Fortuity with a Single Matrix

Yiming Chen
Stanford University

Based on arXiv:2511.00790

Matrix Model for Superstring/M-theory, 2025.12.01

What are some signatures of black holes we can look for in matrix models, with the advancement of various techniques such as Monte Carlo, bootstrap and quantum simulation?

Extensive entropy, exponential decay of two-point function, …

What are some signatures of black holes we can look for in matrix models, with the advancement of various techniques such as Monte Carlo, bootstrap and quantum simulation?

Extensive entropy, exponential decay of two-point function, …

In the context of **supersymmetric (BPS)** black hole microstates, an interesting new feature called “**fortuity**” was proposed.

It is a sharp feature even at finite N.

[Chang, Lin ’24, building on many earlier works]

Lightening review of the fortuity idea

One can classify BPS states in holographic theories into two types:

monotone / fortuitous

Examples:

- 4d $\mathcal{N} = 4$ super Yang-Mills [Chang, Lin]
- 2d D1-D5 CFT [Chang, Lin, Zhang; Hughes, Shigemori]
- Supersymmetric SYK model [Chang, Chen, Sia, Yang]
- Higher spin theories [Kim, Lee, Lee, Oh]
- ……

[2402.10129 by Chi-Ming Chang and Ying-Hsuan Lin]

The classification arises by studying the theory at different N, or different strength of the gravitational coupling G_N.

There is a natural inclusion of the Hilbert space of the smaller N theories into the larger ones.

$$\ldots \subset \mathcal{H}_N \subset \mathcal{H}_{N+1} \subset \ldots \subset \mathcal{H}_\infty$$

In matrix models (singlet sector), this structure becomes most apparent using the multi-trace basis.

e.g. $\mathrm{Tr}[Z^2]$

The Hilbert space becomes smaller due to redundancies of the basis, namely **trace relations**.

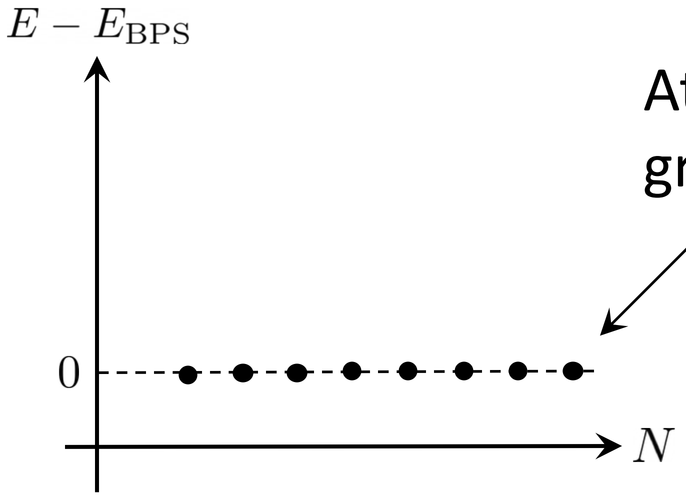
- Monotone type: BPS for any values of N.

$$QO_N = 0, \quad \forall N$$

Example: $\mathrm{Tr}[Z^2]$

- Monotone type: BPS for any values of N.

$$QO_N = 0, \quad \forall N$$

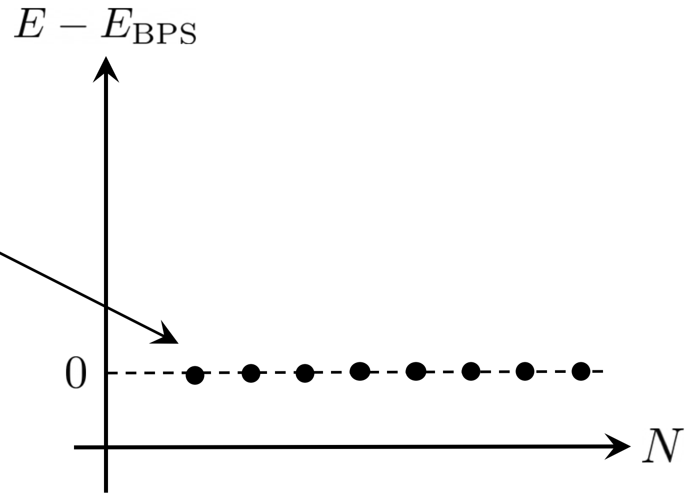


At $N = \infty$, all the BPS states are light graviton excitations.

- Monotone type: BPS for any values of N.

$$QO_N = 0, \quad \forall N$$

We can then follow them until they become heavy objects.



Conjectured to correspond to microstates of horizonless geometries in $\mathrm{AdS}_5 \times S^5$.

[Lin, Lunin, Maldacena; …]

- Fortuitous type: BPS only when N small enough.

Underlying mechanism: **trace relations**

$$QO_N \in \mathcal{I}_{N_*}$$

space of trace relations in the $N = N_*$ theory

- Fortuitous type: BPS only when N small enough.

Underlying mechanism: **trace relations**

$$QO_N \in \mathcal{I}_{N_*}$$

↖ space of trace relations in the $N = N_*$ theory

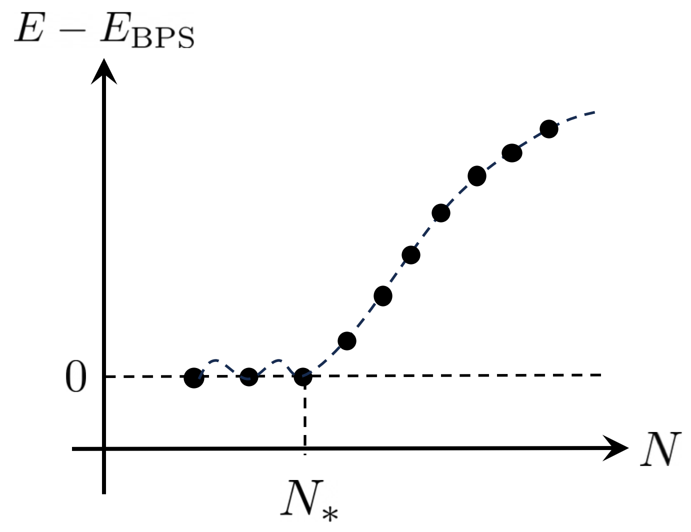
Example of trace relation: $w_1 = \mathrm{Tr}[Z^3],\ w_2 = \mathrm{Tr}[Z^2]\mathrm{Tr}[Z],\ w_3 = \mathrm{Tr}[Z]^3$

They are linearly independent when $N > 2$

But, when $N \leq 2$ $\quad w_3 - 3w_2 + 2w_1 = 0$

- Fortuitous type: BPS only when N small enough.

$$QO_N \in \mathcal{I}_{N_*}$$



Responsible for the black hole entropy in the 1/16 BPS sector of $\mathcal{N} = 4$ SYM.

How to quickly spot fortuity

In different theories, the notion of “Hilbert space inclusion” is different, so the precise definition of fortuity can differ.

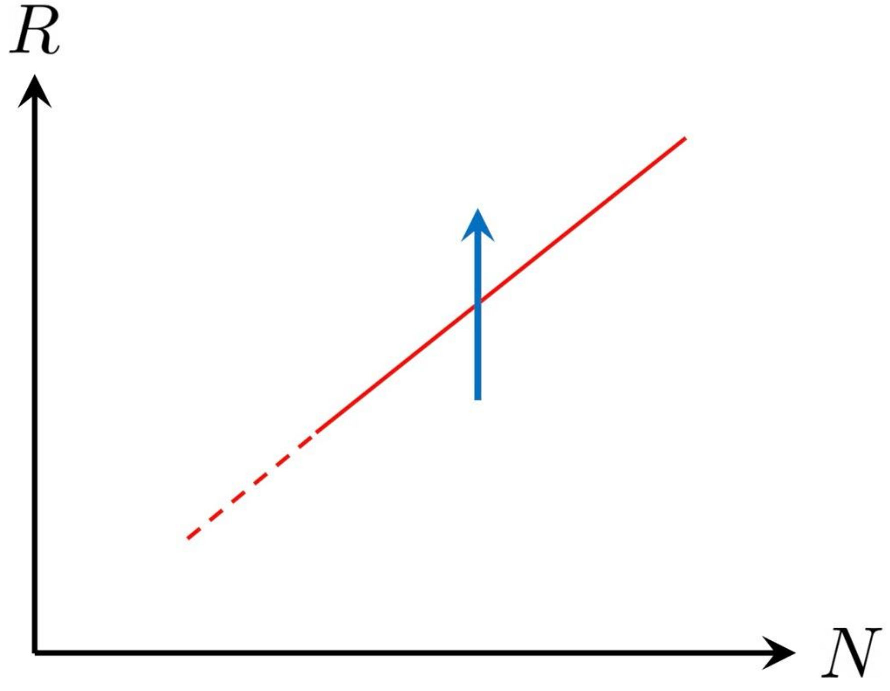
But there is a common trait that help us identify them easily.

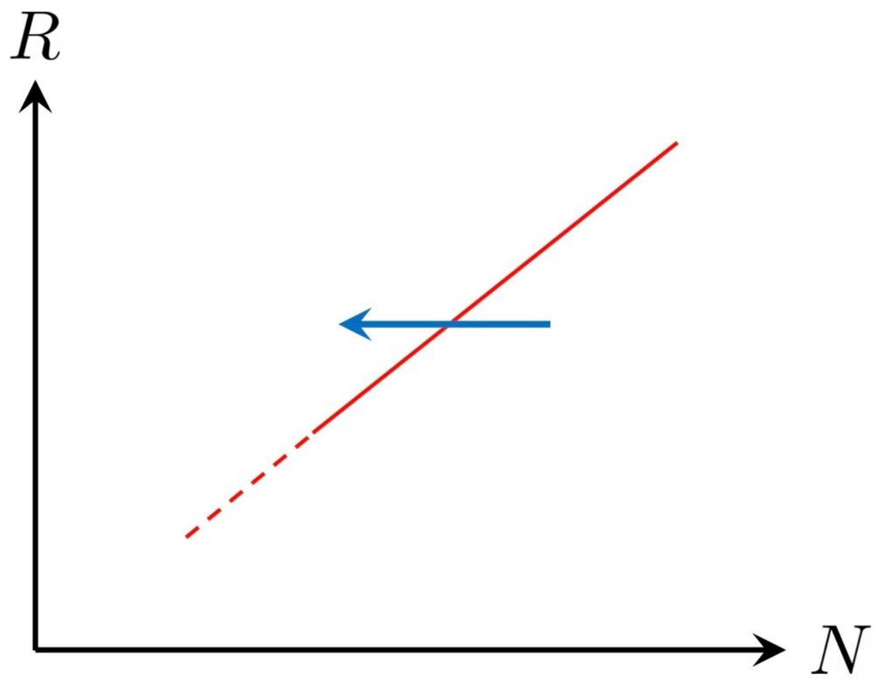
Most notably, they “**concentrate**” – they tend to only exist with quantum numbers that depend explicitly on N.

R-charge concentration

More precisely, after we fix all the charges that commute with supercharge Q, we are left with an R-charge R that does not commute with the supercharge. $[R, Q] = Q$

We expect fortuitous states to concentrate sectors with very specific values of R-charge.


R
N



As N is decreased, we suddenly encounter an exponential number of BPS states.

As N is further decreased, they all exit the physical Hilbert space by becoming "null".

Let me advertise that there is a nice toy model that illustrates all of these features clearly - the $\mathcal{N} = 2$ SUSY SYK Model.

$$\{\psi_i, \bar{\psi}_j\} = \delta_{ij} \qquad i, j = 1, ..., N$$

$$Q = \sum_{i,j,k} C_{ijk} \psi_i \psi_j \psi_k \qquad \text{Hamiltonian} \quad H = \{Q, \bar{Q}\}$$

In this model, the statement of R-charge concentration is sharp. Essentially, BPS states only exist in a **single** R-charge sector once we fix all the commuting charges.

[Fu, Gaiotto, Maldacena, Sachdev; Chang, Chen, Sia, Yang]

Let me advertise that there is a nice toy model that illustrates all of these features clearly - the $\mathcal{N} = 2$ SUSY SYK Model.

$$\{\psi_i, \bar{\psi}_j\} = \delta_{ij} \qquad i, j = 1, ..., N$$

$$Q = \sum_{i,j,k} C_{ijk}\psi_i\psi_j\psi_k \qquad \text{Hamiltonian} \quad H = \{Q, \bar{Q}\}$$

Beside this, the model also demonstrates many other features related to fortuity, such as chaos and the universal low energy **Schwarzian theory** for near BPS black holes.

[Boruch, Heydeman, Iliesiu, Turiaci] [Heydeman, Toldo]

[Heydeman, Shi, Turiaci; ……]

Even though SYK is a nice toy model and helps us understand many universal features of fortuity, it differs from conventional holographic models in that it contains random couplings.

The techniques we use to solve it do not easily generalize to SYM and matrix models.

Even though SYK is a nice toy model and helps us understand many universal features of fortuity, it differs from conventional holographic models in that it contains random couplings.

The techniques we use to solve it do not easily generalize to SYM and matrix models.

Just like you, I like matrix models, so I want a matrix model that exhibits fortuity while being simpler than N=4 SYM.

This is what I will describe to you in the rest of the talk.

Fortuity with a single matrix

We consider an $U(N)$ invariant fermionic matrix model

$$Q = \mathrm{Tr}[\Psi^3] \qquad\qquad \Psi = (\psi_{ij})_{N\times N}$$

One can view it as a very sparse (and non-disorder) version of the SYK model.

We consider an $U(N)$ invariant fermionic matrix model

$$Q = \mathrm{Tr}[\Psi^3] \qquad\qquad \Psi = (\psi_{ij})_{N\times N}$$

One can view it as a very sparse (and non-disorder) version of the SYK model.

This model is exactly solvable. It is useful to introduce a different basis for the fermions.

$$\Psi = \frac{\psi^0}{\sqrt{N}}1 + \sum_{a=1}^{N^2-1}\sqrt{2}\psi^a T^a \qquad Q = \frac{i}{\sqrt{2}}\sum_{abc} f^{abc}\psi^a\psi^b\psi^c$$

The trace mode decouples.

We can write down the following $SU(N)$ generators

$$J^a = -i f^{abc} \psi^b \bar{\psi}^c$$

satisfying

$$[J^a, J^b] = i f^{abc} J^c \qquad C_2 = \sum_{a=1}^{N^2-1} J^a J^a$$

Then we find that

$$H = \{Q, \bar{Q}\} = 3N(N^2 - 1)\,\mathbf{1} - 9\,C_2$$

$$H = \{Q, \bar{Q}\} = 3N(N^2 - 1)\,\mathbf{1} - 9\,C_2$$

The energy of states are purely determined by which $SU(N)$ representation they carry!

Interestingly, in this model, the singlet states have the highest energy.

The lowest energy states, which have zero energy, are those states in the representation with **maximum quadratic Casimir**.

Let me denote this representation by r_*.

Special representation r_*

The representation r_* has the largest Casimir one can get. Intuitively, it must occupy a large number of fermions. The optimal number is around $N^2/2$.

Special representation r_*

The representation r_* has the largest Casimir one can get. Intuitively, it must occupy a large number of fermions. The optimal number is around $N^2/2$.

One can explicitly construct such a representation using some basic knowledge of the $su(N)$ representation theory.

Fermion matrix Ψ transforms under $SU(N)$ as an adjoint.

This becomes manifest with the following form of generators:

$$J_X = \mathrm{Tr}[\Psi[\bar{\Psi}, X]] \qquad [J_X, \Psi] = [X, \Psi]$$

Let me discuss the simple example of $su(3)$.

Consider the following matrices:

$$E_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad E_{13} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad E_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

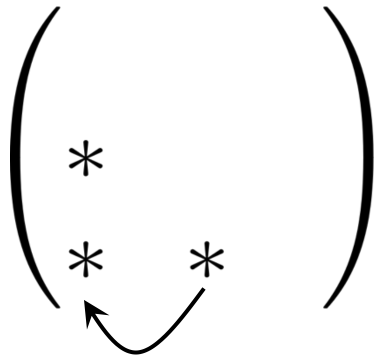
$J_{E_{12}},\ J_{E_{13}},\ J_{E_{23}}$ form the positive-root generators.

A highest weight state is annihilated by all such generators.

Occupy all the fermions in the lower triangle $\psi_{21}\psi_{31}\psi_{32}|0\rangle$

$$\begin{pmatrix} & & \\ * & & \\ * & * & \end{pmatrix}$$

Occupy all the fermions in the lower triangle $\psi_{21}\psi_{31}\psi_{32}|0\rangle$



For example, $[J_{E_{12}}, \psi_{32}] = ([E_{12}, \Psi])_{32} = -\psi_{31}$

Occupy all the fermions in the lower triangle $\psi_{21}\psi_{31}\psi_{32}|0\rangle$

$$\begin{pmatrix} & & \\ * & & \\ * & * & \end{pmatrix}$$

The state is annihilated by all positive-root generators.

One can then act with the negative-root generators to form the full representation.

We are free to "dress” the highest weight state by additional fermions on the diagonal:

$$\begin{pmatrix} * & & \\ * & & \\ * & * & \end{pmatrix}$$

This becomes a highest weight state with a different fermion number.

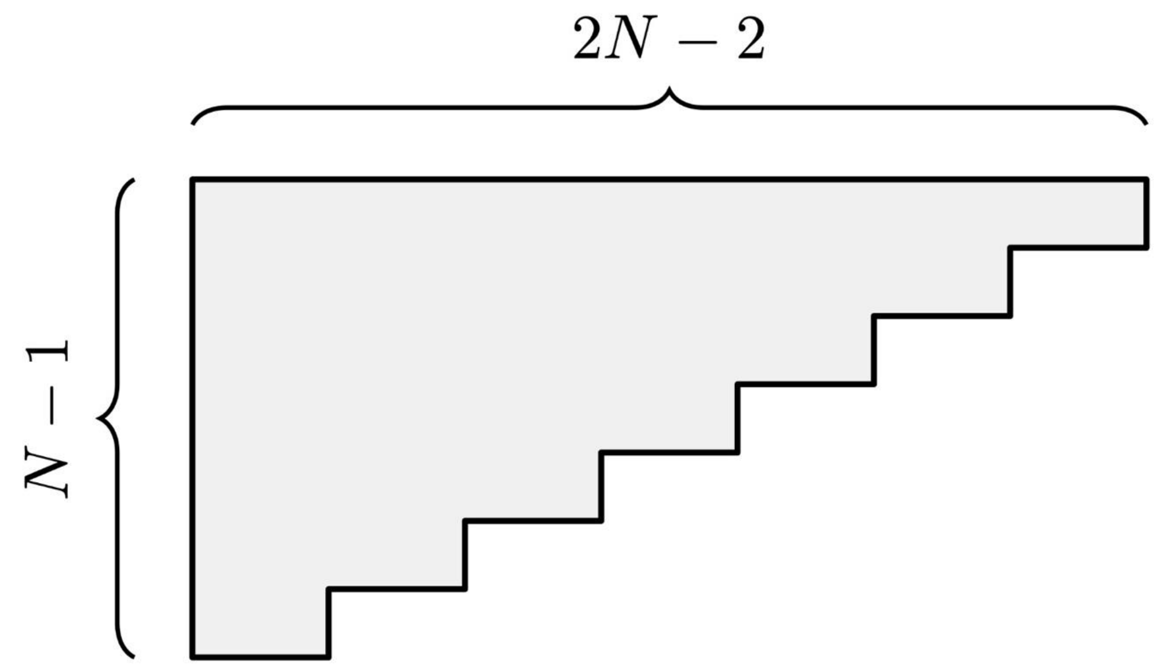
What is said here applies to general $su(N)$. One builds a highest weight state where all the fermions in the lower triangle are occupied and dress it with arbitrary number of fermions in the diagonal.

$$\prod_{\text{any } \ell} \psi_{\ell\ell} \prod_{i>j} \psi_{ij} |0\rangle$$

Starting from the highest weight state, one can then build the entire representation.

We see that such representations always have fermion number that is close to $N^2/2$.

The representation r_* has a "staircase" Young diagram



Very big representation

$$\dim(r_*) = 3^{\frac{N(N-1)}{2}}$$

Fortuity

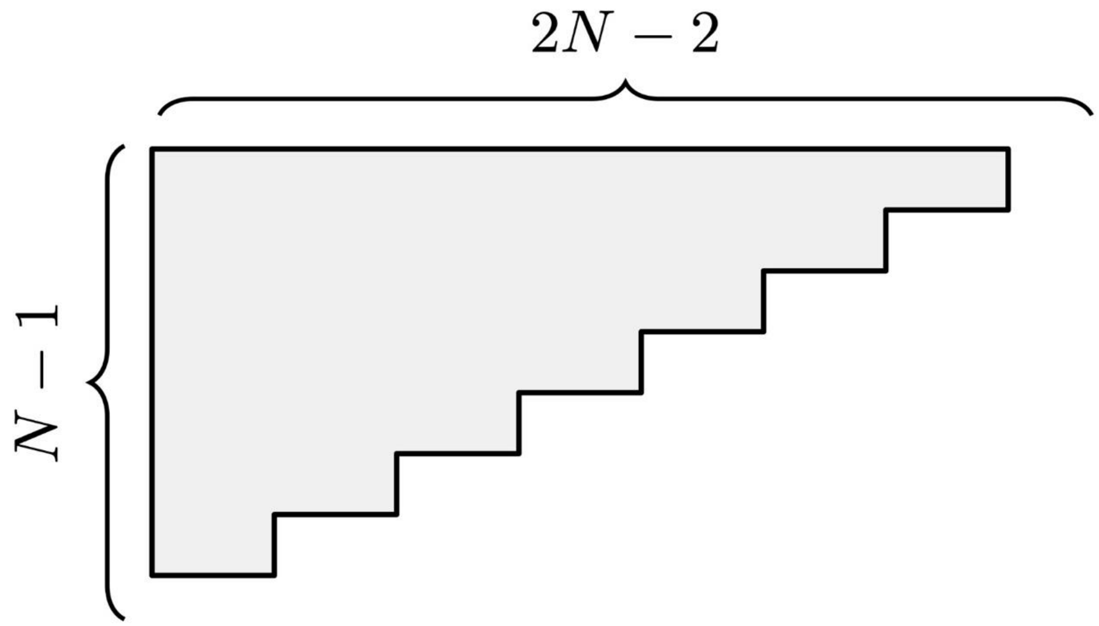
We discussed the representation that hosts the BPS states. Its Young diagram depends on N in an explicit way.

However, if we now imagine **fixing the shape** of the Young diagram while **changing the value of N**, we see pictorially why the states are fortuitous.

Fortuity

We discussed the representation that hosts the BPS states. Its Young diagram depends on N in an explicit way.

However, if we now imagine **fixing the shape** of the Young diagram while **changing the value of N**, we see pictorially why the states are fortuitous.

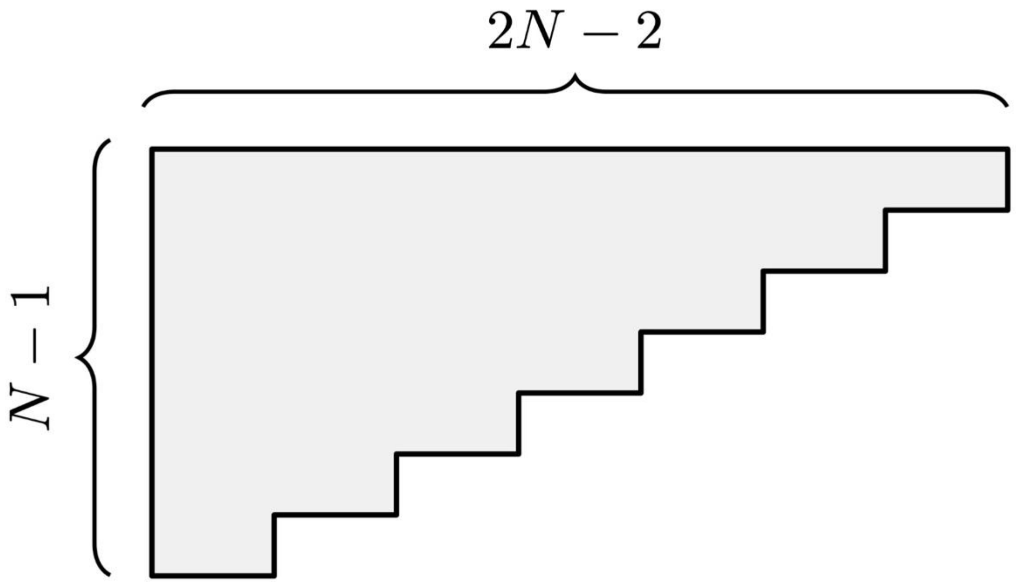


$$Q\,|\psi\rangle \neq 0$$

Fortuity

We discussed the representation that hosts the BPS states. Its Young diagram depends on N in an explicit way.

However, if we now imagine **fixing the shape** of the Young diagram while **changing the value of N**, we see pictorially why the states are fortuitous.

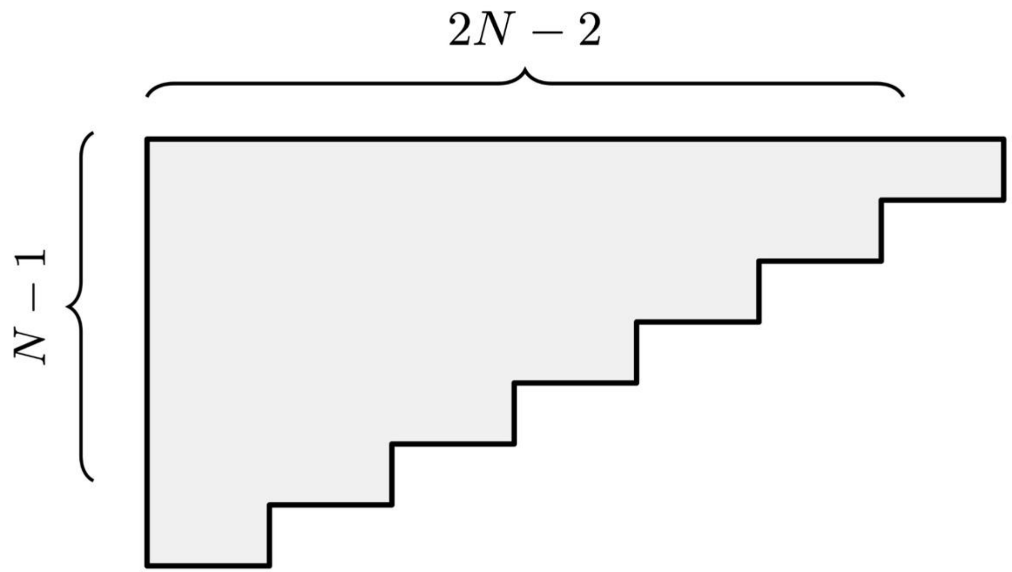


$$Q\,|\psi\rangle = 0$$

Fortuity

We discussed the representation that hosts the BPS states. Its Young diagram depends on N in an explicit way.

However, if we now imagine **fixing the shape** of the Young diagram while **changing the value of N**, we see pictorially why the states are fortuitous.



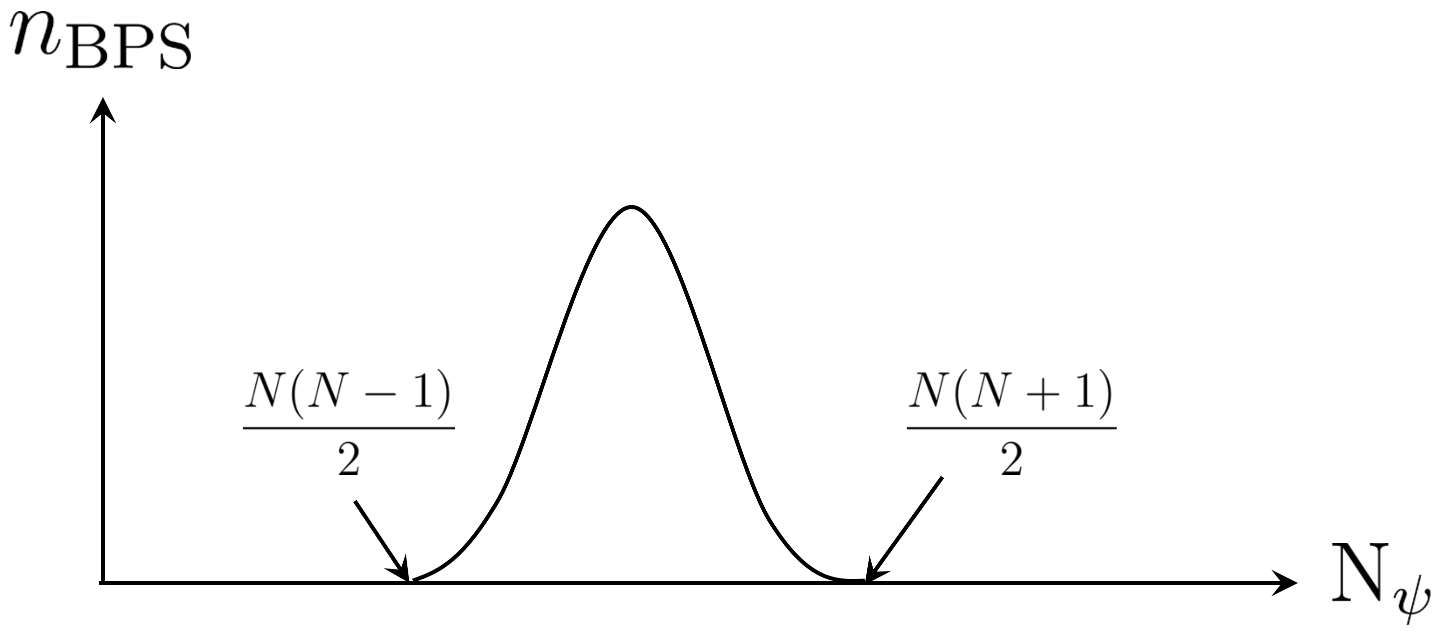
$$|\psi\rangle = 0$$

$$Z_{\mathrm{BPS}}(q) \equiv \mathrm{Tr}_{\mathrm{E}=0}[q^{\mathrm{N}_\psi}] = (1+q)^N \times 3^{\frac{N(N-1)}{2}} \times q^{\frac{N(N-1)}{2}}$$

Cartan part　　dim(r_*)　　r_*

$$Z_{\mathrm{BPS}}(q) \equiv \mathrm{Tr}_{\mathrm{E}=0}[q^{\mathrm{N}_\psi}] = (1+q)^N \times 3^{\frac{N(N-1)}{2}} \times q^{\frac{N(N-1)}{2}}$$

We do have an order N^2 amount of ground state entropy.

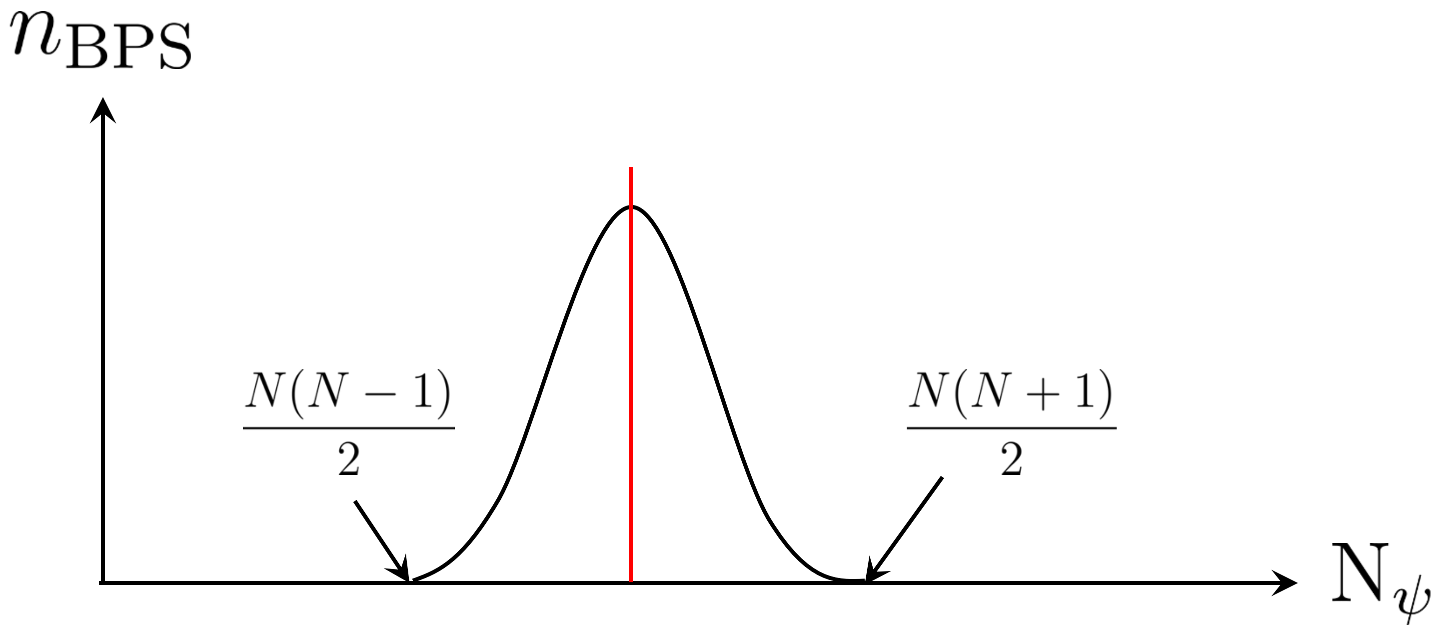


Fortuity, but not quite concentrated

$$Z_{\rm BPS}(q) \equiv {\rm Tr}_{\rm E=0}[q^{\rm N_\psi}] = (1+q)^N \times 3^{\frac{N(N-1)}{2}} \times q^{\frac{N(N-1)}{2}}$$

We do have an order N^2 amount of ground state entropy.

However, the BPS states are not quite sharply concentrated as in SYK.



In some sense, this toy model only captured the “skeleton” of the fortuity phenomenon, but didn’t manage to capture the “meat” – R-charge concentration, quantum chaos, etc.

In some sense, this toy model only captured the “skeleton” of the fortuity phenomenon, but didn’t manage to capture the “meat” – R-charge concentration, quantum chaos, etc.

Of course, this is perhaps as much as what we can get from an exactly solvable model.

In some sense, this toy model only captured the “skeleton” of the fortuity phenomenon, but didn’t manage to capture the “meat” – R-charge concentration, quantum chaos, etc.

Of course, this is perhaps as much as what we can get from an exactly solvable model.

So, what is this model good for?

In some sense, this toy model only captured the “skeleton” of the fortuity phenomenon, but didn’t manage to capture the “meat” – R-charge concentration, quantum chaos, etc.

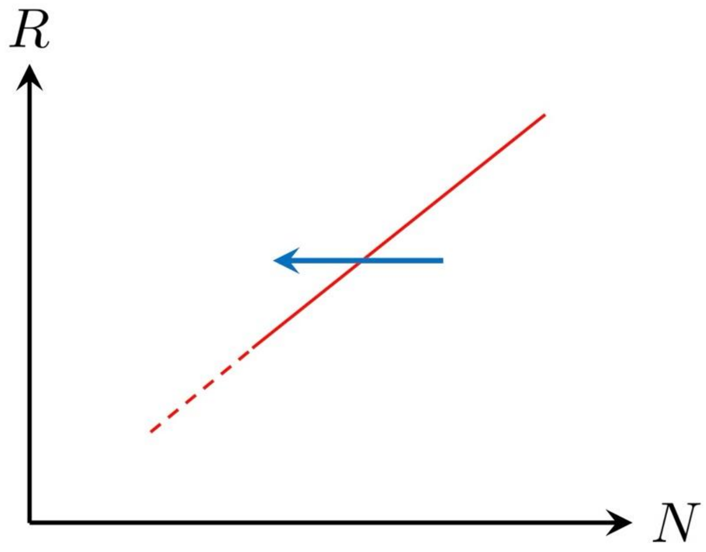
Of course, this is perhaps as much as what we can get from an exactly solvable model.

So, what is this model good for?

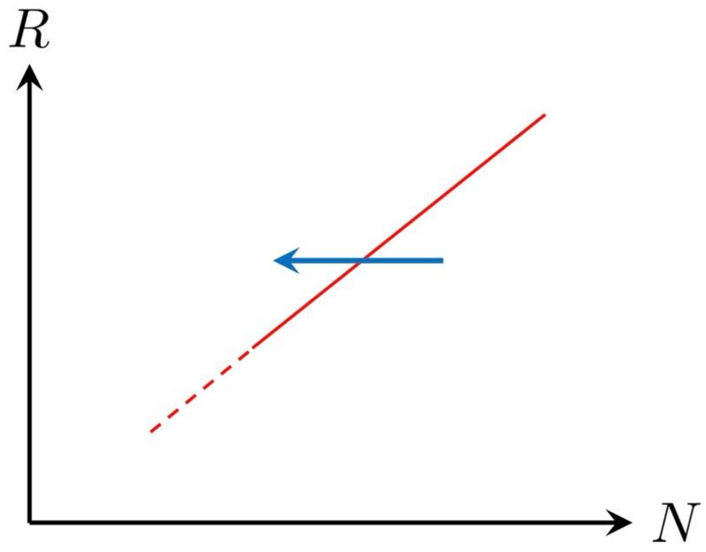
I view it as a steppingstone to test some techniques which can be generalized to more interesting models.

In particular, we can use this simple model to understand aspects of fortuity that are not related to chaos.

As previously mentioned, a striking feature of fortuity is the sudden disappearance of an exponential number of states.



As previously mentioned, a striking feature of fortuity is the sudden disappearance of an exponential number of states.



This seems particularly puzzling from the gravity point of view. We are used to think about an exponential number of states as associated with a classical geometry/saddle point, which should naively be stable against changing N slightly.

The same phenomenon happens in our model, regardless of the fact that it is non-holographic.

All the BPS states in the N-theory will become “null” in the (N-1)-theory.

Microscopically, this is easy to understand from the “oversize” of the Young diagram.

We would like to understand it in a different way that might be generalizable.

$$Z_{\mathrm{BPS}}(q) \equiv \mathrm{Tr}_{\mathrm{E}=0}[q^{\mathrm{N}_\psi}] = (1+q)^N \times 3^{\frac{N(N-1)}{2}} \times q^{\frac{N(N-1)}{2}}$$

$$\approx e^{-\frac{N^2}{2}\log(3q)}$$

More concretely, the BPS partition function comes with an order N^2 exponent.

1. Can one understand it as coming from the saddle point action of some large N integral?

2. If yes, how does the saddle point manage to “disappear” as we decrease N by one?

$$Z_{\mathrm{BPS}}(q) \equiv \mathrm{Tr}_{\mathrm{E}=0}[q^{\mathrm{N}_\psi}] = (1+q)^N \times 3^{\frac{N(N-1)}{2}} \times q^{\frac{N(N-1)}{2}}$$

$$\approx e^{-\frac{N^2}{2}\log(3q)}$$

More concretely, the BPS partition function comes with an order N^2 exponent.

1. Can one understand it as coming from the saddle point action of some large N integral?

2. If yes, how does the saddle point manage to “disappear” as we decrease N by one?

It turns out that the answer to 1 is yes, but in an intricate way such that 2 is possible.

We will look at a closely related quantity, which counts the multiplicity of representation r_* in the Hilbert space.

This can be written in terms of a unitary matrix integral. Such integrals are familiar in the study of large N gauge theories. [Aharony, Marsano, Minwalla, Papadodimas, Raamsdonk]

$$Z_{r_*} = \int \mathcal{D}U \, \chi_{r_*}(U) \exp\left[\sum_{n=1}^{\infty} \frac{(-1)^{n+1} q^n}{n} \left(\mathrm{Tr}U^n \mathrm{Tr}U^{\dagger n} - 1\right)\right]$$

↓ character of rep r_*

We will suppress the $(1+q)^N$ Cartan part in the following. The answer of the integral should be

$$q^{\frac{N(N-1)}{2}} \sim e^{-\frac{N^2}{2}\log\frac{1}{q}}$$

$$Z_{r_*} = \int \mathcal{D}U\, \chi_{r_*}(U) \exp\left[\sum_{n=1}^{\infty} \frac{(-1)^{n+1} q^n}{n} \left(\mathrm{Tr}U^n \mathrm{Tr}U^{\dagger n} - 1\right)\right]$$

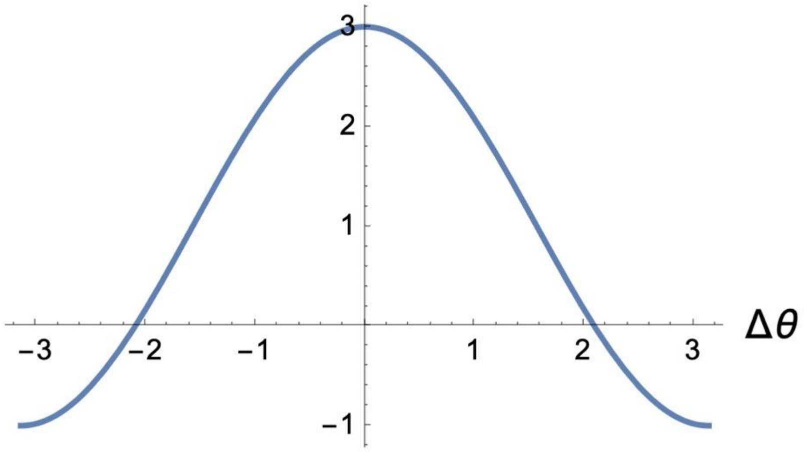
Denote the eigenvalues of U as $e^{i\theta_i}, i = 1, \dots, N$.

We have explicitly

$$\chi_{r_*}(U) = \prod_{i<j} (1 + 2\cos(\theta_i - \theta_j))$$

The character is not positive definite, so it cannot be easily exponentiated and added to the original action.

$$-\int d\theta_1 d\theta_2\, \rho(\theta_1)\rho(\theta_2) \log(1 + 2\cos(\theta_1 - \theta_2))$$?



One can attempt various $i\varepsilon$ prescriptions to make sense of the expression.

$$-\int d\theta_1 d\theta_2 \, \rho(\theta_1)\rho(\theta_2) \log(1 + 2\cos(\theta_1 - \theta_2))$$

Formally this leads to a unitary matrix integral with double trace potential, but with complex coefficients. One can presumably identify complex saddle points.

Understanding the phase structure of this complex matrix model in general seems quite subtle.

$$Z_{r_*} = \int \mathcal{D}U \, \chi_{r_*}(U) \exp\left[\sum_{n=1}^{\infty} \frac{(-1)^{n+1} q^n}{n} \left(\mathrm{Tr}U^n \mathrm{Tr}U^{\dagger n} - 1\right)\right]$$

Instead, we adopt a different strategy.

We can **complexify *q*** such that we **cancel out** the oscillating signs in the character.

$$Z_{r_*} \sim \int_0^{2\pi} \prod_{i=1}^{N} d\theta_i \prod_{i<j} 2(1-\cos\theta_{ij})(1+2\cos\theta_{ij})(1+q^2+2q\cos\theta_{ij}) \qquad \theta_{ij} \equiv \theta_i - \theta_j \, .$$

↓ measure factor

↓ $$(1+qe^{i\theta_i - i\theta_j})(1+qe^{i\theta_j - i\theta_i})$$

$$\psi_{ij}, \, \psi_{ji}$$

$$Z_{r_*} = \int \mathcal{D}U \, \chi_{r_*}(U) \exp\left[\sum_{n=1}^{\infty} \frac{(-1)^{n+1} q^n}{n} \left(\mathrm{Tr}U^n \mathrm{Tr}U^{\dagger n} - 1\right)\right]$$

Instead, we adopt a different strategy.

We can **complexify** $\boldsymbol{q}$ such that we **cancel out** the oscillating signs in the character.

$$Z_{r_*} \sim \int_0^{2\pi} \prod_{i=1}^{N} d\theta_i \prod_{i<j} 2(1-\cos\theta_{ij})(1+2\cos\theta_{ij})(1+q^2+2q\cos\theta_{ij}) \qquad \theta_{ij} \equiv \theta_i - \theta_j \,.$$

$\downarrow$

measure factor

$$q = e^{\pm i\frac{\pi}{3}} \;\rightarrow\; (1+q^2+2q\cos(\theta)) = 2e^{\pm i\frac{\pi}{3}}(1+2\cos\theta_{ij})$$

Therefore, at $q = e^{\pm i\frac{\pi}{3}}$, we simply have the integral

$$Z_{r_*} \sim e^{\pm i N^2 \frac{\pi}{6}} \int_0^{2\pi} \prod_{i=1}^{N} d\theta_i \prod_{i<j} 2(1-\cos\theta_{ij})(1+2\cos\theta_{ij})^2$$

$$= e^{\pm i N^2 \frac{\pi}{6}} \int_0^{2\pi} \prod_{i=1}^{N} d\theta_i \prod_{i<j} |e^{i3\theta_i} - e^{i3\theta_j}|^2$$

The last expression is very familiar.

It is simply the usual measure factor for a unitary matrix integral, but with every θ replaced by 3θ !

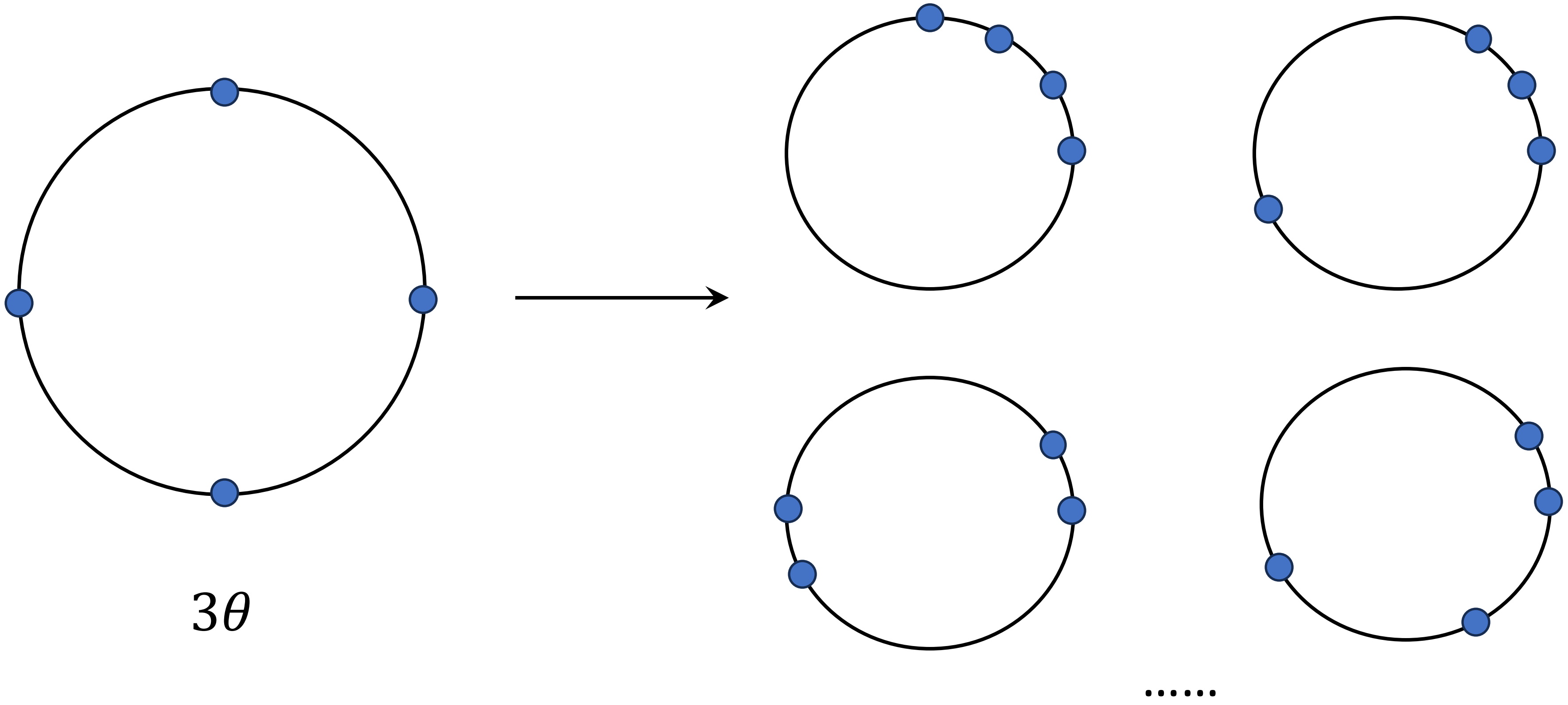
The saddle points corresponds to configurations where $3\theta_i$ are spaced uniformly on the circle.

However, mapping back to the original θ variables, this gives not just a single saddle point, but in fact **a large number of** saddle points which contributes equally.

$$\theta_k = \frac{1}{3}\frac{2\pi k}{n} + \frac{2\pi}{3}\sigma_k, \quad \sigma_k = 0, 1, 2$$

The saddle points corresponds to configurations where $3\theta_i$ are spaced uniformly on the circle.

However, mapping back to the original θ variables, this gives not just a single saddle point, but in fact **a large number of** saddle points which contributes equally.



The number of saddle points, even though large, does not scale as e^{N^2}. It is also well-known that the original uniform saddle point has **zero large N action**.

Therefore, we have

$$Z_{r_*} \sim e^{\pm i N^2 \frac{\pi}{6}} \int_0^{2\pi} \prod_{i=1}^{N} d\theta_i \prod_{i<j} |e^{i3\theta_i} - e^{i3\theta_j}|^2$$

$$\sim e^{\pm i N^2 \frac{\pi}{6}} = e^{-\frac{N^2}{2} \log \frac{1}{q}}$$

The fact that we do not have a single large N saddle point, but rather a variety of them, might be vaguely reflecting the fact the model is non-holographic (namely, we do not have a single classical bulk geometry).

All these saddle points contribute equally.

How do we understand the “disappearance” of these saddle points if we decrease N by one?

To properly formulate this question, we would like to hold fix the representation, while decreasing the number of degrees of freedom.

There is a very natural way of asking this question in our current context.

We simply ask, how many copies of representation r_* we can get if we decouple the last column and the last row of fermions.

There is a very natural way of asking this question in our current context.

We simply ask, how many copies of representation r_* we can get if we decouple the last column and the last row of fermions.

We can do that with the following integral:

$$Z_{r_*,\lambda}(q) \sim \int \prod_{i=1}^{N} d\theta_i \prod_{1\le i<j\le N} (1-\cos\theta_{ij})(1+2\cos\theta_{ij}) \prod_{1\le i<j\le N-1} (1+qe^{i\theta_{ij}})(1+qe^{i\theta_{ji}})$$
$$\times \prod_{i=1}^{N-1} (1+\lambda q e^{i\theta_{iN}})(1+\lambda q e^{i\theta_{Ni}})$$

At $\lambda = 1$ we have our original integral with classical saddle points.

At $\lambda = 0$ we should instead find zero. How does it happen?

$$Z_{r_*,\lambda}(q) \sim \int \prod_{i=1}^{N} d\theta_i \prod_{1\le i<j\le N} (1-\cos\theta_{ij})(1+2\cos\theta_{ij}) \prod_{1\le i<j\le N-1} (1+qe^{i\theta_{ij}})(1+qe^{i\theta_{ji}})$$
$$\times \prod_{i=1}^{N-1} (1+\lambda q e^{i\theta_{iN}})(1+\lambda q e^{i\theta_{Ni}})$$

Let's still try to understand this at the special point $q = e^{\pm i\frac{\pi}{3}}$.

It turns out the saddle points are still there.

For example, one can fix a saddle point and study the integral over the remaining θ_N and find "eigenvalue instanton" configurations that give a large contribution (see paper).

The way that the answer conspires to be zero is that the saddle points no longer add up, they are dressed by prefactors depending on θ_N.

At $\lambda = 0$, we have

$$\prod_{1 \le i < j \le N-1} |e^{i3\theta_i} - e^{i3\theta_j}|^2 \times \prod_{i=1}^{N-1} (1 + 2\cos\theta_{iN})(1 - \cos\theta_{iN})$$

For each θ_i, we sum over different saddle points where it is shifted by $0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

The way that the answer conspires to be zero is that the saddle points no longer add up, they are dressed by prefactors depending on θ_N.

At $\lambda = 0$, we have

$$\prod_{1\leq i<j\leq N-1} |e^{i3\theta_i} - e^{i3\theta_j}|^2 \times \prod_{i=1}^{N-1} (1+2\cos\theta_{iN})(1-\cos\theta_{iN})$$

For each θ_i, we sum over different saddle points where it is shifted by $0, \frac{2\pi}{3}, \frac{4\pi}{3}$.

Using an elementary identity $\sum_{k=0,1,2} \left[1+2\cos\left(\frac{2\pi k}{3}+x\right)\right]\left[1-\cos\left(\frac{2\pi k}{3}+x\right)\right] = 0$

we see that the saddle points cancel out and we get zero!

Summary of the model

- Exponentially large number of zero energy states in the largest representation.

- All such states are fortuitous, though they are not chaotic and do not concentrate sharply.

Summary of the model

- Exponentially large number of zero energy states in the largest representation.
- All such states are fortuitous, though they are not chaotic and do not concentrate sharply.
- They can be identified with a family of large N saddle points in the unitary matrix integral.
- Usually, these saddle points add up. However, if we try to change N to N-1, they cancel out delicately and gives zero.

Getting fortuity in the singlet sector

If one insists on getting fortuitous states in the singlet sector, there is an almost trivial way of doing it using what we already understood.

The price is to introduce two fermionic matrices.

$$Q = \mathrm{Tr}[\Psi_1^3] + \mathrm{Tr}[\Psi_2^3]$$

The Hamiltonian is simply what we had before. Therefore, the zero energy states of the new model live in the tensor product of two copies of r_*.

But we now have $$r_* \otimes r_* = \text{singlet} \oplus \ldots$$

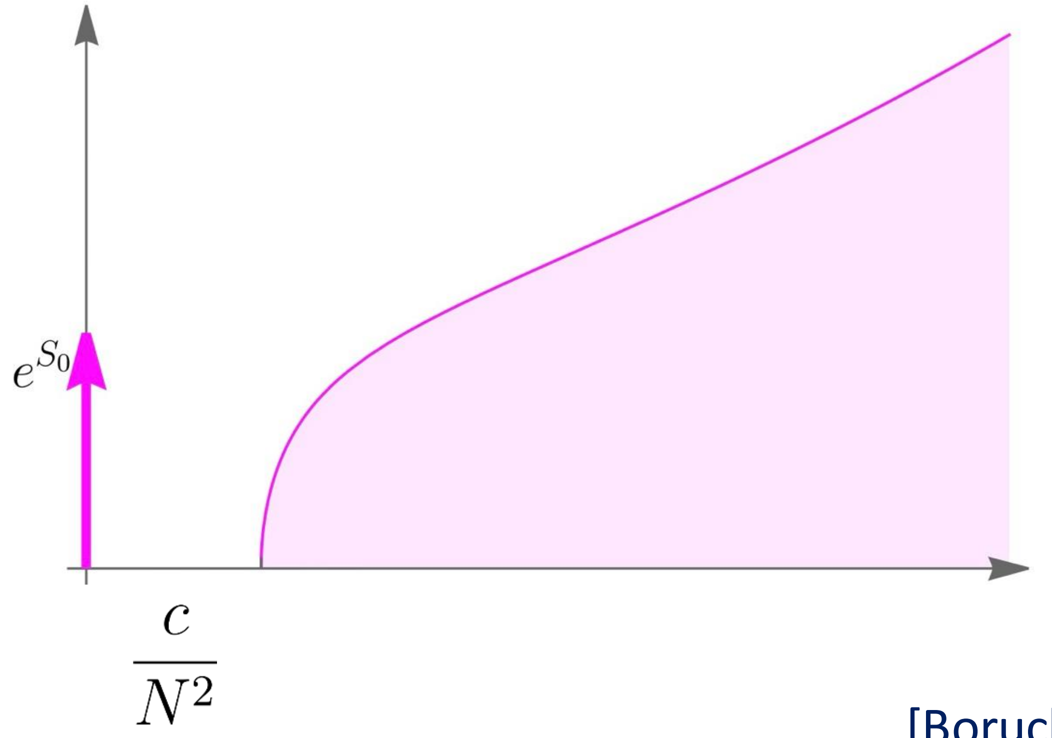
Some open questions

Of particular interest will be to complicate the model slightly more (e.g. using more flavors of fermions), such that the BPS ground states become bona fide black hole microstates.

I believe this is an interesting target for matrix bootstrap, since one can access black hole physics by looking at the ground states.

- Can we bootstrap the distribution of BPS states, showing they concentrate?

- More interestingly, can we bootstrap the gap, which Schwarzian theory gives a particular prediction?



[Stanford, Witten]

[Boruch, Heydeman, Iliesiu, Turiaci]

Coming back to holographic theories/gravity:

- Is there an analogues computation one can do to probe the sensitivity of fortuitous microstates to N?

Coming back to holographic theories/gravity:

- Is there an analogues computation one can do to probe the sensitivity of fortuitous microstates to N?

- Is there a analogues gravity picture of changing N and can we understand the role of D-branes there?

Thank you!