The polarised IKKT model

Matrix Model for Superstring/M-theory workshop
YITP Kyoto

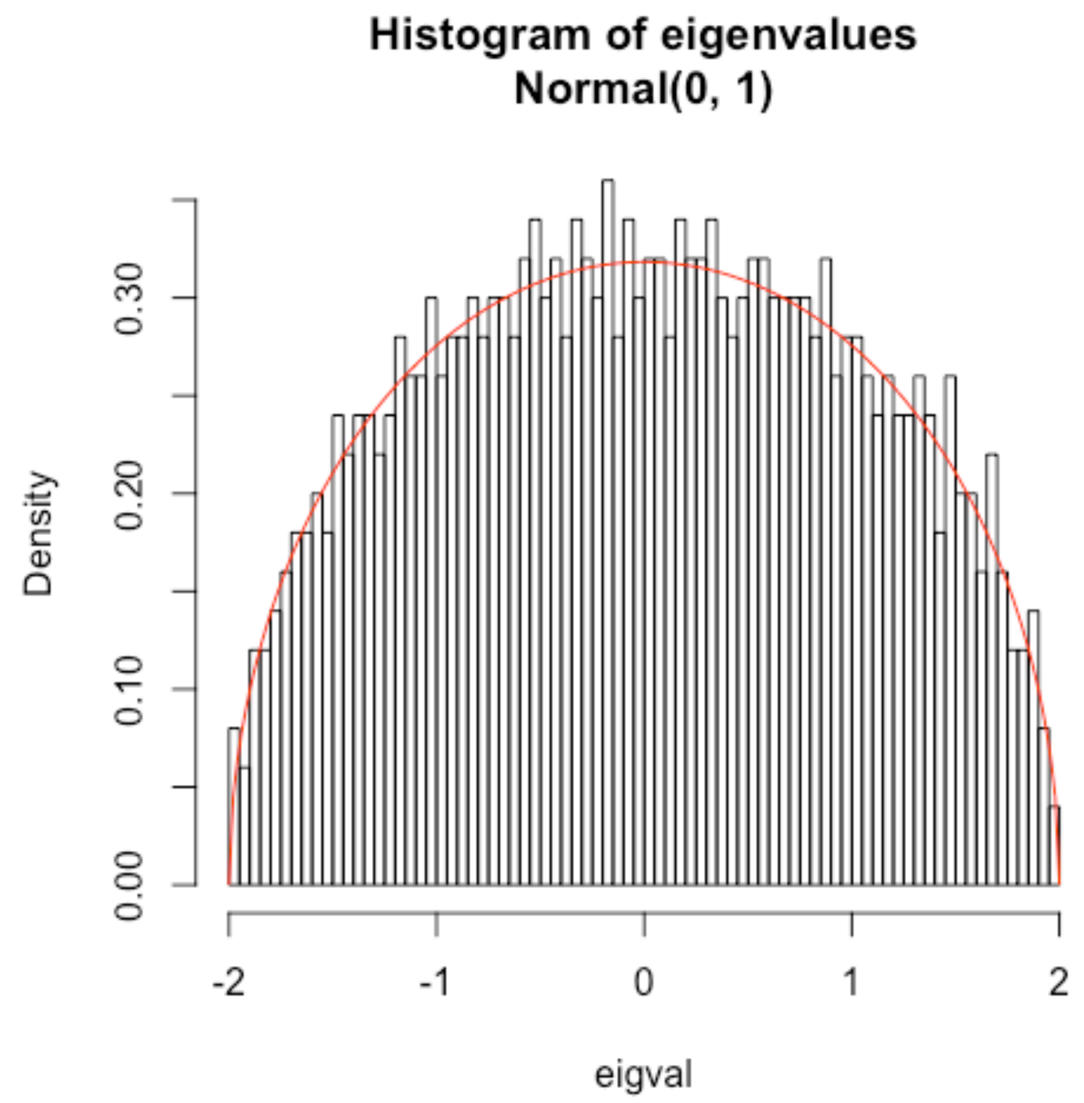
Sean Hartnoll, U of Cambridge. Dec 2025.

Background: Matrices and geometry

- Large N matrices have many connections to geometry:

  - Ex. 1, large N eigenvalues of a random matrix build a Wigner semicircle:

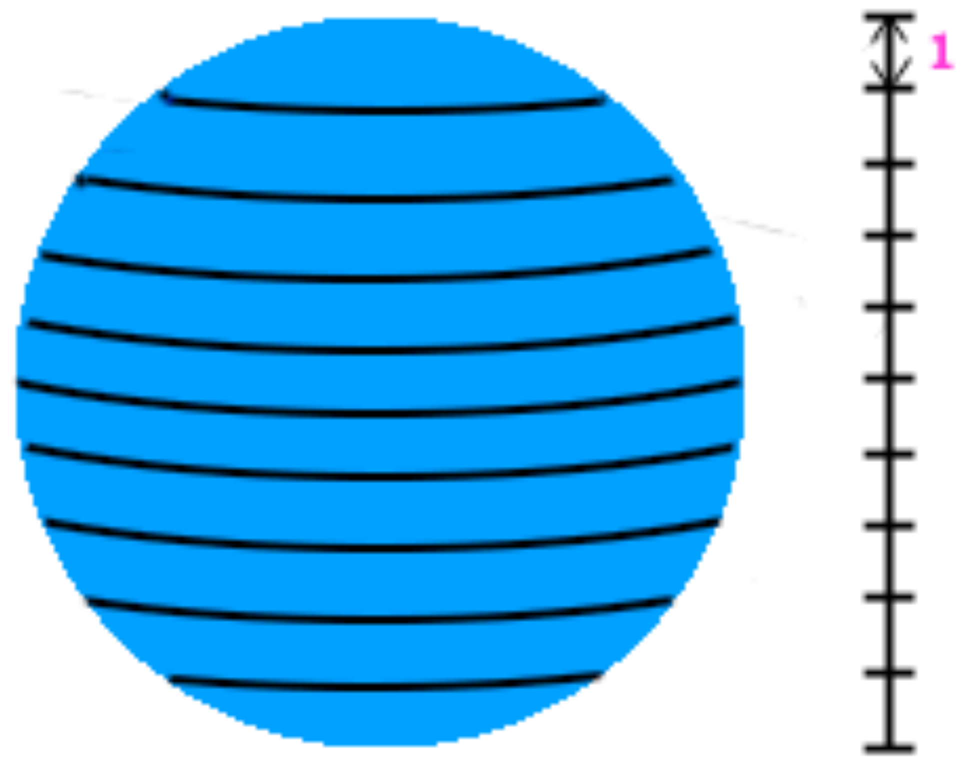
$$\int dM\, e^{-\frac{N}{2}\mathrm{tr}\left(M^2\right)} \quad \Rightarrow$$



  - Ex. 2, the N-dimensional irrep of SU(2) defines a fuzzy sphere:

$$\int dX_i e^{-\nu\, \mathrm{tr}(\epsilon_{ijk}X_k - i[X_i, X_j])^2}$$

$$Z = \begin{pmatrix} \frac{N}{2} & & & \\ & \frac{N}{2}-1 & & \\ & & \ddots & \\ & & & -\frac{N}{2} \end{pmatrix} \qquad X + iY = \begin{pmatrix} 0 & \ddots & & \\ & 0 & \sqrt{n(N-n)} & \\ & & 0 & \ddots \\ & & & 0 \end{pmatrix} \qquad X^2 + Y^2 + Z^2 = \frac{N^2}{4}$$



Background: Holography

- The richest correspondences are holographic:

  SUSY QM/QFT of large N matrices ↔ Emergent dynamical spacetime

- Examples:
  1. AdS/CFT:

$$\int \mathcal{D}A^{ab}(t,\vec{x})e^{-\int dt d^d x \, \mathrm{Tr}\left[F^2+\cdots\right]} \sim \int \mathcal{D}g_{\mu\nu}(x^\rho)e^{-\int dt d^D x \, \sqrt{g}(R+\cdots)}$$

  2. BFSS:

$$\int \mathcal{D}X^{ab}(t)e^{-\int dt \, \mathrm{Tr}\left[\dot{X}^2+[X,X]^2+\cdots\right]} \sim \int \mathcal{D}g_{\mu\nu}(x^\rho)e^{-\int dt d^D x \, \sqrt{g}(R+\cdots)}$$

Motivation: the absence of time

- The QM theories come with a time evolution/Hamiltonian: e^{iHt}
- The Hamiltonian constraint of GR: $\mathcal{H} = 0$
- The Hamiltonian constraint is the GR analogue of the Gauss law in Maxwell theory. In Maxwell theory A_t imposes the constraint $\nabla . E=0$. In GR g_{tt} imposes the Hamiltonian constraint: $K^2 + {}^{(3)}R \sim 0$ (no preferred slicing of space).
- In the canonical models of holography, this tension is resolved by having *H* live on the boundary of space.
- Feels a bit extrinsic to the core structure of GR and inapplicable to e.g. cosmology or infalling observers.

The IKKT model

- There exists a SUSY model of large N matrices with no time:

$$Z = \int dX_\mu d\Psi_\alpha e^{-S_{\text{IKKT}}}$$

$$S_{\text{IKKT}} = \text{Tr}\left(-\frac{1}{4}[X_\mu, X_\nu][X^\mu, X^\nu] + \frac{1}{2}\Psi_\alpha \left(\mathcal{C}\Gamma^\mu\right)_{\alpha\beta}[X_\mu, \Psi_\beta]\right)$$

- 10 *NxN* Hermitian matrices *X*.

[Ishibashi-Kawai-Kitazawa-Tsuchiya '96]

- SO(10) symmetry ('Euclidean' model)
- Today, the emergent time will be Euclidean.
- ∃ an SO(1,9) 'Lorentzian' model too, also interesting, Z is not finite in that case. More work needed.

[see J. Nishimura talk later]

Good news part I

- This integral has been done exactly!

$$Z \propto \sum_{d|N} \frac{1}{d^2}$$

[Green-Gutperle ’97, Moore-Nekrasov-Shatashvili ’98, Krauth-Nicolai-Staudacher ’98]

- NB. There may not be a simple large *N* limit of this expression. May be nicer to sum over N and weight with string coupling:

$$\sum_N e^{-\frac{4\pi N}{g_\mathrm{s}}} Z_N$$

- Most tractable setting in which to actually understand the matrix physics leading to spacetime?

Good news part II

- The IKKT model is the worldvolume theory of N D-instantons [D(-1) branes].
- The backreacted (Euclidean) geometry of such instantons is known!
- In Einstein frame:

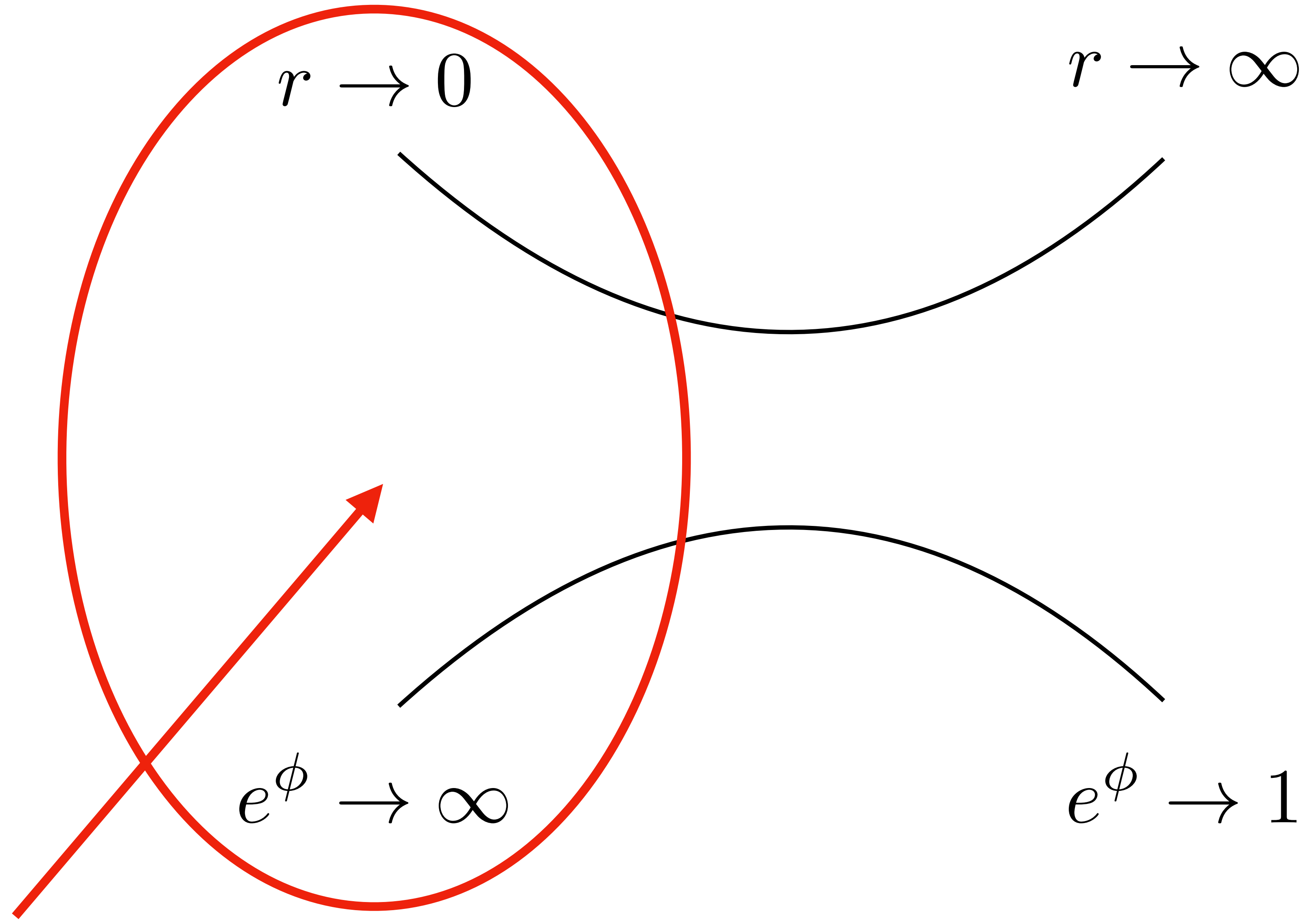
$$ds^2 = d\vec{x}^2$$

$$e^{\phi} = 1 + \frac{\alpha}{r^8}$$

$$C_0 = -e^{-\phi} + \beta$$

[Gibbons-Green-Perry '95]

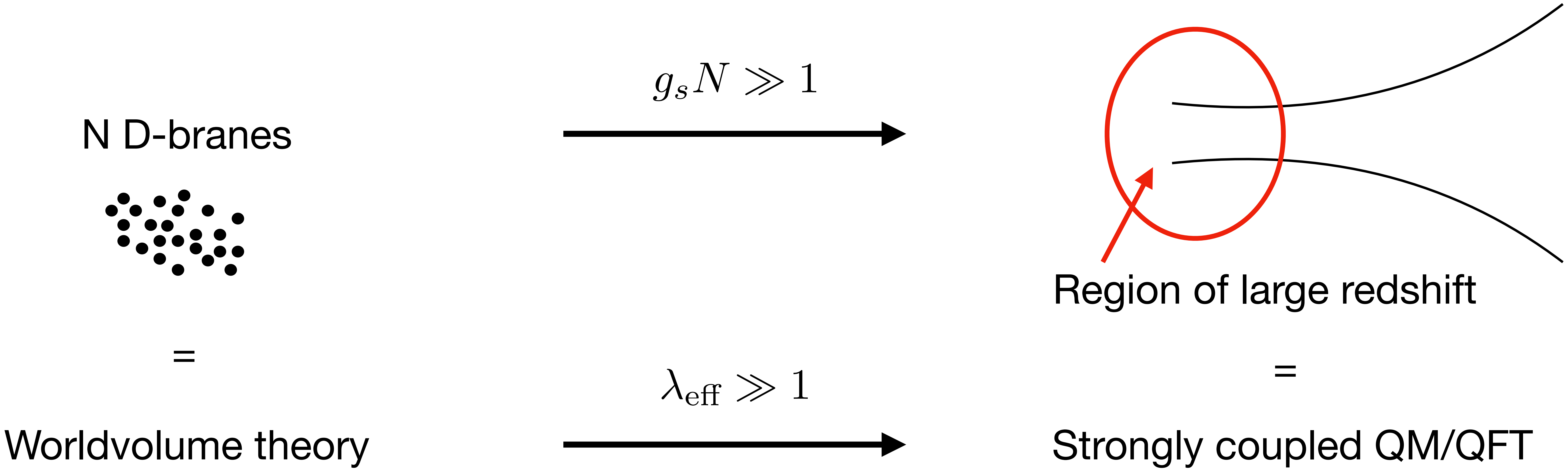
- Wormhole in string frame:



Hyperscaling-violating 'near-horizon' geometry.
Holography à la Biggs-Maldacena '23?

Conceptual difficulty

- Key step in obtaining holographic dualities is the decoupling limit.
- Two descriptions for the IR dynamics of a subset of d.o.f. in string theory:



- In the IKKT model there is (1) no meaningful λ_{eff}

  (2) no notion of IR or redshift

But possibly a familiar difficulty ….

- The internal S^5 dimensions in the most familiar instance of holography are also not associated to a useful notion of redshift.
- We don’t understand the emergence of these dimensions very well.
- The IKKT model can be thought of as a model of holography where all the emergent dimensions are “internal” (even while noncompact).
- In this sense the model forces us to confront difficulties that also arise elsewhere.
- Would be useful to have more handles on the model …

The polarised IKKT model

- In the remainder I will discuss a SUSY deformation of the IKKT model.

$$S_\Omega = S_{\text{IKKT}} + \text{Tr}\left(\frac{\Omega^2}{4^3}X_A^2 + \frac{3\Omega^2}{4^3}X_a^2 + i\Omega X_8[X_9, X_{10}] + \frac{i\Omega}{8}\Psi_\alpha(\mathcal{C}\hat{N})_{\alpha\beta}\Psi_\beta\right)$$

[Bonelli ’02, Martina ’25]

- A close cousin of the Polchinski-Strassler mass deformation of AdS/CFT and of the BMN deformation of BFSS. Breaks SO(10) → SO(3) x SO(7).

- Coupling Ω tunes between a non-backreacting and a backreacting D-brane configuration, corresponding to a weakly and strongly interacting matrix integral. Possibly a good framework for a ‘timeless’ holographic duality.

The weakly coupled limit

- In 2409.18706, Jun Liu and I solved the 'weak coupling' $\Omega \to \infty$ limit of the theory and identified a corresponding non-backreacting D-brane geometry.

- In this limit the integral is dominated by a single saddle point, given by an irreducible fuzzy sphere configuration:

$$[X_a, X_b] = i\frac{3\Omega}{8}\epsilon_{abc}X_c$$

- A 1-loop analysis gives:

$$Z_{\Omega\to\infty} = 2^{3(N^2-1)}\frac{(2\pi)^{\frac{(10N+11)(N-1)}{2}}}{N^{\frac{3}{2}}G(N+1)}\prod_{l=1}^{N-1}\left(\frac{3l+2}{3l+1}\right)^3 e^{\frac{9\Omega^4}{2^{15}}N(N^2-1)}$$

The D-brane configuration

- The corresponding brane configuration, in the same $\Omega \to \infty$ limit, is a spherical Euclidean D1-brane in a background NSNS 3-form flux H = dB $\propto \Omega$.

- The D1-brane carries N units of worldvolume flux *F*. This flux gives the brane N units of D-instanton charge, due to a C_0F coupling.

- The spherical D1-brane can be thought of as N D-instantons that have polarised via the Myers mechanism.

$$S_{D1} = \frac{T_1}{g_s} \int d^2\sigma e^{-\phi} \sqrt{\det(\mathcal{G}_{ij} + \mathcal{B}_{ij} - 2\pi\alpha' \mathcal{F}_{ij})} + \frac{T_1}{g_s} \int \mathcal{C}_0 \left(\mathcal{B} - 2\pi\alpha' \mathcal{F}\right)$$

tension polarisation

The D-brane configuration

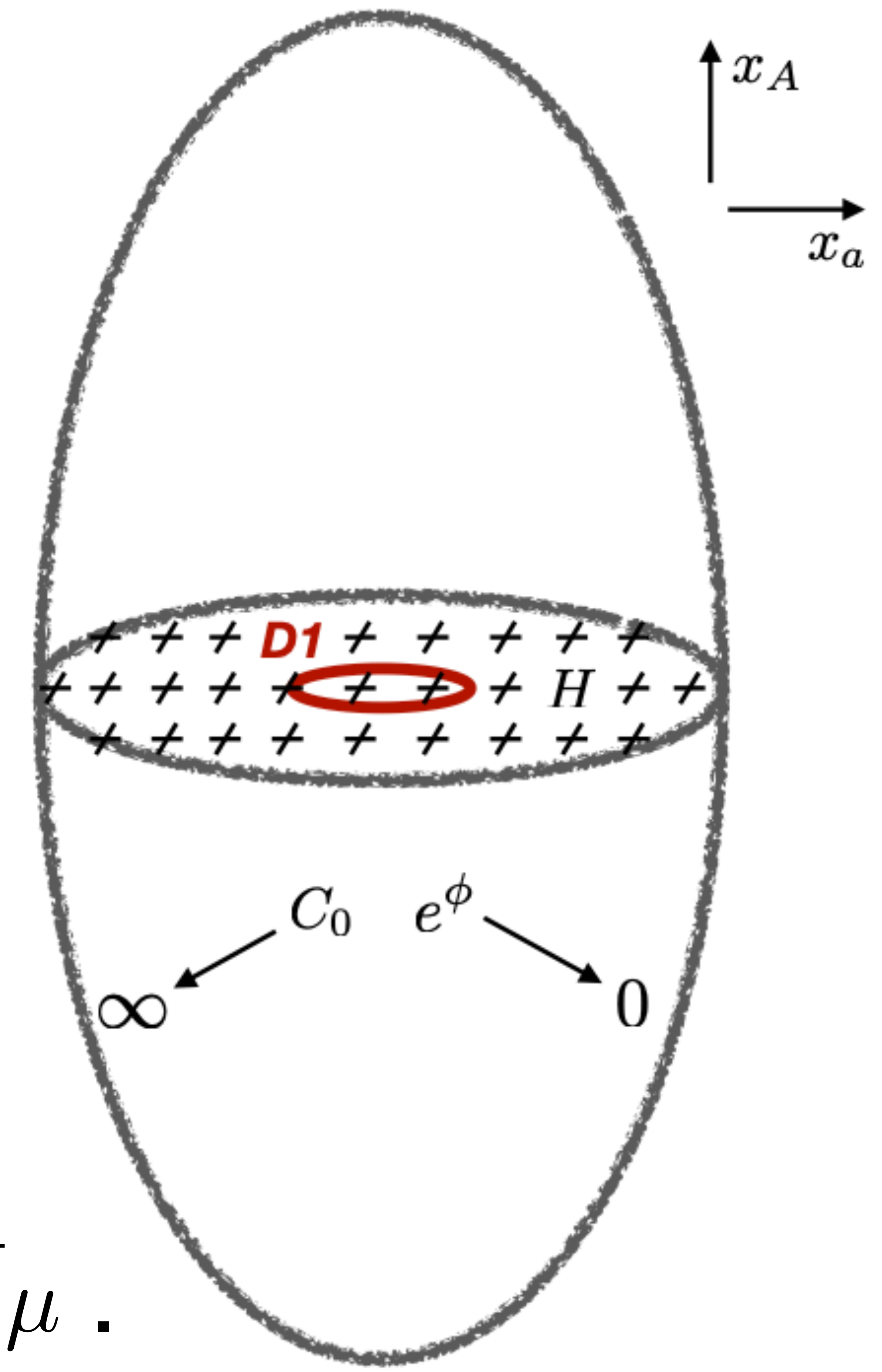
- The following Euclidean ‘cavity’ geometry preserves all the symmetries and SUSY of the matrix integral:

$$ds^2 = d\vec{x}^2$$

$$H = \mu\, dx^8 \wedge dx^9 \wedge dx^{10}$$

$$e^{\phi} = -\frac{1}{C_0} = 1 - \frac{\mu^2}{32}\left(x_A^2 + 3x_a^2\right)$$

[cf. Dasgupta, Sheikh-Jabbari, Van Raamsdonk ’02]



- Polarised D1 is at fixed radius in this background, preserving all SUSY. Matches fuzzy sphere for $\Omega \sim \sqrt{\alpha'}\mu$.

Towards strong coupling

- Established a matrix integral <-> Euclidean D1-brane identification. Can now, as in other holographic setups, move into a parameter regime where the D1-brane backreacts.

- Localisation formula [Komatsu-Martina-Penedones-Vuignier-Zhao 2411.18678]:

$$Z = \sum_{\mathcal{R}} Z_{\mathcal{R}}\,, \qquad Z_{\mathcal{R}} = C_{\mathcal{R}} e^{-S^0_{\mathcal{R}}} \int d\vec{m}_{\mathcal{R}}\, e^{-S^{\text{eff}}_{\mathcal{R}}(\vec{m}_{\mathcal{R}})}$$

Representations of SU(2), decompose into irreps:

$$\sum_M n_M M = N$$

Maximally irreducible representation dominates:

$$S^0_{\mathcal{R}} = -\frac{9\Omega^4}{2^{15}} \sum_M n_M M(M^2-1)$$

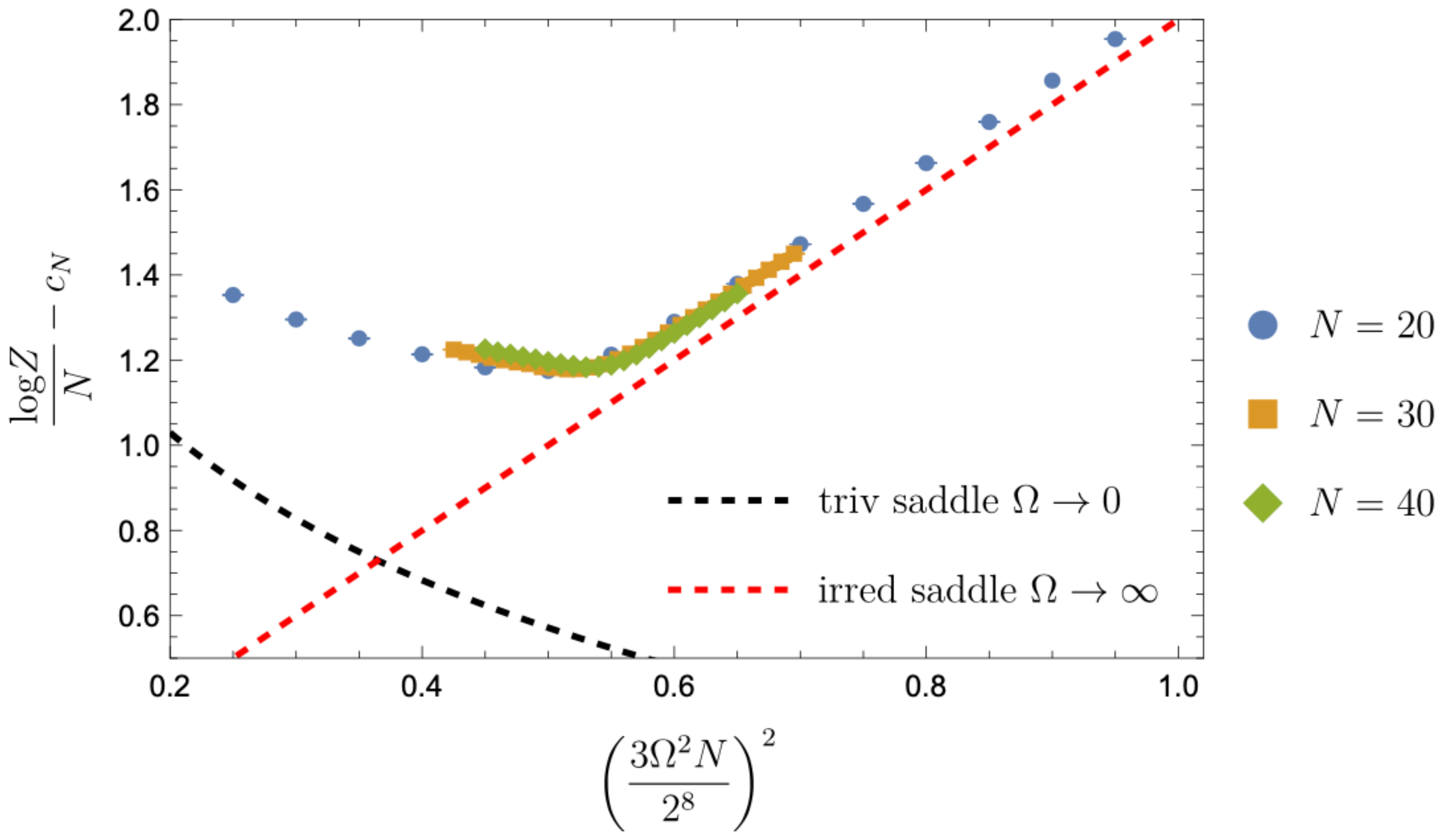
One modulus $m_{\mathcal{R}}$ for each irrep:

$$S^{\text{eff}}_{\mathcal{R}} = \frac{3\,\Omega^4}{2^7} \sum_M \sum_{i=1}^{n_M} M m^2_{Mi}$$

[similar to previous work in BMN: Asano-Ishiki-Okada-Shimasaki ’14]

Phase structure of the theory

- In 2504.06481, Jun Liu and I performed the moduli integrals using Monte Carlo. The sum over representations can be done without approximation:



- Can think of as a statistical physics phase transition between energy (at low 'temperature') and entropy (at high 'temperature').

Phase structure of the theory

- Large Ω solved by saddle point as we saw: max. irrep dominates
- Small Ω solved using canonical ensemble and simplification in moduli integral

(Simplifications found in [2411.18678])

$$Z_{\mathcal{R}} \approx a_N \prod_M \left[\frac{1}{n_M!} \left(\frac{b_M}{\Omega^2} \right)^{n_M} \right]$$

$$\Rightarrow \qquad \widetilde{Z} = a_N \sum_{\{n_M\}} \prod_M \left[\frac{1}{n_M!} \left(\frac{b_M}{\Omega^2} \right)^{n_M} e^{(\alpha_M - \beta M) n_M} \right]$$

$$= a_N \prod_M \exp\left[\frac{b_M}{\Omega^2} e^{\alpha_M - \beta M} \right] \quad \Rightarrow \quad \langle n_M \rangle = \frac{\partial \log \widetilde{Z}}{\partial \alpha_M} \bigg|_{\alpha_M = 0} = \frac{b_M}{\Omega^2} e^{-\beta M}$$

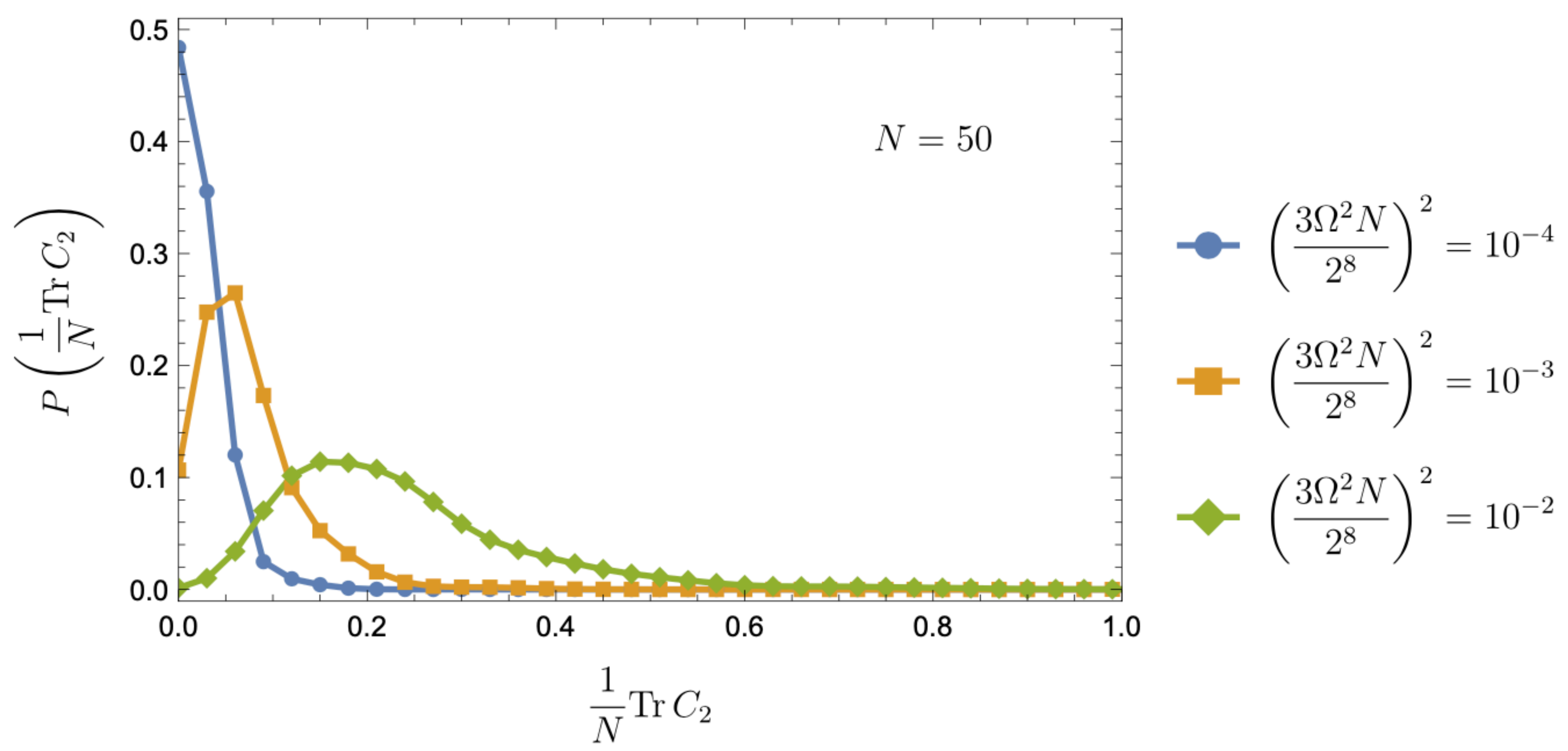
Casimir expectation value

- Characterise representations by Casimir:

$$\frac{1}{N}\mathrm{Tr}_{\mathcal{R}}C_2 \equiv \frac{1}{4N}\sum_M n_M M(M^2-1)$$

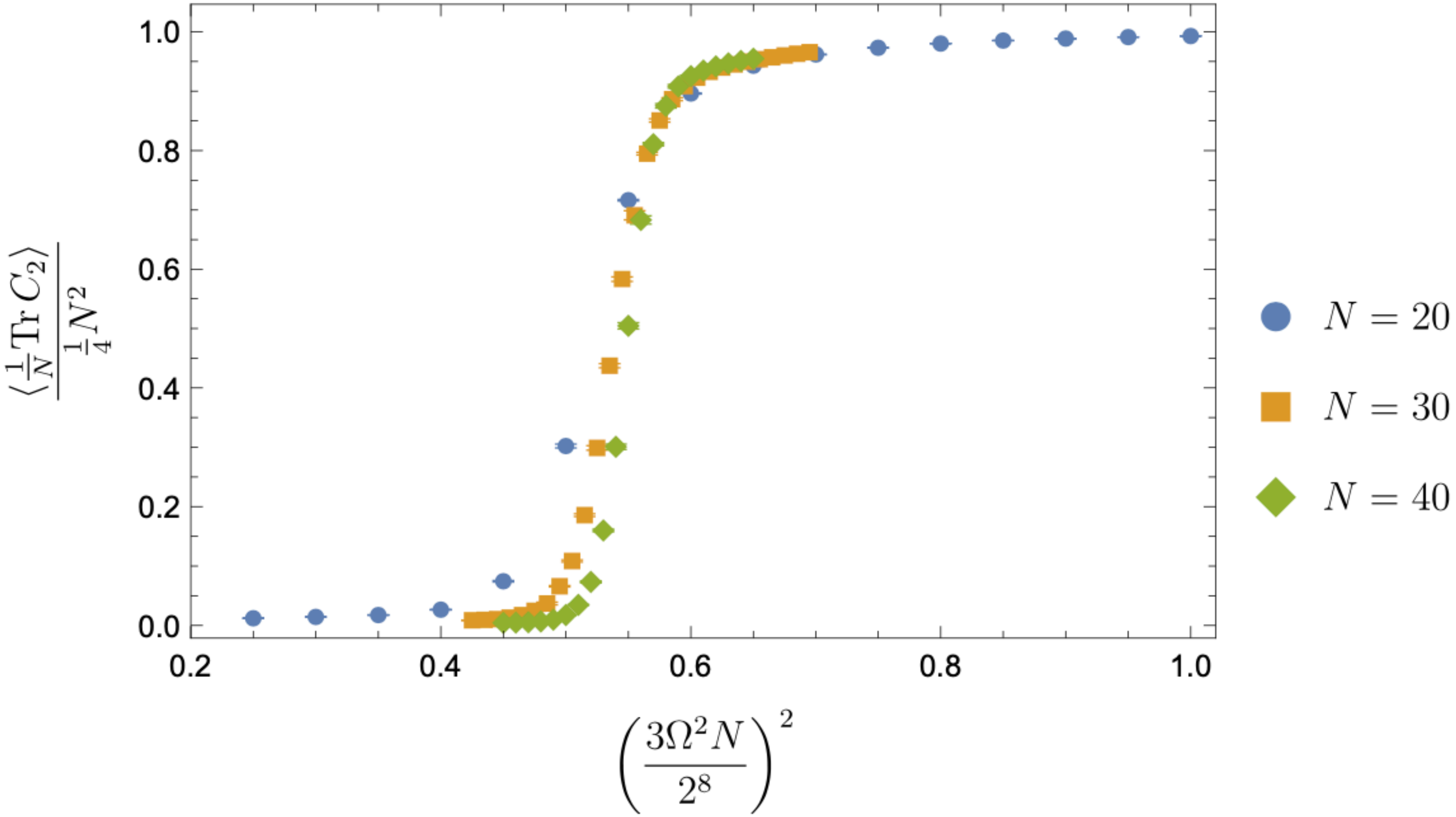
- The Casimir is strongly peaked close to the max. reducible representation:

$$\langle \frac{1}{N}\mathrm{Tr}C_2\rangle \sim \Omega^2 N \qquad \sigma^2_{\frac{1}{N}\mathrm{Tr}C_2} \sim \frac{1}{N}\Omega^2 N$$



Phase structure of the theory

- The first order phase transition is between these highly reducible and irreducible representations:

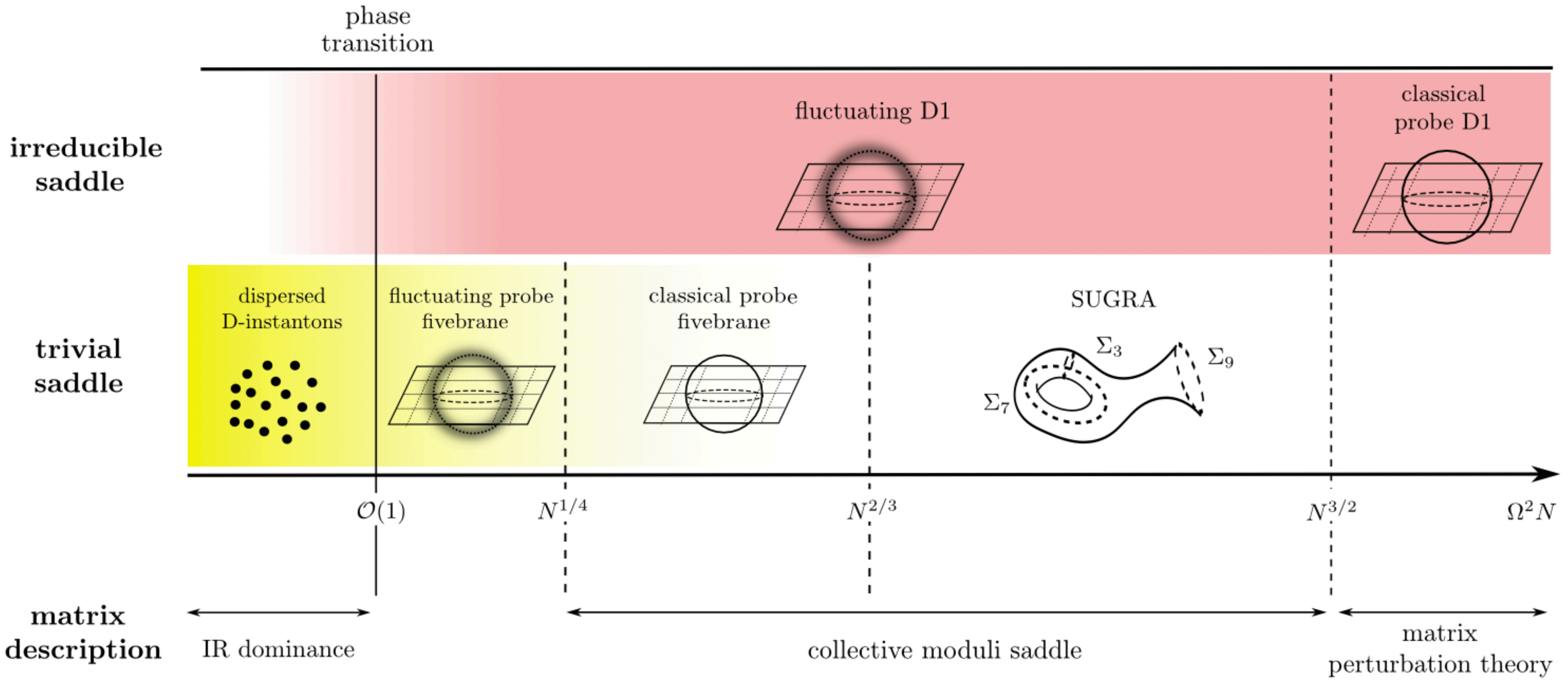


Similar to previous collapse of fuzzy spheres seen in:

[Azuma-Bal-Nagao-Nishimura 04,
Delgadillo-Blando-O'Connor-Ydri 07,
Catterall-van Anders 10,
Asano-Filev-Kovacik-O'Connor 18,
Han-Hartnoll 19,
MCSMC 20,
Joseph-Schaich 24, ….
]

Spacetime picture

- The spacetime interpretation of each saddle depends on Ω:



Spacetime picture

- On both sides of the phase transition, the matrices do not create a gravitating spacetime in the dominant saddle.

- SUGRA [Komatsu-Martina-Penedones-Vuignier-Zhao 2410.18173] solutions are weakly curved and have small dilaton only in the sub-dominant saddle and over a finite range of deformations.

$$ds^2 = F(\rho, z)\left[d\rho^2 + dz^2\right] + G(\rho, z)d\Omega^2_{S^6} + H(\rho, z)d\Omega^2_{S^2}$$

- F, G, H determined by a linear Poisson problem, with sources that depend on the SU(2) representation data n_M. (cf. Lin-Lunin-Maldacena 04).

Spacetime picture

- In 2504.06481 we found a further (sub-dominant) regime given by a Euclidean (p,q) fivebrane in the cavity geometry. (cf. Polchinski-Strassler 00)

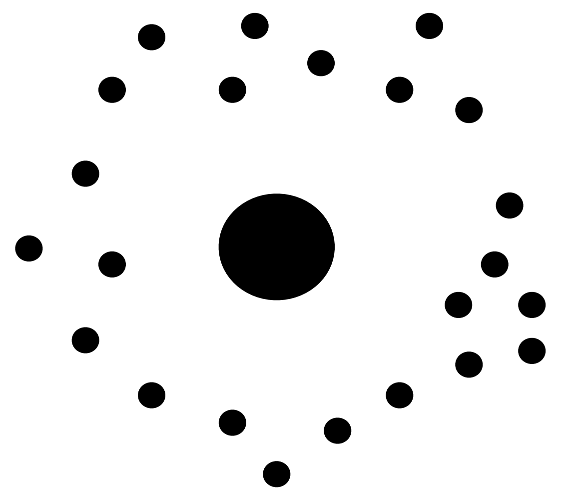
$$\rho_M(x) \equiv \sum_{i=1}^{n_M} \delta(x - m_{Mi}) \sim (\hat{R}^2 - x^2)^2$$

Matches radius of
(p,q) fivebrane
in cavity geometry

[found in 2411.18678, similar to Asano-Ishiki-Shimasaki-Terashima '14 for BMN, one dimensional projection of 6-sphere]

[fivebranes ~ fluctuations of D1 branes]

- Below the transition: 'gas' of well-separated D-instantons with no collective effects (i.e. no emergence). The Ω -> 0 limit is discontinuous and the Ω = 0 IKKT model comes from sub-dominant matrix configurations.



Comments

- The model we are studying has a clear realisation within string theory. This allows matrix configurations to be related to geometries.

- The emergence of semiclassical gravitating geometry from matrices in this model occurs within subdominant saddles.

- Does spacetime emerge in matrix partition functions above a sea of non-geometric configurations (cf. negatives modes in Maldacena '24).

- Within the gravitating saddles, it should be possible to establish a holographic dictionary. In my opinion the most interesting question is to understand how the Hamiltonian constraint (= emergence of time) is encoded. Does $Z = |\Psi|^2$?