Recent Advances in Field Theory for Nuclear Theory

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From Last lecture by former YITP director

He is interested in

1. 格子QCD(HALQCD)
2. (主にハミルトン形式を用いた)量子論
   Quantum theory of Hamilton formalism
3. AdS/CFT
4. 一般相対性理論
   General relativity

One of recent developments
in nuclear physics

= Generalized global symmetries

Hamiltonian Lattice gauge theory

from the viewpoint of
Generalized global symmetries

Summary of this talk

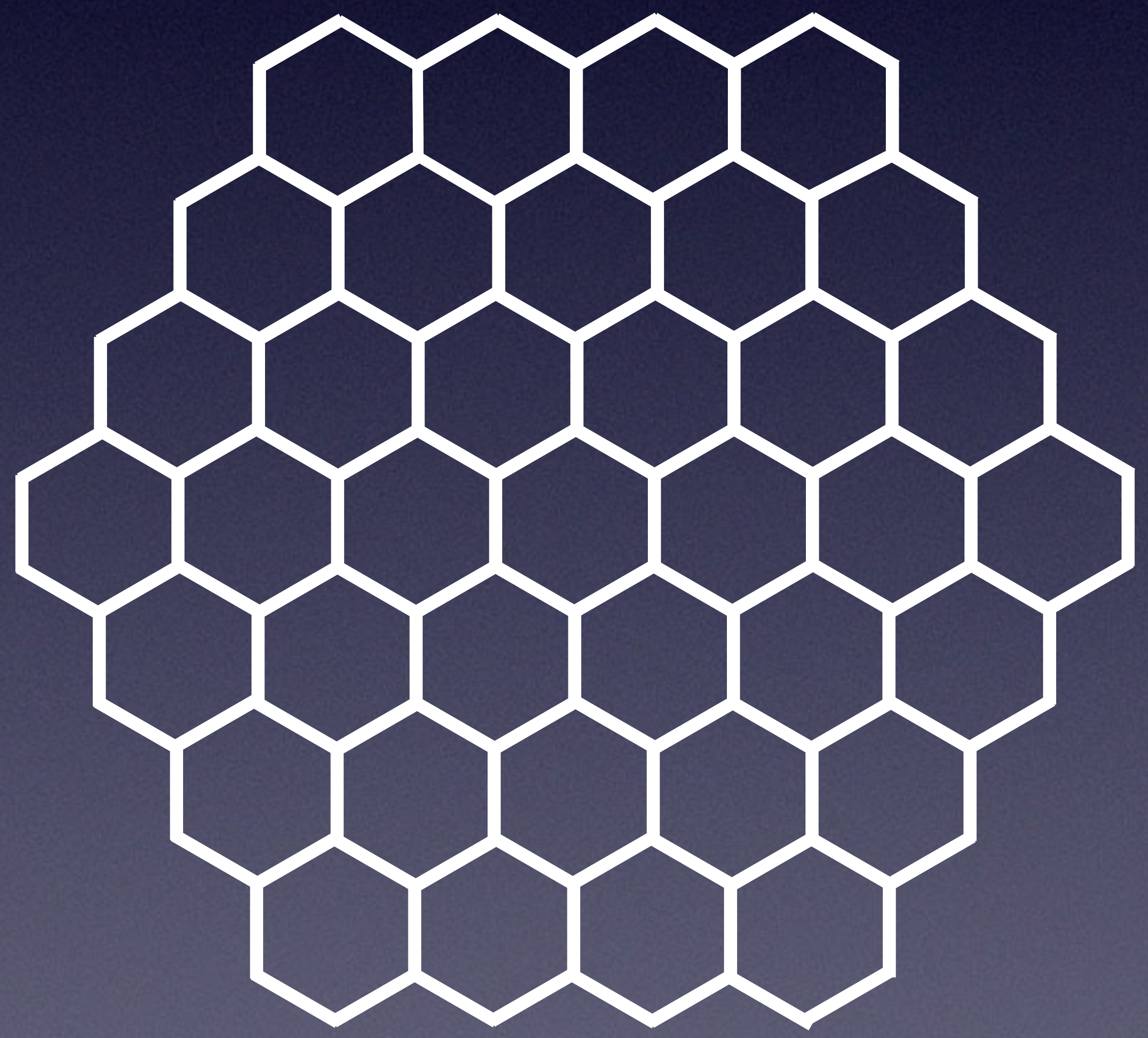
Hamiltonian Lattice gauge theory

$$\rotatebox{90}{$\simeq$}$$

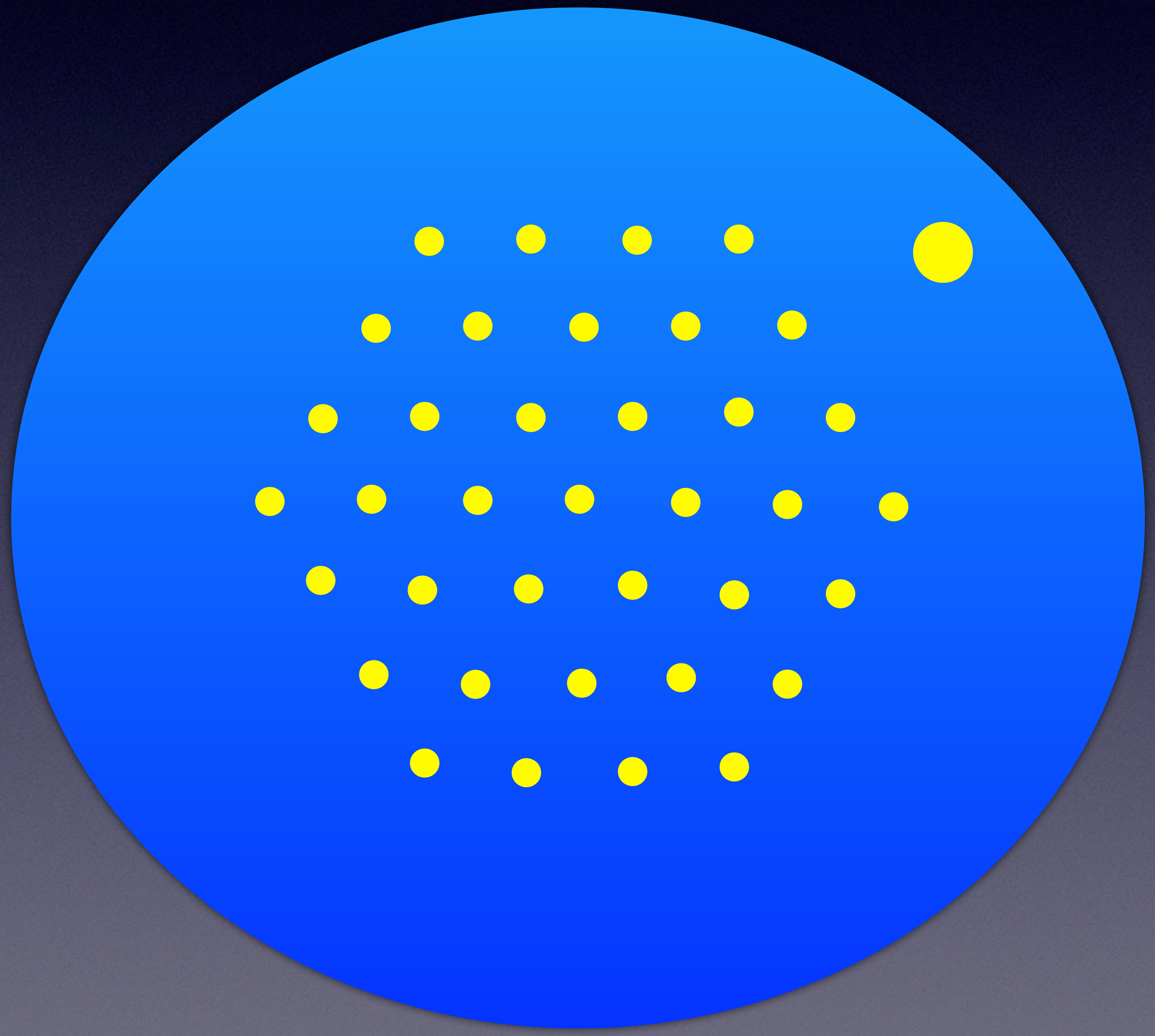
Topological quantum field theory with defects

Hilbert space of physical lattice gauge theory

Hilbert space of topological quantum field theory with defects



$$\simeq$$



n plaquettes lattice system

$(n+1)$ defects on S^2

Advantage:

- Topological calculus can be used
- Dual theory can be taken
  - It will help us to understand confinement mechanism
- Maybe useful for Fault Tolerant quantum computing of gauge theory
  - surface code = $\mathbb{Z}_2$ gauge theory with defects

Outline

- Formalism
  - Kogut-Susskind Hamiltonian formalism
- TQFT with defects as gauge theory
- Summary

Kogut-Susskind Hamiltonian formalism

Kogut, Susskind ('75)

$SU(N)$ gauge theory (Temporal gauge $A_0 = 0$)

Commutation relation $[A_n^i(x), E_{mj}(x')] = i\delta_{nm}\delta_j^i\delta(x - x')$

Gauge field Electric field

Hamiltonian $H = \int d^3x \left(\frac{g^2}{2} E^2(x) + \frac{1}{2g^2} B^2(x) \right)$

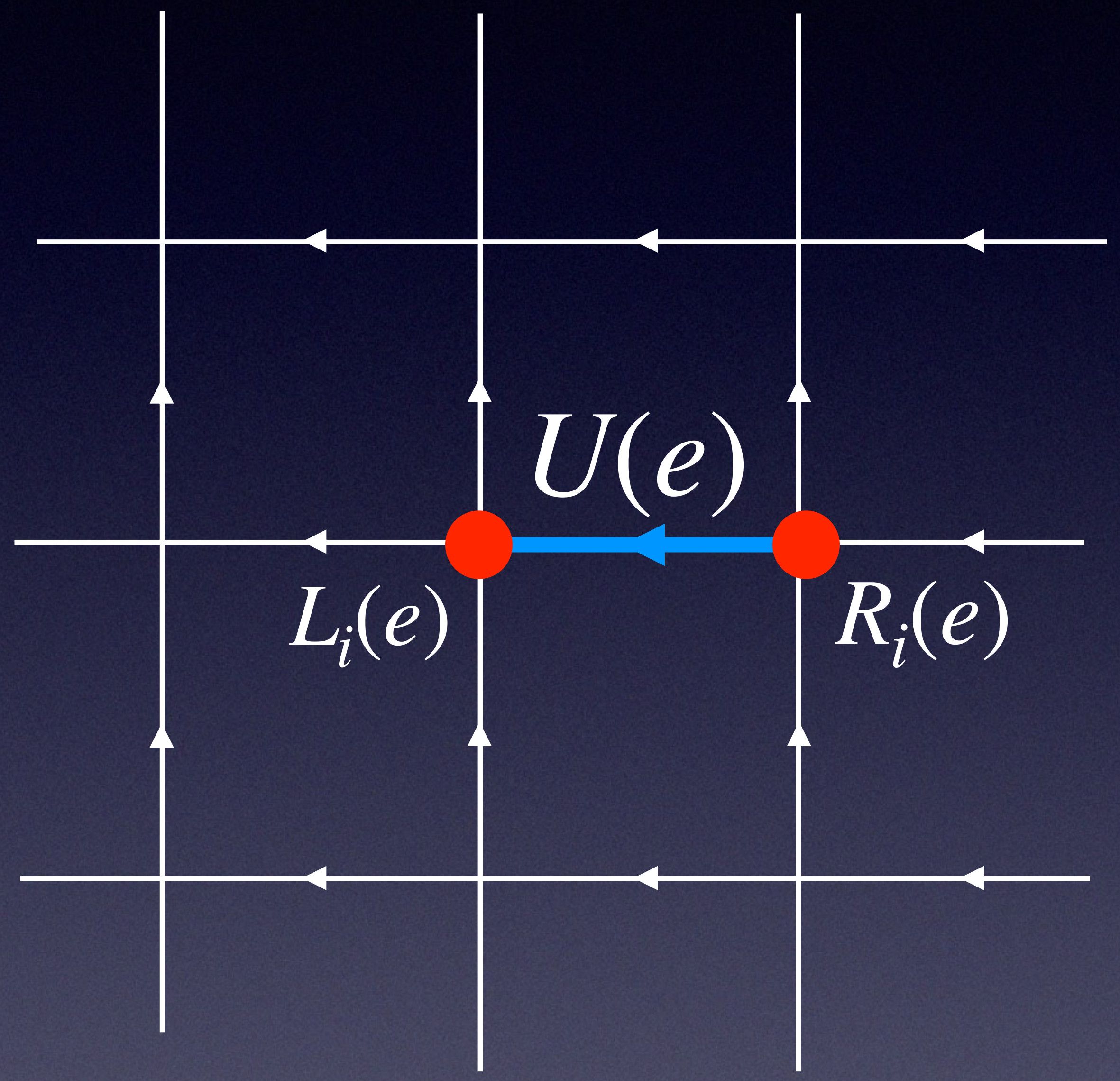
Magnetic field $B_l^i = \frac{1}{2}\epsilon_{lnm}(\partial_m A_n^i - \partial_m A_n^i + f_{jk}^i A_m^j A_n^k)$

Gauss law constraint $(\boldsymbol{D} \cdot \boldsymbol{E})^i \,|\, \Psi_{\text{phys}}\rangle = 0$

Kogut-Susskind Hamiltonian formalism

Kogut, Susskind, Phys. Rev. D 11, 395 (1975)

Time is continuous, space is discretized



$e^{i\int A} \rightarrow U(e)$:Link variable $\in \mathrm{SU}(N)$ on edge e

$L_i(e), R_i(e)$:Left and right electric fields $\in \mathrm{su}(N)$

$L_i(e)$ and $R_i(e)$ are not independent

$$[U_{\mathrm{adj}}(e)]_i^j L_j(e) = R_i(e) \quad \Rightarrow \quad R_i^2(e) = L_i^2(e) =: E_i^2(e)$$

Commutation relation

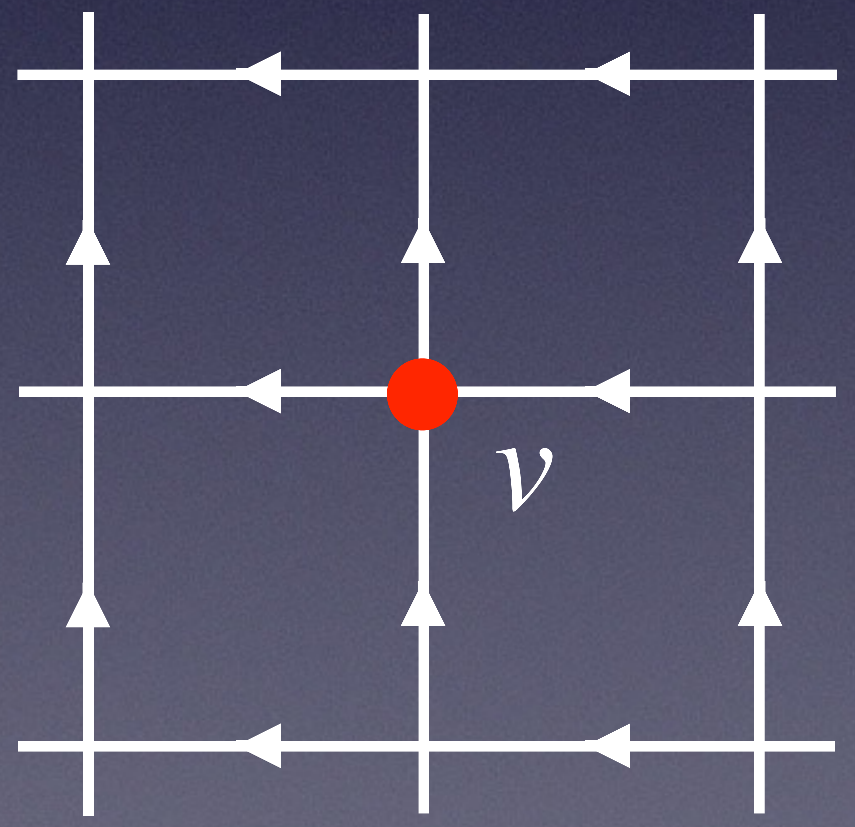
$$[A_n^i(x), E_{mj}(x')] = i\delta_{nm}\delta_j^i\delta(x - x')$$

$$[R_i(e), U(e')] = U(e)T_i\delta_{e,e'}$$

$$[L_i(e), U(e')] = T_iU(e)\delta_{e,e'}$$

$$[L_i(e), L_j(e')] = -\mathrm{i}f^k{}_{ij}L_k(e)\delta_{e,e'}$$

$$[R_i(e), R_j(e')] = \mathrm{i}f^k{}_{ij}R_k(e)\delta_{e,e'}$$



Gauss law constraint $(D \cdot E)_i |\Psi_{\mathrm{phys}}\rangle = 0$

$$G_i(v)|\Psi_{\mathrm{phys}}\rangle = 0 \quad G_i(v) = \left(\sum_{e\in E|\mathrm{s}(e)=v} R_i(e) - \sum_{e\in E|\mathrm{t}(e)=v} L_i(e)\right)$$

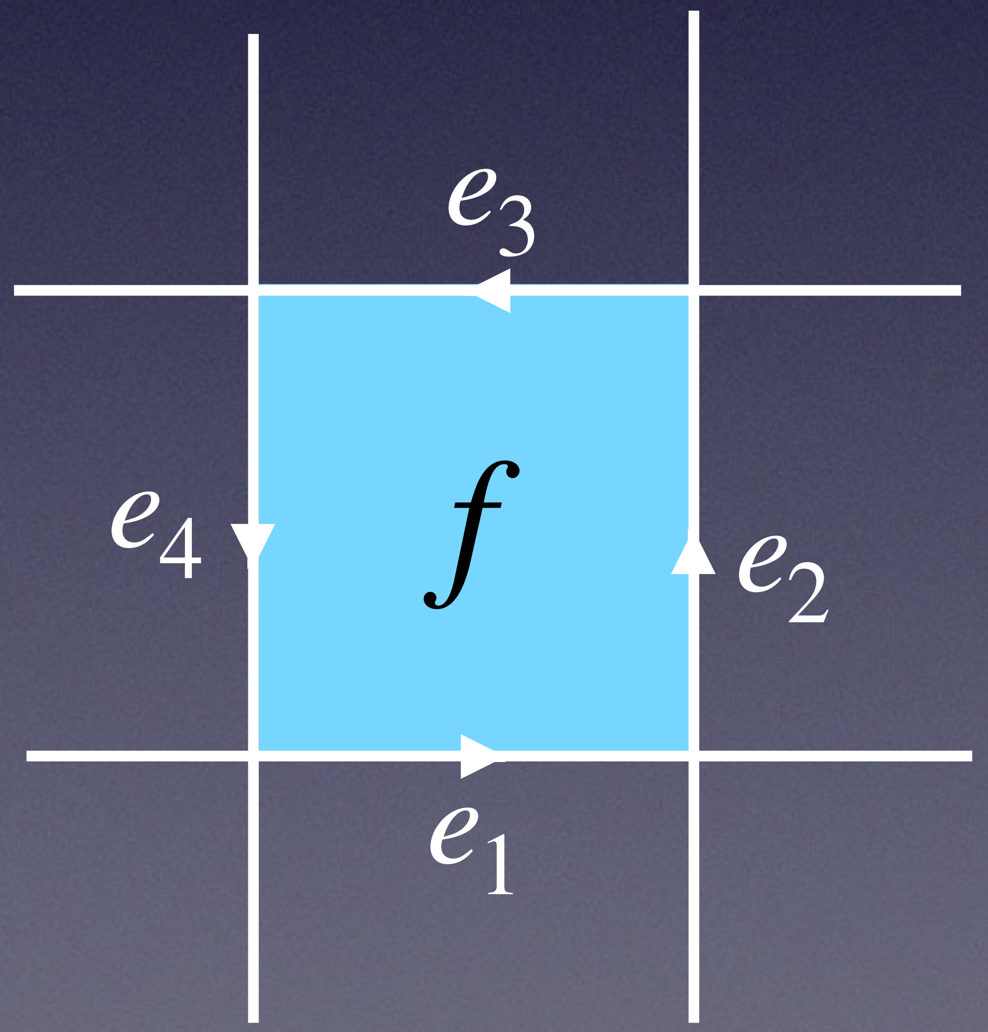
E:set of edges, s,t: source and target functions

Hamiltonian

$$H = J \sum_{e \in E} (E(e))^2 - \frac{K}{2} \sum_{f \in F} (\mathrm{tr}U(f) + \mathrm{tr}U^\dagger(f))$$

$e \in E$:set of edges

$f \in F$:set of faces



$$U(f) := U(e_4)U(e_3)U(e_2)U(e_1)$$

Two types of bases:

G basis: Eigenstates of Wilson lines

$$[U_a]^m_n \,|\, g\rangle = [\rho_a]^m_n(g) \,|\, g\rangle \qquad g \in \mathrm{SU}(2)$$

Wilson line with Rep a

representation matrix

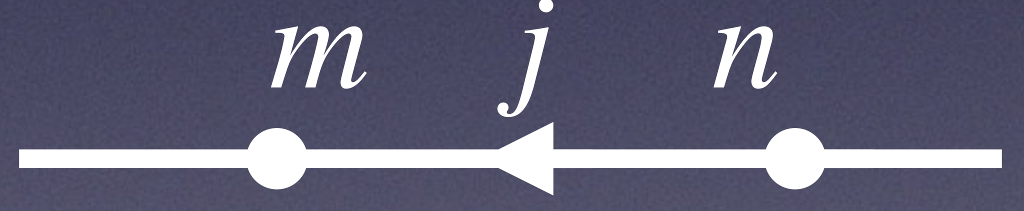
Rep basis: Eigenstates of Electric fields (for $SU(2)$)

$$R_i^2(e) \,|\, j, m, n\rangle = C_2(j) \,|\, j, m, n\rangle$$

Casimir $C_2(j) = j(j+1)$

$$R_3(e) \,|\, j, m, n\rangle = n \,|\, j, m, n\rangle$$

$$L_3(e) \,|\, j, m, n\rangle = m \,|\, j, m, n\rangle$$



$|\, j, m, n\rangle$

A state can be generated by Wilson line

$$\sqrt{d_a}[U_a]^{m_a}_{n_a}|0,0,0\rangle = |j_a, m_a, n_a\rangle$$

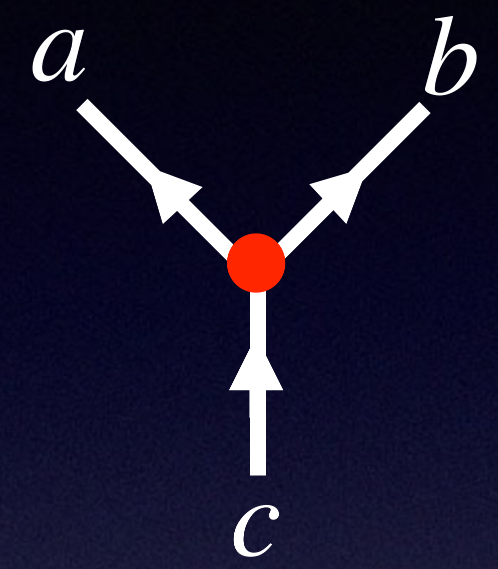
$d_a = 2j_a + 1$ **Quantum dimension**

Graphical representation: $\uparrow a \Leftrightarrow \sqrt{d_a}[U_a]^{m_a}_{n_a} \Leftrightarrow |j_a, m_a, n_a\rangle$

operator **state**

Physical state: Spin network

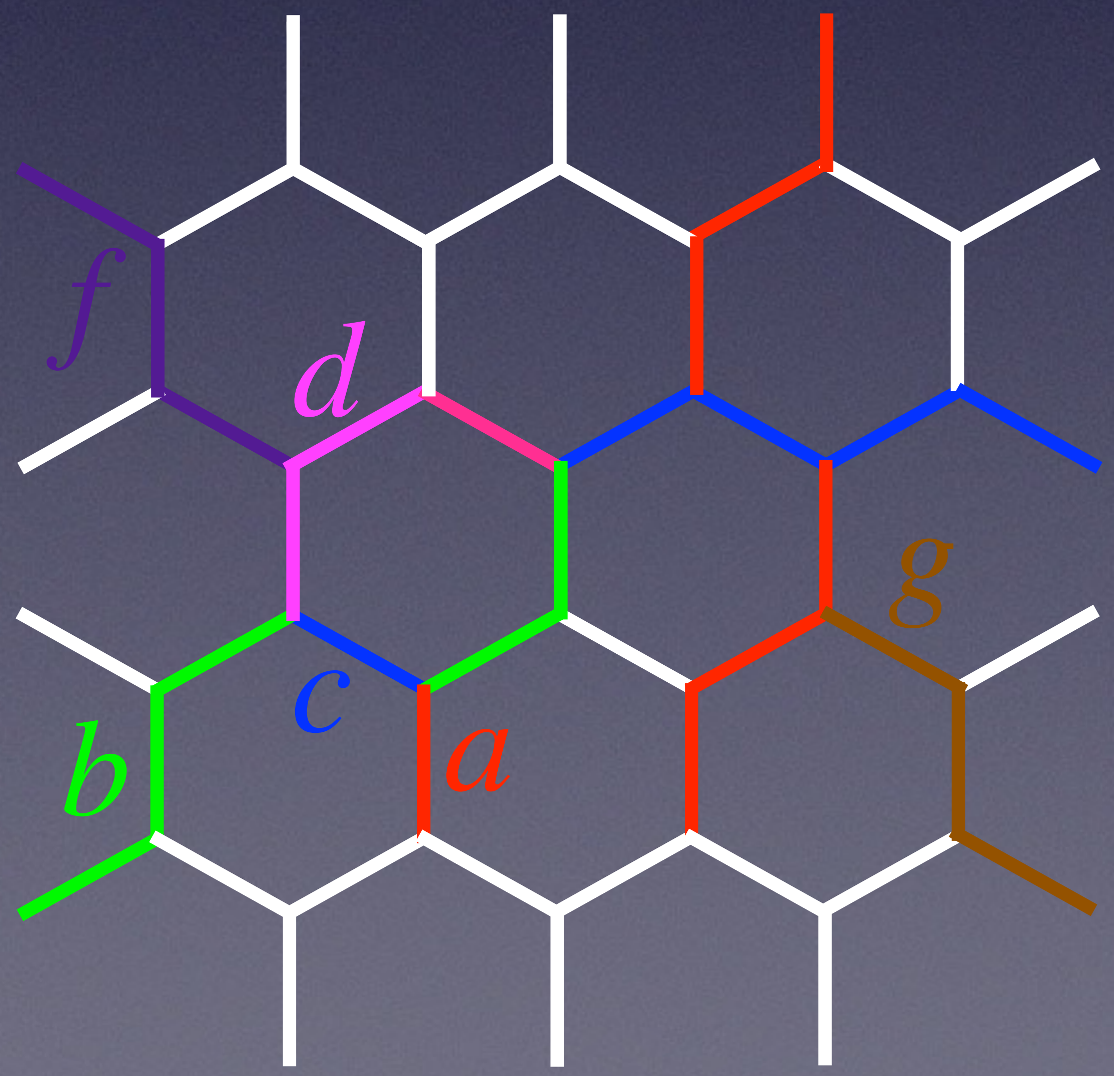
Gauss law constraint: $SU(2)$ invariant of each vertex



$$(L_i(c) - R_i(a) - R_i(b))\,|\Psi_{\mathrm{phys}}\rangle = 0$$

$$\Rightarrow \sum_{n_a,n_b,m_c} \frac{1}{\sqrt{d_c}} \langle j_a n_a\, j_b n_b \,|\, j_c m_c\rangle \,|j_a, m_a, n_a\rangle\, |j_b, m_b, n_b\rangle\, |j_c, m_c, n_c\rangle$$

Clebsch–Gordan coefficients



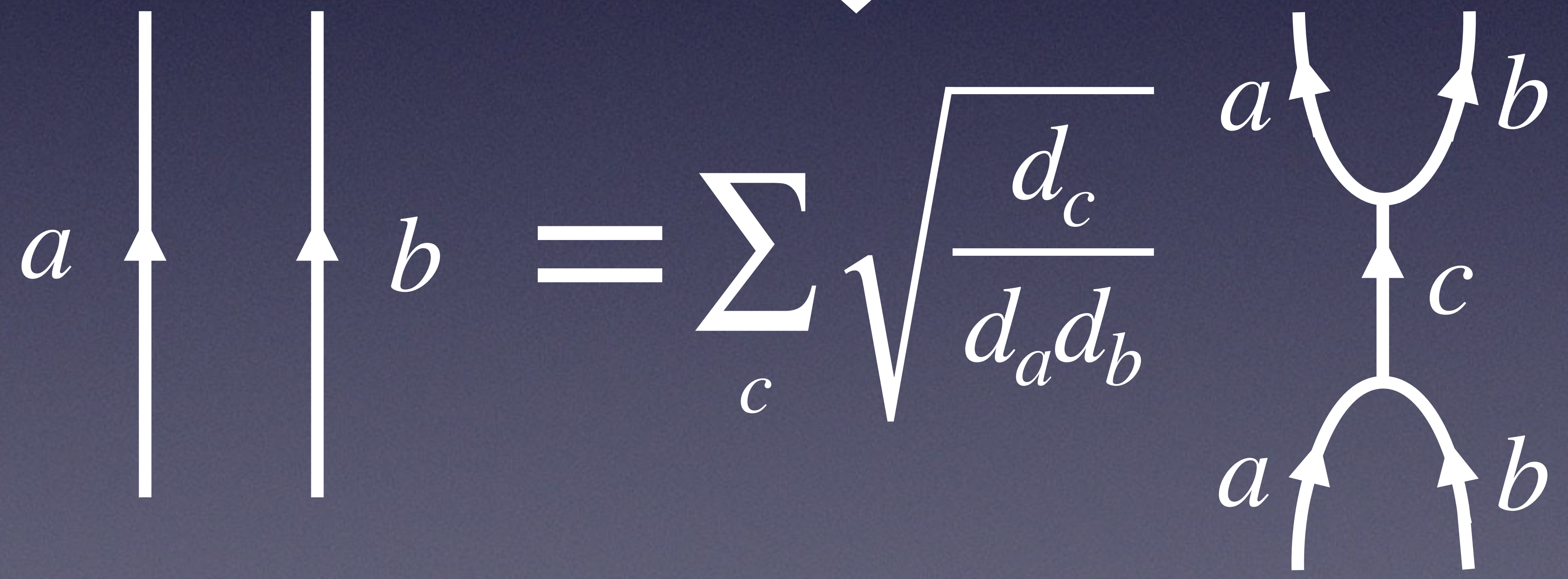
Physical state: spin network with three vertices

Angular momentum labels on the edges

At each vertex the labels satisfy the triangle inequality

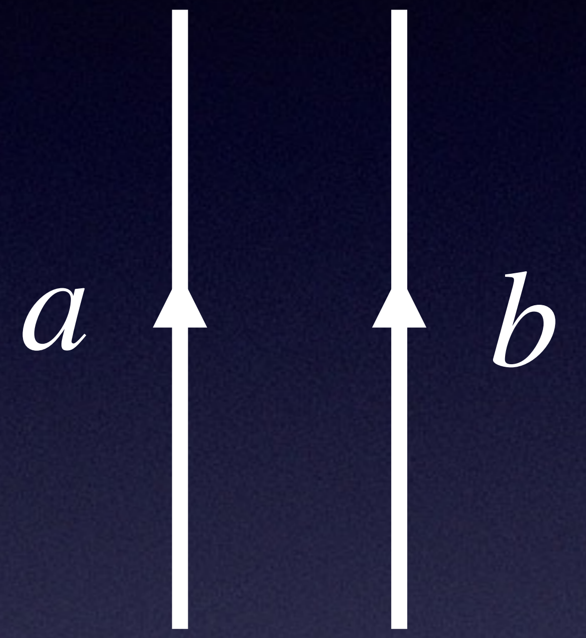
If the composition rules of the Wilson line are known, the action of an operator on a state is determined:

$$[U_a]^{m_a}_{n_a}[U_b]^{m_b}_{n_b} = \sum_{j_c,m_c,n_c} \langle j_a m_a\, j_b m_b \,|\, j_c, m_c\rangle\langle j_c, n_c \,|\, j_a n_a\, j_b n_b\rangle [U_c]^{m_c}_{n_c}$$

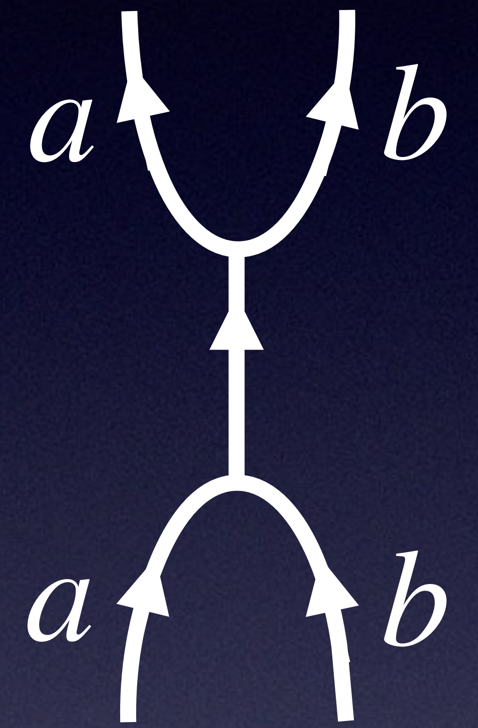


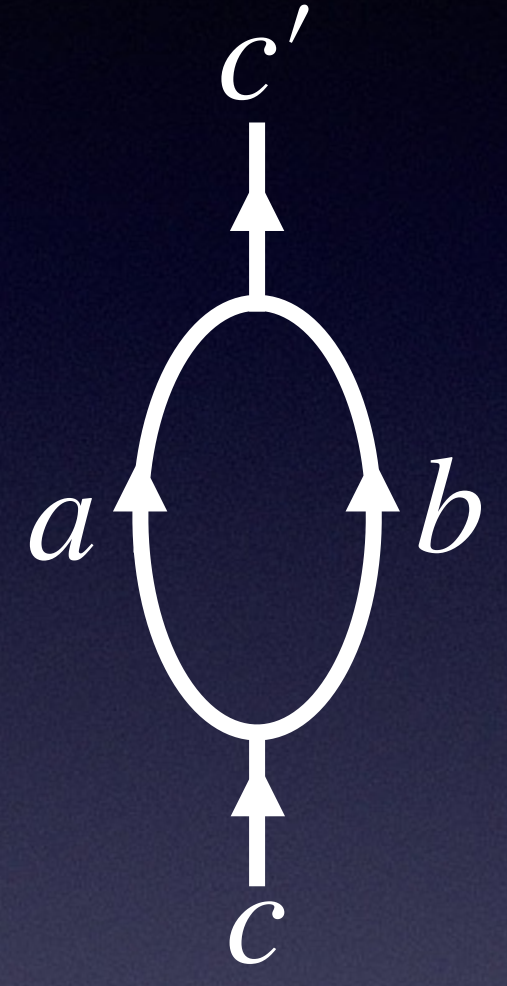
Algebra of Networks

Fusion rule $a \times b = \sum_c N_{ab}^c c$ $\quad N_{ab}^c = \delta_{abc} = \begin{cases} 1 & |j_a - j_b| \le j_c \le j_a + j_b, j_a + j_b + j_c \in \mathbb{Z} \\ 0 & \text{else} \end{cases}$

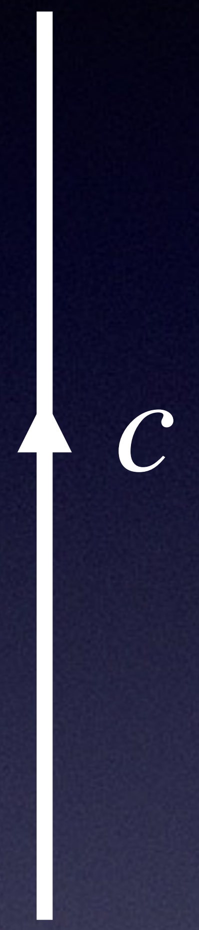


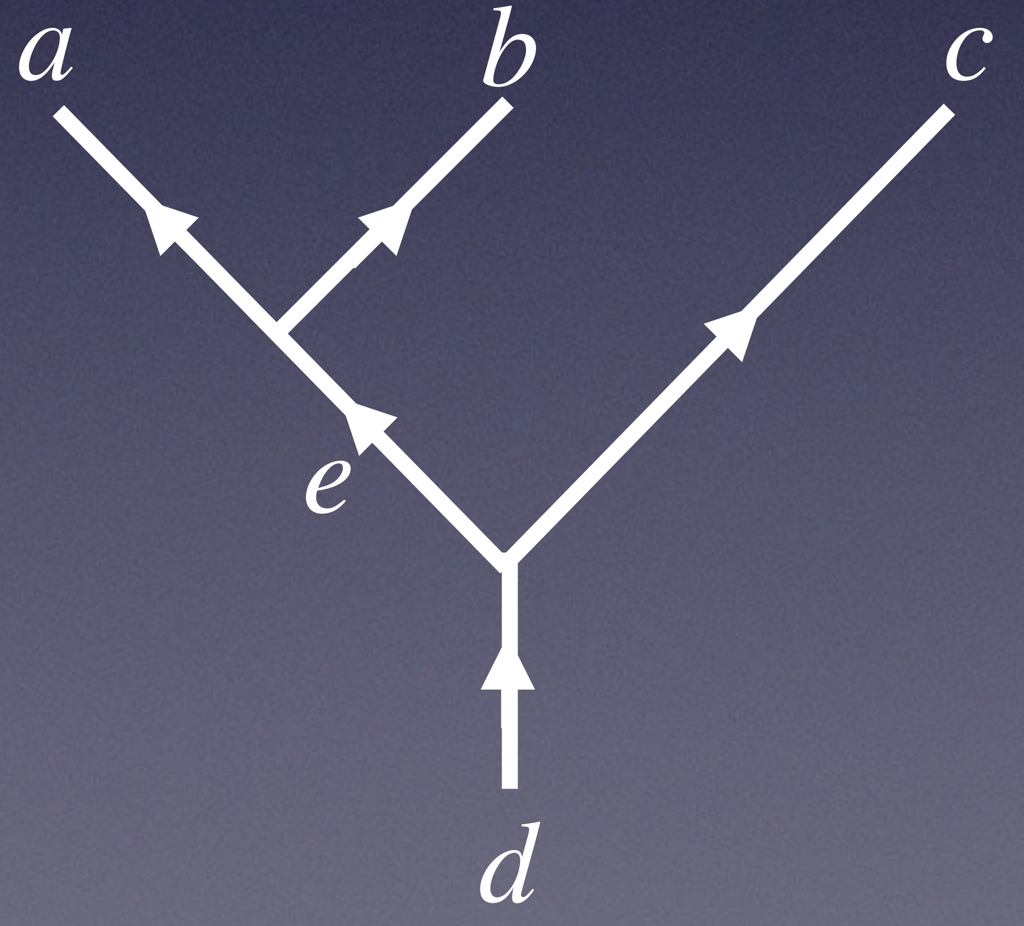
$$= \sum_c \sqrt{\frac{d_c}{d_a d_b}}$$



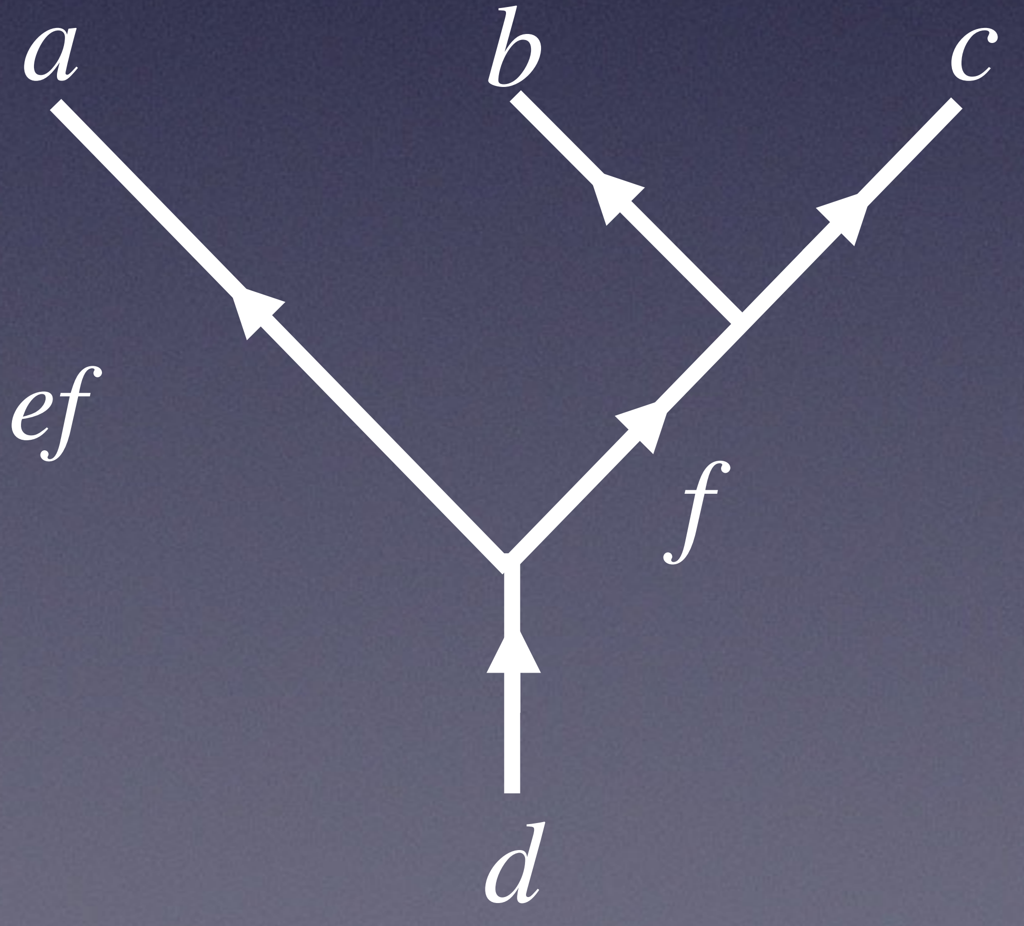


$$= \delta_c^{c'} \sqrt{\frac{d_a d_b}{d_c}}$$





$$= \sum_f [F_d^{abc}]_{ef}$$



Wigner 6-j symbol

$$[F_d^{abc}]_{ef} = (-1)^{j_a + j_b + j_c + j_d} \sqrt{d_e d_f} \begin{Bmatrix} j_a & j_b & j_e \\ j_c & j_d & j_f \end{Bmatrix}$$

+consistency condition

SU(2) Yang-Mills theory

Hamiltonian

$$H = J\sum_{e\in E}(E_i(e))^2 - K\sum_{f\in F}\mathrm{tr}U(f)$$

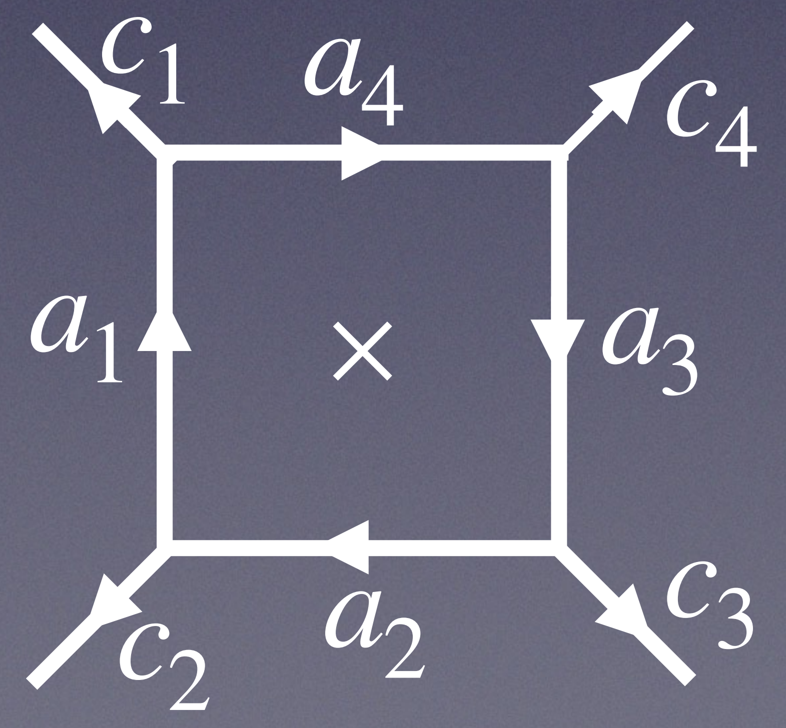
set of edges   set of faces

Action on a state

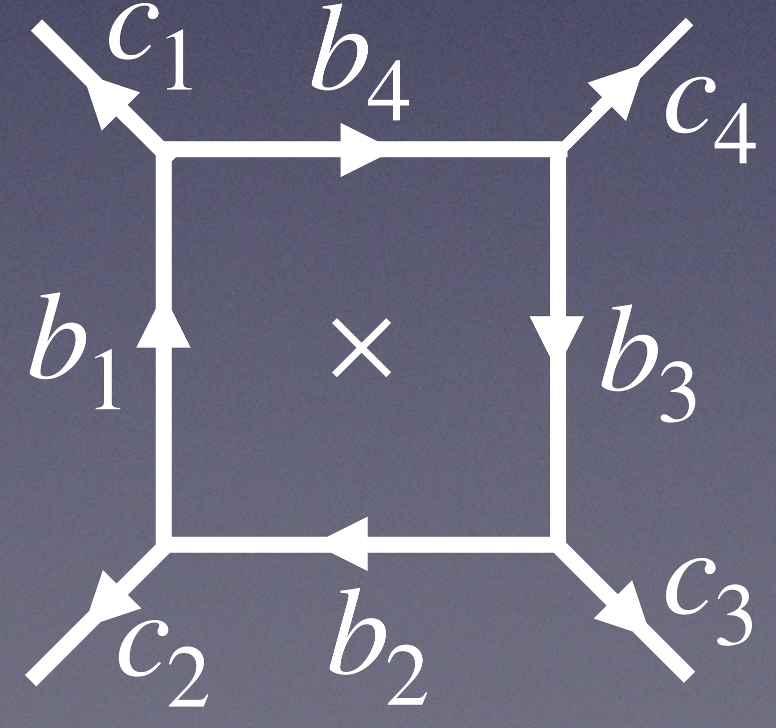
E_i^2

$= C_2(a)$

$\mathrm{tr}U$

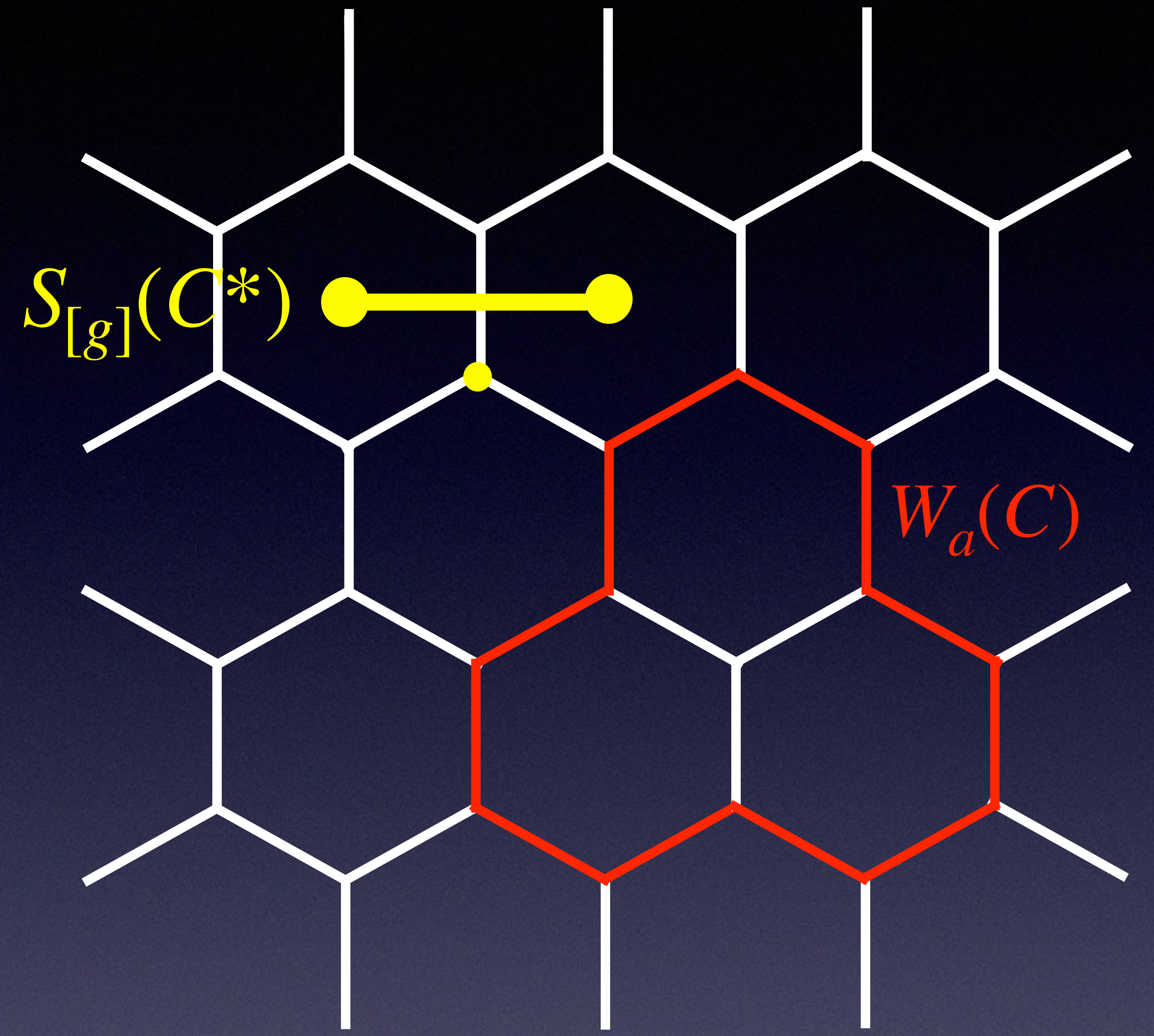


$$= \prod_{i=1}^{4}\sum_{b_i}[F^{c_i a_{i-1}\frac{1}{2}}_{b_i}]_{a_i b_{i-1}}$$



cf. Levin, Wen, Phys. Rev. B 71 (2005) 045110

Gauge invariant operators



Wilson loop: $W_a(C) = \mathrm{tr} \prod_{l \in C} U_a(l)$

a **: representation**

Gukov-Witten operator $S_{[g]}(C^*)$ **with boundary:**

e.g., shortest one is

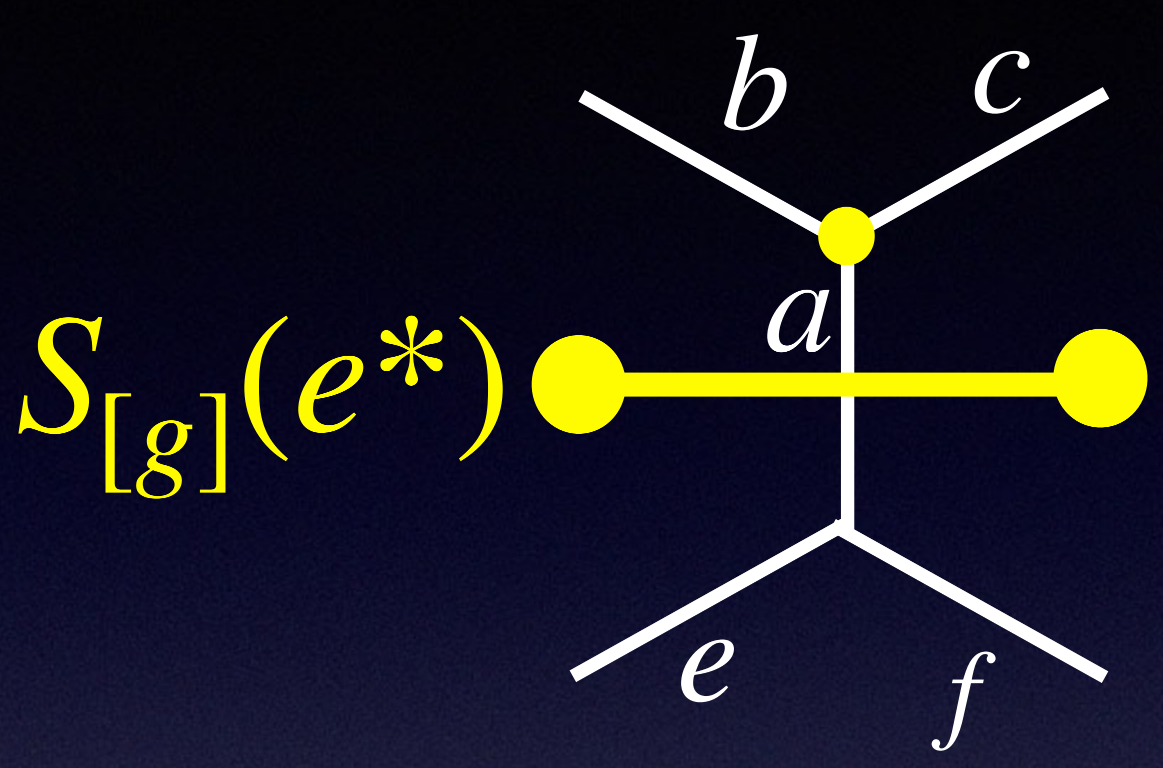
$$S_{[g]}(e^*) = \int_{e^{it_k\theta^k} \in \mathscr{C}_g} d\theta e^{iR_k(v)\theta^k}$$

$[g = e^{it_k\theta^k}]$ **: representative of conjugacy class** $\mathscr{C}_g = \{hgh^{-1} \,|\, h \in G\}$

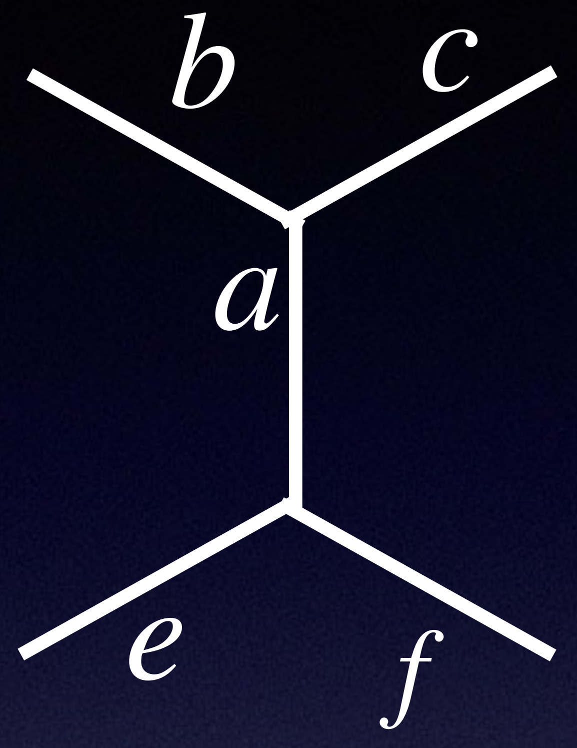
If $[g]$ is a center element, $S_{[g]}$ is 't Hooft operator

If $[g]$ is a center element and path is closed loop,
$S_{[g]}$ is symmetry operator for 1 form symmetry

Action of $S_{[g]}(e^*)$ on Hilbert space



$$= \frac{\chi_a(g)}{d_a}$$



$\chi_a(g)$ **:Character**

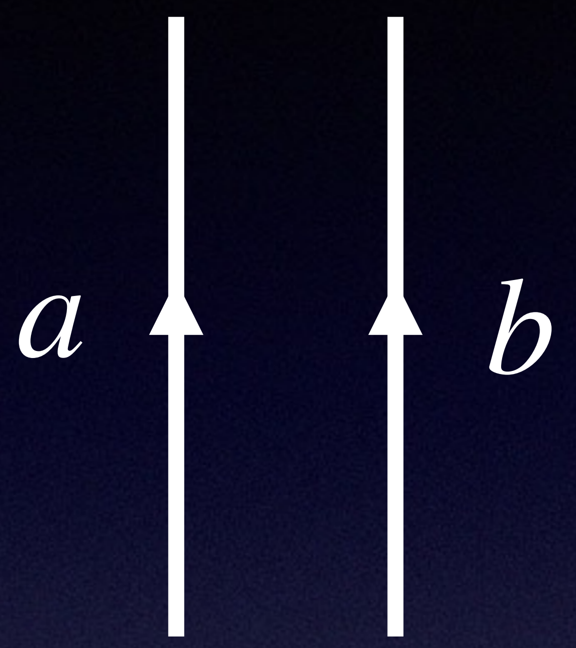
e.g., $U(1) : e^{in_a\theta_g}$ $\qquad SU(2) : \dfrac{\sin \frac{2j_a+1}{2}\theta}{\sin \frac{\theta}{2}}$

d_a **:dimension of representation**

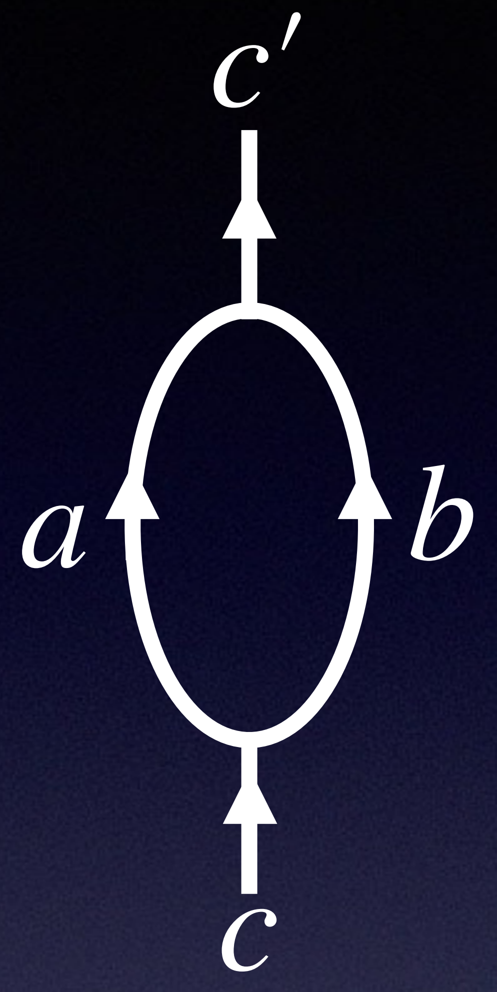
e.g., $d_a = 1 \quad d_a = 2j_a + 1$

TQFT formulation for Lattice gauge theory

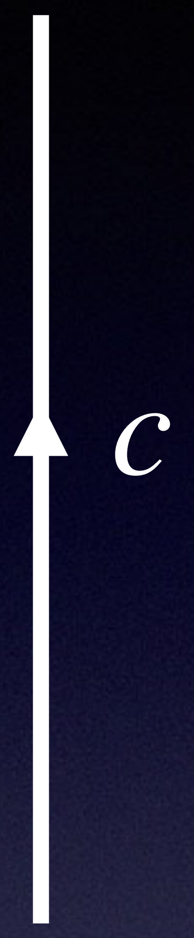
Topological algebra

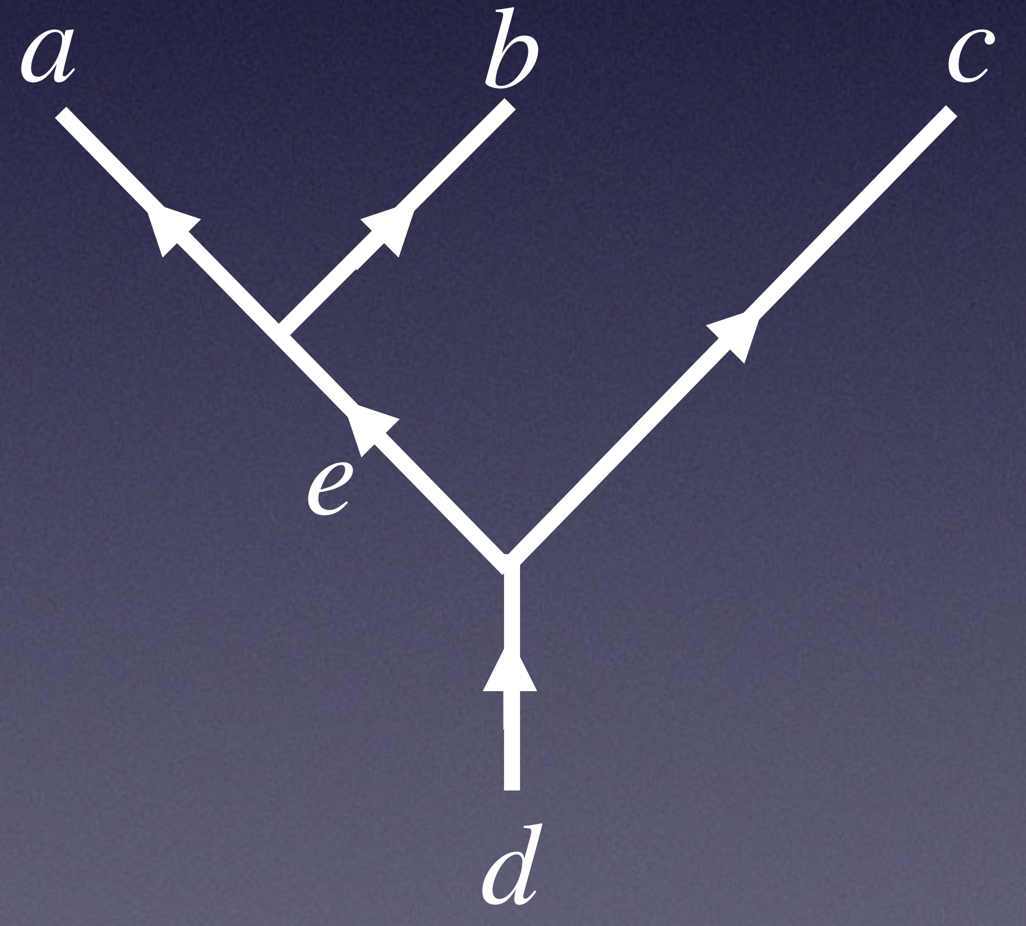


$$= \sum_c \sqrt{\frac{d_c}{d_a d_b}}$$



$$= \delta_c^{c'} \sqrt{\frac{d_a d_b}{d_c}}$$





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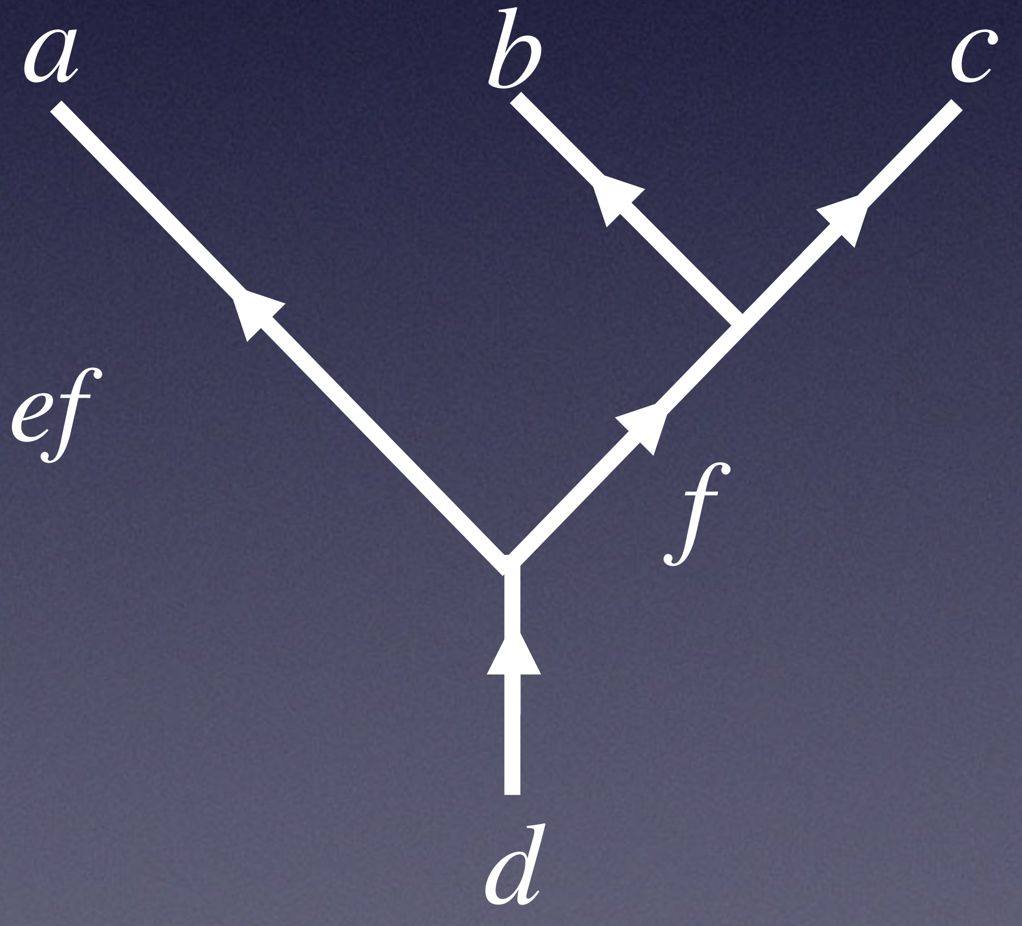

Wigner 6-j symbol

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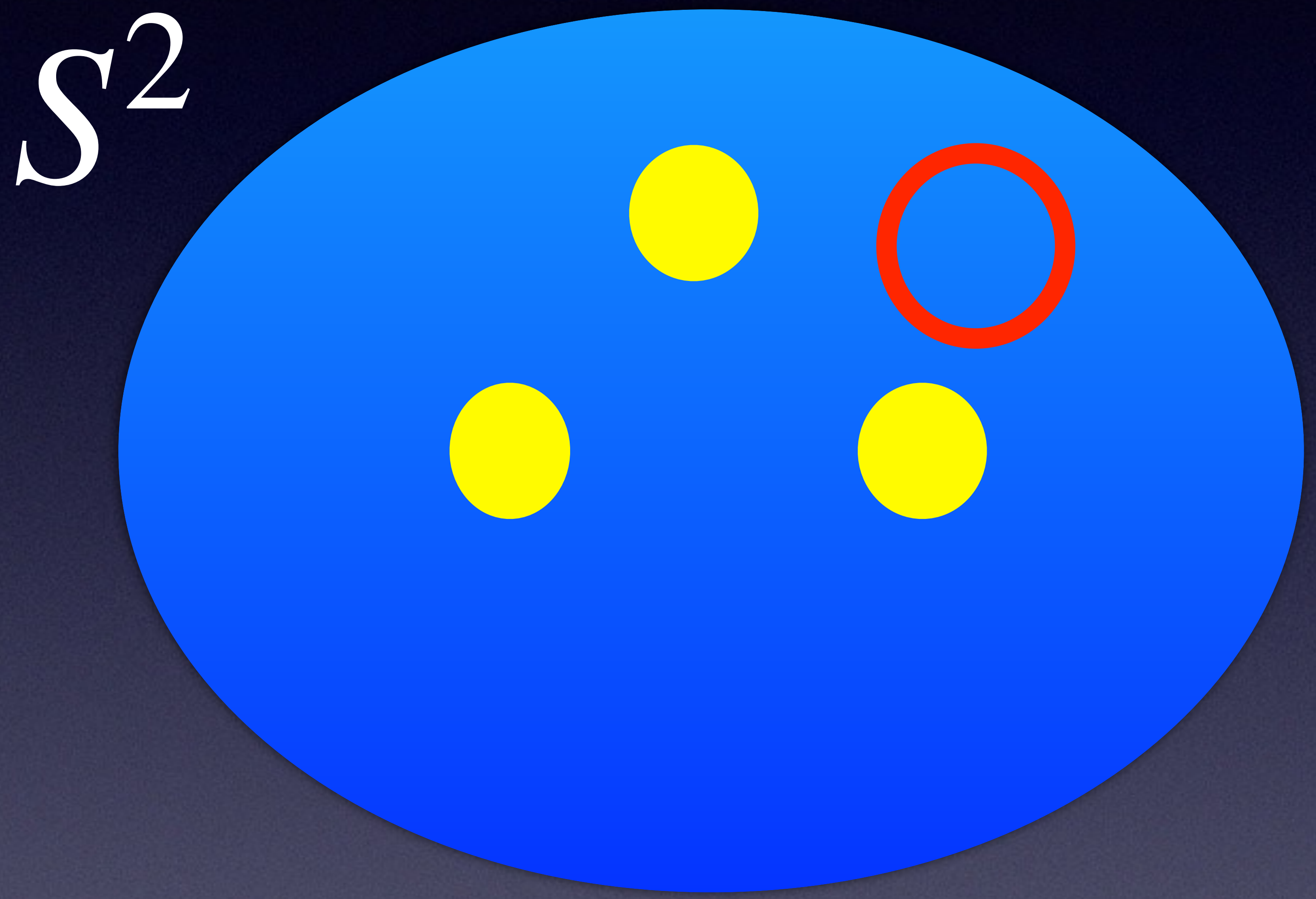
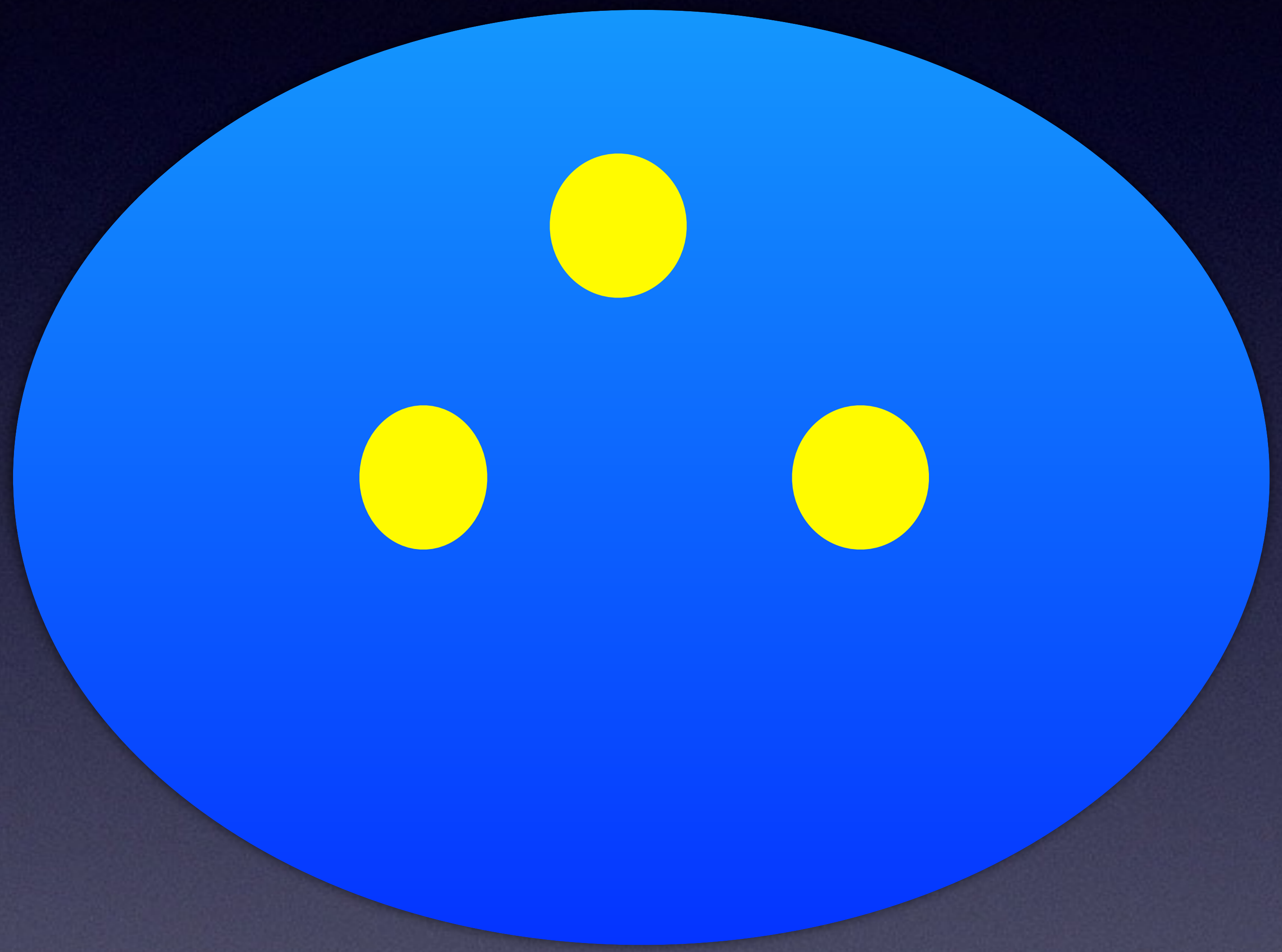
+consistency condition

TQFT calculas

Consider



$\simeq$

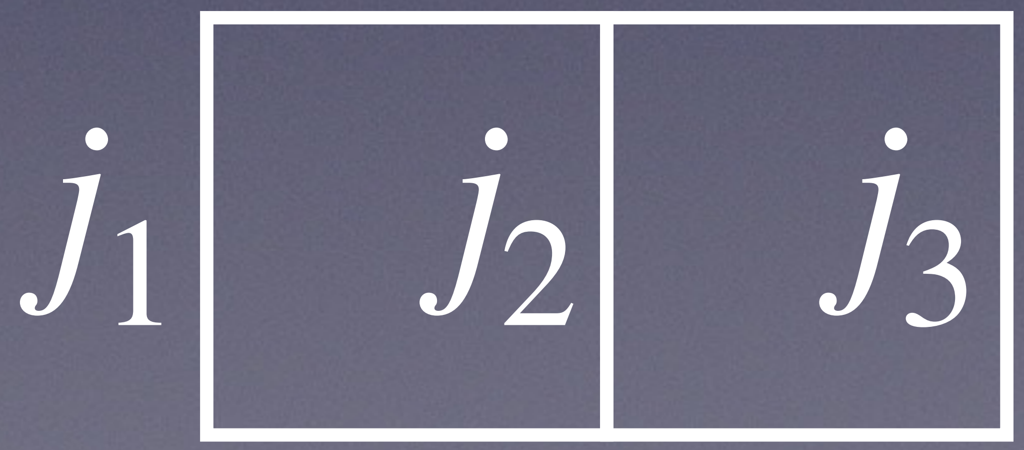


TQFT calculas

TQFT calculas

Basis of Hilbert space

This is nothing but the basis of two plaquettes



$|j_1, j_2, j_3\rangle$

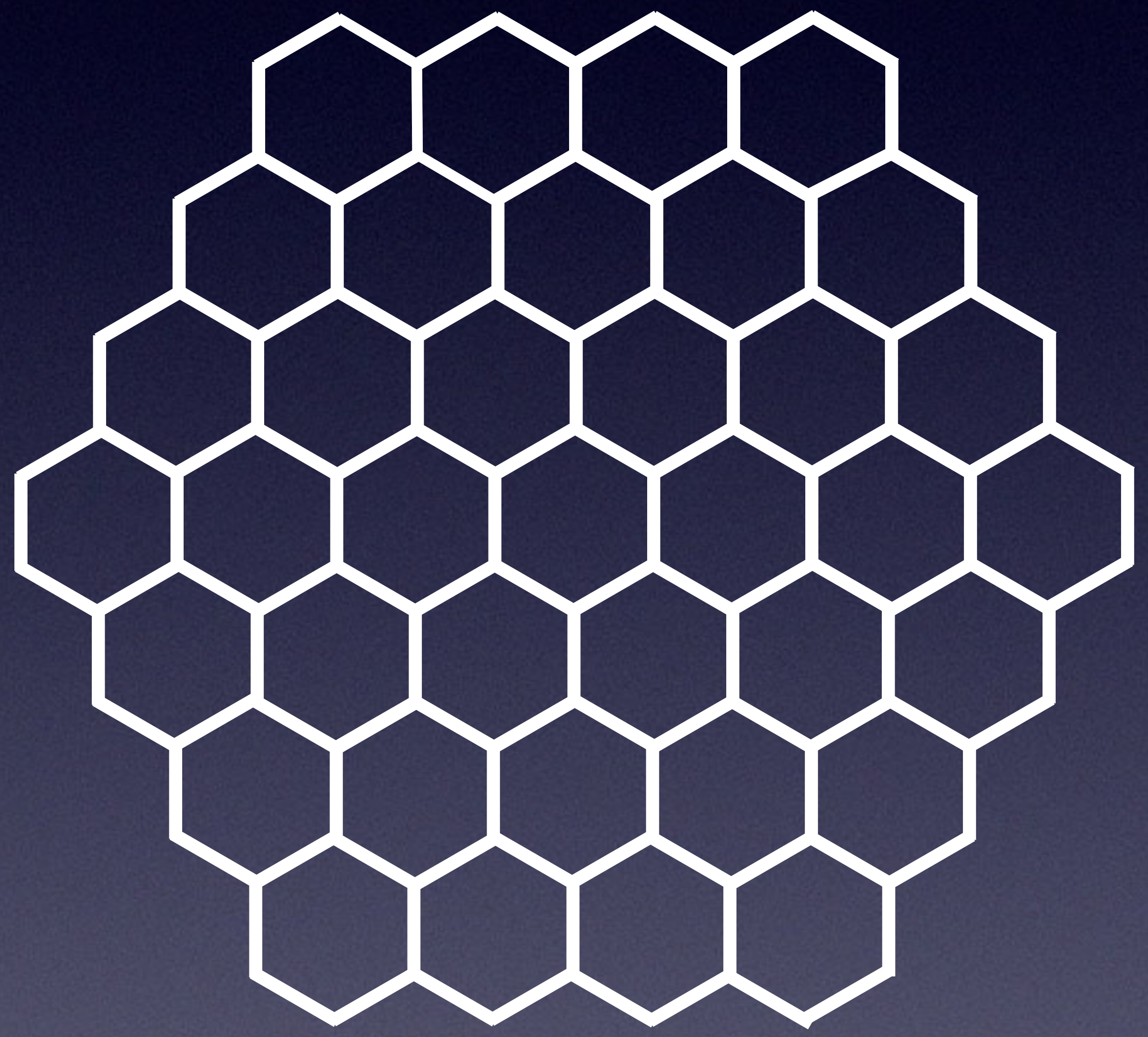
with $|j_1 - j_2| \leq j_3 \leq j_1 + j_2, j_1 + j_2 + j_3 \in \mathbb{Z}$

More generally

$(n+1)$ **defects on** S^2

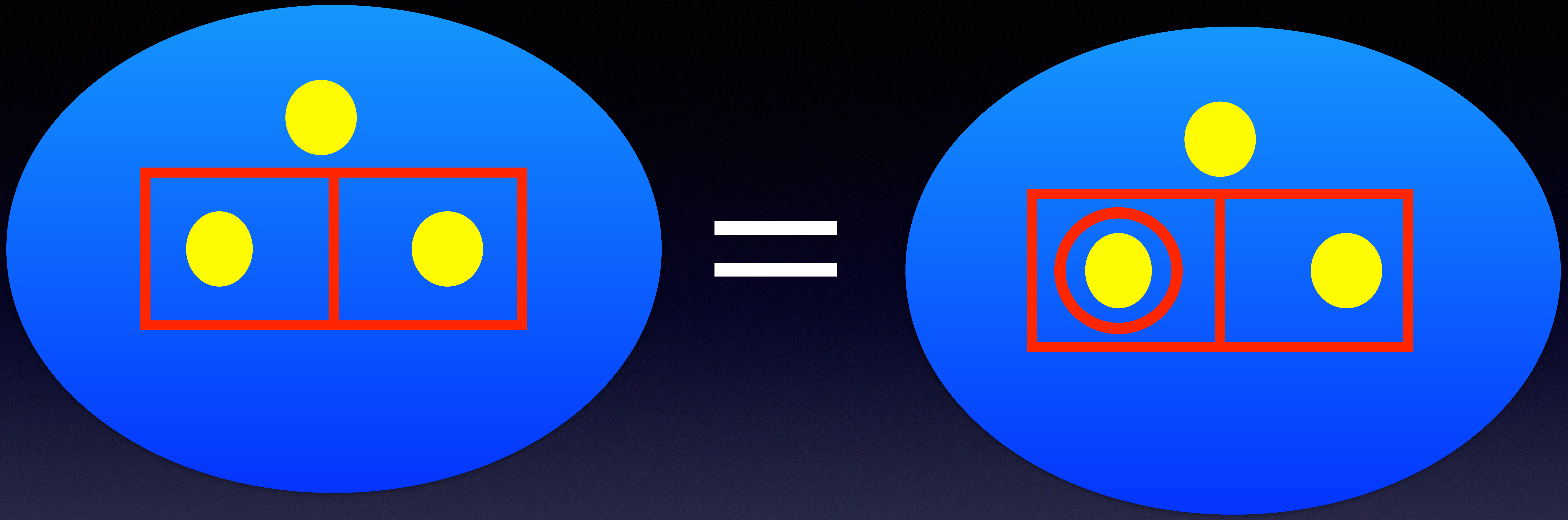
$\simeq$

n **plaquettes lattice system**



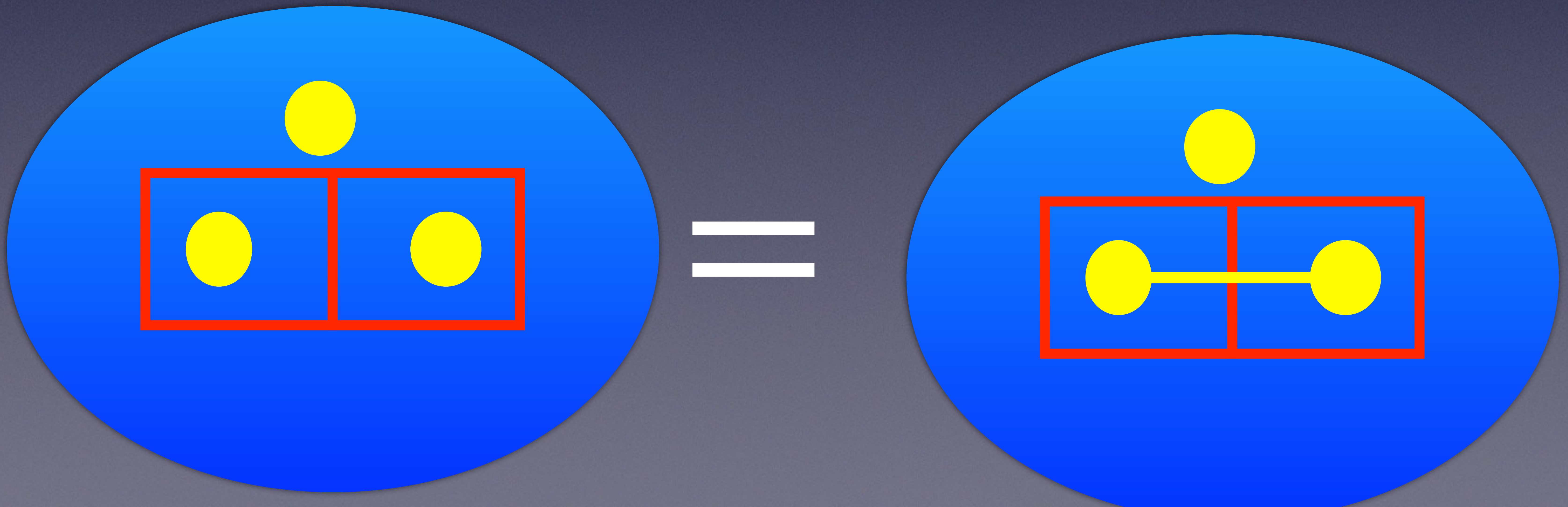
Action of operators

Action of
Wilson loop
on left plauqette $\mathrm{tr}U_a$



Evaluate
using topological algebra

Action of
GW operator
on middle link $S_{[g]}$



Casimir invariant and character

For Lie group

$$-\Delta_G \chi_a(g) = C_2(a) \chi_a(g)$$

Laplacian (E^2) Casimir

Example: $SU(2)$

$$\Delta_G = \frac{\partial^2}{\partial \theta^2} + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \frac{\partial}{\partial \theta} \qquad \chi_{j_a}(g) = \frac{\sin\frac{2j_a+1}{2}\theta}{\sin\frac{\theta}{2}} \qquad g = e^{i\theta t_3} \quad \Rightarrow \quad -\Delta_G \chi_{j_a}(g) = j(j+1)\chi_{j_a}(g)$$

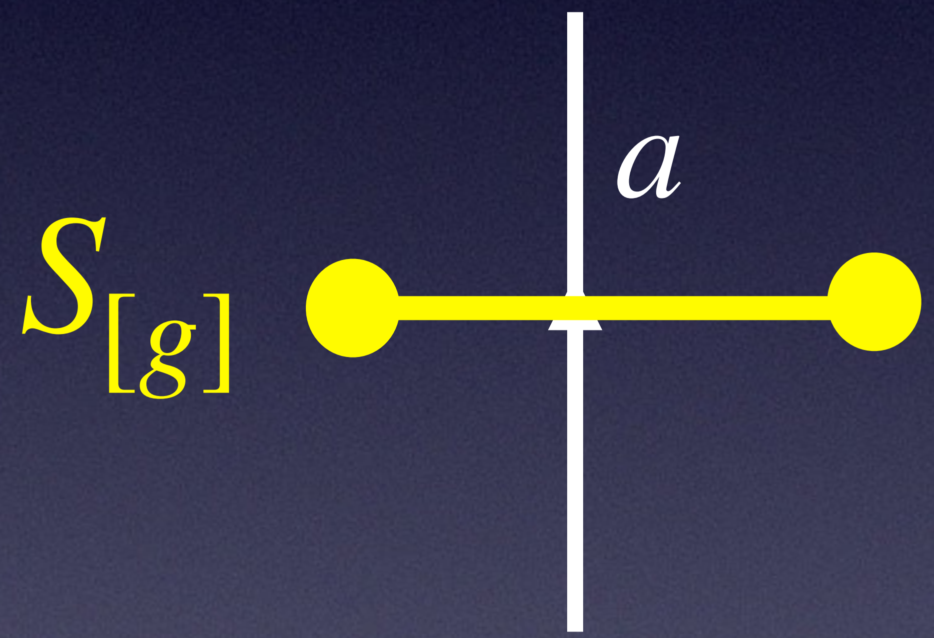
We can replace E^2 by $E^2 \simeq \frac{6}{\epsilon^2}(1 - S_{[e^{i\epsilon t^3}]}(e^*))$

Modified $\mathrm{SU}(2)$ Yang-Mills theory

Hamiltonian $$H = -J \sum_{e^* \in E^*} S_{e^{it^3\epsilon}}(e^*) - K \sum_{f \in F} \mathrm{tr}U_{1/2}(f)$$

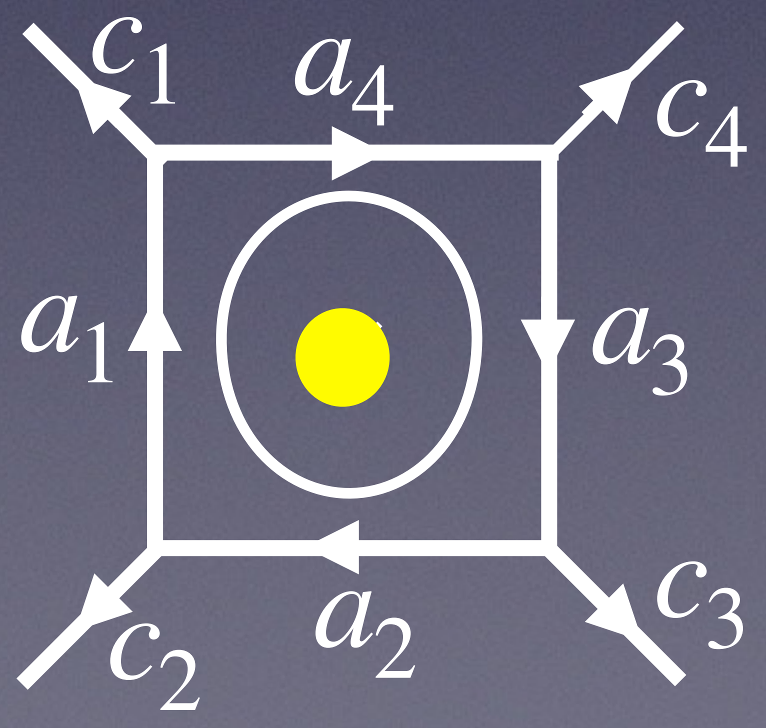
set of dual link   set of faces

Action on a state

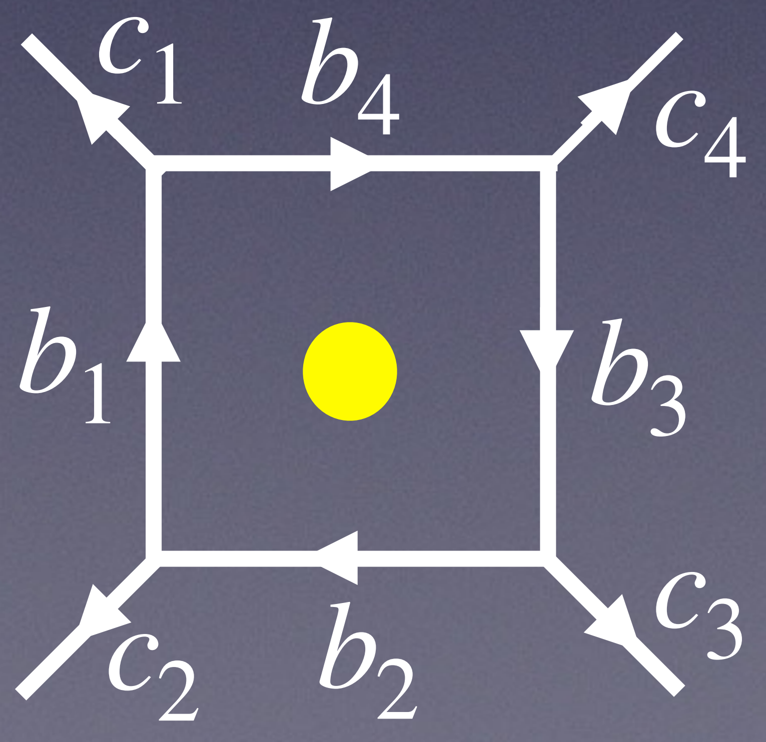


$$= \frac{\chi_a(g)}{d_a}$$

$\mathrm{tr}U_d$



$$= \prod_{i=1}^{4} \sum_{b_i} [F^{c_i a_{i-1} d}_{b_i}]_{a_i b_{i-1}}$$

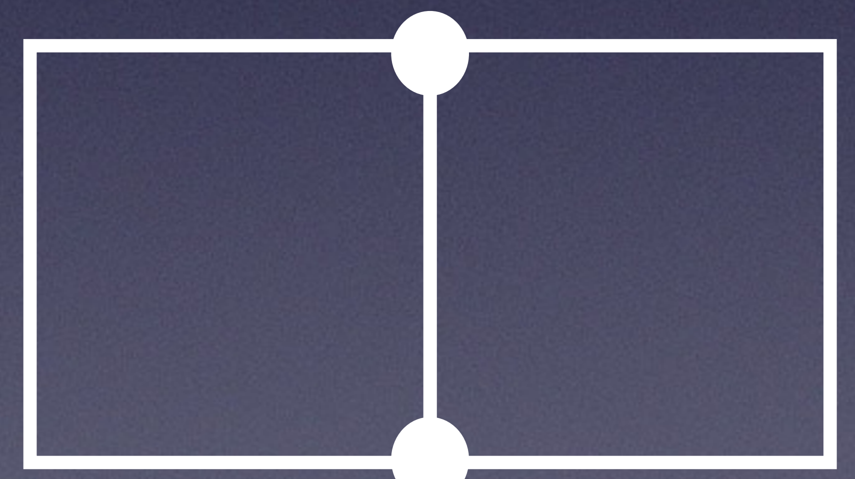


Hilbert space with matter field

matter field = source of electric flux

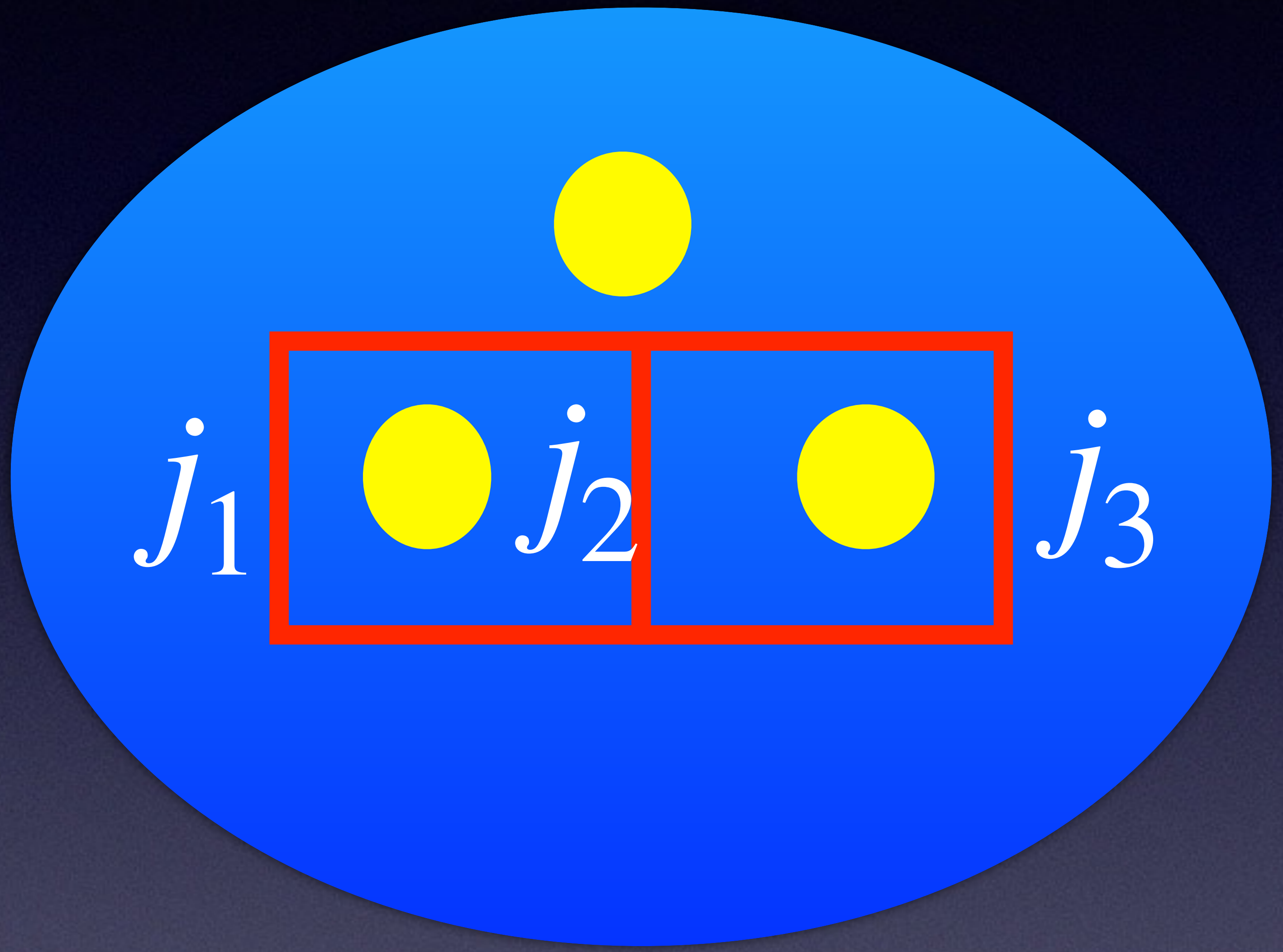
- defect where a Wilson line can terminate
- defect where a GW line can terminate

$\simeq$

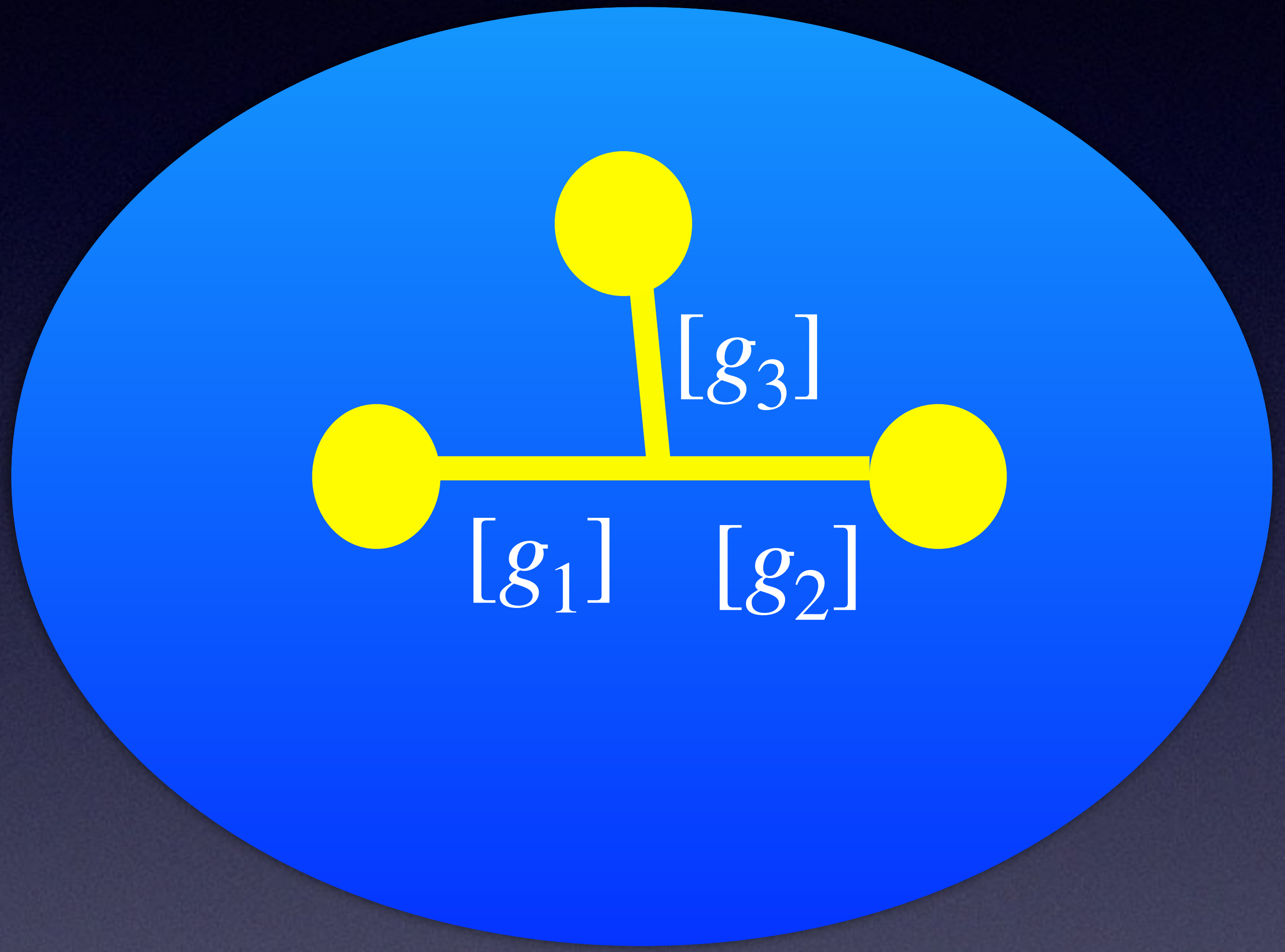


Dual Hilbert space

Wilson line network **Gukov-Witten(GW) network**



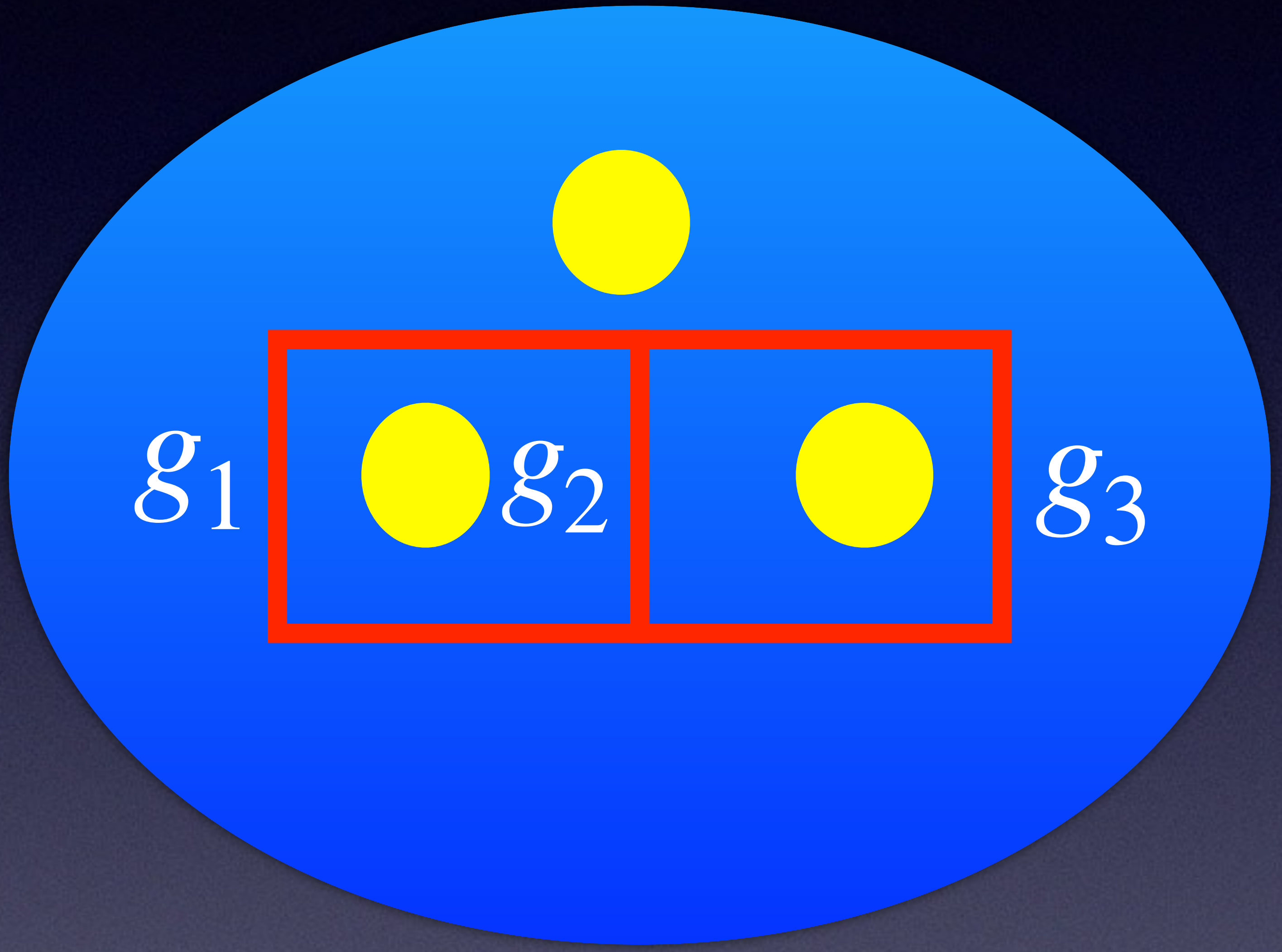
$\simeq$



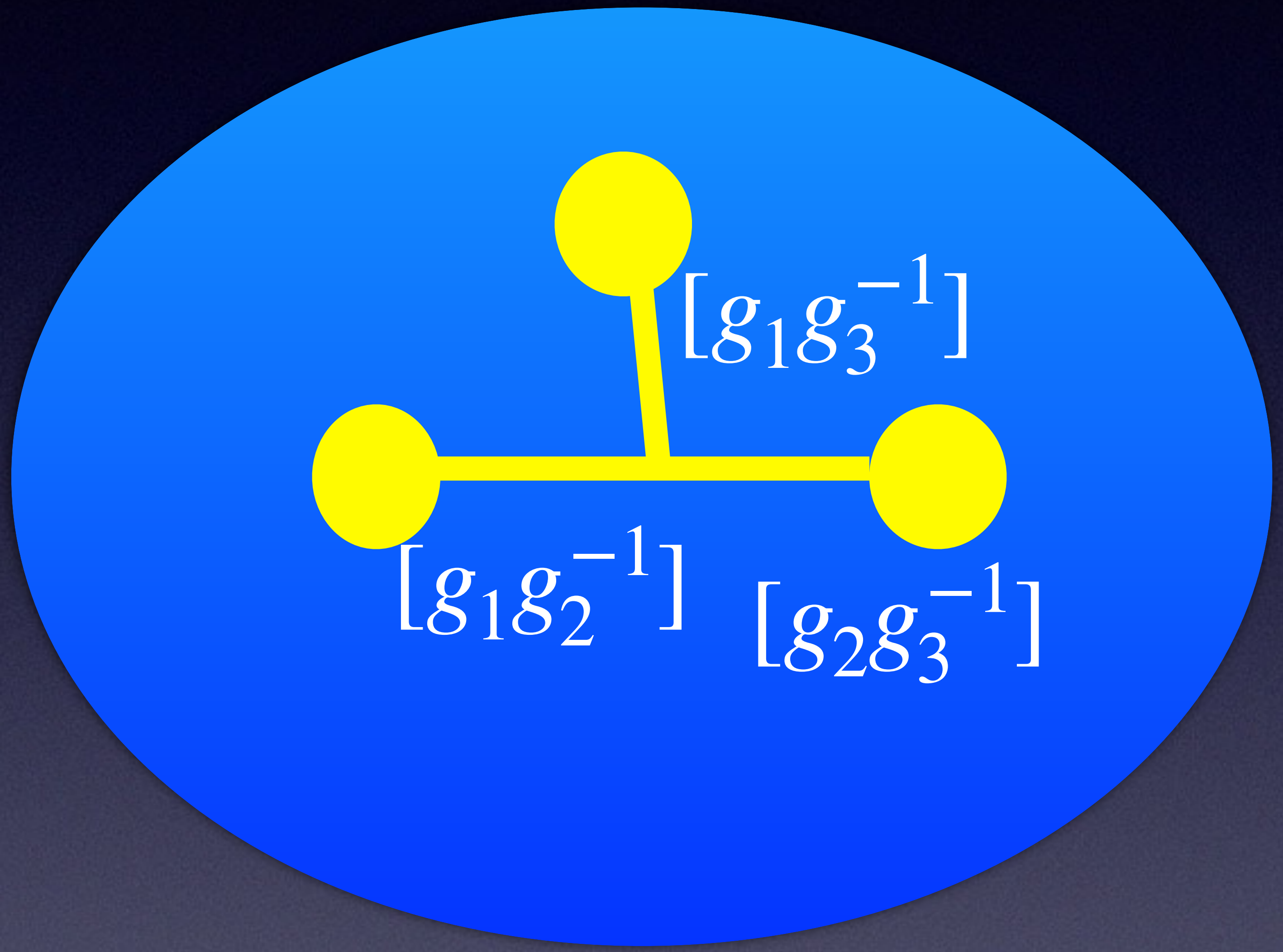
$|j_1, j_2, j_3\rangle$

$|[g_1], [g_2], [g_3]\rangle$

Correspondence between G basis and GW network basis



$|g_1, g_2, g_3\rangle$



$|[g_1 g_2^{-1}], [g_2 g_3^{-1}], [g_1 g_3^{-1}]\rangle$

Confinement and deconfinement phase

$$H = -J \sum_{e^* \in E^*} S_{e^{it^3 \epsilon}}(e^*) - K \sum_{f \in F} \mathrm{tr} U_{1/2}(f)$$

Strong coupling limit

$$H = -J \sum_{e^* \in E^*} S_{e^{it^3 \epsilon}}(e^*)$$

Trivial state for S_g $\quad S_g \,|\Omega_{\mathrm{sc}}\rangle = |\Omega_{\mathrm{sc}}\rangle$

Weak coupling limit

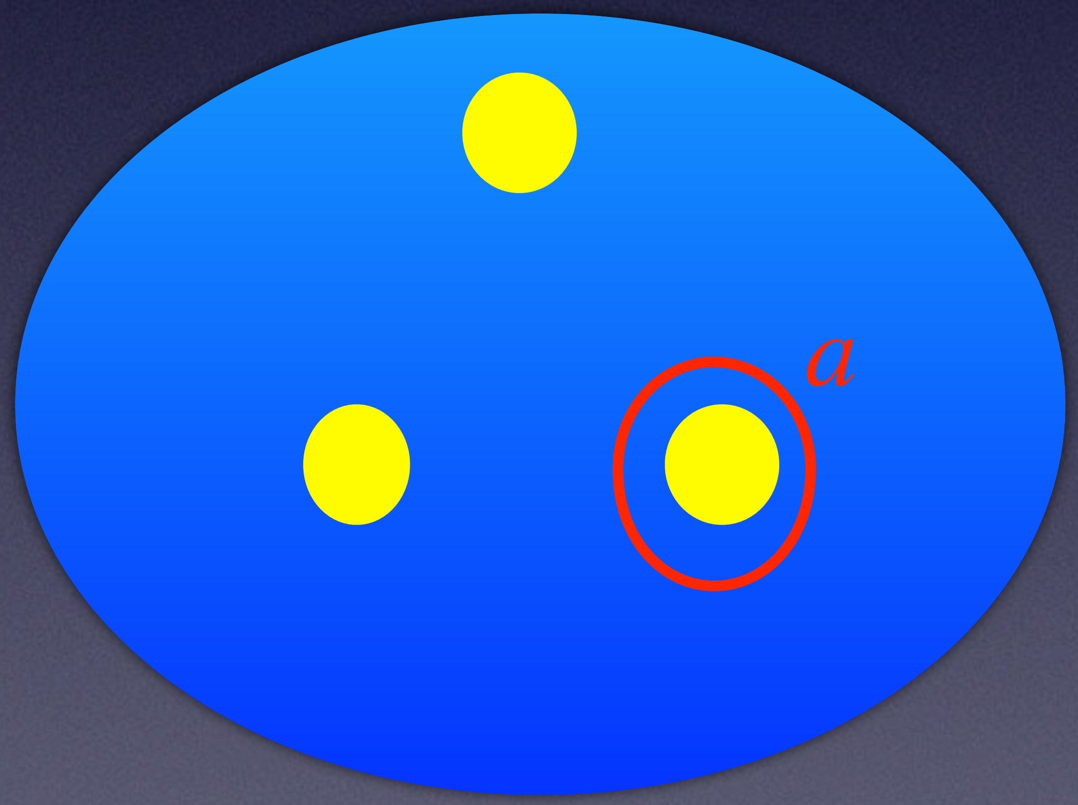
$$H = -K \sum_{f \in F} \mathrm{tr} U_{1/2}(f)$$

Trivial state for $\dfrac{1}{d_a}\mathrm{tr} U_a$ $\quad \dfrac{1}{d_a}\mathrm{tr} U_a \,|\Omega_{\mathrm{wc}}\rangle = |\Omega_{\mathrm{wc}}\rangle$

Confinement and deconfinement phase in Rep basis

We write $|\Omega_{\rm sc}\rangle =$

$\mathrm{tr}U_a|\Omega_{\rm sc}\rangle =$

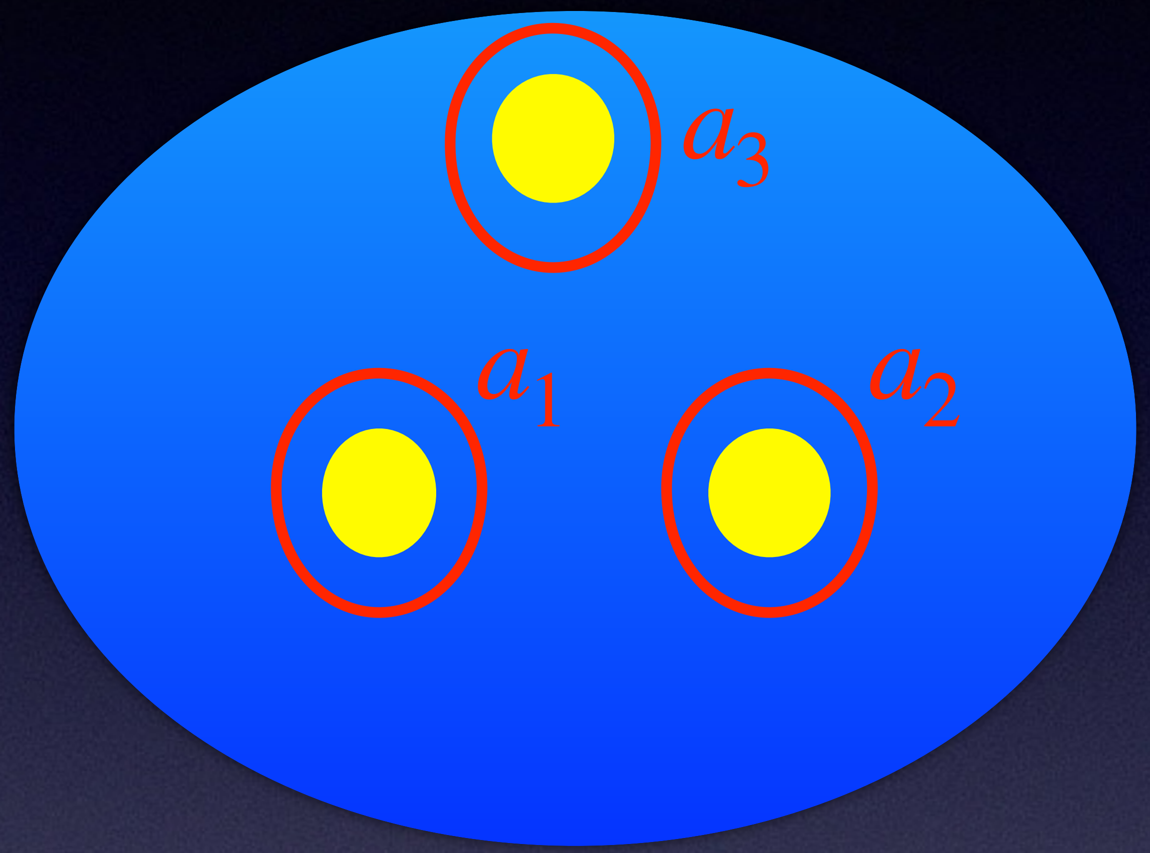


which is orthogonal to $|\Omega_{\rm sc}\rangle$

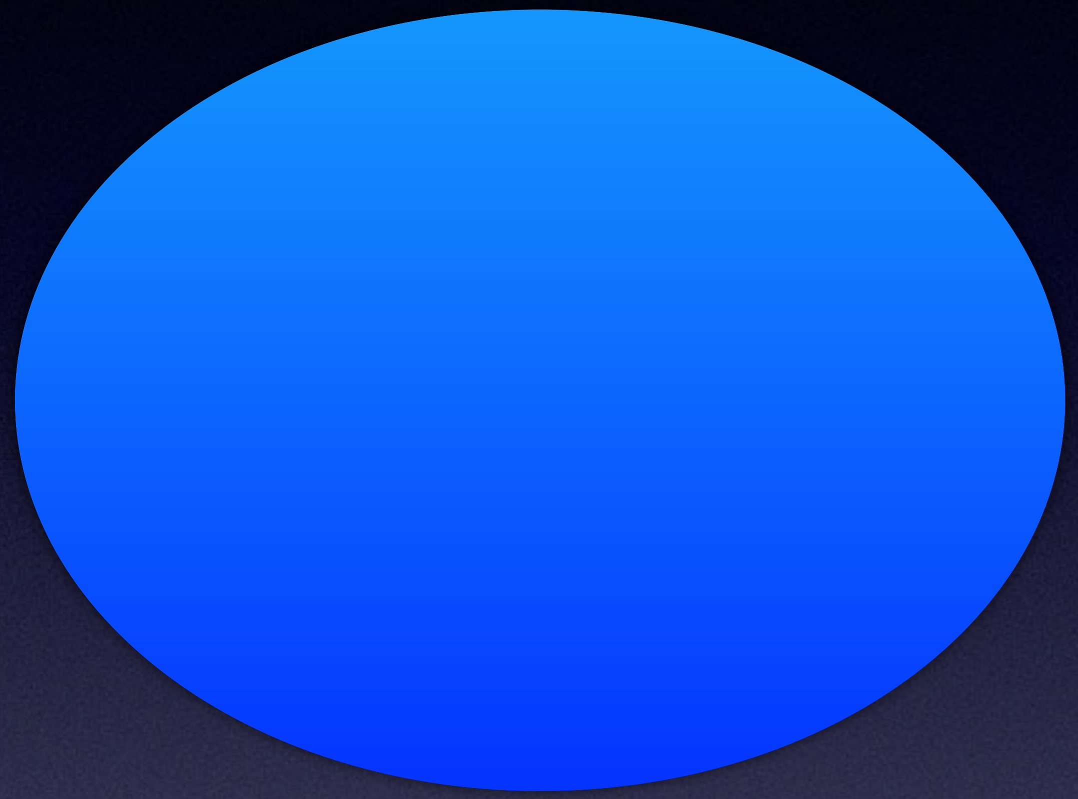
$\Rightarrow$ $\langle\Omega_{\rm sc}|\mathrm{tr}U_a|\Omega_{\rm sc}\rangle = 0$ confinement

Confinement and deconfinement phase in Rep basis

$$|\Omega_{\mathrm{wc}}\rangle = \mathcal{N} \sum_{a_1,a_2,a_3} d_{a_1} d_{a_2} d_{a_3}$$

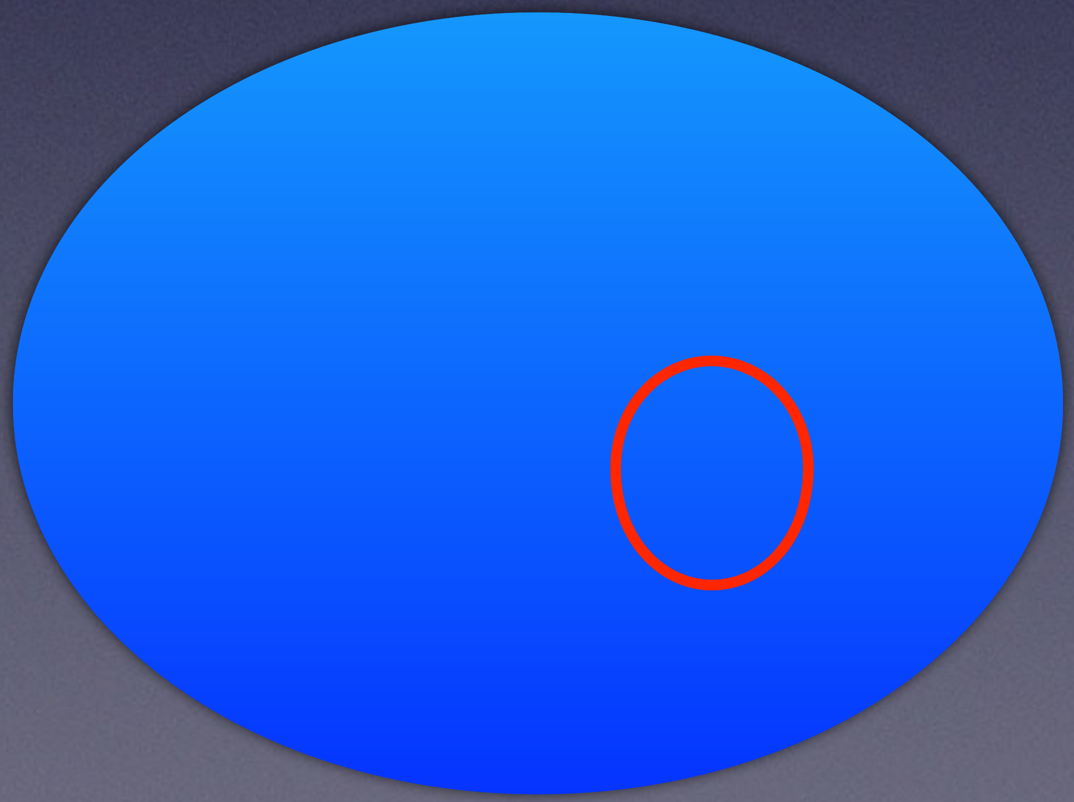


$=$

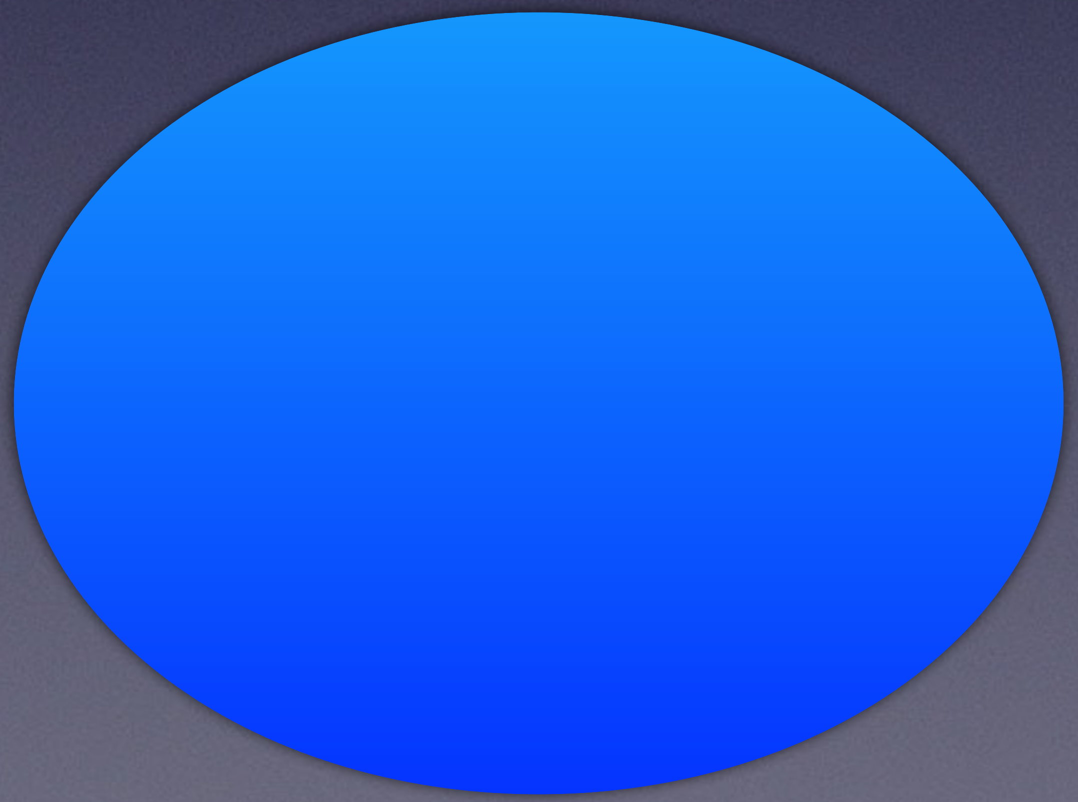


Stringnet condensation

$$\frac{1}{d_a}\mathrm{tr}U_a|\Omega_{\mathrm{wc}}\rangle =$$



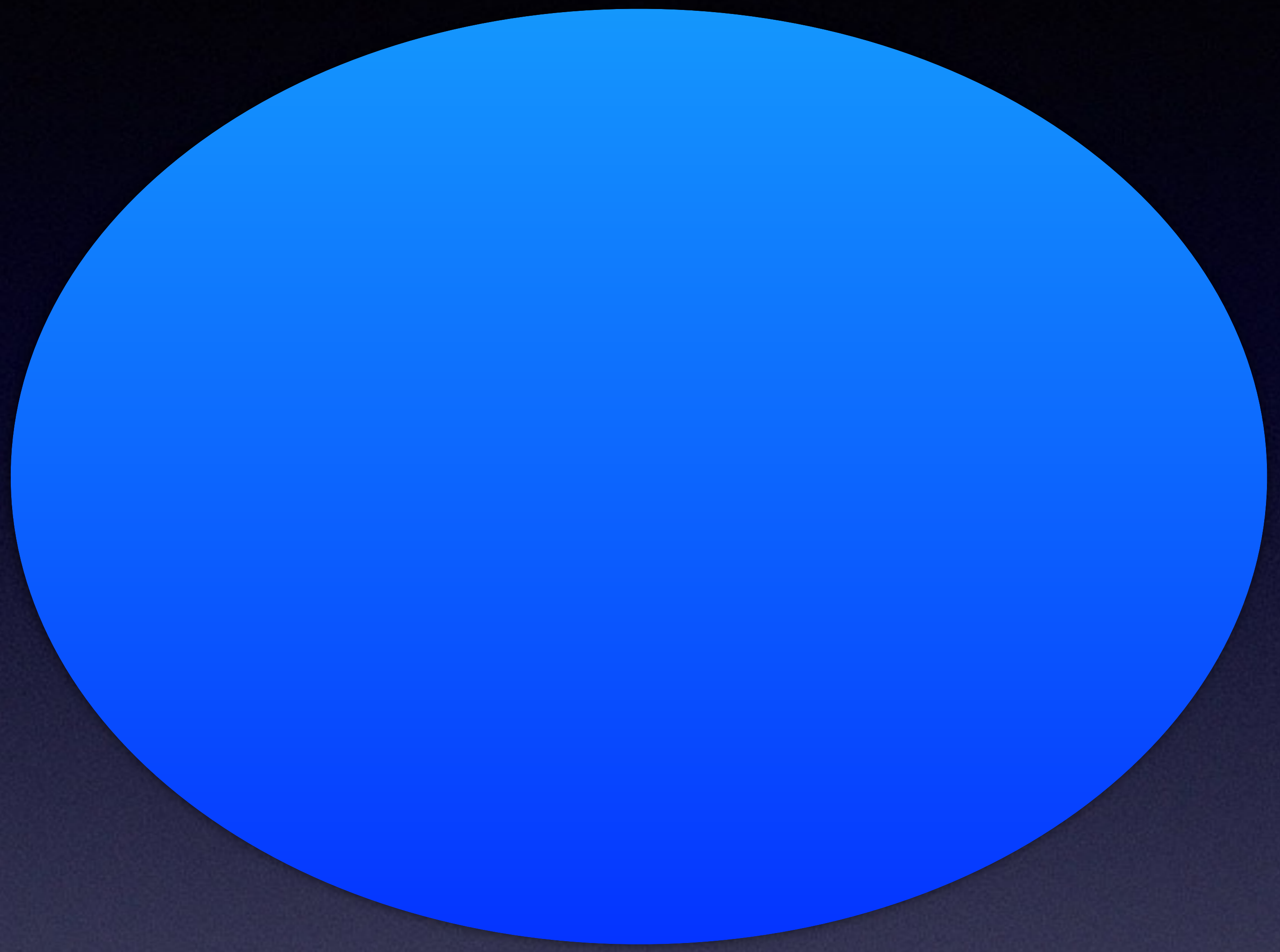
$=$

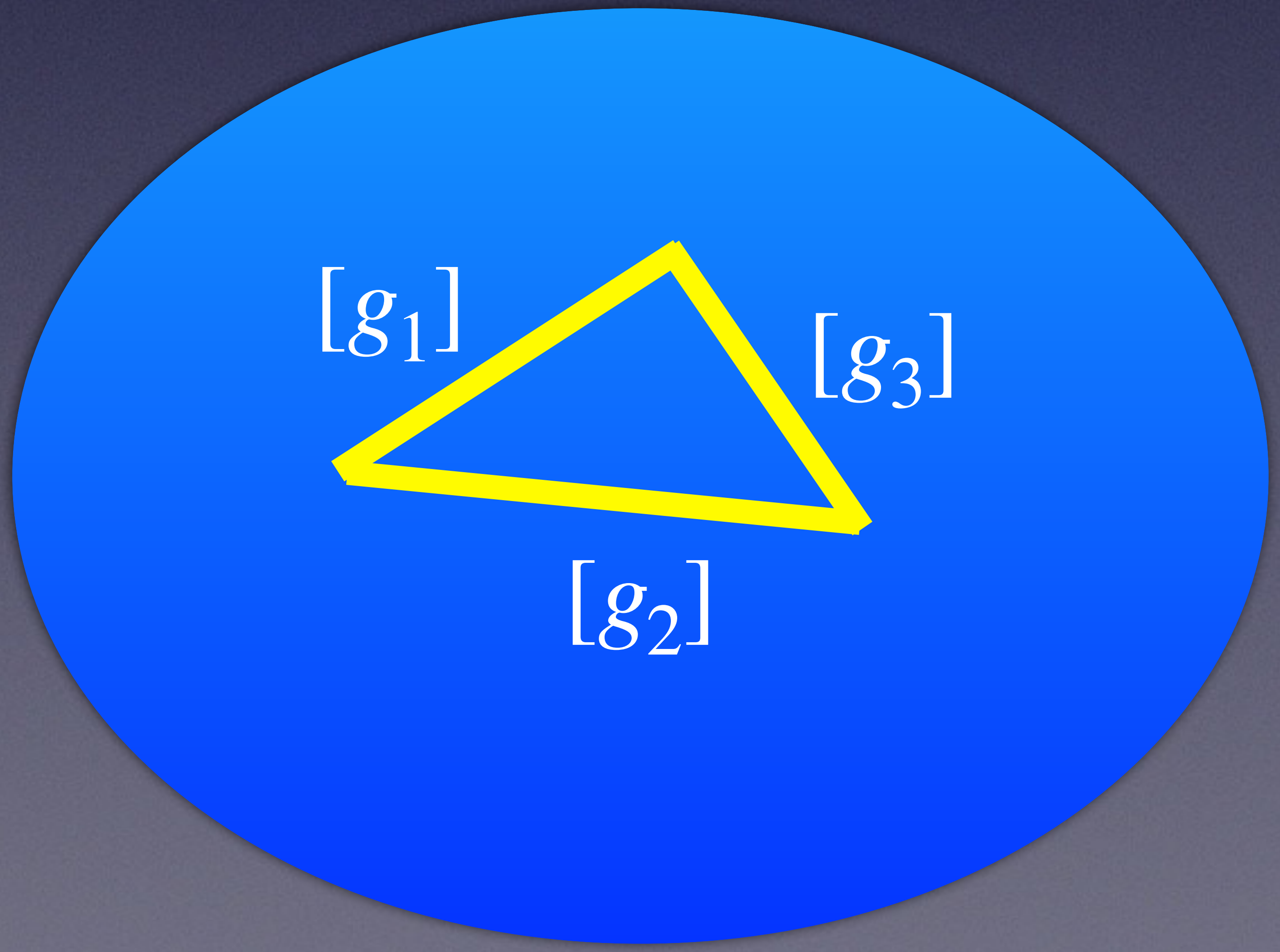


$$= |\Omega_{\mathrm{wc}}\rangle$$

⇒ $\langle\Omega_{\mathrm{sc}}|\mathrm{tr}U_a|\Omega_{\mathrm{sc}}\rangle = 1$ deconfinement

Confinement and deconfinement phase in Gukov-Witten network basis

We write $|\Omega_{\mathrm{wc}}\rangle =$ 

$$|\Omega_{\mathrm{st}}\rangle = \mathcal{N} \sum_{[g_1],[g_2],[g_3]} \prod_i \frac{|\mathscr{C}_{[g_1]}|}{|G|}$$ 

magnetic charge condensation

Confinement and deconfinement phase in Gukov-Witten network basis

$|\Omega_{\mathrm{wc}}\rangle =$ 

$S_g |\Omega_{\mathrm{wc}}\rangle =$  which is orthogonal to $|\Omega_{\mathrm{wc}}\rangle$

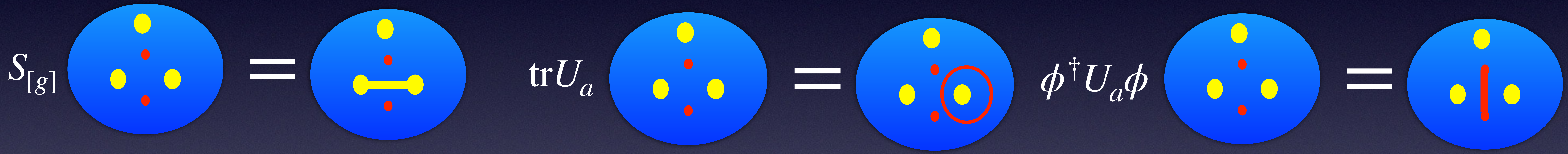
disorder parameter
’t Hooft operator
g is center

$\Rightarrow \langle \Omega_{\mathrm{wc}} | S_g | \Omega_{\mathrm{wc}} \rangle = 0$ deconfinement

Analogy with Higgs condensation

$$H = -J \sum_{e^* \in E^*} S_{e^{it3_\epsilon}}(e^*) - K \sum_{f \in F} \mathrm{tr} U_a(f) - \lambda \sum_{e \in E} \phi^\dagger U_a \phi - \lambda_e \sum_{v \in V} \Pi^2(v)$$

$$|\phi| = 1$$



Strong λ limit (Deep Higgs phase)

$$H = -\lambda \sum_{e \in E} \phi^\dagger U_a \phi \qquad \phi^\dagger U_a \phi \, |\Omega_{\mathrm{Hggs}}\rangle = |\Omega_{\mathrm{Hggs}}\rangle$$

Confinement and ‘Center vortex’

Expectation value of Wilson loop for large loop C

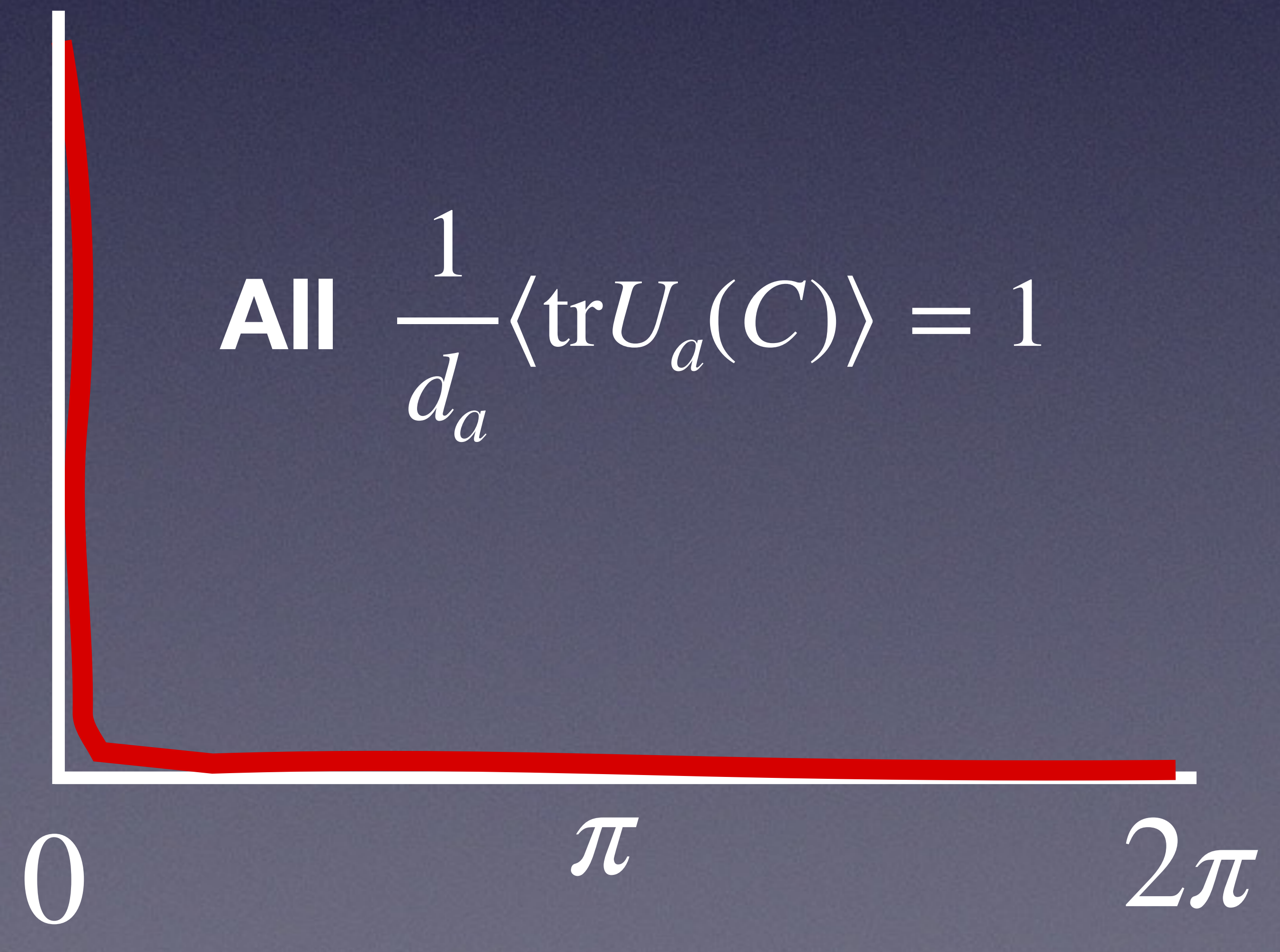
$$\frac{1}{d_a}\langle \mathrm{tr} U_a(C)\rangle = \int d[g] P_C([g]) \frac{1}{d_a}\chi_a([g]) = \int_0^{2\pi} d\theta P_C(\theta) \cos\frac{\theta}{2}$$

for SU(2)

Possible distributions

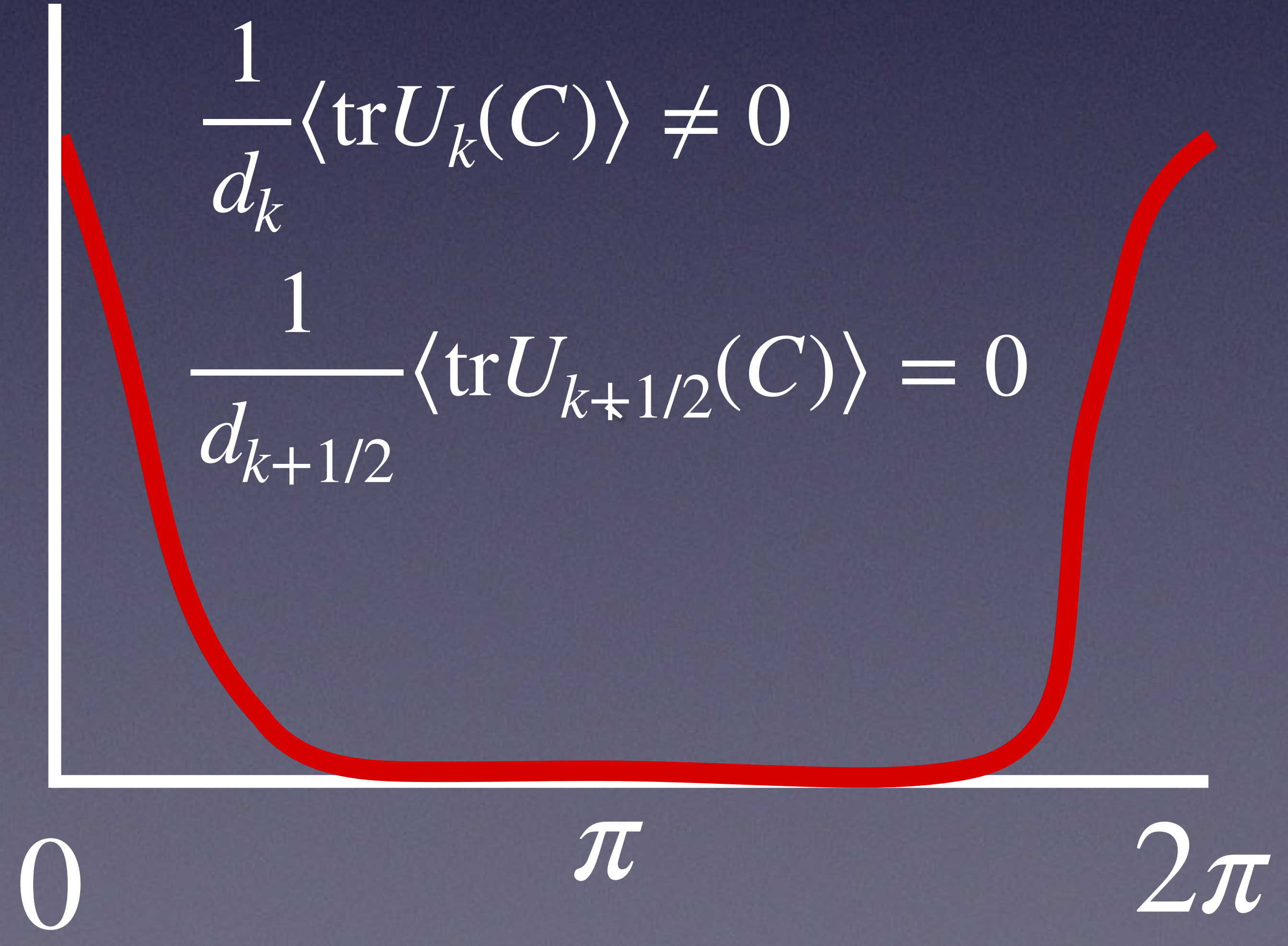
(complete) deconfinement

$P_C(\theta)$ **localize at** $\theta = 0$



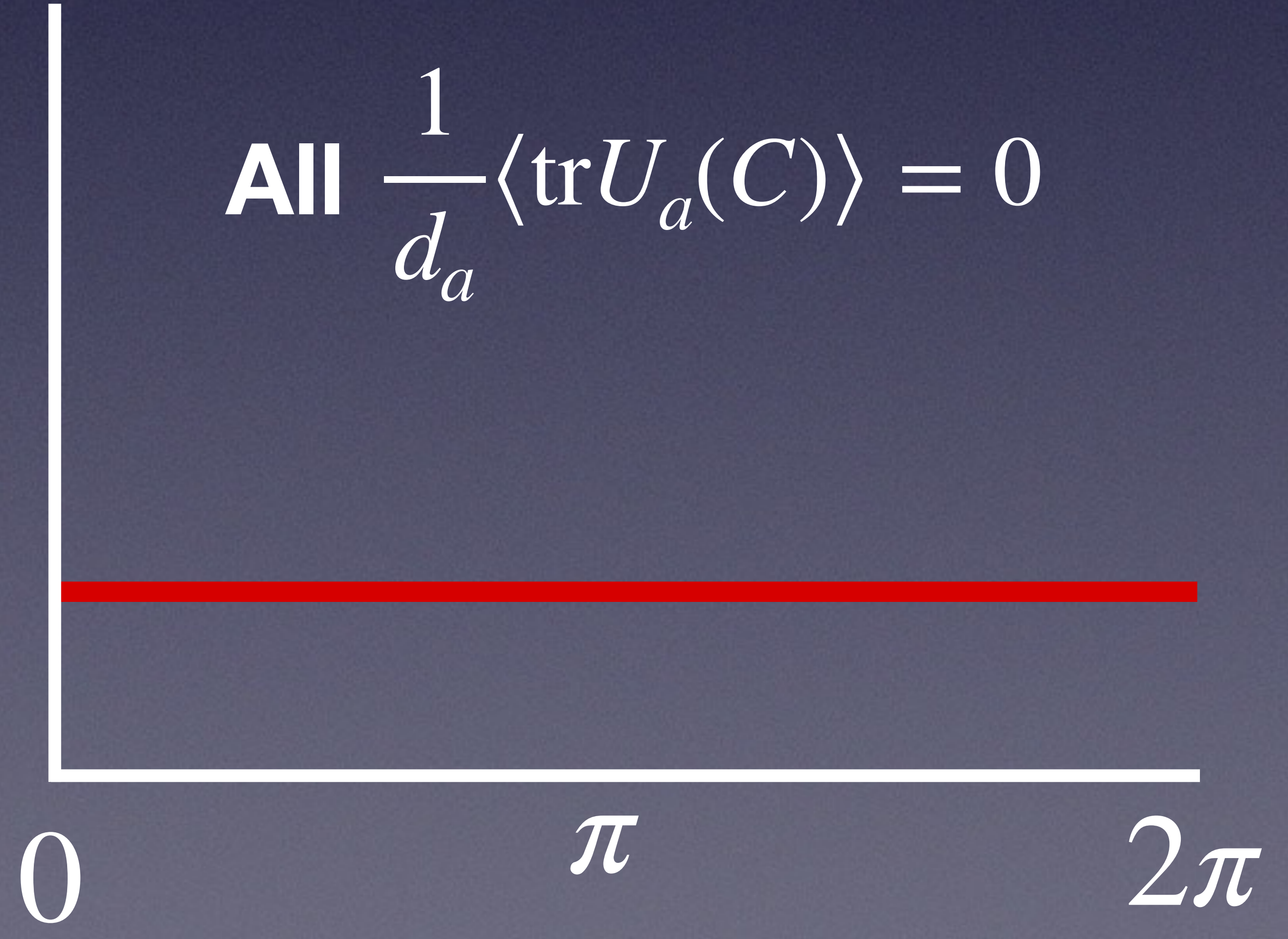
Confinement

$P_C(\theta)$ 2π **periodic**



(complete) confinement

$P_C(\theta)$ **flat distribution**



Existence of center vortex implies 2π periodicity

Summary

Lattice gauge theory in Hamiltonian formalism

$$\cong$$

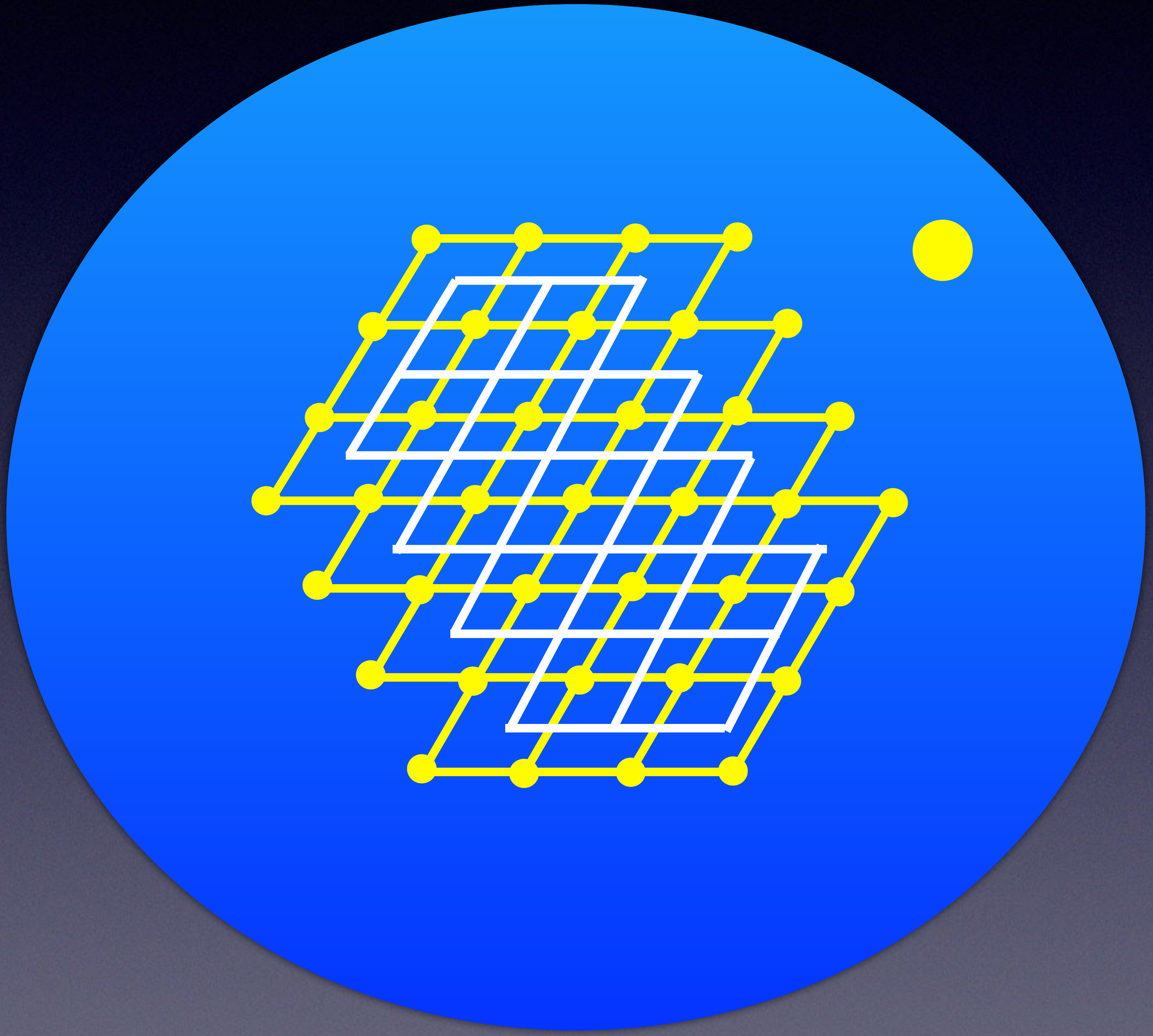
Topological quantum field theory with defects

Outlook

Higher dimensions, asymptotic free, confinement mechanism,…

Choice of Hamiltonian determine shape of lattice

honeycomb lattice



square lattice

Deconfinement limit is unstable

$$H = H_0 + H_1$$

$$H_0 = -K\sum_{f\in F}\mathrm{tr}U_a(f)$$ **Ground state** $|\Omega_{\mathrm{wc}}\rangle$

$$H_1 = J\sum_{e\in E}E^2(e)$$

First order perturbation

$$\langle\Omega_{\mathrm{wc}}|H_1|\Omega_{\mathrm{wc}}\rangle = J\sum_{e\in E}E^2(e) \sim Vj_{\max}^2 \to \infty$$

Confinement? or Coulomb phase?

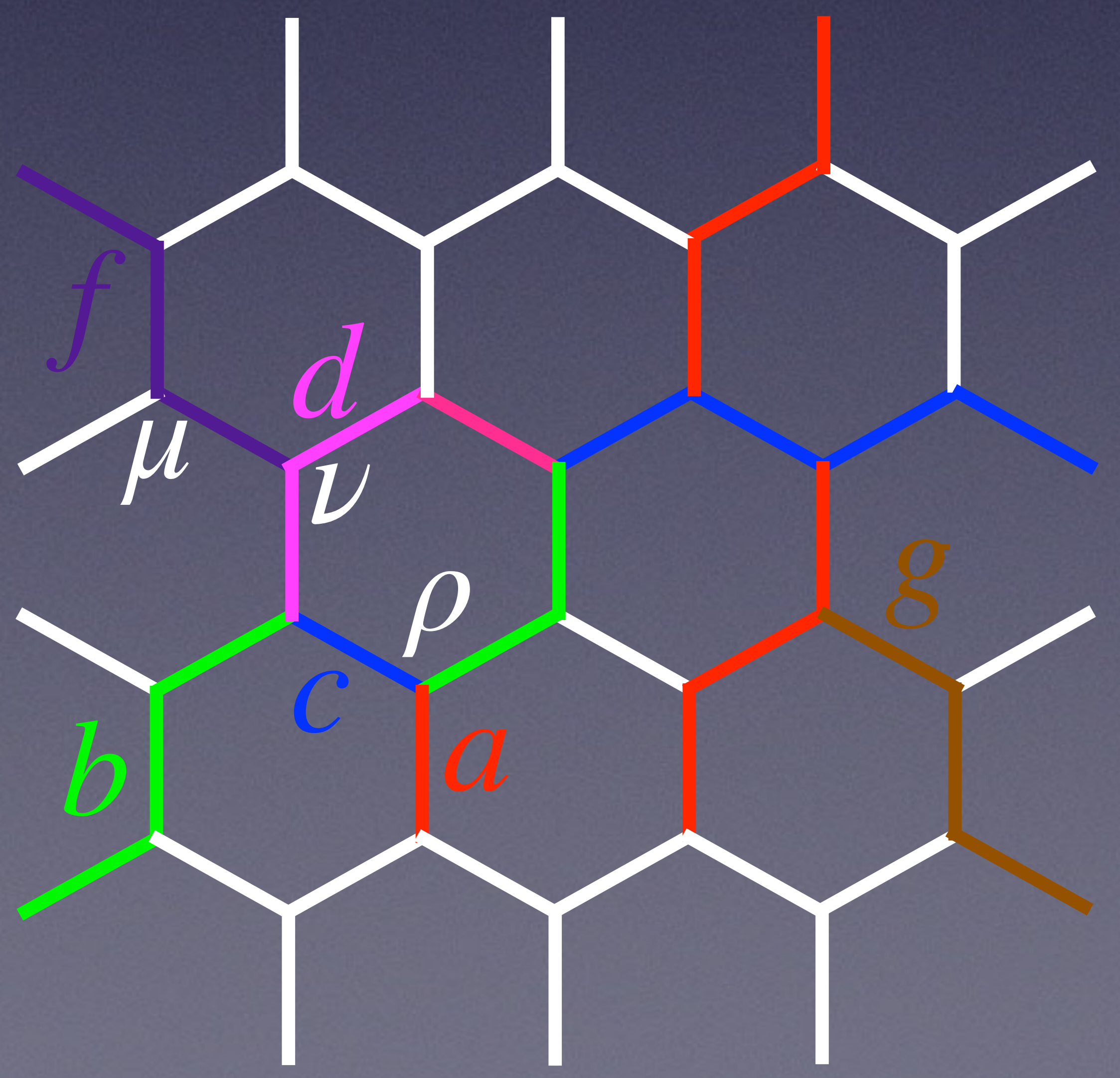
Regularization:

$SU(3) \to SU(3)_k$:Quantum deformation

which preserve properties of gauge symmetry

Solving Gauss law constraint:

Physical states are network of Wilson lines

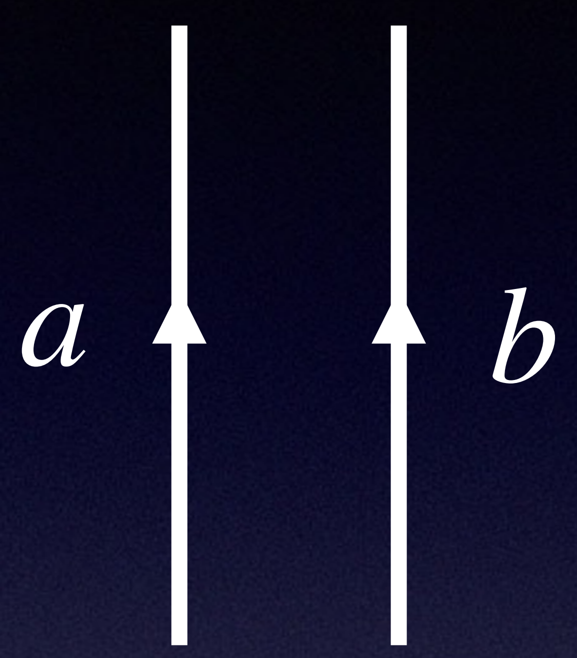


$a, b, c, \cdots$: representation of Wilson lines, e.g., fundamental, adjoint,…

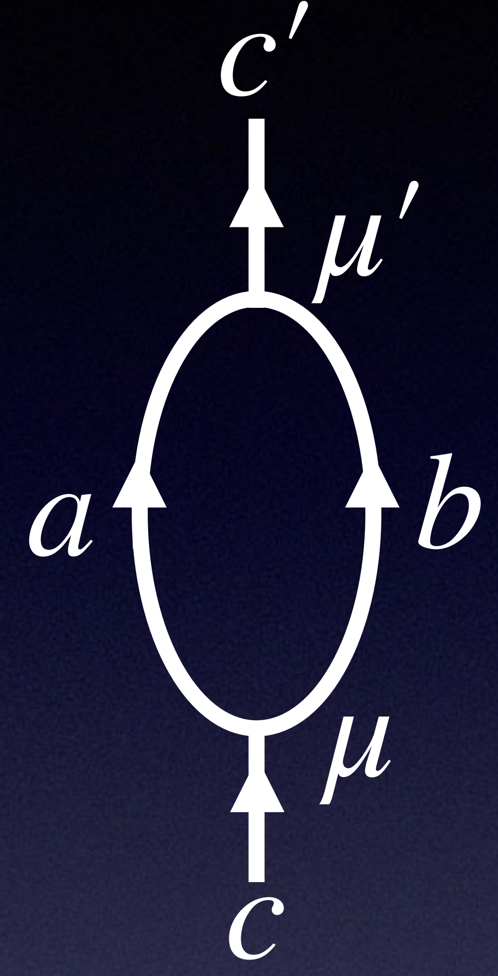
$\mu, \nu, \rho, \cdots \in N_{ab}^{c}$: Multiplicity index

with fusion rule $a \times b = \sum_c N_{ab}^{c}$

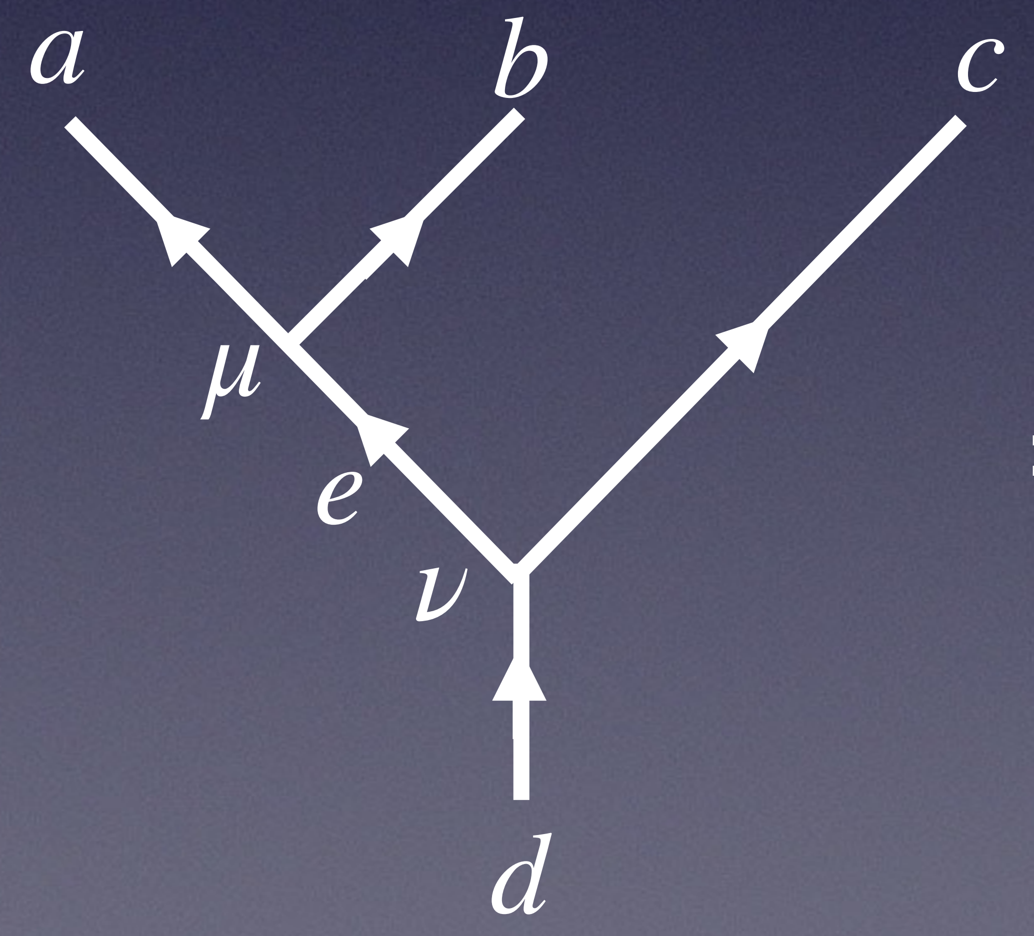
Algebra of Wilson lines for $\mathrm{SU}(3)_k$



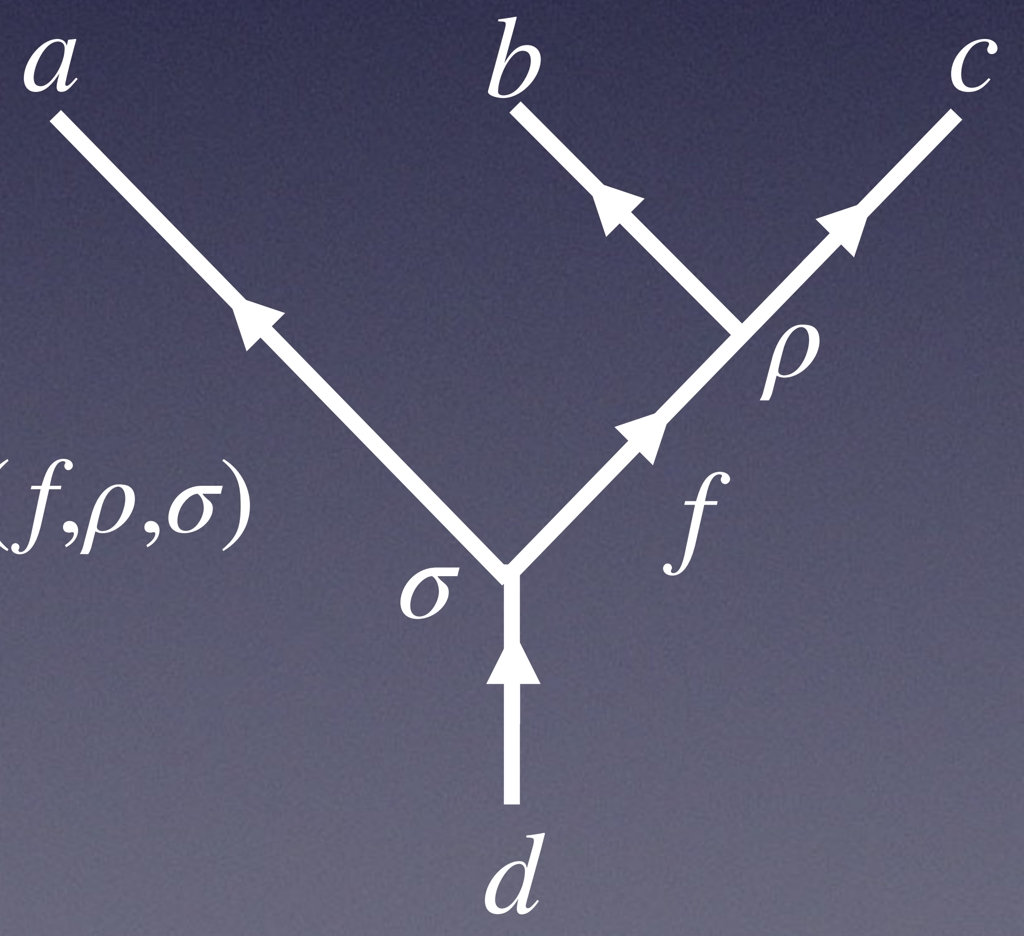
$$= \sum_{c,\mu} \sqrt{\frac{d_c}{d_a d_b}}$$



$$= \delta_c^{c'} \delta_\mu^{\mu'} \sqrt{\frac{d_a d_b}{d_c}}$$



$$= \sum_{f,\rho,\sigma} [F_d^{abc}]_{(e,\mu,\nu)(f,\rho,\sigma)}$$



+consistency condition

$\mathrm{SU}(3)_k$ Yang-Mills theory

Hamiltonian $$H = \frac{1}{2}\sum_{e\in C_1}(E(e))^2 - \frac{K}{2}\sum_{f\in C_2}(\mathrm{tr}U(f) + \mathrm{tr}U^\dagger(f))$$

Action on a state

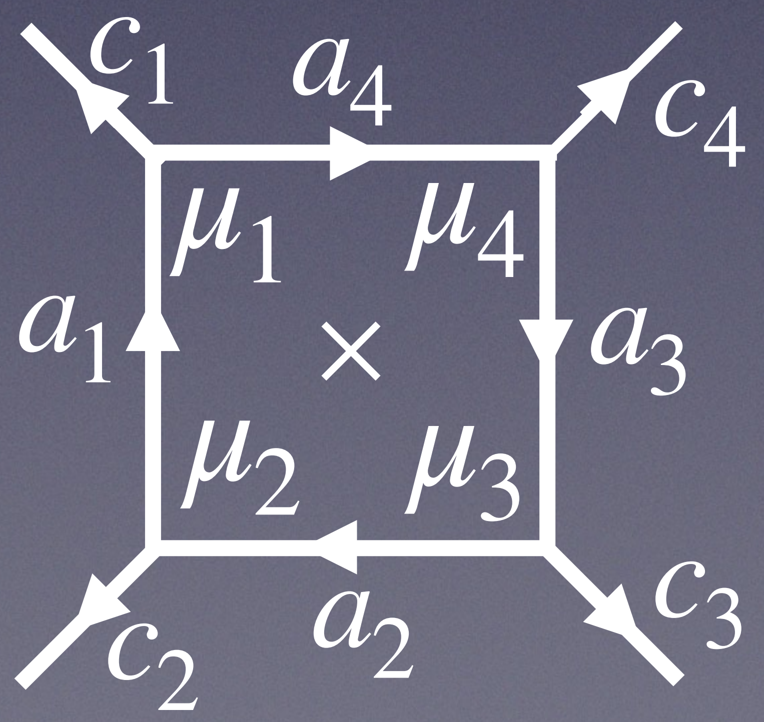
Electric fields

E_i^2 

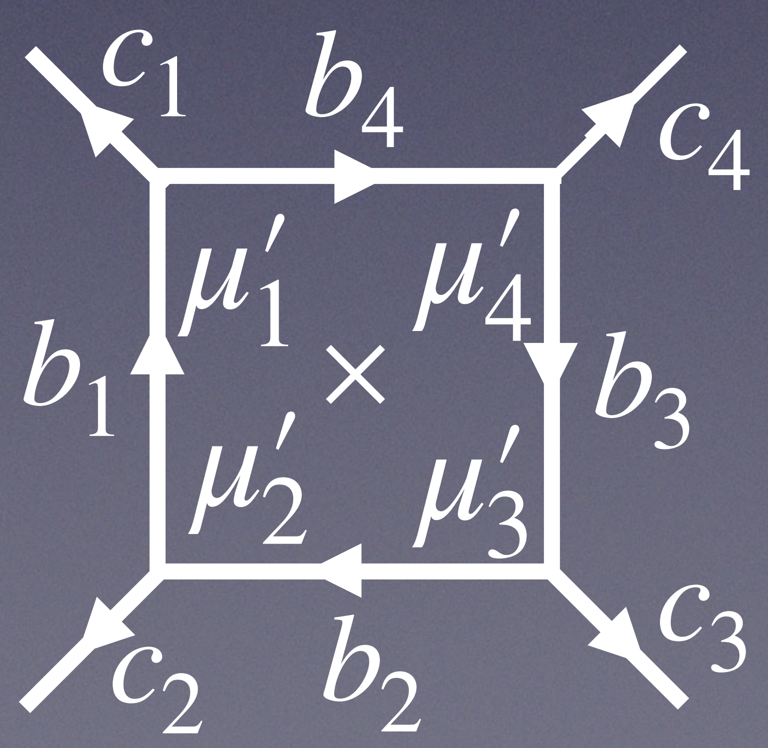
$= C_2(a)$ 

Casimir

Wilson loop

$\mathrm{tr}U$ 

$$= \prod_{i=1}^{4}\sum_{b_i}[F^{c_i a_{i-1}\frac{1}{2}}_{b_i}]_{(a_i,\mu_i,\mu'_{i+1})(b_{i-1},\mu_{i-1},\mu'_i)}$$



Application

Confinement-deconfinement phase transition in mean field approximation for $\mathrm{SU}(3)_k$ in (2+1) dimensions

Variational ansatz for wave function

Dusuel, Vidal, Phys. Rev. B 92 (2015) 12, 125150, Zache, González-Cuadra, Zoller, 2304.02527, Hayata, YH, JHEP 09 (2023) 126

$$|\Psi\rangle = \prod_{f\in\mathcal{F}} \sum_{a_f} \psi(a_f)\, \mathrm{tr}\, U_{a_f}(f)\, |0\rangle$$

We minimize the energy expectation value

open boundary condition, infinite volume limit

$$E = \min_{\psi} \langle\Psi|\, H\, |\Psi\rangle$$

We can calculate observables for given wave function

Hayata, YH, JHEP 09 (2023) 126

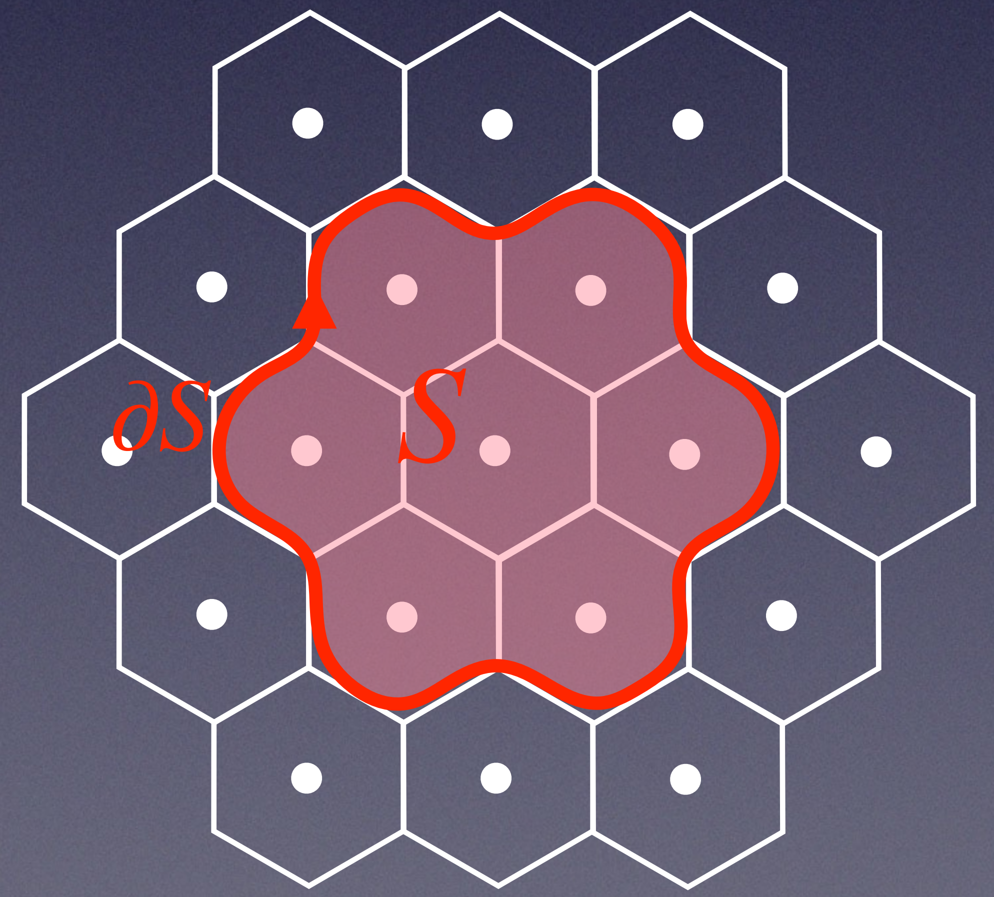
Energy density

$$h = \frac{1}{V}\langle H\rangle = \sum_{a,b,c} C_2(c) N^c_{\bar{a}b} \frac{d_c}{d_a d_b} |\psi(a)|^2 |\psi(b)|^2 - \frac{K}{2}\sum_{a,b} \psi^*(a) \left(N^a_{(1,0)b} + N^a_{(0,1)b} \right) \psi(b)$$

$C_2(a)$ Casimir invariant, d_a: quantum dimensions, N^c_{ab}: multiplicity

Wilson loop

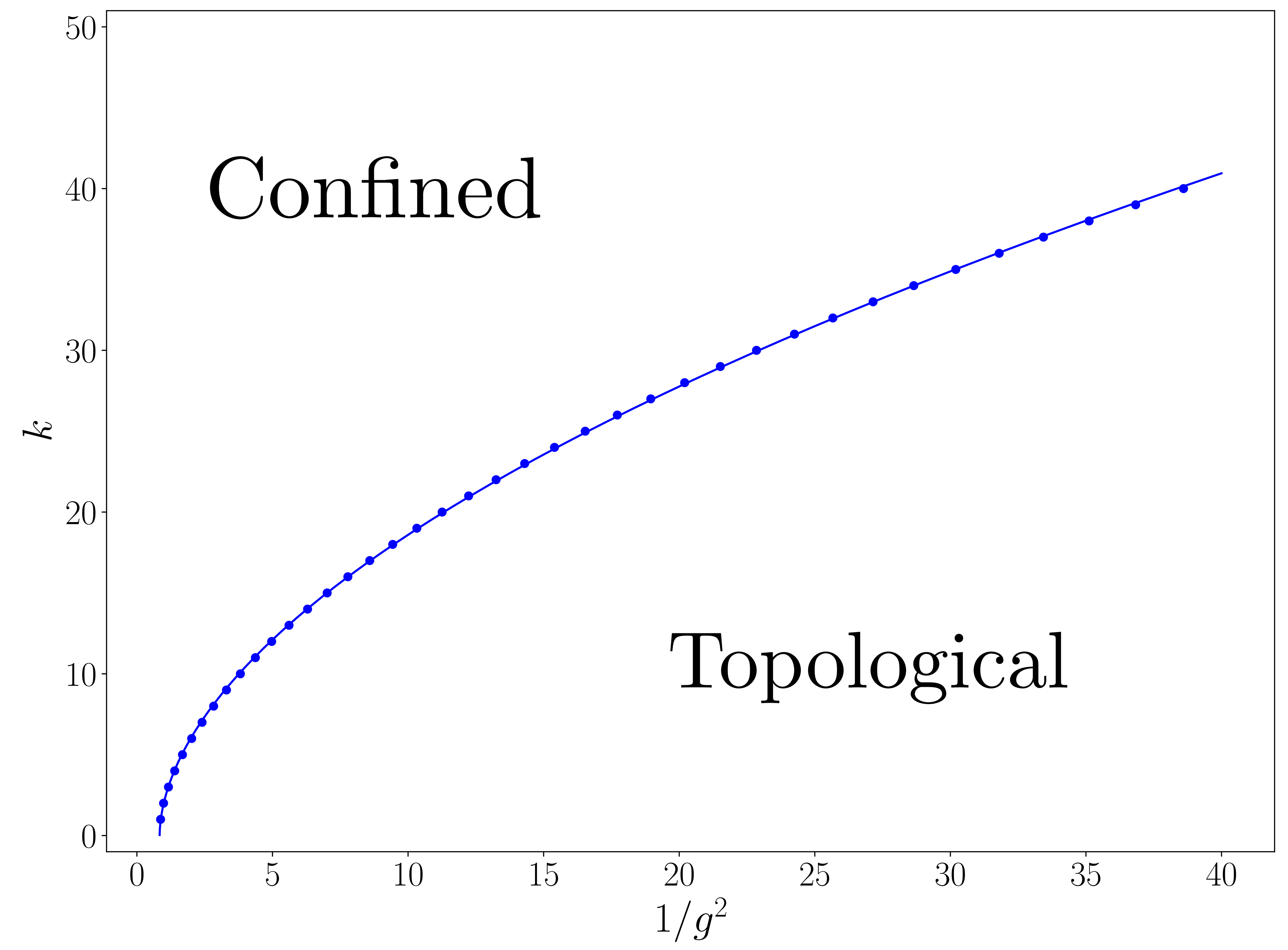
$$\langle \mathrm{tr}\, U_d(\partial S)\rangle = d_d \exp(-|S|\sigma_d)$$

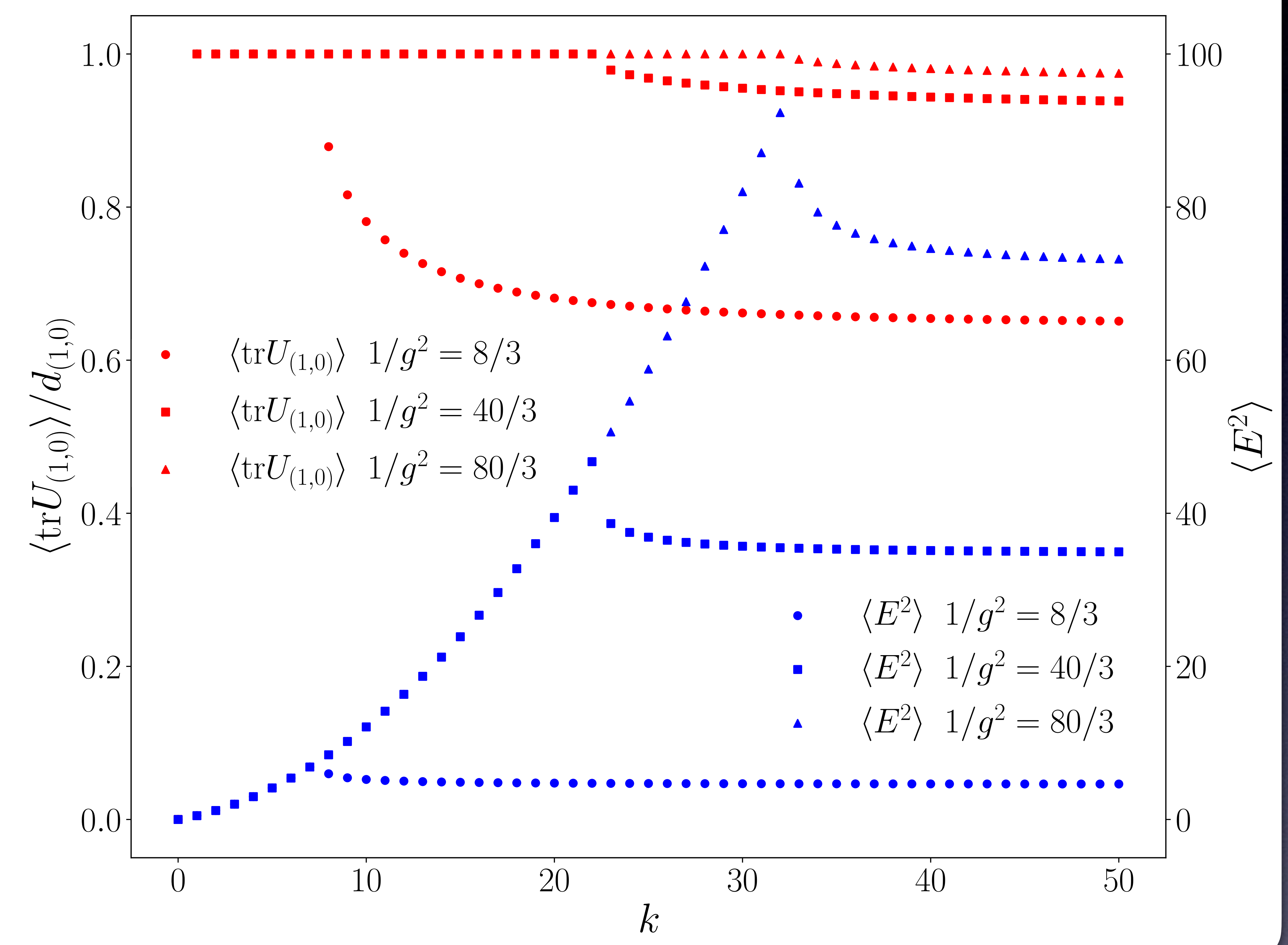


String tension $\sigma_d := \ln \dfrac{d_d}{\sum_{a,b} N^a_{db}\psi^*(a)\psi(b)}$

Numerical results

Numerical results





Topological phase:

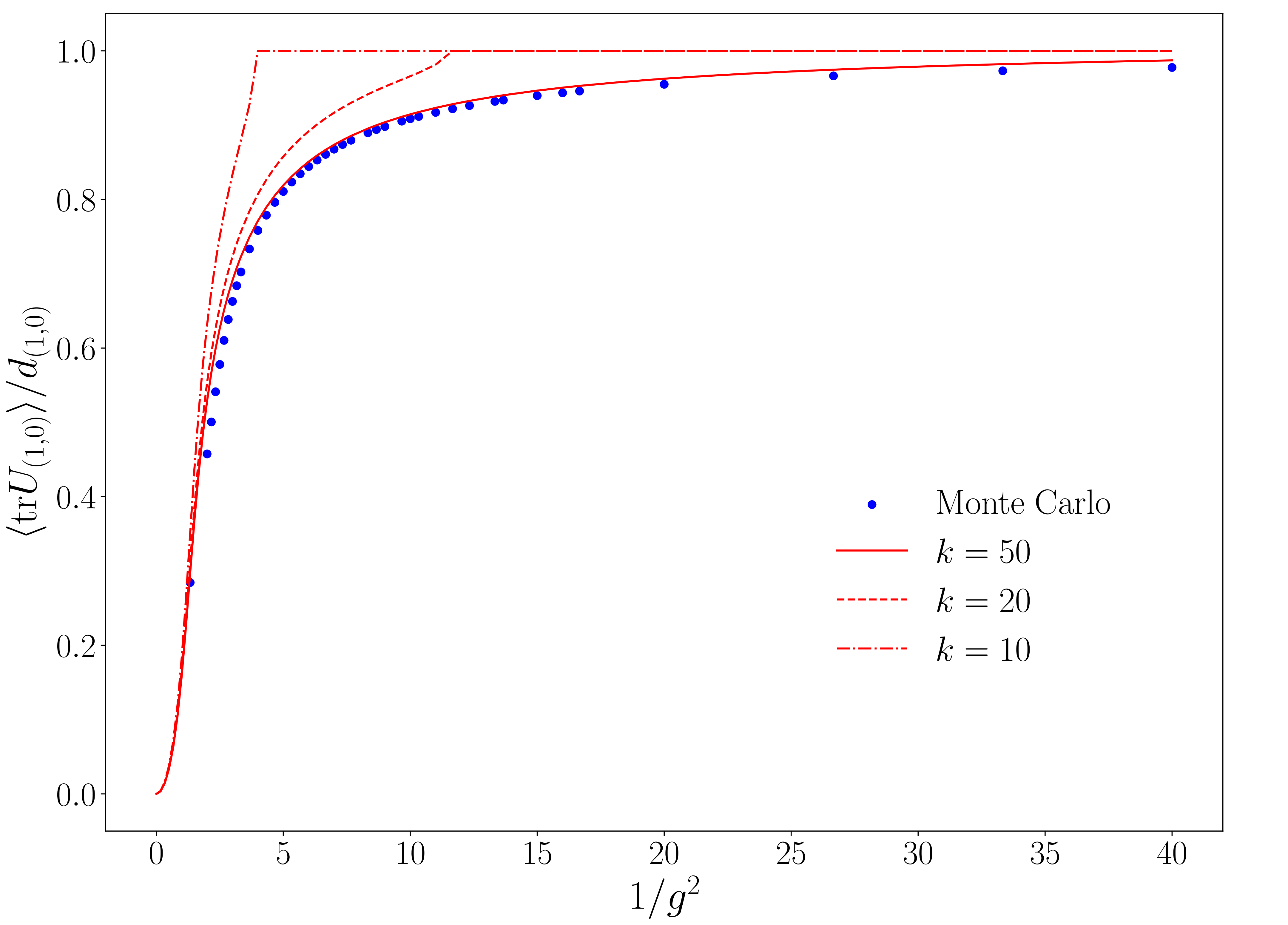
String-net condensation: $\psi(a) \sim d_a$

where string tension vanishes

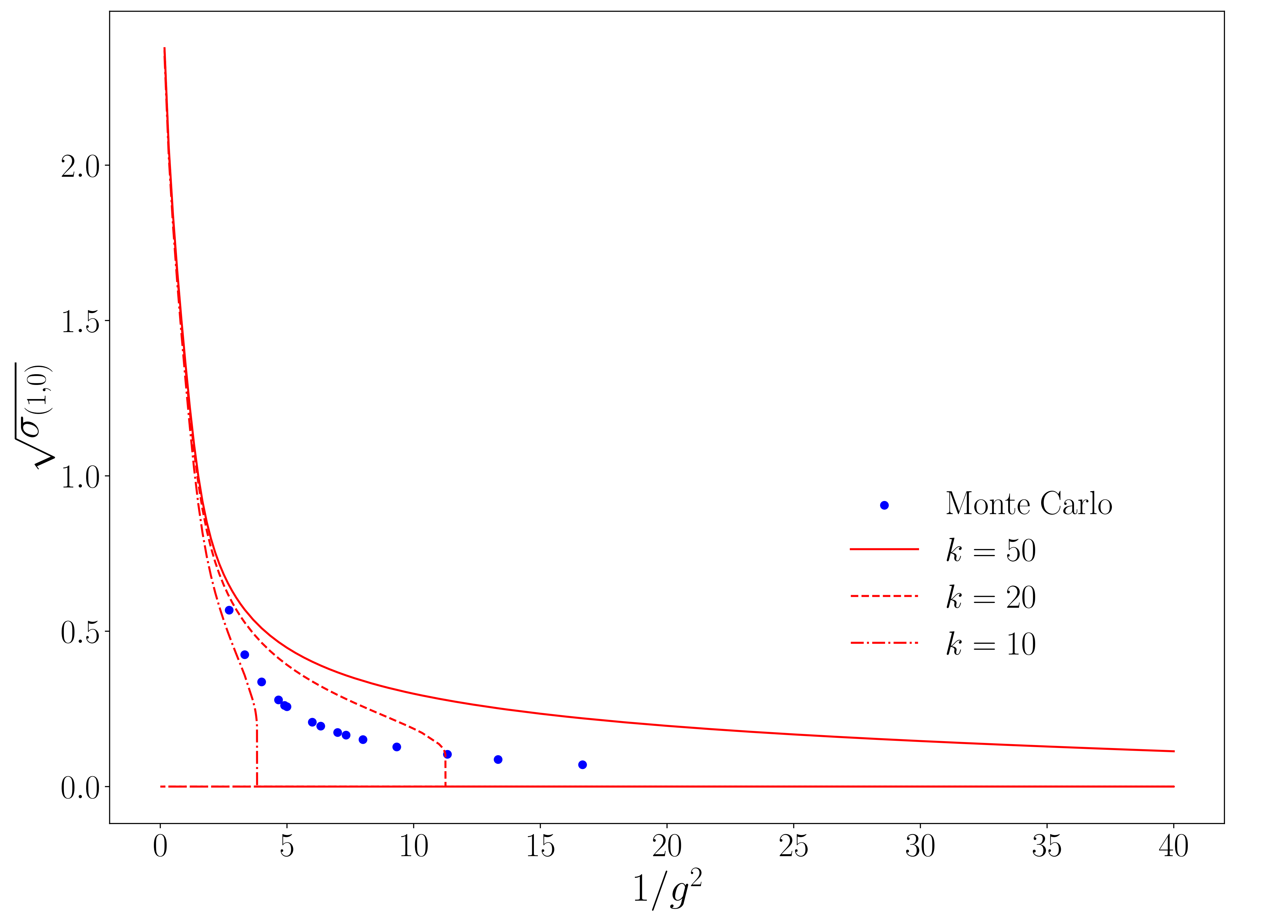
Phase transition occurs

Comparison with Monte-Carlo simulation

Plaquette (small Wilson loop)



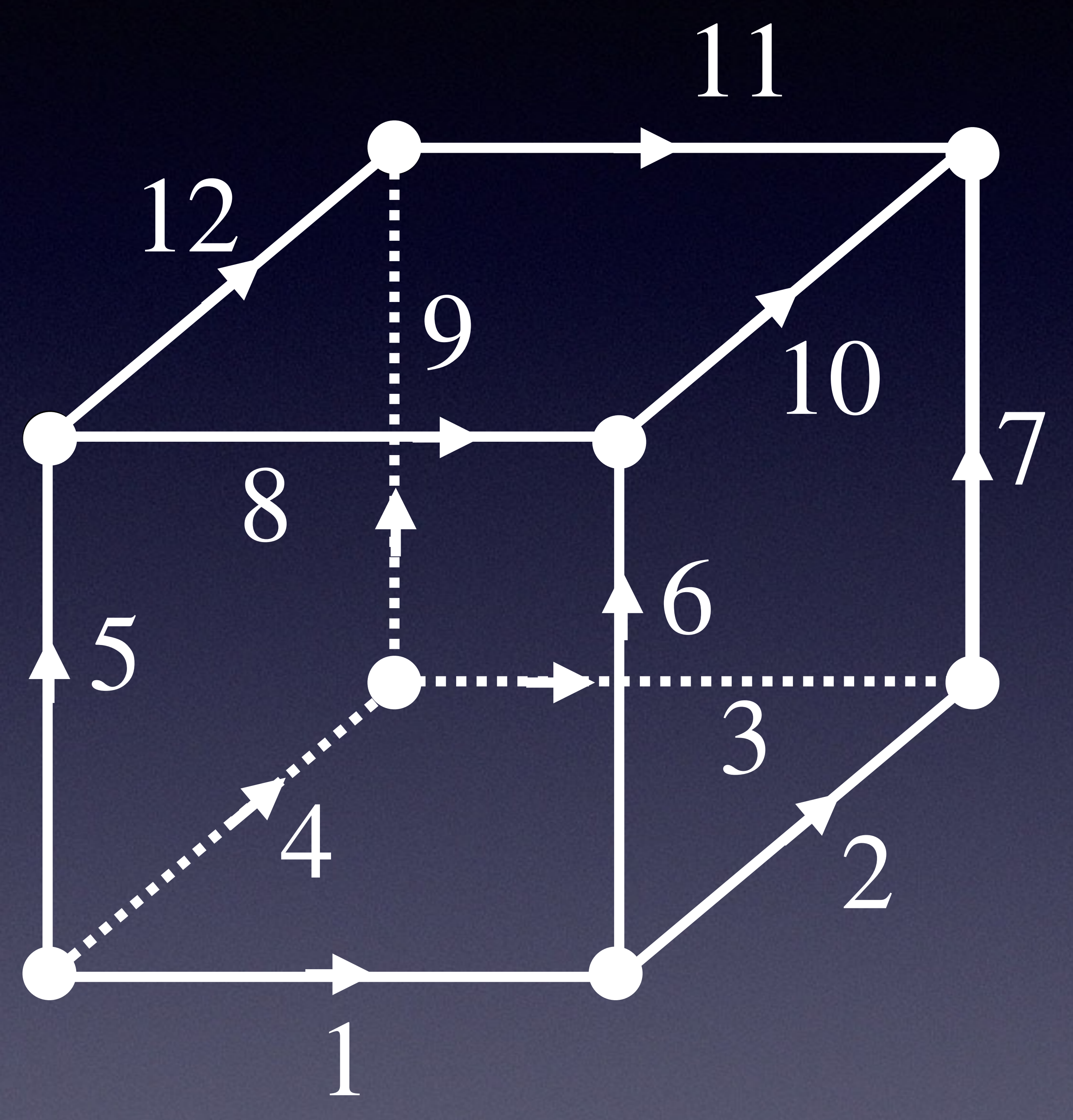
String tension



Good agreement for large k!

Thermalization on a small lattice

Small lattice system

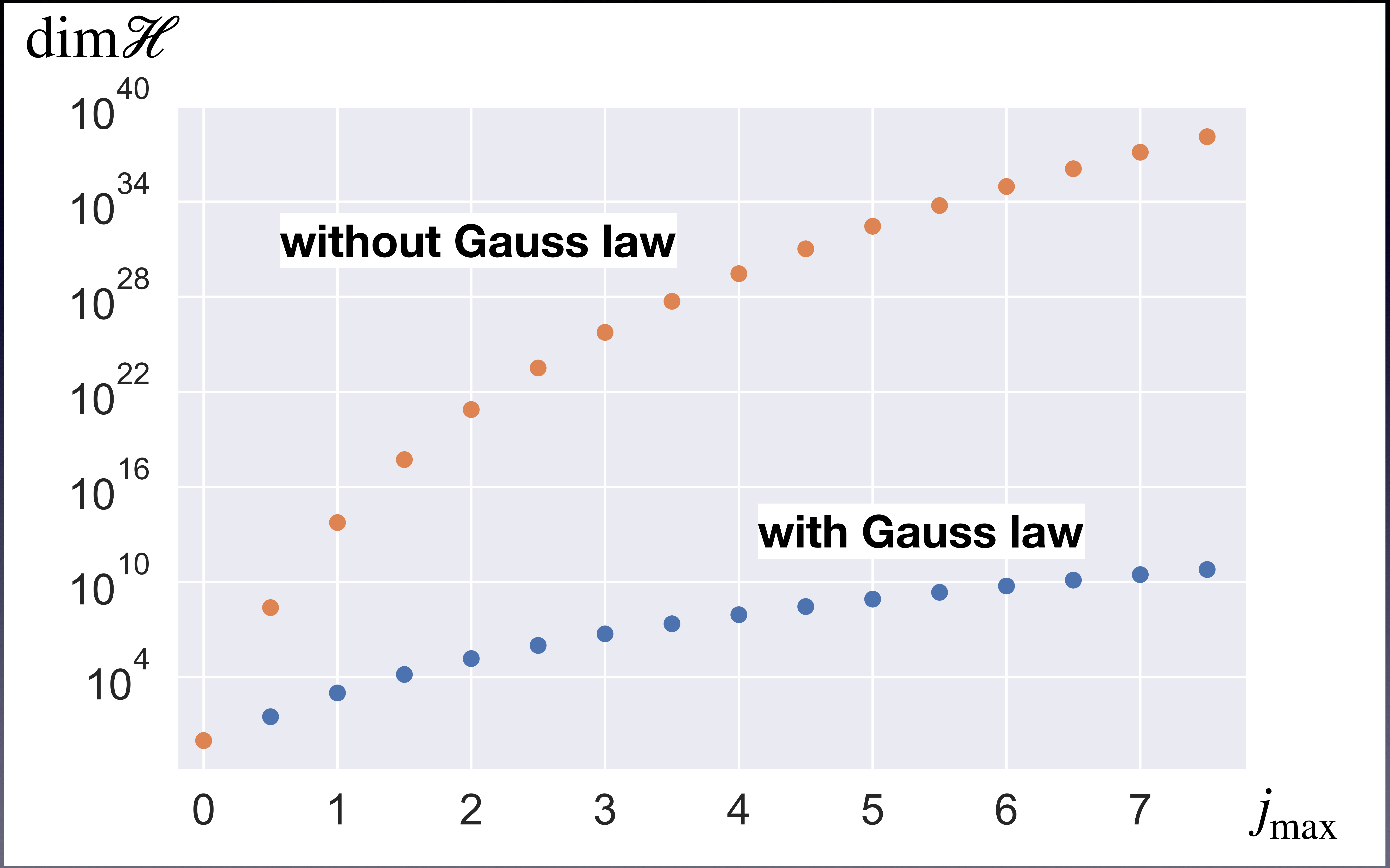


Basis

$$|j_1, \cdots, j_{12}\rangle = |j_1, j_2, j_6\rangle\,|j_2, j_3, j_7\rangle\,|j_3, j_4, j_8\rangle\,|j_1, j_4, j_5\rangle$$
$$|j_6, j_9, j_{10}\rangle\,|j_7, j_{10}, j_{11}\rangle\,|j_8, j_{11}, j_{12}\rangle\,|j_5, j_9, j_{12}\rangle$$

Cutoff $j_i \leq j_{\max} = k/2$

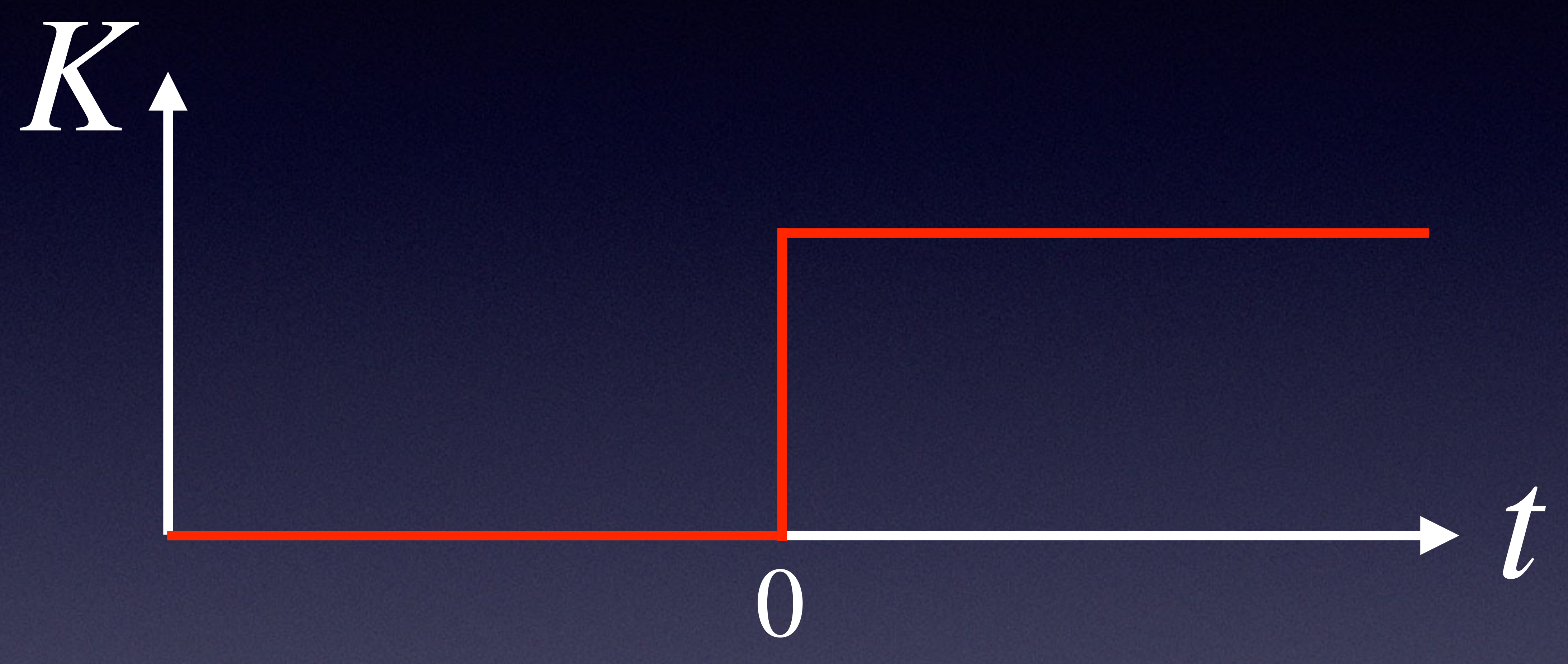
Dimension of Hilbert space



We employ $j_{\max} = 4$: $\dim\mathscr{H} = 87{,}426{,}119$

Setup

In order to mimic heavy ion collision experiments, the interaction quenching

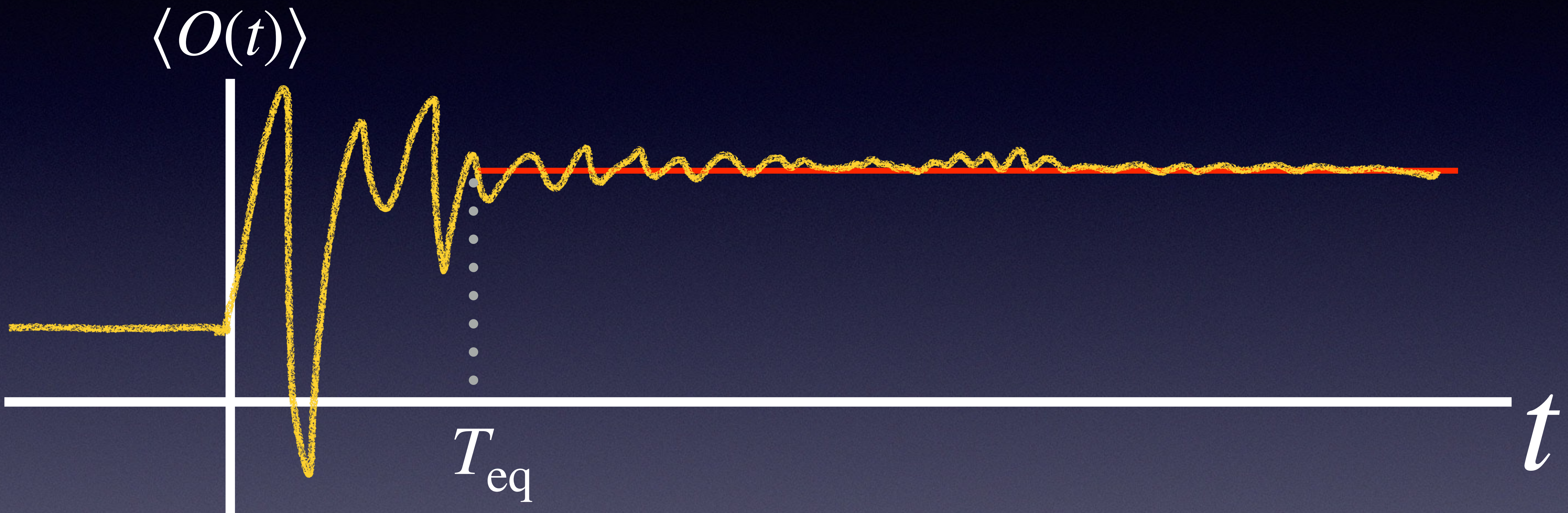


$$t < 0 \ |\mathrm{Vac}\rangle_{K=0}$$

$$t \geq 0 \ |\Psi(t)\rangle = e^{-\mathrm{i}Ht}|\mathrm{Vac}\rangle_{K=0}$$

Expected behavior

for an operator O $\quad \langle O(t) \rangle := \langle \Psi(t) \,|\, O \,|\, \Psi(t) \rangle$



$$\langle O(t) \rangle \simeq \langle O(t) \rangle_{\mathrm{eq}} ?$$

Temperature and Canonical Ensemble

Energy is fixed by an initial condition

$$E = \langle H \rangle = \langle \Psi(t) \,|\, H \,|\, \Psi(t) \rangle$$

(Independent of time)

For a given energy,
a canonical distribution that reproduces
the expected value can be defined

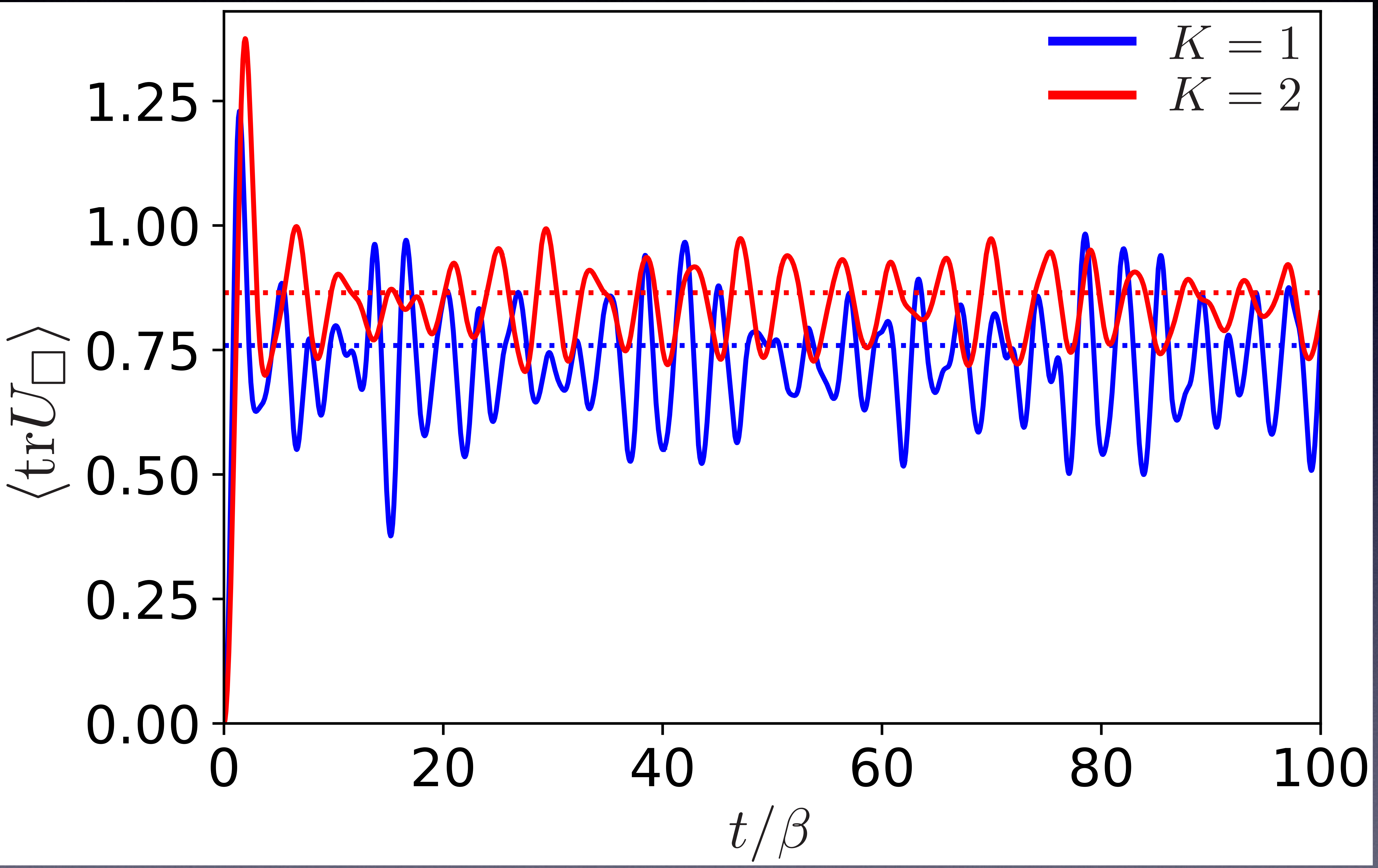
$$E = \langle H \rangle_{\mathrm{eq}} := \mathrm{tr} \rho_{\mathrm{eq}} H \quad \textbf{with} \quad \rho_{\mathrm{eq}} = \frac{e^{-\beta H}}{\mathrm{tr} e^{-\beta H}}$$

Numerical results

Expected value of Wilson loop

Strong coupling (low temperature $T < E_1$)

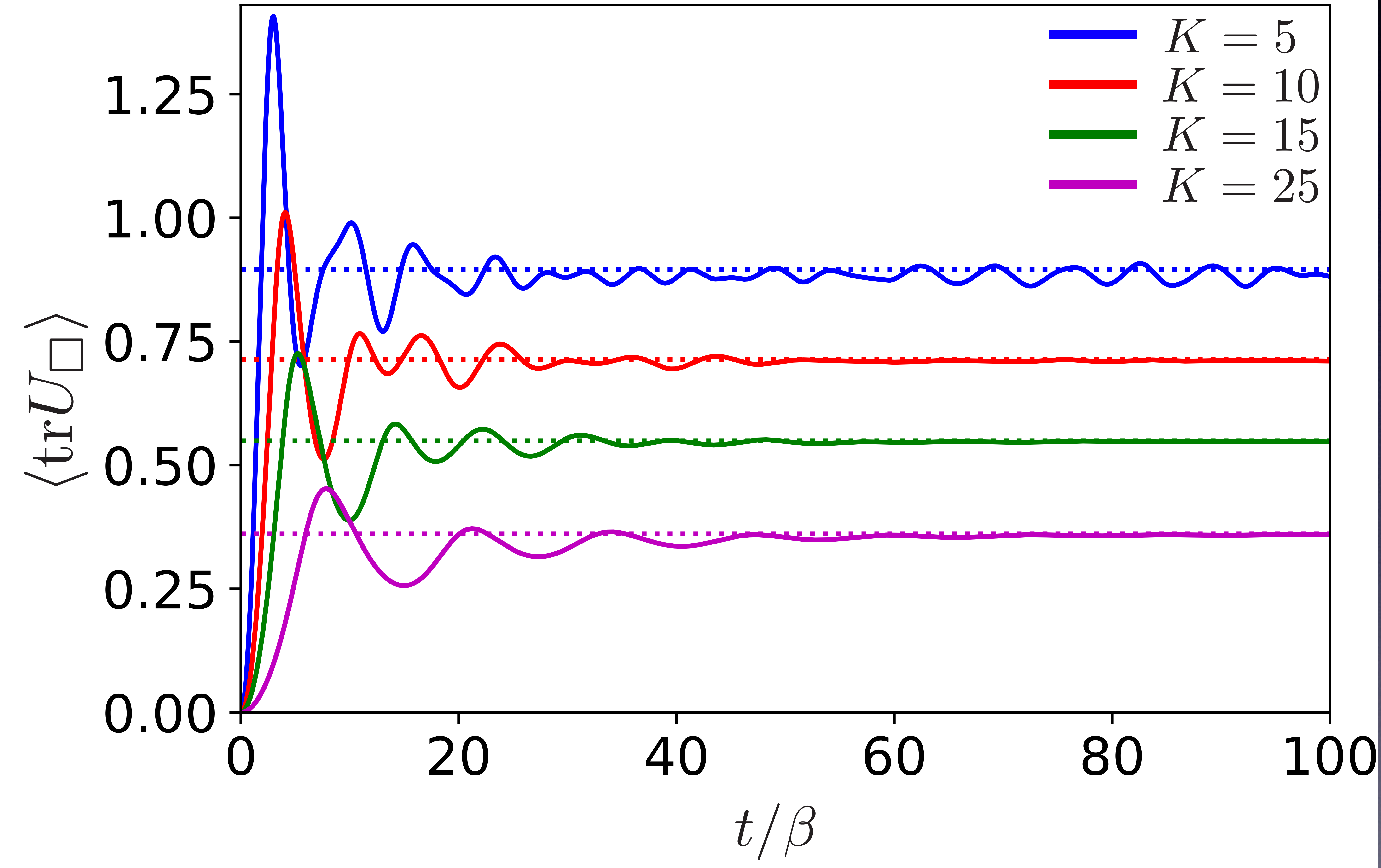
first excited energy



Fluctuations are not small.

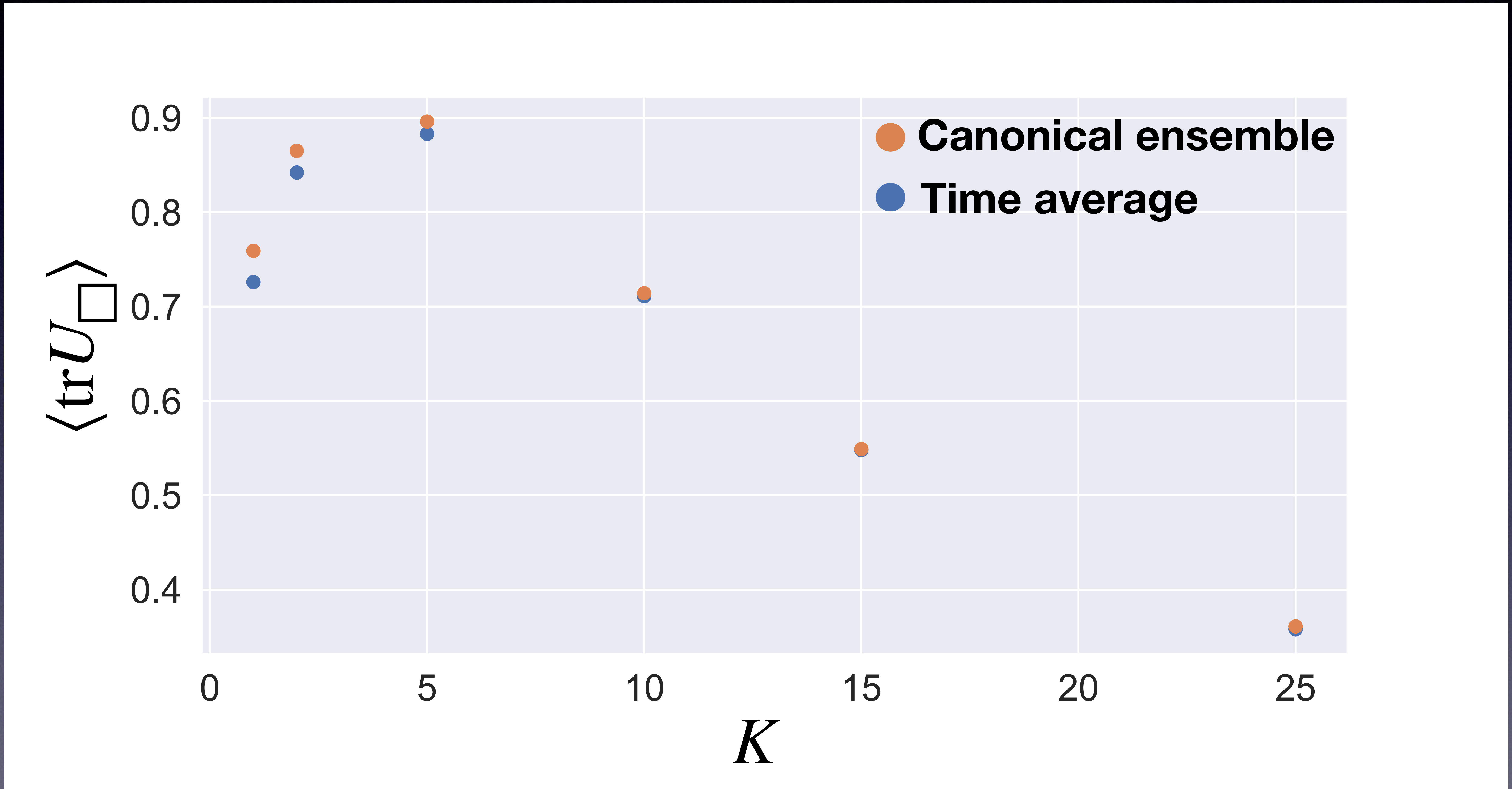
Expected value of Wilson loop

Weak coupling (high temperature $T > E_1$)



Steady state observed

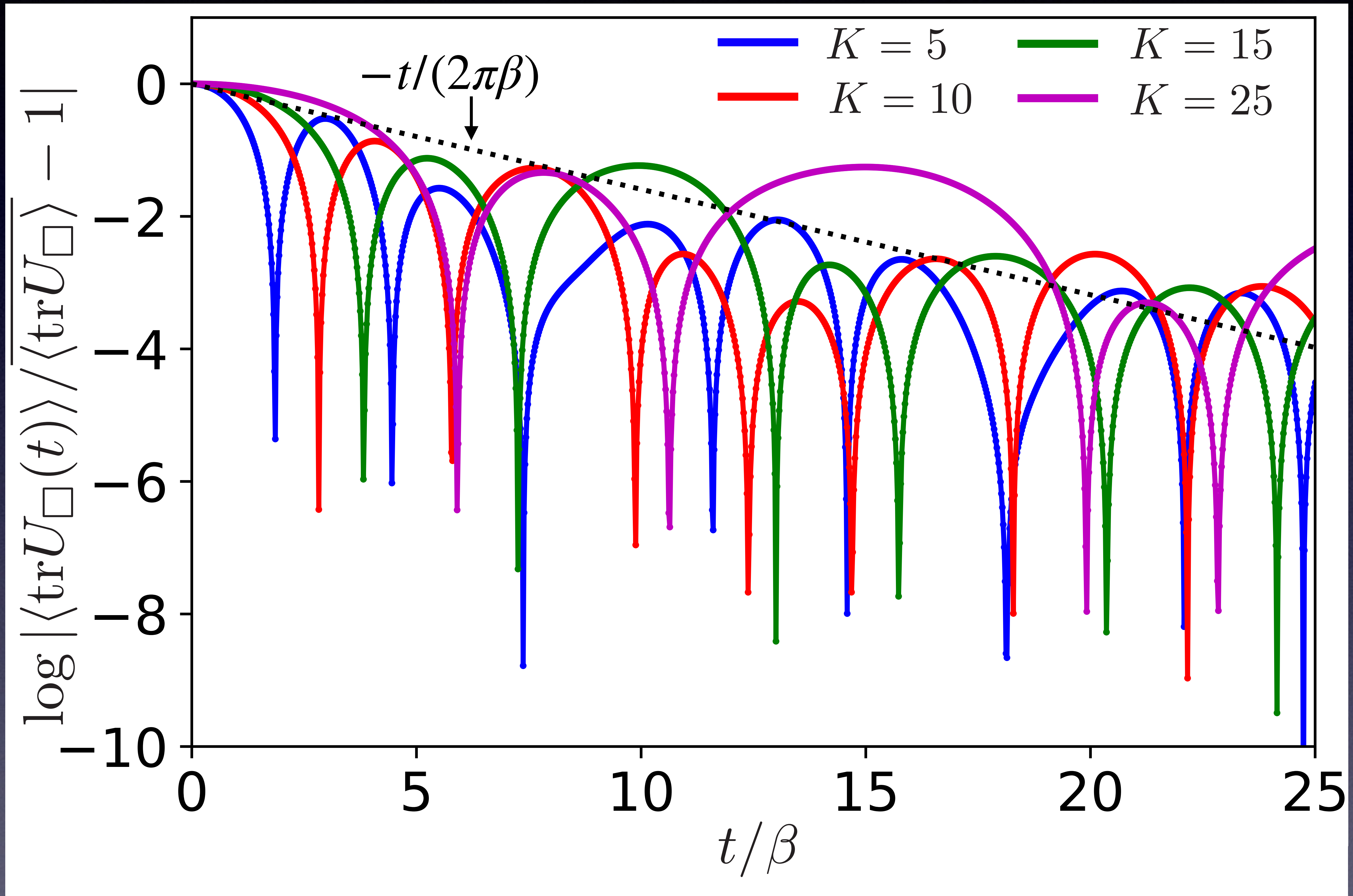
Long-time average vs canonical ensemble



Difference is less than 1% for $K > 5$

Relaxation to equilibrium

$$\langle \mathrm{tr} U_\square(t) \rangle - \overline{\langle \mathrm{tr} U_\square \rangle} \sim e^{-t/\tau_{\mathrm{eq}}}$$

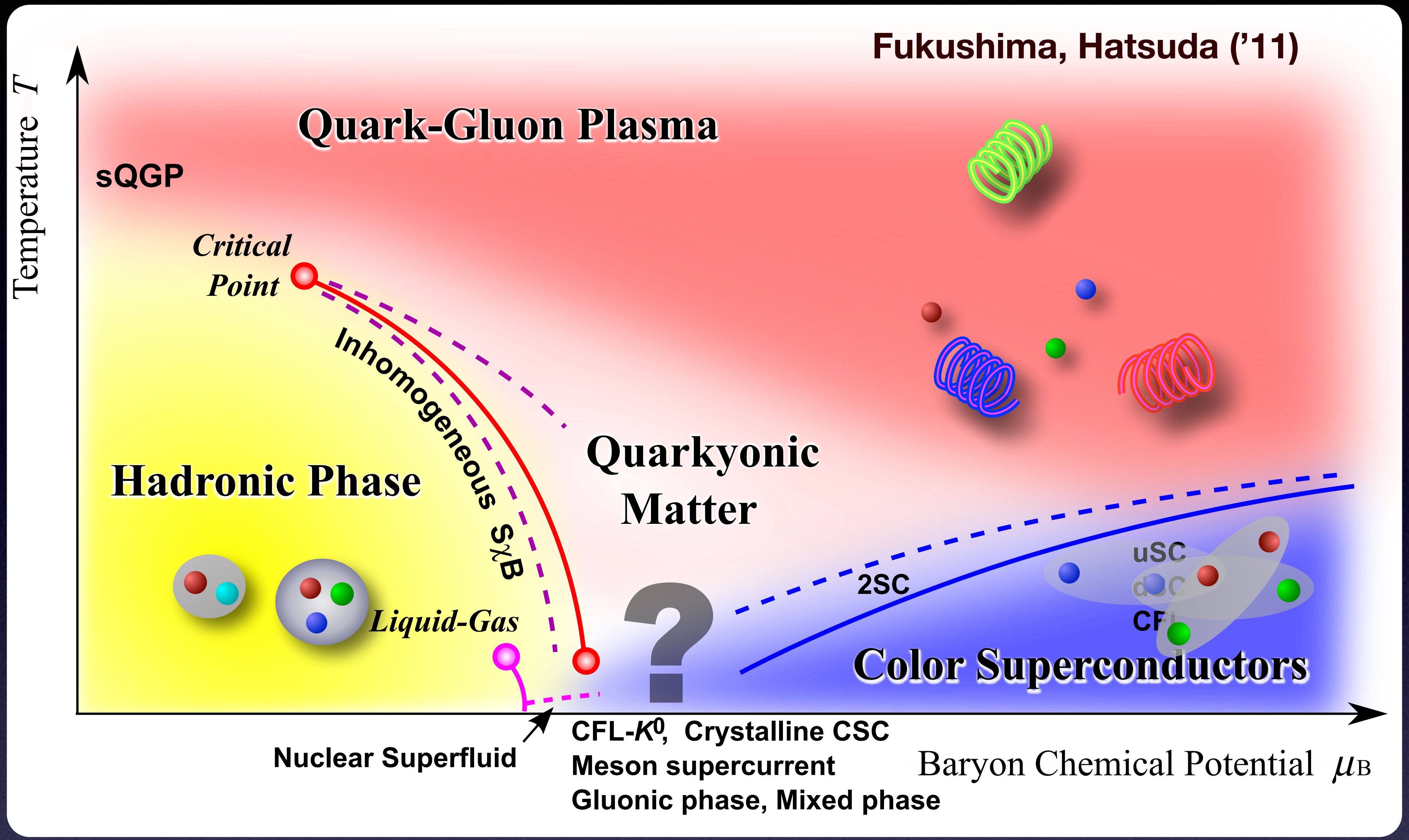


Close to Boltzmann time $2\pi\beta$.

Goldstein, Hara, Tasaki, New J. Phys. 17 (2015) 045002

QCD_2 at finite density

QCD at finite density



- What is the equation of state for QCD at finite density?
- How does the quark distribution function change when transitioning from baryonic matter to quark matter?
- What kind of phase is realized? An inhomogenous phase?

QCD_2

Properties of (1+1) dimensions

- Gauge fields are nondynamical
- Hilbert space is finite dimensional in Open Boundary Condition(OBC)

(dimensionless)QCD_2 Hamiltonian

$$J = \frac{ag_0}{2}, w = \frac{1}{2g_0 a}, m = m_0/g_0$$ **We use $g_0 = 1$ unit**

$$H/g_0 = J \sum_{n=1}^{N-1} E_i^2(n)$$ **Electric field term**

$$+w \sum_{n=1}^{N-1} \left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1) \right)$$

Hopping term

$$+m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n)$$ **Mass term**

Elimination of Link variables U

Sala, Shi, Kühn, Bañuls, Demler, Cirac, Phys. Rev. D 98, 034505 (2018)
Atas, Zhang, Lewis, Jahanpour, Haase , Muschik , Nature Commun. 12, 6499 (2021)

$$\Theta\chi(n)\Theta^\dagger := U(n-1)U(n-2)\cdots U(1)\chi(n)$$

$$\Theta H\Theta^\dagger = J\sum_{n=1}^{N-1}\left(\sum_{m=1}^{n}\chi^\dagger(m)T_i\chi(m)\right)^2$$ **Electric fields term**

$$+w\sum_{n=1}^{N-1}\left(\chi^\dagger(n+1)\chi(n)+\chi^\dagger(n)\chi(n+1)\right)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **mass term**

As a variational ansatz of wave function

- We employ a matrix product state

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$ **:** $D \times D$ **matrix**

- Optimize the wave function by density matrix renormalization group technique

$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

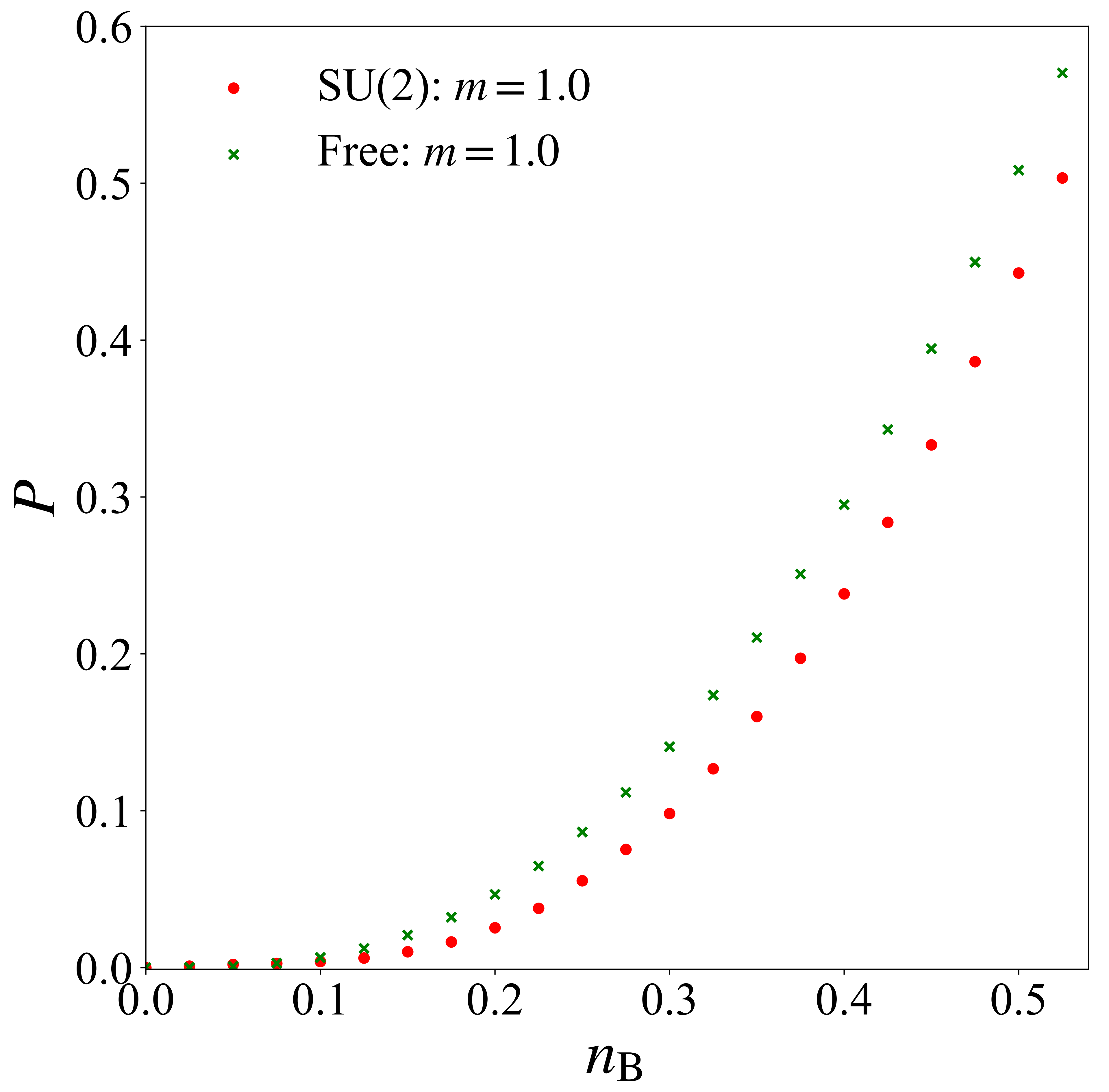
We employ iTensor

Numerical results

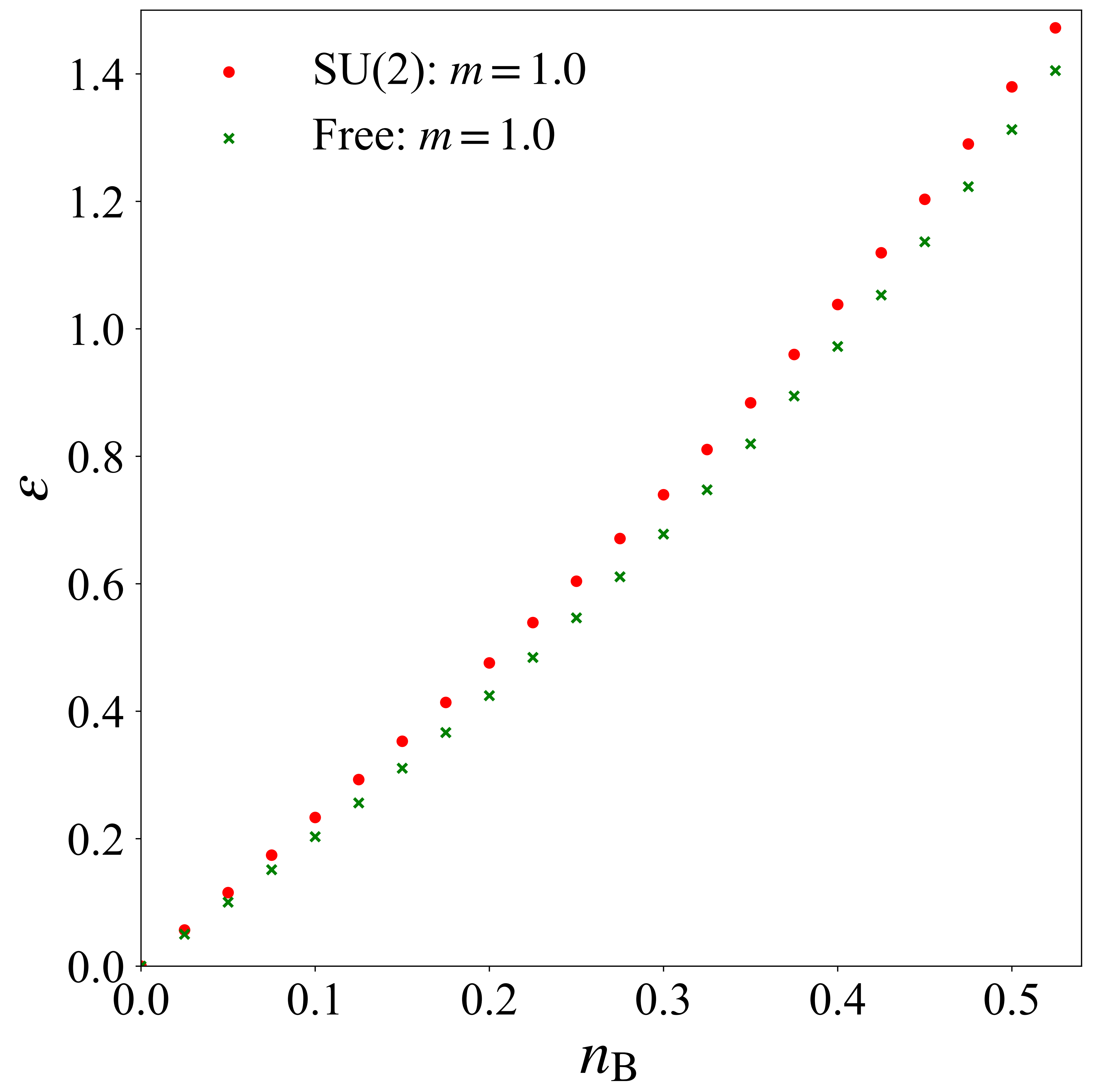
Color SU(2), 1 flavor, vacuum

$$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$$

Pressure

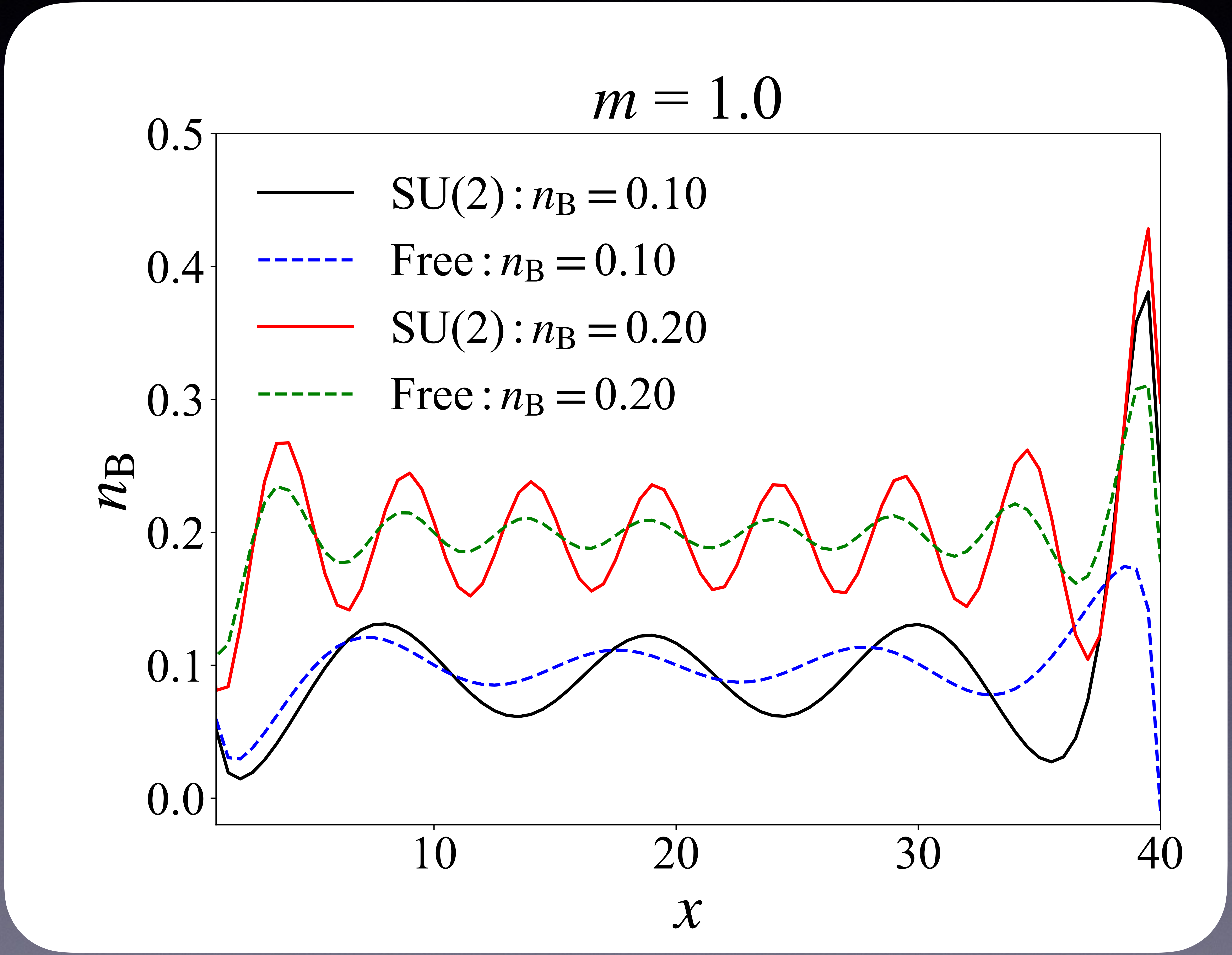


Energy density



Inhomogeneous phase (density wave)

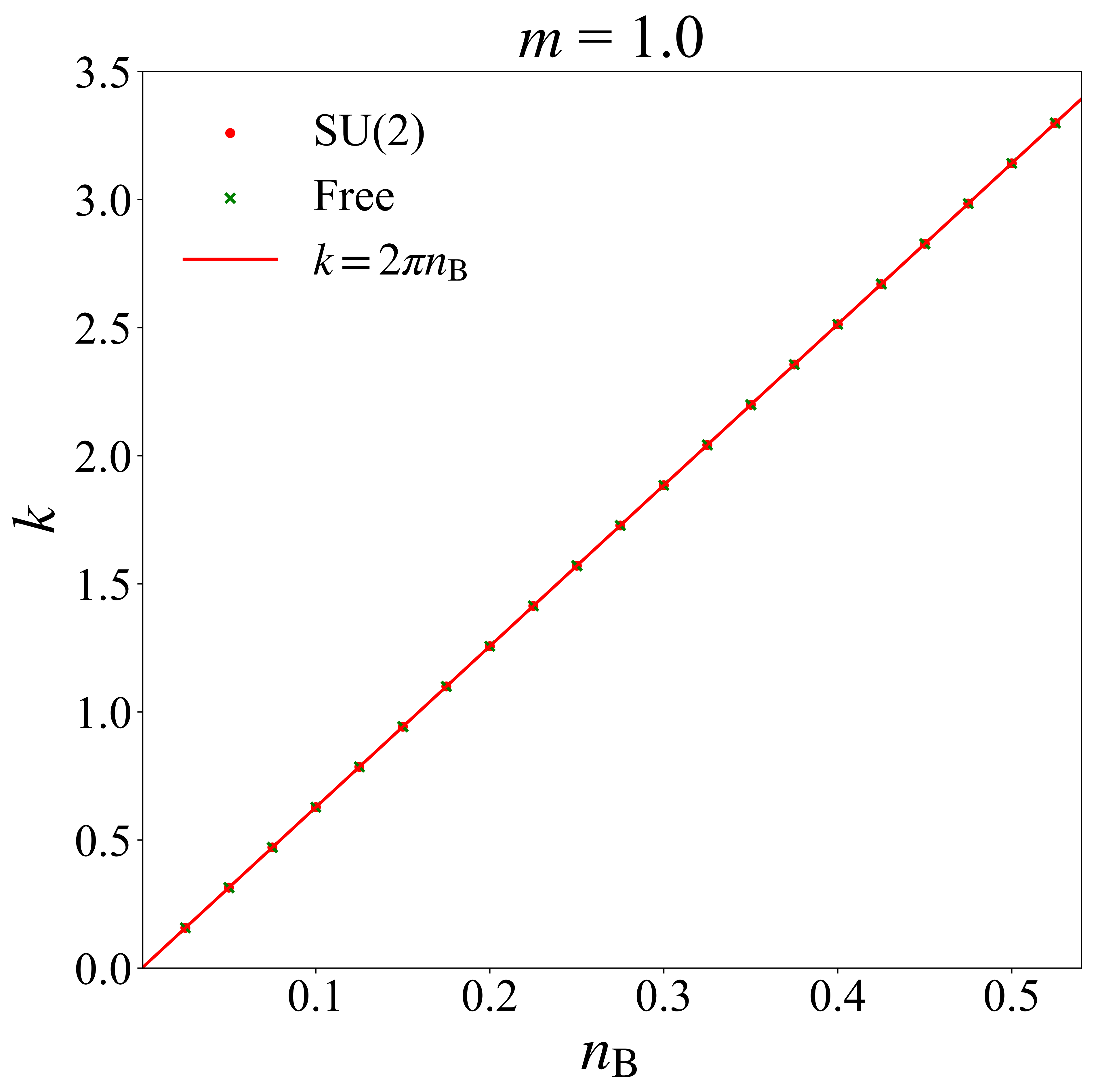
$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$



Wave number dependence

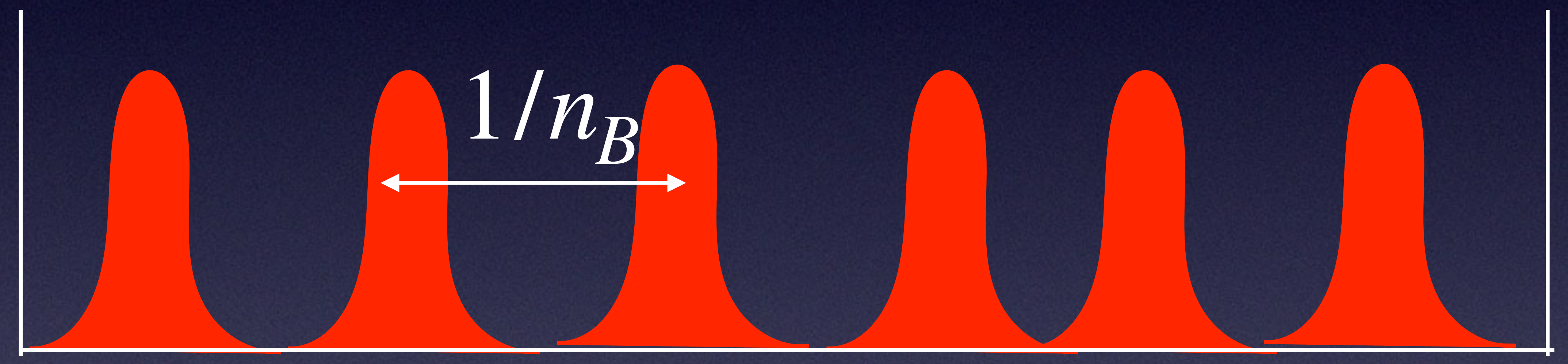
$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$

Wave number dependence



Hadronic picture

If hadron interactions are repulsive



distance $1/n_B \Rightarrow k = 2\pi n_B$

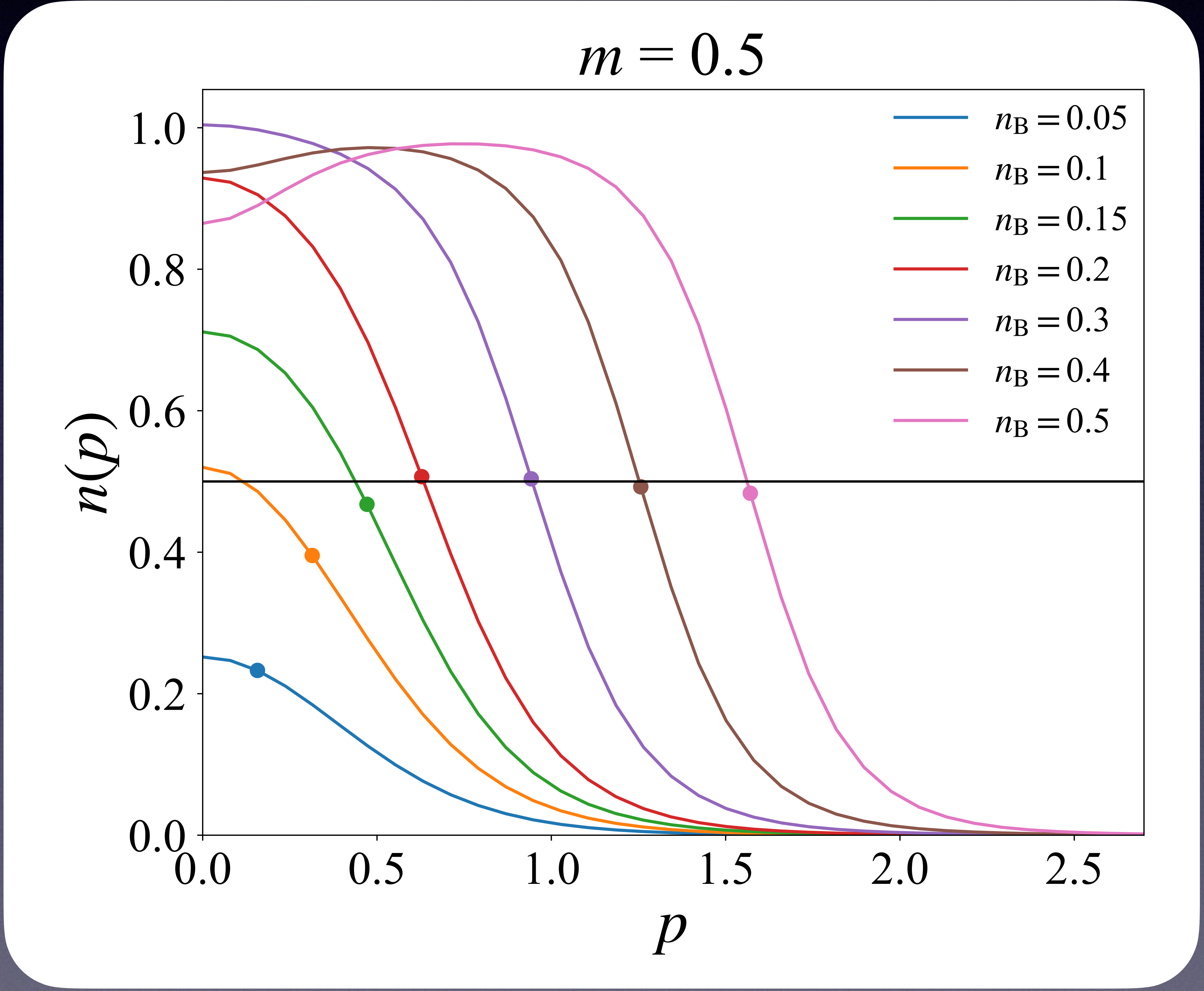
Quark picture

If interactions between quarks
Fermi surface is unstable

$\Rightarrow$**density wave** $k = 2p_{\rm F} = 2\pi n_{\rm B}$

Quark distribution function

$J = 1/8 \quad w = 2 \quad V = 60 \qquad \dim\mathscr{H} = 2^{480}$



- **Low density**
  No Fermi sea
- **High density**
  Fermi-sea
  +BCS like pairing
  (density wave)

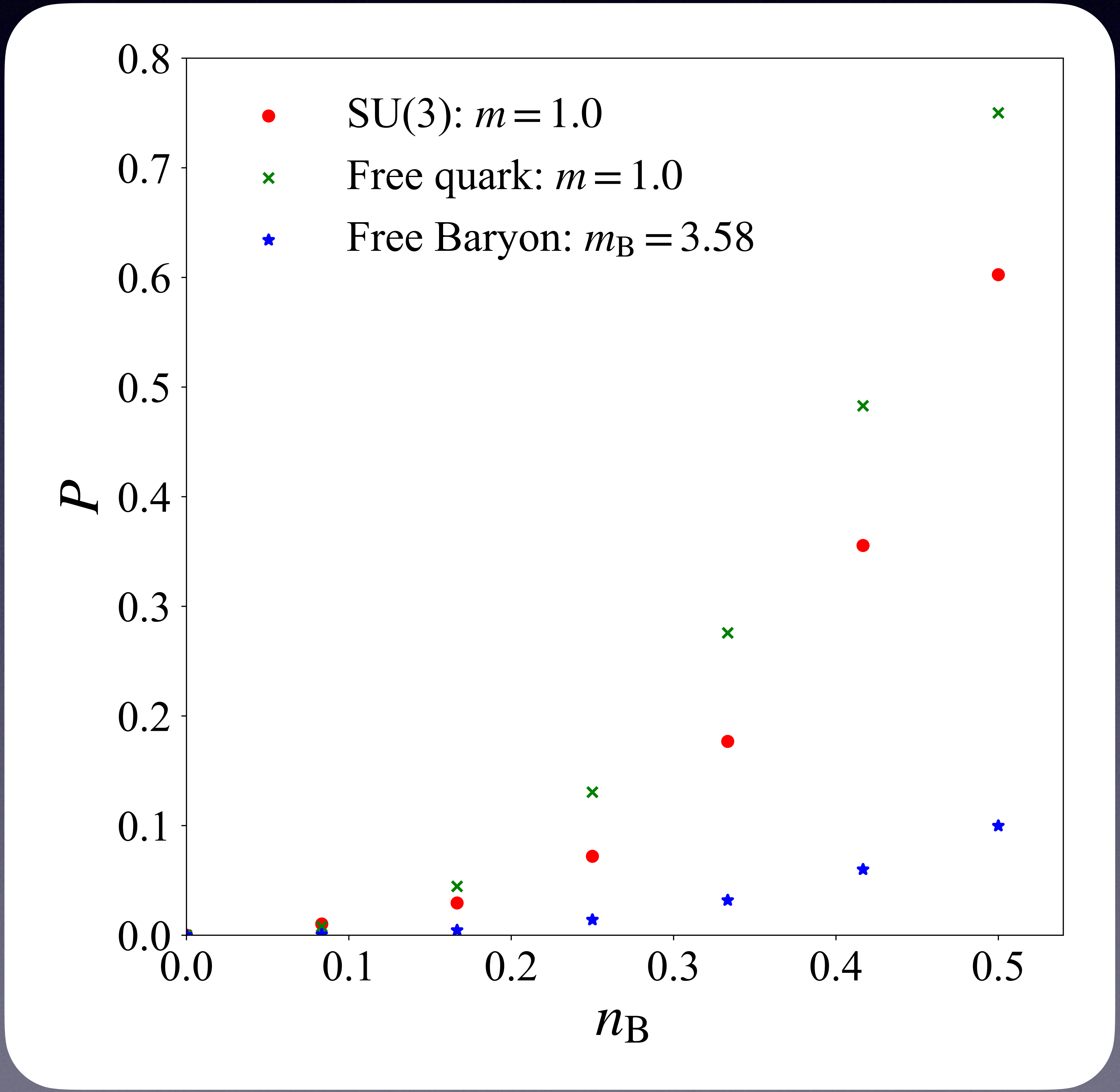
baryon quark transition around $n_B \sim 0.2$

SU(3) QCD with $N_f = 1$

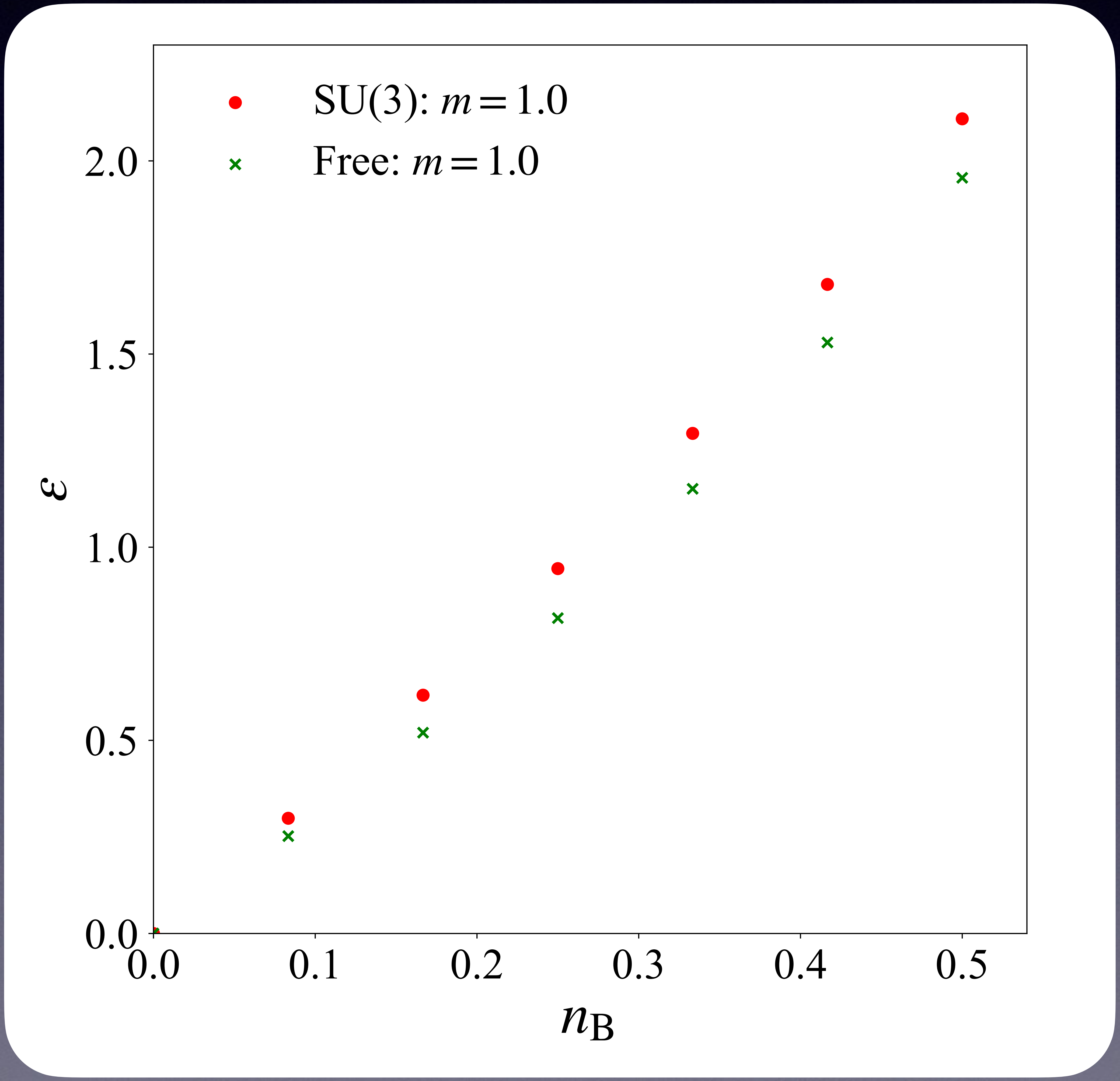
Color SU(3), 1 flavor

$$J = 1/8 \;\; w = 2 \;\; V = 12 \;\; \dim\mathscr{H} = 2^{144}$$

Pressure



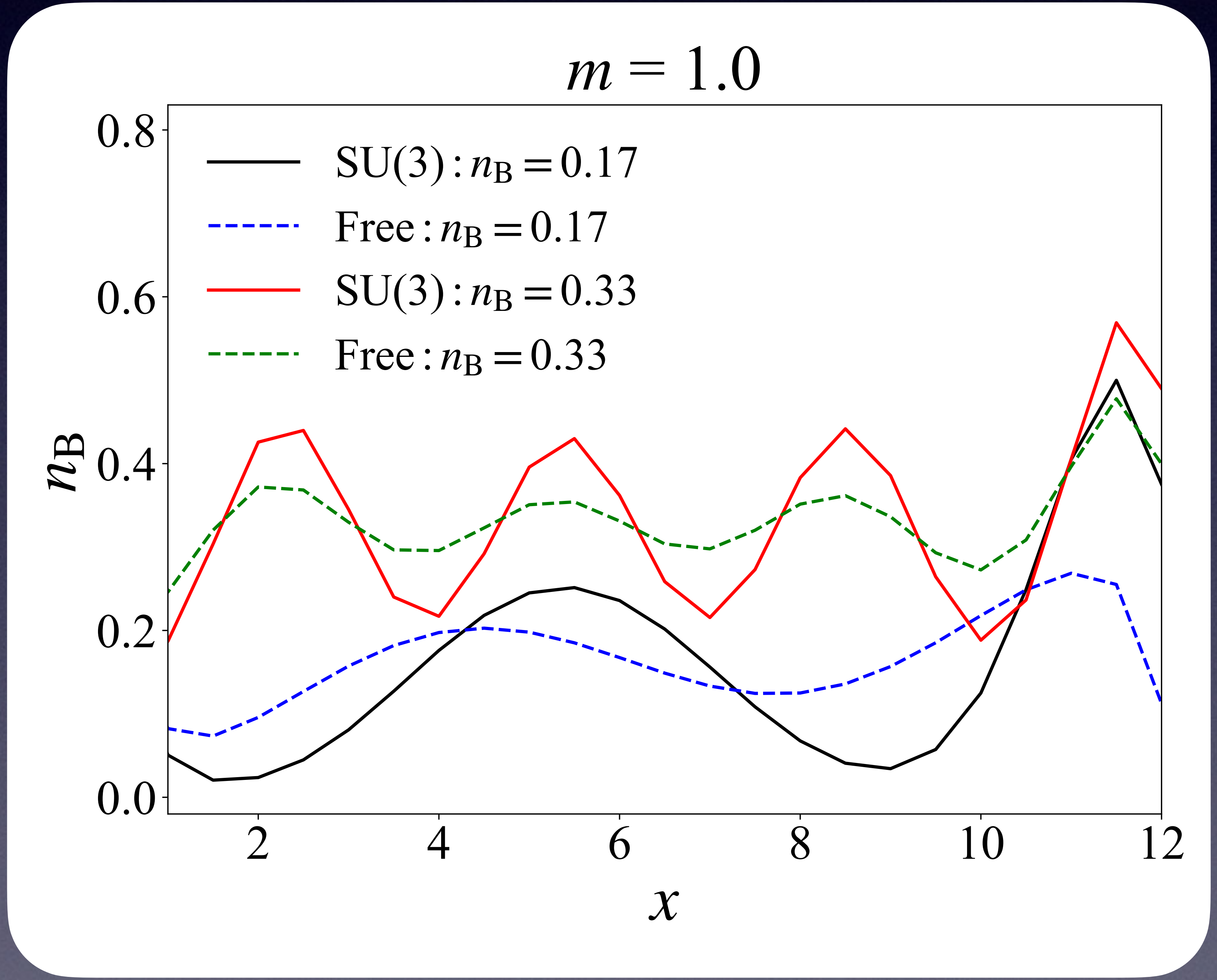
Energy density



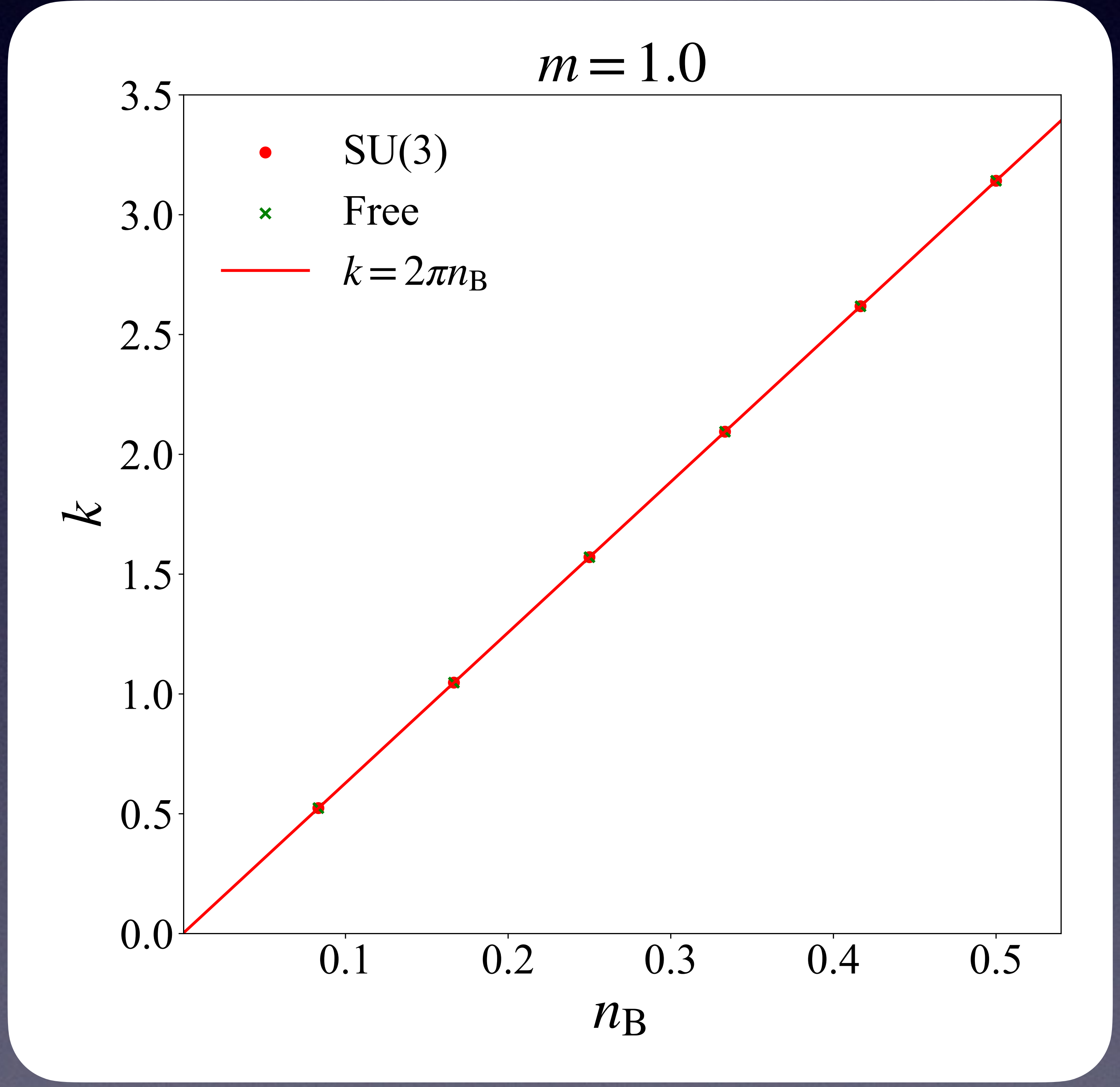
Color SU(3), 1 flavor

$$J = 1/8 \quad w = 2 \quad V = 12 \quad \dim\mathscr{H} = 2^{144}$$

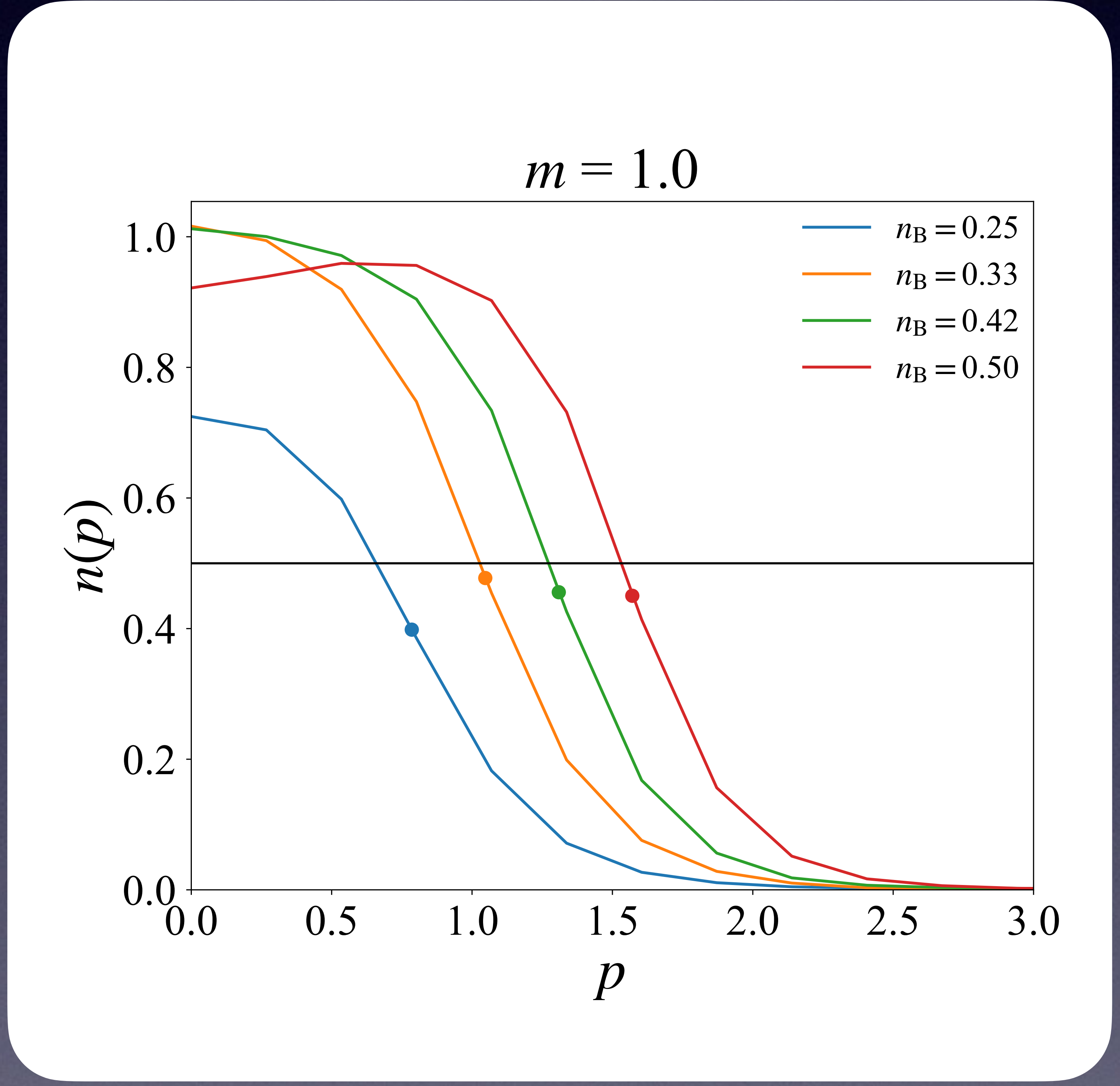
density wave



Wave number dependence



Quark distribution



Baryon quark transition around $n_B = 0.3$?

Summary

- **Formalism**

  Kogut-Susskind Hamiltonian formalism

- **Application**

  $\mathrm{SU}(3)_k$ **gauge theory in** $(2+1)$ **dimensions**

  Confinement-topological phase transition

  Thermalization of Yang-Mills theory in (3+1)-dimensional small systems

  Relaxation time of thermalization

  $\tau_{\mathrm{eq}} \sim 2\pi/T$ Boltzmann time

  QCD_2 **at finite density**

  baryon quark transition, inhomogeneous phase