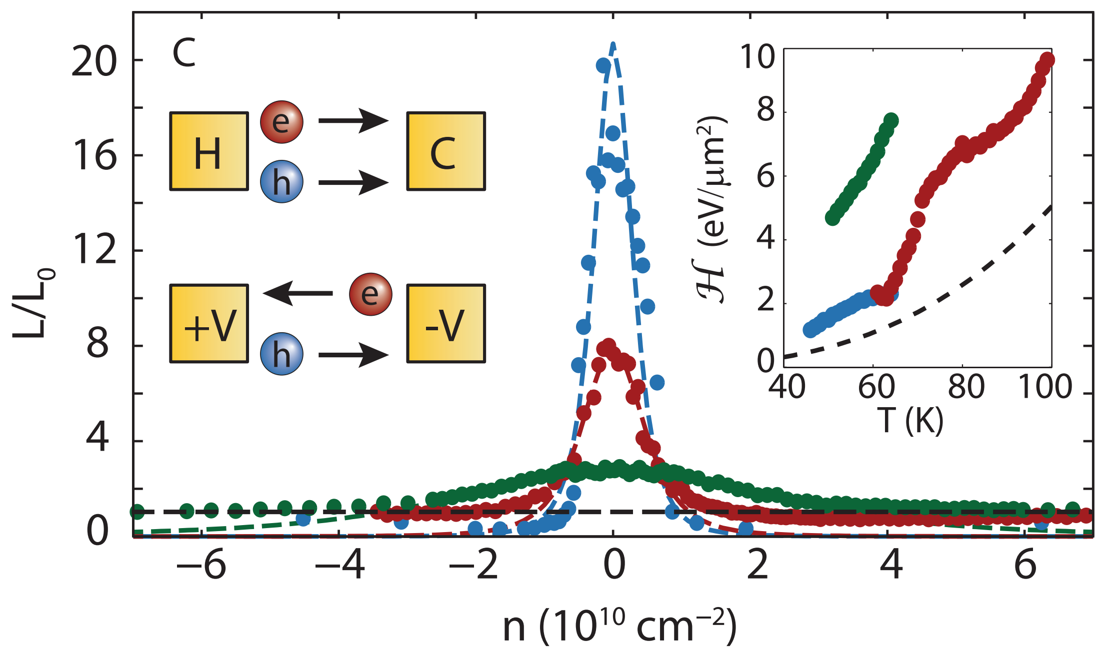


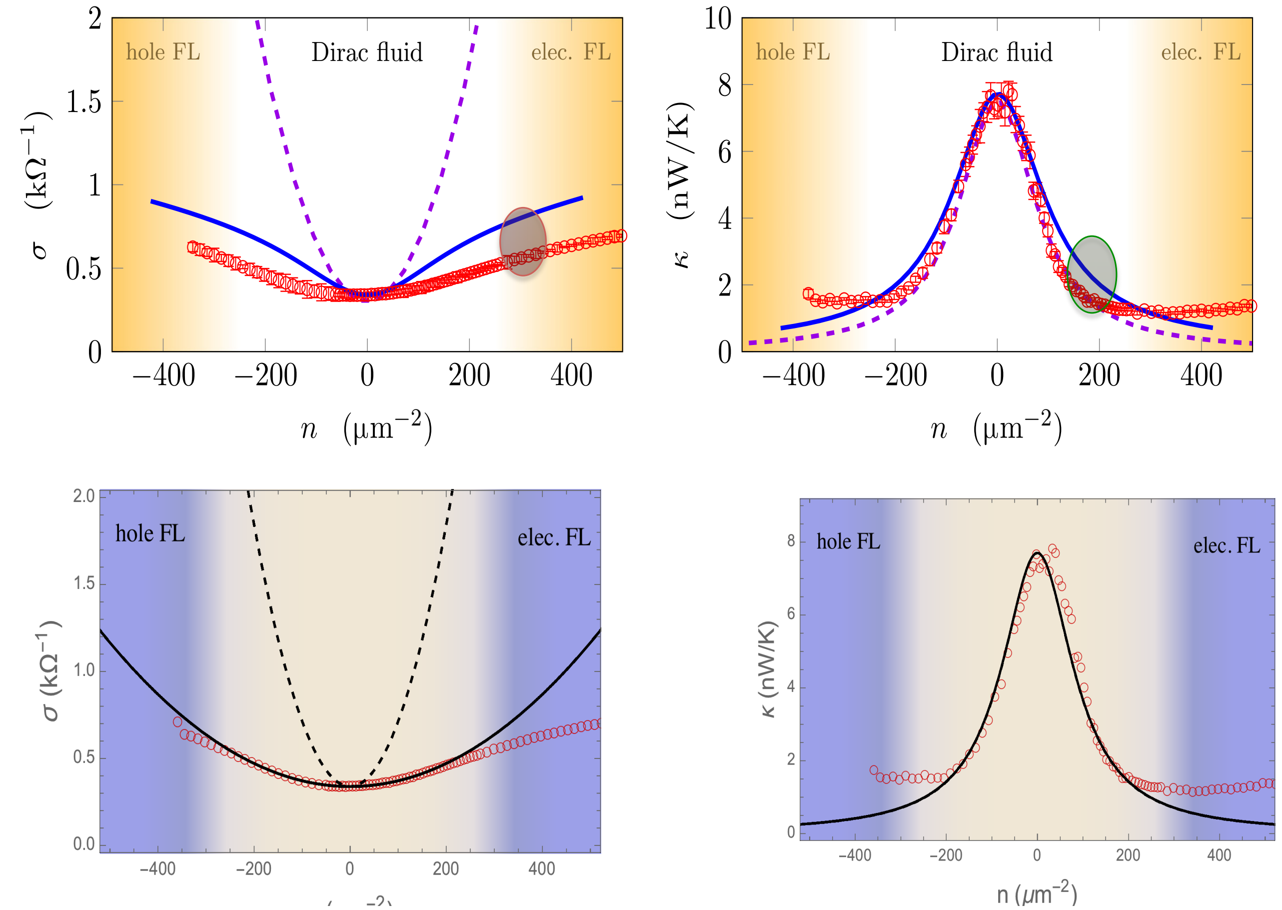
Phys.Rev.Lett. 118 (2017) no.3, 036601
Editors' Suggestion

Transport anomaly in Graphene

Summary: Holography provides a modeling tool for hydrodynamics.



4 March 2016



Why it works for graphene?

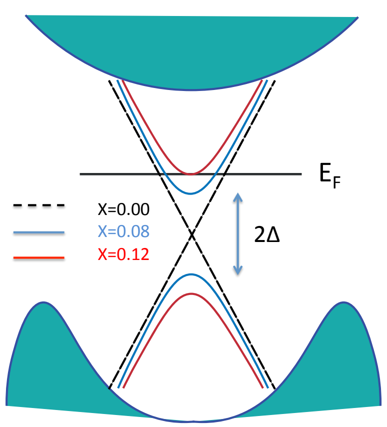
Small Fermi surface (little screening) +

masslessness (space filling brane)

ii) Dirac material & Surface of Topological Insulators

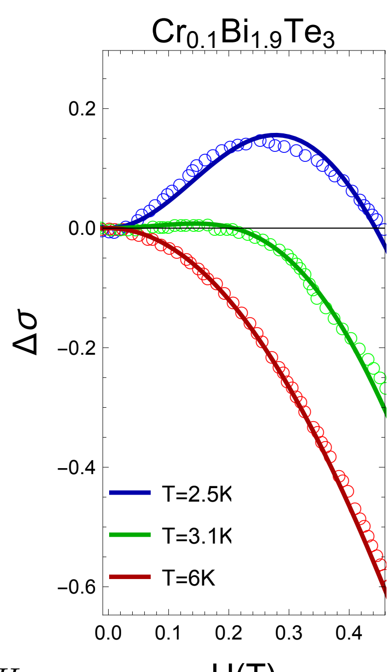
[1703.07361, prB, rapid comm 서윤석,송근호,SJS]

Theory fits
not only for Cr doped Bi_2Te_3 but also Mn doped Bi_2Se_3

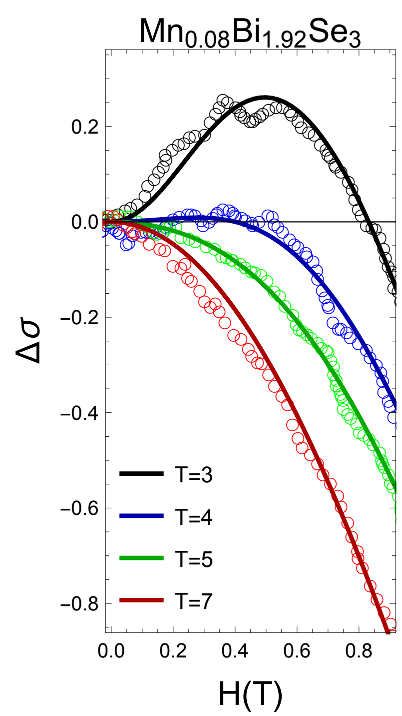


$$\sigma_{xx} = \frac{(\mathcal{F}+\mathcal{G}^2)(\mathcal{F}-H^2)}{\mathcal{F}^2+H^2\mathcal{G}^2}$$

$$\mathcal{F} = r_0^2(\alpha^2+\lambda^2)+(1+\theta^2)H^2 - q\,\theta H$$
$$\mathcal{G} = q - \theta H.$$



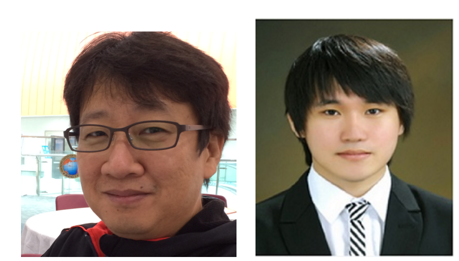
2019.03.27@KHU



Strong Correlation Effects on Surfaces of Topological Insulators via Holography

Yunseok Seo, Geunho Song and Sang-Jin Sin
Department of Physics, Hanyang University, Seoul 04763, Korea.

Published in Phys.Rev. B96 (2017) no.4, 041104 (rapid communications)



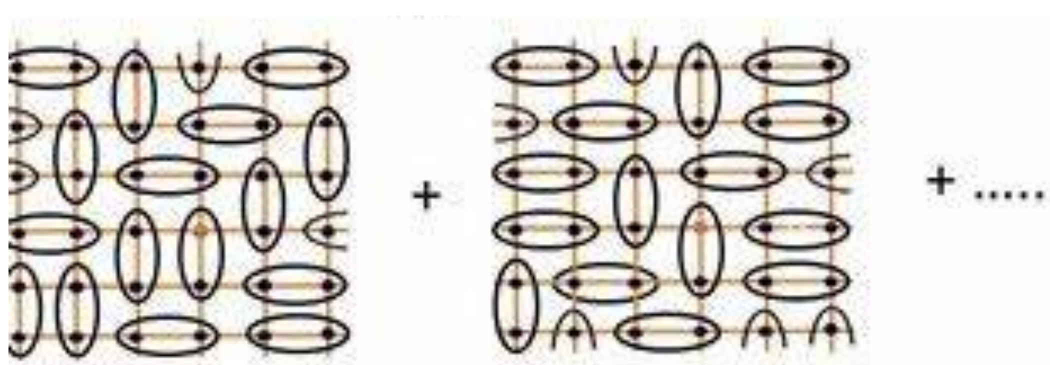
+SJS

$$\sigma_{ij}(B,T,n_{imp})$$

Small Fermi Surfaces and Strong Correlation Effects in Dirac Materials with Holography
Y. Seo, G. Song, C. Park + SJS
Published in JHEP 1710 (2017) 204

$$\kappa_{ij}(B,T,n_{imp})$$

Why it works in Condensed Matter?

- Relativistic massless theory for graphene.
- Where is large N in U(1) gauge theory?
- Origin of SU(3) = degeneracy if 3 color state.
- SIS: degenerate ground state:
  ex:RVB.
- 

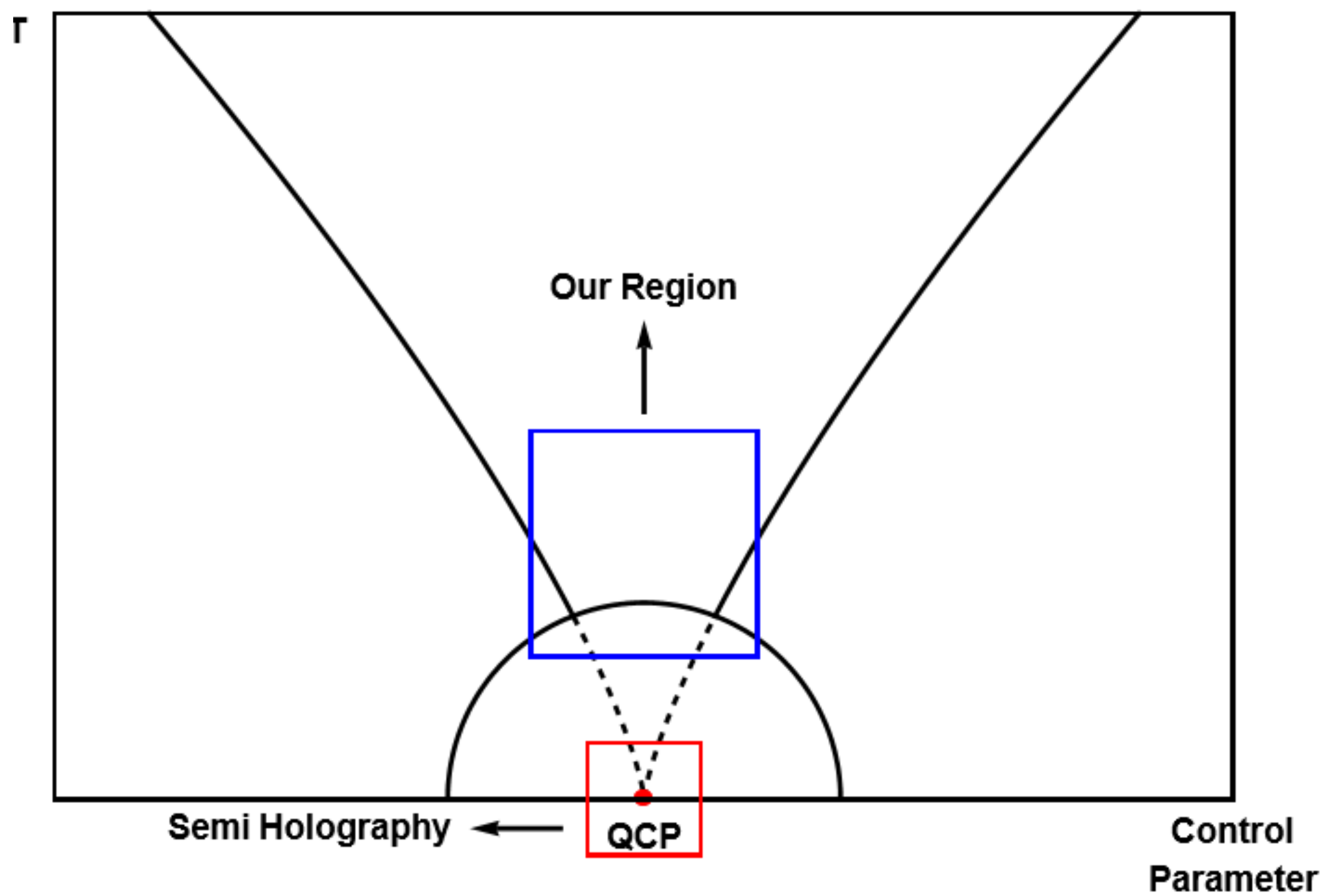
Why it works (ii)

1. Ground state degeneracy
   => Fluctuation/Frustration/Entagnlement
2. => Long distance entanglement at boundary
   bdy theory should not be local!
3. No control over the theory?
   Bulk locality?

Prescription: Model the system by Bulk Local Theories!

Real condensed matter physics by AdS/CFT?

Beyond Graphene? Off the QCP. Need a scale.
Beyond transport? ARPES data. => Need fermion.



Scale from Symmetry breaking

Theory of dynamical symmetry breaking
= Mean field theory:

Study of condensation and
fermion gaps and their roles out of 4-fermion int.

Universal structure & problem of MFT

1. The mean field theory has universal structure :

$$(\bar{c}_k \Gamma^i c_{-k}) \cdot (\bar{c}_k \Gamma^i c_{-k}) \to \bar{\Delta}^i \cdot (\bar{c}_k \Gamma^i c_{-k}) + hc - \bar{\Delta}^i \Delta^i,$$

2. Depending on the type of instability of FS and the gap, we have BCS, CDW, SDW, Kondo lattice,... chap11-20.

3. Universal Problem: New ground state is caused by the instability of the old vacuum. Need relevant operator. It happens for large enough coupling at low energy of relevant op., where the calculation is not valid in general.

4. Need a formalism to overcome the usual MFT, To discuss all above phenomena in the same fashion.

II. Holographic Mean field theory

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Mean field theory for strongly coupled systems: Holographic approach

- Supalert Sukrakarn(Hanyang U.), Taewon Yuk(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Nov 3, 2023)
  - Published in: *JHEP* 06 (2024) 100 • e-Print: 2311.01897 [hep-th]

The emergence of strange metal and topological liquid near quantum critical point in a solvable model

- Eunseok Oh(Hanyang U.), Taewon Yuk(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Mar 15, 2021)
  - Published in: *JHEP* 11 (2021) 207 • e-Print: 2103.08166 [hep-th]

Ginzberg-Landau-Wilson theory for Flat band, Fermi-arc and surface states of strongly correlated systems

- Eunseok Oh(Hanyang U.), Yunseok Seo(GIST, Gwangju), Taewon Yuk(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Aug 6, 2020)
  Published in: *JHEP* 01 (2021) 053 • e-Print: 2007.12188 [hep-th]

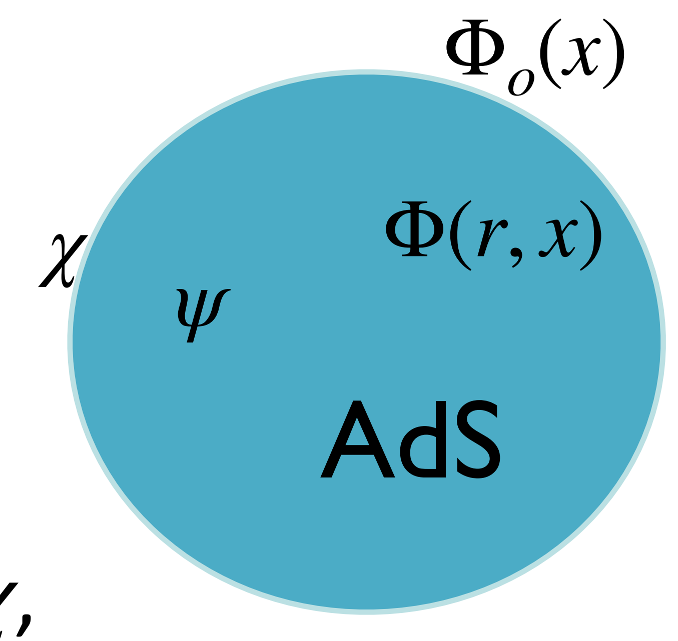
< T.Yuk

S. Sukrakarn >

Effect of Order in holographic theory

I. Theory of Condensation = $(\Phi_M, A_\mu, g_{\mu\nu})$

II. Theory of gap = fermion in the bulk.



Order : $\langle \bar{\chi}\Gamma^A\chi \rangle \neq 0$,

Dictionary:

1. First consider : Spin n tensor Φ_A dual to $\bar{\chi}\Gamma^A\chi$,

 & find the configuration of Φ_A + gravity.

2. Then, consider ψ *dual to* χ^* (the gauge singlet version of χ),

and add $\Phi_A \cdot \bar{\psi}\Gamma^A\psi$ to $\mathscr{L}_0$. Index $A = (\mu_1\mu_2\cdots\mu_n)$

$$\mathscr{L} = \bar{\psi}(\gamma^\mu i\partial_\mu - m)\psi + \Phi_A \cdot \bar{\psi}\Gamma^A\psi.$$ (But Φ_A, ψ)

—> Study $\psi(z,x)$ in the fixed ($g_{\mu\nu}$, Φ) to get spectrum of χ.

Holographic Mean field theory
Bulk locality=> almost unique in leading order.

$$S_{total} = S_\psi + S_{bdy} + S_{g,\Phi} + S_{int},$$

$$S_\psi = i\int d^dx \sum_{j=1}^{2} \sqrt{-g}\ \bar\psi^{(j)}\left(\not{D} - m^{(j)}\right)\psi^{(j)},$$

$$S_{bdy} = \frac{i}{2}\int_{bdy} d^{d-1}x\sqrt{-h}\left(\bar\psi^{(1)}\psi^{(1)} \pm \bar\psi^{(2)}\psi^{(2)}\right),$$

$$S_{g,\Phi} = \int d^dx\sqrt{-g}\left(R - 2\Lambda + |D_M\Phi_I|^2 - m_\Phi^2|\Phi|^2\right),$$

$$S_{int} = \int d^dx\sqrt{-g}\left(\bar\psi^{(1)}\Phi\cdot\Gamma\psi^{(2)} + h.c\right) \qquad I = (\mu_1\mu_2\cdots\mu_n)$$

where Φ_I is order parameter field, $\bar\psi^{(1)}\Phi\cdot\Gamma\psi^{(2)}$ is constructed by considering all possible Lorentz symmetry.

For known Φ, this is a Mean field Theory for fermion.

Results 1: Gap or no-gap

Among 16 possible Γ^A,

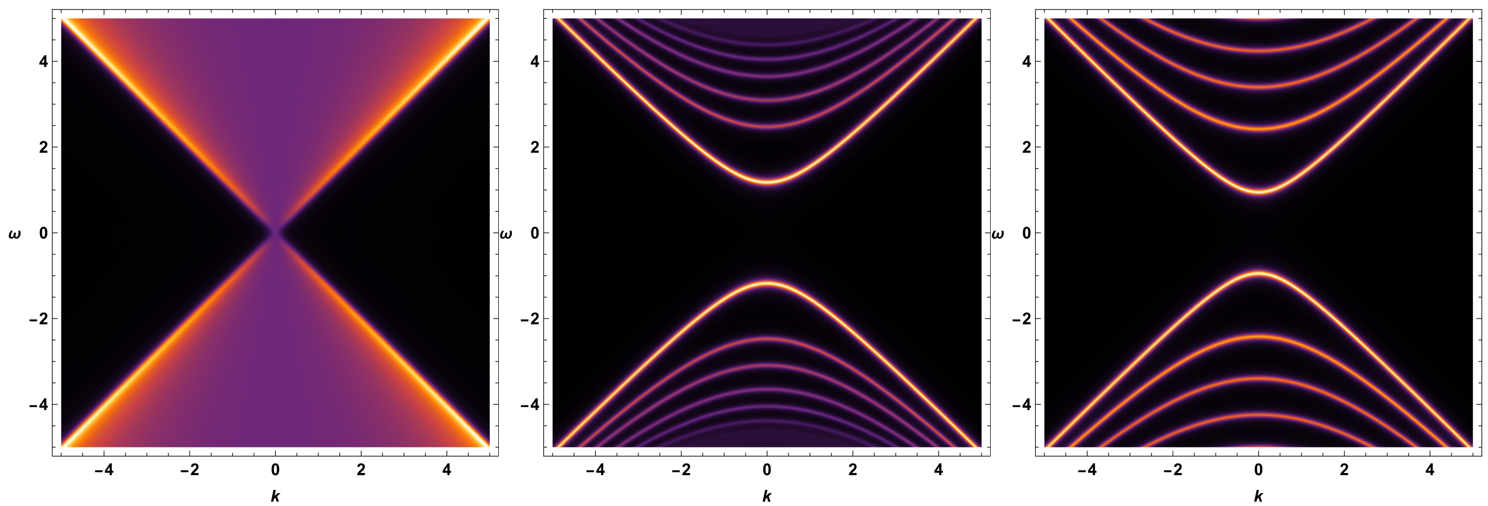
$\mathbf{1}, \Gamma^5, \Gamma^{rt}$ *produce s-wave gap*

$\Gamma^i, i = x, y$ *produce p-wave gap* with/without flat band

Anything else: non-gap

$$\mathrm{Tr}\,\mathbb{G}_{M_0}^{(SS)} = 4\omega \frac{\sqrt{\boldsymbol{k}^2 - \omega^2 + M_0^2}}{\boldsymbol{k}^2 - \omega^2 - i\epsilon}. \qquad \mathrm{Tr}\,\mathbb{G}_{M_0}^{(SA)} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2 - \omega^2 + M_0^2}},$$

Spectral features: gap or not



(a) $(\Delta_1, \Delta_2) = (0, 0)$ (b) $(\Delta_1, \Delta_2) = (2, 0)$ (c) $(\Delta_1, \Delta_2) = (0, 2)$

Figure 1: The gap structure (GS) with different Φ configuration. GS are similar for different power of z in the leading term of Φ. We used $T = 0.01$, $\mu = 0$.

Notice also: localized band

$$\frac{k-\omega}{\sqrt{k^2-\omega^2}}$$

$$\frac{1}{x+i\epsilon} = P\frac{1}{x} - i\pi\delta(x)$$

Results 2. Classifying the quantum fluid by singularity of G

1. Branch cut type: most of them. $\Phi_A \rightarrow B_{\mu\nu\ldots}$

- B_u, B_t, B_{ti}, B_{ui} B_{xy} (AdS_5)
- $B_u, B_t, B_{ti}, B_{ui}, B_{5t}, B_{5u}$ (AdS_4)

$$\mathbb{G}(k)^{(SA)}_{B^{(-1)}_{xy}} = \frac{1}{2\mathcal{K}_{xy+}\mathcal{K}_{xy-}} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix},$$

$$\mathcal{K}_{xy\pm} = \sqrt{(b_{xy} \pm |\boldsymbol{k}_\perp|)^2 + k_z^2 - \omega^2},$$

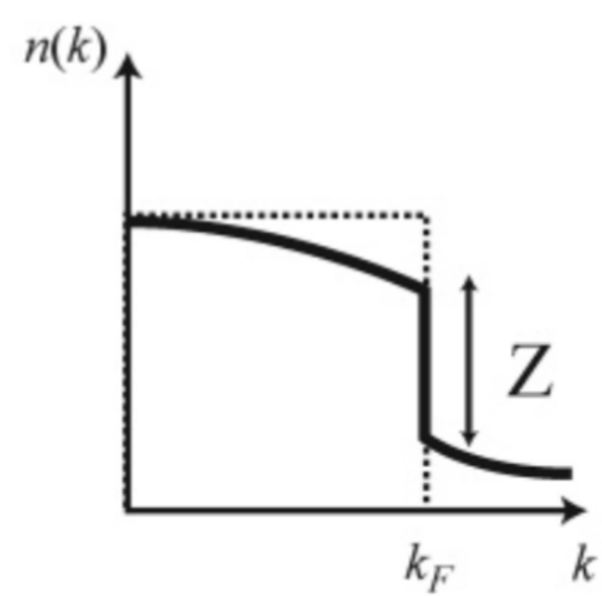
—> New type of Non-Fermi liquid

* Observation: CFT singularity structure remains even after scaling sym. Is broken.

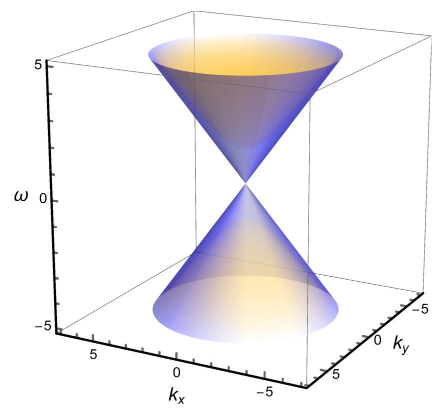
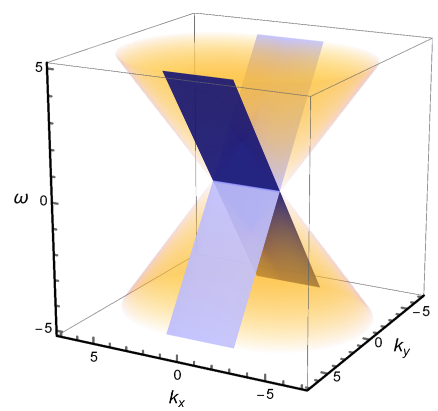
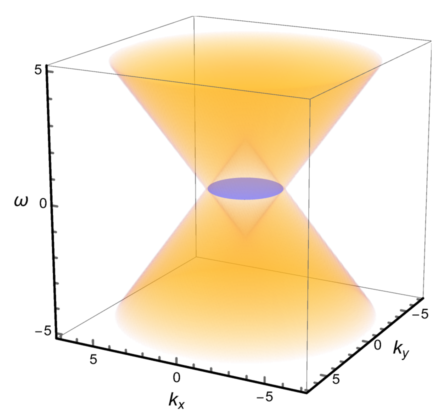
* Traditional way:

$3 \; ways \; to \; NFL: i) \; \Sigma \sim \omega^a \; with \; a < 1; \quad ii) Z \rightarrow 0, \quad iii) \; flat \; band$

$$G(k,\omega) = \frac{Z}{\omega - \epsilon_k - \Sigma} + less \; singular$$



2. Pole type G_R in AdS$_4$

$\Phi_{(SS)}, \Phi_{5(SA)}$		$TrG_R(k_\mu) = \dfrac{4\omega\sqrt{\vec{k}^2 - \omega^2 + M^2}}{\vec{k}^2 - \omega^2}$
$B_{x(SA)}, B_{x5(SS)}$		$TrG_R(k_\mu) = \dfrac{2\omega}{b(k_y^2 - \omega^2)}\left[(b + k_x)\epsilon_- + (b - k_x)\epsilon_+\right]$; $\epsilon_\pm = \sqrt{(b \pm k_x)^2 + k_y^2 - \omega^2}$
$B_{xy(SS)}, B_{tu(SA)}$		$TrG_R(k_\mu) = -\dfrac{2}{b\omega}\left[(b + \lvert\vec{k}\rvert)\epsilon_- + (b - \lvert\vec{k}\rvert)\epsilon +\right]$; $\epsilon_\pm = \sqrt{(b \pm \lvert\vec{k}\rvert)^2 - \omega^2}$

3. Holographic Flat band (AdS_5): multi layered graphene

$$TrG_\Phi \simeq \frac{4\omega|M|}{\vec{k}^2-\omega^2},$$ Point: Usual Cone

$$TrG_{B_i} \simeq \frac{4\omega(b^2-k_i^2)}{\vec{k}_\perp^2-\omega^2}\Theta(b^2-k_i^2),$$ Flat Line

$$TrG_{B_{jk}} \simeq \frac{4\omega(b^2-k_\perp^2)}{\vec{k}_i^2-\omega^2}\Theta(b^2-\vec{k}_\perp^2),$$ Flat Disk

$$TrG_{B_{tu}} \simeq -\frac{2(b^2-\vec{k}^2)}{\omega}\Theta(b^2-\vec{k}^2)$$ Flat Ball

θ-fnt from branch cut

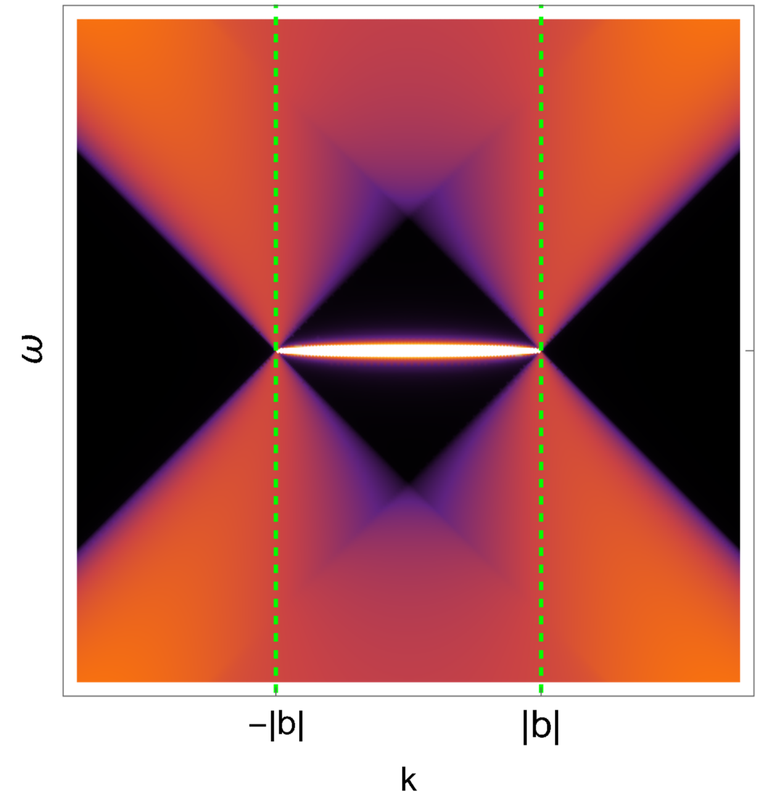


Figure: Holographic Flat band

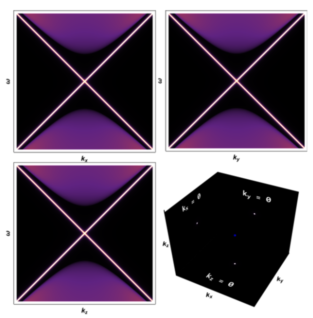
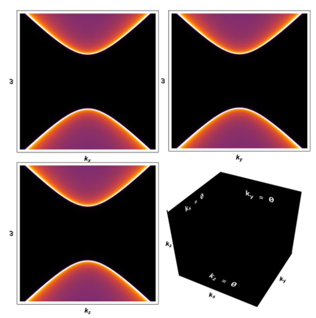
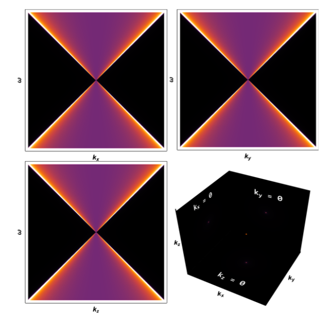
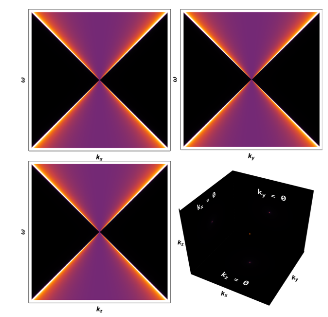
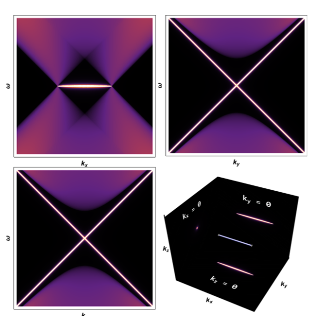
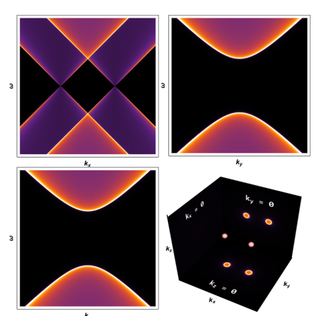
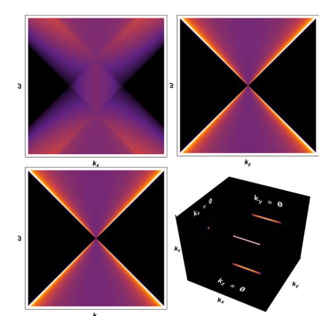
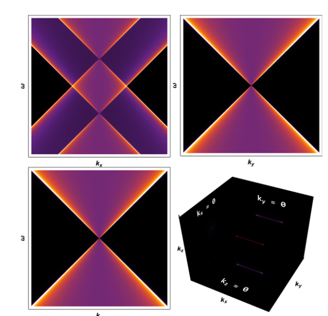
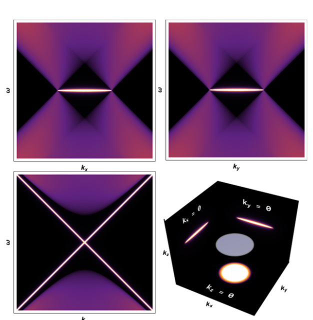
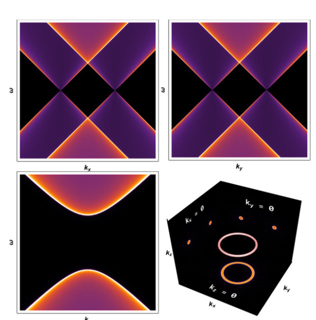
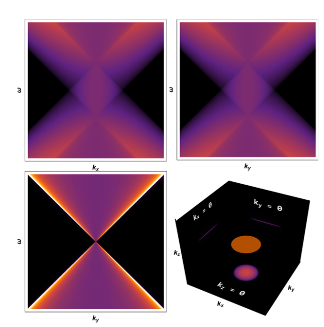
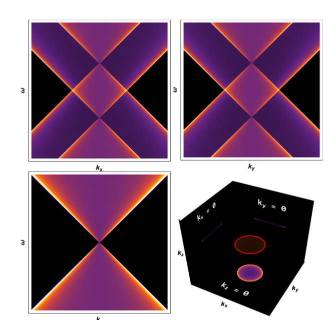
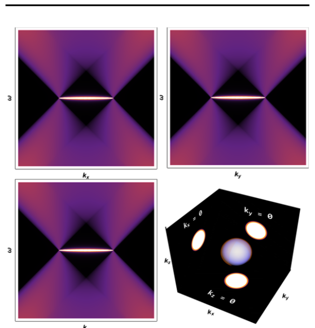
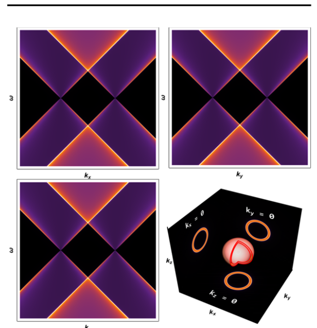
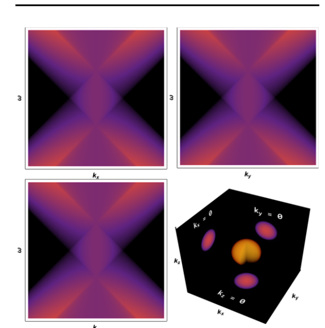
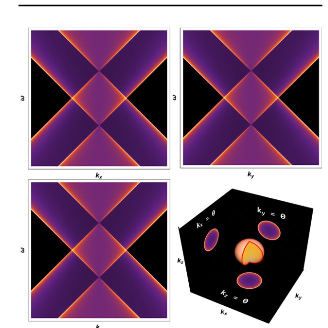
b=symmetry breaking scale

! Flat band of 0,1,2,3 dim, only from Pole type Gr.

Inv Green Functions

Interactions	**Trace of analytic Green's functions** $(\mathbf{AdS}_4)$	**Features/Classification**
M_0/M_{05}	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_0} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_{50}} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}$	Gapful/s-wave gap
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_0} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_{50}} = 4\omega\frac{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}{\boldsymbol{k}^2-\omega^2-i\epsilon}$	Topological liquid
B_x/B_{5x}	$\mathrm{Tr}\,G^{(SS)}_{B_x^{(0)}} \equiv \mathrm{Tr}\,G^{(SA)}_{B_{5x}^{(0)}} = \frac{2\omega}{\sqrt{(b-k_x)^2+k_y^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+k_x)^2+k_y^2-\omega^2}}$	Shifting cones/p-wave gap
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_x^{(0)}} \equiv \mathrm{Tr}\,G^{(SS)}_{B_{5x}^{(0)}} = \frac{2\omega}{b}\Big[\frac{(b+k_x)\sqrt{(b-k_x)^2+k_y^2-\omega^2}+(b-k_x)\sqrt{(b+k_x)^2+k_y^2-\omega^2}}{k_y^2-\omega^2-i\epsilon}\Big]$	1D flat band
B_{xy}/B_{tu}	$\mathrm{Tr}\,G^{(SA)}_{B_{xy}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tu}^{(-1)}} = \frac{2\omega}{\sqrt{(b-\boldsymbol{k})^2-\omega^2}} + \frac{2\omega}{\sqrt{(b+\boldsymbol{k})^2-\omega^2}}$	Nodal ring
(anti-symmetric)	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{xy}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tu}^{(-1)}} = -\frac{2}{b}\Big[\frac{(b+\lvert\boldsymbol{k}\rvert)\sqrt{(b-\boldsymbol{k})^2-\omega^2}+(b-\lvert\boldsymbol{k}\rvert)\sqrt{(b+\boldsymbol{k})^2-\omega^2}}{\omega+i\epsilon}\Big]$	2D flat band
B_u	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_u^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_u^{(0)}} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2}}$	QCP
B_{ux}/B_{5u}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{ux}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{5u}^{(-1)}} = 4\omega\frac{b^2+\boldsymbol{k}^2-\omega^2+f_+f_-}{f_+f_-(f_++f_-)}\ ;\ f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-k_y^2}\Big)^2}$	Filled nodal line
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{ux}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{5u}^{(-1)}} = 4\omega\frac{(f_++f_-)\sqrt{\omega^2-k_y^2}-b(f_+-f_-)}{\sqrt{\omega^2-k_y^2}(b^2+\boldsymbol{k}^2-\omega^2+f_+f_-)}\ ;\ f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-k_y^2}\Big)^2}$	Non-singular segment
B_t/B_{5t}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_t^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{5t}^{(0)}} = 2\Big(\frac{b+\omega}{\sqrt{\boldsymbol{k}^2-(b+\omega)^2}} - \frac{b-\omega}{\sqrt{\boldsymbol{k}^2-(b-\omega)^2}}\Big)$	Filled nodal ring
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_t^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{5t}^{(0)}} = \frac{2}{b}\Big[\sqrt{\boldsymbol{k}^2-(b-\omega)^2} - \sqrt{\boldsymbol{k}^2-(b+\omega)^2}\Big]$	Non-singular disk

Spectral features

Order p. & Dims	*Flat bands*	*Semi-metals*	*Order p. & Dims*	*Nonsingular*	*ω-shiftings*
Φ $d_{eff}=0$	SS, (figure 2) 	SA, (figure 2) 	B_u $d_{eff}=0$	SS,SA 	SS,SA 
B_x $d_{eff}=1$	SA, (figure 5) 	SS, (figure 5) 	B_{ux} $d_{eff}=1$	SA, (figure 6) 	SS, (figure 6) 
B_{xy} $d_{eff}=2$	SS, (figure 7) 	SA, (figure 7) 	B_{tz} $d_{eff}=2$	SS, (figure 8) 	SA, (figure 8) 
B_{tu} $d_{eff}=3$	SA, (figure 4) 	SS, (figure 4) 	B_t $d_{eff}=3$	SA, (figure 3) 	SS, (figure 3) 

Summary of Holo mean field theory: order type/ spectral features/ singularity type

- Scalar, pseudo scalar : s-wave gap
- Spatial vector gives p-wave gap.
- Temporal vector : nodal ring (2d AdS4), nodal shell(3d AdS5)
- Flat band : 1,2,3 dim by B_x, B_{xy}, B_{tr},
- Symmetric 2-tensor: D-wave

Implications

Lattice=$10^3 eV$, Transport =IR data. 0.1eV
So, electrons can not see the lattice!
They can see only repeated structure or protected by the symmetry: Condensation is creation of new order although it is expressed as violation of symmetry by the OLD ground state!

Lattice <=> Symmetry breaking order.
Interaction => Entanglement
Duality:
Qm or Cl depends on the excitation you are looking for

11. Applications

Mean field theory and holographic Kondo lattice

- Young-Kwon Han(Hanyang U.), Debabrata Ghorai(Hanyang U.), Taewon Yuk(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Jul 2, 2024)
  - e-Print: 2407.01978 [hep-th]

Observation of Kondo condensation in a degenerately doped silicon metal

- Hyunsik Im, Dong Uk Lee, Yongcheol Jo, Jongmin Kim, Yonuk Chong et al. (Jan 21, 2023)
  - Published in: *Nature Phys.* 19 (2023) 5, 676-681 • e-Print: 2301.09047 [cond-mat.str-el]

ABC-stacked multilayer graphene in holography

- Jeong-Won Seo(Hanyang U.), Taewon Yuk(Hanyang U.), Young-Kwon Han(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Aug 31, 2022)
  - Published in: *JHEP* 11 (2022) 017 • e-Print: 2208.14642 [hep-th]

Classes of holographic Mott gaps

- Debabrata Ghorai(Hanyang U.), Taewon Yuk(Hanyang U.), Young-Kwon Han(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Apr 16, 2024)
  - Published in: *JHEP* 10 (2024) 062 • e-Print: 2404.10412 [hep-th]

Encoding the lattice in the holography

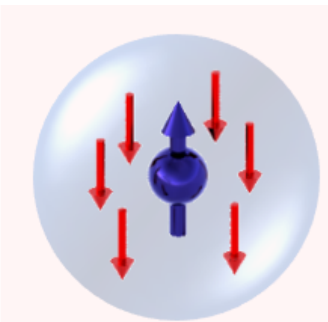
- Taewon Yuk(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Jan 15, 2024)
  - Published in: *Phys.Rev.D* 110 (2024) 10, 106017 • e-Print: 2401.07498 [hep-th]

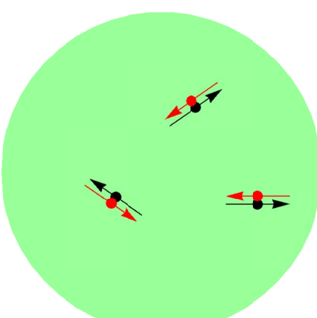
< T.Yuk
.Seo >

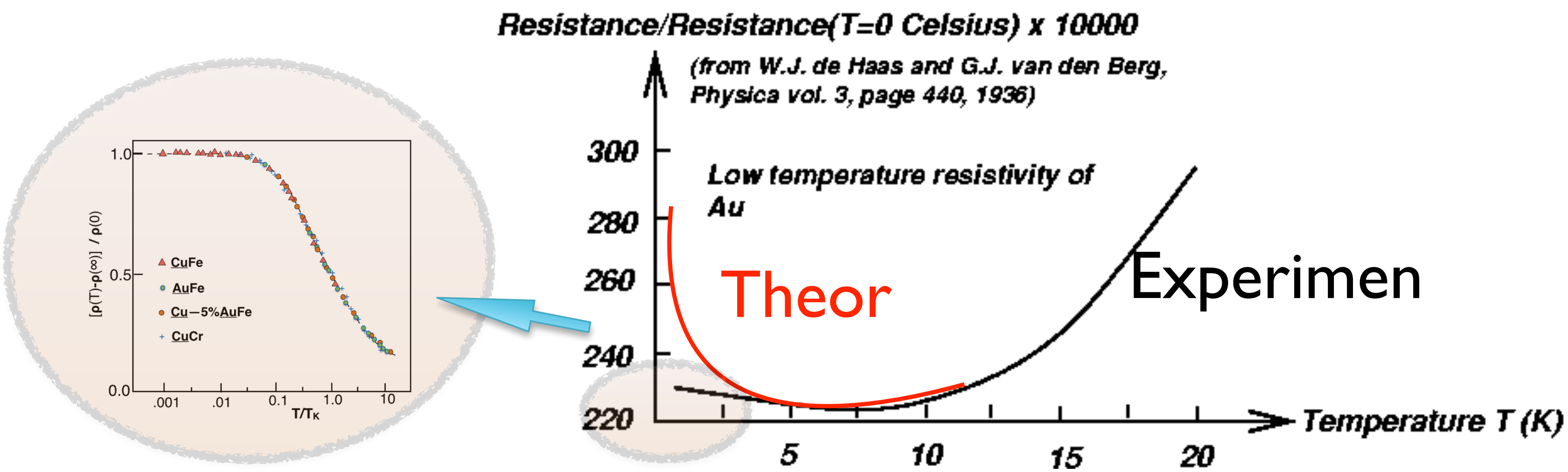
<YK Han
D. Ghorai >

application I : multi-Kondo

- Single Kondo: spin impurity in conductor.



- 



K.Wilson: RG

Kondo: $\rho(T) = \rho_0 + aT^2 + bT^5 + c_m \ln \frac{\mu}{T},$

Holographic single Kondo effect : Issue : log T

J. Erdmenger, R. Myer, A. Obanon, A. Karch,…..: impurity=AdS defect in AdS

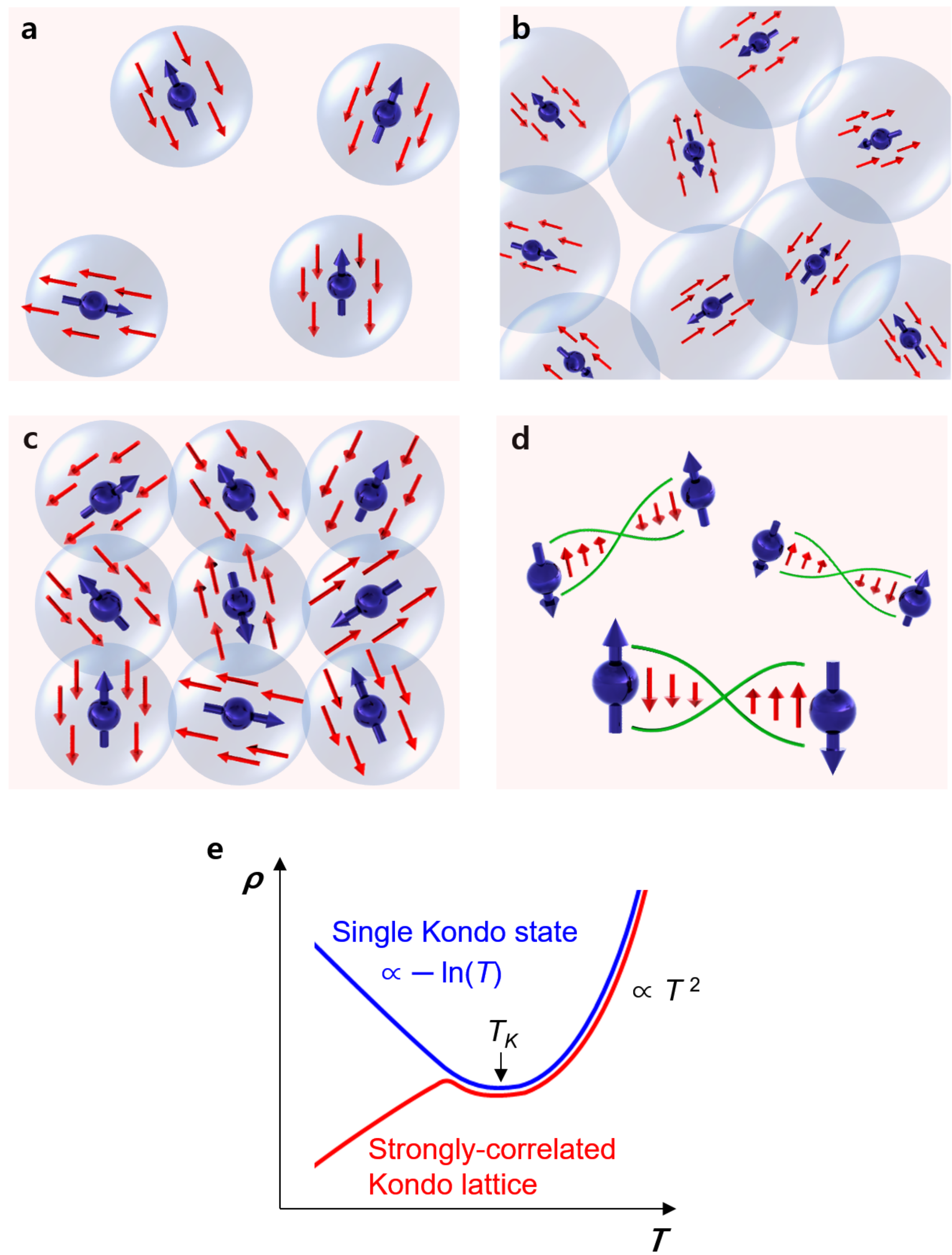
Multi Kondo : Random vs regular impurities

single Kondo

Random imp.
: gap

Kondo Lattice
heavy fermion,
Kondo insulator

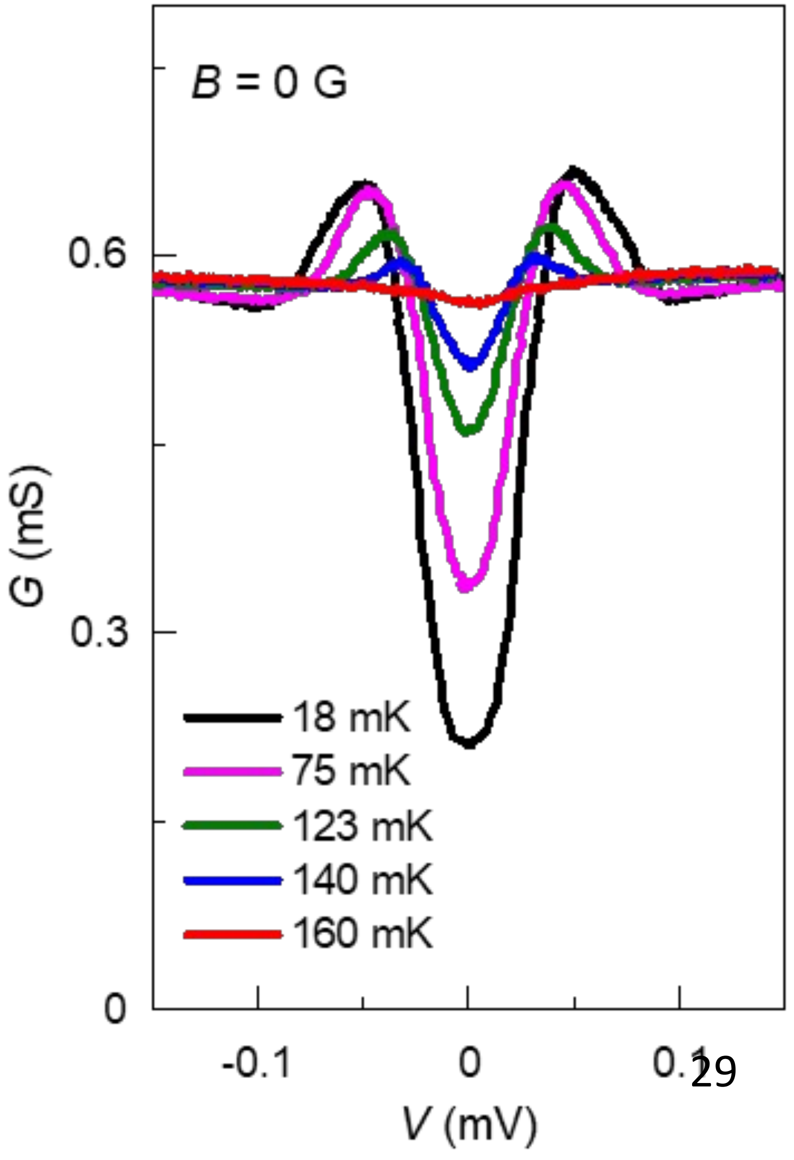
RKKY
weak coupling



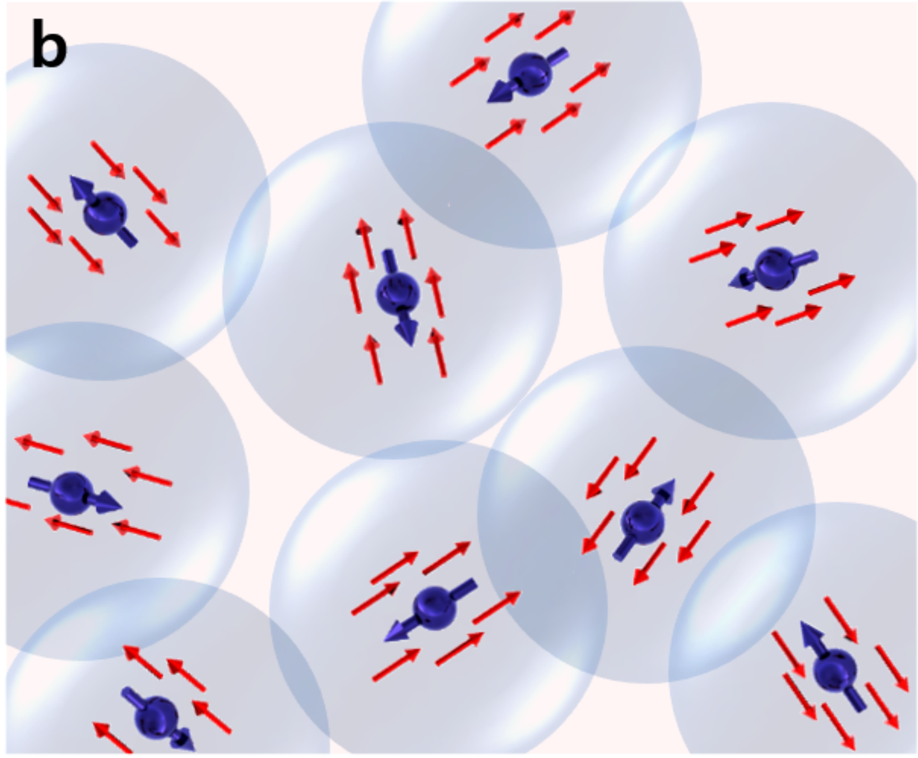
Random Kondo

10 years ago,
H. Im et.al found a tiny gap in extremely high P-doped Si
A Dirty sample

S-wave gap



Kondo-Condensation



Kondo cloud

$$f^\dagger_\alpha c_\beta = 0 \oplus 1 = \Phi + A_\mu$$

Phenomena says
S-wave gap
=> spin 0
Discard vector.

For Kondo condensation: scalar-fermion

$$S = S_\Phi + S_\psi + S_{bdy},$$

$$S_\Phi = \int d^{d+1}x\sqrt{-g}\left(D_\mu\Phi_I^2 - m_\Phi^2\Phi^2\right), \qquad S_{bdy} = i\int_{\partial M} d^dx\sqrt{-h}\bar{\psi}\psi,$$

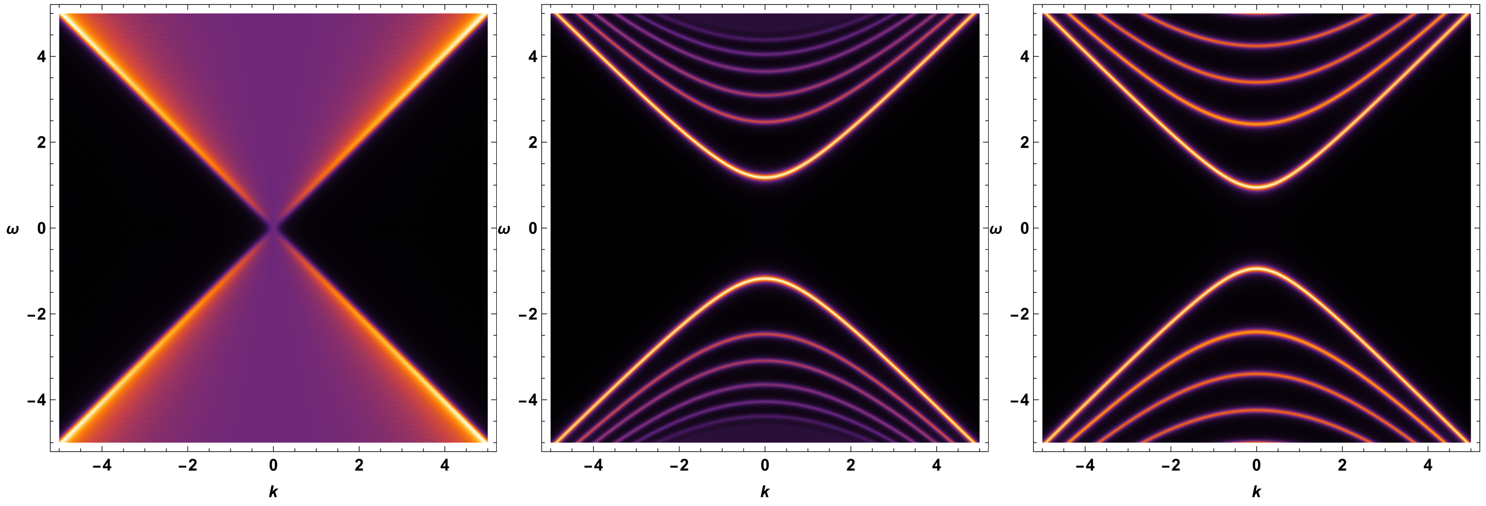
$$S_\psi = \int d^{d+1}\sqrt{-g}x\ i\bar{\psi}\left(\Gamma^\mu\mathcal{D}_\mu - (m + g\Phi)\right)\psi,$$

$D_\mu = \partial_\mu$ => V(Φ) is needed

$$\Phi = M_0 z + M_1 z^2$$

$$m_\Phi^2 = -2$$

$$M_0 M_1 = 0$$

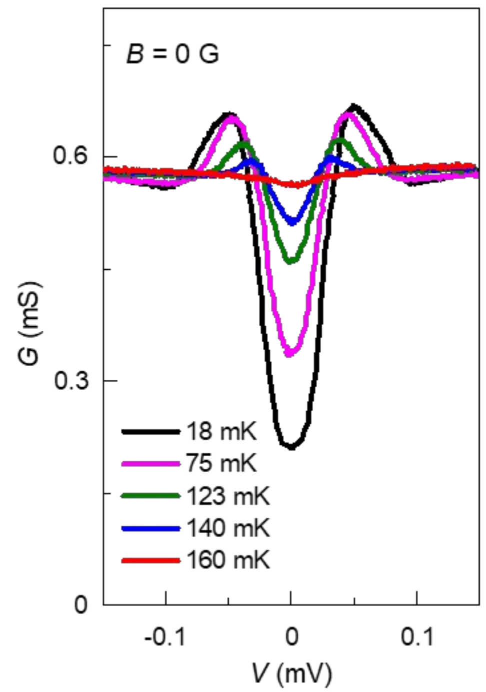


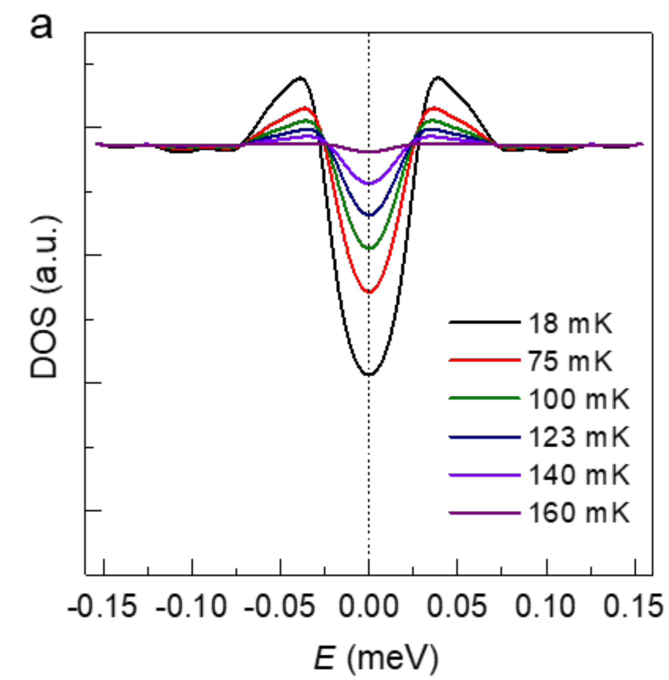
(a) $(\Delta_1, \Delta_2) = (0, 0)$ (b) $(\Delta_1, \Delta_2) = (2, 0)$ (c) $(\Delta_1, \Delta_2) = (0, 2)$

Figure 1: The gap structure (GS) with different Φ configuration. GS are similar for different power of z in the leading term of Φ. We used $T = 0.01$, $\mu = 0$.

Spectral functions and DOS

Exp vs Theory





nature physics

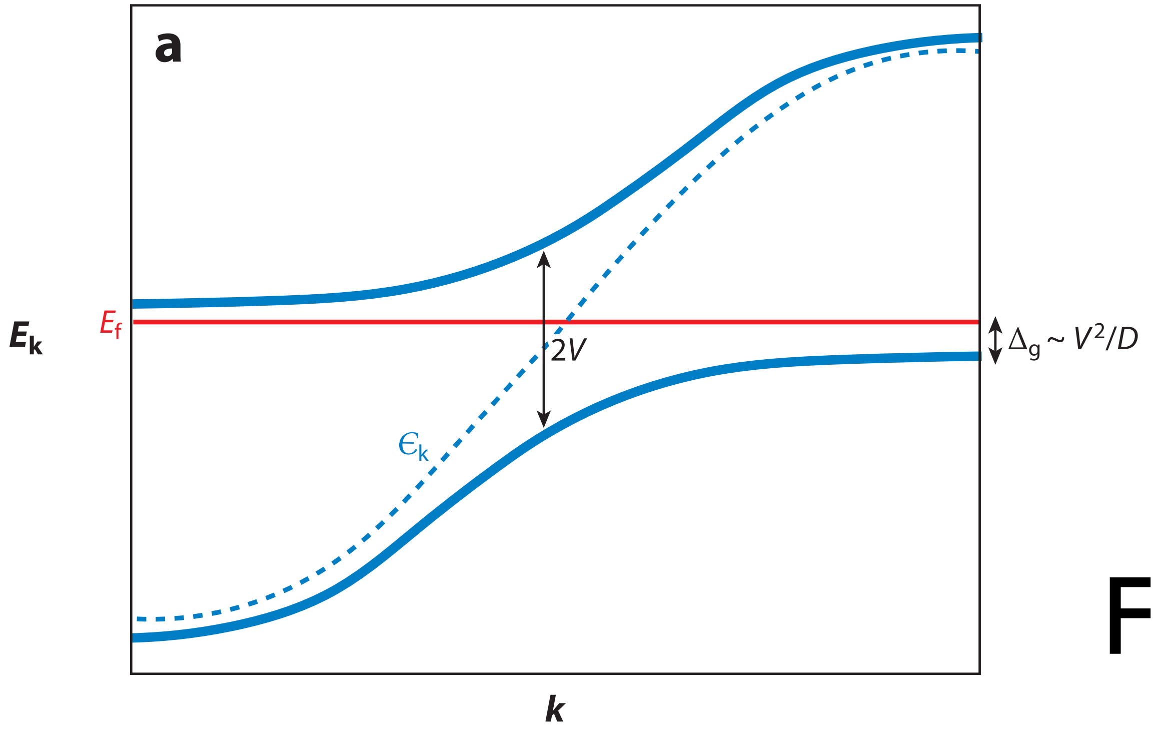
Article

https://doi.org/10.1038/s41567-022-01930-3

Observation of Kondo condensation in a degenerately doped silicon metal

Hyunsik Im[1,2], Dong Uk Lee[3], Yongcheol Jo[1], Jongmin Kim[1], Yonuk Chong[4], Woon Song[5], Hyungsang Kim[1], Eun Kyu Kim[3], Taewon Yuk[3], Sang-Jin Sin[3], Soonjae Moon[3], Jonathan R. Prance[6], Yuri A. Pashkin[6] & Jaw-Shen Tsai[2,7]

Kondo lattice



Hybridization of
Flat band +dispersive band

Essence of the Kondo Lattice physics:

1. Heavy fermion, FL

2. Luttinger volume up.

On a larger length scale, a very slow coherent motion.

Both are explained from the

MFT for the Kondo lattice

$$\mathcal{L} = \psi^\dagger \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu \right) \psi + \chi^\dagger \left(i\frac{\partial}{\partial t} - \lambda \right) \chi$$
$$+ \frac{g_l}{2}(\psi^\dagger\psi)^2 - g_s(\psi^\dagger\psi)(\chi^\dagger\chi) - g_v(\psi^\dagger\vec{\sigma}\psi)\cdot(\chi^\dagger\vec{\sigma}\chi).$$

Kondo int.

Using the Fierz identity,

$$\mathcal{L} = \psi^\dagger \left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu \right) \psi + \chi^\dagger \left(i\frac{\partial}{\partial t} - \lambda \right) \chi$$
$$+ \frac{g_l}{2}(\psi^\dagger\psi)^2 + g_s'(\psi^\dagger\chi)(\chi^\dagger\psi) + g_v'(\psi^\dagger\vec{\sigma}\chi)\cdot(\chi^\dagger\vec{\sigma}\psi), \qquad g_s' := \frac{g_s + 3g_v}{2}, \quad g_v' := \frac{g_s - g_v}{2}.$$

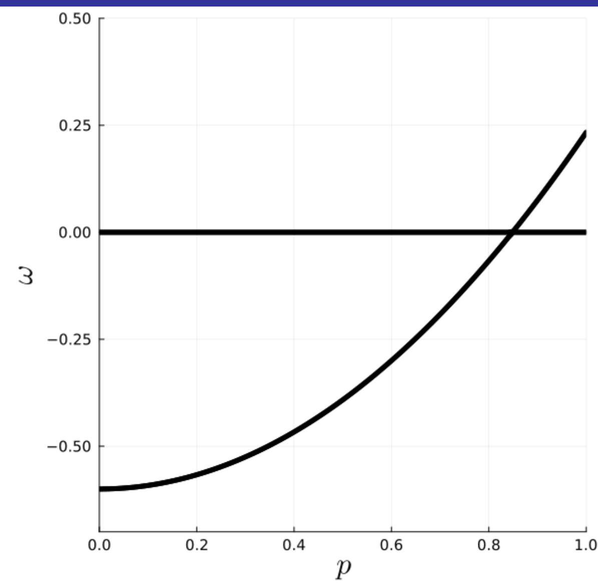
$$\mathcal{L}_{\mathrm{MF}} = \Psi^\dagger D \Psi - U,$$

$$\Psi^\dagger := \begin{pmatrix} \psi^\dagger & \chi^\dagger \end{pmatrix}, \quad \Psi := \begin{pmatrix} \psi \\ \chi \end{pmatrix}, \qquad \langle\psi^\dagger\psi\rangle \equiv -\frac{M}{g_l}, \quad \langle\psi^\dagger\chi\rangle \equiv \frac{\Delta_s}{g_s'}, \quad \langle\psi^\dagger\vec{\sigma}\chi\rangle \equiv \frac{\vec{\Delta}_v}{g_v'},$$

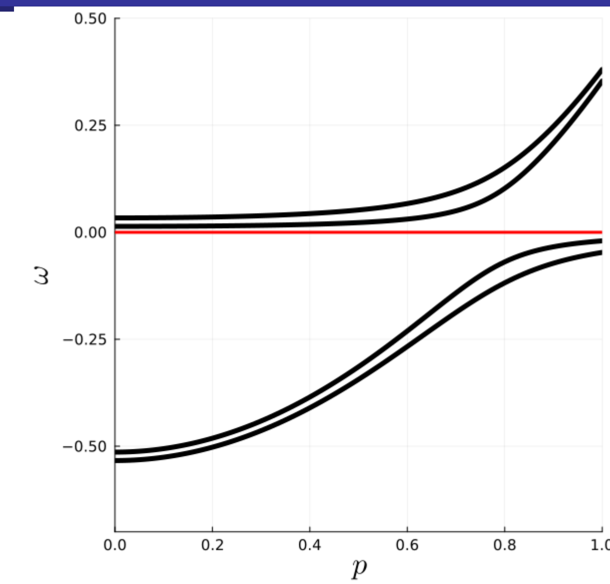
$$D := \begin{pmatrix} i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} + \mu - M & \Delta_s^* + \vec{\sigma}\cdot\vec{\Delta}_v^* \\ \Delta_s + \vec{\sigma}\cdot\vec{\Delta}_v & i\frac{\partial}{\partial t} - \lambda \end{pmatrix},$$

$$U := \frac{M^2}{2g_l} + \frac{|\Delta_s|^2}{g_s'} + \frac{|\vec{\Delta}_v|^2}{g_v'}.$$

MFT for the Kondo lattice (continued)



(a) $\omega(p)$ without condensation.

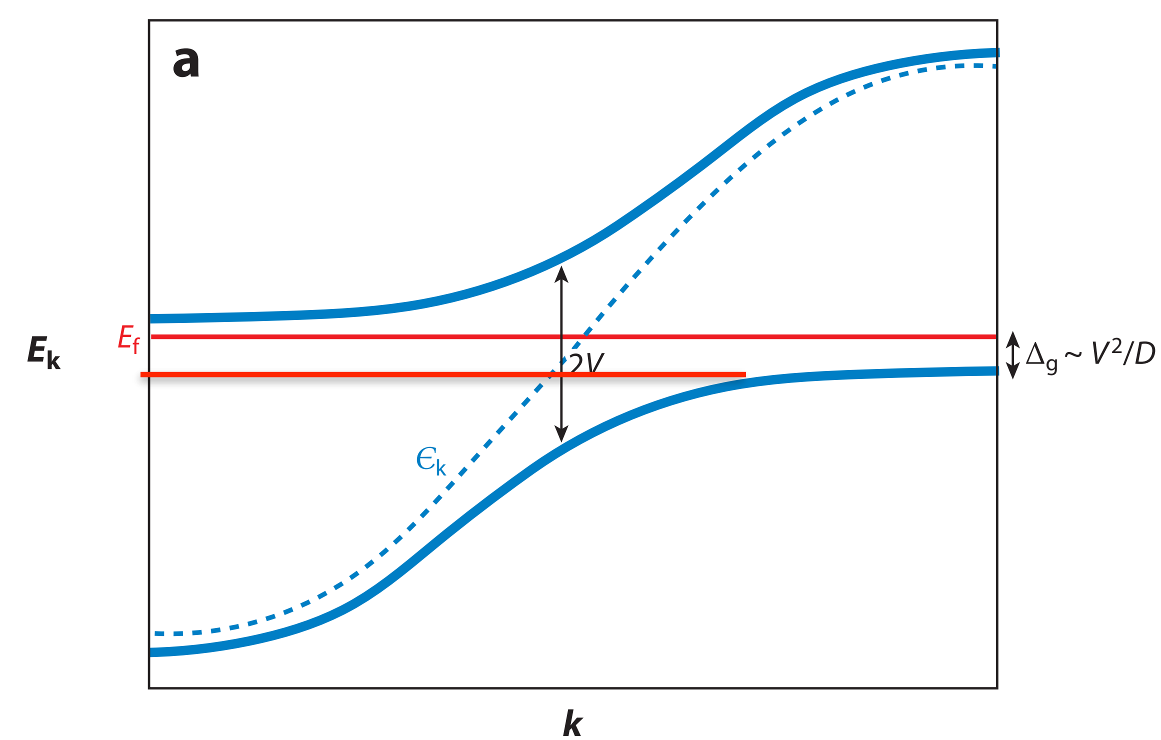


(b) $\omega(p)$ with condensation.

Schematic presentation of the hybridization

$T_K \sim V^2/D$: 1 – *Kondo Temp* .

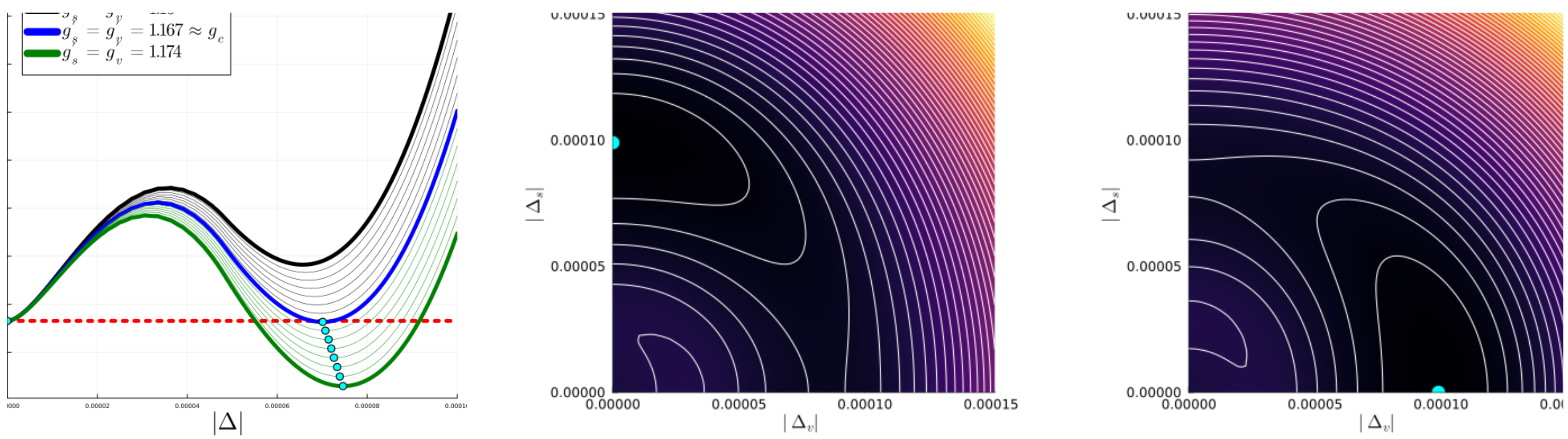
FS in gap-> K insulator, otherwise Heavy Fermion w/ larger FS



MFT for the Kondo lattice

$$\Omega = U + \frac{1}{V}\sum_{|\vec{p}|<\Lambda}\sum_{i=1}^{4}\left\{-\frac{1}{2}|\omega_i(\vec{p})| - \frac{1}{\beta}\ln\left[1+e^{-\beta|\omega_i(\vec{p})|}\right]\right\}$$

$$= U - \frac{1}{4\pi^2}\int_0^{\Lambda} \mathrm{d}p p^2 \sum_{i=1}^{4}|\omega_i(p)| - \frac{1}{2\pi^2\beta}\int_0^{\Lambda}\mathrm{d}p p^2\sum_{i=1}^{4}\ln\left[1+e^{-\beta|\omega_i(p)|}\right],$$



(a) Ω versus $|\Delta|$. (b) Ω with strong $g'_s > g'_v > g_c$. (c) Ω with strong $g'_v > g'_s >$

Condensation only for large enough coupling!

Validity of MFT?

Holographic Kondo Lattice

YoungKwon Han

$$S_{\text{tot}} = S_{\text{bg}} + S_{\text{spin}},$$

$$S_{\text{bg}} = S_{\text{bg,bdy}} + \int d^4x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

$$+ \int d^4x\sqrt{-g}[-(\partial_\mu\Phi_{\text{s}})(\partial^\mu\Phi_{\text{s}}) - m_{\text{s}}^2\Phi_{\text{s}}^2 - (\partial_\mu\Phi_{\text{ps}})(\partial^\mu\Phi_{\text{ps}}) - m_{\text{ps}}^2\Phi_{\text{ps}}^2],$$

$$S_{\text{spin}} = S_{\text{spin,bdy}} + \sum_{j=1}^{2}\int d^4x\sqrt{-g}\,i\bar{\psi}^{(j)}\left[\frac{1}{2}\left(\overrightarrow{\not{D}}^{(j)} - \overleftarrow{\not{D}}^{(j)}\right) - m_j\right]\psi^{(j)}$$

$$+ \int d^4x\sqrt{-g}\begin{pmatrix}\bar{\psi}^{(1)}\\ \bar{\psi}^{(2)}\end{pmatrix}^T\begin{pmatrix}g_1\Phi_{\text{ps}}\cdot\Gamma^5 & V\Phi_{\text{s}}\cdot i\mathbb{I}_4\\ V\Phi_{\text{s}}\cdot i\mathbb{I}_4 & g_2\Phi_{\text{s}}\cdot i\mathbb{I}_4\end{pmatrix}\begin{pmatrix}\psi^{(1)}\\ \psi^{(2)}\end{pmatrix},$$

$$S_{\text{spin,bdy}} = \frac{1}{2}\int d^3x\sqrt{-h}[\bar{\psi}^{(1)}(i\mathbb{I}_4)\psi^{(1)} + \bar{\psi}^{(2)}\Gamma^{\underline{\text{xy}}}\psi^{(2)}],$$

$$\not{D}^{(j)} = \Gamma^a e_a{}^B\left(\partial_B + \frac{1}{4}\omega_{Bcd}\Gamma^{cd} - iq_jA_B\right), \qquad h = gg^{uu},$$

$$L = 1, \qquad \bar{\psi}^{(j)} = \psi^{(j)\dagger}\Gamma^{\underline{\text{t}}},$$

$$\Gamma^{\underline{\text{t}}} = \sigma_1\otimes i\sigma_2 = \begin{pmatrix}0 & i\sigma_2\\ i\sigma_2 & 0\end{pmatrix}, \qquad \Gamma^{\underline{\text{x}}} = \sigma_1\otimes\sigma_1 = \begin{pmatrix}0 & \sigma_1\\ \sigma_1 & 0\end{pmatrix},$$

$$\Gamma^{\underline{\text{y}}} = \sigma_1\otimes\sigma_3 = \begin{pmatrix}0 & \sigma_3\\ \sigma_3 & 0\end{pmatrix}, \qquad \Gamma^{\underline{\text{u}}} = \sigma_3\otimes\sigma_0 = \begin{pmatrix}\sigma_0 & 0\\ 0 & -\sigma_0\end{pmatrix}$$

$$\Gamma^5 = i\Gamma^{\underline{\text{t}}}\Gamma^{\underline{\text{x}}}\Gamma^{\underline{\text{y}}}\Gamma^{\underline{\text{u}}}, \qquad \Gamma^{ab} = \frac{1}{2}[\Gamma^a, \Gamma^b].$$

Role of interaction terms

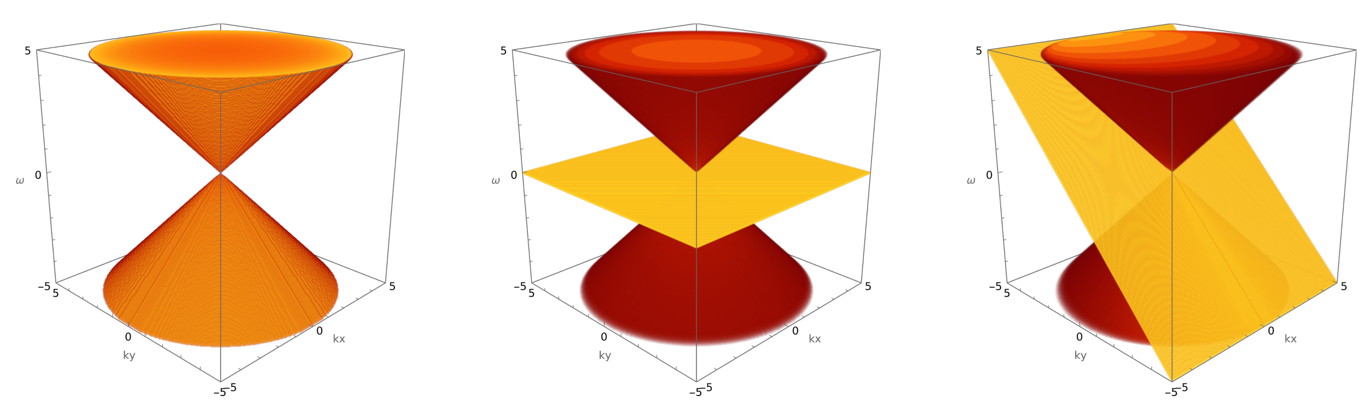
Eq. Of Motion

$$S_{\text{spin}} = S_{\text{spin,bdy}} + \sum_{j=1}^{2} \int d^4x \sqrt{-g}\, i\bar{\psi}^{(j)} \left[\frac{1}{2}\left(\overrightarrow{\not{D}}^{(j)} - \overleftarrow{\not{D}}^{(j)}\right) - m_j\right] \psi^{(j)}$$

$$S_{\text{spin,bdy}} = \frac{1}{2}\int d^3x \sqrt{-h}[\bar{\psi}^{(1)}(i\mathbb{I}_4)\psi^{(1)} + \bar{\psi}^{(2)}\Gamma^{\underline{xy}}\psi^{(2)}]$$

Gives Dirac cone
(H. Liu et.al)

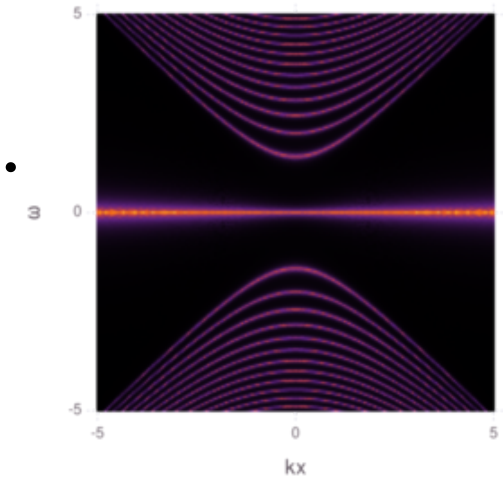
Gives Flat band
(Tong+Laia)

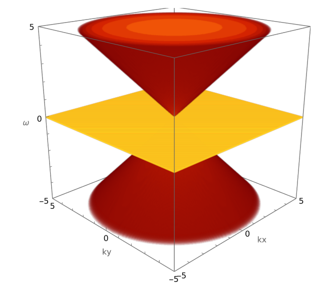


(a) Standard ($\Gamma = i\mathbb{I}_4$) (b) $\Gamma = \Gamma^{\underline{xy}}$ (c) $\Gamma = \Gamma^{5\underline{x}}$

Role of interaction terms

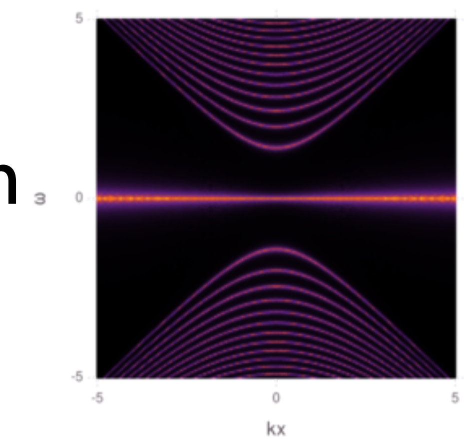
Gives hyperbolic spec.



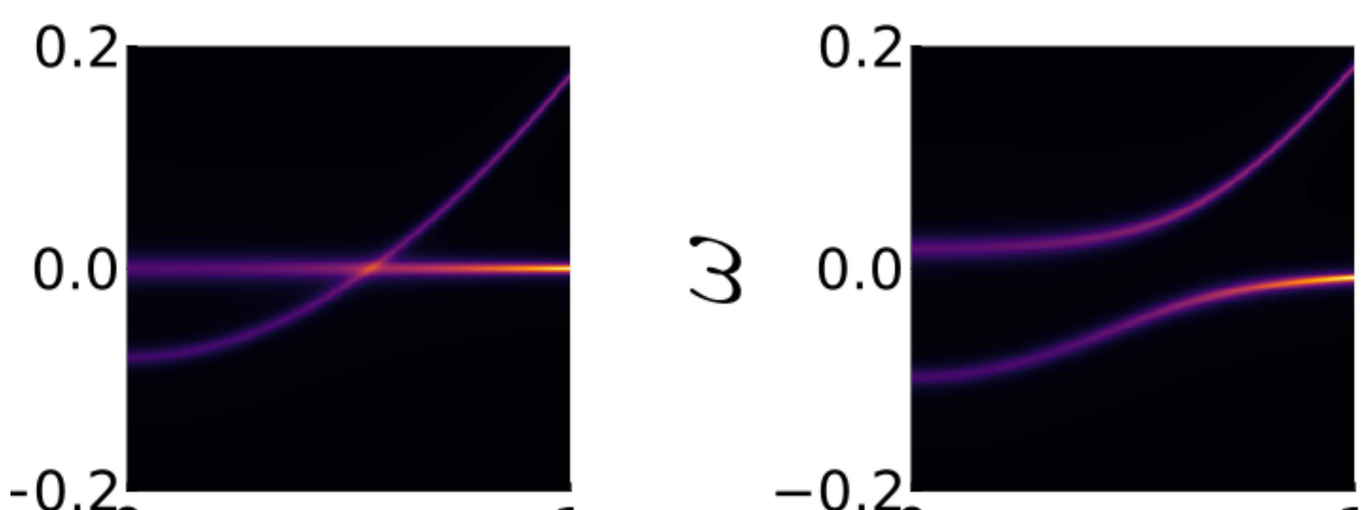


$$+\int \mathrm{d}^4x\sqrt{-g}\begin{pmatrix}\bar{\psi}^{(1)}\\ \bar{\psi}^{(2)}\end{pmatrix}^T\begin{pmatrix}g_1\Phi_{\mathrm{ps}}\cdot\Gamma^5 & V\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4\\ V\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4 & g_2\Phi_{\mathrm{s}}\cdot i\mathbb{I}_4\end{pmatrix}\begin{pmatrix}\psi^{(1)}\\ \psi^{(2)}\end{pmatrix},$$

Separating flat band from Dirac cone.



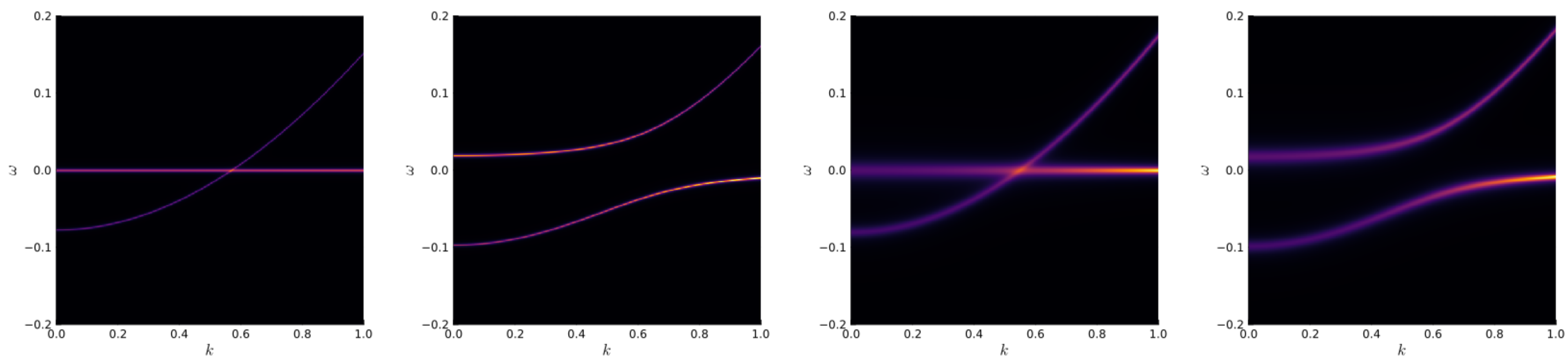
Making Hybridization to compete the goal



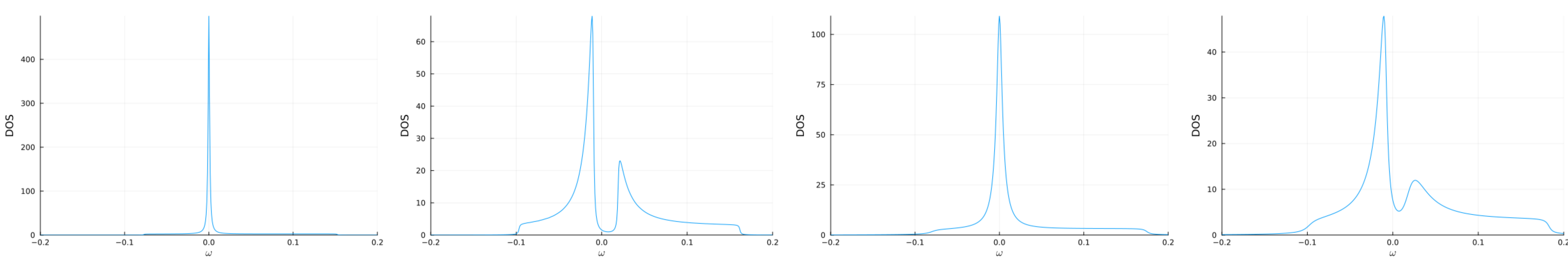
Holographic Kondo Lattice

$$\left[\begin{pmatrix} \overrightarrow{\not{D}} - m_1 & 0 \\ 0 & \overrightarrow{\not{D}} - m_2 \end{pmatrix} - i \begin{pmatrix} g_1 \Phi_{\rm ps} \cdot \Gamma^5 & V \Phi_{\rm s} \cdot i\mathbb{I}_4 \\ V \Phi_{\rm s} \cdot i\mathbb{I}_4 & g_2 \Phi_{\rm s} \cdot i\mathbb{I}_4 \end{pmatrix}\right] \begin{pmatrix} \psi^{(1)} \\ \psi^{(2)} \end{pmatrix} = 0.$$

$T/\mu \sim 0,$ $T/\mu \sim 1,$

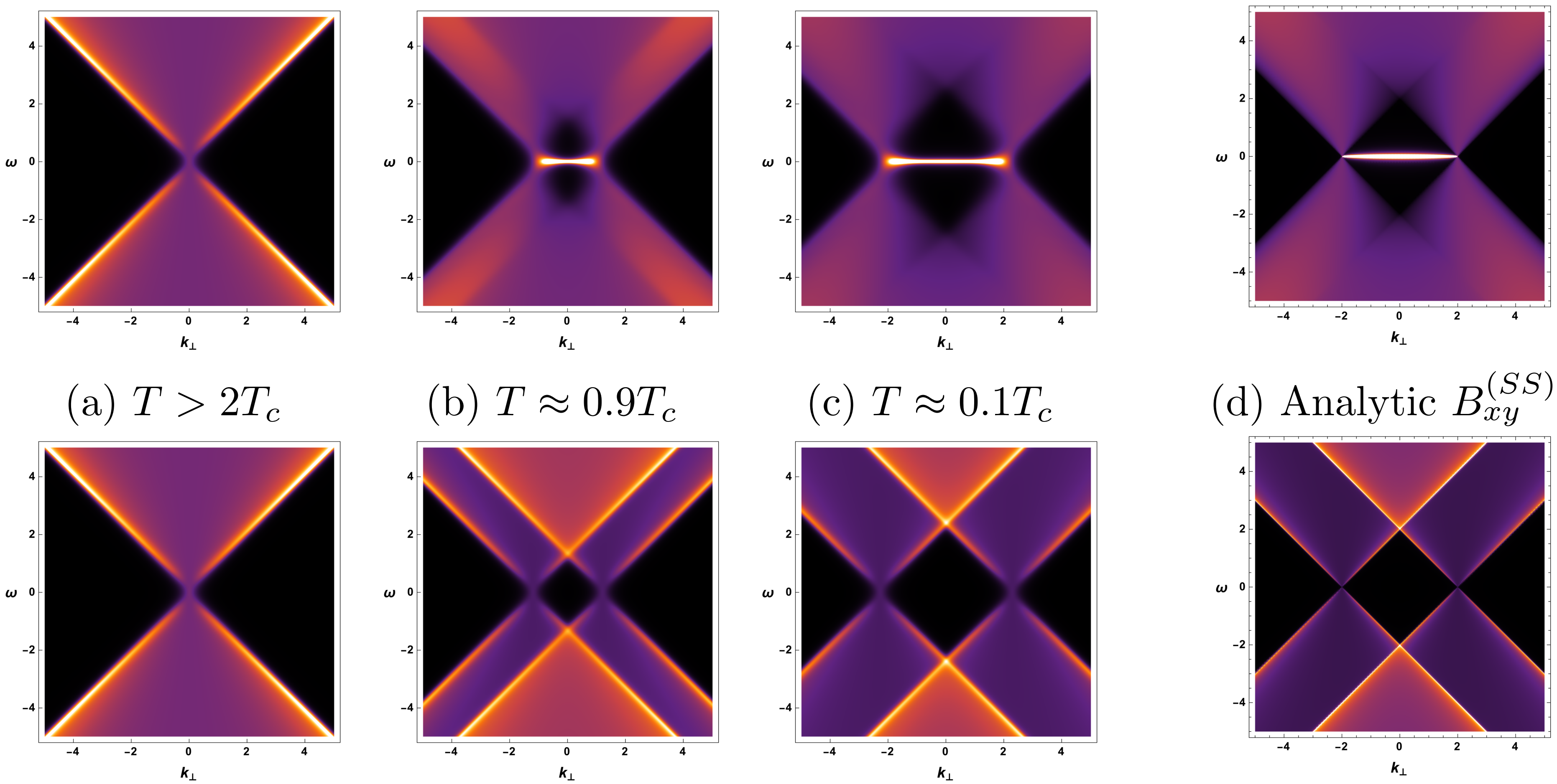


(a) $A(\omega, k), V = 0.$ (b) $A(\omega, k), V = 0.5.$ (c) $A(\omega, k), V = 0.$ (d) $A(\omega, k), V = 0.5.$



(e) $g(\omega), V = 0.$ (f) $g(\omega), V = 0.5.$ (g) $g(\omega), V = 0.$ (h) $g(\omega), V = 0.5.$

results with back reaction: T-evolution



(a) $T > 2T_c$ (b) $T \approx 0.9T_c$ (c) $T \approx 0.1T_c$ (d) Analytic $B_{xy}^{(SS)}$

(e) $T > 2T_c$ (f) $T \approx 0.9T_c$ (g) $T \approx 0.1T_c$ (h) Analytic $B_{xy}^{(SA)}$

Figure 12: Backreacted fermions spectral functions (SFs) by fixing the order parameter $\langle \mathcal{O}_1 \rangle \approx 2.0$ at $T \approx 0.1T_c$. (a,e) fermions SF with B_{xy} interaction type at above T_c. In this regime, the symmetry is restored so that $\langle \mathcal{O}_1 \rangle = 0$. (b,f) SFs where the order parameter is of the order of temperature ($T \sim T_c \sim \langle \mathcal{O}_1 \rangle$). (c,g) SFs where the order parameter is much bigger than temperature ($\langle \mathcal{O}_1 \rangle \gg T$). (d,h) SFs generated by our analytic results given in the section 4.

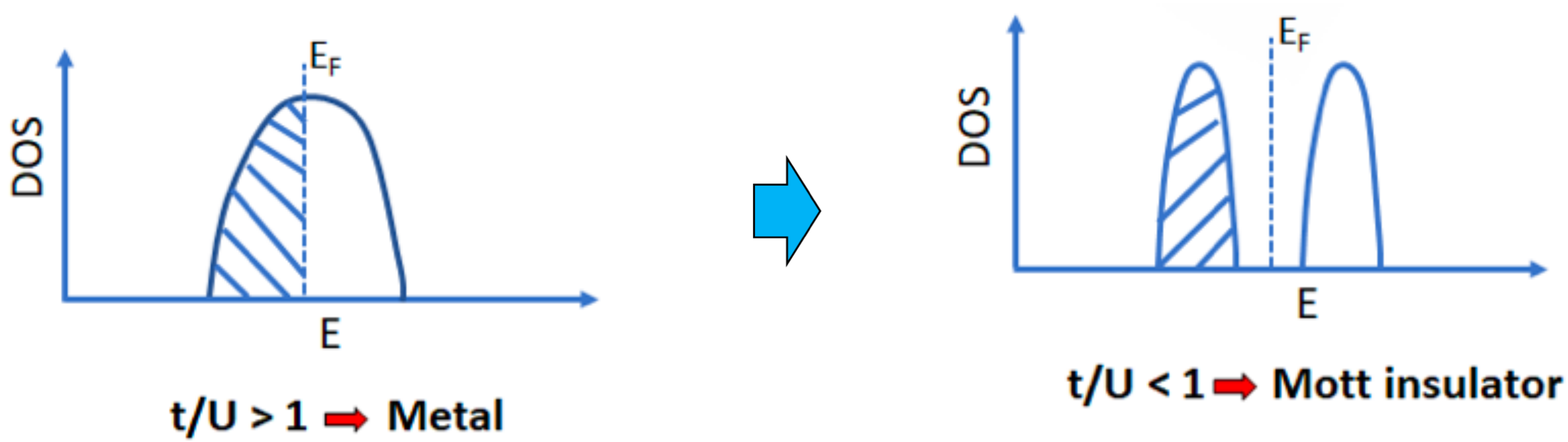
Application 2. Classifying the Mott gap

- Mott gap : gap without order, due to the Coulomb int.
  Hubbard model.

$$H_{\text{Hubbard}} = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

  No symmetry breaking, No condensation.

- Previously Phillips group used Pauli term $F_{\mu\nu}\bar{\psi}\Gamma^{\mu\nu}\psi$ for it.



Order Gap vs Mott gap

- Order : gap by condensation by spon.sym.breaking.
- Mott gap: gap by coulomb interaction without sym. breaking.

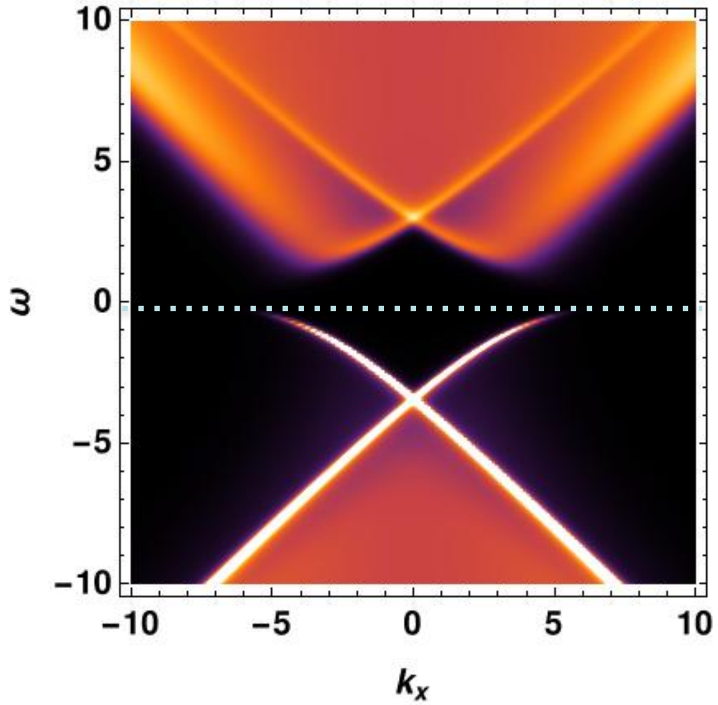
- In holography,
  if a matter field has source only or condensation only => Order.
  if a matter field has both source and condensation => induced gap

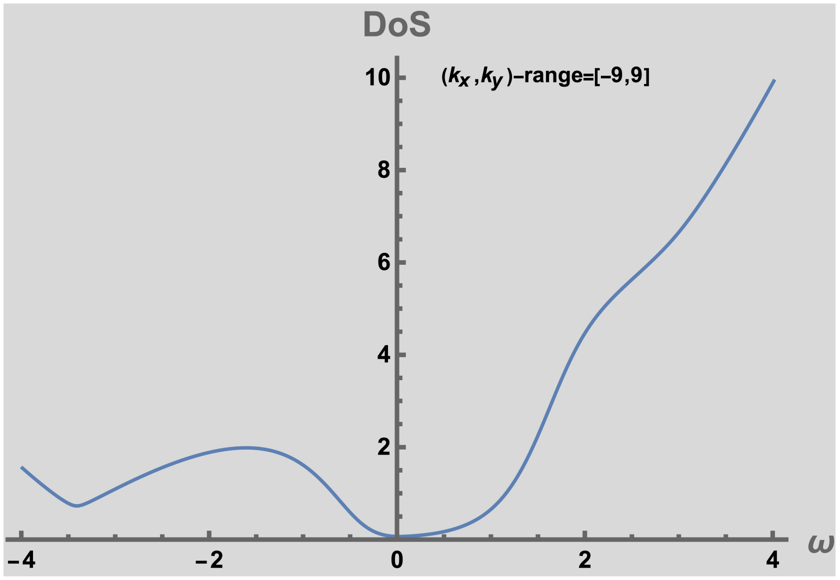
- We treat the Mott gap as the induced gap.

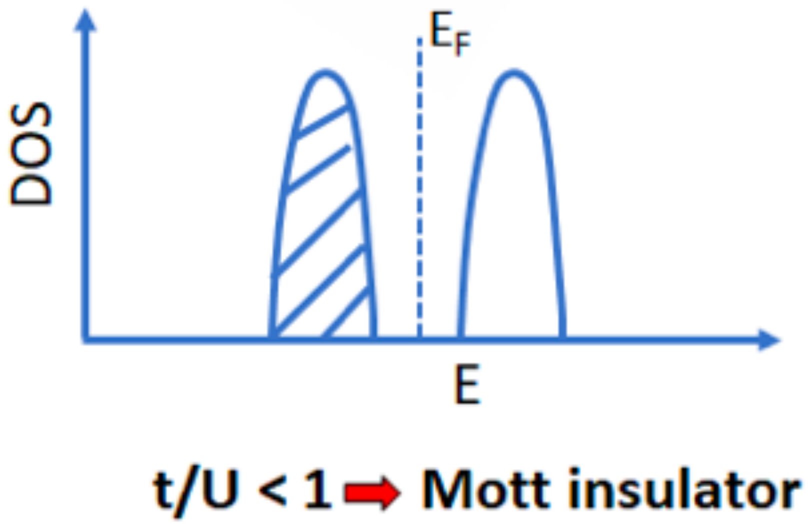
- So, holography can handle the Mott phonomena
  as a hMFT=analogue of Landau theory.
  This is a surprise!

Pauli term and asymmetry

- $\mathcal{L}_f = i\bar{\psi}(\Gamma^\mu D_\mu - m_f - i\frac{p}{2}F_{\mu\nu}\Gamma^{\mu\nu})\psi$
- Phillips et.al







Spectral fn   DoS: asymmetric gap

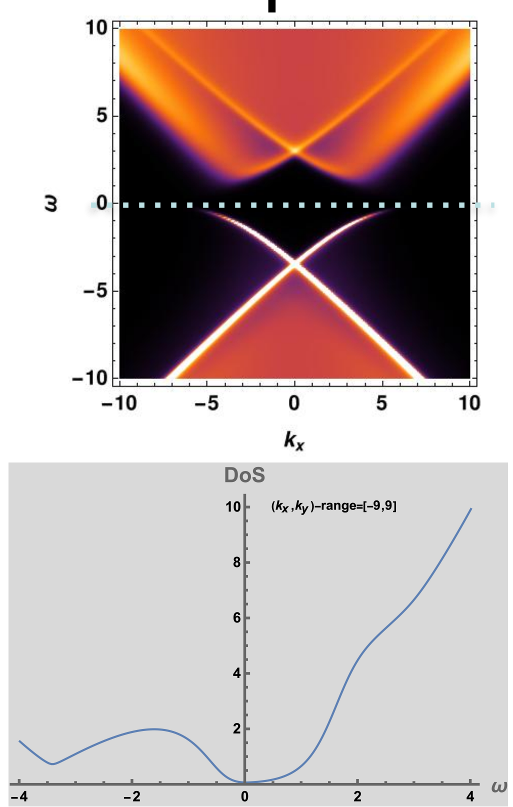
Our proposal

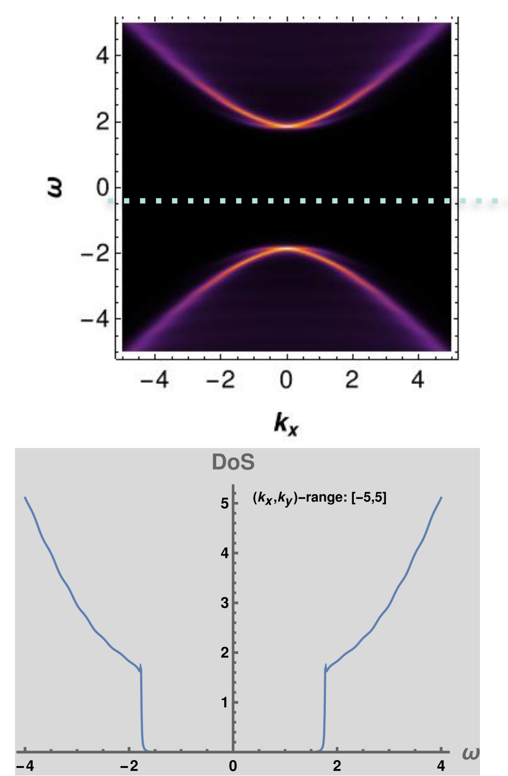
- No order parameter. Use only density.
- Want more canonical symmetric gap

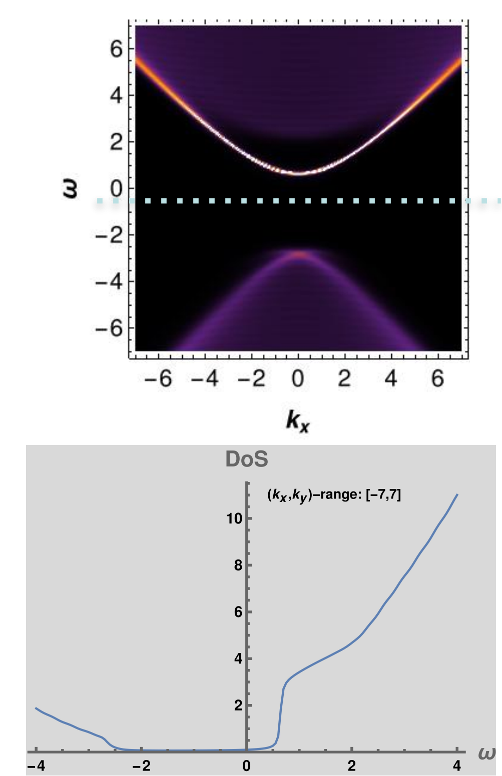
Gauge	$i\eta\bar{\psi}L_{int}\psi$			
Field	Gapless	Gap	Flat Band	Effect of q
$A = A_t(z)dt$	$\Gamma^z F^2$, $i\Gamma^{5z}F^2$	$F^2(\eta > 0), iF^2\Gamma^5$, $iF_{\mu\nu}\Gamma^{\mu\nu}$	$F_{\mu\nu}\Gamma^{\mu\nu}\Gamma^5$	Shifting & Bending

Phillips et.al

WO/W minimal int.







(a) For $F_{\mu\nu}\Gamma^{\mu\nu}$ at $T = 0, q = 1$ (b) For F^2 at $T = 0.025\mu, q = 0$ (c) For F^2 at $T = 0.025\mu, q = 1$

Application 3. Topology in interacting system

- Topological Hamiltonian Method and Eigenvectors $(\omega = 0)$

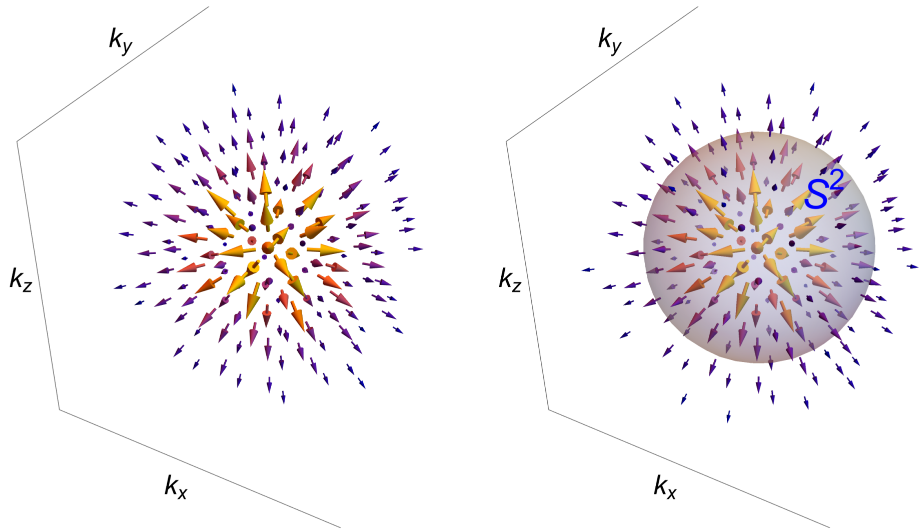
$$\mathcal{H}_t(\boldsymbol{k}) = -\mathbb{G}^{-1}(0, \boldsymbol{k})$$

where eigenvector of H_t and H share the same eigenvector, $|n\rangle$.

$$\mathcal{F}_c = \nabla \times \langle n | \partial_{\boldsymbol{k}} | n \rangle \tag{2}$$

- Alternative method: "Cubic of Green's function"

$$\mathcal{F}_c = \frac{1}{3!} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \epsilon_{\mu\nu\rho c} \mathrm{Tr}\big[\mathbb{G}(\partial_\mu \mathbb{G}^{-1})\mathbb{G}(\partial_\nu \mathbb{G}^{-1})\mathbb{G}(\partial_\rho \mathbb{G}^{-1})\big] \tag{3}$$



Monopole Number:

$$C_n = \oint \mathcal{F}_c \cdot dS = i \oint \nabla \times \langle n | \partial_{\boldsymbol{k}} | n \rangle \cdot dS$$

Critical case ($\Phi = 0$)

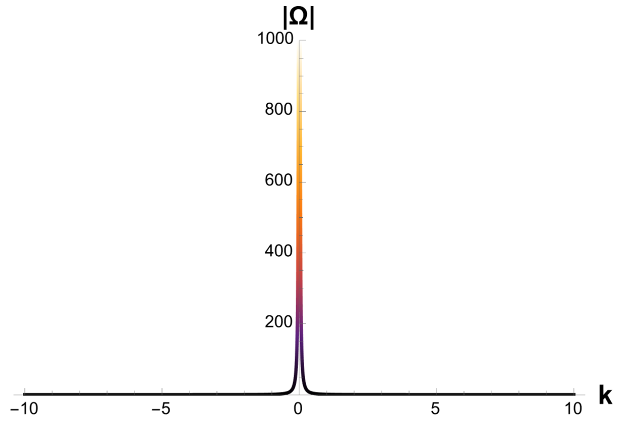
$$\mathcal{A}^{11} = \mathcal{A}^{22} = \frac{|\boldsymbol{k}| - k_x}{2|\boldsymbol{k}|(k_y^2 + k_z^2)}(0, -k_z, k_y)^{\mathrm{T}} \tag{5.1}$$

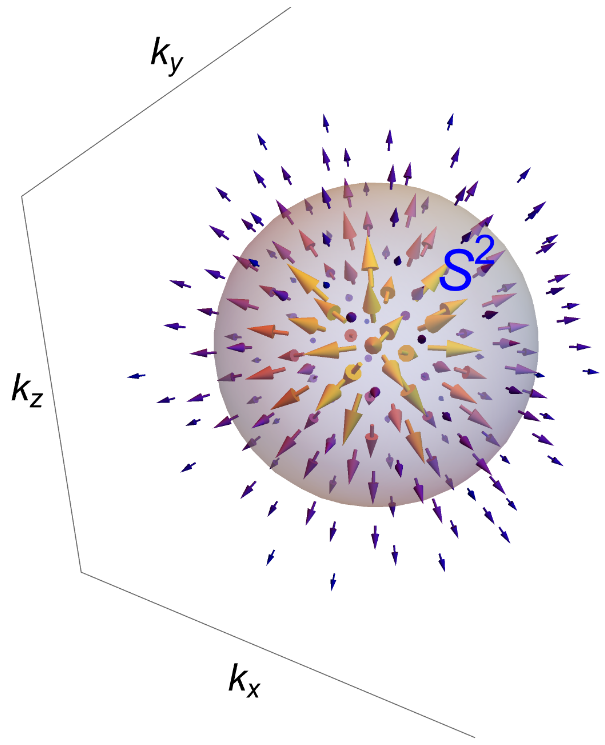
$$\mathcal{A}^{33} = \mathcal{A}^{44} = \frac{|\boldsymbol{k}| + k_x}{2|\boldsymbol{k}|(k_y^2 + k_z^2)}(0, -k_z, k_y)^{\mathrm{T}} \tag{5.2}$$

$$\mathcal{A}^{13} = \mathcal{A}^{24} = \mathcal{A}^{31*} = \mathcal{A}^{42*} = \frac{\sqrt{\boldsymbol{k}^2 - k_x^2}}{2\boldsymbol{k}^2(k_y^2 + k_z^2)}(-i(k_y^2 + k_z^2), ik_xk_y + |\boldsymbol{k}|k_z, ik_xk_z - |\boldsymbol{k}|k_y)^{\mathrm{T}} \tag{5.3}$$

F=dA+A^A =>for Abelian case, denote F= $\boldsymbol{\Omega}$

$$\boldsymbol{\Omega} = \frac{1}{\boldsymbol{k}^{3/2}}(k_x, k_y, k_z)^{\mathrm{T}} \qquad \mathrm{flux} = \int_{\mathcal{S}} \boldsymbol{\Omega} \cdot d\boldsymbol{S} = 2\pi$$





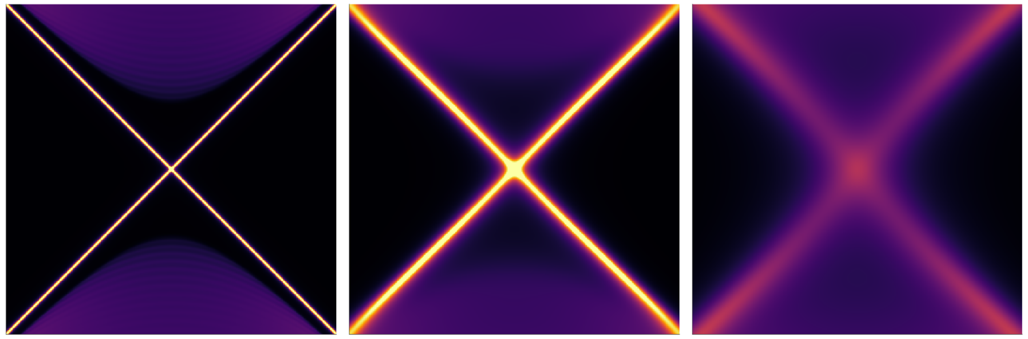
Topological Liquid : scalar order without gap

$$S_\psi = \int d^5x \sum_{j=1}^{2} \sqrt{-g}\; \bar{\psi}^{(j)} \Big(\frac{\overrightarrow{\not{D}} - \overleftarrow{\not{D}}}{2} - m^{(j)} \Big) \psi^{(j)}, \tag{5}$$

$$S_{g,\Phi} = \int d^5x \sqrt{-g} \Big(R - 2\Lambda - \nabla_M \Phi^2 - m_\Phi^2 |\Phi|^2 \Big) \tag{6}$$

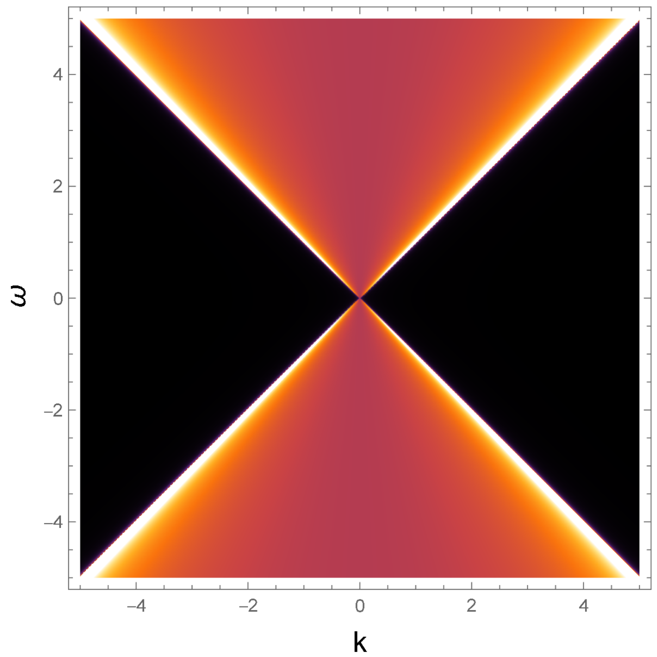
$$S_{int} = \int d^5x \sqrt{-g} \Big(i\Phi \bar{\psi}^{(1)} \psi^{(2)} + h.c \Big). \tag{7}$$

where $\not{D} = \Gamma^M D_M$, $D_M = (\partial_M - iqA_M + \frac{1}{4}\omega_{M\alpha\beta}\Gamma^{\alpha\beta})$

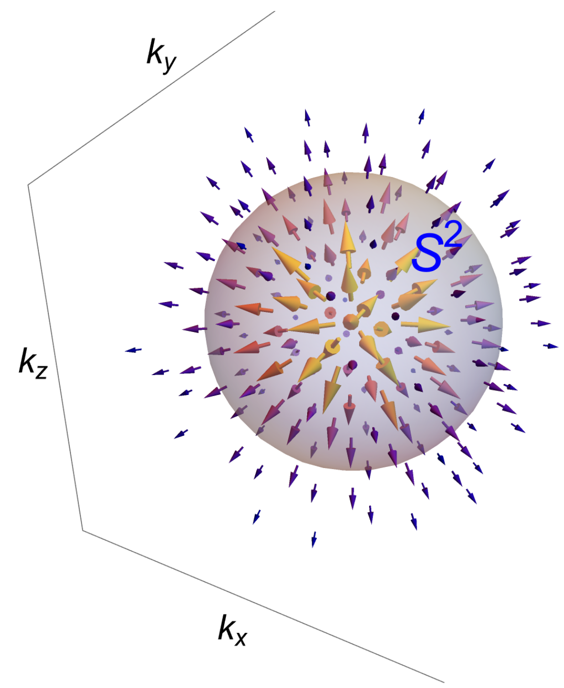


Scalar Interaction case(SS quantization)

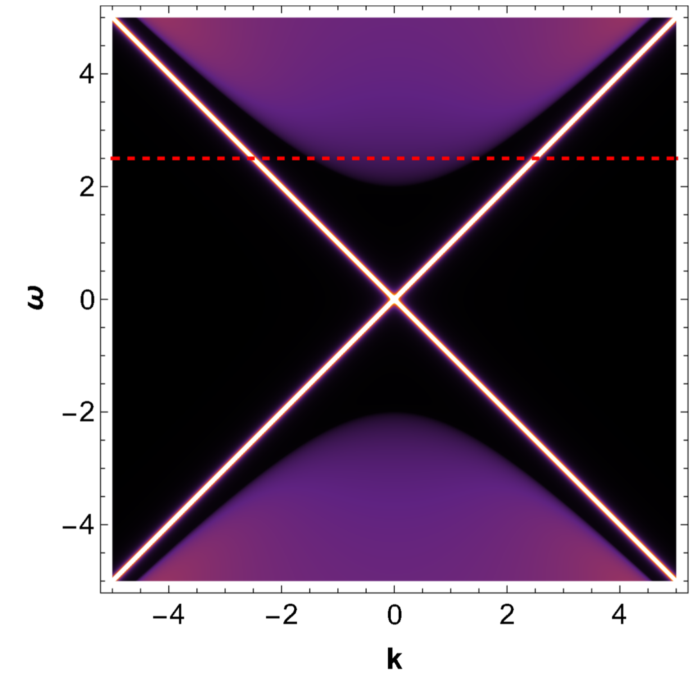
Spectrum is pole type, differ from critical case.



Spectrum of free Fermion



Berry curvature for both cases



Spectrum of scalar coupled Fermion

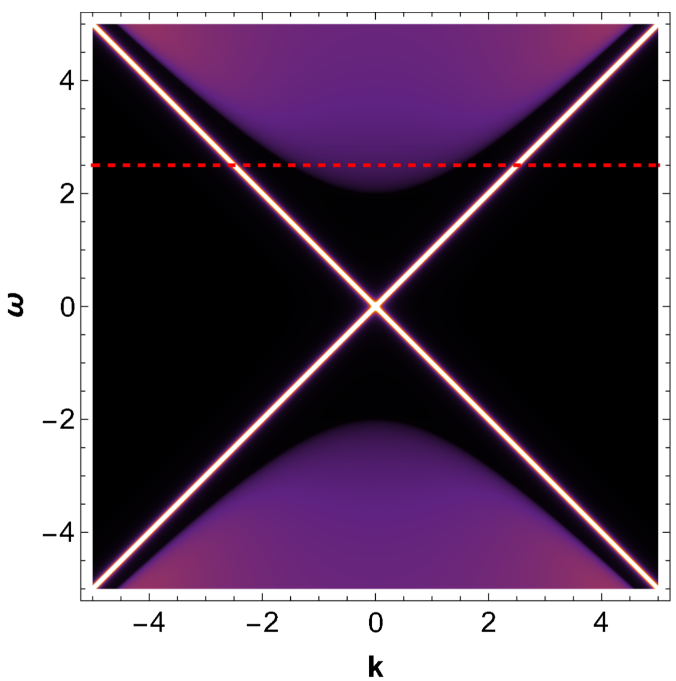
However, Berry Curvature is Identical to critical case.

The same Dirac monopole

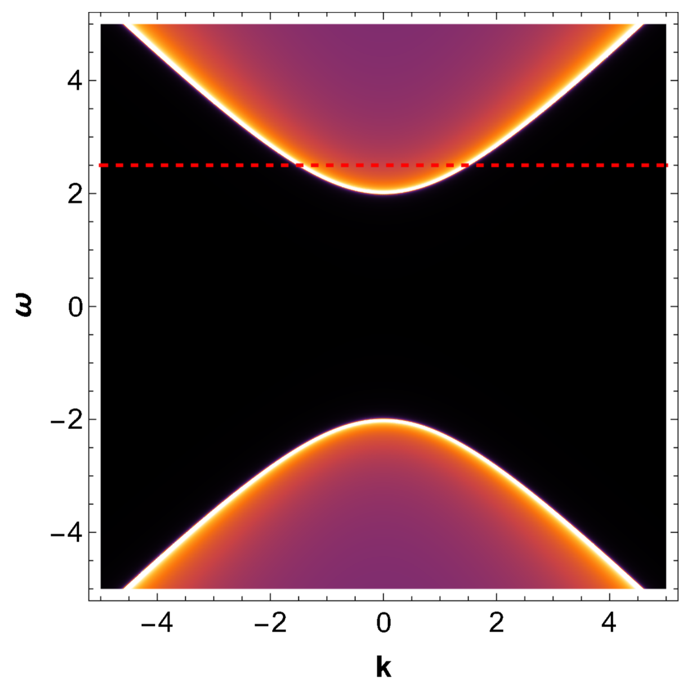
Scalar Interaction case (SA quantization)

- Gapped spectrum
- Trivial topology

$$\mathrm{Tr}\, \mathbb{G}_{M_0}^{(SA)} = \frac{4\omega}{\sqrt{\boldsymbol{k}^2 - \omega^2 + M_0^2}},$$



(a) $M_0^{(SS)}, \omega$-$\boldsymbol{k}$



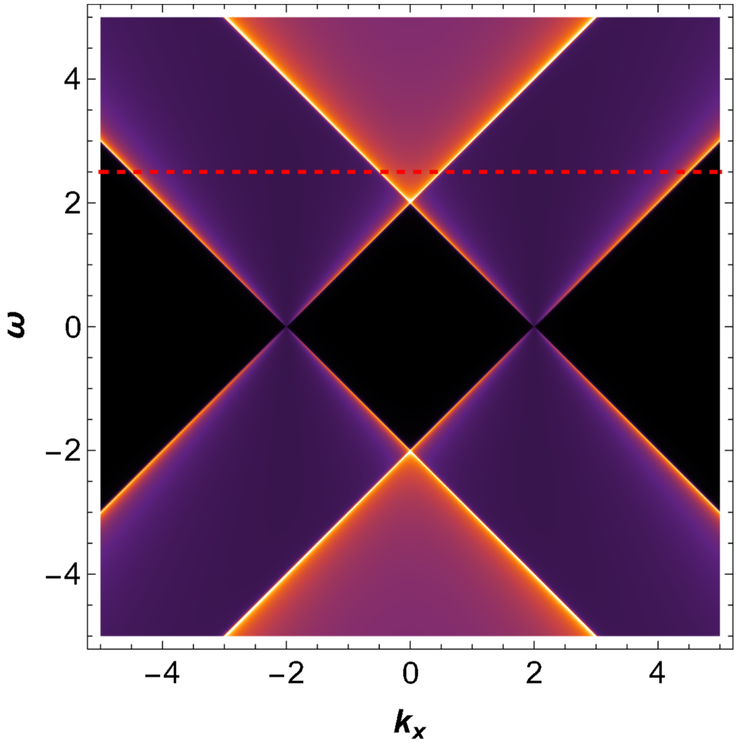
(b) $M_0^{(SA)}, \omega$-$\boldsymbol{k}$

Vector Interaction : Separated Dirac monopole

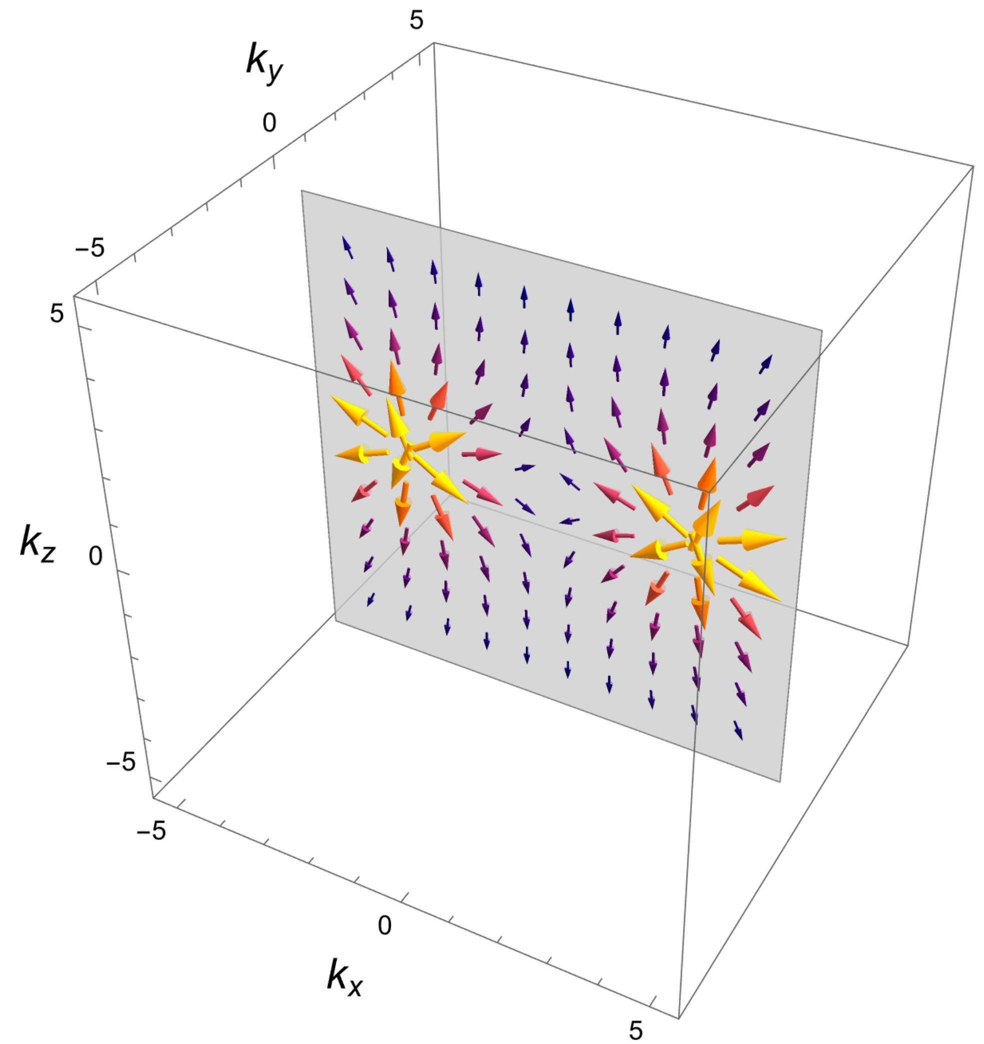
Berry curvature

$$\mathbf{\Omega} = \frac{1}{2\left((b_x+k_x)^2+k_y^2+k_z^2\right)^{3/2}}(k_x+b_x,k_y,k_z)^{\mathrm{T}} + \frac{1}{2\left((b_x-k_x)^2+k_y^2+k_z^2\right)^{3/2}}(k_x-b_x,k_y,k_z)^{\mathrm{T}}$$

Spectrum



(a) $B_x^{(0)(SS)}, \omega\text{-}k_x$



(b) Berry curvature on k_z-k_x plane

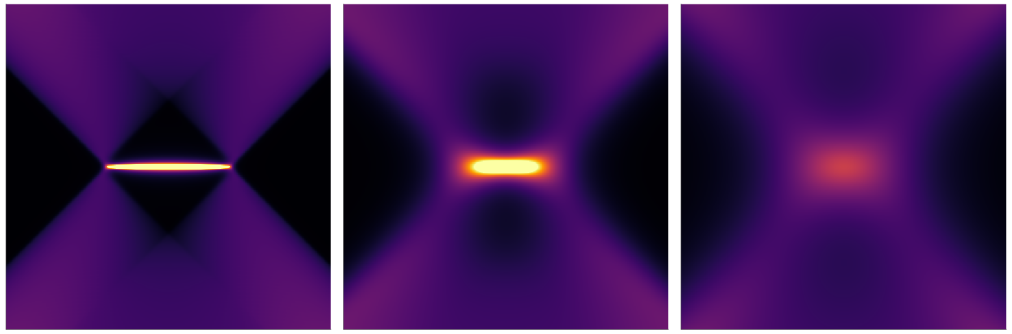
B_{xy} case

$$S_\psi = \int d^5x \sum_{j=1}^{2} \sqrt{-g}\, \bar{\psi}^{(j)} \left(\frac{\overrightarrow{\not{D}} - \overleftarrow{\not{D}}}{2} - m^{(j)} \right) \psi^{(j)}, \quad (8)$$

$$S_{g,B_{\mu\nu}} = \int d^5x \sqrt{-g} \left(R - 2\Lambda - |D_M \Phi_I|^2 - m_\Phi^2 |\Phi|^2 \right), \quad (9)$$

$$S_{int} = \int d^5x \sqrt{-g} \left(B_{\mu\nu} \bar{\psi}^{(1)} \Gamma^{\mu\nu} \psi^{(2)} + h.c \right). \quad (10)$$

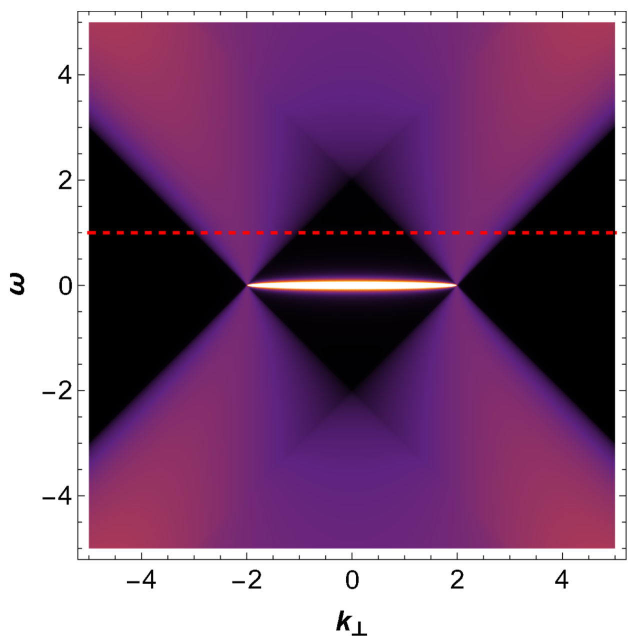
where $\not{D} = \Gamma^M D_M$, $D_M = (\partial_M + \frac{1}{4}\omega_{M\alpha\beta}\Gamma^{\alpha\beta})$, and $B = B_{xy}(u)\, dx \wedge dy$



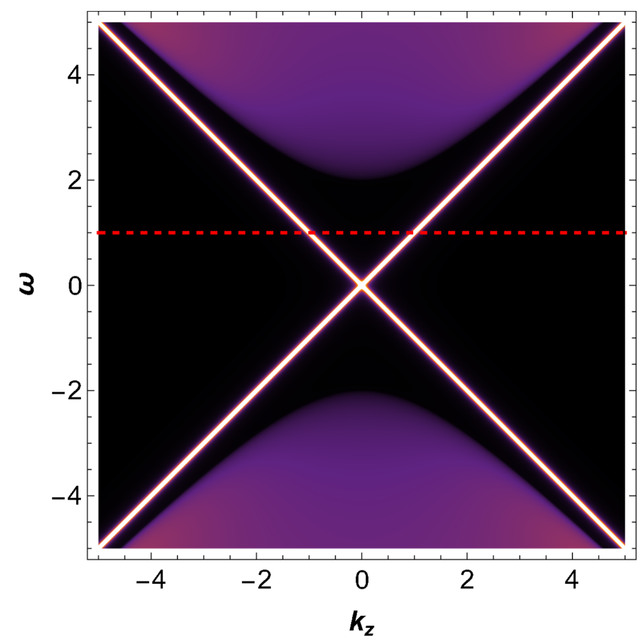
Topology of Flat band

Spectrum =2d Disk

$$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B^{(-1)}_{xy}} = \frac{2\omega}{b}\Big[\frac{(b+|\boldsymbol{k}_\perp|)\sqrt{(b-|\boldsymbol{k}_\perp|)^2+k_z^2-\omega^2}+(b-|\boldsymbol{k}_\perp|)\sqrt{(b+|\boldsymbol{k}_\perp|)^2+k_z^2-\omega^2}}{k_z^2-\omega^2-i\epsilon}\Big].$$

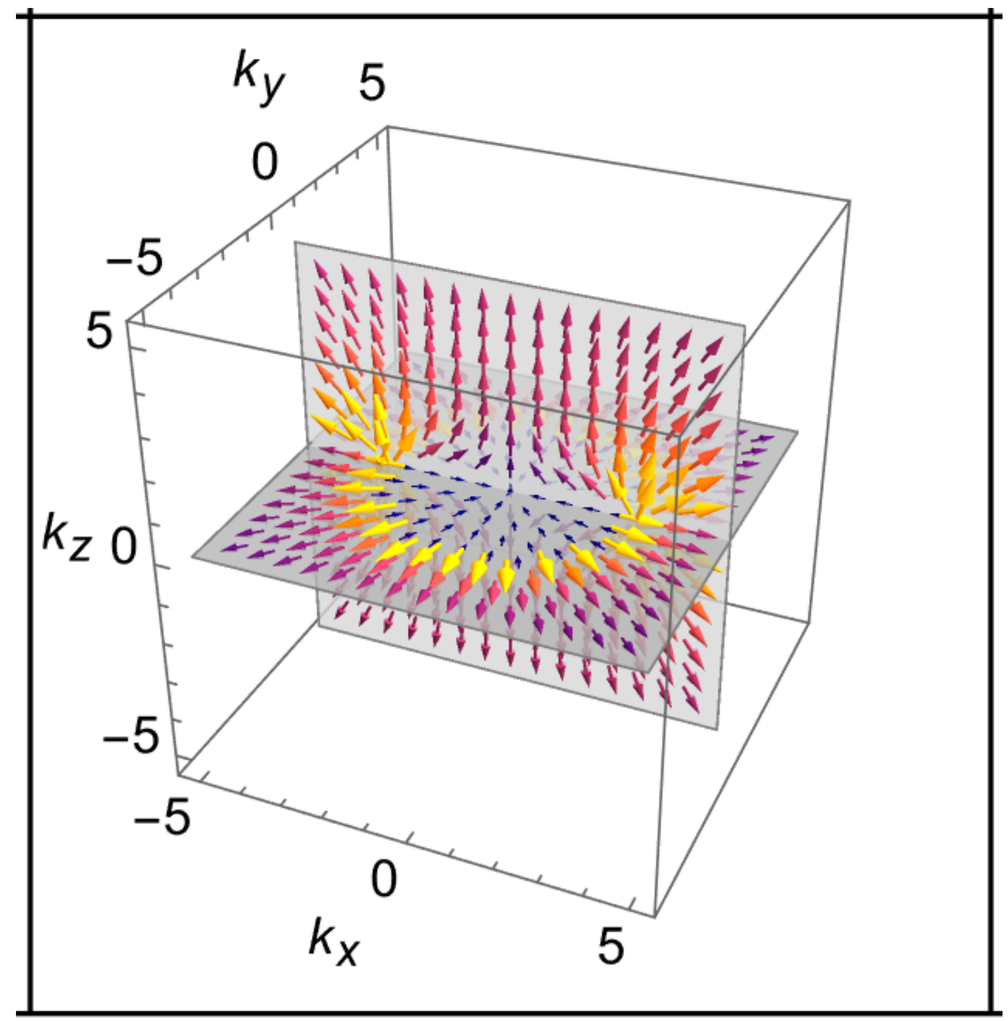


(e) $B^{(-1)(SS)}_{xy}, \omega\text{-}\boldsymbol{k}_\perp$



(f) $B^{(-1)(SS)}_{xy}, \omega\text{-}k_z$

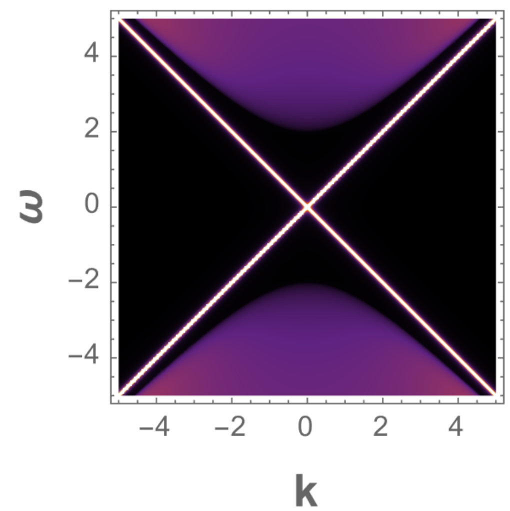
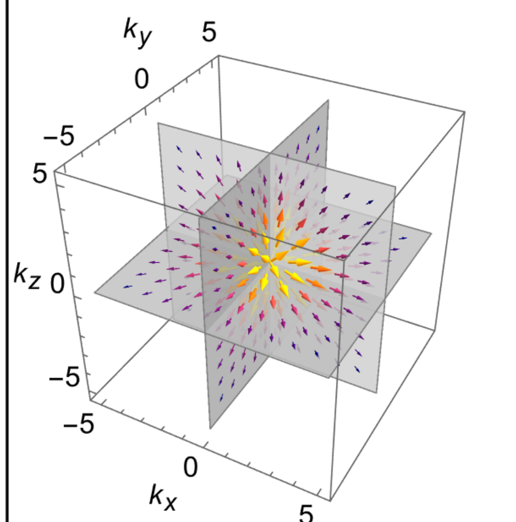
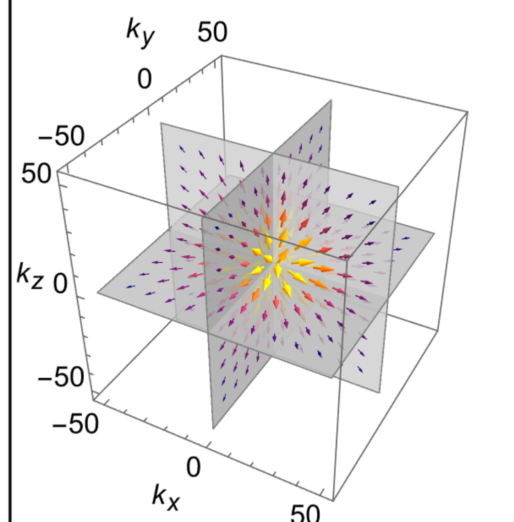
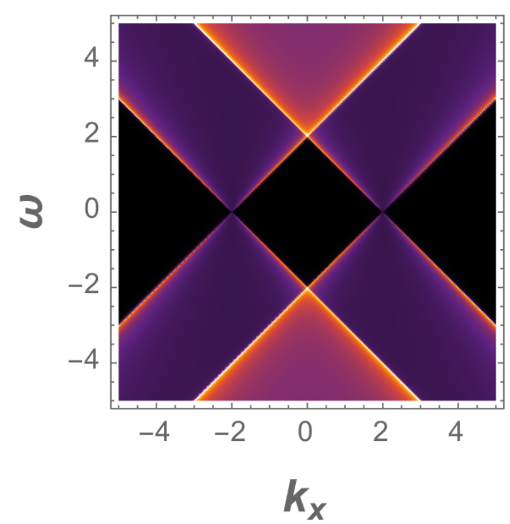
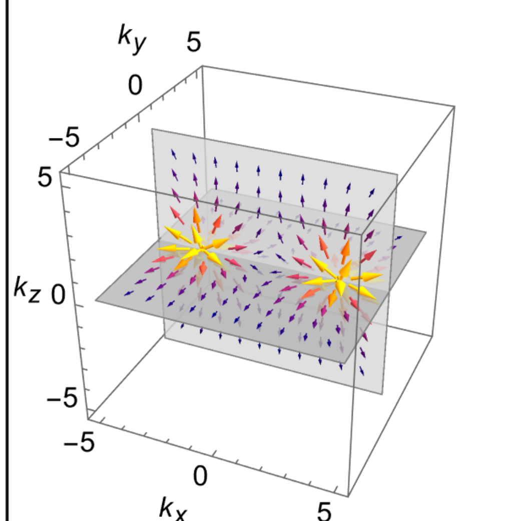
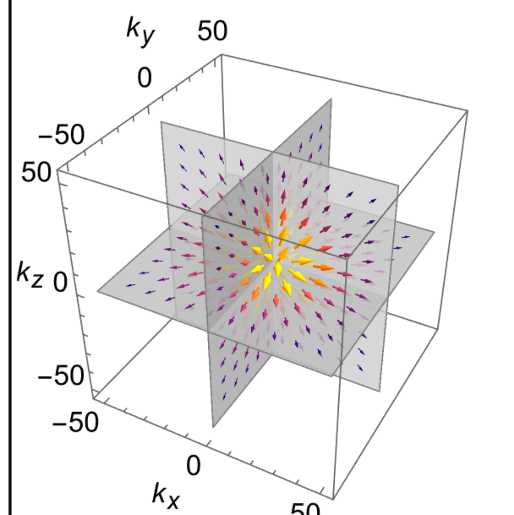
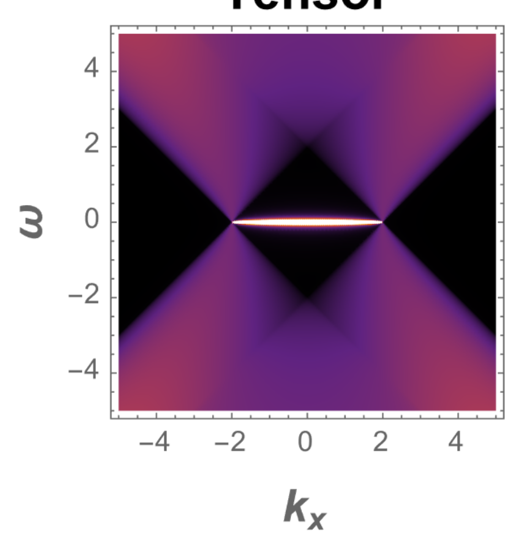
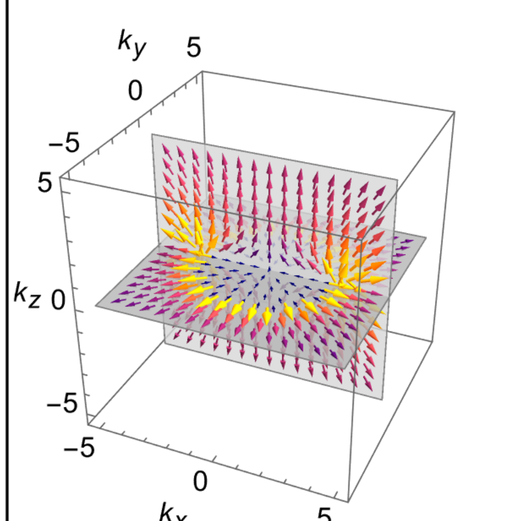
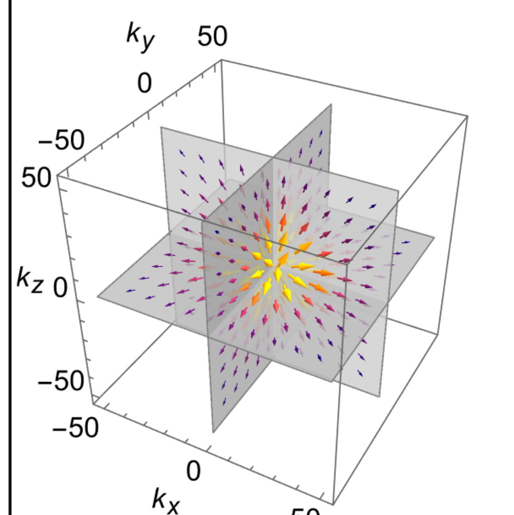
Berry curvature=monopole Ring



$$\boldsymbol{\Omega} = \frac{k_z(f_- - f_+)^2(\boldsymbol{k}^2-b^2)}{4\sqrt{2}|\boldsymbol{k}_\perp|f_-f_+\big((\boldsymbol{k}^2-b^2)(\boldsymbol{k}^2-b_{xy}^2-f_-f_+)\big)^{3/2}}\begin{pmatrix}\big(|\boldsymbol{k}_\perp|(f_- - f+)+b_{xy}(f_-+f_+)\big)\sin\theta\\ \big(|\boldsymbol{k}_\perp|(f_- - f+)+b_{xy}(f_-+f_+)\big)\cos\theta\\ \frac{4k_z(b_{xy}^2+\boldsymbol{k}_\perp^2+k_z^2)\big((\boldsymbol{k}_\perp^2+k_z^2)(f_--f_+)+b_{xy}|\boldsymbol{k}_\perp|(f_-+f_+)\big)}{(\boldsymbol{k}_\perp^2+k_z^2-b_{xy}^2)(f_--f_+)^2}\end{pmatrix}$$

$$f_\pm = \sqrt{(b_{xy}\pm\boldsymbol{k}_\perp)^2+k_z^2}\ \text{and}\ \boldsymbol{k}_\perp = \sqrt{k_x^2+k_y^2}.$$

Summary (AdS_5 or 3d topology)

Interaction types/SF	Near the origin	Far from the origin	
Scalar 			Single monopole
Vector 			Separated monopole
Tensor 			Monopole ring

AdS_4 : scalar vs pseudo-scalar

the scalar $\Gamma \cdot \Phi = iM_0$ Green's function is given by

$$\mathbb{G} = \begin{pmatrix} \frac{k_x+\omega}{-M_0+\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} & \frac{k_y}{M_0-\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} \\ \frac{k_y}{M_0-\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} & \frac{k_x+\omega}{M_0-\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}} \end{pmatrix}$$

$$\mathrm{Tr}\,\mathbb{G} = \frac{2\omega}{-M_0+\sqrt{k_x^2+k_y^2+M_0^2-\omega^2}}$$

Spectrum-> gap (g>0)
Topological Liquid (g<0)

But in both case
$\Omega_{xy} = 0$

the 1-flavor with psudo scalar $\Gamma \cdot \Phi = \Gamma^5 M_5$ can give a gap

$$\mathbb{G} = \frac{1}{\sqrt{k_x^2+k_y^2+M_5^2-\omega^2}} \begin{pmatrix} k_x+\omega & -k_y+iM_5 \\ -k_y-iM_5 & -k_x+\omega \end{pmatrix},$$

$$\mathrm{Tr}\,\mathbb{G} = \frac{2\omega}{\sqrt{k_x^2+k_y^2+M_5^2-\omega^2}}$$

Spectrum-> gap

$$\boldsymbol{\Omega} = \frac{M_5}{2(k_x^2+k_y^2+M_5^2)^{3/2}}$$

$$c_1 = \frac{1}{2\pi}\int F = 1$$

Topology in finite temperature

1. Non-interacting (single particle) theory:
   Finite temperature is ensemble average.
   Each band has its own topological number c_n .
   Therefore the topological number = average of c_n :

$$c(T) = \sum p_n(T) c_n$$

   Actually Uhlmann defined a T-dependent c.

Q: But does it make sense for a topology to be dependent on T, a continuous deformation?

Q: What holography says about it?

Monopole number at Finite T in holography

Method 1: A & F are T-independent, though G depends on T.
Method 2: GdG^{-1} depends on T.

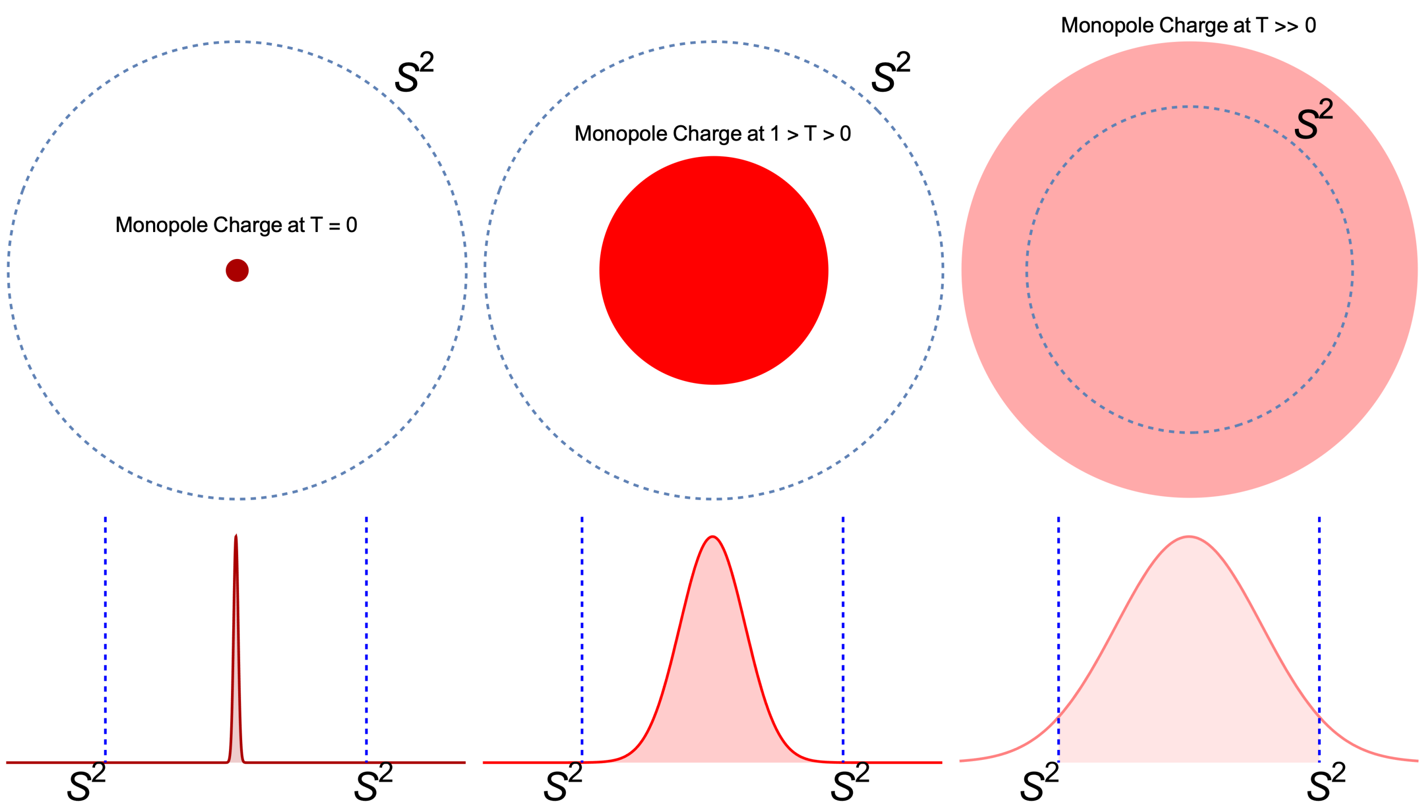


Figure: Monopole charge with increasing of temperature, with a fixed sphere surface

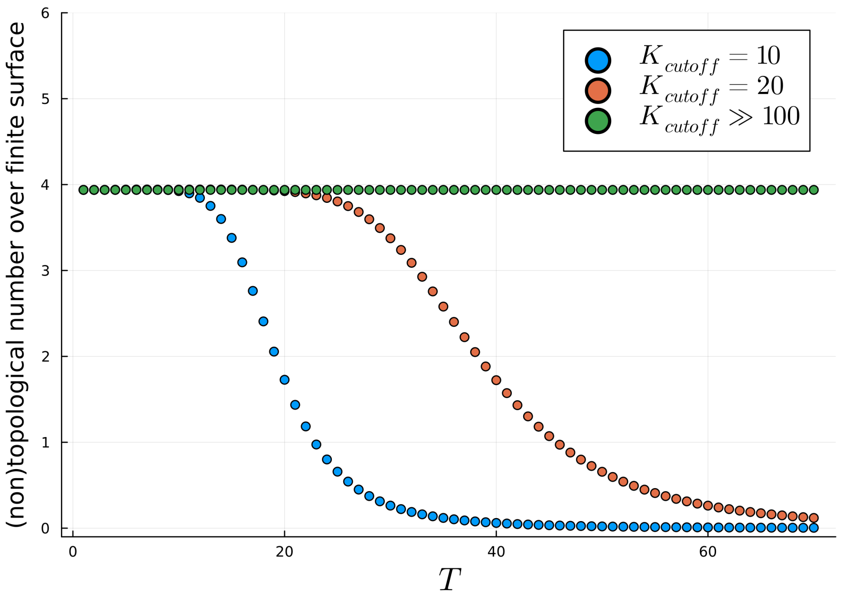


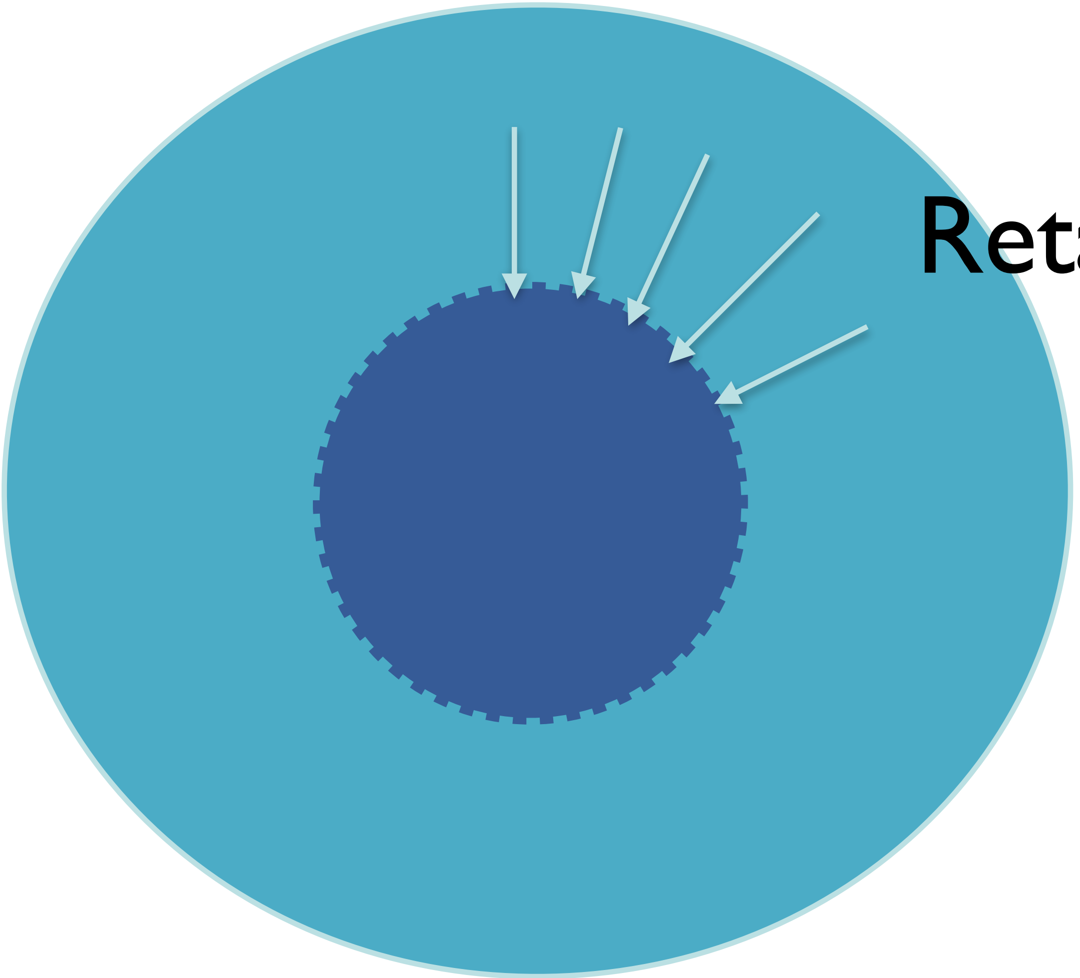
Figure: monopole numbers over the evolution of temperature by various integration sphere radius.

Flux over Large enough Surface => temperature independent result.

Observation

1. In holography, $c_1(T) = c_1(0)$.

2. Why this happen?
In AdS/CFT dictionary,
finite temperature ~ black hole ~ (a pure) state!



Retarded = inflating BC

Other applications

1. BCS and s-p-d-wave gaps
2. Strange metal
2. Topology and interaction/temperature
3. Fermi-Liquid as a topological liquid

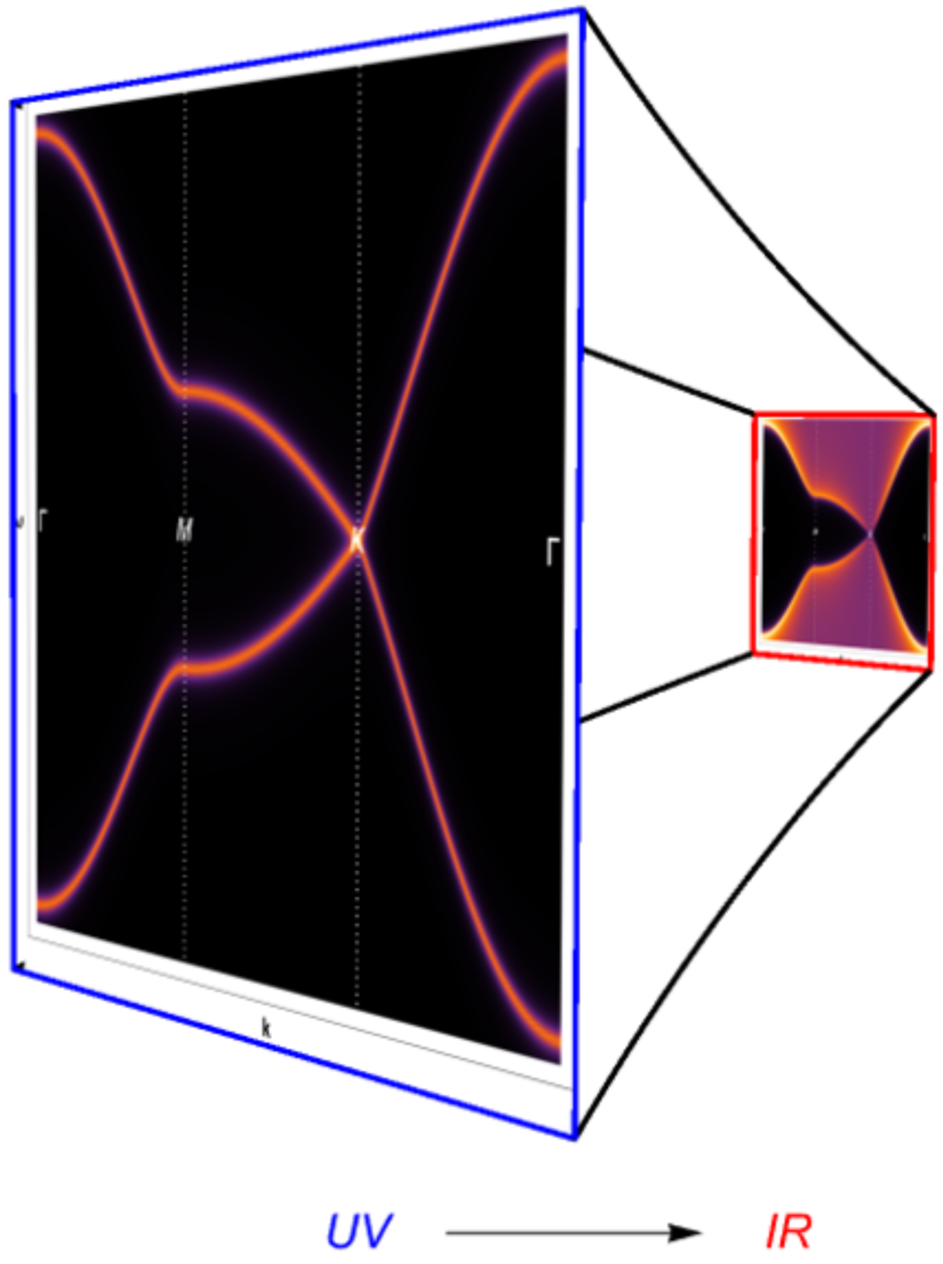
……

Application 4. How to encode the lattice in holography?

- To encode the lattice use localized basis (Wannier basis).
- Fourier transform ==> Tight binding model.
  for n orbital => H(k,w) : matrix of n x n.
- Schroedinger eq has the form of Dirac eq.
  -> Embed this Dirac eq in 2+1 into Dirac eq in AdS4.
- $$\left[\tilde{\Gamma}^z \partial_z - m - i\tilde{\Gamma}^t \left\{ i\mathcal{D}_t - \mathcal{G}\mathcal{H}(k_x, k_y) \right\}\right] \zeta = 0,$$

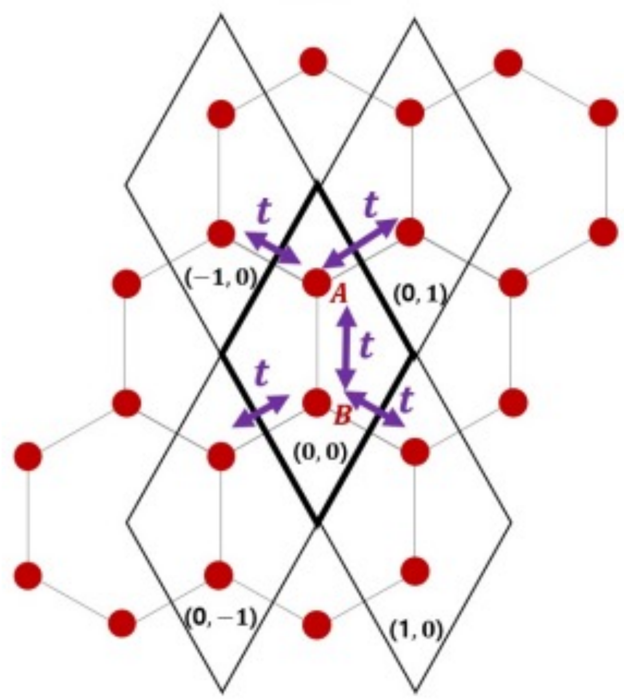
Physical meaning: RG running of Dirac eq in 2+1

$$\left[\tilde{\Gamma}^z \partial_z - m - i\tilde{\Gamma}^t \left\{ i\mathcal{D}_t - \mathcal{G}\mathcal{H}(k_x, k_y) \right\}\right] \zeta = 0,$$



Two band case: I. Graphene

- Graphene's tight-binding Hamiltonian and its eigenvalue are given by



https://drive.google.com/file/d/1PXvC9tJVcN391NngWw1bPWPyTvOok8T-/view

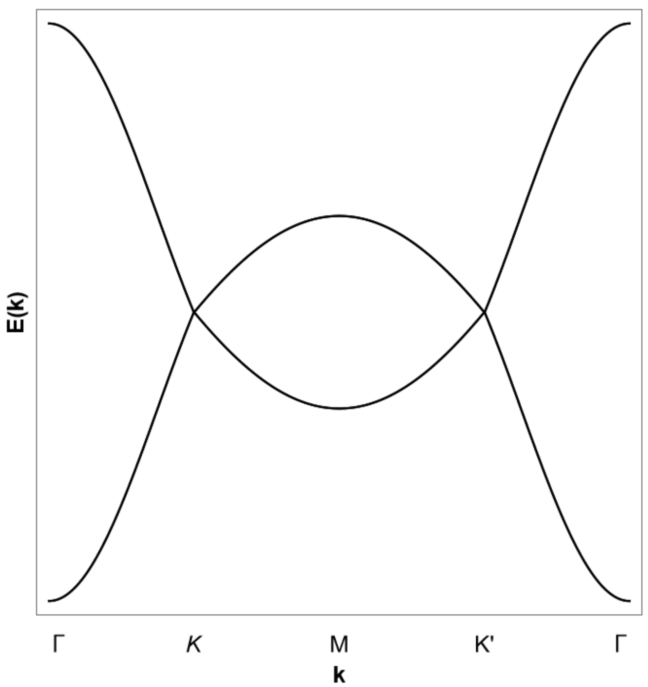
$$H = \begin{bmatrix} 0 & 1 + 2e^{3iak_x/2}\cos(\frac{\sqrt{3}}{2}ak_y) \\ 1 + 2e^{-3iak_x/2}\cos(\frac{\sqrt{3}}{2}ak_y) & 0 \end{bmatrix}.$$

$$E = \pm\sqrt{3 + 2\cos(\sqrt{3}ak_y) + 4\cos(\frac{3ak_x}{2})\cos(\frac{\sqrt{3}ak_y}{2})}.$$

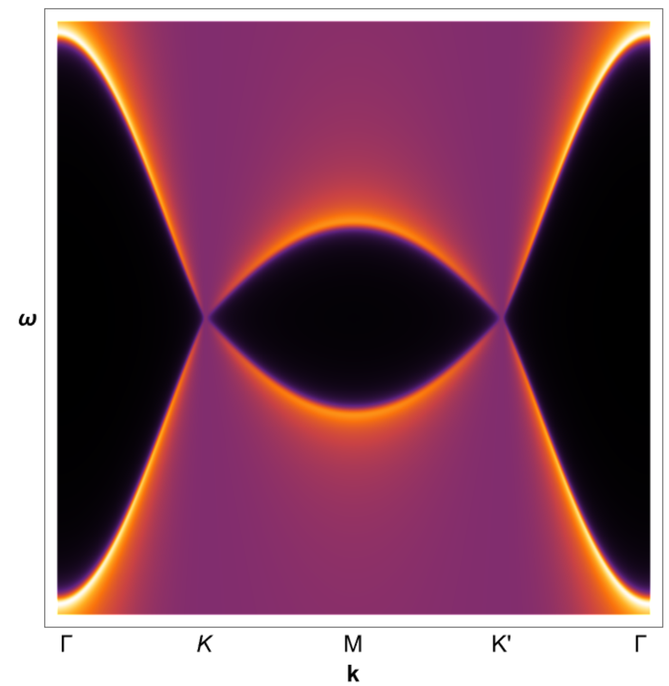
$$TrG = \frac{2\omega}{\sqrt{\epsilon^2 - \omega^2}}, \quad \epsilon^2 = 3 + 2\cos(\sqrt{3}ak_y) + 4\cos(\frac{3ak_x}{2})\cos(\frac{\sqrt{3}ak_y}{2})$$

Graphene spectrum

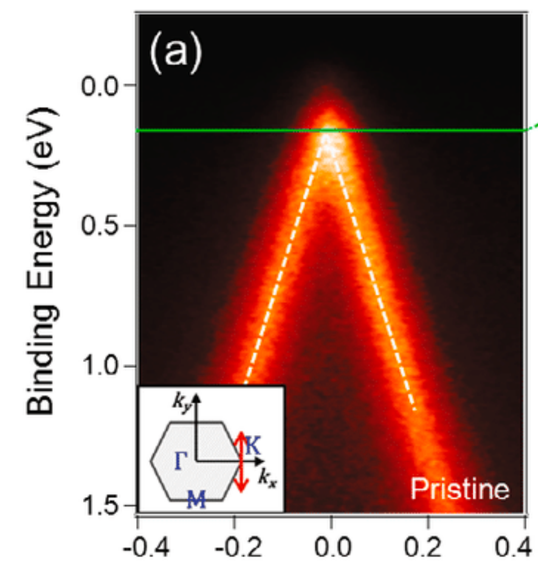
$$TrG = \frac{2\omega}{\sqrt{\epsilon^2 - \omega^2}}, \quad \epsilon^2 = 3 + 2\cos(\sqrt{3}ak_y) + 4\cos(\frac{3ak_x}{2})\cos(\frac{\sqrt{3}ak_y}{2})$$

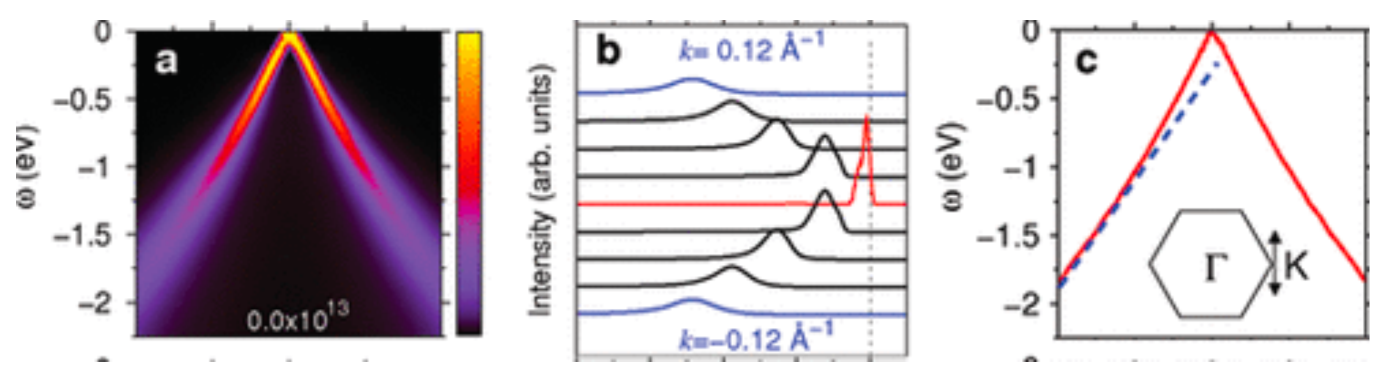


(a) Dispersion Relation of the Graphene

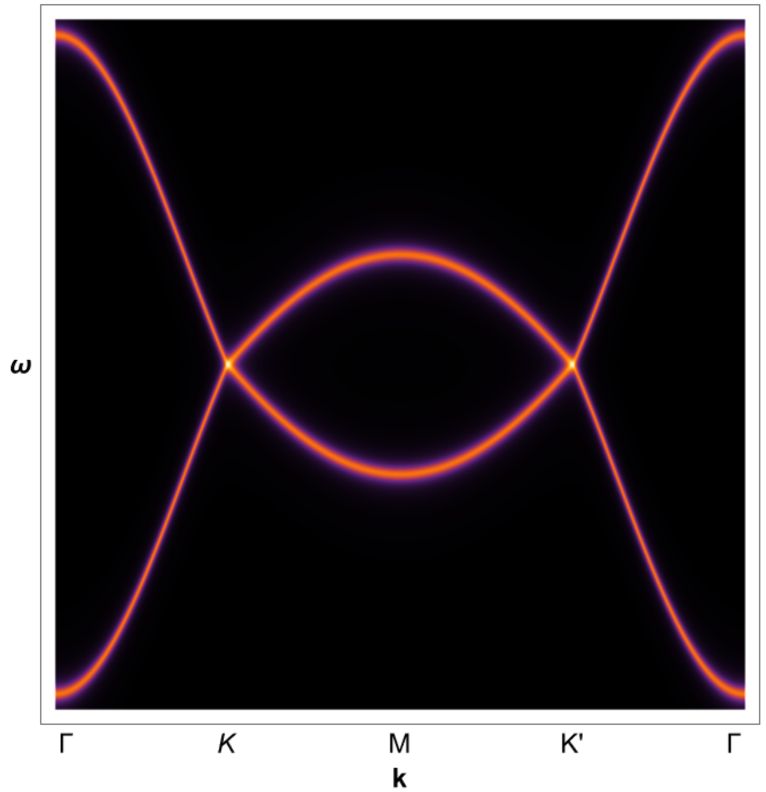


(b) Spectral Density of the Graphene

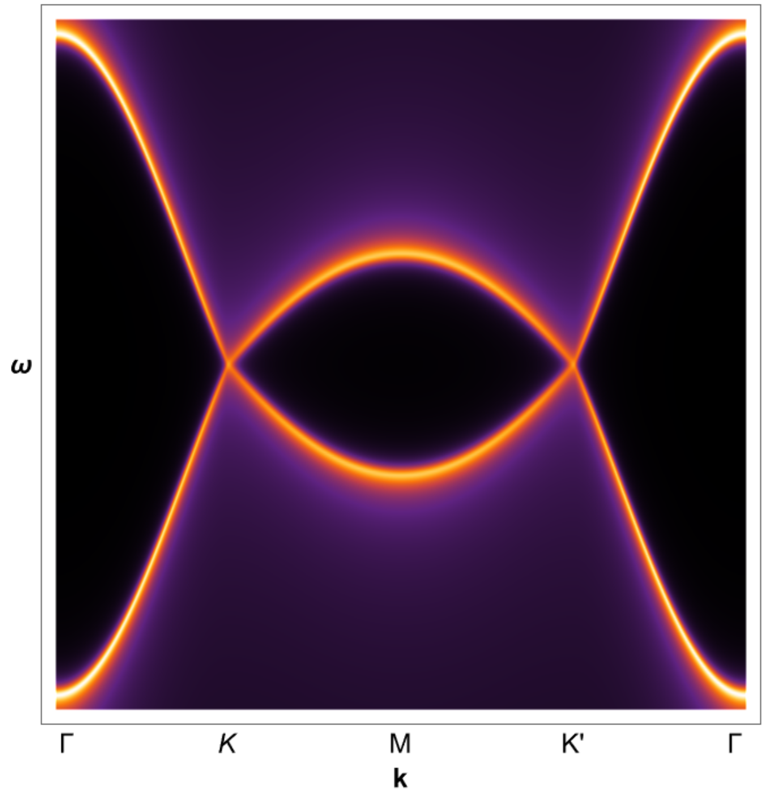




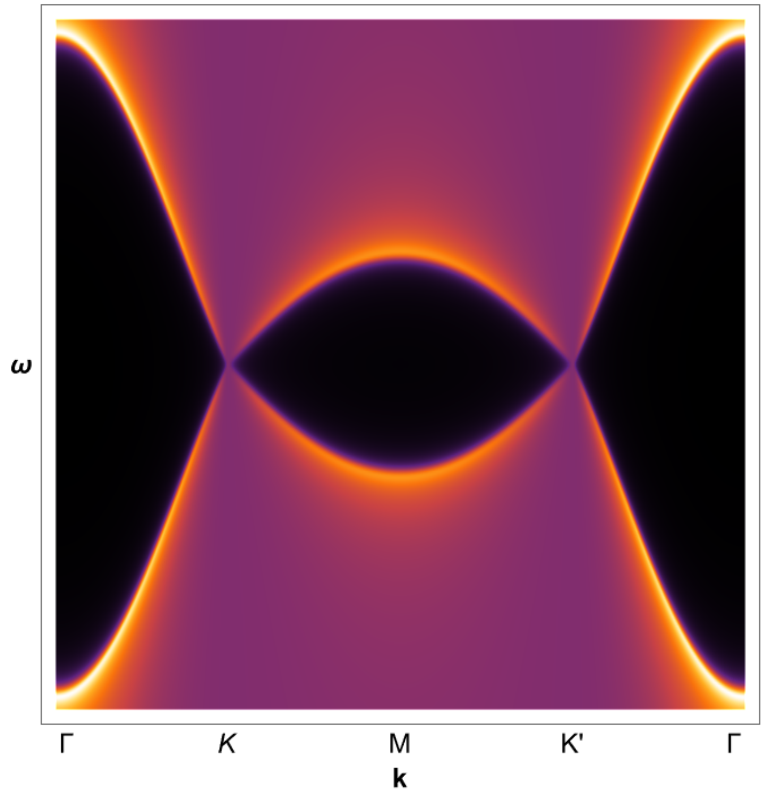
Graphene spec as coupling grow.



(a) $m \simeq 0.5$



(b) $m = 0.25$



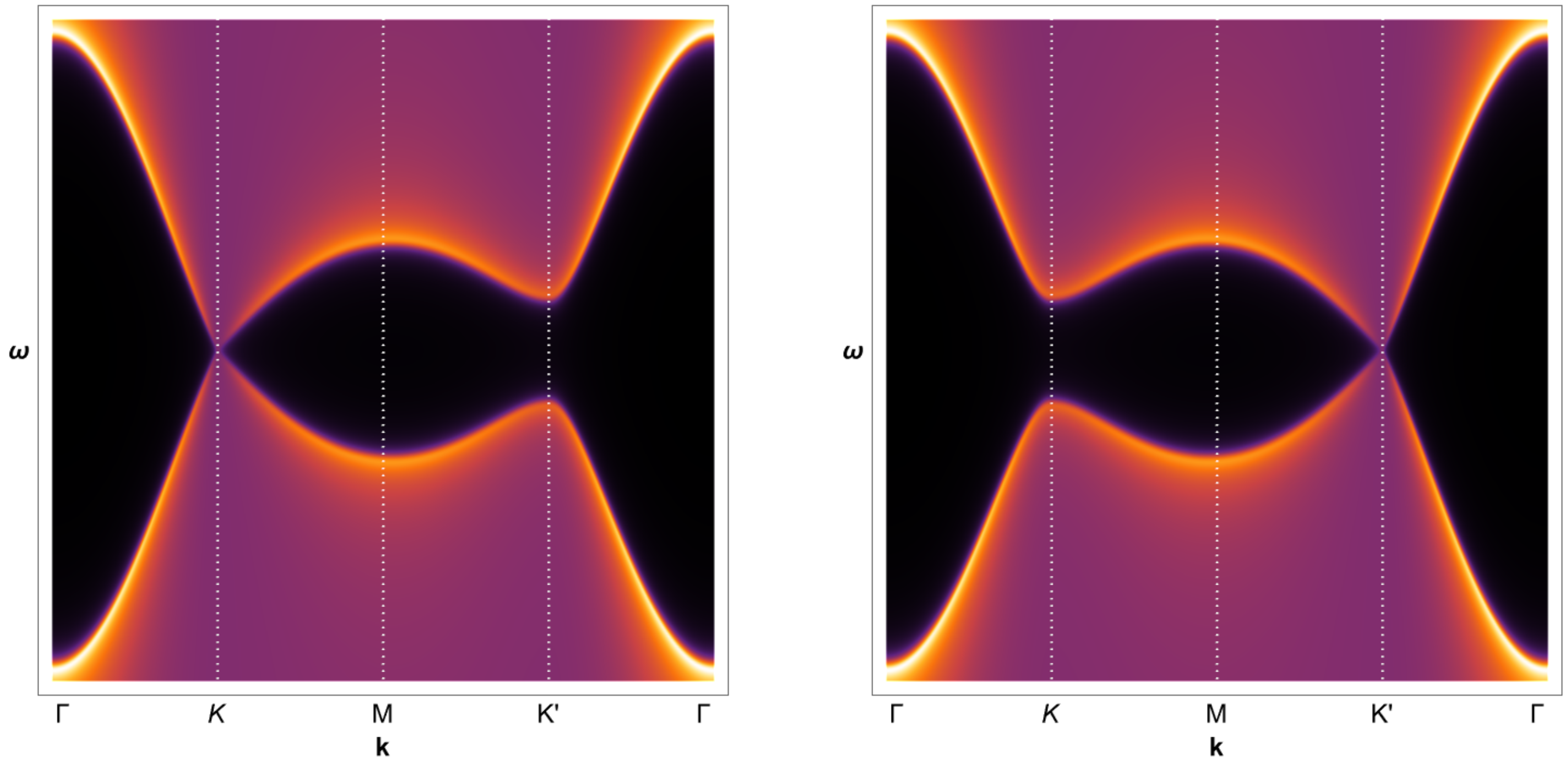
(c) $m = 0$

Figure 3: Graphene's SD changed by m with $t = 1$, $t_2 = 0$ and $\lambda_v = 0$. As we change the value of m from $\frac{1}{2}$ to 0, there is a transition from a simple pole to a branch-cut type pole.

Haldane Model

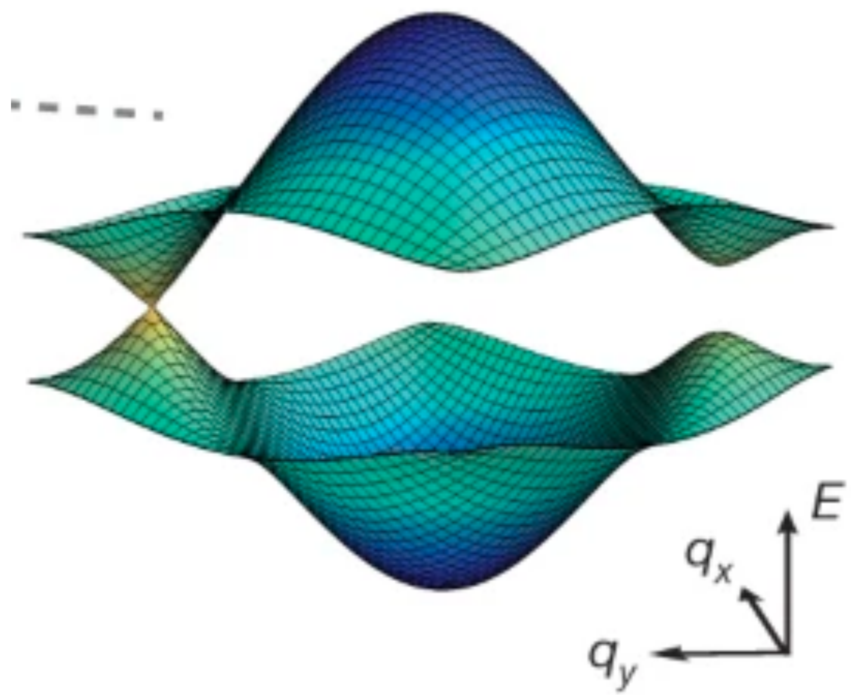
$$H_{TB} = t \sum_{\langle ij \rangle} c_i^\dagger c_j + t_2 \sum_{\langle\langle ij \rangle\rangle} e^{i\nu_{ij}\phi} c_i^\dagger c_j + \lambda_v \sum_i \xi_i c_i^\dagger c_i.$$

$$\nu_{ij} = \mathrm{sign}(\hat{d}_i \times \hat{d}_j)_z = \pm 1, \quad (i,j) \in \{1,2\}$$



(a) $m = 0$ (b) $m = 0.25$

Figure 4: Haldane model SD



Conclusion

1. Holographic Mean field theory = classifying the order
   I) spin of the order parameter:
   ii) Singularity of the Green fct. (Pole vs branch cut)
   iii) Features of band: 0,1,2,3 dim Flat band. nodal ring, nodal shell….

2. surprise: Effect of the lattice =symmetry breaking.

3. Mottness can be discussed in parallel to order.
   Gap without order. Possible due to the conformal structure.
   Mass term => $\bar{\psi}F^2\psi$

4. Future: Strange metallicity from the <ER=EPR>

Thank you

Prescription: Model the system by Bulk Local Theories!

Why it works?

1. Large N= large degeneracy in the ground state.
   => Fluctuation/Frustration/Entagnlement
2. Duality between CL and Quantum (Missing step)
3. Long distance entanglement at boundary
   <=> Non-locality in the bdy.
   Bulk locality is very plausible!
4. Strange metallicity from the entanglement. <ER=EPR>