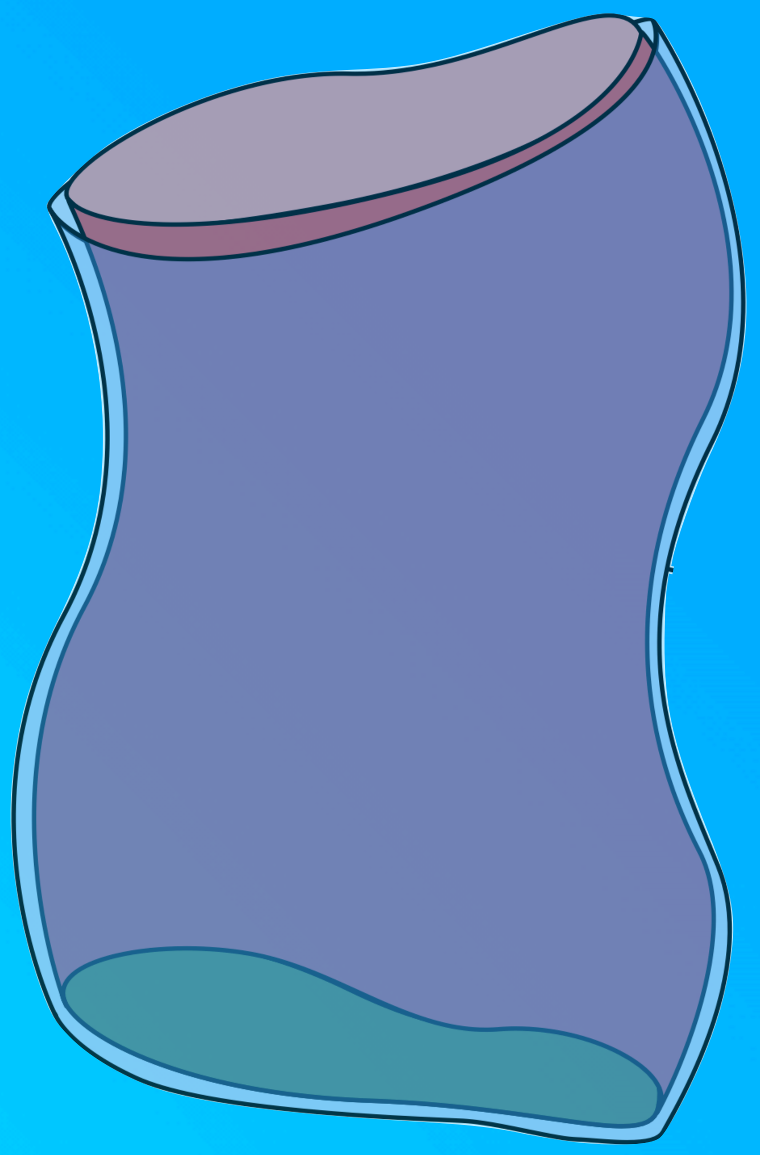
QUANTUM GRAVITY AND INFORMATION IN EXPANDING UNIVERSE - YITP - FEB 2025

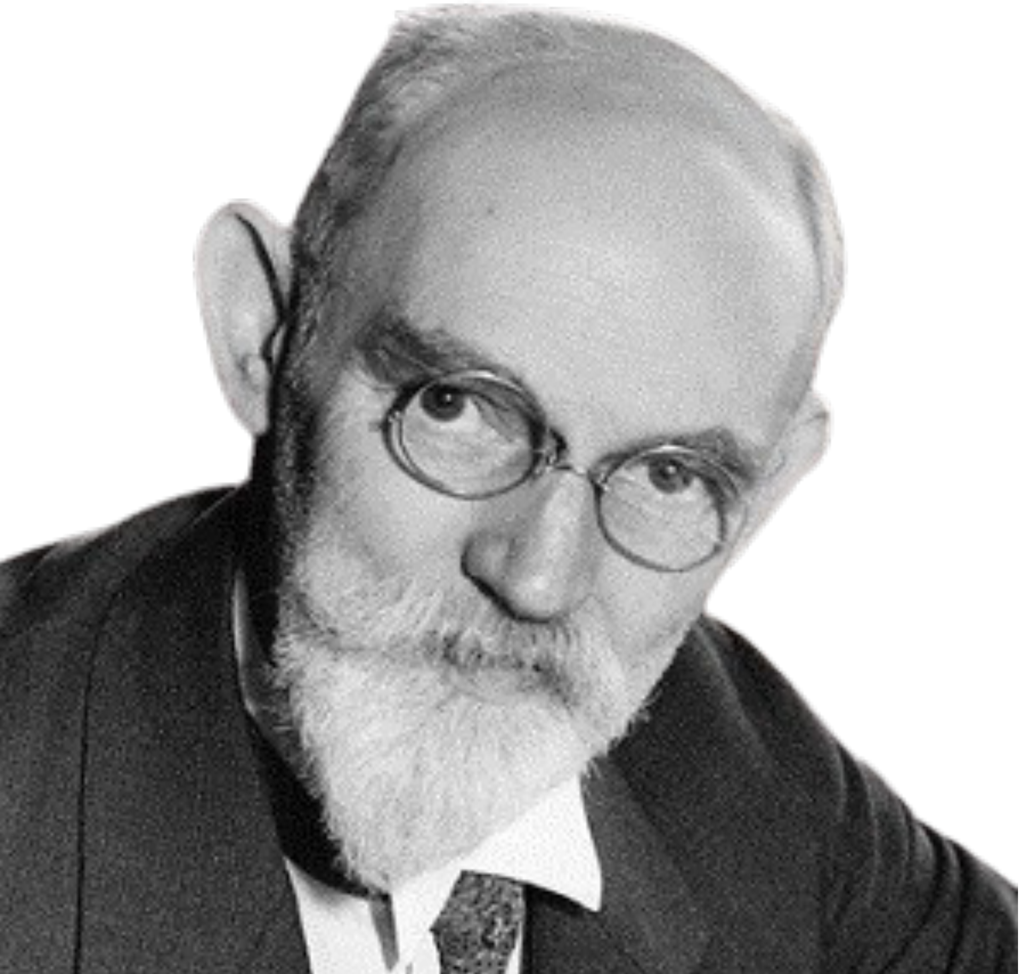
TIMELIKE BOUNDARIES IN AN EXPANDING UNIVERSE

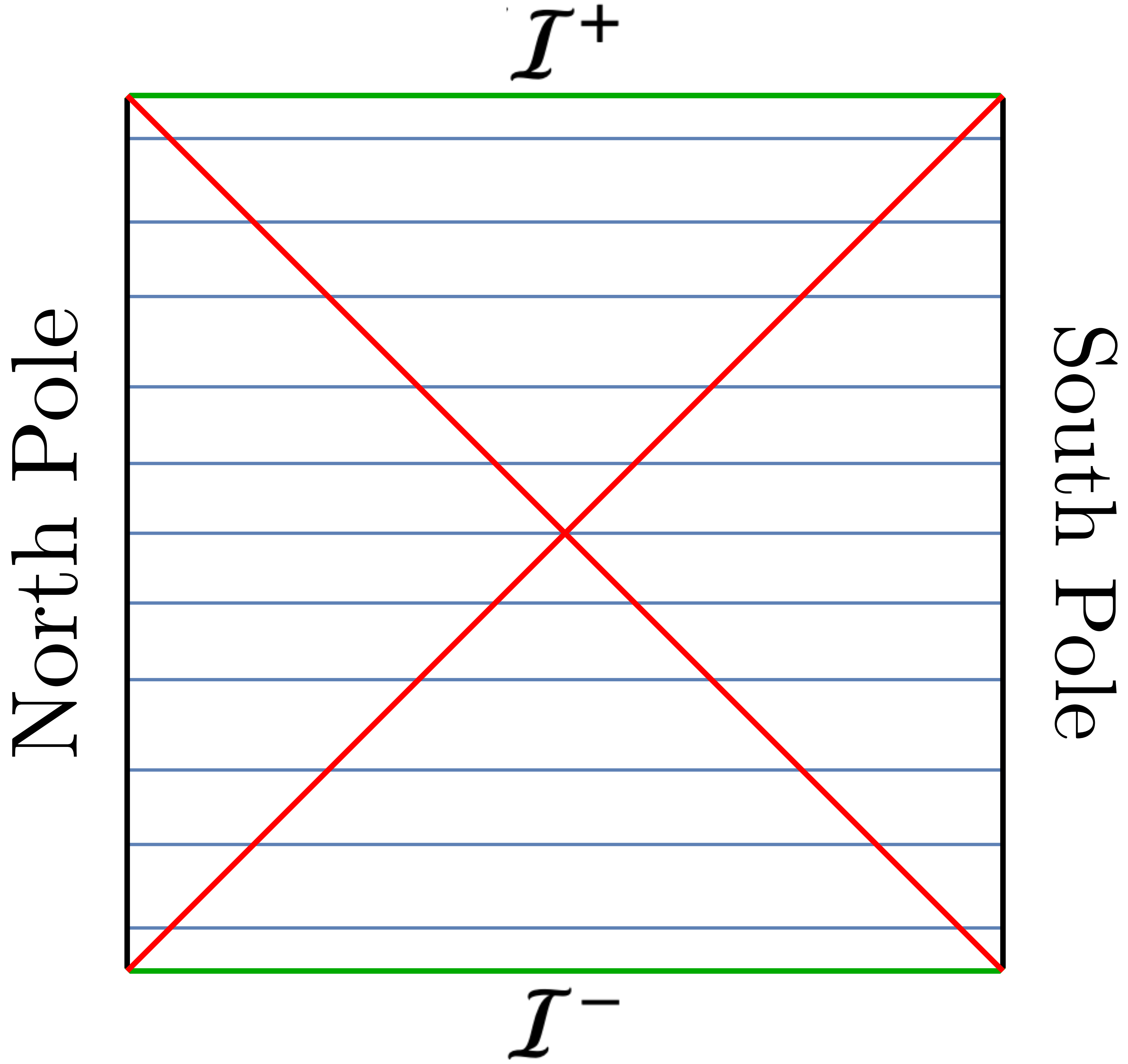


DAMIÁN A. GALANTE
KING'S COLLEGE LONDON

BASED ON 2310.08648, 2402.04305, 2412.16305,
WITH D. ANNINOS, R. ARIAS AND C. MANEERAT

DE SITTER SPACE

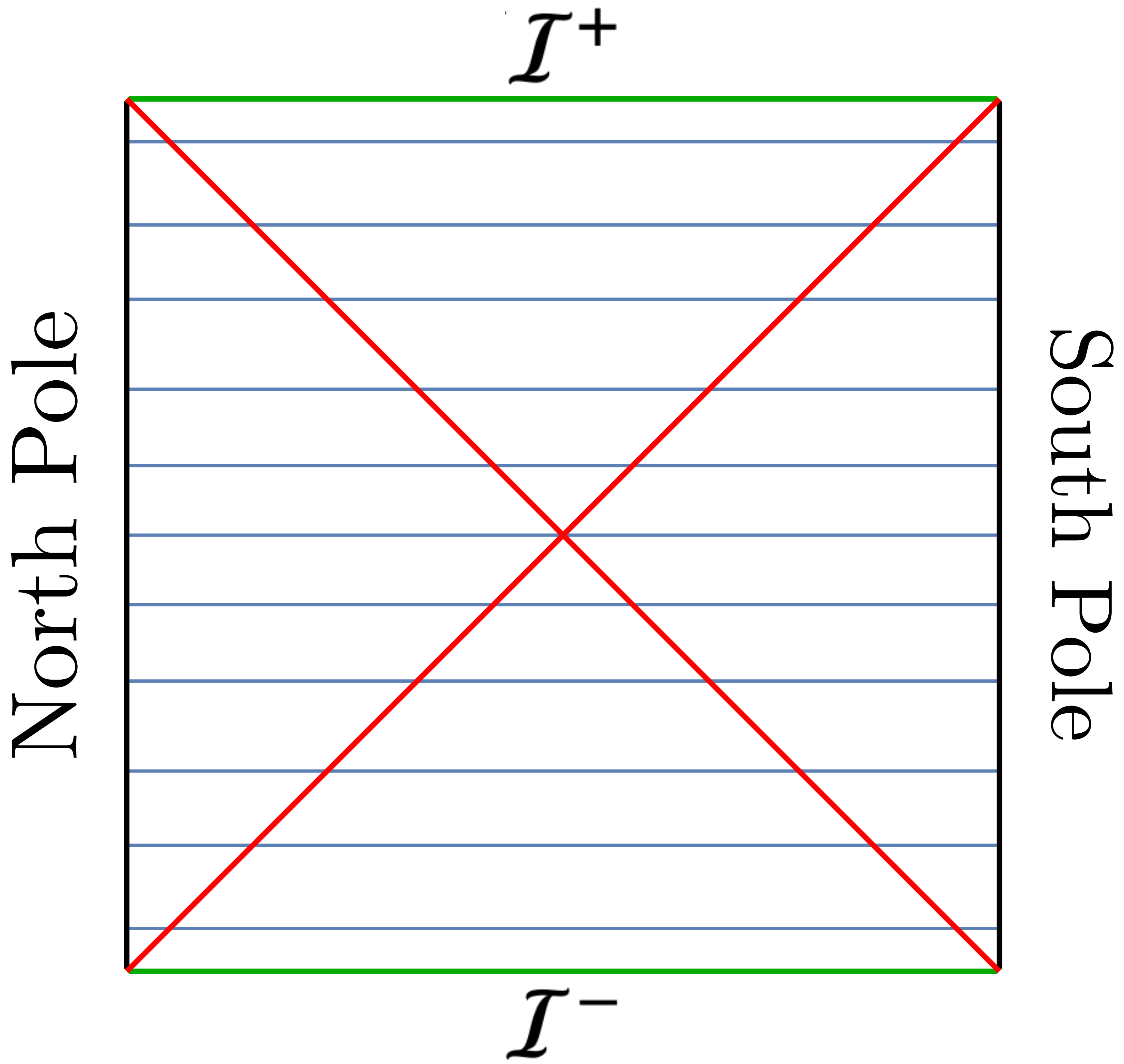


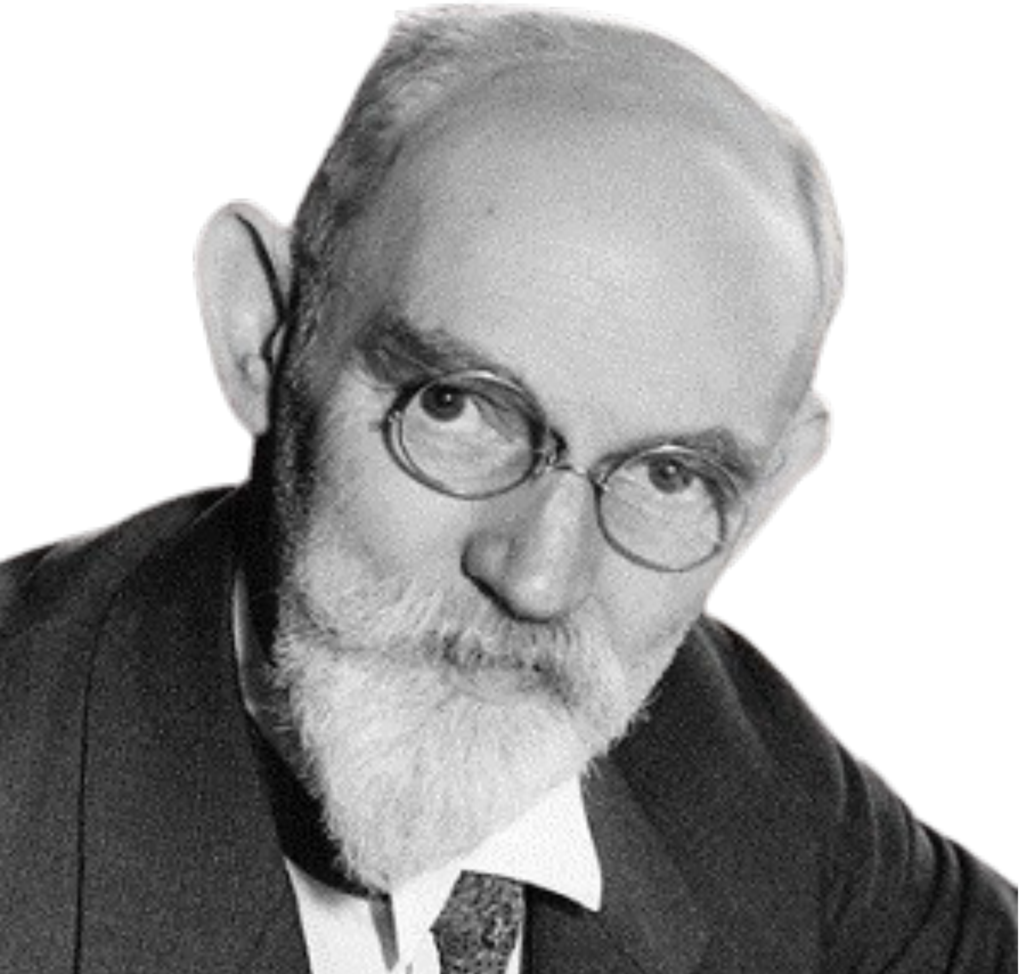


DE SITTER SPACE

The global

$$ds^2 = -d\tau^2 + \cosh^2\tau \, d\Omega_3^2$$

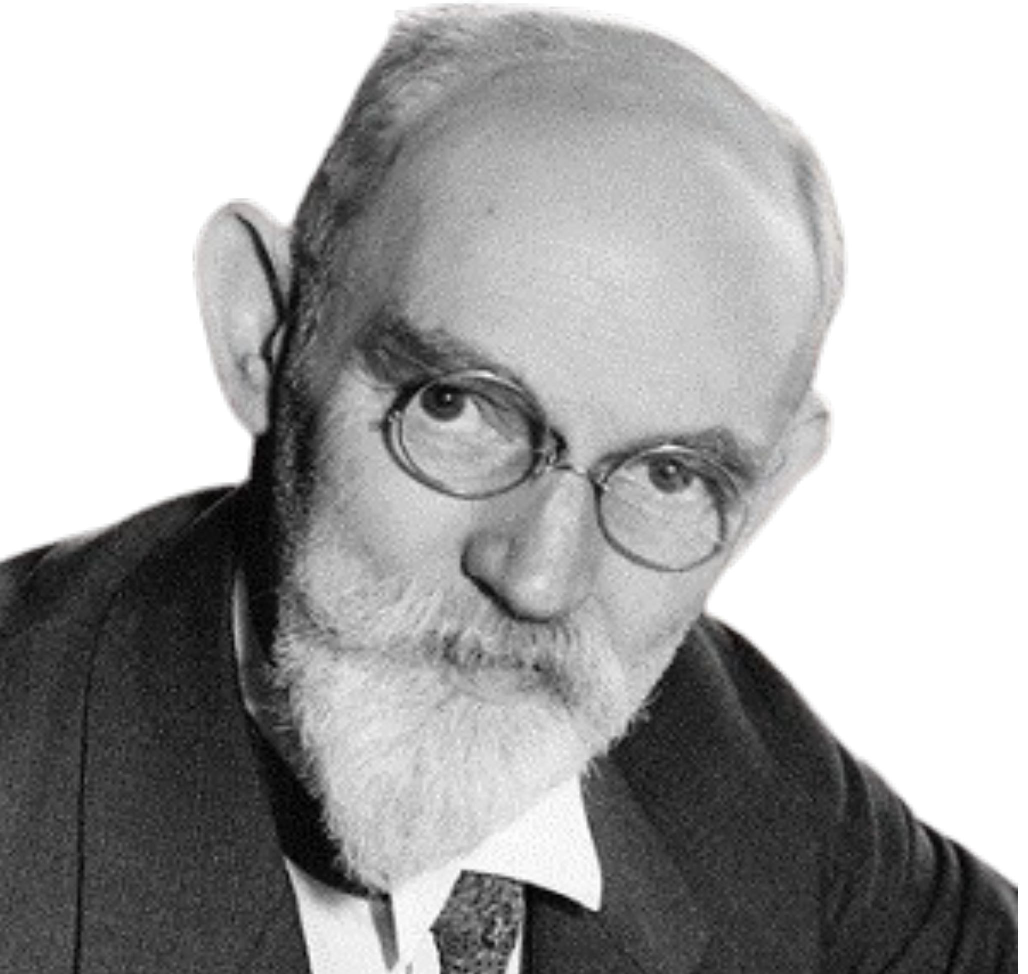


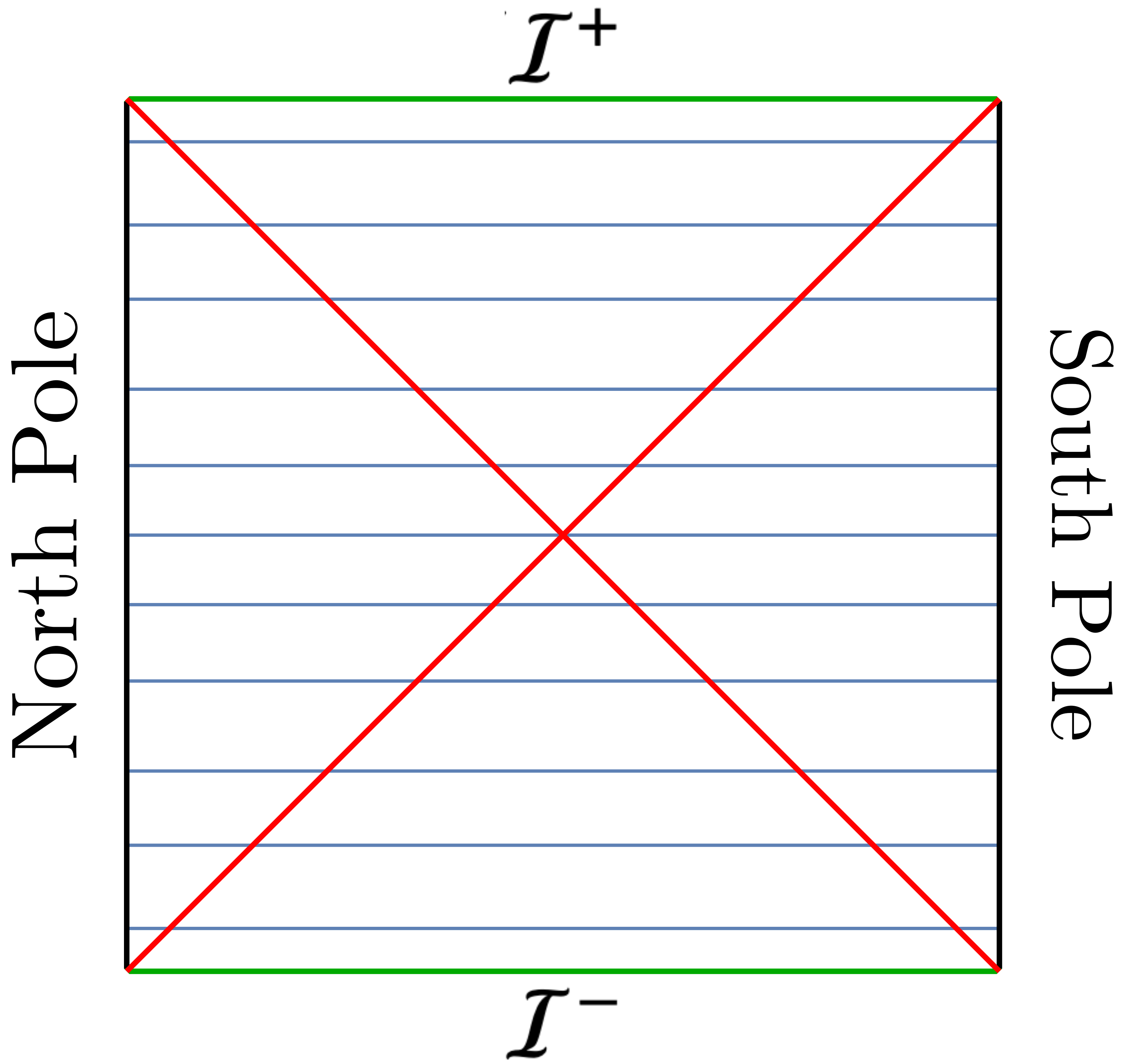


DE SITTER SPACE

The static patch

$$ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_2^2$$





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DE SITTER SPACE

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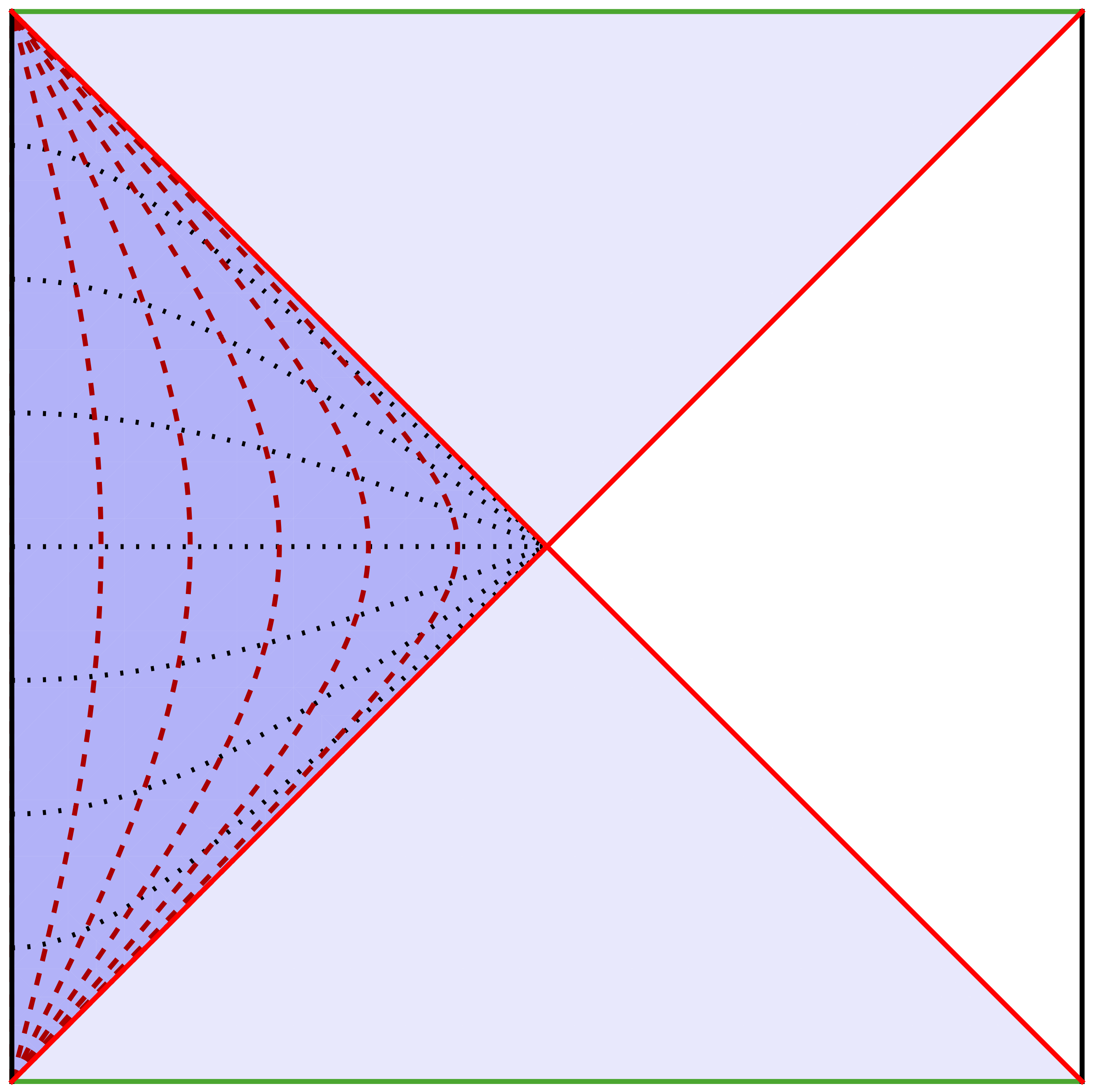
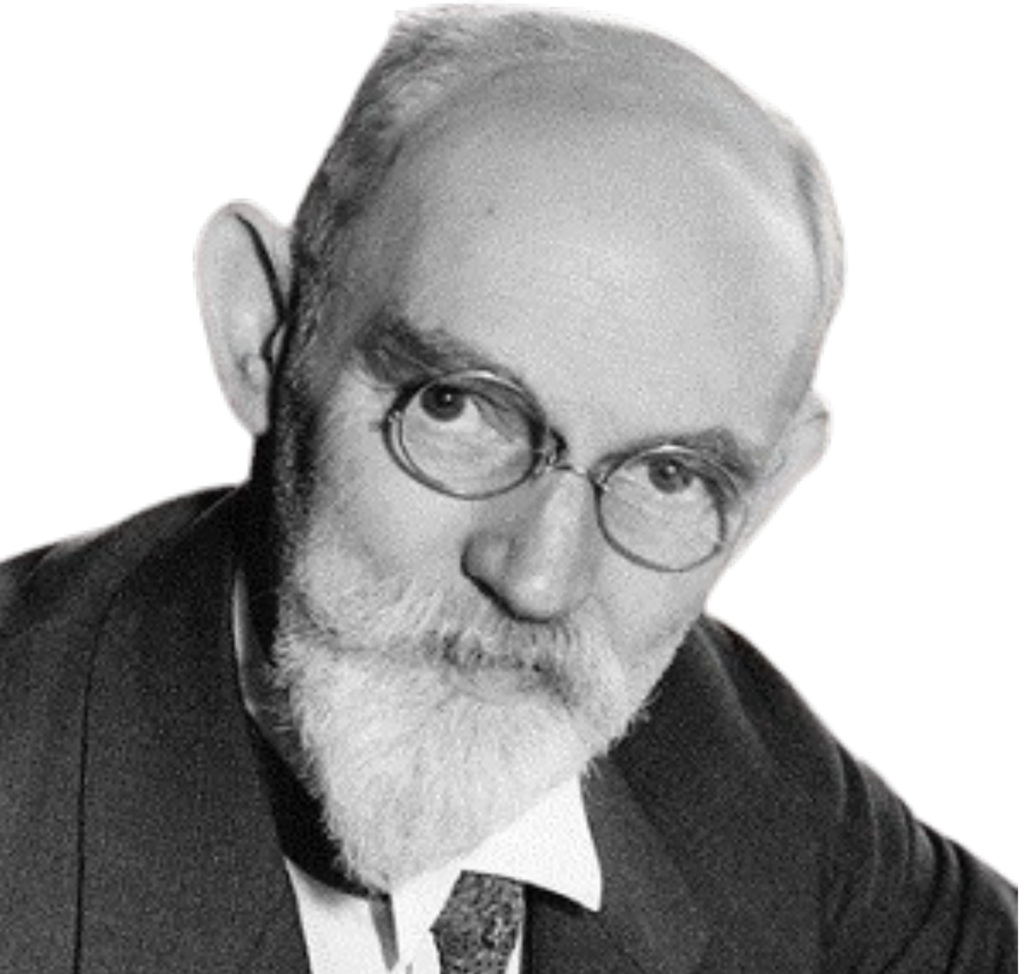
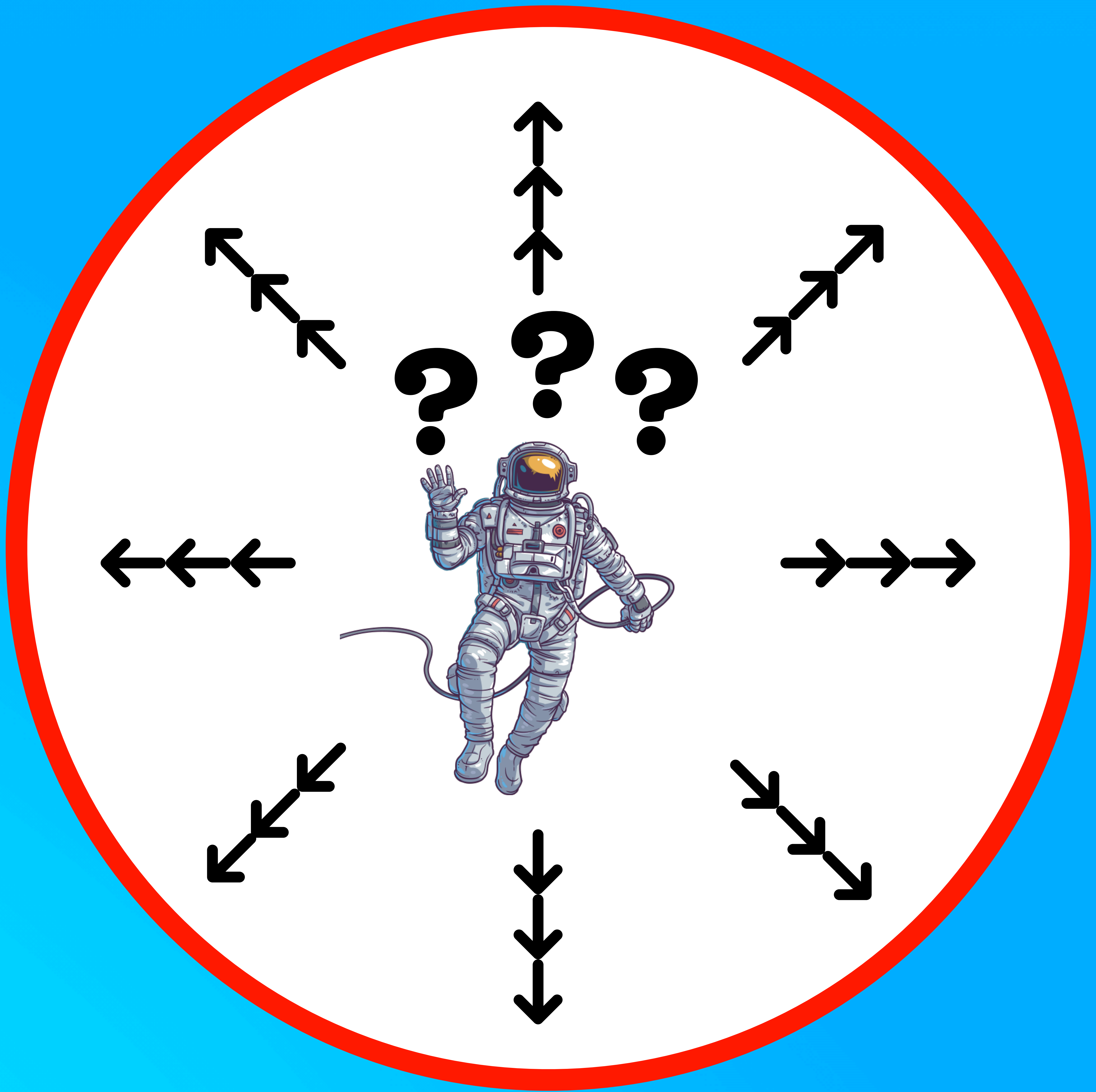






HOW CAN WE DESCRIBE PHYSICS INSIDE THE DS STATIC PATCH?

DE SITTER ENTROPY?
QUANTUM INFORMATION IN EXPANDING SPACETIMES?

THE ANALYTIC CONTINUATION TO EUCLIDEAN SPACE OF BOTH THE STATIC AND THE GLOBAL PATCH IS THE 4-SPHERE.

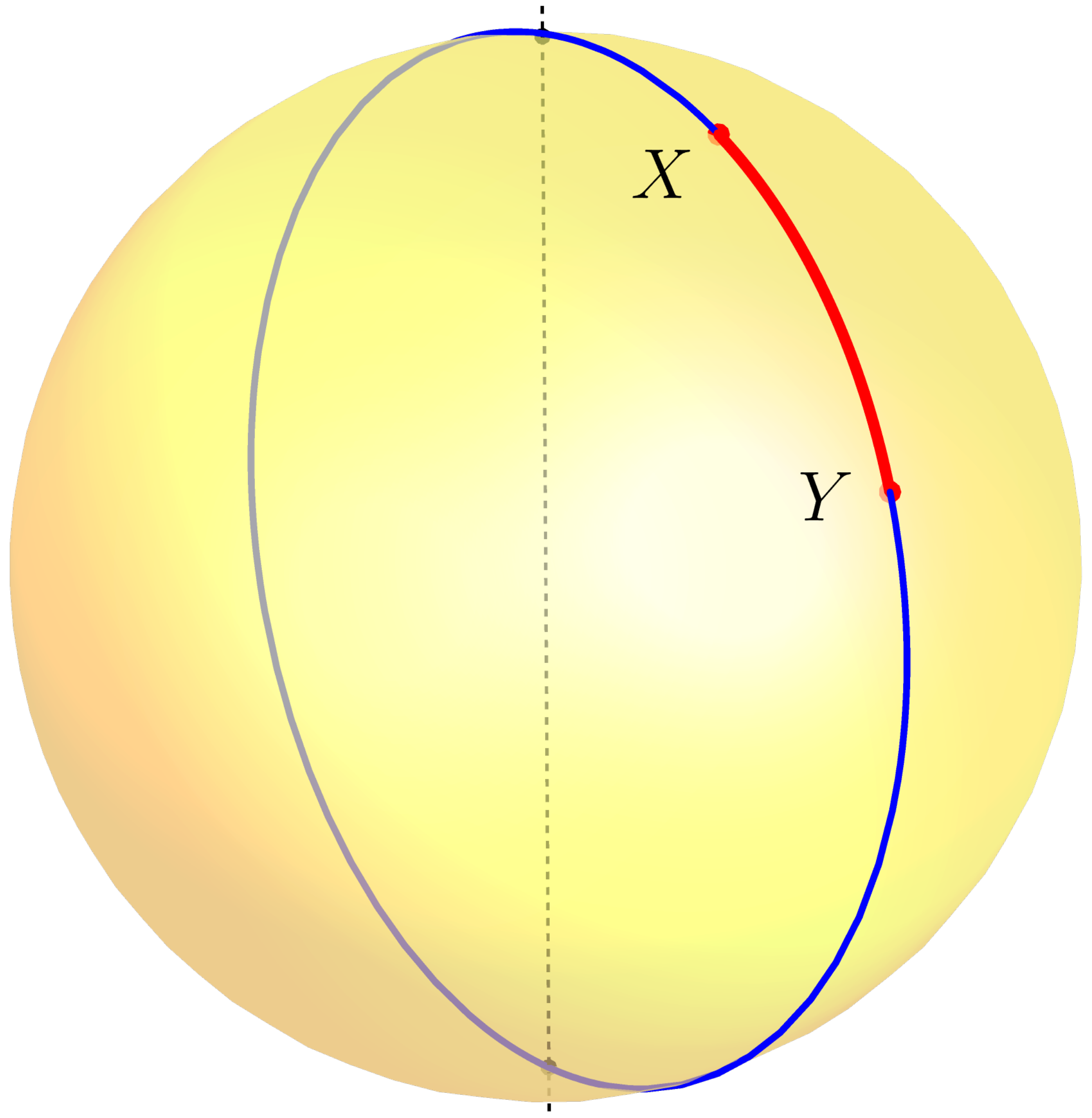
$$ds^2 = d\tau_E^2 + \cos^2 \tau_E \, d\Omega_3^2$$

$$ds^2 = (1 - r^2)dt_E^2 + \frac{dr^2}{1 - r^2} + r^2 d\Omega_2^2$$

- **Gibbons and Hawking (1977) argued that similar to black holes**

$$S_{GH} = \log Z[S^4] = \frac{A_H}{4G_N} = \frac{3\pi}{\Lambda G_N} \,.$$

- **Famously,** $S_{GH} \approx 10^{122}$.

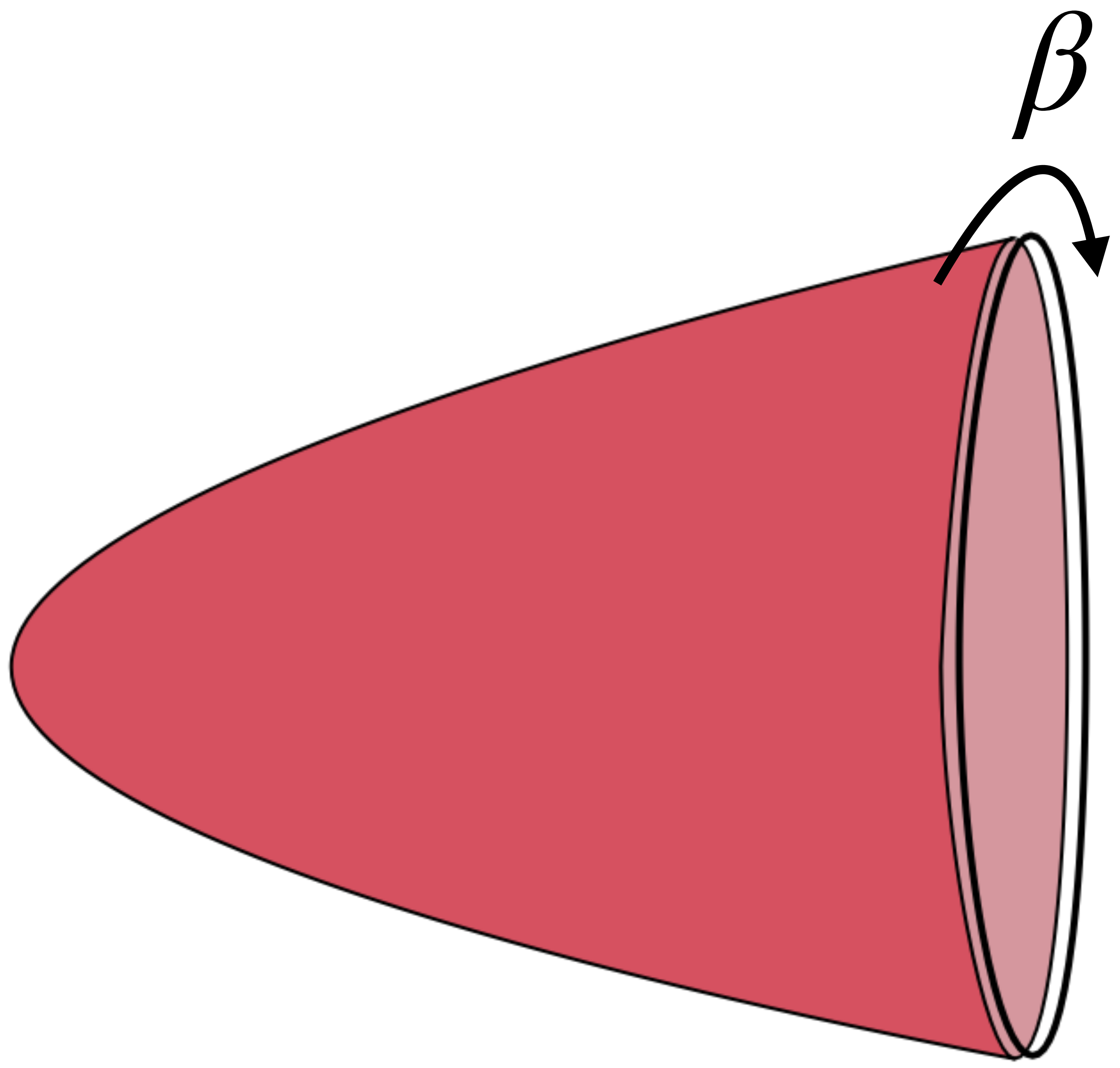


[SEE MY MODAVE LECTURES 2306.10141]

A BLACK HOLE ANALOGY…

THE BLACK HOLE IN FLAT SPACE HAS NEGATIVE SPECIFIC HEAT

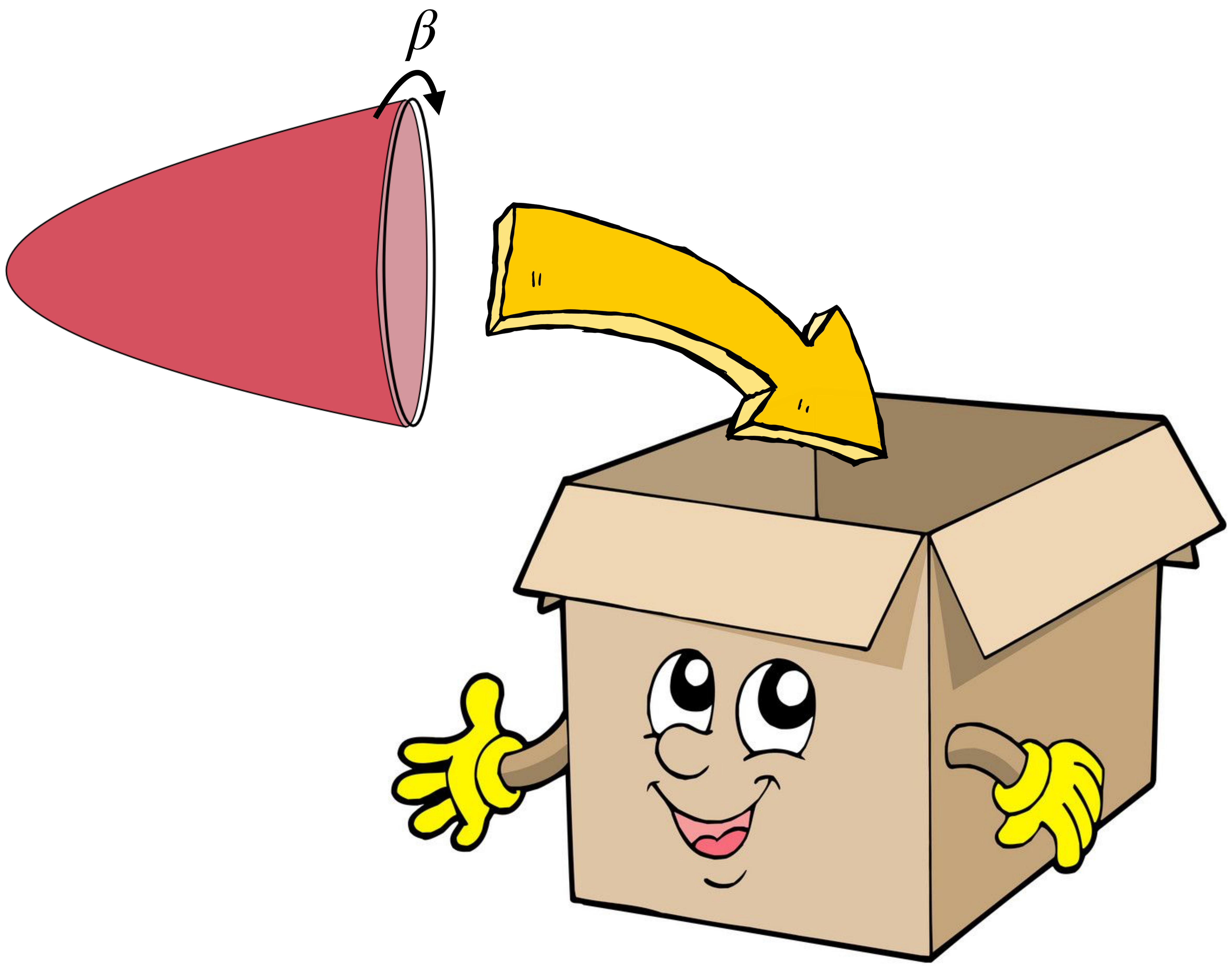
[TALK BY TAKAYANAGI]



A BLACK HOLE ANALOGY…

ONE IDEA: PUT THE BLACK HOLE INSIDE AN ANTI DE SITTER BOX

[HAWKING-PAGE '83, …, WITTEN '98, …]



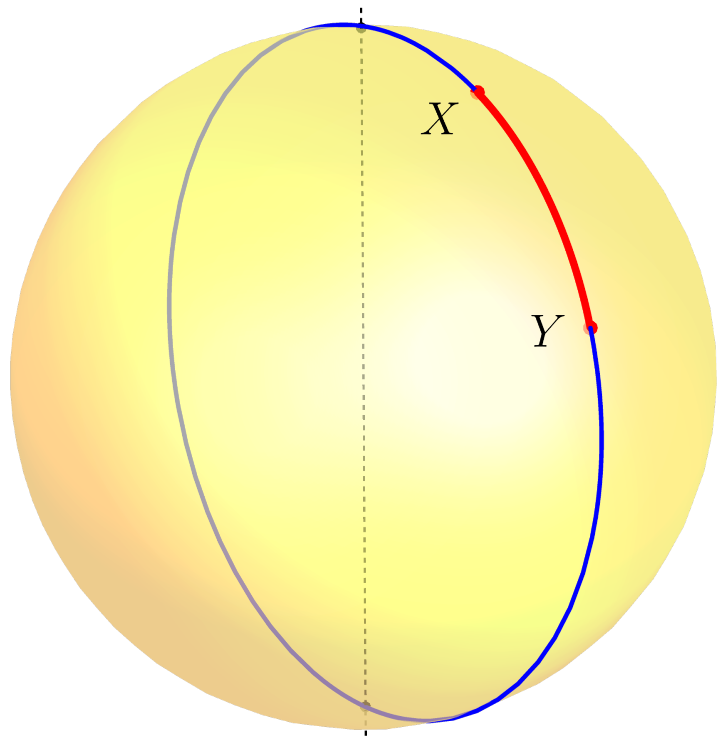
$$C = \frac{\ell_{AdS}^{d-2}}{G_N} \frac{1}{\beta^{d-2}} > 0!$$

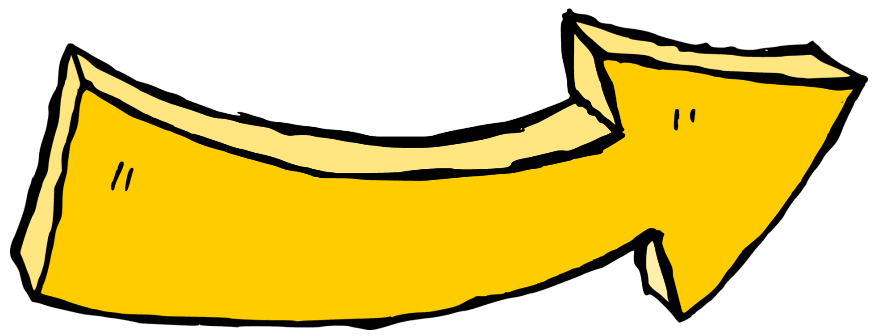
- POSITIVE SPECIFIC HEAT
- CFT RESULT

CAN WE DO THE SAME WITH THE COSMOLOGICAL HORIZON?

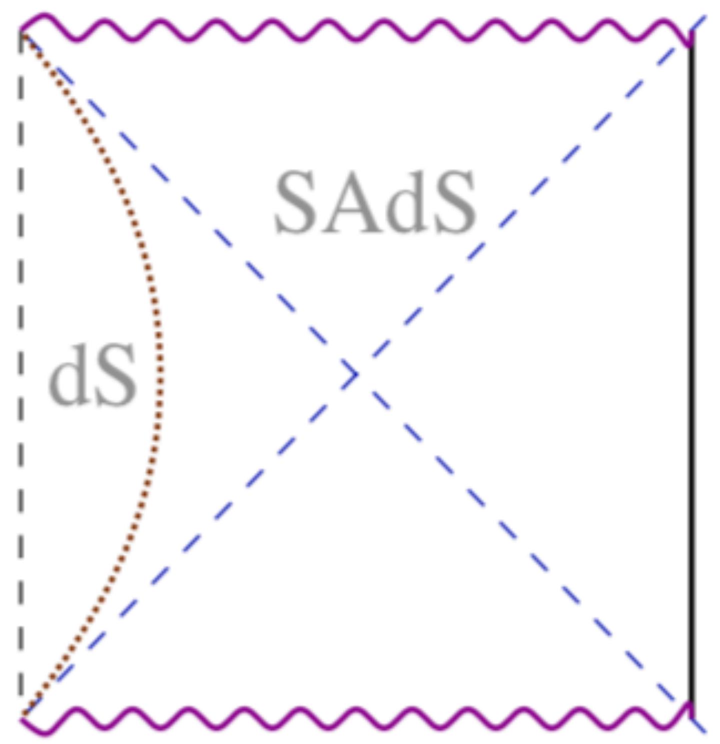
ONE IDEA: EMBED (A PATCH OF) DE SITTER INSIDE ANTI DE SITTER

[FREIVOGEL, HUBENY, MALONEY, MYERS, RANGAMANI, SHENKER, '05]

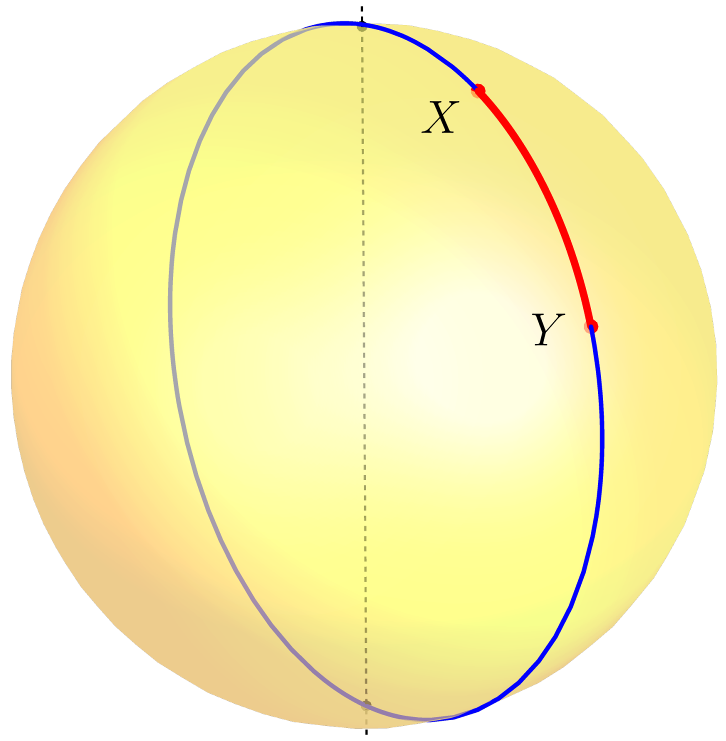


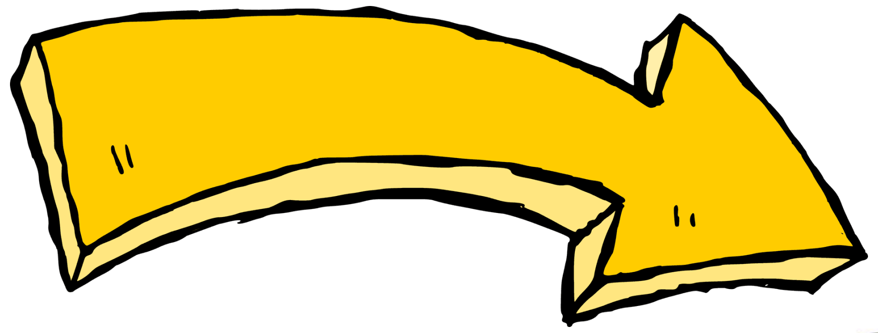


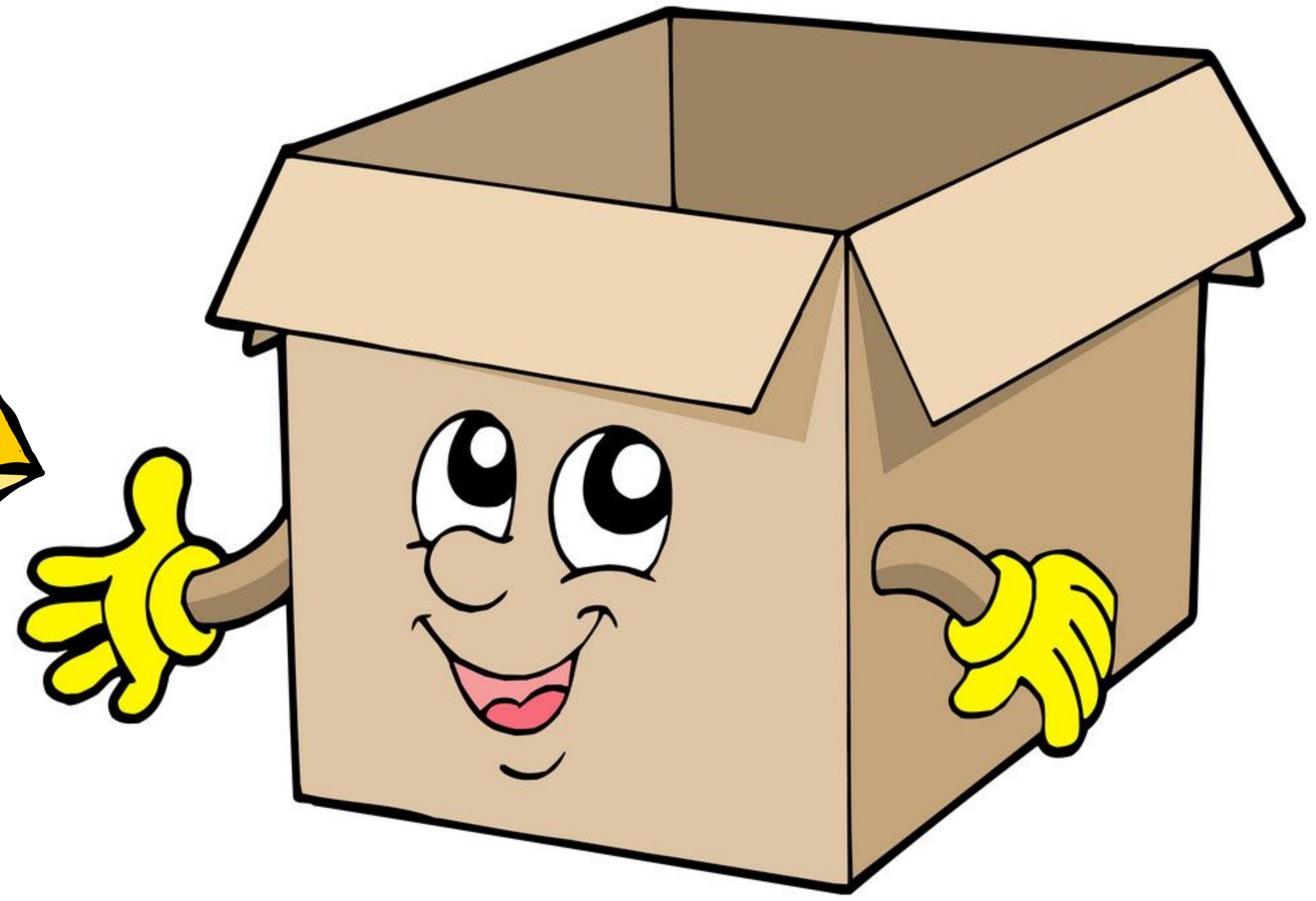
OBSTRUCTION:
DE SITTER BUBBLE HIDES INSIDE BLACK HOLE HORIZON!

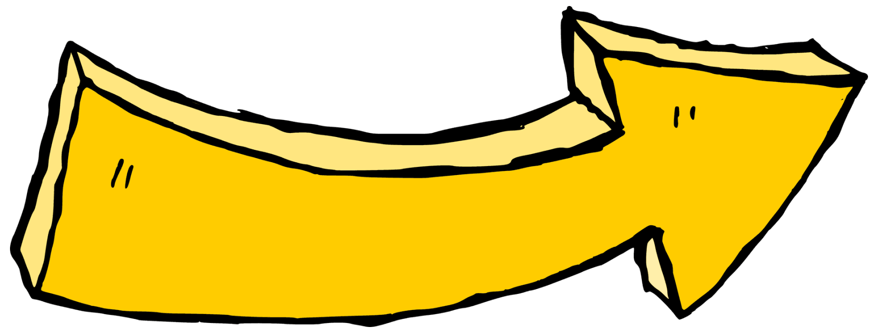


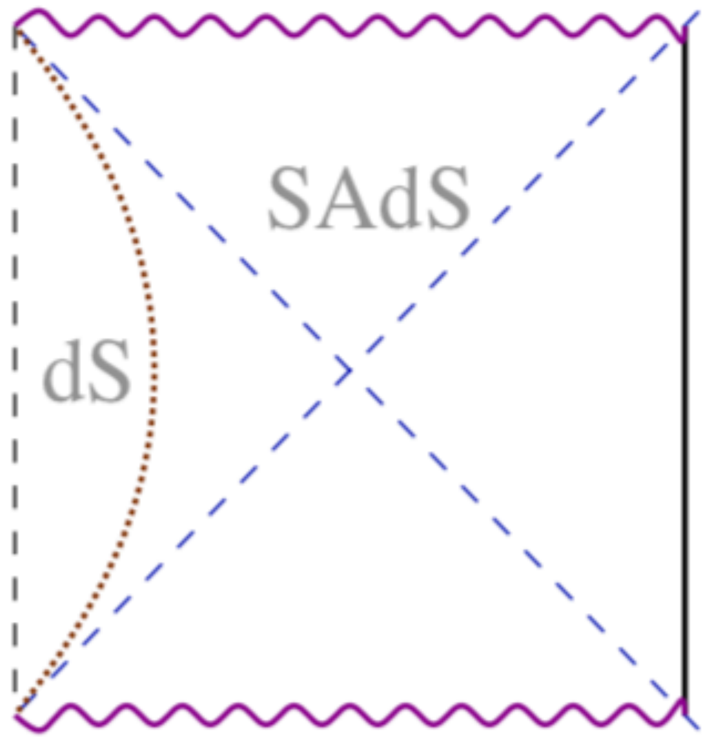
[TALK BY ZENONI]









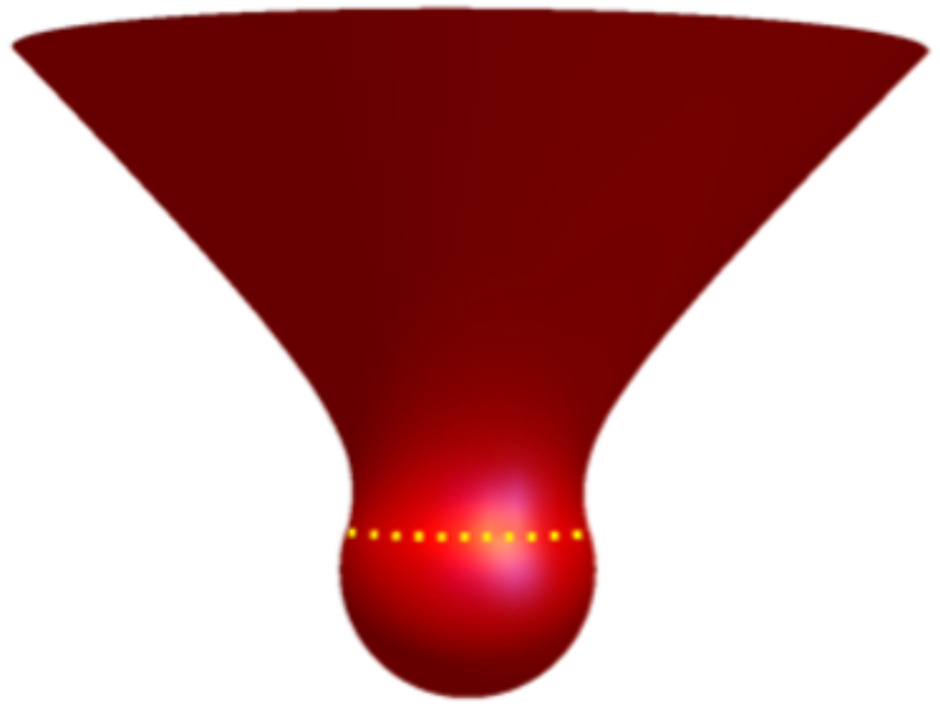


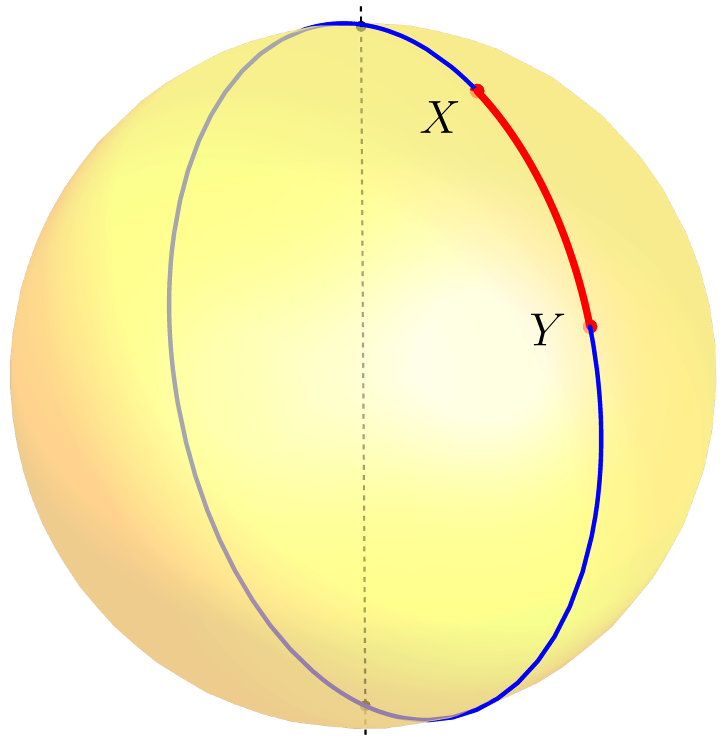
- **This obstruction comes from matter needed to support the geometry violating the null-energy condition**

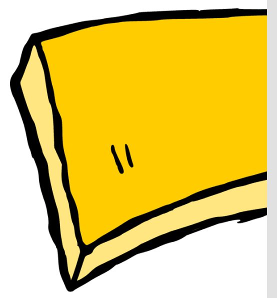
[ANNINOS, DAG, '20]

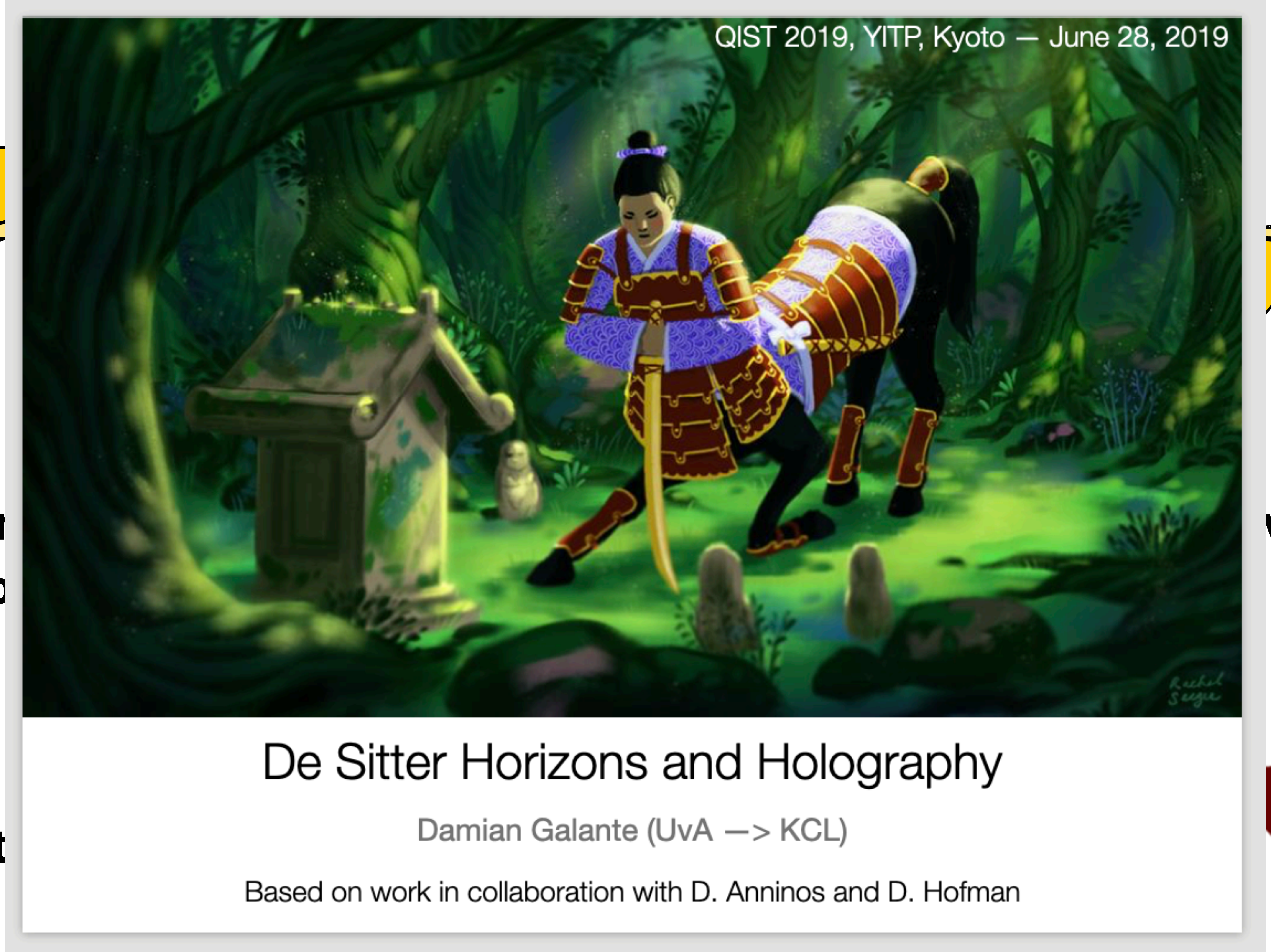
- **Note however, this is not true in two-dimensional spacetimes.**

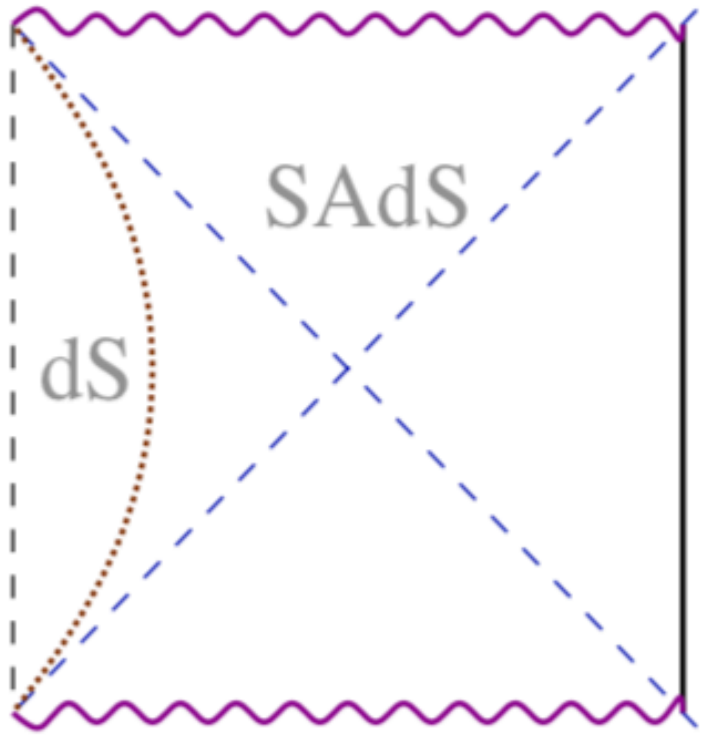
[ANNINOS, DAG, HOFMAN, '18, + ...]





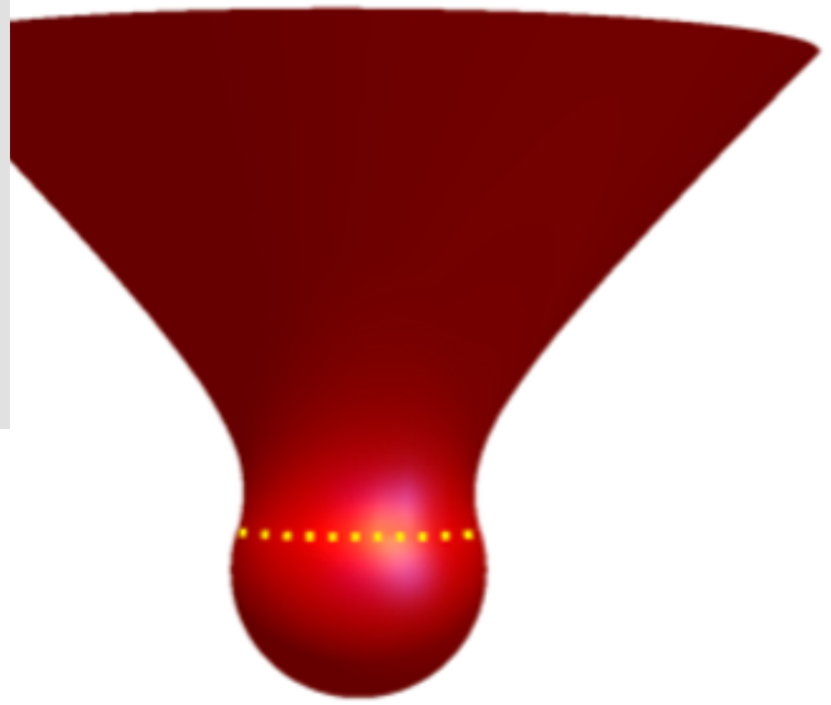






- **This obstructio[...] violating the null-energy conditio[...]**

[ANNINOS, DAG, '20]

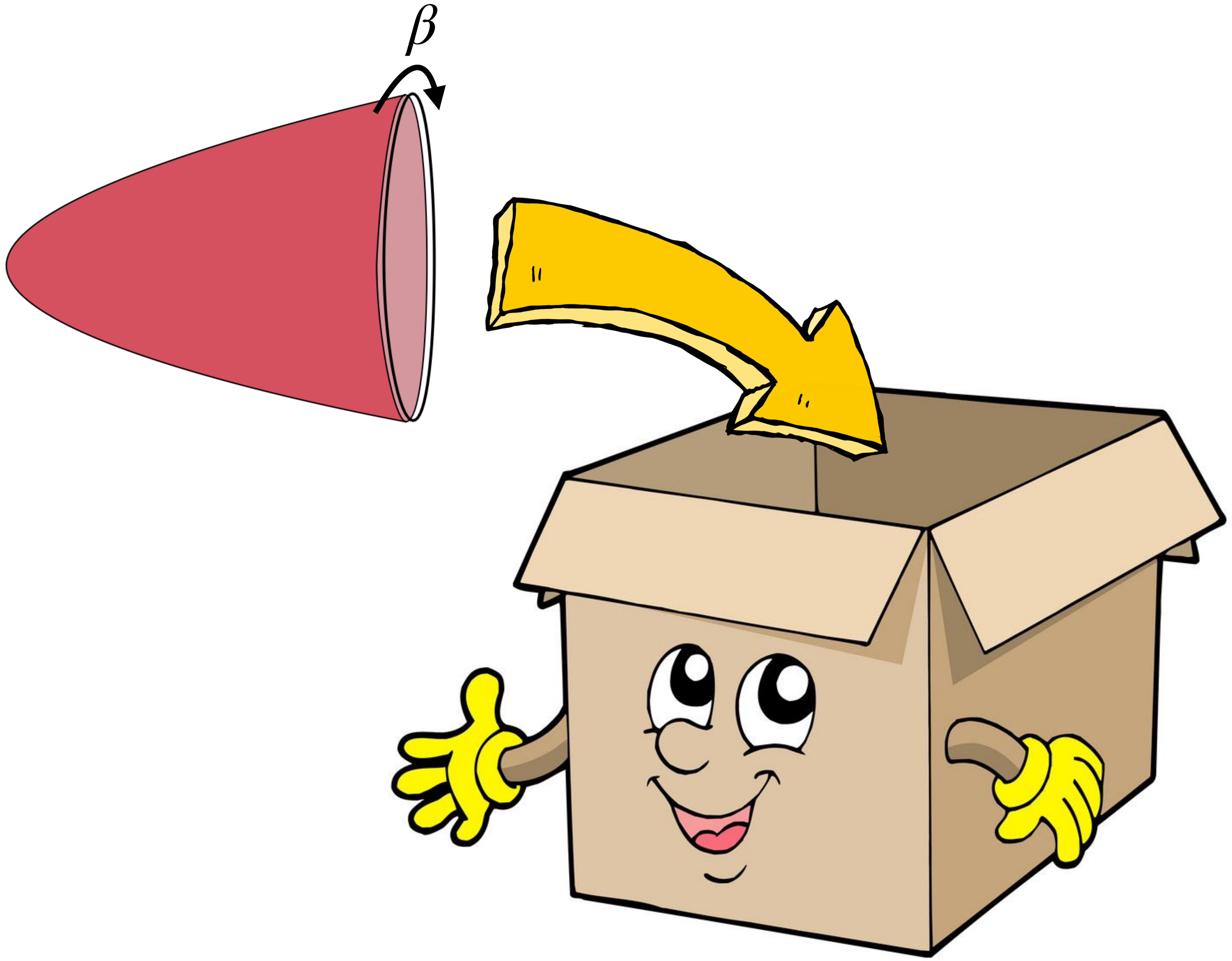


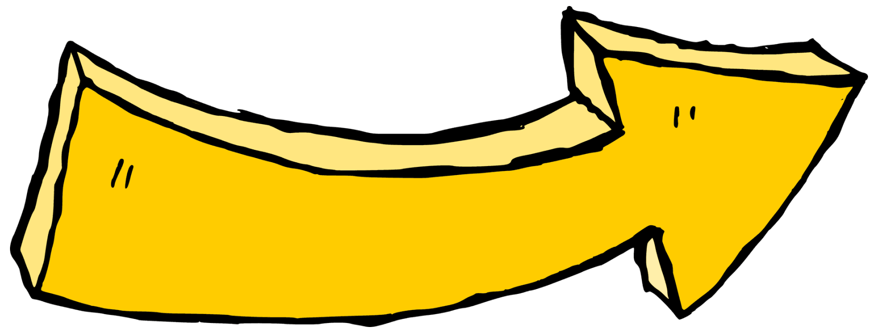
- **Note however, t[...]**

BACK TO THE BLACK HOLE ANALOGY…

SECOND IDEA: PUT THE BLACK HOLE INSIDE A FINITE DIRICHLET BOX

[GIBBONS-HAWKING ’77, YORK, ’80'S]





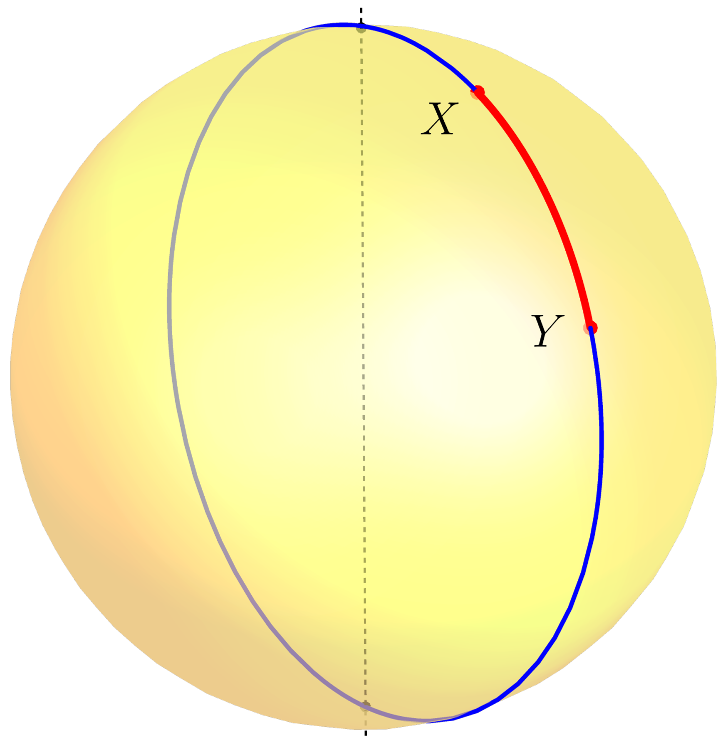
$$C > 0!$$

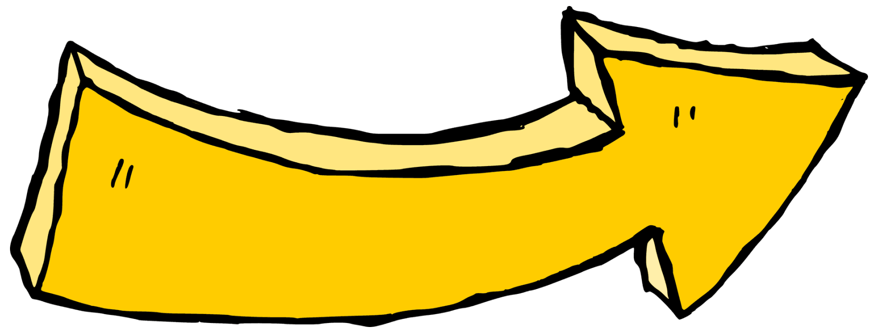
- **POSITIVE SPECIFIC HEAT**
- ~~**CFT RESULT**~~

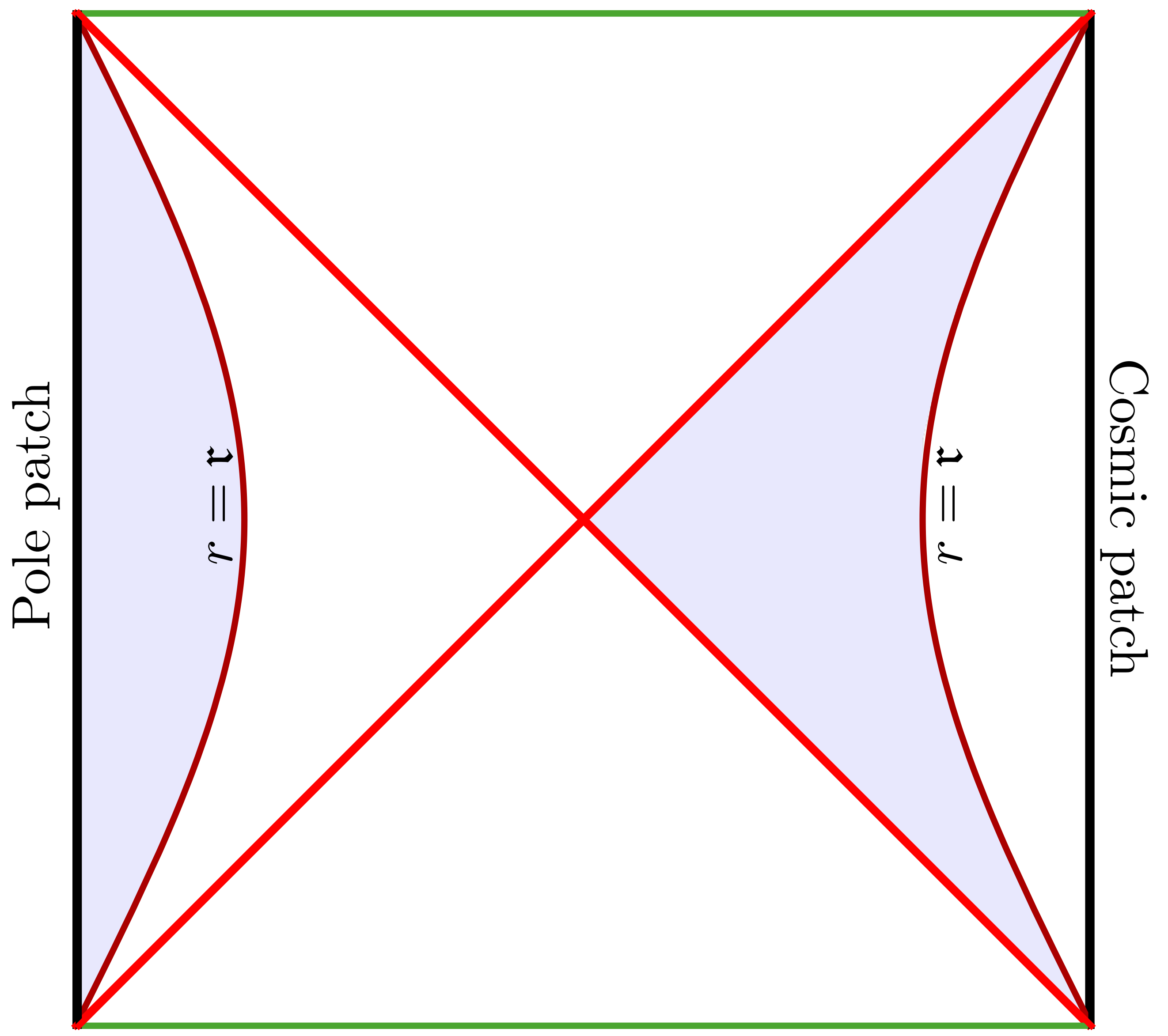
CAN WE DO THE SAME WITH THE COSMOLOGICAL HORIZON?

SECOND IDEA: CONSTRUCT (A PATCH OF) DE SITTER INSIDE A DIRICHLET BOUNDARY

[HAYWARD, '90, HUANG-WANG '01,
DRAPER-FARKAS, '22, BANIHASHEMI-JACOBSON, '22]

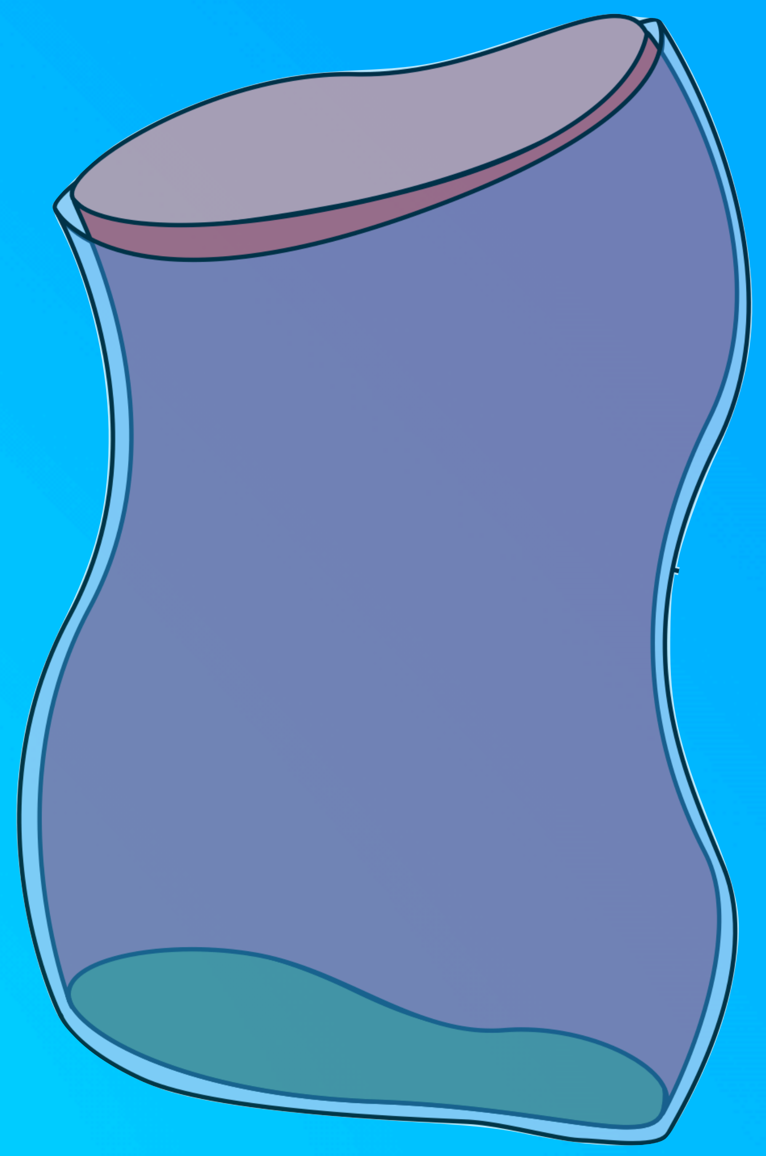






$$C < 0!$$

THIS TALK: NEW DEVELOPMENTS REGARDING SPACETIMES WITH FINITE BOUNDARIES



OUTLINE

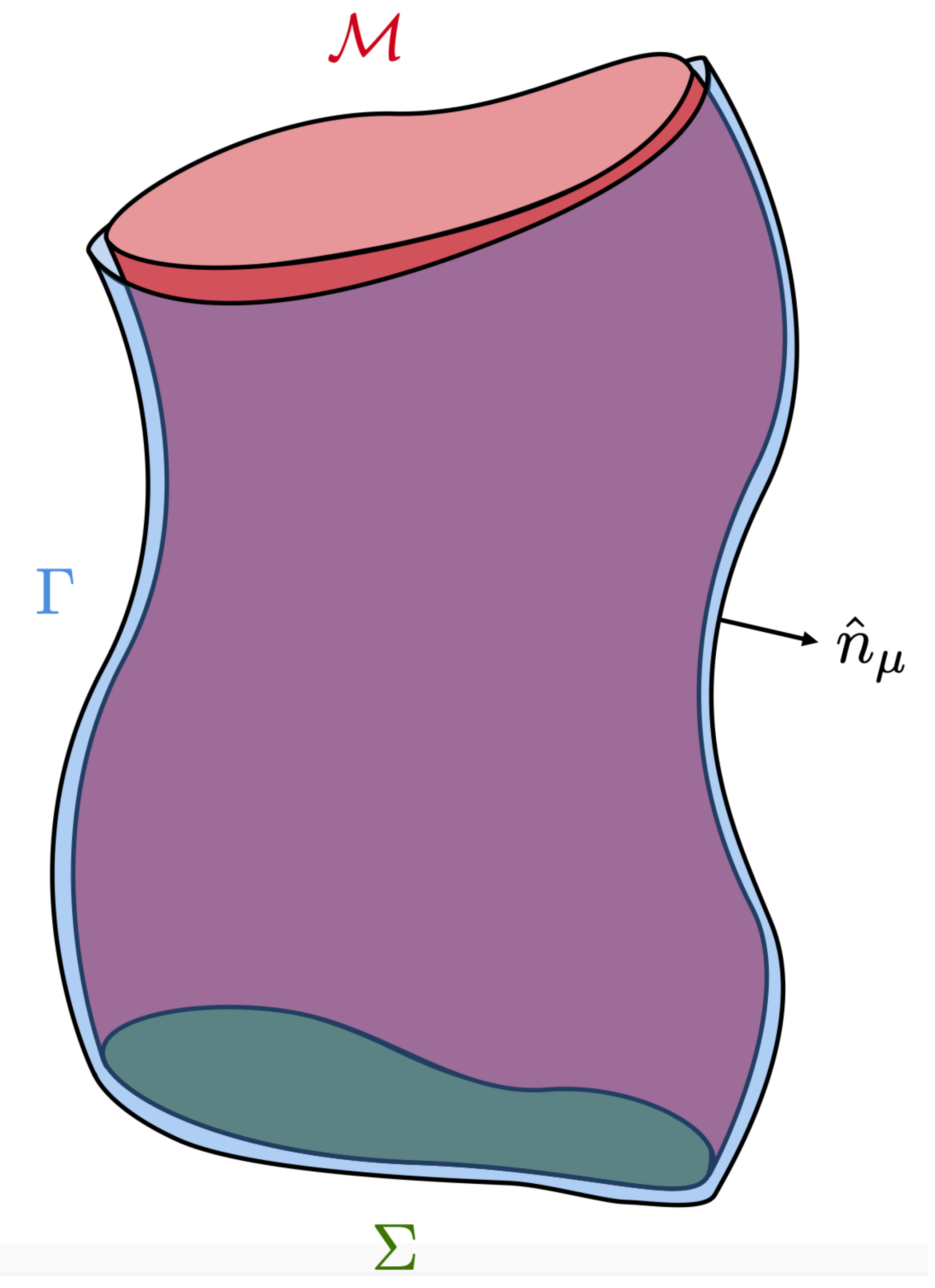
The initial boundary value problem in General Relativity

Conformal boundary conditions

Applications to the static patch of de Sitter space

THE INITIAL BOUNDARY VALUE PROBLEM

- Solutions to general relativity in four spacetime dimensions.
- Vacuum solutions.
- (With or) Without a cosmological constant.
- In manifolds with non-trivial (but finite) boundary.
- The inside and/or the outside solution.



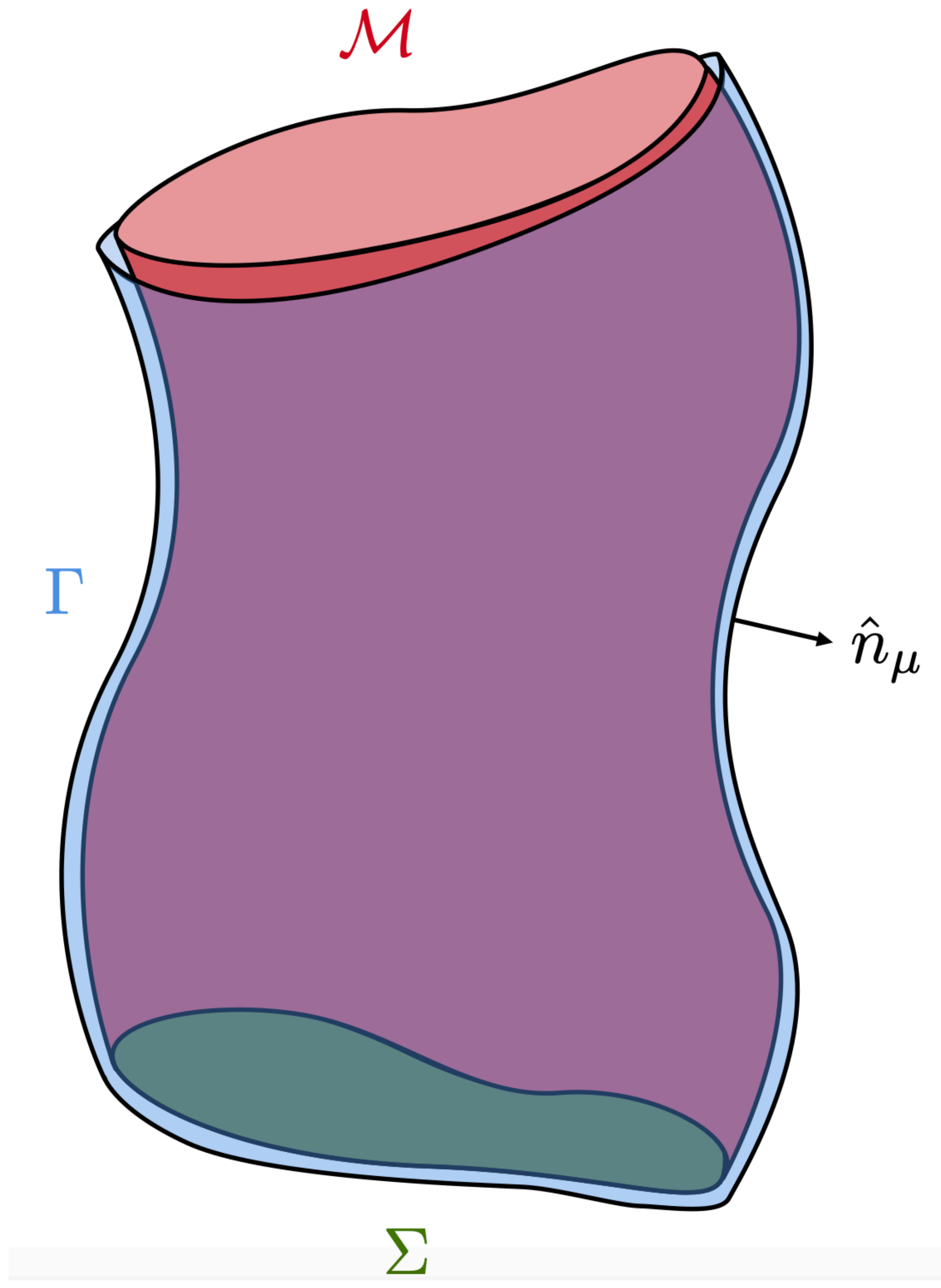
Initial Conditions on Σ : $\left[g_{ij}, K_{ij}\right]$

QUESTION: WHAT ARE THE ALLOWED BOUNDARY CONDITIONS ON Γ?

WELL-POSEDNESS OF THE IBVP IN GENERAL RELATIVITY

The IBVP is said to be well-posed if there is:

1. Existence of solutions provided generic boundary data.
2. Uniqueness of solutions provided generic boundary data
3. (Stability of solutions at late times)



Initial Conditions on Σ : $\left[g_{ij}, K_{ij}\right]$

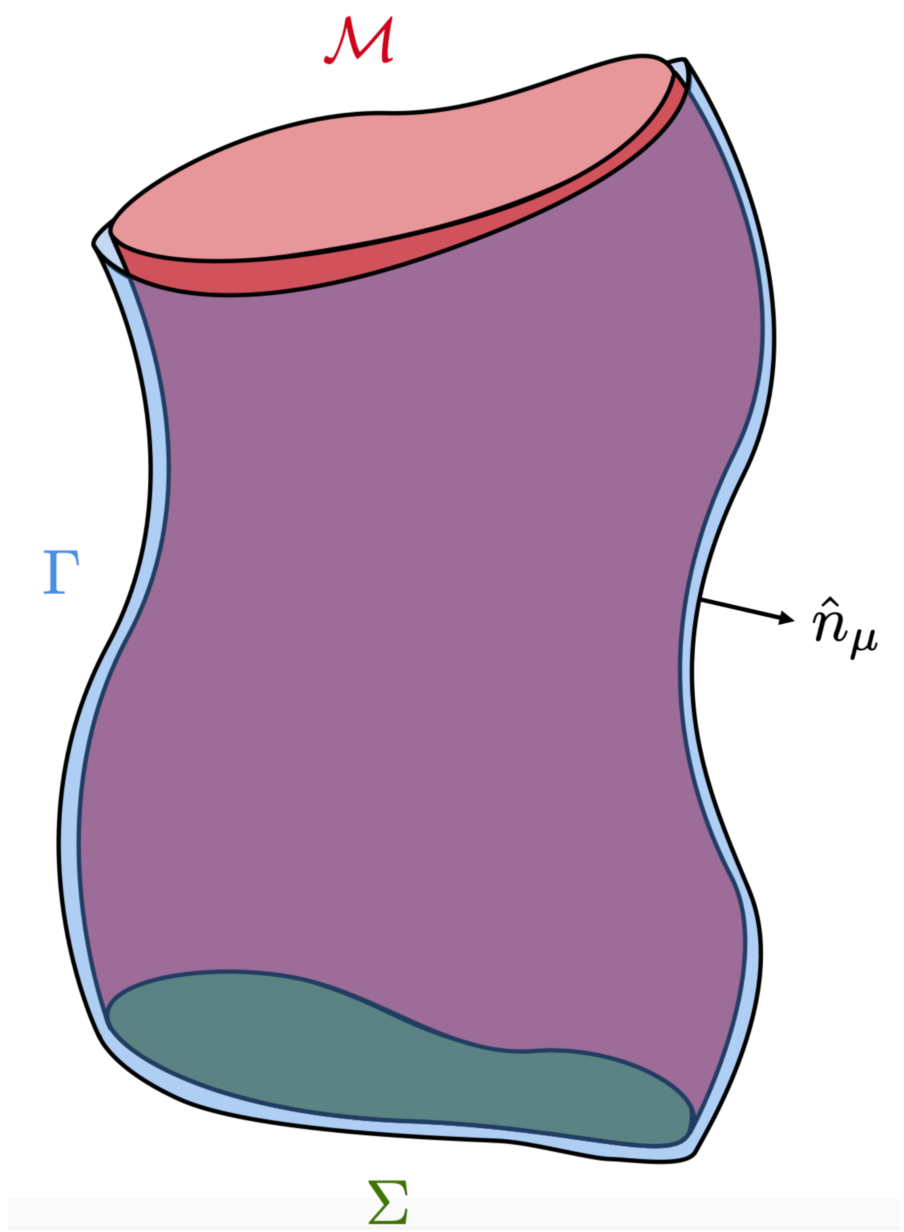
QUESTION: BOUNDARY CONDITIONS ON Γ THAT MAKE THE IBVP WELL-POSED?

THE INITIAL BOUNDARY VALUE PROBLEM

THEOREM [AN, ANDERSON '21]: THE INITIAL BOUNDARY VALUE PROBLEM IN GENERAL RELATIVITY WITH DIRICHLET (OR NEUMANN) BOUNDARY CONDITIONS IS NOT WELL POSED!!

Initial Conditions on $\Sigma : \left[g_{ij}, K_{ij}\right]$

Boundary Conditions on $\Gamma : \left[g_{mn}\right]$



1. Existence
2. Uniquenes

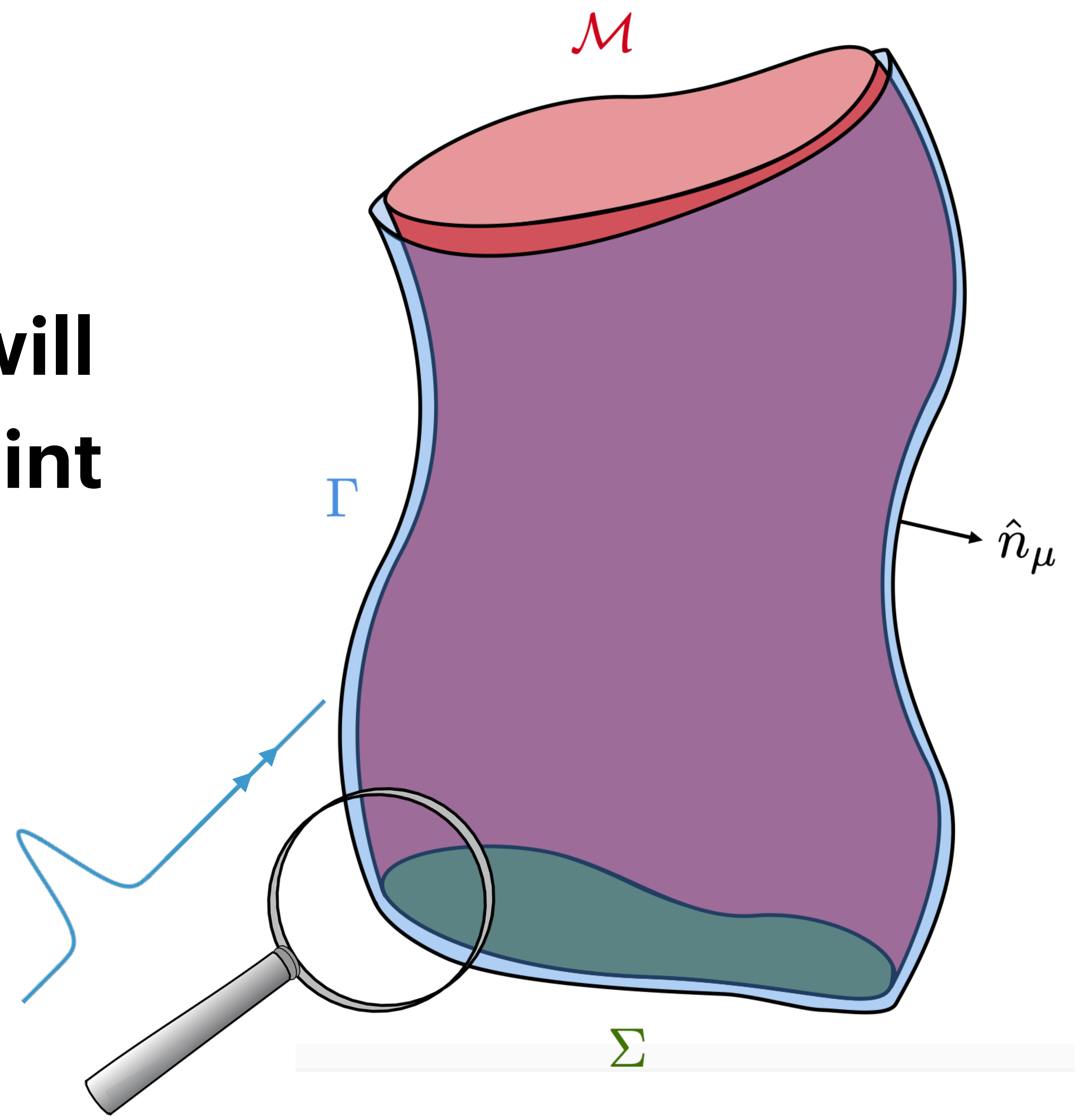
ISSUES WITH THE DIRICHLET PROBLEM

[An-Anderson '21, Anninos-DAG-Maneerat '23]

EXISTENCE

A generic induced metric g_{mn} will not satisfy the Einstein constraint equation at the boundary

$$\mathscr{R} + K^2 - K_{mn}K^{mn}\,|_{\Gamma} = 0$$



UNIQUENESS

- Dirichlet boundary conditions are not enough to fix all the components of the metric.
- Local diffeomorphisms that do not satisfy the boundary condition become physical perturbations!

COMMENTS ON THE DIRICHLET PROBLEM

- The initial value problem in general relativity is well-posed.

[Choquet-Bruhat '52, Choquet-Bruhat & Geroch '69]

- The IBVP with Dirichlet boundary conditions is well-posed for scalar, Maxwell and Yang-Mills theory.

[Sarbach-Tiglio '12, Witten '18]

- Note that there are solutions with particular Dirichlet boundary conditions (e.g., with spherical symmetry) that do not suffer from these problems.

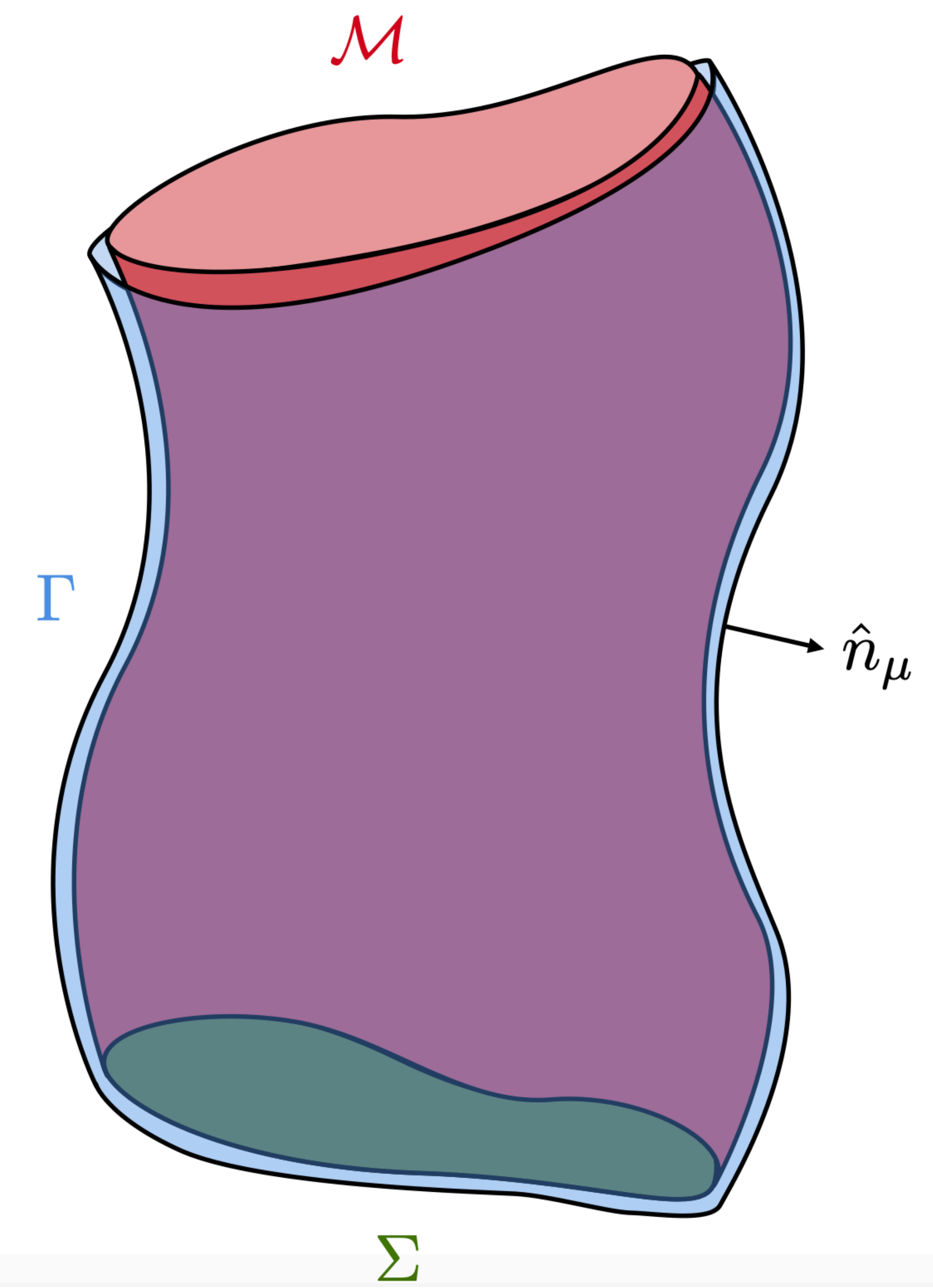
[Anninos-DAG-Maneerat '23, An-Anderson, to appear]

- These can be used for dS holography.

[Silverstein, Torroba '24]

CONFORMAL BOUNDARY CONDITIONS

CONJECTURE [AN, ANDERSON '21]:
CONFORMAL BOUNDARY CONDITIONS LEAD TO WELL-POSEDNESS.



Initial Conditions on Σ : $\left[g_{ij}, K_{ij}\right]$

Conformal Boundary Conditions on Γ : $\left[[g_{mn}], \mathrm{tr}\, K_{mn}\right]$

COMMENTS ON CONFORMAL BOUNDARY CONDITIONS

- First proposed by York for spacelike surfaces.
  [York, '87]
- Reappeared in the context of fluid-gravity.
  [Bredberg-Strominger, '11, Anninos-Anous-Bredberg-Ng, '11]
- Proven to be well-posed in Euclidean signature.
  [Anderson '08, Witten '18]
- Existence is hard to prove in Lorentzian signature.
  [An, Anderson, to appear]
- Uniqueness* can be shown at a linearised level.
  [Anninos-DAG-Mühlmann, '22]

A COMMENT ON THE ADS BOUNDARY

[Anninos-Arias-DAG-Maneerat '24]

- **The IBVP in AdS is famously well-posed.**

[Friederich, '95]

- **But what are the boundary conditions?**
- **The metric near the AdS boundary is given by**

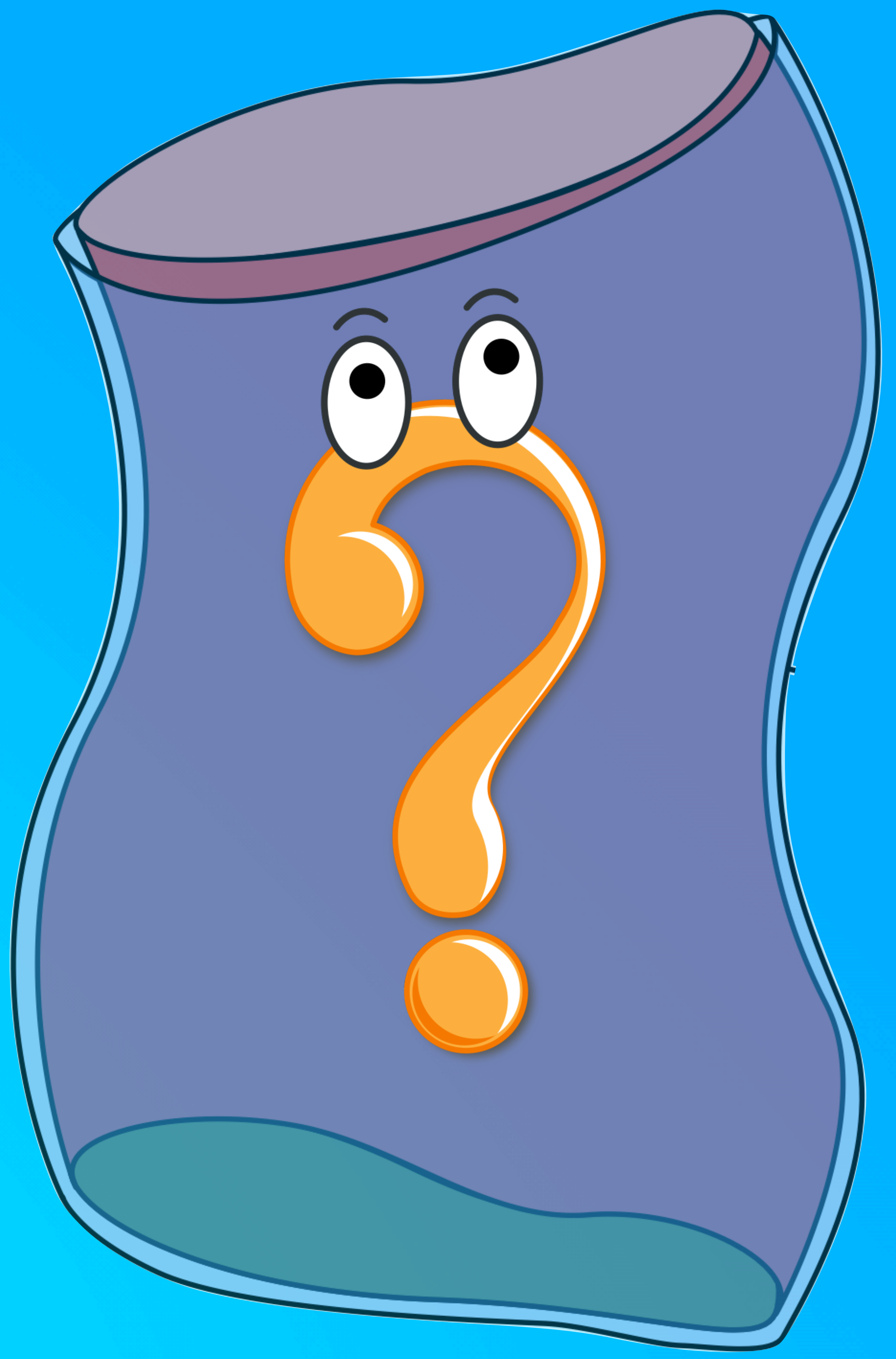
$$\frac{ds^2}{\ell^2} = d\rho^2 + \left(e^{2\rho} g^{(0)}_{mn}(x^m) + g^{(2)}_{mn}(x^m) + e^{-\rho} g^{(3)}_{mn}(x^m) + \ldots\right) dx^m dx^n$$

- **The trace of the extrinsic curvature at the boundary**

$$K_{mn} \approx e^{2\rho} g^{(0)}_{mn} \ell \to K = \frac{d-1}{\ell}$$

[Fournodavlos-Smulevici, '20,'21]

SUMMARY: IF WE WANT AN IBVP THAT IS WELL-POSED, BETTER USE CONFORMAL BOUNDARY CONDITIONS.



OUTLINE

- The initial boundary value problem in General Relativity
- Conformal boundary conditions
- Applications to the static patch of de Sitter space

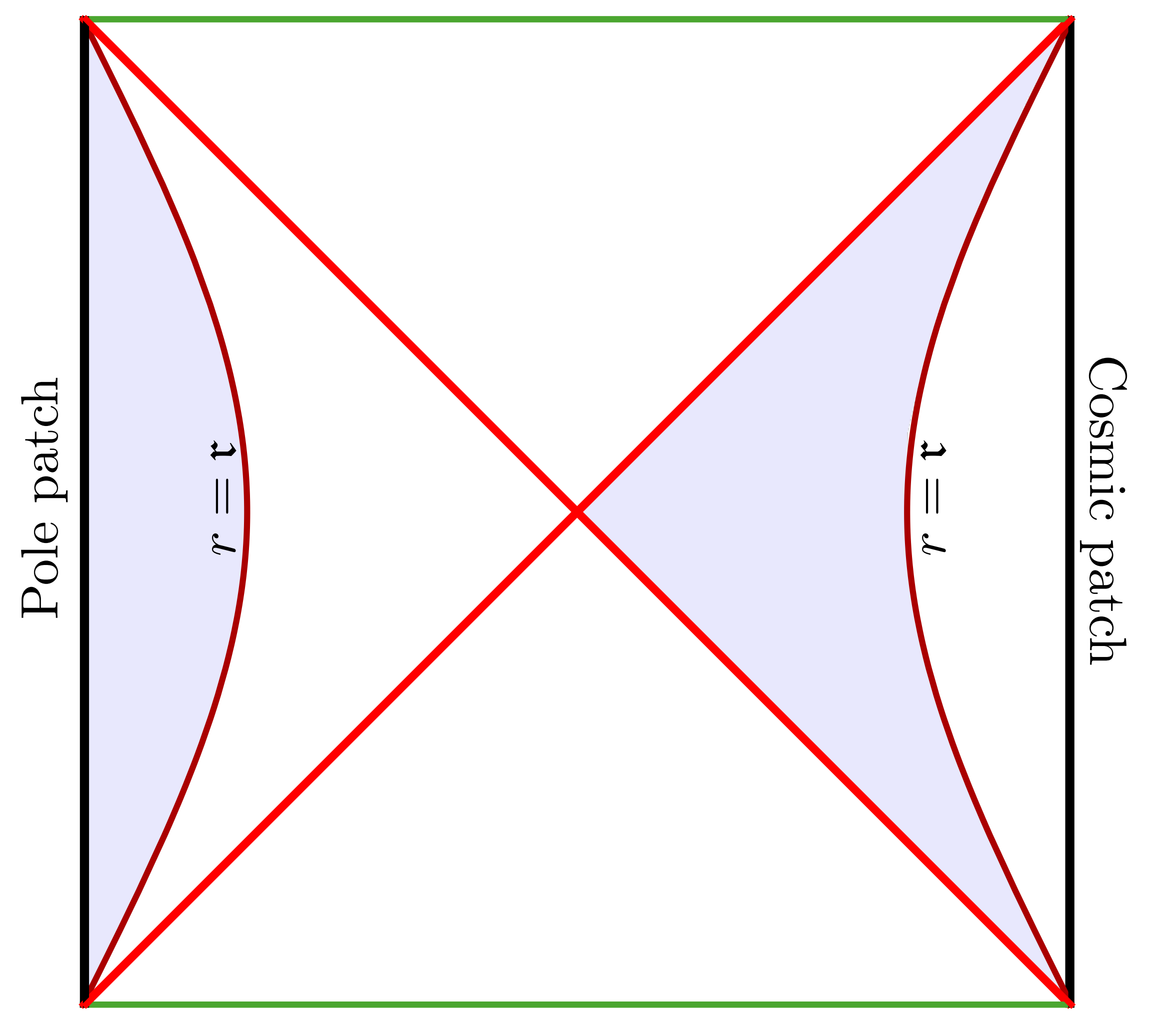
CONFORMAL BOUNDARY CONDITIONS IN DE SITTER

- One possibility to describe physics in the static patch is to consider a static patch with a finite timelike boundary.
- When the boundary is very small, it becomes an approximation for to the observer's worldline.

[Anninos, Hartnoll, Hofman, '12; …; Chandrasekaran, Longo, Penington, Witten, '21]

- When the boundary is very close to the horizon, we can connect to stretched horizon holography ideas.

[…, Susskind '21, Shaghoulian '21, …]



EUCLIDEAN CONFORMAL THERMODYNAMICS

- **We follow the same philosophy as in Gibbons-Hawking-York.**
- **In the standard Dirichlet problem, they define the canonical ensemble.**

$$\mathscr{Z}_{\text{can.}}(\beta, \mathfrak{r}) \equiv \sum_{g^*_{\mu\nu}} e^{-I_E[g^*_{\mu\nu}]}$$

- **With conformal boundary conditions, instead:**

$$\mathscr{Z}_{\text{can.}}(\beta, \mathfrak{r}) \to \mathscr{Z}(\tilde{\beta}, K)$$

EUCLIDEAN CONFORMAL THERMODYNAMICS

- **The gravitational action needs to be modified in order for the variational principle to be well-defined,**

$$I_E = -\frac{1}{16\pi G_N}\int_{\mathscr{M}} d^D x\sqrt{\det g_{\mu\nu}}\,(R-2\Lambda) - \frac{1}{(D-1)8\pi G_N}\int_{\Gamma} d^{D-1}x\sqrt{\det g_{mn}}\,K\,,$$

- **Note that this differs from the standard Gibbons-Hawking-York term.**
- **One can further generalise this to an arbitrary coefficient.**

[Liu, Santos, Wiseman, '24]

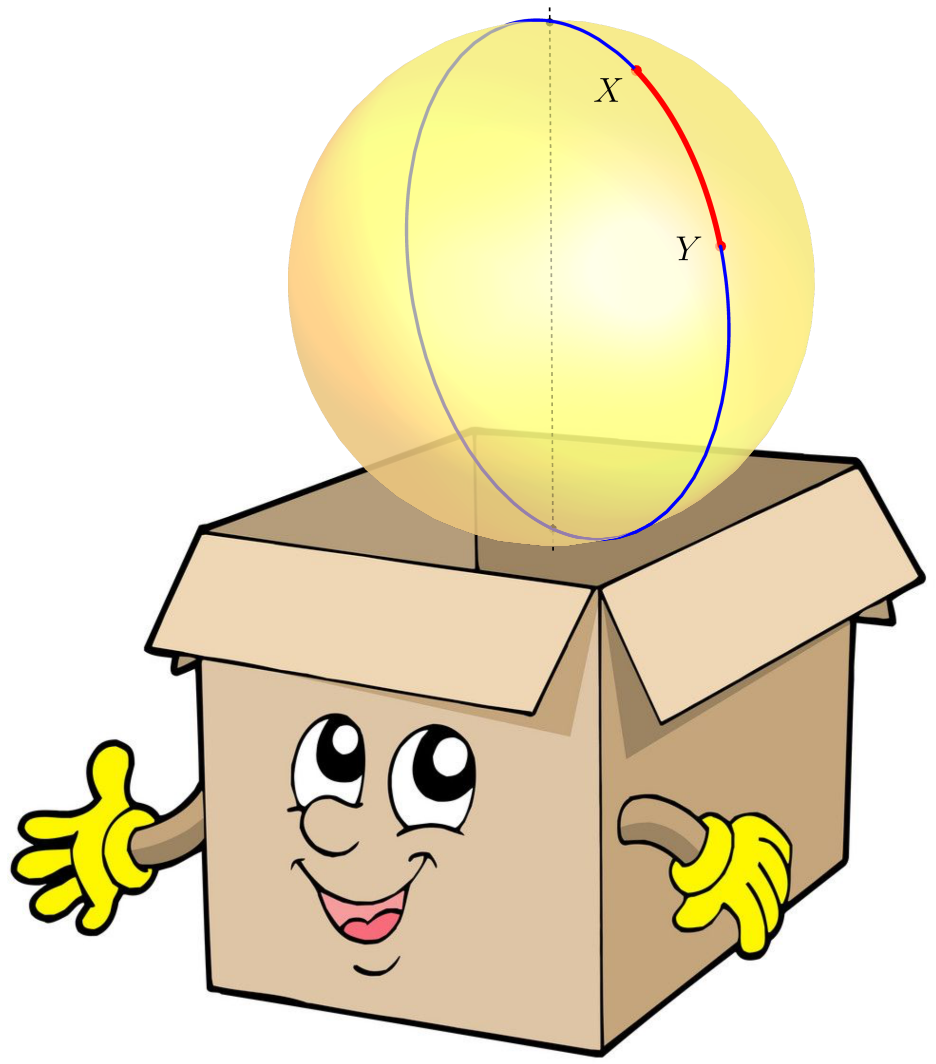
- **With this action we can study thermodynamic quantities:**

$$E_{\text{conf}} \equiv -\,\partial_{\tilde\beta}\Big|_K \log\mathscr{Z}\,,\qquad \mathscr{S}_{\text{conf}} \equiv \left(1-\tilde\beta\partial_{\tilde\beta}\right)\Big|_K \log\mathscr{Z}\,,\qquad C_K \equiv \tilde\beta^2\partial^2_{\tilde\beta}\Big|_K \log\mathscr{Z}\,.$$

EUCLIDEAN CONFORMAL THERMODYNAMICS

$$d = 3$$

DIRICHLET



$$C < 0!$$

CONFORMAL

$$C_K = \frac{2\pi^2 \mathfrak{c}_{\text{conf}}}{3\tilde{\beta}} > 0\,.$$

- **Positive specific heat at all temperatures!**
- **Conformal result in 2d!**

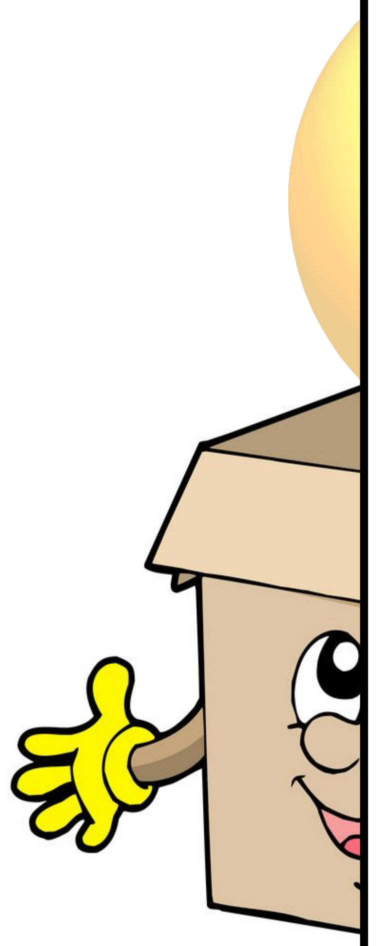
$$\mathfrak{c}_{\text{conf}} \equiv \frac{3\ell}{4G_N}\left(\sqrt{K^2\ell^2 + 4} - K\ell\right)$$

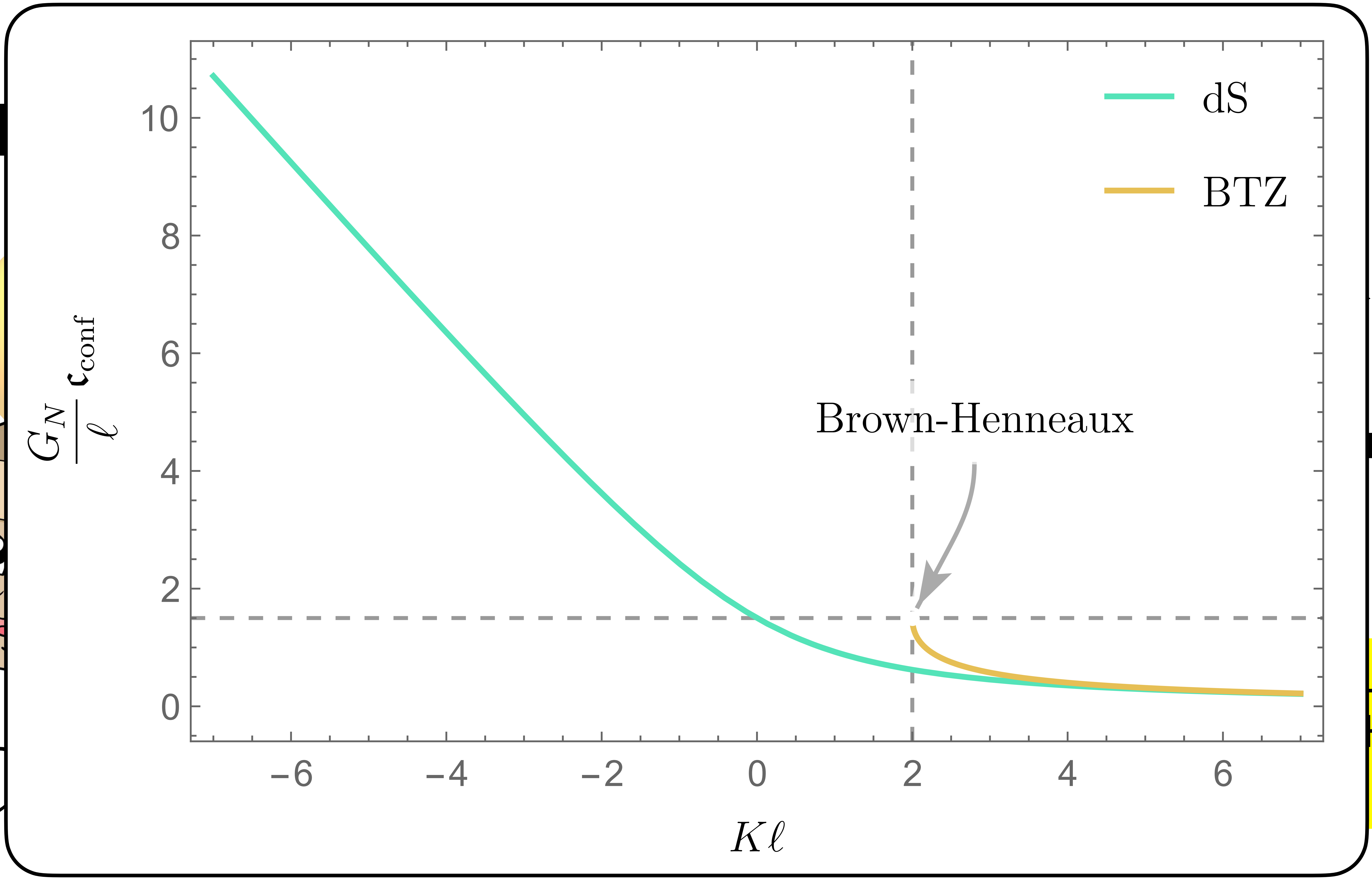
[Coleman-Mazenc-Shyam-Silverstein-Soni-Torroba-Yang, '21]

EUCLIDEAN CONFORMAL THERMODYNAMICS

$d = 3$

DIRICHLET





$0\,.$

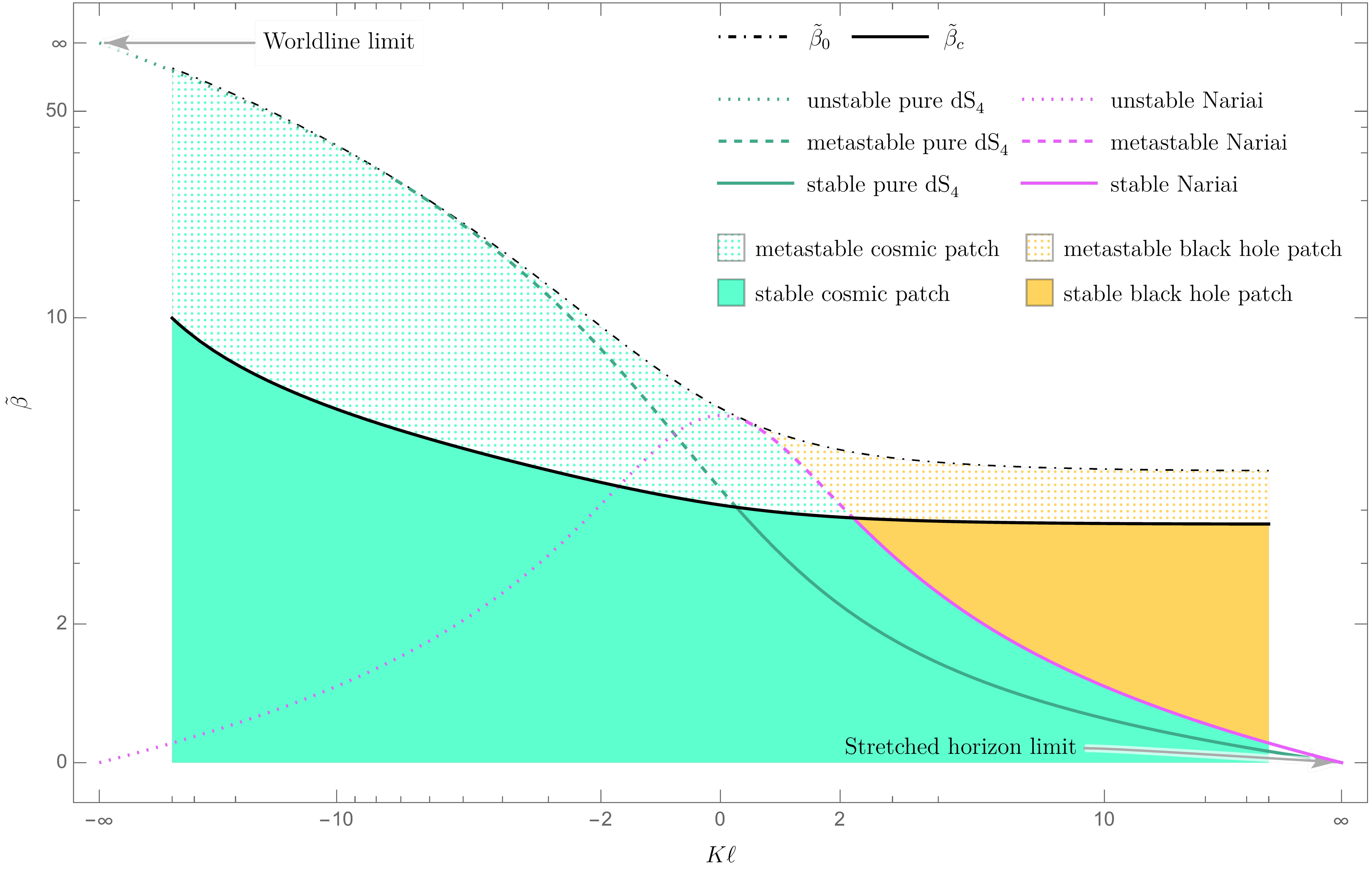
mperatures!

C … $4 - K\ell\big)$

[Coleman-Mazenc-Shyam-Silverstein-Soni-Torroba-Yang, '21]

EUCLIDEAN CONFORMAL THERMODYNAMICS

$d = 4$



EUCLIDEAN CONFORMAL THERMODYNAMICS

$d = 4$

- **In the large temperature limit, we get a very similar looking formula to the D=3,**

$$C_K \to \frac{32\pi^3\ell^2}{81\tilde{\beta}^2 G_N}\left(\sqrt{K^2\ell^2+9}-K\ell\right)^2 > 0\,, \qquad \text{as} \quad \tilde{\beta} \to 0$$

- **Moreover, in the large-K limit, this becomes**

$$C_K = \frac{8\pi^3}{G_N K^2 \tilde{\beta}^2}$$

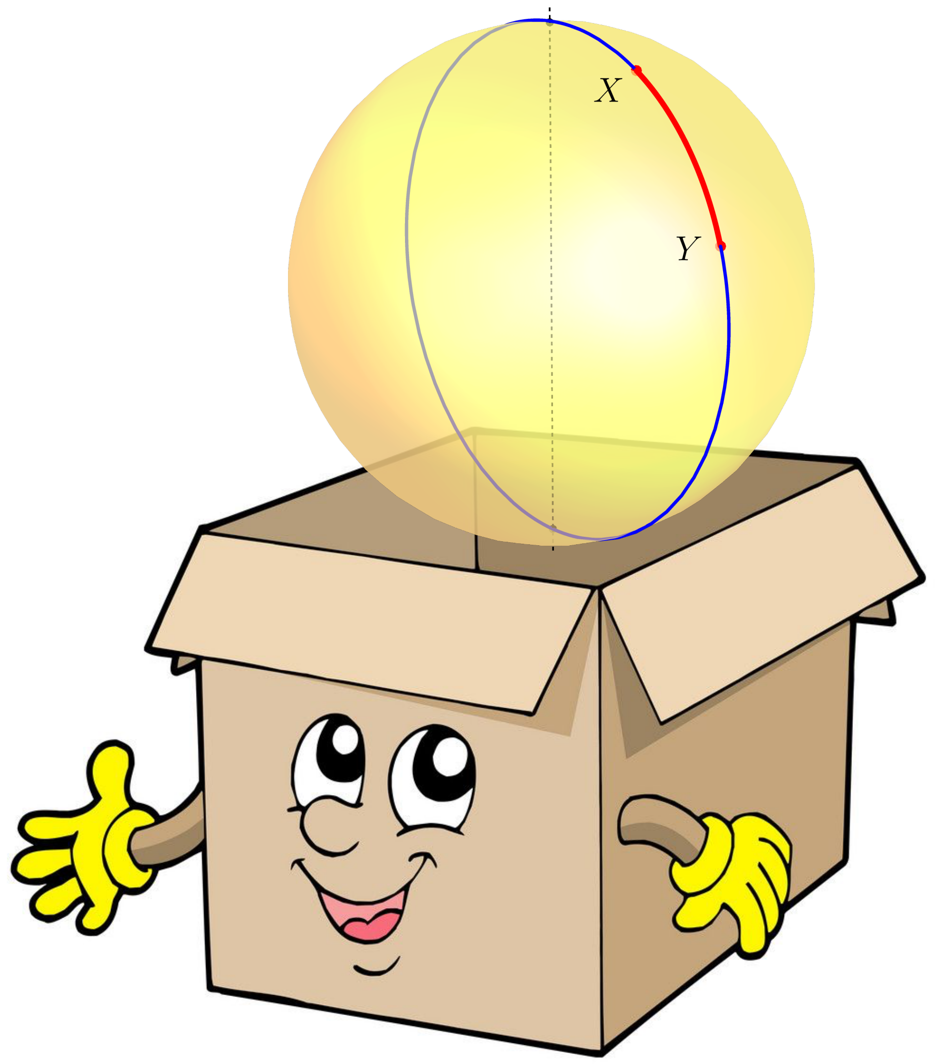
- **This is the specific heat of a black hole in asymptotically Minkowski space with conformal boundary conditions!**

$$N_{d.o.f.} = \frac{8\pi^3}{G_N K^2}$$

EUCLIDEAN CONFORMAL THERMODYNAMICS

$$d = 4$$

DIRICHLET



$$C < 0!$$

CONFORMAL

- At large temperatures and close to the cosmological horizon,

$$C_K = \frac{N_{\text{d.o.f.}}}{\tilde{\beta}^2} > 0\,.$$

- Conformal result in 3d!

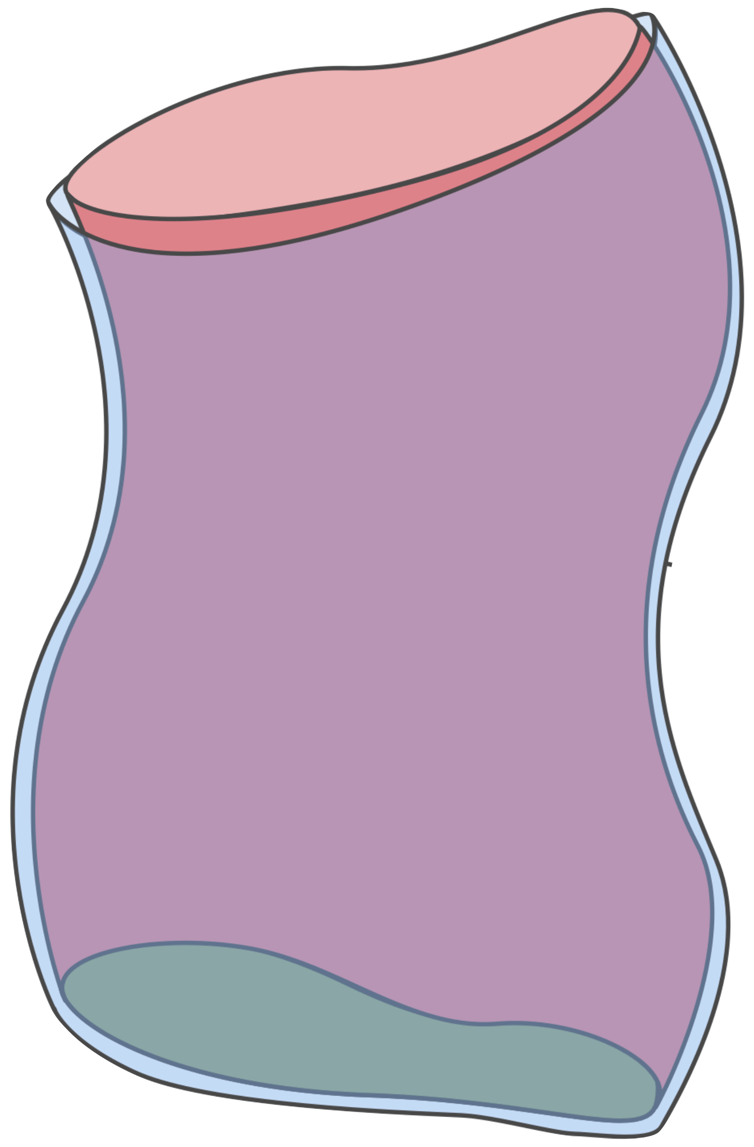
$$N_{\text{d.o.f.}} = \frac{32\pi^3\ell^2}{81 G_N}\left(\sqrt{K^2\ell^2+9} - K\ell\right)^2$$

SUMMARY

- New (quasi-local) way of studying gravity in finite regions of spacetimes.

- Independent of the sign of the cosmological constant.

- Suggestive thermodynamic (Euclidean) behaviour including cosmic patches with positive specific heat.



OUTLOOK

To do list:

- Wobbly observatories.
- Two-dimensional observatories.
- Adding matter, rotation, charge, etc.
- The "boundary dual" to K?
- Relation to flat space holography.
- Relation to AdS/CFT at finite cutoff.
- The fate of the growing modes.
- The role of spherical symmetry.
- Patching and gluing?
- Off-shell Euclidean stability.
- …

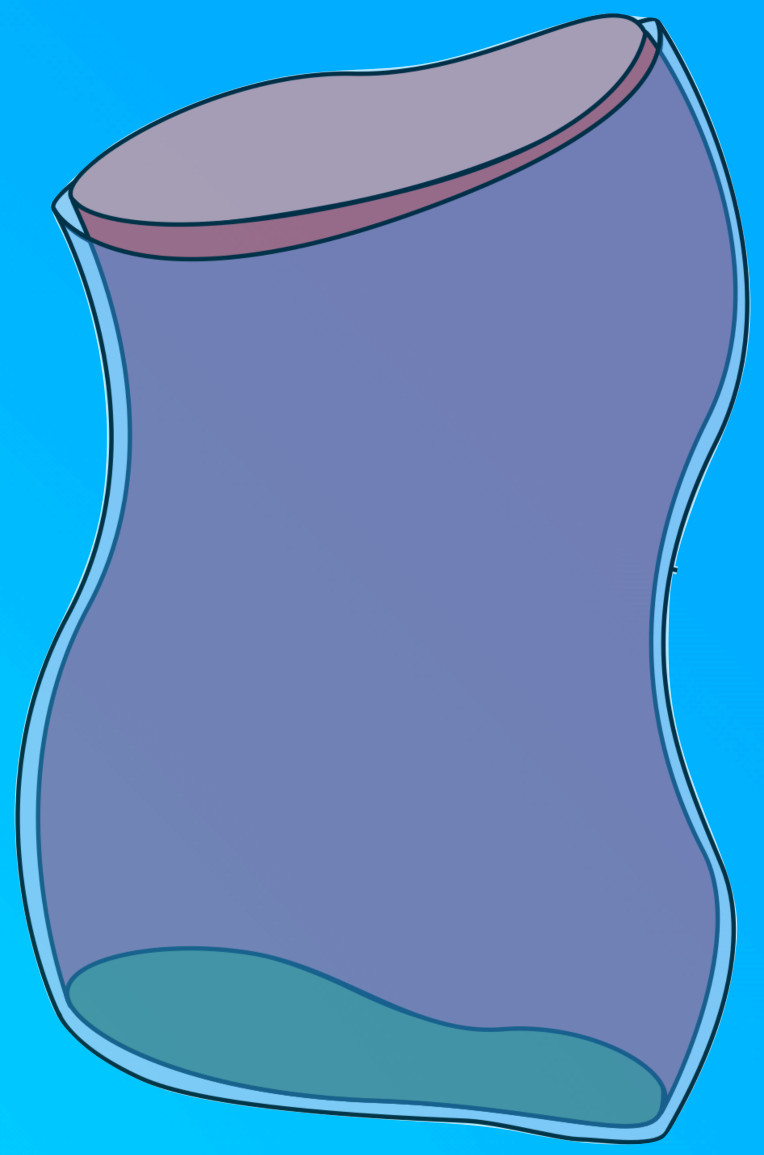
QUANTUM GRAVITY AND INFORMATION IN EXPANDING UNIVERSE

Is there a microscopic dual of these gravitational observatories?

Do they exist in Quantum Gravity?

What is the quantum information content of gravitational observatories?

TIMELIKE BOUNDARIES IN AN EXPANDING UNIVERSE



THANK YOU!