Simulating the Euclidean and Lorentzian IKKT matrix model with deformations

Jun Nishimura (KEK, SOKENDAI)

invited talk at the workshop “Matrix Model for Superstring/M-theory”

Yukawa Institute for Theoretical Physics, Kyoto University

Dec 1 – 5, 2025

Ref.) Chou, JN, Wang, Phys.Rev.Lett. 135 (2025) 22, 221601,
Chau, Chou, JN, Wang, in preparation
Asano, JN, Piensuk, Yamamori, Phys.Rev.Lett. 134 (2025) 4, 041603
Anagnostopoulos, Azuma, Hirasawa, Karydis, J.N., Tsuchiya, Yamamori, work in progress

0. Introduction

IKKT (or type IIB) matrix model

Ishibashi-Kawai-Kitazawa-Tsuchiya,
Nucl.Phys.B 498 (1997) 467, hep-th/9612115 [hep-th]

- a nonperturbative formulation of superstring theory
  (all the vacua in the ``string landscape'' included as saddle points)

$$S_{\rm b} = -\frac{1}{4} N \, {\rm tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$
$$S_{\rm f} = -\frac{1}{2} N \, {\rm tr}(\Psi_\alpha (\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

(0-dimensional reduction of 4D $\mathcal{N}=4$ SYM)

$N \times N$ Hermitian matrices

SO(9,1) Lorentz symmetry

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector
$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = {\rm diag}(-1, 1, \cdots, 1)$ is used to raise and lower indices.

- Unlike AdS/CFT, not only space but also time emerge as the eigenvalue distribution of the 10 bosonic matrices.

maximal SUSY (incl. translation : $A_\mu \mapsto A_\mu + \alpha_\mu \mathbf{1}$)

the Euclidean IKKT model

"Wick rotation" : $A_0 = -iA_{10}$

Aoki-Iso-Kawai-Kitazawa-Tada ('98)
Hotta-JN-Tsuchiya ('98)

$$Z_{\mathrm{E}} = \int dA\, d\Psi\, e^{-(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA\, e^{-S_{\mathrm{b}}}\, \mathrm{Pf}\mathcal{M}(A)$$

$S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu})^2$ positive semi-definite!

$F_{\mu\nu} = -i\,[A_\mu, A_\nu]$: Hermitian

Euclidean model is well defined without any cutoff.

Krauth-Nicolai-Staudacher ('98), Austing-Wheater ('01)

$\mathrm{Pf}\mathcal{M}(A)$: complex valued

⇒ Fluctuation of the phase becomes milder for lower dimensional configs.

A possible mechanism for SSB of SO(10) J.N.-Venizzi ('00)

Emergence of 3d space suggested by complex Langevin simulation.

Anagnostopoulos, et al. JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]

SUSY deformation of the Euclidean IKKT model

Euclidean IKKT model → polarized IKKT model

SUSY deformation
(preserving 16 SUSY)

parameter: Ω

$O(A^2)$ bosonic mass term
$O(A^3)$ Myers term
$O(\bar{\Psi}\Psi)$ fermionic mass term

dominant configurations

3-dimensional "fuzzy ball"
[SSB : SO(10)→ SO(3)]

dominant configurations

fuzzy sphere (at large Ω)
commuting matrices (at small Ω)
(diverges as $\Omega \to 0$)

- Dual supergravity solutions identified similarly to the BMN model.
- The original Euclidean model is **not** retrieved in the $\Omega \to 0$ limit.

S.A. Hartnoll, J. Liu, JHEP 03 (2025) 060 [arXiv: 2409.18706 [hep-th]]
S. Komatsu, A. Martina, J. Penedones, A. Vuignier and X. Zhao, arXiv: 2410.18173 [hep-th]

Plan of the talk

0. Introduction
1. Euclidean IKKT model with SUSY deformation
2. Lorentzian IKKT model with SUSY-like deformation
3. Summary and discussions

1. Euclidean IKKT model with SUSY deformation

Ref.) Chou, JN, Wang, Phys.Rev.Lett. 135 (2025) 22, 221601,

Chau, Chou, JN, Wang, in preparation

See Cheng-Tsung Wang's poster and Ronny Chau's poster on Wednesday.

SUSY-deformation of the Euclidean model

SUSY deformation (preserves 16 SUSY but SO(10)→SO(7) × SO(3))

$$Z = \int dA\, d\Psi\, e^{-(S_{\mathrm{b}}+S_{\mathrm{f}}+S_{\Omega})}$$

$$S_{\Omega} = \frac{\Omega^2}{4^3}\left\{3\sum_{i=1}^{3}\mathrm{Tr}\,(A_i)^2 + \sum_{a=4}^{10}\mathrm{Tr}\,(A_a)^2\right\} + i\frac{\Omega}{3}\sum_{i,j,k=1}^{3}\epsilon^{ijk}\mathrm{Tr}(A_iA_jA_k) - \frac{\Omega}{8}\mathrm{Tr}\left(\overline{\Psi}\Gamma^1(\Gamma^2)^{\dagger}\Gamma^3\Psi\right)$$

- Holographic dual has been identified.

  S.A. Hartnoll, Jun Liu, JHEP 03 (2025) 060 [arXiv: 2409.18706 [hep-th]]
  S. Komatsu, A. Martina, J. Penedones, A. Vuignier and X. Zhao, arXiv: 2410.18173 [hep-th]

- Solvable by SUSY localization technique.

  S. Komatsu, A. Martina, J. Penedones, A. Vuignier and X. Zhao, arXiv:2411.18678 [hep-th]

$$Z = \sum_{\mathcal{R}} Z_{\mathcal{R}}\,,$$
$$Z_{\mathcal{R}} = C_{\mathcal{R}} e^{-S^0_{\mathcal{R}}} \int d\vec{m}_{\mathcal{R}}\, e^{-S^{\mathrm{eff}}_{\mathcal{R}}(\vec{m}_{\mathcal{R}})}$$

$\Omega \to \infty$: fuzzy sphere dominates
$\Omega \to 0$: singular ($Z \to \infty$)

MC simulation of this localized model at N=40 → phase transition at $\frac{3\,\Omega_c^2 N}{2^8} = \mathcal{O}(1)$

S.A. Hartnoll, J. Liu, SciPost Phys. 19 (2025) 099, arXiv:2504.06481 [hep-th]

- Here we simulate the matrix model directly.

  Probe the spacetime emerging from the matrix configuration at each Ω.

saddle points in the N=2 SUSY deformed model

Ansatz:

$$A_1 = a\frac{\sigma_1}{2}\ ,\quad A_2 = b\frac{\sigma_2}{2}\ ,\quad A_3 = c\frac{\sigma_3}{2}$$
$$A_a = 0 \qquad (a = 4, \cdots, 10)$$

general up to symmetry
SU(2) × SO(7) × SO(3)

$$S = \frac{1}{4}\left(a^2b^2 + b^2c^2 + c^2a^2\right) + \frac{3\Omega^2}{2^7}\left(a^2 + b^2 + c^2\right) - \frac{\Omega}{2}abc - \log\left(2abc + \frac{\Omega}{4}\left(a^2 + b^2 + c^2\right) - \frac{\Omega^3}{64}\right)^8$$

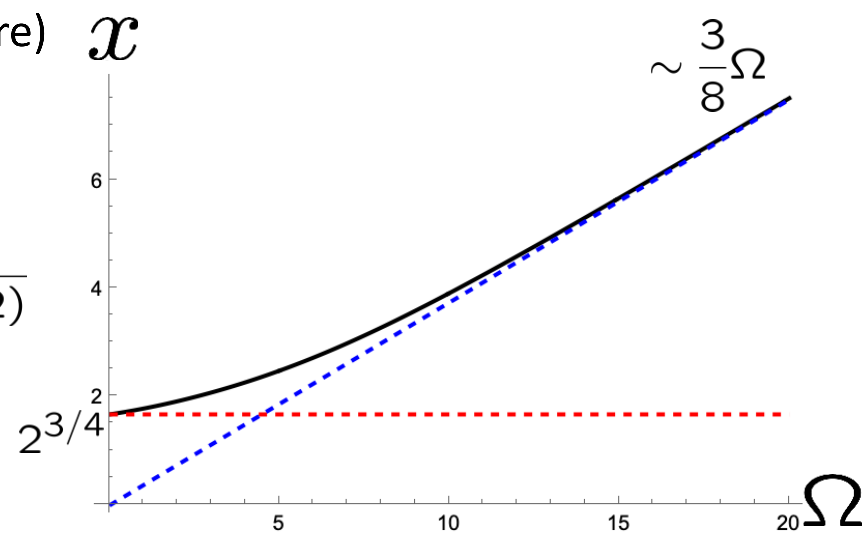
Pfaffian

$\Omega = 0$ (original Euclidean IKKT)

$$a = b = c = 2^{3/4} \quad \text{(fuzzy sphere)}$$

$\Omega \neq 0$ assuming $a = b = c (= x)$

$$0 = x^3 + \frac{3\Omega^2}{2^6}x - \frac{\Omega}{2}x^2 - \frac{256\,x}{(8x - \Omega)(4x + \Omega)}$$



Fuzzy sphere at large Ω is smoothly connected to the solution at Ω=0.

We can prove that this is true at any N for the irrep. of SU(N) algebra.
If true also for reducible rep., we may understand the emergence of "fuzzy 3d ball" at Ω=0.

1-loop effective theory at small Ω

- 1-loop perturbative expansion around commuting configurations

  - original Euclidean IKKT model:
    complications due to fermionic zero modes

    H.Aoki, S.Iso, H.Kawai, Y.Kitazawa, T.Tada, Prog.Theor.Phys. 99 (1998) 713 [hep-th/9802085]

  - SUSY-deformed IKKT model:
    Fermionic zero modes become massive and decouple.

    S. Komatsu, A. Martina, J. Penedones, A. Vuignier and X. Zhao, arXiv:2411.18678 [hep-th]

$$A_\mu = \mathrm{diag}(x_\mu^{(1)}, \cdots, x_\mu^{(N)})$$

$$Z_{1-\mathrm{loop}} = \int dx \exp\left[-\frac{\Omega^2}{2^6}\sum_{p=1}^{N}\left\{3\sum_{i=1}^{3}(x_i^{(p)})^2 + \sum_{a=4}^{10}(x_a^{(p)})^2\right\}\right]$$

⇒ dominant configs. diverge as $\frac{1}{\Omega}$

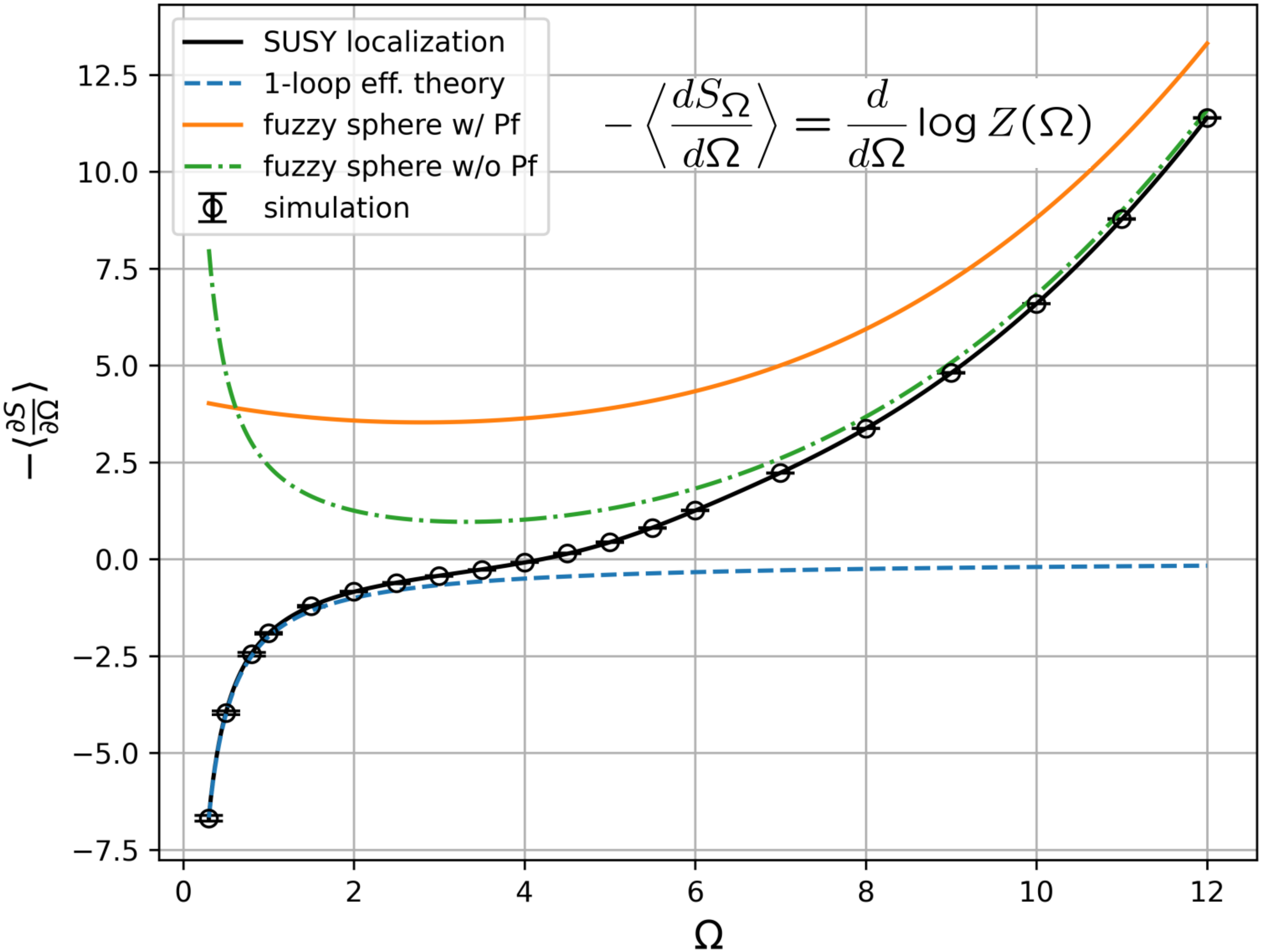
bosonic fluctuations yield $\{\prod_{p<q}(x_\mu^{(p)} - x_\mu^{(q)})^2\}^{-8}$

fermion fluctuations yield $\{\prod_{p<q}(x_\mu^{(p)} - x_\mu^{(q)})^2\}^{8}$

} cancel each other (due to SUSY)

comparison with the localization results

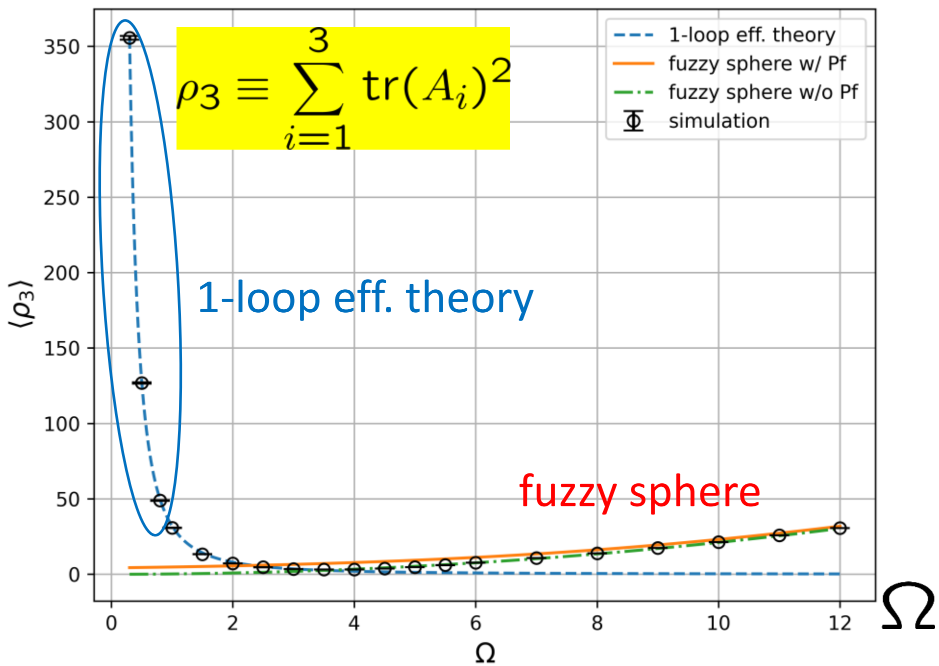
$N = 2$

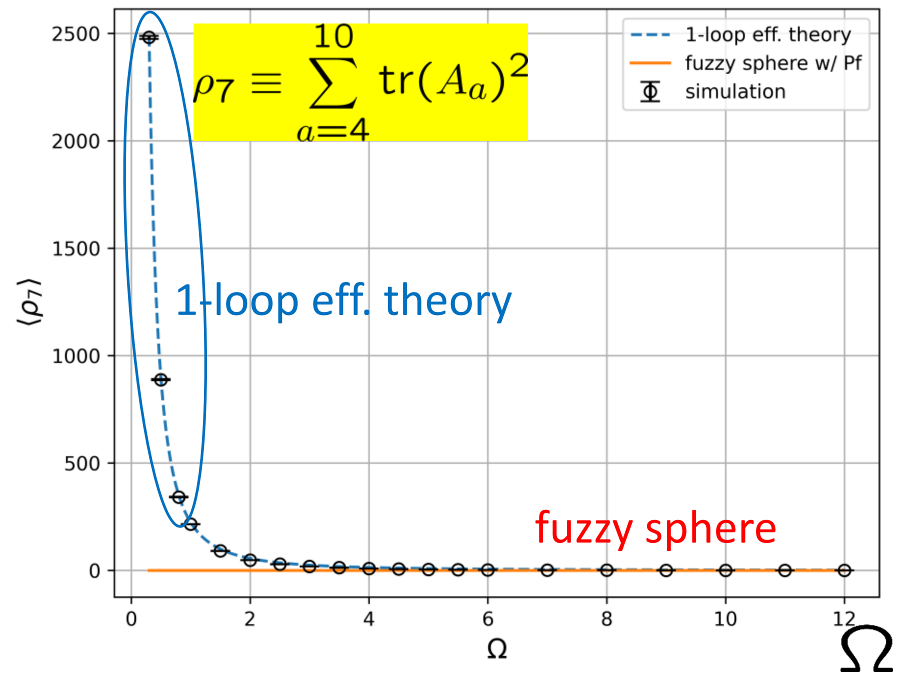


Simulation result agrees with the localization result for any Ω.

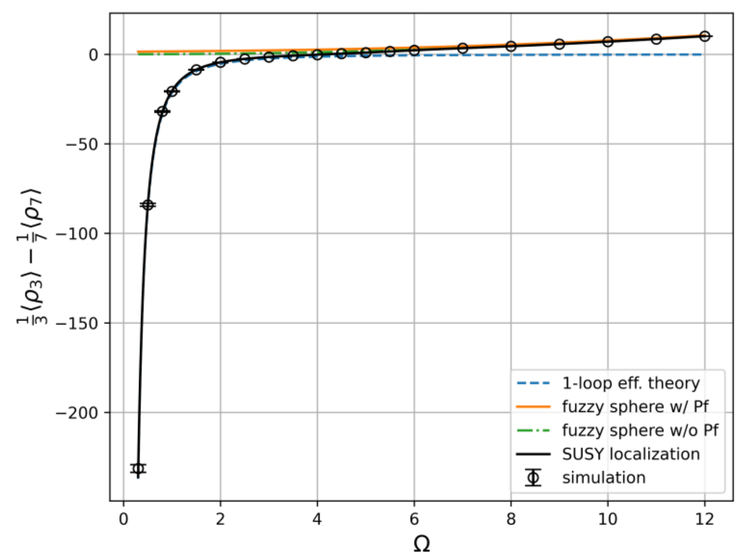
various quantities (inaccessible by localization!)

$N = 2$



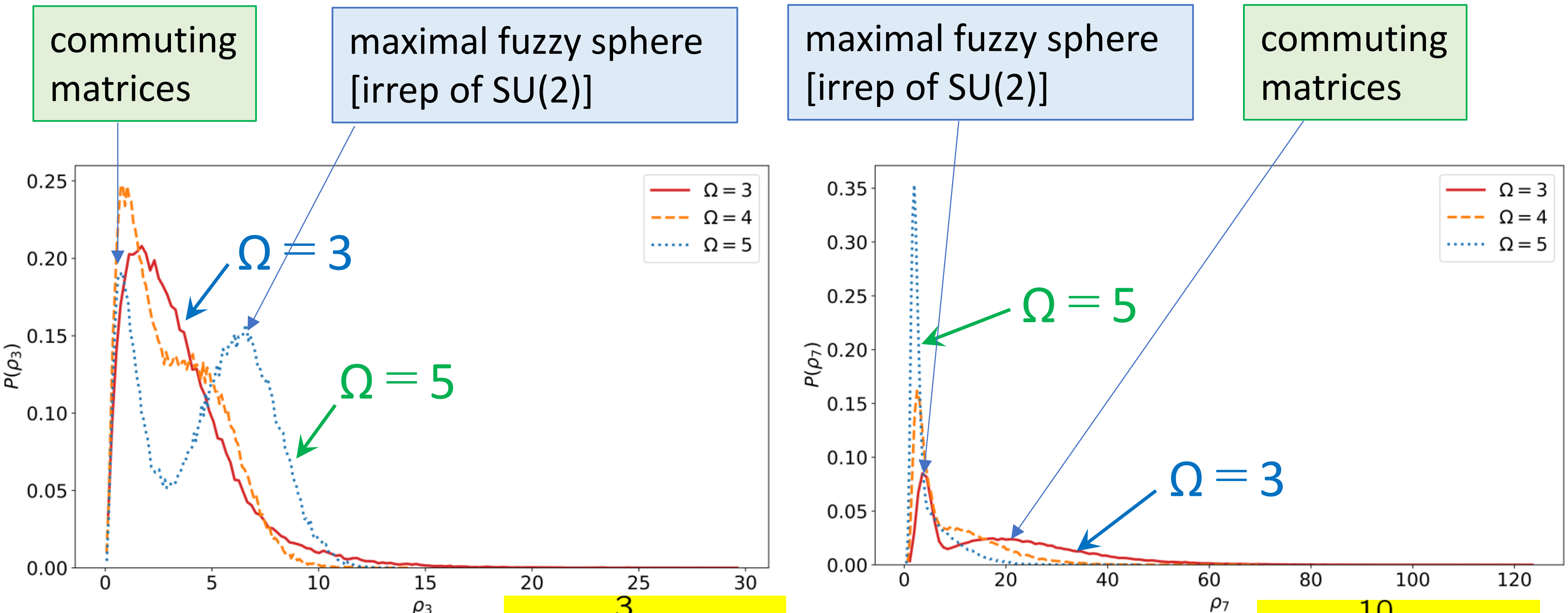


The linear combination $\frac{1}{3}\langle \rho_3 \rangle - \frac{1}{3}\langle \rho_7 \rangle$ agrees with the localization result.



transition around $\Omega \sim 4$

$N = 2$

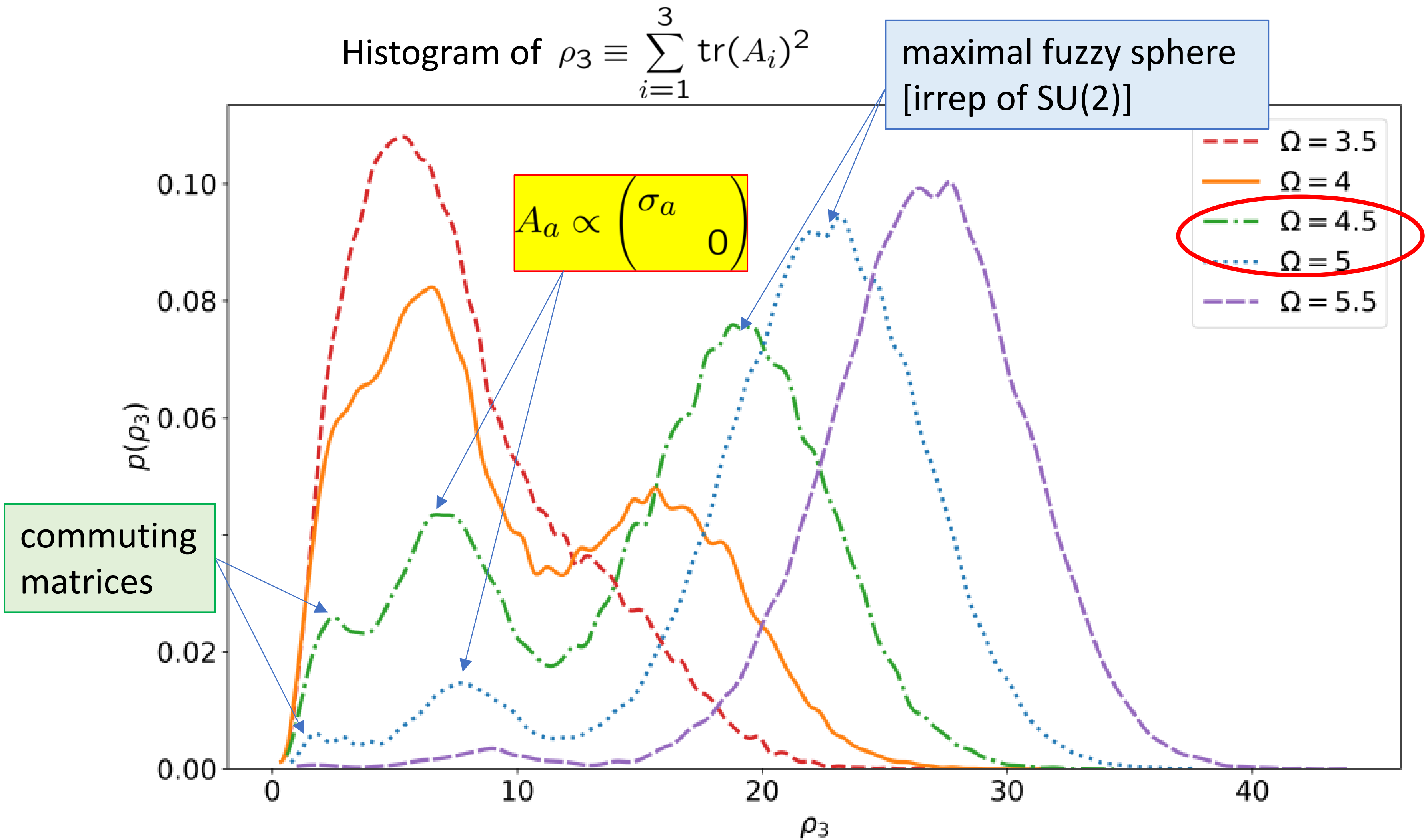


$$\rho_3 \equiv \sum_{i=1}^{3} \mathrm{tr}(A_i)^2$$

$$\rho_7 \equiv \sum_{a=4}^{10} \mathrm{tr}(A_a)^2$$

Two peaks corresponding to the fuzzy sphere and commuting matrices clearly identified at Ω～4.

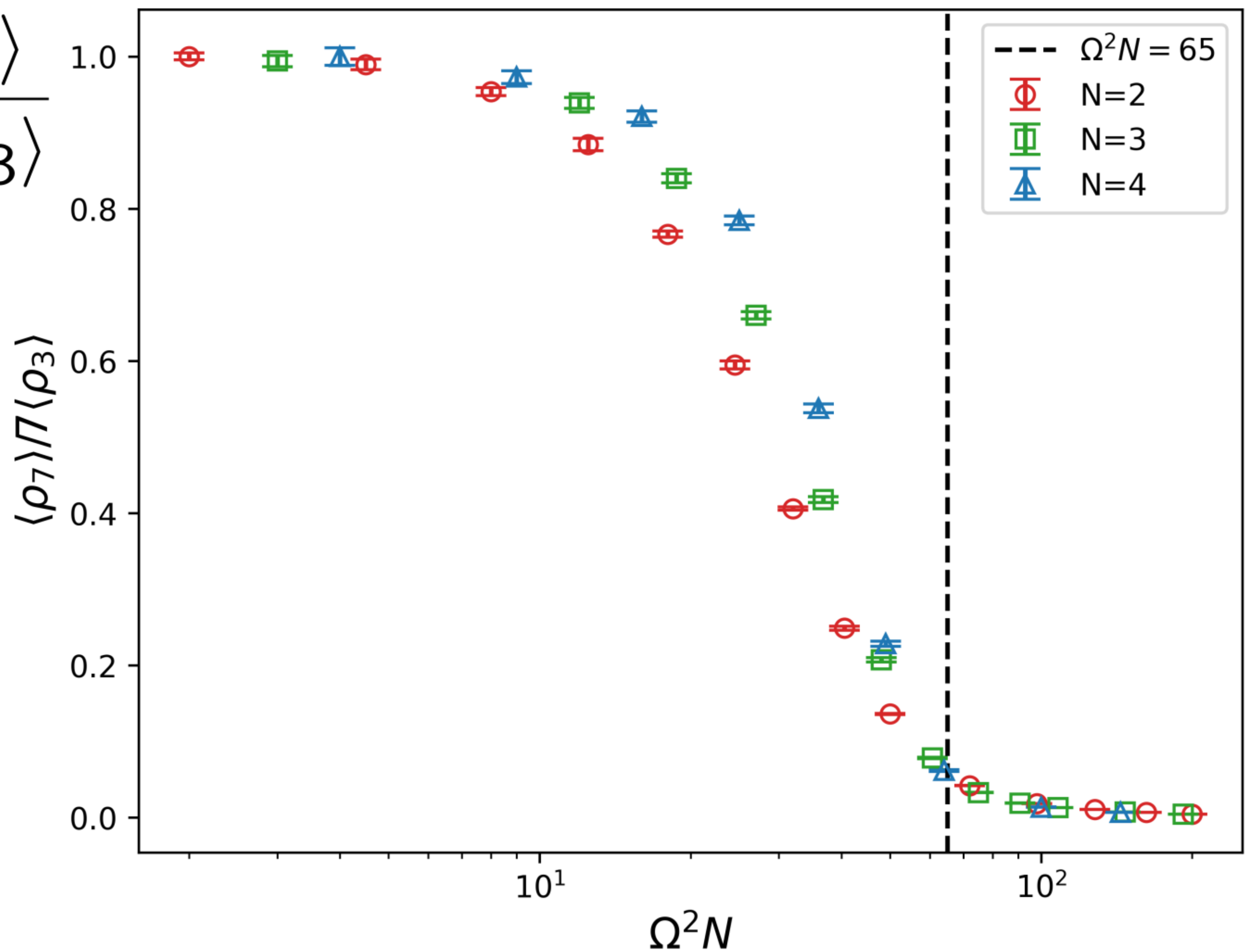
N=3 results : reducible representation



Reducible representations of SU(2) algebra appear as subdominant saddles in the intermediate region of Ω.

an "order parameter" for the transition

$$\frac{\langle \rho_7 \rangle}{7\langle \rho_3 \rangle}$$



$\Omega^2 N$

Scaling behavior is seen for N=2,3,4.
The transition becomes sharper with increasing N.

2. Lorentzian IKKT model with SUSY-like deformation

2.1 Defining the Lorentzian IKKT model

regularizing the Lorentzian model

- Unlike the Euclidean model,
  the Lorentzian model is NOT well defined as it is.

$$Z = \int dA\, d\Psi\, e^{i(S_{\mathrm{b}}+S_{\mathrm{f}})} = \int dA\, e^{iS_{\mathrm{b}}} \mathrm{Pf}\,\mathcal{M}(A)$$

pure phase factor

polynomial in A

real valued

- Wick rotation

(Yuhma Asano '19, private communication)

$$S_{\mathrm{b}} \mapsto \tilde{S}_{\mathrm{b}} = N e^{i\frac{\pi}{2}s}\left\{\frac{1}{2}e^{-i\pi k}\mathrm{tr}\,[\tilde{A}_0,\tilde{A}_i]^2 - \frac{1}{4}\mathrm{tr}\,[\tilde{A}_i,\tilde{A}_j]^2\right\}$$

on the worldsheet

in the target space

This corresponds to deforming the integration contour in the Lorentzian model.

$$\begin{cases} A_0 = \mathrm{e}^{i\frac{\pi}{8}s - i\frac{\pi}{2}k}\tilde{A}_0 = \mathrm{e}^{-i\frac{3\pi}{8}u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{\pi}{8}s}\tilde{A}_i = \mathrm{e}^{i\frac{\pi}{8}u}\tilde{A}_i \end{cases}$$

$$s = k (= u)$$

Path deformed theory is well-defined for $0 < u \leq 1$

(Yuhma Asano '19, private communication)

$$e^{iS_b(A)} = e^{-S(\tilde{A})} \qquad \begin{cases} A_0 = e^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = e^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases} \qquad \tilde{F}_{\mu\nu} = -i[\tilde{A}_\mu, \tilde{A}_\nu]$$

$$S(\tilde{A}) \sim 2e^{i\frac{\pi}{2}(1-u)}\mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)}\mathrm{tr}\,(\tilde{F}_{ij})^2$$

positive real part for $0 < u \leq 1$

$\mathrm{Re}\, S(\tilde{A}) \geq 0$

$S(\tilde{A})$: real positive at $u = 1$ (Euclidean).

According to Cauchy's theorem,
$\langle \mathcal{O}(e^{-i\frac{3}{8}\pi u}\tilde{A}_0, e^{i\frac{1}{8}\pi u}\tilde{A}_i)\rangle_u$ is independent of u.

⇒ If we define the Lorentzian model by taking the $u \to +0$ limit,

$$\langle \mathcal{O}(A_0, A_i)\rangle_L = \langle \mathcal{O}(e^{-i\frac{3\pi}{8}}\tilde{A}_0, e^{i\frac{\pi}{8}}\tilde{A}_i)\rangle_E$$

confirmation of the equivalence by CL simulation

Anagnostopoulos-Azuma-Hirasawa-J.N.-Papadoudis-Tsuchiya, in preparation

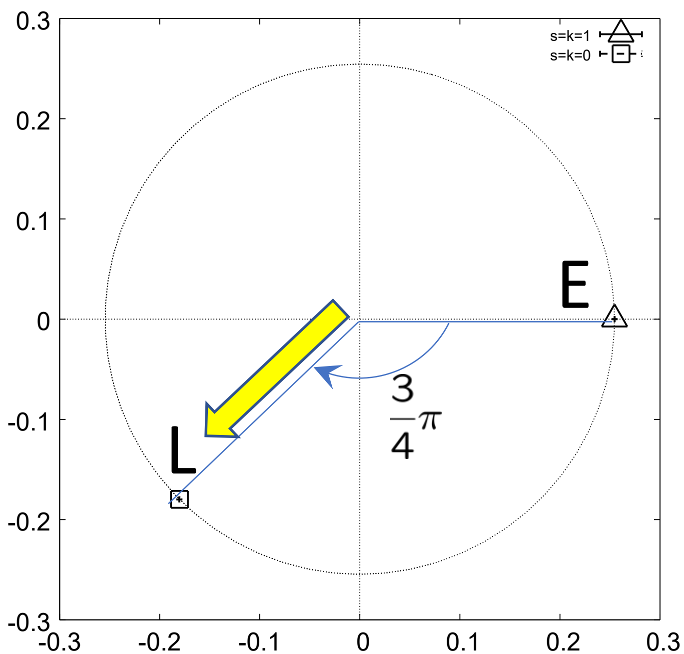
(9+1)D bosonic model

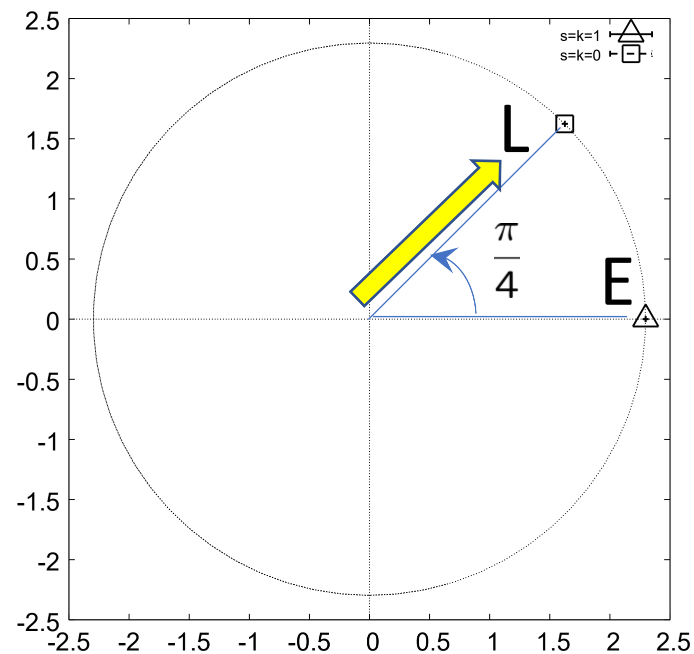
$$\left\langle \frac{1}{N}\mathrm{tr}(A_0)^2 \right\rangle_{\mathrm{L}} = e^{-\frac{3\pi}{4}i} \underline{\left\langle \frac{1}{N}\mathrm{tr}(\tilde{A}_0)^2 \right\rangle_{\mathrm{E}}}$$

real positive

$$\left\langle \frac{1}{N}\mathrm{tr}(A_i)^2 \right\rangle_{\mathrm{L}} = e^{\frac{\pi}{4}i} \underline{\left\langle \frac{1}{N}\mathrm{tr}(\tilde{A}_i)^2 \right\rangle_{\mathrm{E}}}$$

real positive





The emergent space-time is complex and has Euclidean signature.

introducing a Lorentz invariant mass term

$$Z = \int dA\, e^{i(S_{\rm b}+S_{\rm m})} \mathrm{Pf}\mathcal{M}(A)$$

Anagnostopoulos-Azuma-Hirasawa-J.N.-Papadoudis-Tsuchiya, in preparation

$$S_{\rm m} = \frac{1}{2} N\gamma \left\{ \mathrm{tr}(A_0)^2 - \mathrm{tr}(A_i)^2 \right\}$$

$\gamma > 0$

$$e^{i(S_{\rm b}(A)+S_{\rm m}(A))} = e^{-S(\tilde{A})}$$

$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u} \tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u} \tilde{A}_i \end{cases}$$

$$\tilde{F}_{\mu\nu} = -i[\tilde{A}_\mu, \tilde{A}_\nu]$$

positive real part for $0 < u \leq 1$

$$S(\tilde{A}) \sim 2e^{i\frac{\pi}{2}(1-u)} \mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)} \mathrm{tr}\,(\tilde{F}_{ij})^2 + \gamma e^{-i\frac{\pi}{2}(1+\frac{3}{2}u)} \mathrm{tr}\,(\tilde{A}_0)^2 + \gamma e^{i\frac{\pi}{2}(1+\frac{1}{2}u)} \mathrm{tr}\,(\tilde{A}_i)^2$$

negative real part for $0 < u \leq 1$

One cannot define the model by contour deformation any longer.

Equivalence to the Euclidean model can be avoided.

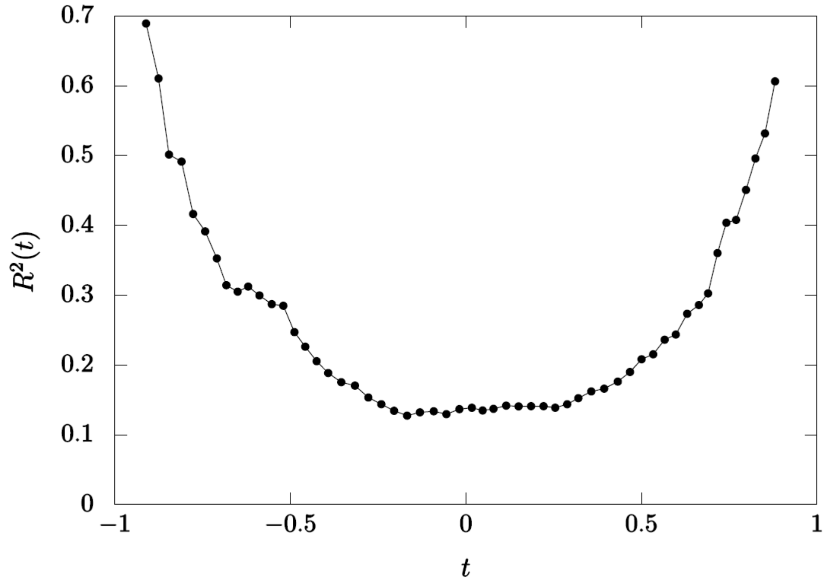
classical solutions

Eq. of motion : $[A^\nu, [A_\nu, A_\mu]] - \gamma A_\mu = 0$

$\gamma < 0$: trivial solutions only
$\gamma = 0$: diagonal solutions only
$\gamma > 0$: expanding solutions

Note: The space-time dimensionality is not determined at the classical level.

Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob, *PTEP* 2020 (2020) 4, 043B10



Our proposal: 1) $N \to \infty$, 2) $\gamma \to +0$

- Expanding space-time is entropically favored.
- (3+1)-dim. space-time is favored by the Pfaffian. (Pf vanishes for 2d configs.)

fixing the Lorentz symmetry

Asano, JN, Piensuk, Yamamori: Phys.Rev.Lett. 134 (2025) 4, 041603
See Ashutosh Tripathi's poster on Wednesday for related work

- The partition function is divergent due to Lorentz symmetry.

noncompact group!

$$Z = \int dA\, e^{i(S_{\rm b}+S_{\rm m})}$$

Nontrivial saddle points have flat directions, along which the integration diverges.

minimize: $\mathrm{tr}\,(A_0)^2$ w.r.t. Lorentz tr.

$$\begin{pmatrix} A'_0 \\ A'_j \end{pmatrix} = \begin{pmatrix} \cosh\sigma & \sinh\sigma \\ \sinh\sigma & \cosh\sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_j \end{pmatrix}$$

$\mathrm{tr}\,(A_0 A_j) = 0$ for all j

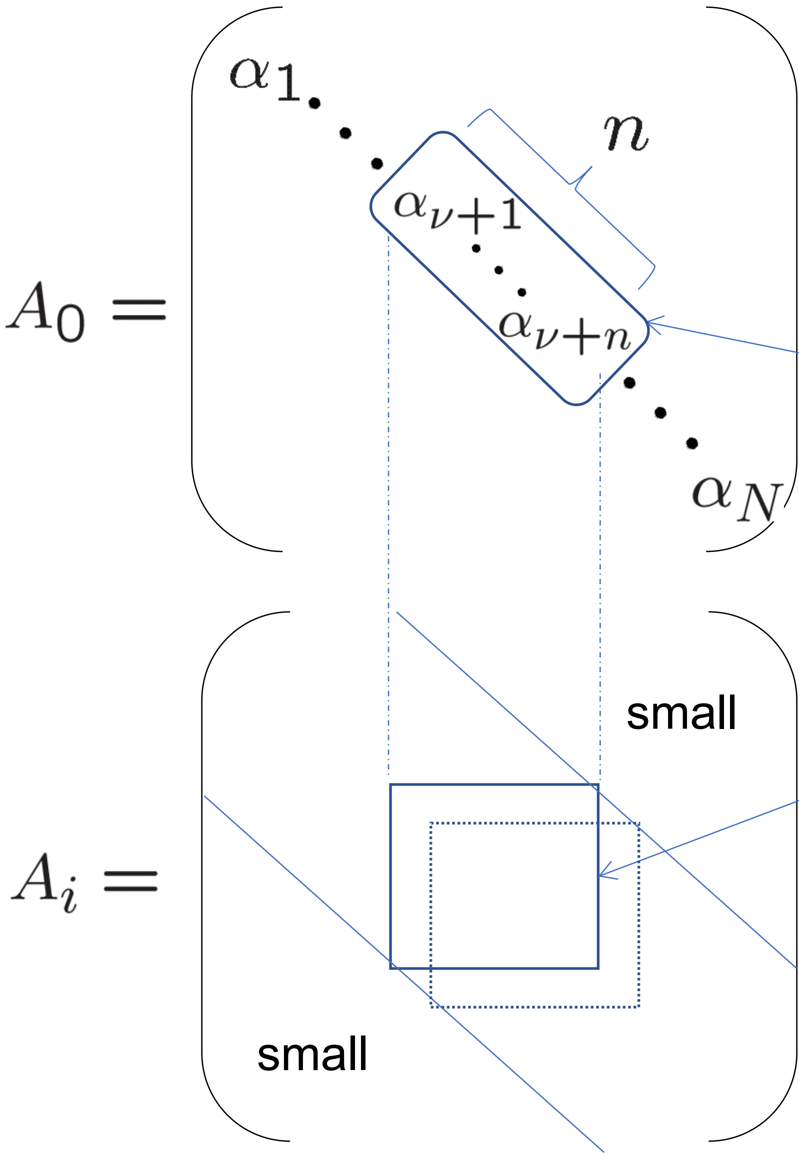
$$Z = \int dA\, e^{i(S_{\rm b}+S_{\rm m})} \Delta_{\rm FP}[A] \prod_{j=1}^{d} \delta(\mathrm{tr}\,(A_0 A_j))$$

$$\Delta_{\rm FP}[A] = \det \Omega \ , \quad \Omega_{ij} = \mathrm{tr}\,(A_0)^2 \delta_{ij} + \mathrm{tr}\,(A_i A_j)$$

sign problem → complex Langevin method

extracting time-evolution from the Lorentzian model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]



$$\alpha_1 < \cdots < \alpha_N$$

definition of “time”

$$t_\nu = \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t_\nu)$ at time t_ν

A_i has a band diagonal structure

non-trivial dynamical property

defining "time" of the IKKT model in complex Langevin simulation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

Fixing the U(N) symmetry: $A_\mu \mapsto U A_\mu U^\dagger$

$$Z = \int dA_0 \, dA_i \, e^{-S} = \int d\alpha \, dA_i \, \Delta^2(\alpha) \, e^{-S}$$

$$A_0 = \mathrm{diag}(\alpha_1, \cdots, \alpha_N)$$

$$\alpha_1 < \alpha_2 < \cdots < \alpha_N$$

$$\Delta(\alpha) = \prod_{a<b} (\alpha_a - \alpha_b)$$: van der Monde determinant

We make the change of variables

$$\alpha_1 = 0 \ , \quad \alpha_2 = e^{\tau_1} \ , \quad \alpha_3 = e^{\tau_1} + e^{\tau_2} \ , \quad \cdots \ , \quad \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \ ,$$

to introduce the "time ordering" respecting holomorphicity.

complex Langevin equation

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

the effective action

$$S_{\rm eff} = -i\,N\left\{\frac{1}{2}{\rm tr}\,[A_0, A_i]^2 - \frac{1}{4}{\rm tr}\,[A_i, A_j]^2\right\} - \log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a$$

complex Langevin equation

$$\begin{cases} \dfrac{d\tau_a}{dt} = -\dfrac{\partial S_{\rm eff}}{\partial \tau_a} + \eta_a \\ \dfrac{d(A_i)_{ab}}{dt} = -\dfrac{\partial S_{\rm eff}}{\partial (A_i)_{ba}} + (\eta_i)_{ab} \end{cases}$$

Gaussian noise (real) — η_a

Gaussian noise (Hermitian) — $(\eta_i)_{ab}$

τ_a : complex variables, A_i : general complex matrices.

some tricks to make the CLM work

Anagnostopoulos-Azuma-Hirasawa-J.N.-Papadoudis-Tsuchiya, in prep.

- Complex Langevin method fails
  when the Dirac operator has near-zero modes.

  To avoid this, we add a mass term to the fermionic action.

$$S_{\mathrm{f}} = \frac{1}{2}\mathrm{tr}\left(\bar{\Psi}_\alpha(\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta] + i\, m_{\mathrm{f}} \bar{\Psi}_\alpha(\Gamma_7 \Gamma_8^\dagger \Gamma_9)_{\alpha\beta}\Psi_\beta\right)$$

$$\mathrm{Pf}\mathcal{M}(A) \in \mathbb{R}$$

$m_{\mathrm{f}} = \infty$: bosonic
$m_{\mathrm{f}} = 0$: SUSY

CLM fails at small m_{f}, though.

- To mimic the SUSY deformation, we also introduce anisotropy in the bosonic mass term.

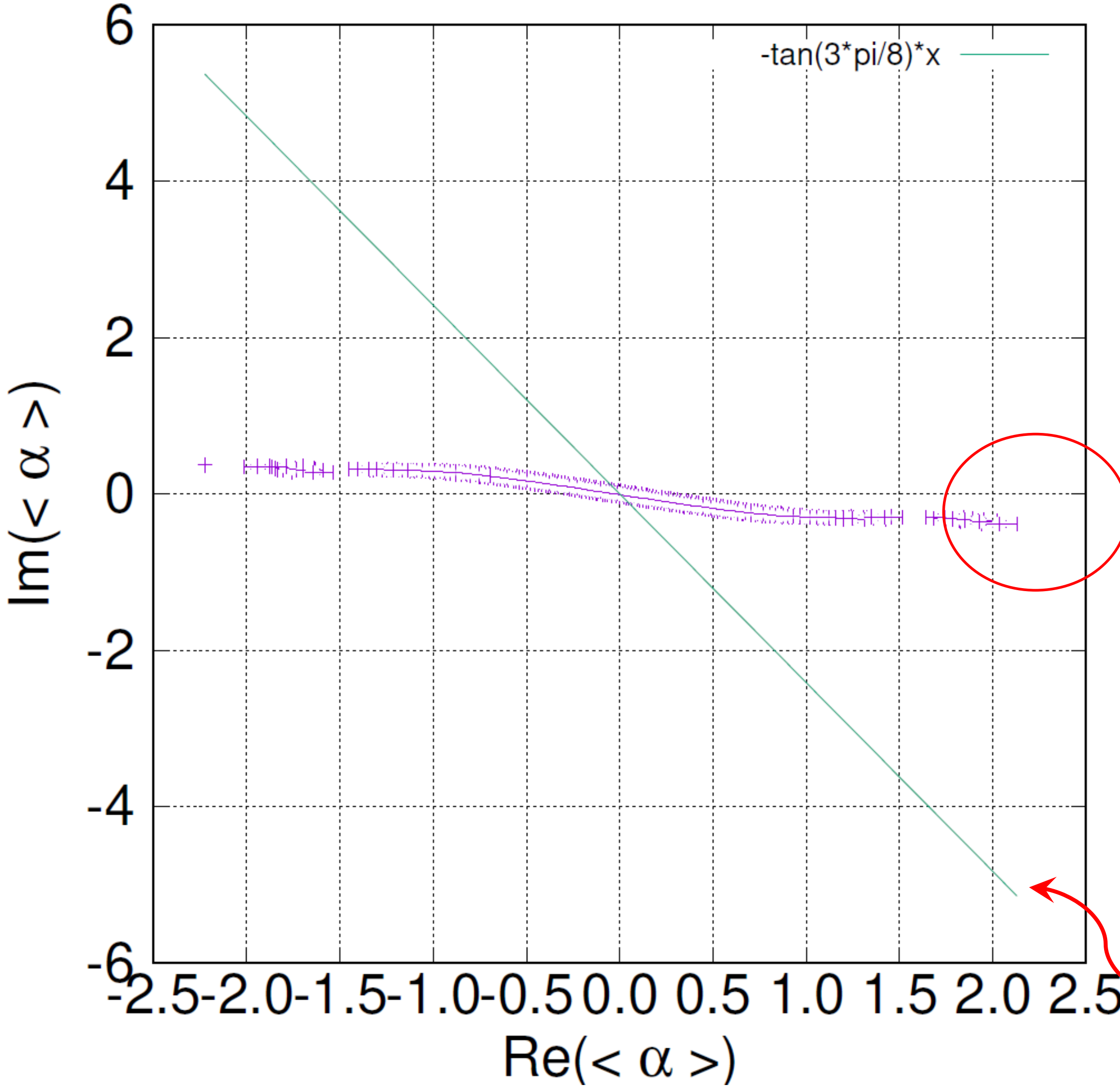
$$S_{\mathrm{m}} = \frac{N\gamma}{2}\left\{\mathrm{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \mathrm{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^{9} \mathrm{tr}(A_I)^2\right\}$$

2.2 Results of complex Langevin simulations

Anagnostopoulos, Azuma, Hirasawa, Karydis, J.N., Tsuchiya, Yamamori, work in progress

eigenvalues of A_0

$N = 128,\ \gamma = 4,\ m_f = 5,\ \xi = 10,\ \tilde{d} = 5$



$\Delta\alpha_i \in \mathbb{R}$

(Lorentzian)

Real time emerges at late times.

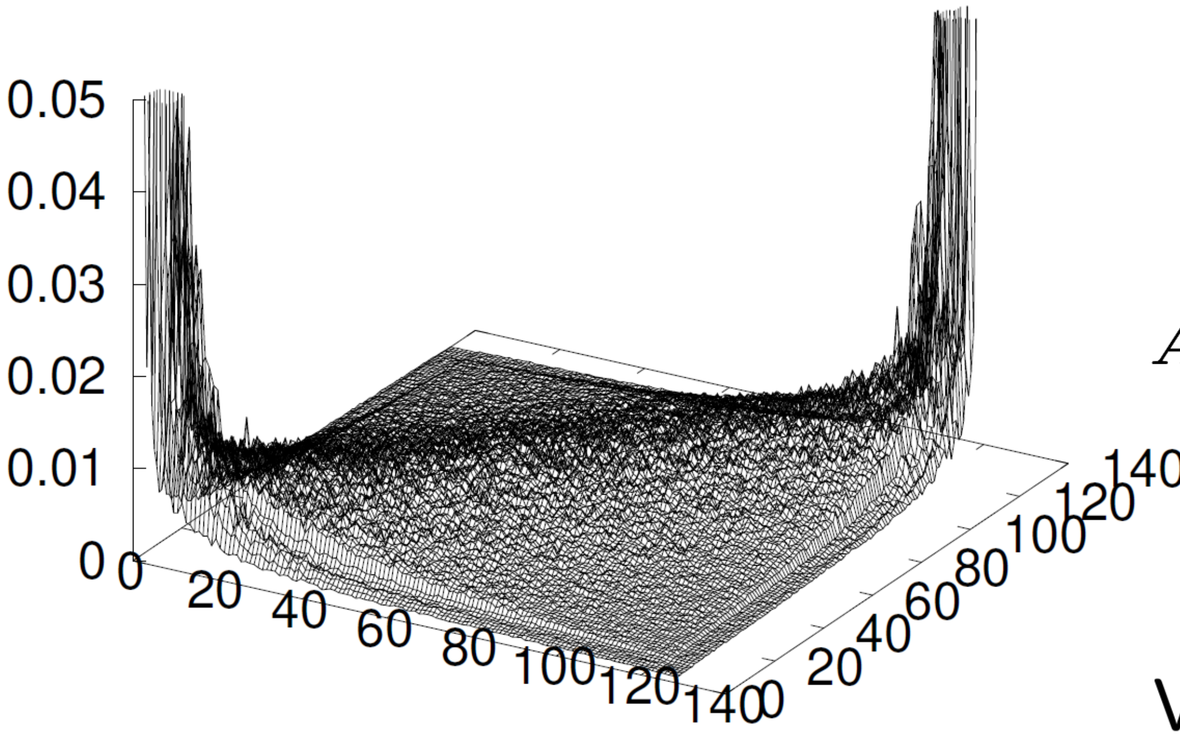
$\Delta\alpha_i \propto e^{-i\frac{3\pi}{8}}$

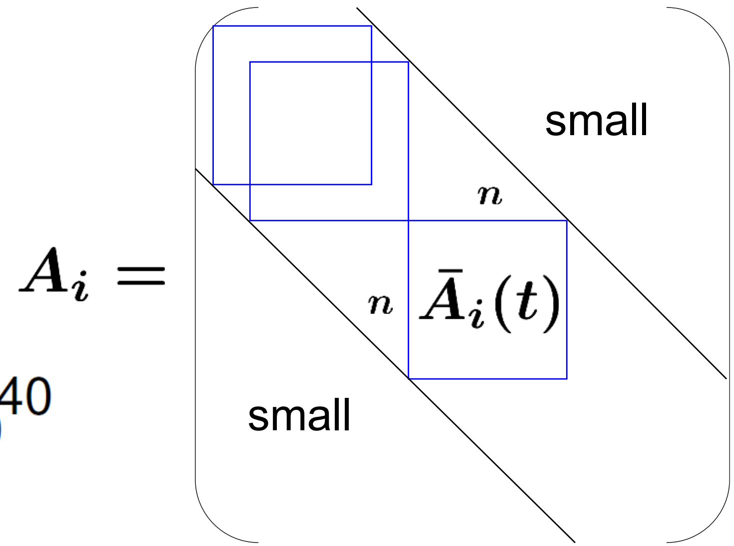
(Euclidean)

band-diagonal strucure

$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9} |(A_i)_{pq}|^2$$

$N = 128$, $\gamma = 4$, $m_f = 5$, $\xi = 10$, $\tilde{d} = 5$





We choose $n = 6$.

emergence of band-diagonal structure

important for extracting the time evolution from matrix configs.

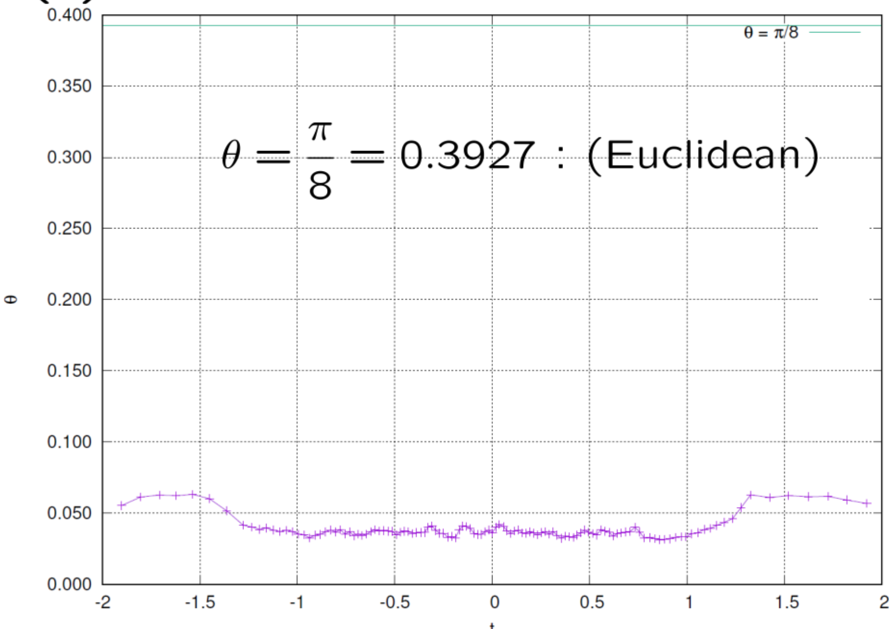
the time evolution of space

$N = 128,\ \gamma = 4,\ m_f = 5,\ \xi = 10,\ \tilde{d} = 5$

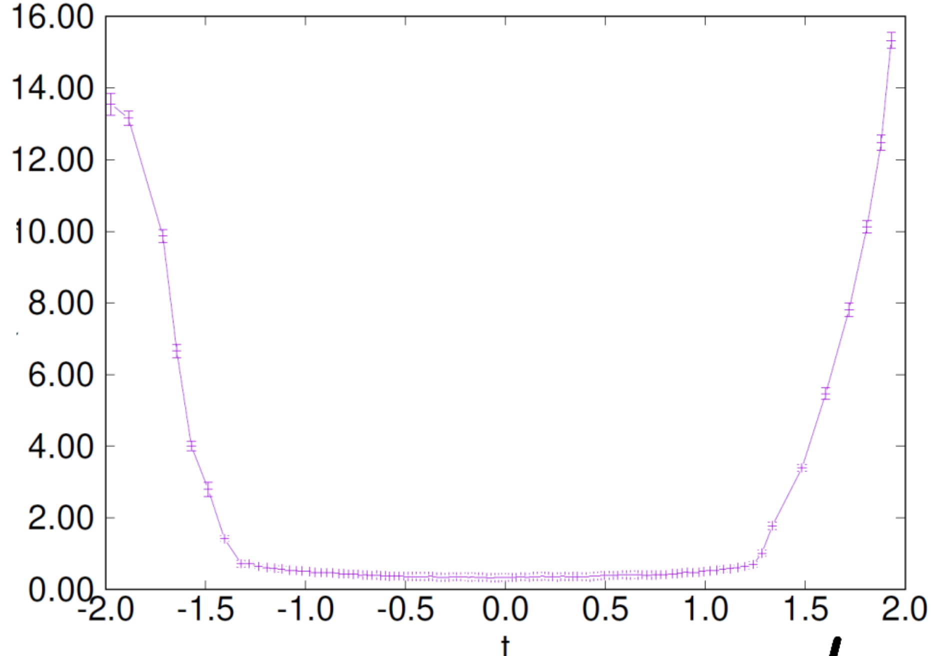
$$R^2(t) = \left\langle \frac{1}{n}\mathrm{tr}\left(\bar{A}_i(t)\right)^2 \right\rangle = e^{2i\theta_s(t)}|R^2(t)|$$

$$t_\rho = \sum_{\nu=1}^{\rho} |\bar{\alpha}_{\nu+1} - \bar{\alpha}_\nu|$$

$\theta_s(t)$



$|R^2(t)|$

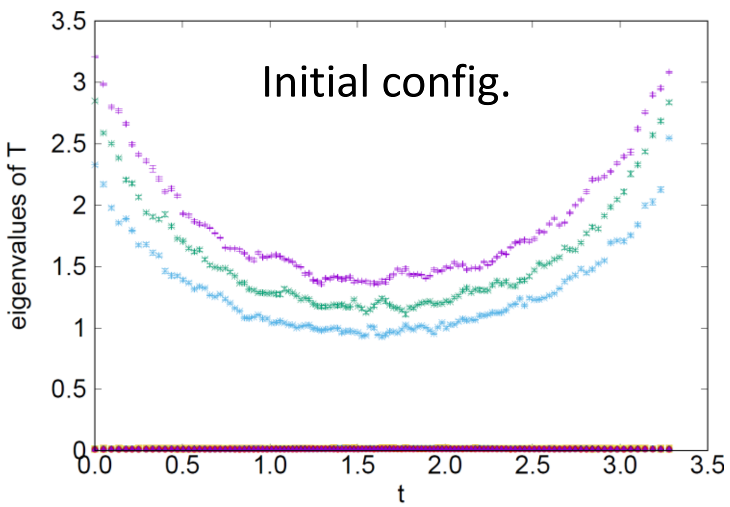


emergence of real space
at late times

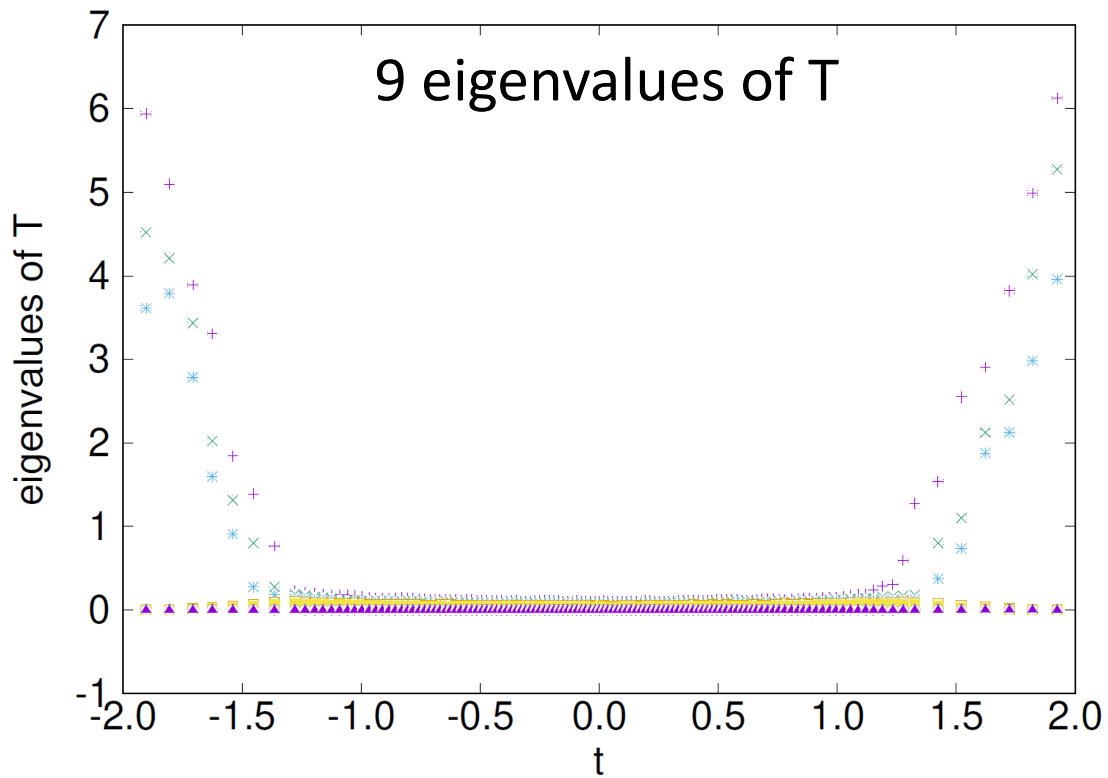
expanding behavior
at late times

emergence of 3d expanding space-time

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left(\bar{A}_i(t)\bar{A}_j(t)\right)$$



$N = 128,\ \gamma = 4,\ m_f = 5,\ \xi = 10,\ \tilde{d} = 5$

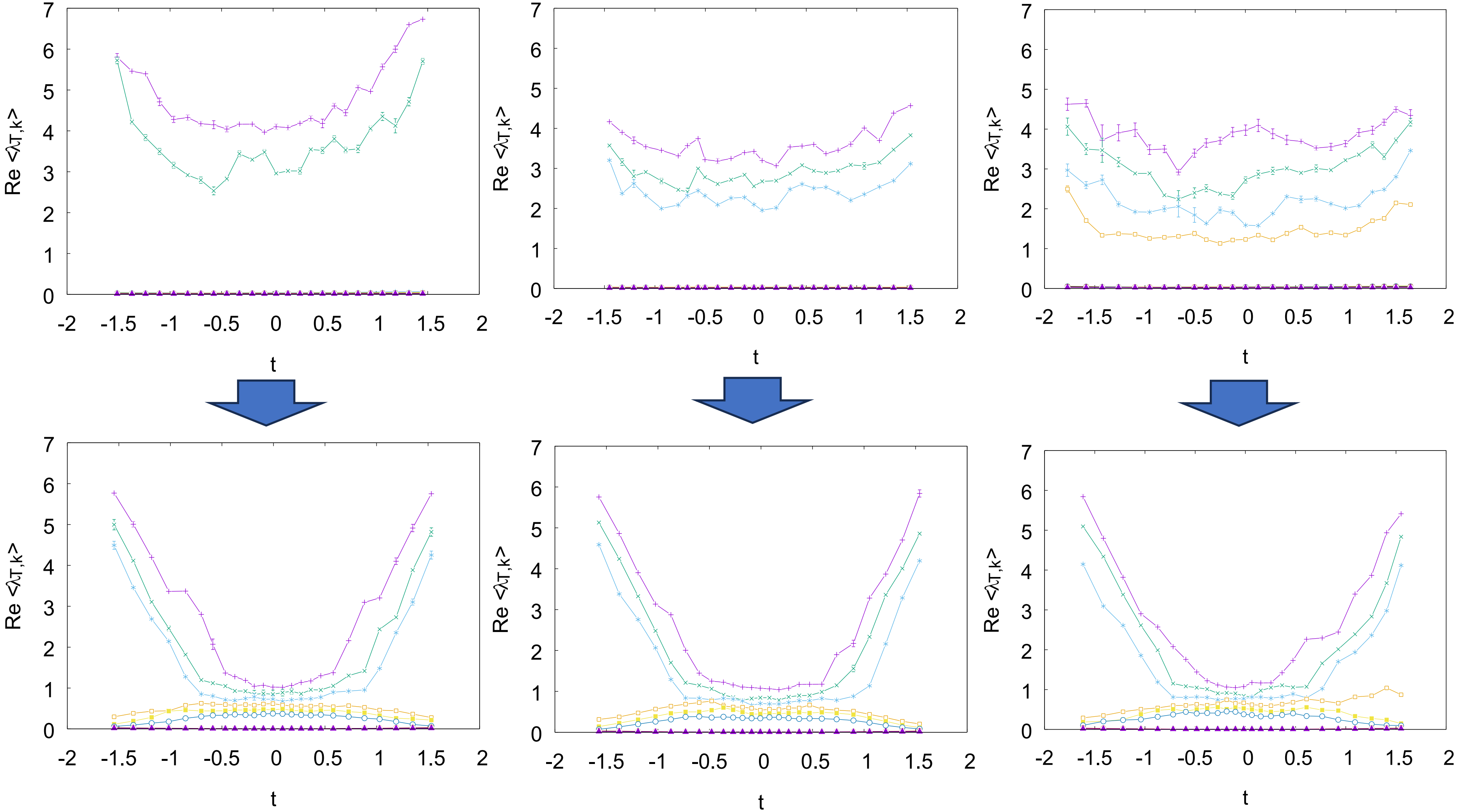


3 out of 9 directions start to expand at some point in time.
(SSB of rotational symmetry)

dependence on the initial config.

$$N = 32,\ \gamma = 6,\ m_f = 2,\ \xi = 10,\ \tilde{d} = 6$$

Initial configs.



We always obtain 3d expanding behaviors after thermalization.

towards SUSY-deformed model

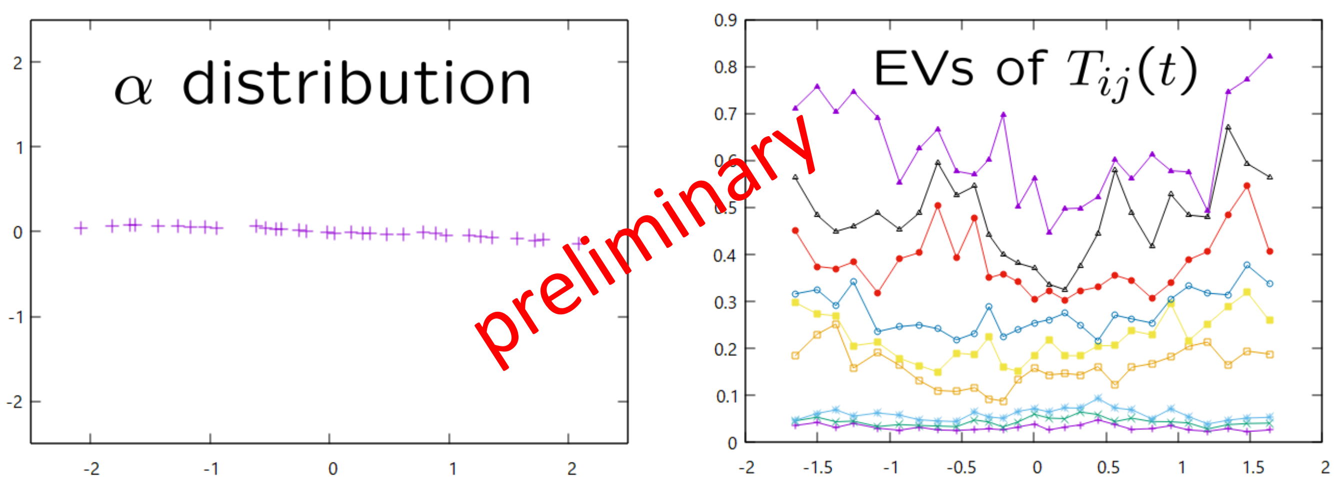
SUSY deformation of Lorentzian model

add the Myers term : $S_{\text{Myers}} = -iN\mu \text{Tr}(A_7[A_8, A_9]) \in \mathbb{R}$

and set $\gamma = -\frac{\mu^2}{32},\ m_f = \frac{\mu}{4},\ \xi = 3,\ \tilde{d} = 6$

- For real μ $(\gamma < 0)$, the only classical solution will be trivial (A=0).
- We target $N = 32,\ \mu = 16i,\ \gamma = 8,\ m_f = 4i,\ \xi = 3,\ \tilde{d} = 6$.

$$N = 32,\ \mu = 16i,\ \gamma = 8,\ m_f = 4e^{0.8\pi i/2},\ \xi = 6,\ \tilde{d} = 6$$



5. Summary and discussions

Summary

Euclidean IKKT model with SUSY deformation

- We performed careful MC sim. samping all the dominant configs. correctly.
- From the histograms of various observables, we identified the spacetime emerging from the matrix configuration at each Ω .
- Understood why the original IKKT model is not retrieved in the $\Omega \to 0$ limit.
- Conjecture : the concentric fuzzy spheres explain the "fuzzy 3d ball" at $\Omega=0$.

Lorentzian IKKT model

Lorentz invariant mass term ➡ Expanding spacetimes (w/ various dim.) appear as classical solutions.

Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob, *PTEP* 2020 (2020) 4, 043B10

- Complex Langevin method applied to the model with SUSY-like deformations.
- Evidence for the emergence of (3+1)-dimensional expanding spacetime.

Future prospects

- Euclidean model (with SUSY deformation)
  - Simulations at larger N (sign problem → Lefschetz thimble method)
  - understanding the mechanism for the emergence of “fuzzy 3d ball”

- Lorentzian model
  - Lorentz inv. mass deformation v.s. SUSY deformation (universality?)
  - Does (3+1)-dimensional expanding space-time emerge at large N?
  - Understanding the mechanism for the emergence of 3d space

The Pfaffian prefers collapsed configurations, but it becomes **zero** for configurations with not more than 2 extended directions.

Krauth-Nicolai-Staudacher (’98)
JN-Vernizzi (’00)