Duality Orbits of DLCQs and Holography

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Matrix Model for superstring / M-theory

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VILLUM FONDEN

A BPS Road to Holography

2410.03591 w/ Chris Blair,
Johannes Lahnsteiner, Niels Obers

also recent works w/

Eric Bergshoeff
Stefano Baiguera
Jan de Boer
Joaquim Gomis
Kevin Grosvenor
Troels Harmark
Yang Lei
Dawid Maskalaniec
Luca Romano
Utku Zorba

classify BPS decoupling limits of string theory

- fundamental d.o.f.: matrix theories
- backreactions: holographic constructions

how to classify BPS decoupling limits? "DLCQ"

- target space dynamics: non-Lorentzian geometric techniques
- bulk geometry via generalized $T\bar{T}$ deformation

(non-)Lorentzian quantum gravity from string worldsheet

Guided by a BPS Road...

Discrete Light-Cone Quantization of M-Theory

[Seiberg '97] [Sen '97] [Susskind '97]

M-theory on a spatial circle 

energy spectrum for Kaluza-Klein states $\quad E = \sqrt{\frac{N^2}{R_0^2} + P_i P_i} \qquad N \in \mathbb{Z}$

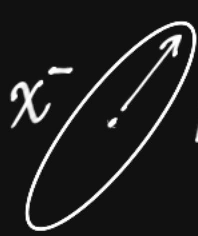
BPS states: KK momenta $\quad E = \frac{N}{R_0}$

a limit zooming in on BPS states is motivated by a heuristic "infinite boost"

$$\begin{cases} x^- = \omega (x^{10} - x^0) \sim x^- + 2\pi \omega R_0 \\ x^+ = \frac{1}{\omega} x^0 \qquad \text{no periodicity} \end{cases} \qquad \omega = \frac{e^\theta}{\sqrt{2}} \rightsquigarrow \text{rapidity}$$

$$\Rightarrow \; ds_{11}^2 = 2\, dx^- dx^+ + \omega^{-2} (dx^-)^2 + dx^i dx^i$$

"infinite boosts" $\begin{cases} \theta \to \infty \qquad \omega \to \infty \qquad \text{DLCQ} \\ \theta \to -\infty \qquad \omega \to 0\,? \end{cases}$

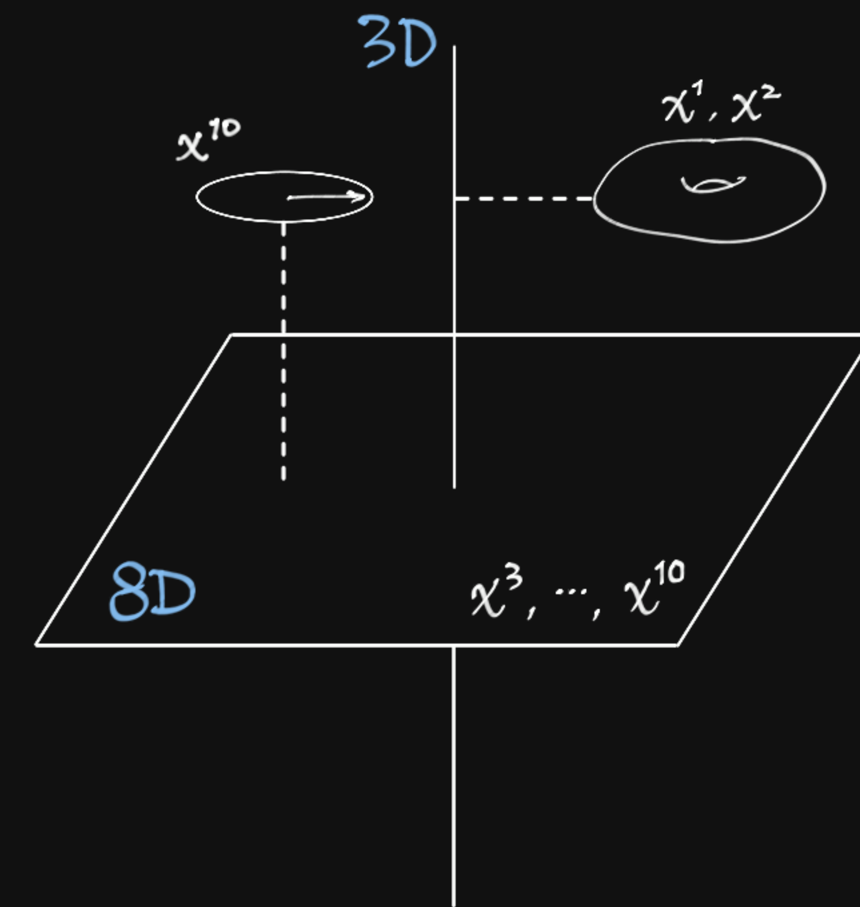


$R = \omega R_0 \to$ finite $\qquad \varepsilon = \frac{R}{2N} P_i P_i$

[Blair, Obers, ZY '25] [Gomis, ZY, WIP]

U-Duality

[Ebert, ZY '23]
[Blair, Lahnsteiner, Obers, ZY '23]



$$ds_{11}^2 = 2\,dx^- dx^+ + \omega^{-2}(dx^-)^2 + dx^i dx^i$$

U-dual $\updownarrow$

$R = \omega R_0 \longrightarrow$ finite

$$ds_{11}^2 = \omega^{\frac{4}{3}}\left(-dt^2 + dx_1^2 + dx_2^2\right) + \omega^{-\frac{2}{3}}\left(dx_3^2 + \cdots + dx_{10}^2\right)$$

$$A_3 = -\omega^2\, dt \wedge dx^1 \wedge dx^2$$

[Gopakumar, Minwalla, Seiberg, Strominger '00]

generalization to arbitrary backgrounds

$$G_{MN} = \omega^{\frac{4}{3}}\, \tau_M{}^a \tau_N{}^b \eta_{ab} + \omega^{-\frac{2}{3}}\, E_M{}^i E_N{}^i \qquad dt \to \tau_\mu{}^a dx^\mu$$

$$A_3 = -\omega^2\, \tau^0 \wedge \tau^1 \wedge \tau^2 + a_3 \qquad dx^i \to E_\mu{}^i dx^\mu$$

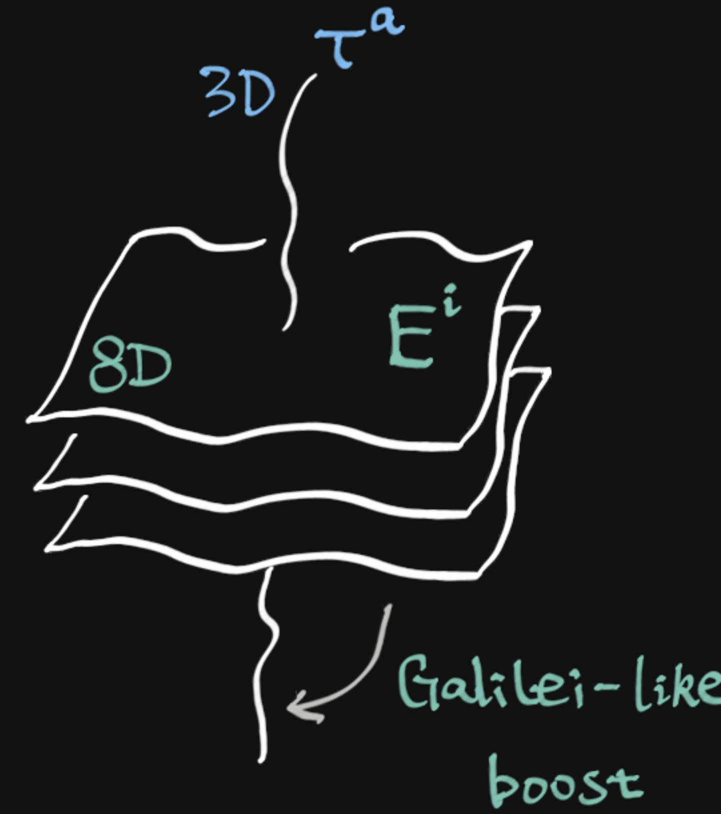
$\omega \to \infty$ limit of M-theory: non-rel. M-theory $\rightsquigarrow$ Galilei-like

$\omega \to 0$: Carroll-like geometry

Probe Membrane

$$G_{MN} = \omega^{\frac{4}{3}}\, \tau_M{}^a \tau_N{}^b \eta_{ab} + \omega^{-\frac{2}{3}}\, E_M{}^i E_N{}^i \qquad M = 0, \cdots, 10$$

$$A_3 = -\omega^2\, \tau^0 \wedge \tau^1 \wedge \tau^2 + a_3$$



bosonic sector of M2-brane

$$S_{M2} = -\int d^3\sigma \sqrt{-\det G_{\alpha\beta}} - \int A_3$$

non-rel. M-theory $\omega \to \infty$: BPS decoupling limit

induced worldvolume metric $\tau_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N \tau_M{}^a \tau_N{}^b \eta_{ab}$

$$S_{M2} = -\int d^3\sigma \sqrt{-\tau}\, \tau^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N E_M{}^i E_N{}^i - \int a_3$$

Galilei-like boost: $\delta \tau_M{}^a = 0, \quad \delta E_M{}^i = v^i{}_a \tau_M{}^a$

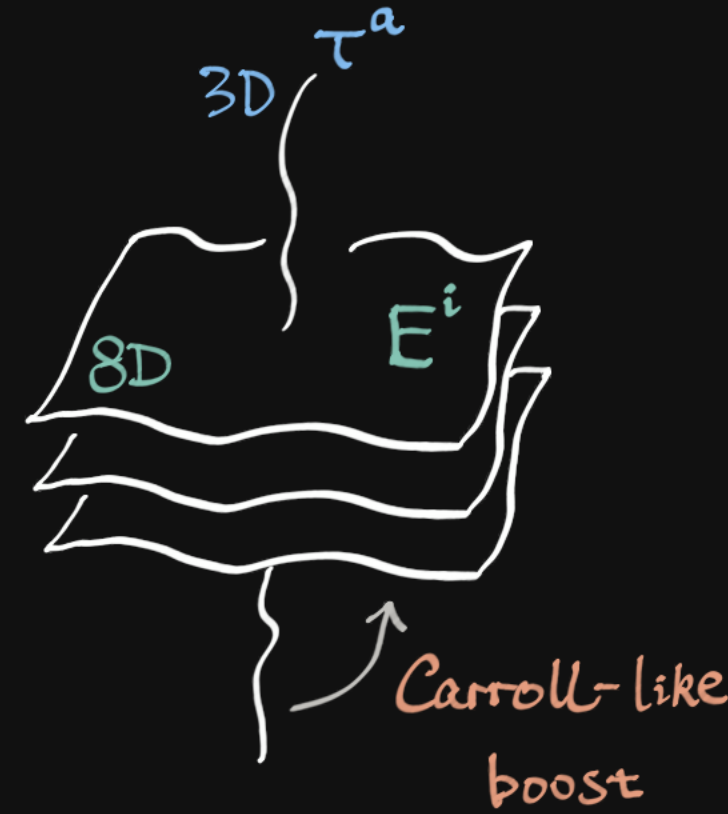
$$\delta a_3 = -3\, v^{ia} E^i \wedge \tau^b \wedge \tau^c \epsilon_{abc}$$

Probe Membrane

[de Boer, Harmark, Obers, ZY, WIP]
[Gomis, ZY, WIP]

$$G_{MN} = \omega^{\frac{4}{3}} \tau_M{}^a \tau_N{}^b \eta_{ab} + \omega^{-\frac{2}{3}} E_M{}^i E_N{}^i \qquad M = 0, \cdots, 10$$

$$A_3 = -\omega^2 \tau^0 \wedge \tau^1 \wedge \tau^2 + a_3$$



bosonic sector of M2-brane

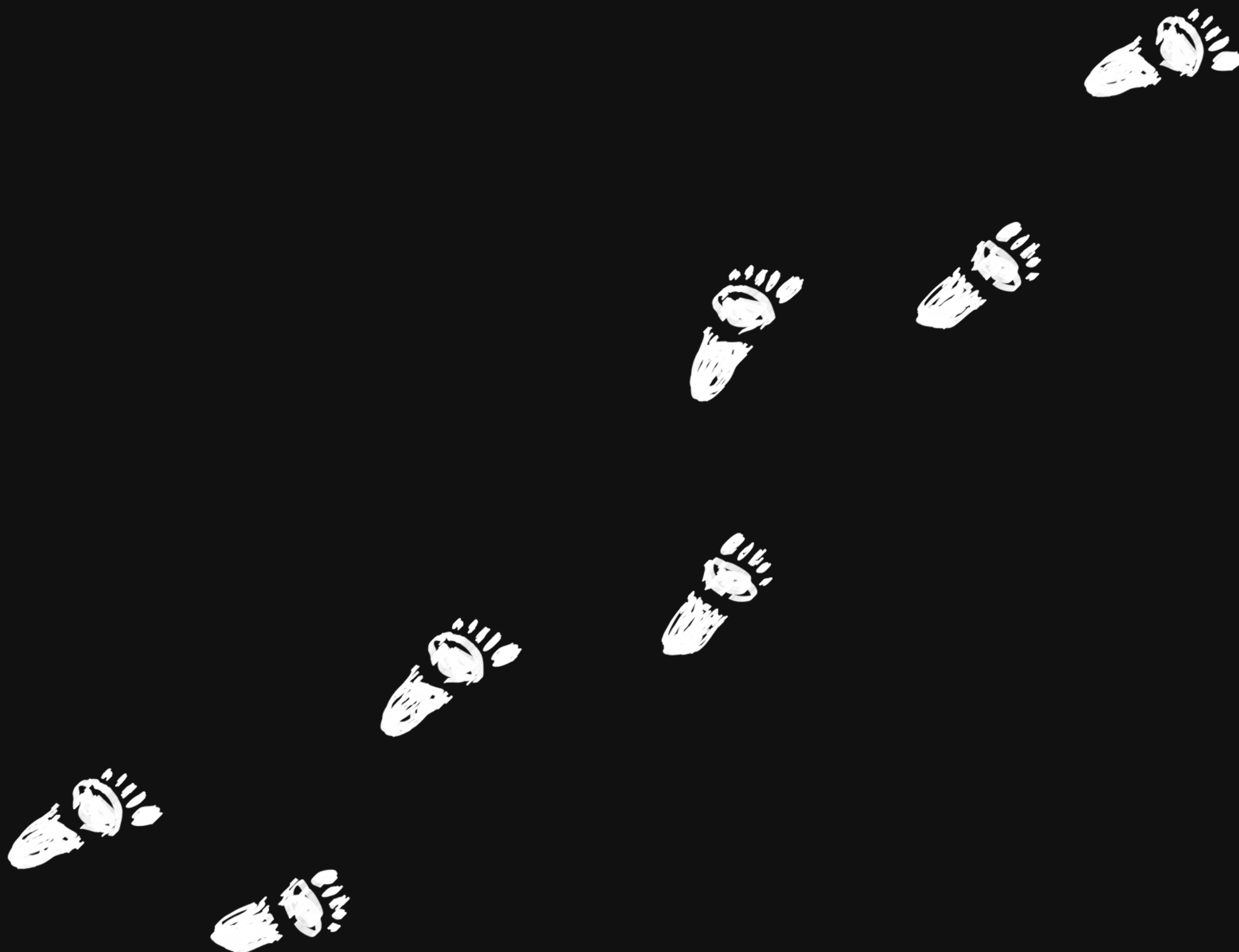
$$S_{M2} = -\int d^3\sigma \sqrt{-\det G_{\alpha\beta}} - \int A_3$$

Carroll M-theory ? $\omega \to 0$

induced worldvolume metric $\tau_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N \tau_M{}^a \tau_N{}^b \eta_{ab}$

$$S_{M2} = -\frac{1}{\sqrt{2}} \int d^3\sigma \left[\sqrt{-\left(\epsilon^{\alpha\beta\gamma} \tau_\alpha{}^a E_\beta{}^i E_\gamma{}^j\right)^2} + \epsilon^{\alpha\beta\gamma} E_\alpha{}^i E_\beta{}^j E_\gamma{}^k \lambda_{ijk} \right] - \int a_3$$

Carroll-like boost: $\delta \tau_M{}^a = v^a{}_i E_M{}^i , \quad \delta E_M{}^i = 0$



...To Holography...

Backreaction and AdS_4/CFT_3

non-rel. M-theory $\omega \to \infty$

$$ds_{11}^2 = \omega^{\frac{4}{3}}(-dt^2 + dx_1^2 + dx_2^2) + \omega^{-\frac{2}{3}}(dx_3^2 + \cdots + dx_{10}^2)$$

$$A_3 = -\omega^2 dt \wedge dx^1 \wedge dx^2$$

dilatation symmetry in $\omega \to \infty$ generated by $\omega \to \omega \Delta(x^\mu)$

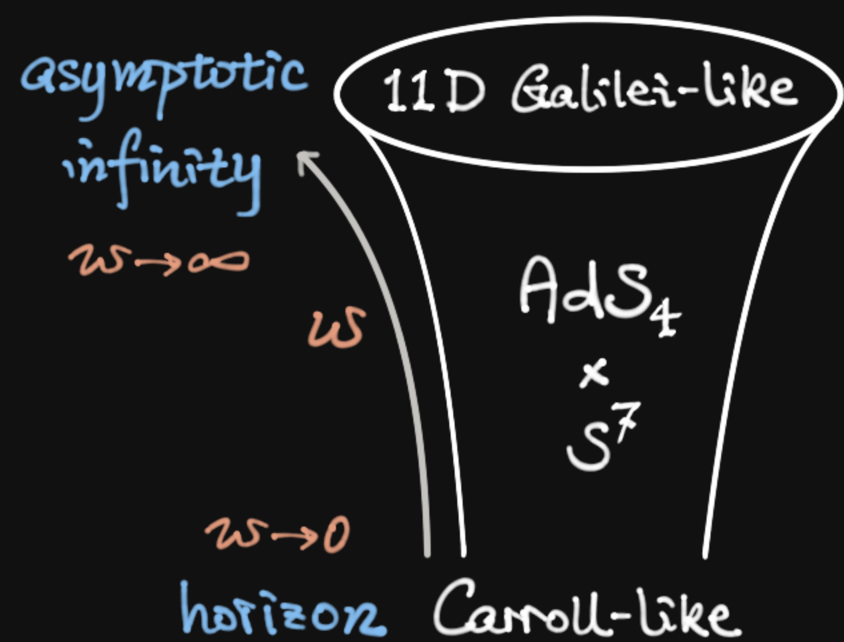
backreact M2: $\Box \omega^{-2} \sim \delta(x^i)$ $\quad i = 3, \cdots, 10 \quad \Rightarrow \quad \omega = \frac{r^3}{\ell^3}, \quad r^2 = x_3^2 + \cdots + x_{10}^2$

deform away from non-Lorentzian corner

bulk $AdS_4 \times S^7$ geometry

$$ds_{11}^2 = \frac{r^4}{\ell^4}(-dt^2 + dx_1^2 + dx_2^2) + \frac{\ell^2}{r^2}(dx_3^2 + \cdots + dx_{10}^2)$$

$$A_3 = -\frac{r^6}{\ell^6} dt \wedge dx^1 \wedge dx^2$$



[Blair, Obers, ZY '25]

BPS v.s. Near-Horizon Limit

$$\boxed{\begin{aligned}(t,x^1,x^2)&\to \omega^{\frac{2}{3}}(t,x^1,x^2)\\ (x^3,\cdots,x^{10})&\to \omega^{-\frac{1}{3}}(x^3,\cdots,x^{10})\end{aligned}}$$

$\omega\to\infty$: BPS limit

field theory description:

ABJM as N membranes

in non-rel. M-theory

non-rel. M-theory $\omega\to\infty$

$$ds_{11}^2=\omega^{\frac{4}{3}}(-dt^2+dx_1^2+dx_2^2)+\omega^{-\frac{2}{3}}(dx_3^2+\cdots+dx_{10}^2)$$

$$A_3=-\omega^2 dt\wedge dx^1\wedge dx^2$$

M2-brane geometry

$$ds_{11}^2=H^{-\frac{2}{3}}(-dt^2+dx_1^2+dx_2^2)+H^{\frac{1}{3}}(dx_3^2+\cdots+dx_{10}^2)$$

$$A_3=-H^{-1}dt\wedge dx^1\wedge dx^2 \qquad H=1+\frac{l^6}{r^6} \qquad r^2=x_3^2+\cdots+x_{10}^2$$

$\xrightarrow{\omega\to\infty}$ bulk $AdS_4\times S^7$ geometry

near-horizon limit

$$ds_{11}^2=\frac{r^4}{l^4}(-dt^2+dx_1^2+dx_2^2)+\frac{l^2}{r^2}(dx_3^2+\cdots+dx_{10}^2)$$

$$A_3=-\frac{r^6}{l^6}dt\wedge dx^1\wedge dx^2$$

And Back Via TT...

Matrix String Perspective

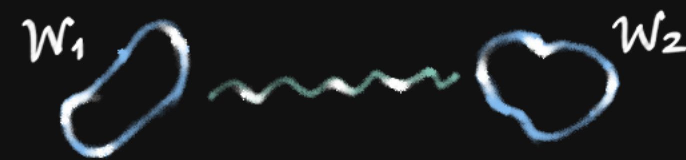
DLCQ M-theory $\omega\to\infty$	non-rel. M-theory $\omega\to\infty$
$ds_{11}^2 = 2dx^- dx^+ + \omega^{-2}(dx^-)^2 + dx^i dx^i + dx^{10} dx^{10}$ $\quad i = 2,\cdots,9$	$ds_{11}^2 = \omega^{\frac{4}{3}}(-dt^2 + dx_1^2 + dx_2^2) + \omega^{-\frac{2}{3}}(dx_3^2 + \cdots + dx_{10}^2)$ $A_3 = -\omega^2 dt \wedge dx^1 \wedge dx^2$
compactify x^{10} DLCQ of IIA x^- $\quad R = \omega R_0 \to$ finite $S_{F1} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(2\partial_\alpha X^- \partial^\alpha X^+ + \partial_\alpha X^i \partial^\alpha X^i)$	compactify x^2 & x^{10}: BPS limit of IIB $ds_{10}^2 = \omega^2(-dt^2 + dx_1^2) + dx^i dx^i$ $B = -\omega^2 dt\wedge dx^1 \qquad e^{\Phi} = \omega\, g_s$ T-dual [Bergshoeff, Gomis, ZY '18] x^1 $\quad \frac{1}{R}$ IIB non-rel. string theory $S_{F1} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(\partial_\alpha X^i \partial^\alpha X^i + \lambda\bar{\partial}X + \bar{\lambda}\partial\bar{X})$ 2nd quantization: Matrix String Theory

Matrix String Perspective

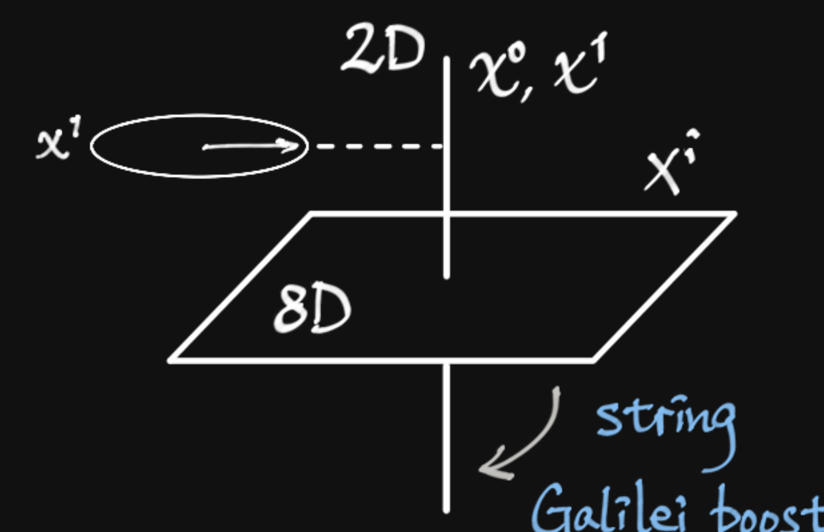
[Klebanov, Maldacena '00] [Gomis, Ooguri '00]
[Danielsson, Guijosa, Kruczenski '00]

IIB non-rel. string theory $S_{F1} = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\partial_\alpha X^i \partial^\alpha X^i + \lambda \bar{\partial} X + \bar{\lambda} \partial \bar{X}\right)$

asymptotic states are winding strings



Newton-like instantaneous force



generalization to curved spacetime : $dx^a \to \tau^a, \; dx^i \to E^i$

β-function : 10D non-Lorentzian supergravity

[Gomis, Oh, ZY '19] [Gallegos, Gürsoy, Zinnato '19]
[Bergshoeff, Gomis, Rosseel, Şimşek, ZY '19]

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matrix string theory : $\mathcal{N}=8$ S_N orbifold QFT

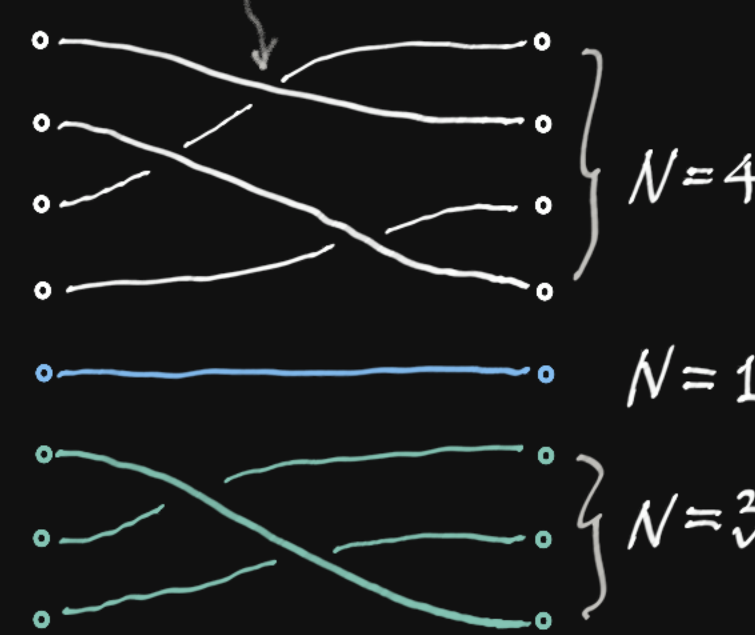
free theory : 8 N×N matrices $X^i = U x^i U^{-1}$

$\downarrow$

$U \in U(N)$ diagonal matrix

interactions

twisted sectors



[Motl '97] [Dijkgraaf, Verlinde, Verlinde '97]

$T\bar{T}$ Deformation

[Blair '20]
[Blair, Lahnsteiner, Obers, ZY '24]

IIB non-rel. string theory $S_{F1} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(\partial_\alpha X^i \partial^\alpha X^i + \lambda \bar{\partial} X + \bar{\lambda} \partial \bar{X})$

marginal deformation $-\frac{1}{4\pi\alpha'}\int d^2\sigma\, \omega^{-2}\lambda\bar{\lambda}$ $\quad \beta(\omega^{-2}) \sim D_{[i}\tau_{j]}\, D_{[i}\bar{\tau}_{j]}$

BPS limit of superalgebra: extension of Galilei-like symmetry $D_{[\mu}\tau_{\nu]}{}^{a} = 0$

$\Rightarrow \beta(\omega^{-2}) = 0$ at all loop [ZY '21]

susy constraints

[Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek '20 '21]

Nambu-Goto action for non-rel. string

$$\boxed{S_{NG} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\, \sqrt{-\tau}\; \tau^{\alpha\beta}\, \partial_\alpha X^i\, \partial_\beta X^i}$$

$$\tau_{\alpha\beta} = -\partial_\alpha X^0 \partial_\beta X^0 + \partial_\alpha X^1 \partial_\beta X^1$$

static gauge + flat spacetime:

$$S_{NG} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\, \partial_\alpha X^i \partial^\alpha X^i + \omega^{-2}\, T\bar{T} + \cdots$$

[Blair, Lahnsteiner, Obers, ZY '24]
[Harmark, Lahnsteiner, Obers '25]

Bulk Geometry from $T\bar{T}$

field theory from BPS decoupling limit:

IIB on $\begin{cases} ds_{10}^2 = \omega^2(-dt^2 + dx_1^2) + dx^i dx^i \\ B = -\omega^2 dt \wedge dx^1 \qquad e^{\Phi} = \omega\, g_s \end{cases} \xrightarrow{\omega\to\infty}$ Matrix String Theory

first quantization: IIB non-rel. string theory

$$S_{F1} = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left(\partial_\alpha X^i \partial^\alpha X^i + \lambda\bar{\partial}X + \bar{\lambda}\partial\bar{X}\right) \qquad i = 2,\cdots,9$$

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current-current deformation $-\frac{1}{4\pi\alpha'}\int d^2\sigma\, \omega^{-2}\lambda\bar{\lambda}$ from backreacting F1

$$\Box\, \omega^{-2} \sim \delta(x^i) \Rightarrow \omega^{-2} = \frac{\ell^6}{r^6}$$

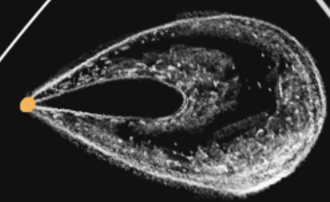
bulk geometry: $ds_{10}^2 = \frac{r^6}{\ell^6}(-dt^2 + dx_1^2) + dx^i dx^i$

$B = -\frac{r^6}{\ell^6} dt \wedge dx^1 \qquad e^{\Phi} = \frac{r^3}{\ell^3} e^{\varphi}$

$(t, x^1) \to \omega(t, x^1)$
$g_s \to \omega g_s$
$\xleftarrow[\text{near-horizon limit}]{\omega\to\infty}$ String soliton geometry

Undo BPS Limit: Brane Generalizations of $T\bar{T}$

non-rel. M-theory ABJM



F1-string BPS limit

$$ds_{10}^2 = \omega^2(-dt^2+dx_1^2) + dx^i dx^i$$
$$B = -\omega^2\, dt\wedge dx^1 \qquad e^{\Phi} = \omega\, g_s$$

IIB non-rel. string theory

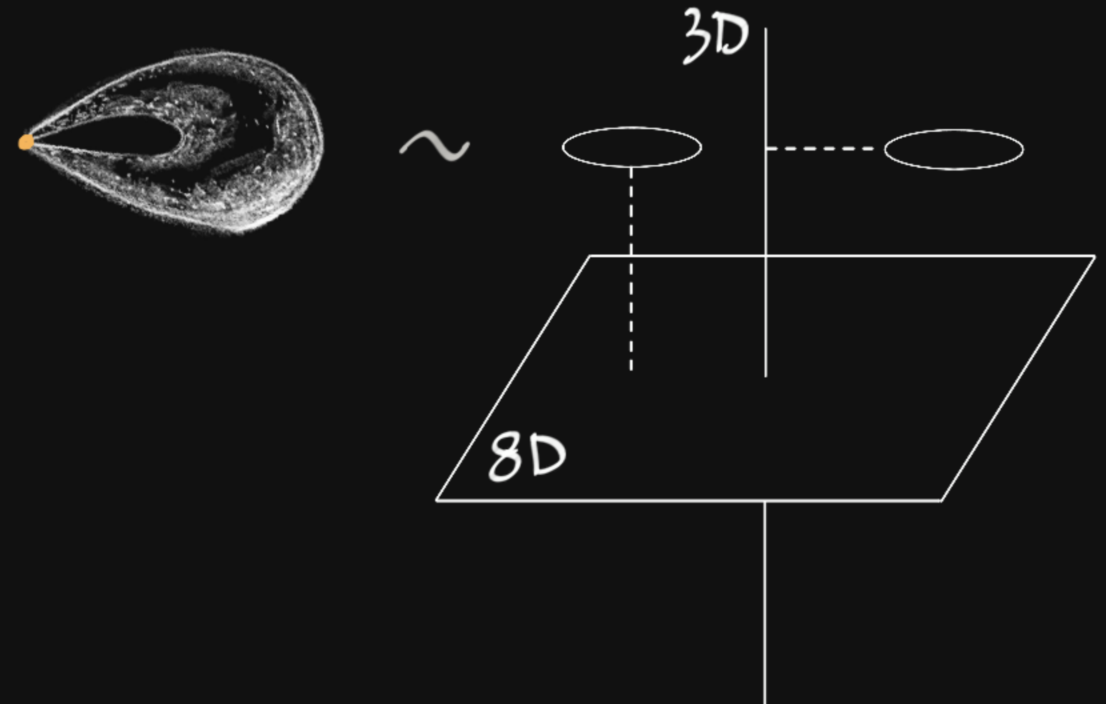
matrix string theory

Matrix 1-brane Theory

D1-string BPS limit

$$ds_{10}^2 = \omega(-dt^2+dx_1^2) + \frac{1}{\omega}\, dx^i dx^i$$
$$C_2 = -\omega^2 g_s^{-1}\, dt\wedge dx^1 \qquad e^{\Phi} = \frac{g_s}{\omega}$$

T-dual on torus



Matrix p-brane Theory

MOT: BFSS

M3T: $\mathcal{N}=4$ SYM

$$ds_{10}^2 = \omega(-dt^2+\cdots+dx_p^2) + \frac{1}{\omega}\, dx^i dx^i$$
$$C_2 = -\frac{\omega^2}{g_s}\, dt\wedge\cdots\wedge dx^p \qquad e^{\Phi} = \omega^{\frac{p-3}{2}} e^{\varphi}$$

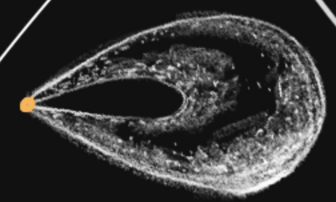
Dp-brane BPS limit

[Gomis, Ooguri '00]
[Danielsson, Guijosa, Kruczenski '00]

[Blair, Lahnsteiner, Obers, ZY '23]
[Gomis, ZY '23]

Undo BPS Limit: Brane Generalizations of $T\bar{T}$

non-rel. M-theory



F1-string BPS limit

$$\boxed{\begin{array}{l} ds_{10}^2 = \omega^2(-dt^2+dx_1^2) + dx^i dx^i \\ B = -\omega^2\, dt\wedge dx^1 \qquad e^{\Phi} = \omega\, g_s \end{array}}$$

IIB non-rel. string theory

matrix string theory

Matrix 1-brane Theory

D1-string BPS limit

$$\boxed{\begin{array}{l} ds_{10}^2 = \omega(-dt^2+dx_1^2) + \frac{1}{\omega}\, dx^i dx^i \\ C_2 = -\omega^2 g_s^{-1}\, dt\wedge dx^1 \qquad e^{\Phi} = \frac{g_s}{\omega} \end{array}}$$

T-dual on torus

Matrix p-brane Theory

$$\boxed{\begin{array}{l} ds_{10}^2 = \omega\left(-dt^2+\dots+dx_p^2\right) + \frac{1}{\omega}\, dx^i dx^i \\ C_2 = -\frac{\omega^2}{g_s}\, dt\wedge\dots\wedge dx^p \qquad e^{\Phi} = \omega^{\frac{p-3}{2}} e^{\varphi} \end{array}}$$

Dp-brane BPS limit

fundamental d.o.f. : Dp-brane

general flow eqn. (no gauge field)

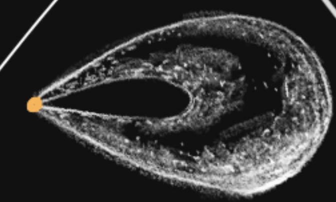
$$\boxed{\frac{\partial\mathcal{L}}{\partial\lambda} = \frac{1}{2\lambda^2}\left\{\mathrm{tr}(\mathbb{1}-\lambda T) - (p-1)\, \det{}^{\frac{1}{p-1}}(\mathbb{1}-\lambda T) - 2\right\} \qquad \lambda = \omega^{-2}}$$

[Blair, Lahnsteiner, Obers, ZY '24]

$p=2$: same form even with gauge field

Undo BPS Limit: Brane Generalizations of $T\bar{T}$

non-rel. M-theory $\quad \frac{\partial \mathcal{L}}{\partial \lambda} = \frac{1}{4}\left[\operatorname{tr} T^2 - (\operatorname{tr} T)^2 + 2\lambda \det T\right]$



$\frac{\partial \mathcal{L}}{\partial \lambda} = -\frac{1}{2}\det T$

IIB non-rel. string theory

matrix string theory

Matrix 1-brane Theory

$\frac{\partial \mathcal{L}}{\partial \lambda} = -\frac{1}{2\lambda^2}\left(\operatorname{tr} T + \sqrt{-\det F}\sqrt{-\operatorname{tr} T + \lambda \det T}\right)$

T-dual on torus

Matrix p-brane Theory

M$_p$T

MOT: BFSS matrix theory

$$\boxed{\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{1}{2\lambda^2}\left\{\operatorname{tr}(\mathbb{1} - \lambda T) - (p-1) \det^{\frac{1}{p-1}}(\mathbb{1} - \lambda T) - 2\right\} \quad \lambda = \omega^{-2}}$$

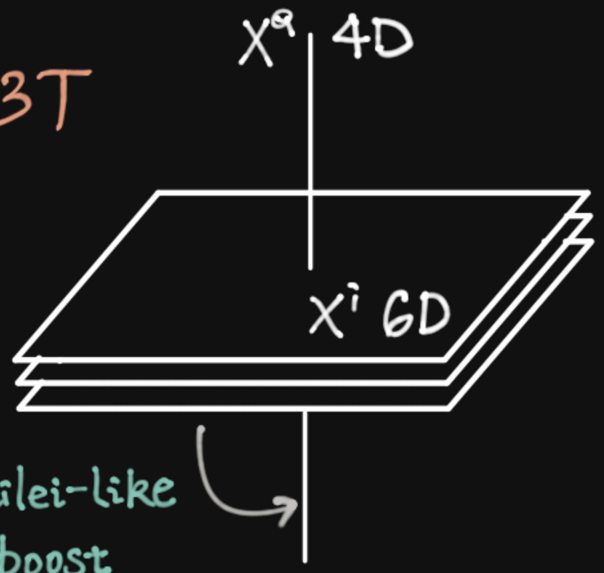
[Blair, Lahnsteiner, Obers, ZY '24]

More Applications

AdS_5/CFT_4

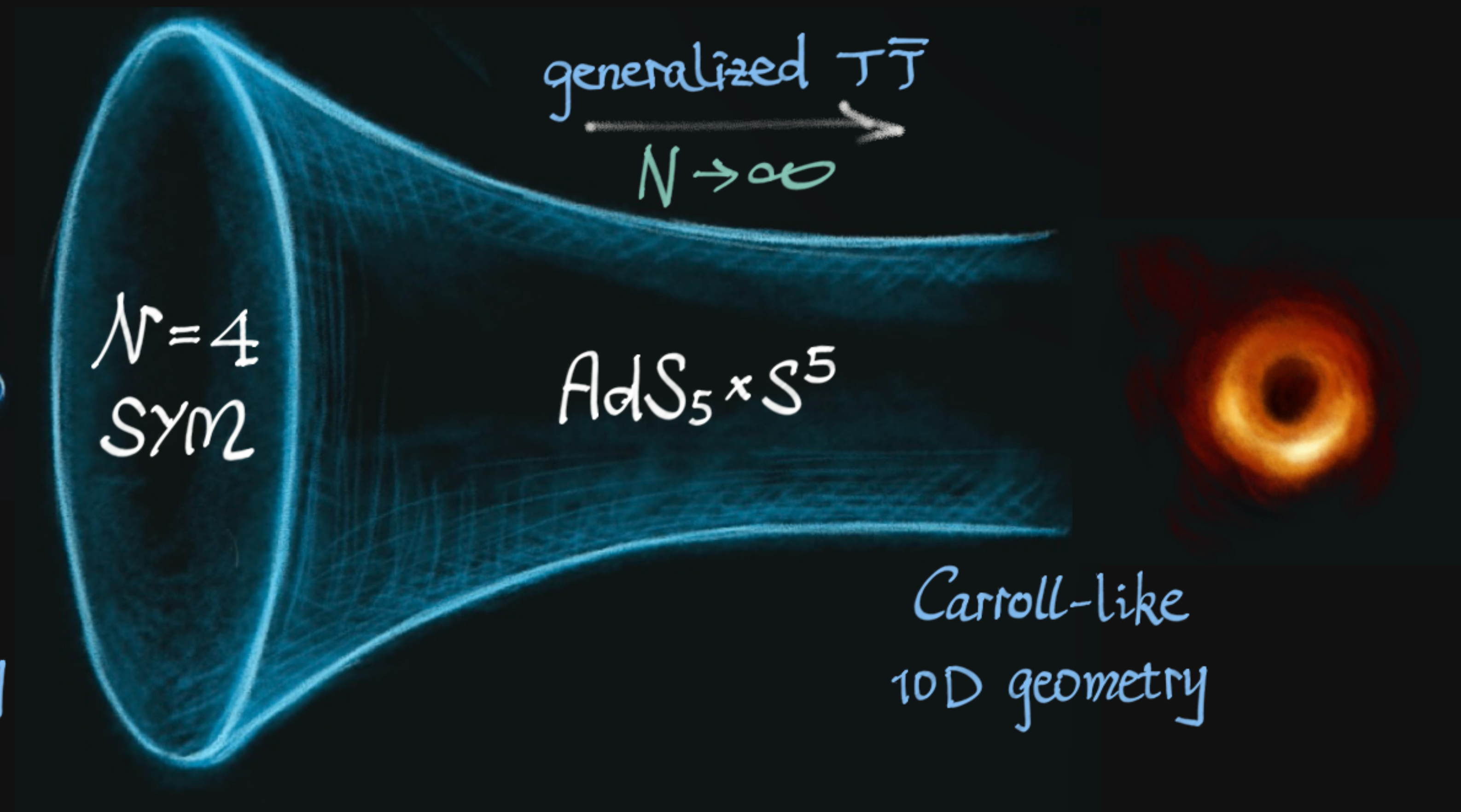
[Blair, Lahnsteiner, Obers, ZY '24]
[Guijosa '25]

M3T



Galilei-like boost

Galilei-like 10D geometry



Carroll-like 10D geometry

$$S_{D3} = -\frac{1}{g_s\alpha'^2}\int d^4\sigma\sqrt{-\det\left(\omega\,\eta_{\alpha\beta}+\frac{1}{\omega}\,\partial_\alpha X^i\partial_\beta X^i + F_{\alpha\beta}\right)} + \frac{\omega^2}{g_s\alpha'^2}\int d^4\sigma$$

$i=4,\cdots,9$

$$\xrightarrow[\text{BPS}]{\omega\to\infty} -\frac{1}{g_s\alpha'^2}\int d^4x\left(\frac{1}{2}\,\partial_a X^i\partial^a X^i + \frac{1}{4}\,F_{ab}F^{ab}\right)$$ in static gauge

non-abelianization $\longrightarrow$ $\mathcal{N}=4$ SYM

Galilei-like boost $\begin{cases}\delta X^a = 0\\ \delta X^i = v^i{}_a X^a\end{cases}$

Morita Equivalence in Matrix Theory

[Blair, Lahnsteiner, Obers, ZY '23]
[Gomis, ZY '23]

BPS decoupling limit zooming on a background D-instanton

$$ds^2 = \frac{1}{\omega}\, dx^\mu dx^\nu g_{\mu\nu}$$

$$C_0 = \omega^2 e^{-\varphi} + c_0$$

$$e^{\Phi} = \frac{i}{\omega^2} e^{\varphi} \xrightarrow[\omega \to \infty]{} \text{IIB}^* \text{ superstring theory}$$

M(-1)T

F1-string : tensionless limit

$$S_{F1} = -\frac{T}{2}\int d^2\sigma\, \partial_\tau X^\mu \partial_\tau X^\nu g_{\mu\nu}$$

non-vibrating

[Isberg, Lindström, Sundborg, Theodoridis '93]

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fundamental d.o.f. : D-instantons

IKKT matrix theory $S \sim \mathrm{tr}\left([X_\mu, X_\nu][X^\mu, X^\nu] + \text{fermions}\right)$

[Ishibashi, Kauai, Kitazawa, Tsuchiya '96]

compactify on a 2-torus

$$\left.\begin{aligned} X_0 + 2\pi R_0 &\sim U_0 X_0 U_0^{-1} \\ X_1 + 2\pi R_1 &\sim U_1 X_1 U_1^{-1} \end{aligned}\right\} U_0 U_1 = e^{2\pi i \theta} U_1 U_0$$

[Connes, Douglas, Schwarz '97] [Douglas, Hull '97]

Morita equivalence

$$\boxed{\tilde{\theta} = \frac{\alpha\theta + \beta}{\gamma\theta + \delta} \qquad \alpha\delta - \beta\gamma = 1}$$

$SL(2,\mathbb{Z})$ T-dual

on non-Riemannian manifold

Morita Equivalence in Matrix Theory

[Bergshoeff, Grosvenor, Lahnsteiner, ZY, Zorba '22 '23]
[Ebert, ZY '23]
[Blair, Lahnsteiner, Obers, ZY '25]

start with standard T-duality on d-torus: $O(d,d;\mathbb{Z})$ transformation

$$\mathcal{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G \end{pmatrix} \xrightarrow{d=2} \frac{G_{ij}}{\sqrt{G}} \otimes \frac{1}{\rho_2}\begin{pmatrix} |\rho|^2 & \rho_1 \\ \rho_1 & 1 \end{pmatrix} \qquad \begin{aligned} \rho &= \rho_1 + i\rho_2 \\ &= \theta + i\sqrt{G} \end{aligned} \qquad B_{ij} = \theta\,\epsilon_{ij}$$

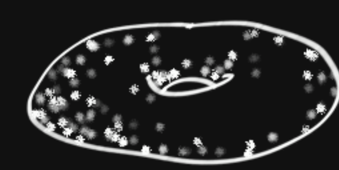
D-instanton BPS Limit

$$ds^2 = \frac{1}{\omega}\, dx^\mu dx^\nu g_{\mu\nu}$$

$$C_0 = \omega^2 e^{-\varphi} + c_0 \qquad \rho = \theta + \frac{i\sqrt{\det g_{ij}}}{\omega}$$

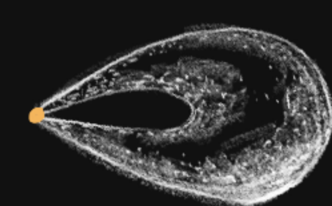
$$e^{\Phi} = \frac{i}{\omega^2} e^{\varphi}$$

real torus



hidden torus

$\omega \to \infty$



$\boxed{\text{systematically derive Maldacena-Russo holography } \theta \neq 0}$

[Maldacena, Russo '99]

non-linear polynomial realization of $SL(2,\mathbb{Z})$

$$\widetilde{O} = \sum_n \kappa^n O_n \qquad \kappa = \frac{\gamma\sqrt{g}}{\gamma\theta + \delta}$$

$\gamma\theta + \delta = 0$

$\gamma\theta + \delta \neq 0$ $\boxed{M(-1)T}$ $\boxed{M1T}$ $\gamma = 0$

10D Lorentzian geometry $\gamma \neq 0$ 10D Galilei-like geometry

[Maskalaniec, ZY, Zorba, WIP]

Target Space Sugra in BPS Limits

M(-1)T: dynamics captured by IKKT on D-instantons

Is there an analog of β-function as in non-rel. string theory?

fundamental string is non-vibrating $S_{F1} = -\frac{T}{2}\int d^2\sigma\, e\, e^{\alpha}{}_0\, e^{\beta}{}_0\, \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$

tensionless string

* phase space action

$$S = \int d^2\sigma \left(P_\mu \partial_\tau X^\mu - \frac{\chi}{2T} P_\mu P_\nu g^{\mu\nu} - \rho P_\mu \partial_\sigma X^\mu \right)$$

* set $\rho = 1$: ambitwistor string theory $S = \int d^2\sigma\, P_\mu \bar\partial X^\mu$, $\mathcal{H} = P_\mu P_\nu g^{\mu\nu} \sim 0$

* rel. sugra e.o.m. from susy anomalies $S = \int d^2\sigma \left(P_\mu \bar\partial X^\mu + \psi_\mu \bar{D} \psi^\mu \right)$ curved βγ

supercurrents $G \sim 0$, $\hat{G} \sim 0$ $\quad G(z)\,\hat{G}(w) \sim \frac{\mathcal{H}}{z-w} + \text{anomalies}$

[Adamo, Casali, Skinner '14]

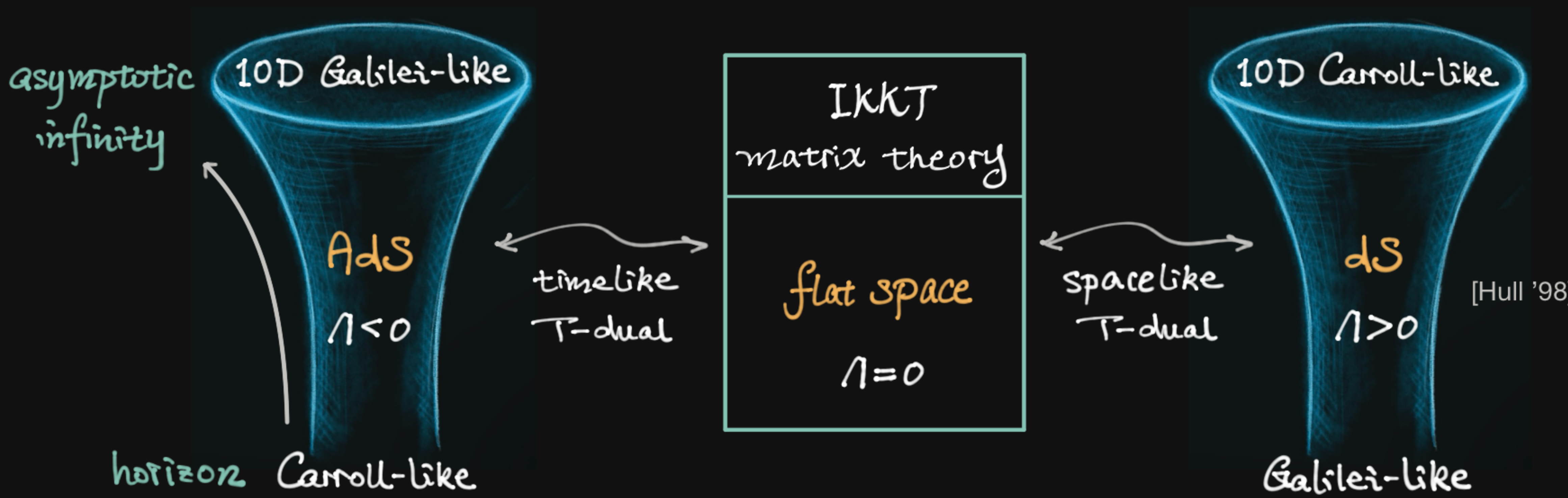
MpT: T-duals of M(-1)T

BFSS: M0T & Newton-like sugra

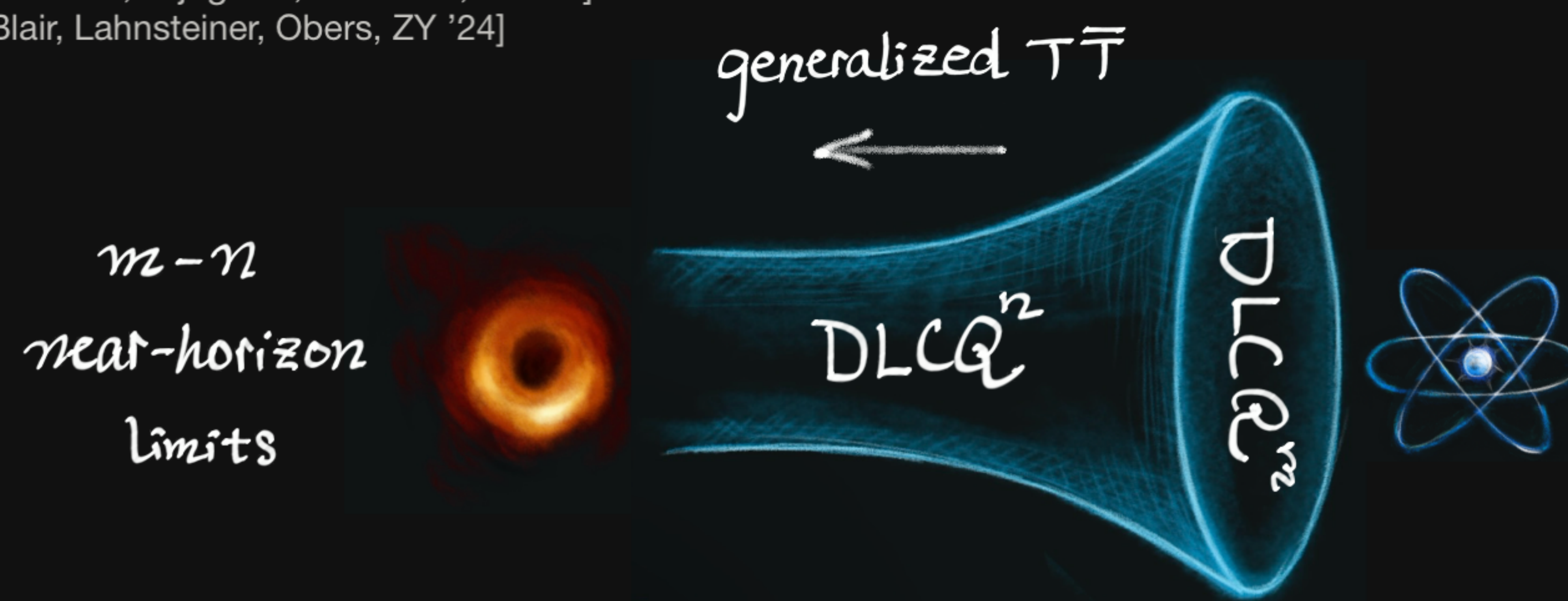
Galilei-like & Carroll-like sugra from string worldsheet theories?

Potential Holographic Constructions

[Blair, Obers, ZY '25]



[De Boer, Dijkgraaf, Harmark, Obers]
[Blair, Lahnsteiner, Obers, ZY '24]



non-Lorentzian holography from top-down

[Lambert, Smith '24] [Blair, Lahnsteiner, Obers, ZY '24 '25] [Harmark, Lahnsteiner, Obers '25] ...

Global Coordinates and Spin Matrix Theory

$AdS_5 \times S^5$ in global coordinates

[Baiguera, Harmark, Lei, ZY, WIP]

$$ds^2 = -(\omega+1)\,dt^2 + \omega R^2 d\Omega_3^2 + \frac{dr^2}{\omega+1} + \frac{1}{\omega} r^2 d\Omega_5^2$$

$$C_4 = \frac{\omega^2}{g_s} dt \wedge R^3 d\Omega_3 + \text{dual part} \qquad \omega = \frac{r^2}{R^2}$$

sending a probe D3-brane to infinite r (i.e. infinite ω) is not "free"

confining potential $\quad C_4 = -\frac{1}{2}\omega\, dt \wedge R^3 d\Omega_3$

further BPS structures are required : rotating D3-branes

$\longrightarrow$ (dual) giant graviton!

near-BPS excitations : Spin Matrix Theory [Harmark, Orselli '14]

PSU(2,2|4) global symmetry $\longrightarrow$

$E \to J_1 + J_2$	$SU(2)$	$E \to S_1 + S_2 + J_1 + J_2 + J_3$	
$E \to S_1 + J_1$	$SU(1,1)$		$PSU(1,2\|3)$

DLCQ of Heterotic String

[Bergshoeff, Grosvenor, Romano, ZY '25]

$$S = \frac{1}{4\pi}\int d^2z \left[\frac{2}{\alpha'} \partial X^\mu \bar\partial X^\nu \mathcal{H}_{\mu\nu} + \mathrm{tr}(\psi \bar\nabla \psi) + \text{righ-moving fermions for susy} \right]$$

$$\mathcal{H}_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu} + \frac{\alpha'}{4}\mathrm{tr}(A_\mu A_\nu)$$ anomaly-free at one-loop

DLCQ: $\mathcal{H}_{yy} = 0$

T-dual in y to heterotic non-rel. string $\quad \mathcal{E}_{\mu\nu} = E_\mu{}^i E_\nu{}^i + b_{\mu\nu} + \frac{1}{2}\mathrm{tr}(a_\mu a_\nu)$

$$S = \frac{1}{4\pi}\int d^2z \Big\{ \frac{2}{\alpha'} \partial X^\mu \bar\partial X^\nu \mathcal{E}_{\mu\nu} + \mathrm{tr}(\psi\bar\partial\psi - i\psi a_\mu \bar\partial X^\mu \psi)$$
$$+ \lambda\bar\partial X^\mu \tau_\mu + \bar\lambda\left[\partial X^\mu \bar\tau_\mu + i\,\mathrm{tr}(\lambda a \lambda)\right] \Big\}$$

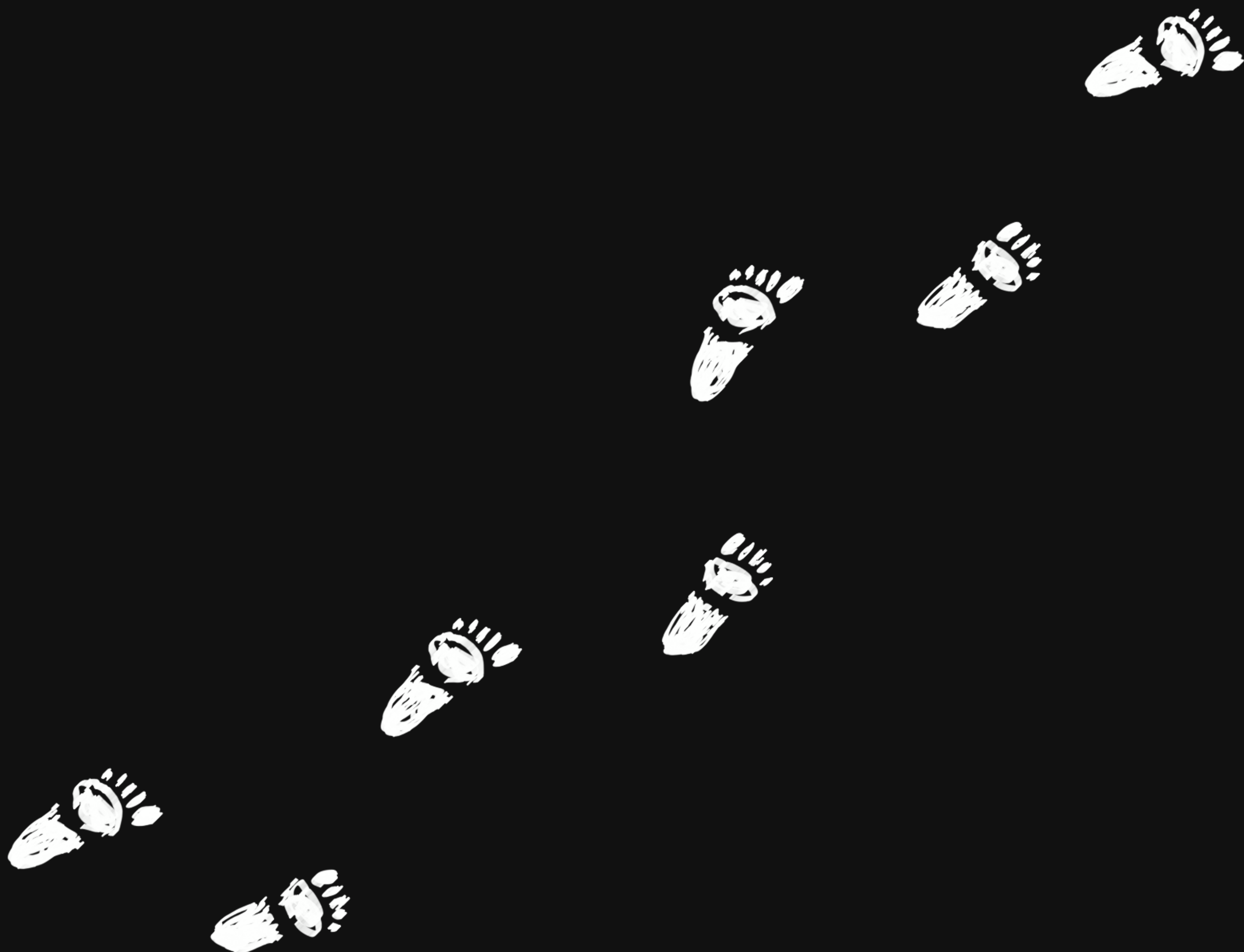
BPS decoupling limit? $T\bar{T}$ deformation! $\quad S_{\text{def.}} = \frac{1}{4\pi}\int d^2z\, \omega^{-2}\lambda\bar\lambda$

$$ds^2 = (-\omega^2 \bar\tau_\mu \tau_\nu + \mathcal{E}_{\mu\nu})\, dx^\mu dx^\nu \qquad A_\mu = \omega^2 a \tau_\mu + u_\mu$$

circumvent highly involved sugra manipulation

[Bergshoeff, Romano '23]

second quantization: heterotic matrix string



Duality Orbits of DLCQs

From DLCQ to DLCQ^2

[Blair, Lahnsteiner, Obers, ZY '23]
[Gomis, ZY '23]

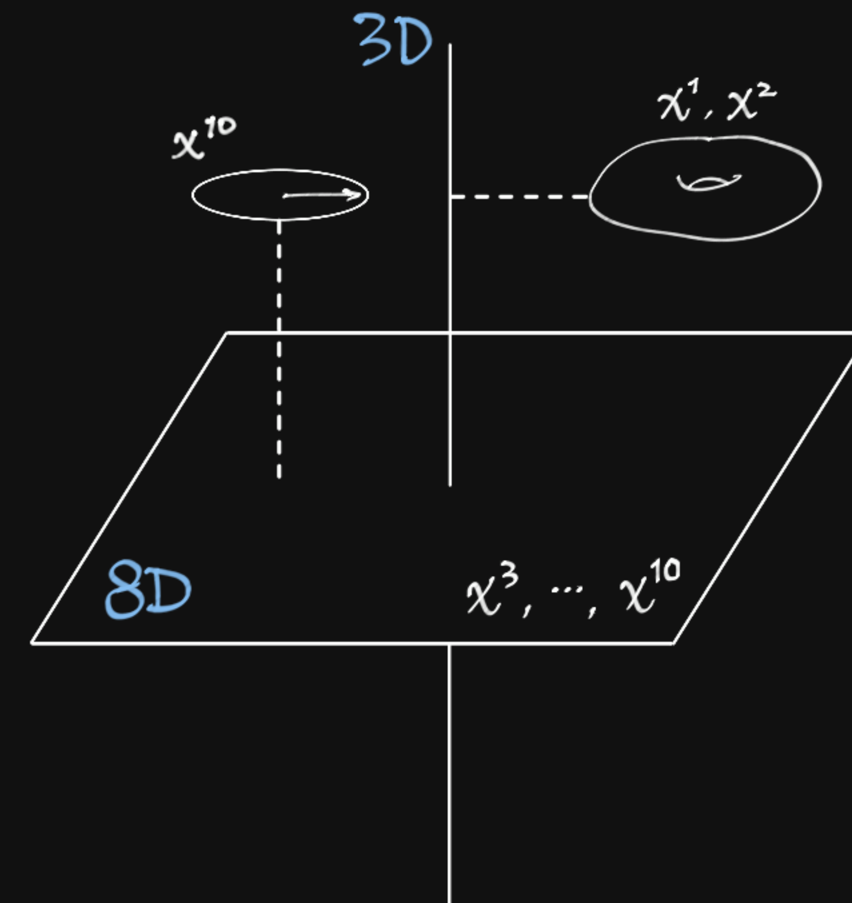
DLCQ of M-theory

$\boxed{\text{fundamental d.o.f.} \quad M2 \quad \tfrac{1}{2}\text{BPS}}$

$$ds^2_{11} = 2\,dx^- dx^+ + \omega_1^{-2}(dx^-)^2 + dx^i dx^i$$

U-dual $\updownarrow$

$R = \omega_1 R_0 \longrightarrow$ finite



non-rel. M-theory

$$ds^2_{11} = \omega_1^{\frac{4}{3}}\left(-dt^2 + dx_1^2 + dx_2^2\right) + \omega_1^{-\frac{2}{3}}\left(dx_3^2 + \cdots + dx_{10}^2\right)$$

$$A_3 = -\omega_1^2\, dt \wedge dx^1 \wedge dx^2$$

DLCQ^2 of M-theory

$$ds^2_{11} = \omega_1^{\frac{4}{3}}\left[2\,dx^- dx^+ + \omega_2^{-2}(dx^-)^2 + dx_2^2\right] + \omega_1^{-\frac{2}{3}}\left(dx_3^2 + \cdots + dx_{10}^2\right)$$

$$A_3 = -\omega_1^2\, dt \wedge dx^1 \wedge dx^2$$

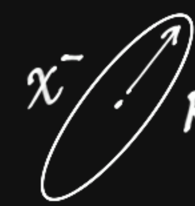
From DLCQ^2 to DLCQ^3

[Blair, Lahnsteiner, Obers, ZY '23]
[Gomis, ZY '23]
[de Boer, Harmark, Obers, ZY, WIP]

DLCQ^2 of M-theory

$$ds_{11}^2 = \omega_1^{\frac{4}{3}}\left[2\,dx^- dx^+ + \omega_2^{-2}(dx^-)^2 + dx_2^2\right] + \omega_1^{-\frac{2}{3}}\left(dx_3^2 + \cdots + dx_{10}^2\right)$$

$$A_3 = -\,\omega_1^2\, dt \wedge dx^1 \wedge dx^2$$



$R = \omega_2 R_0 \longrightarrow$ finite

U-dual $\updownarrow$

multicritical M-theory

$\boxed{\text{fundamental d.o.f.} \quad \text{M2-M2} \quad \tfrac{1}{4}\,\text{BPS}}$

$$ds_{11}^2 = -\,\omega_1^{\frac{4}{3}}\omega_2^{\frac{4}{3}}dt^2 + \omega_1^{\frac{4}{3}}\left(dx_1^2 + dx_2^2\right) + \omega_2^{\frac{4}{3}}\left(dx_3^2 + dx_4^2\right) + \omega_1^{-\frac{2}{3}}\,\omega_2^{-\frac{2}{3}}\left(dx_5^2 + \cdots + dx_{10}^2\right)$$

$$A_3 = -\,\omega_1^2\, dt \wedge dx^1 \wedge dx^2 - \omega_2^2\, dt \wedge dx^3 \wedge dx^4$$

$\omega_1,\ \omega_2 \longrightarrow \infty$

[de Boer, Harmark, Obers, ZY, WIP]

DLCQ^3 M-Theory

DLCQ^2 orbit: BPS decoupling limit zooming in on 2 orthogonal membranes

	0	1	2	3	4	…
M2	×	×	×			
M2	×			×	×	

Compactify + U-dual generate all BPS^2 decoupling limits

What is DLCQ^3 ? magnetic dual via string theory

	0	1	2	3	4	…
D2	×	×	×			
D2	×			×	×	

T-dualize x^5

	0	1	2	3	4	5	…
D3	×	×	×			×	
D3	×			×	×	×	
3rd DLCQ	×		x^0-x^5			×	

non-rel. F1

	0	1	2	3	4	5	…
D2	×	×	×				
D2	×			×	×		
F1	×					×	

DLCQ^3 orbit:

	0	1	2	3	4	5	6	…
M2	×	×	×					
M2	×			×	×			
M2	×					×	×	

fundamental d.o.f.

M2-M2-M2 $\frac{1}{8}$BPS

Duality Orbits of DLCQs

[de Boer, Harmark, Obers, ZY, WIP]

	0	1	2	3	4	$\cdots$	$2n-1$	$2n$	$2n+1$ $\cdots$
M2	×	×	×						
M2	×			×	×				
$\vdots$	$\vdots$								
M2	×						×	×	

$0 \leq n \leq 5$

fundamental d.o.f.

$\underbrace{\text{M2} - \cdots - \text{M2}}_{n}$ $\frac{1}{2^n}$ BPS

U-dual invariant BPS mass formulae [Dijgkraaf, de Boer, Harmark, Obers]

relation to fortuity?

quantum validity, backreaction, exotic branes ...

holographic constructions via generalized $T\bar{T}$

AdS_5/CFT_4: $DLCQ^0/DLCQ^1$

$\text{DLCQ}^n/\text{DLCQ}^m$ correspondence

AdS_3/CFT_2: $DLCQ^0/DLCQ^2$

Thank You!