Standard Model from IIB Matrix Model

Matrix Model for Superstring/M-theory
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Hikaru Kawai (NTU, NITEP)

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(1) P.-M. Ho, W. Piensuk, and W.-H. Shao , to appear,
(2) P.-M. Ho, H . C. Steinacker, e-Print: 2509.06646.

Matrices as bi-local fields

1. IIB matrix model

Ishibashi-HK-Kitazawa-Tsuchiya

$$S_{\mathrm{IIB}} = \alpha\, Tr\left(-\frac{1}{4}\left[A_\mu, A_\nu\right]^2 - \frac{1}{2}\bar{\psi}\Gamma^\mu\left[A_\mu, \psi\right]\right)$$

A_μ and ψ are Hermitian operators on a vector space V.

$V = \mathbb{C}^N$

$\Rightarrow$ A_μ and ψ are $N \times N$ matrices

$(A_\mu)_{ij}\,,(\psi_s)_{ij}$

$V = \{\varphi(x) : \mathbb{R}^d \to \mathbb{C}\}$ the space of functions on $\mathbb{R}^d$

$\Rightarrow$ A_μ and ψ are bilocal fields

$$A_\mu(x, y) = \langle x|A_\mu|y\rangle\,,\ \psi_s(x.y) = \langle x|\psi_s|y\rangle$$

In the case $V = \{\, \varphi(x) \colon \mathbb{R}^d \to \mathbb{C}\}$, the action becomes

$$S = -\frac{\alpha}{2} \int d^d x d^d y d^d z d^d w$$
$$\{A_\mu(x,y) A_\nu(y,z) A_\mu(z,w) A_\nu(w,x)$$
$$- A_\mu(x,y) A_\nu(y,z) A_\nu(z,w) A_\mu(w,x)\}$$
$$+ \int d^d x d^d y d^d z$$
$$\{\bar{\psi}(x,y) \Gamma^\mu A_\mu(y,z) \psi(z,x)$$
$$- \bar{\psi}(x,y) \Gamma^\mu \psi(y,z) A_\mu(z,x)\}.$$

Diffeomorphism invariance

This is invariant under the diffeomorphisms on $\mathbb{R}^d$:

$$A_\mu(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} A_\mu(\varphi(x),\varphi(y)),$$

$$\psi(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} \psi(\varphi(x),\varphi(y)),$$

where $J(x) = \det(\partial_\alpha \varphi^\beta(x))$.

It is because

$$\begin{aligned} &\int d^d y \cdots \phi(x,y)\phi(y,z)\cdots \\ &\mapsto \int d^d y \cdots \phi\big(\varphi(x),\varphi(y)\big)\, J(y)\, \phi\big(\varphi(y),\varphi(z)\big)\cdots \\ &\quad = \int d^d y' \cdots \phi(x',y')\phi(y',z')\cdots . \end{aligned}$$

This a small subset of the $\mathrm{SU}(N)$ invariance:

$$|x\rangle \mapsto J(x)^{\frac{1}{2}} |\varphi(x)\rangle \ \sim \ \text{permutation of } |i\rangle$$

IIB matrix model as pre-geometric theory

We can view this action as a pre-geometric theory, i.e., a theory that, when a certain background is introduced, becomes an ordinary bi-local field on curved spacetime.

It is natural to expect that the low-energy effective theory of this action is a field theory involving gravity.

Roughly speaking, this is true, but we have to be careful about unitarity (or positivity).

This is because the vector and spinor indices of A and ψ are like Lorentz indices which are not transformed under the diffeomorphisms.

$$A_\mu(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} A_\mu(\varphi(x),\varphi(y))$$

$$\psi_s(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} \psi_s(\varphi(x),\varphi(y))$$

However, the IIB matrix model has no built-in local Lorentz invariance.

Therefore the situation is similar to an action written in terms of the vierbein $e_a^{\ \nu}$ but without local Lorentz symmetry.

In general, such theory is not unitary because the negative norm states $e_a^{\ 0}$ cannot be removed.

Constraints to guarantee unitarity

In fact, as we will see, one simple way to guarantee unitarity is to impose the following constraints on the bilocal fields.

$$\int d^d y \, A_\mu(x,y) = 0, \quad \int d^d y \, \psi_s(x,y) = 0 \, .$$

If we do so, the symmetry is reduced from the diffeomorphisms to the volume preserving diffeomorphisms (VPDs)

$$J(x) = \det\left(\partial_\alpha \varphi^\beta(x)\right) = 1 \, ,$$

because these constraints are invariant under the diffeomorphisms only when $J = 1$.

Meaning of the constraints

$\leftarrow \phi(x,y) = \langle x|\phi|y\rangle$

$$\int d^d y\, \phi(x,y) = 0 \;\Leftrightarrow\; \phi|k=0\rangle = 0$$

Here $|k\rangle$ is the momentum eigenstates:

$$|k\rangle = \int d^d x \exp(i\, k\cdot x)\,|x\rangle$$

$$\Rightarrow\; |k=0\rangle = \int d^d y\,|y\rangle\,.$$

These constraints correspond to remove $|k=0\rangle$ from the Hilbert space $V = \{\varphi:\ \mathbb{R}^d \to \mathbb{C}\}$.

In other words, we replace

$$V = \big\langle |k\rangle \big\rangle_{k\in\mathbb{R}^d} \text{ with } V' = \big\langle |k\rangle \big\rangle_{k\in\mathbb{R}^d-\{0\}}\,.$$

This change seems very minor, but it imposes some nontrivial constraints.

formal Taylor expansion with respect to $\hat{p}_\mu$

= differential operator

$$
\begin{aligned}
\phi &= a(\hat{x}) + \frac{1}{2}\left(\hat{p}_\mu a^\mu(\hat{x}) + a^\mu(\hat{x})\hat{p}_\mu\right) \\
&\quad + \frac{1}{2}\left(\hat{p}_\mu \hat{p}_\nu a^{\mu\nu}(\hat{x}) + a^{\mu\nu}(\hat{x})\hat{p}_\mu \hat{p}_\nu\right) + \cdots \\
&= a(x) + \frac{i}{2}\left(\partial_\mu a^\mu(x) + a^\mu(x)\partial_\mu \right) \\
&\quad + \frac{i^2}{2}\left(\partial_\mu \partial_\nu a^{\mu\nu}(x) + a^{\mu\nu}(x)\partial_\mu \partial_\nu\right) + \cdots
\end{aligned}
$$

$$
\phi\,|k=0\rangle = 0 \iff
\begin{cases}
a(x) - \frac{1}{2}\,\partial_\mu \partial_\nu a^{\mu\nu}(x) + \cdots = 0, \\
\partial_\mu a^\mu(x) - \partial_\mu \partial_\nu \partial_\lambda a^{\mu\nu\lambda}(x) + \cdots = 0\,.
\end{cases}
$$

More precisely,
suppose we expand A_μ and ψ_s as differential operators

$$A_\mu = a_\mu(x) + \frac{i}{2}\left(\partial_\nu a_\mu^{\ \nu}(x) + a_\mu^{\ \nu}(x)\partial_\nu\ \right) + \frac{i^2}{2}\left(\partial_\nu\partial_\lambda a_\mu^{\ \nu\lambda}(x) + a_\mu^{\ \nu\lambda}(x)\partial_\nu\partial_\lambda\right) + \cdots$$

From physical intuition, we expect that the higher rank tensors become massive while the lower rank tensors remain massless because of the diffeomorphism symmetry.

Then, we need gauge symmetry or some constraints to eliminate negative norm states for massless fields.

We will show that the constraints indeed fulfill this role.

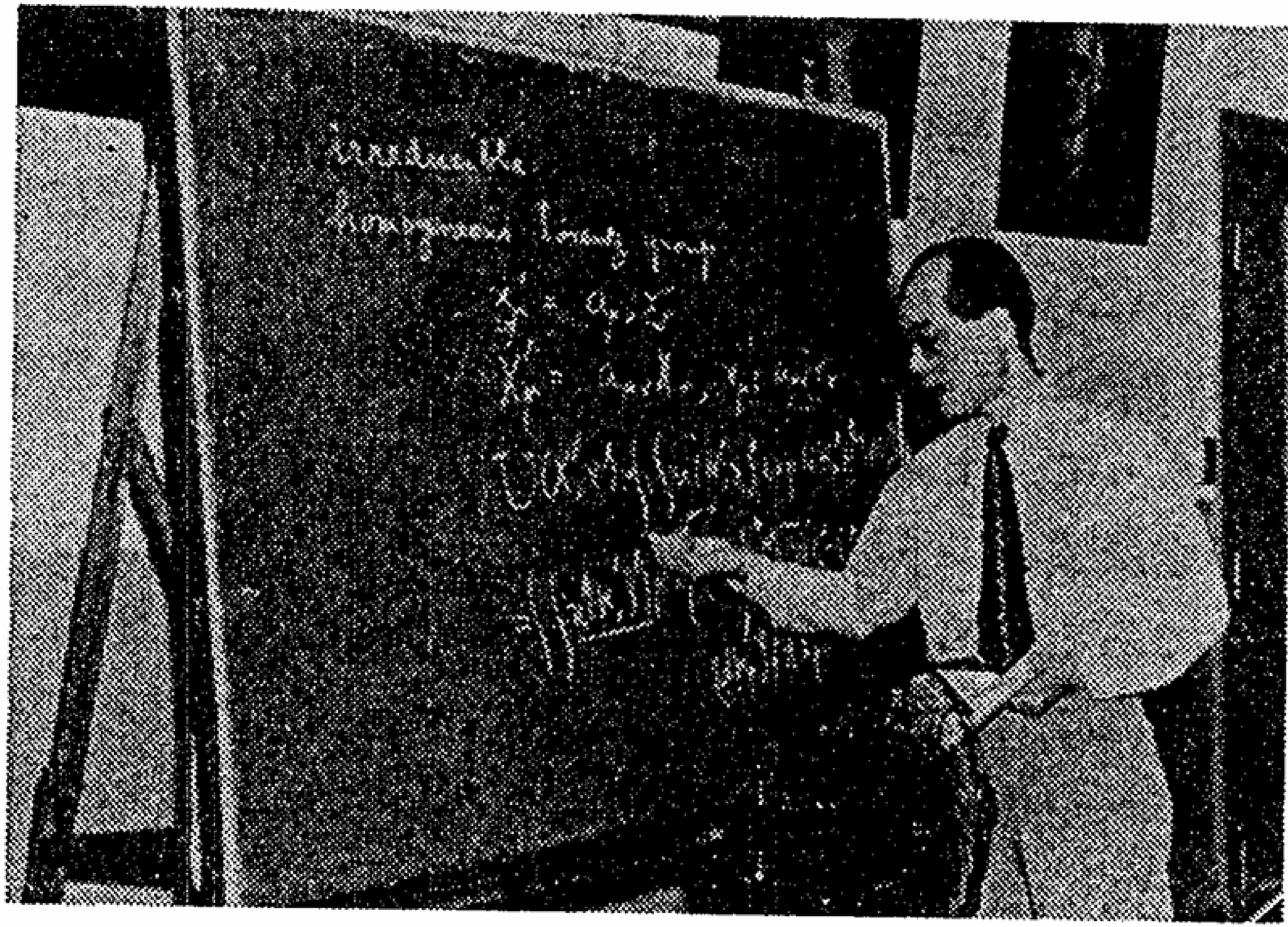
Yukawa and bi-local field

For some reason, Yukawa did not like renormalization theory and instead tried to understand elementary particles by extended fundamental object.

He tried many kinds of nonlocal fields.
Bilocal field, Tetralocal field, .. , Elementary domain

Phys, Rev. 77 (1950) 219, 80 (1950) 1047, …

$$\left(\frac{\partial^2}{\partial X^{\mu 2}} - m^2\right)\psi(X,\xi) = 0,$$
$$(\xi^2 - \lambda^2)\psi(X,\xi) = 0,$$
$$\xi^\mu \frac{\partial}{\partial X^\mu}\psi(X,\xi) = 0.$$



京大における講演 (1950)

JPS journal, vol 37, no.4 (1982), p.276   By O. Hara

Fluctuations around diagonal configurations

2. Commuting background

In the case $V = \mathbb{C}^N$, one of the simplest classical solutions is given by the diagonal matrices:

$$A_\mu^{(0)} = \begin{cases} P_\mu \ (\mu = 1 \sim d), \ \ (P_\mu)_{ij} = p_\mu^i \delta_{ij} \ \ \text{(diagonal)}, \\ 0 \ (\ \mu = d + 1 \sim 10) \end{cases}$$

$$\psi^{(0)} = 0.$$

Let’s translate this to bilocal fields.

Corresponding solution in $V = \{\varphi(x) : \mathbb{R}^d \to \mathbb{C}\}$

Using the momentum representation

$$|k\rangle = \int d^d x \exp(ik \cdot x)|x\rangle,$$

we introduce the following identification:

$$|i\rangle \in \mathbb{C}^N \leftrightarrow |k = p^i\rangle .$$

Then the diagonal solution

$$\left\langle i \middle| A_\mu^{(0)} \middle| j \right\rangle = \begin{cases} p_\mu^i\, \delta_{ij} & (\mu = 1 \sim d) \\ 0 & (\mu = d+1 \sim 10) \end{cases}, \qquad \langle i|\psi^{(0)}|j\rangle = 0$$

becomes

$$\left\langle k \middle| A_\mu^{(0)} \middle| k' \right\rangle = \begin{cases} k_\mu \delta(k - k') & (\mu = 1 \sim d) \\ 0 & (\mu = d+1 \sim 10) \end{cases}, \ \langle k|\psi^{(0)}|k'\rangle = 0.$$

By introducing $\hat{x}^{\mu}$ and $\hat{p}_{\mu}$ as the ordinary QM

$$[\hat{x}^{\mu}, \hat{p}_{\nu}] = i\delta^{\mu}{}_{\nu},$$

we can express

$$A_{\mu}^{(0)} = \begin{cases} \hat{p}_{\mu}\ (\mu = 1 \sim d) \\ 0 \quad (\mu = d + 1 \sim 10) \end{cases}$$

$$\psi^{(0)} = 0\,.$$

In terms of the coordinate representation,

$$\left\langle x \middle| A_{\mu}^{(0)} \middle| x' \right\rangle = \begin{cases} -i\, \partial_{\mu}\delta(x - x'\,) & (\mu = 1 \sim d) \\ \quad 0 & (\mu = d + 1 \sim 10) \end{cases}$$

$$\langle x | \psi^{(0)} | x' \rangle = 0.$$

$A_\mu^{(0)}$ = Flat spacetime background

Commutator with $A_\mu^{(0)}$ is the derivative with respect to the center of mass coordinate of the bilocal fields.

In fact,

$$\left\langle x \right| \left[A_\mu^{(0)}, \phi \right] \left| x' \right\rangle = \langle x | [\hat{p}_\mu, \phi] | x' \rangle = -i \left(\frac{\partial}{\partial x^\mu} + \frac{\partial}{\partial x'^\mu} \right) \langle x | \phi | x' \rangle.$$

$$= -i \frac{\partial}{\partial X^\mu} \phi \left(X + \frac{\xi}{2}, X - \frac{\xi}{2} \right),$$

where X and ξ are the CM and the relative coordinates

$$X = \frac{1}{2}(x + x'),\ \xi = x - x'.$$

We can regard $A_\mu^{(0)}$ as a flat space background of the pre-geometric action.

3. Quenched reduced model and bilocal field

Parisi, Gross-Kitazawa, Bhanot-Heller-Neuberger

If

(1) the diagonal elements of A_μ are quenched uniformly in $\mathbb{R}^d$ (with some cut off),

and

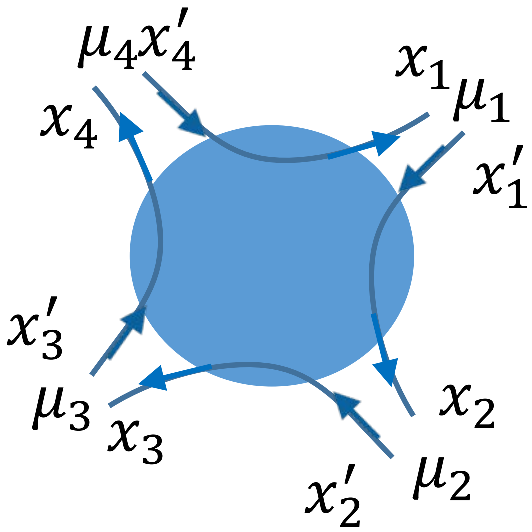
(2) only planar diagrams are considered,

then

the YM-type matrix model is equivalent to the corresponding d-dim YM theory.

More precisely, L pt. Green's functions of the MM and the d-dim YM are related as

$$\left\langle \langle x_1|A_{\mu_1}|x_1'\rangle \cdots \langle x_L|A_{\mu_L}|x_L'\rangle \right\rangle_{\mathrm{MM}}$$
$$= \delta(\xi_1 + \cdots + \xi_L)\, G(X_1, \mu_1, \cdots, X_L, \mu_L)$$



$$\langle A^{a_1}_{\mu_1}(X_1) \cdots A^{a_L}_{\mu_L}(X_L)\rangle_{\mathrm{YM}}$$
$$= \mathrm{tr}(t^{a_1} \cdots t^{a_L})\, G(X_1, \mu_1, \cdots, X_L, \mu_L)$$

<u>correspondence</u>

$$\delta(\xi_1 + \cdots + \xi_L) = \mathrm{tr}(t^{a_1} \cdots t^{a_L})$$

Roughly speaking,

N^2 DOF of adjoint rep. ⇔ continuous DOF of ξ

In particular, there is no negative norm state in the bi-local field representation of IIB MM:

Degeneracy of N^2 particles is resolved by the dynamics of diagonal elements.

For example, $\langle A_i \rangle \; (i = d + 1 \,..\, 10) \quad SU(N) \; \to U(1)^N$

If gravity is there, it must arise from the collective motion of diagonal elements.

But its microscopic mechanism might be complicated.

However, as we shall see, the graviton does indeed appear as a type of Nambu-Goldstone particle.

Graviton as Nambu-Goldstone boson

5. Graviton and NG boson

Sometimes it is said that gravitons can be regarded as NG bosons.

e.g. P. R. Phillips, "Is the Graviton a Goldstone Boson?," Phys.Rev. 146 (1966) 966.

The argument is general, but for simplicity let's consider the Einstein-Hilbert action.

(1) The original action is diffeomorphism invariant.

(2) If we consider the harmonic gauge, the action is invariant under global $GL(d)$:

$$g_{\mu\nu} \to g'_{\mu\nu} = M_\mu{}^{\mu'} M_\nu{}^{\nu'} g_{\mu'\nu'} \text{ , for } M_\mu^\nu \in GL(d).$$

(3) A typical vacuum is given by $g_{\mu\nu}(x) = \eta_{\mu\nu}$.

This vacuum breaks $GL(d)$ to $SO(d-1,1)$.

(4) The other vacua $g_{\mu\nu}(x) = \mathbf{const}$ are obtained from $\eta_{\mu\nu}$ by $GL(d)$.

(5) Therefore the vacuum moduli is given by

$GL(d)/SO(d-1,1) \cong$ rank 2 symmetric tensor ,

(6) We have corresponding NG bosons.

This is nothing but $h_{\mu\nu}$, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

6. VPD in MM and GR

(1) The original action is invariant under

$$SO(d-1,1)\otimes\mathrm{VPD}.$$

(2) If we consider a Lorenz type gauge $\left[A_\mu^{(0)}, A_\mu\right] = 0$,
the action is invariant under global

$$SO(d-1,1)\otimes SL(d).$$

(3) We assume that a typical vacuum has a translationally and rotationally invariant VEV of A_μ:

$$\left\langle\langle x|A_\mu|y\rangle\right\rangle = \frac{\partial}{\partial x^\mu} f\left((x-y)^2\right).$$

This vacuum breaks

$SO(d-1,1)\otimes SL(d)$ to diagonal $SO(d-1,1)$.

(4) We assume that the other vacua are obtained from

$$\left\langle \langle x|A_\mu|y\rangle \right\rangle = \frac{\partial}{\partial x^\mu} f\left((x-y)^2\right)$$

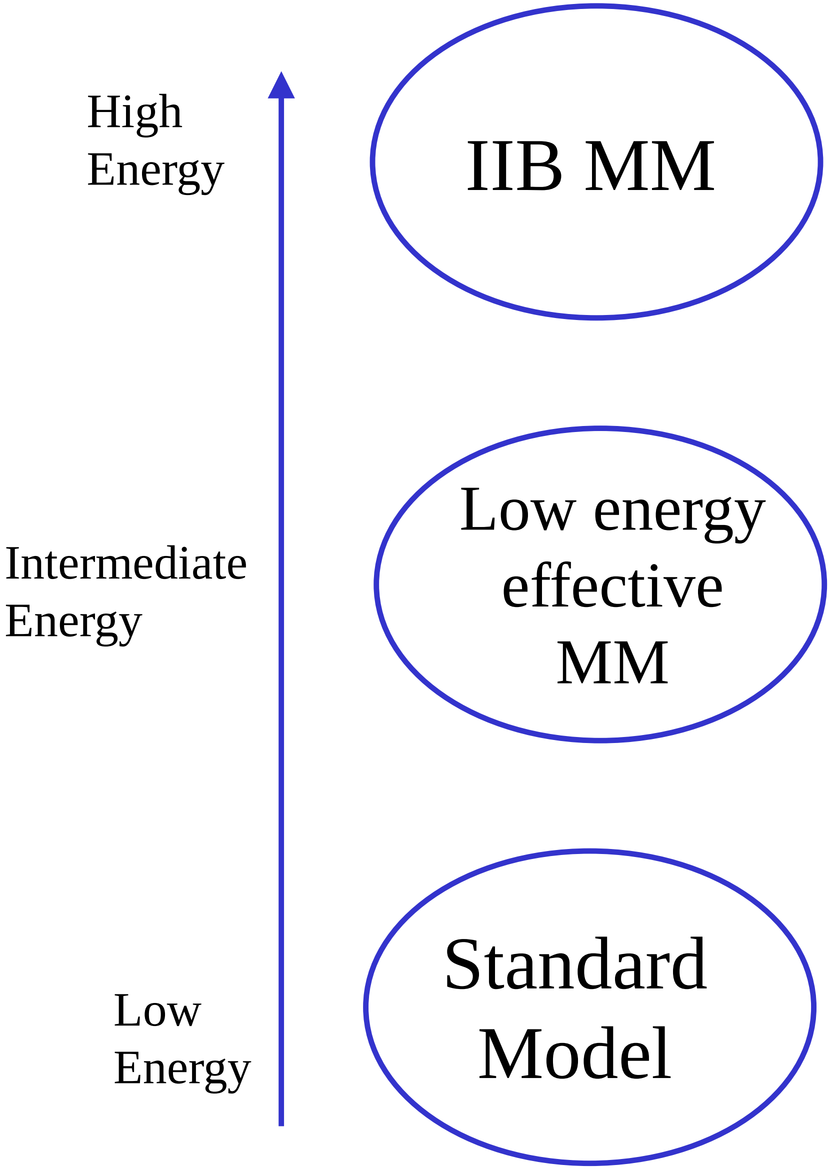
by $SO(d-1,1)\otimes SL(d)$.

(5) Then the vacuum moduli is given by

$$SO(d-1,1)\otimes SL(d)/SO(d-1,1) \cong SL(d)$$

(6) We have corresponding NG bosons that consist of traceless symmetric and anti-symmetric tensors.

Low energy effective theory



Working Hypnoses

Low energy behavior is described by a small IIB MM where matrices are regarded as elements of a Lie algebra.

7. Excitations around vacuum

Suppose the emergent space is a flat space, e.g. $\mathbb{R}^d$.

We express the bi-local fields as a formal power series of derivatives:

real

$$\langle x|A_\mu|y\rangle = \sum_{n=0}^{\infty} a_\mu^{(n)\,\nu_1\cdots\nu_n}(X)\, i^n \partial_{\nu_1}\cdots\partial_{\nu_n}\delta(\xi)\,,$$

$$\langle x|\Psi_s|y\rangle = \sum_{n=0}^{\infty} \psi_s^{(n)\,\nu_1\cdots\nu_n}(X)\, i^n \partial_{\nu_1}\cdots\partial_{\nu_n}\delta(\xi).$$

Physically, we expect only lower spin states are massless.

$a_\mu^{(0)}(x)$: gauge fields

${a^{(1)}}^{\nu}_{\mu}(x)$: vierbein

$\psi_s^{(0)}(x)$: quark-lepton fields

If we keep only two terms in A_μ , we have

$$\langle x|A_\mu|y\rangle = a_\mu(X)\delta(\xi) + i a_\mu{}^\nu(X)\partial_\nu\delta(\xi).$$

The condition for unitarity becomes

$$0 = \int dy\, \langle x|A_\mu|y\rangle = a_\mu(x) + i\partial_\nu a_\mu{}^\nu(x)$$

$$\Leftrightarrow \;\; a_\mu = 0, \quad \partial_\nu a_\mu{}^\nu = 0$$

In terms of differential operator,

$$A_\mu = i\, a_\mu{}^\nu(x)\partial_\nu\, , \partial_\nu a_\mu{}^\nu = 0$$

Each of A_μ is an infinitesimal VPD.

Minimal model

We can consider a simplified matrix model where the matrix algebra is replaced to the set of infinitesimal VPD's on $\mathbb{R}^d$.

$$\mathrm{End}(V) \implies \mathcal{S} = \{v^\mu \partial_\mu , \partial_\mu v^\mu = 0\}$$

Because it forms a Lie algebra, we can consider

EOM $\left[A_\nu, [A_\mu, A_\nu]\right] = 0$

Gauge transformation

$$\delta_v A_\mu = [v,\ A_\mu],\ v = v^\mu \partial_\mu , \partial_\mu v^\mu = 0$$

We can regard this as the low energy effective theory around the flat spacetime.

Unitarity of the minimal model

8. Weak field expansion

In order to check the unitarity of the minimal model, we start with weak field expansion:

$$A_\mu = a_\mu{}^\nu(x)\partial_\nu, \;\; a_\mu{}^\nu = \delta_\mu{}^\nu + c_\mu^\nu \,, \;\; \partial_\nu c_\mu^\nu = 0 \,.$$

Then the EOM $\left[A_\nu,[A_\mu,A_\nu]\right] = 0$ becomes

$$[A_\mu,\; A_\nu]^\rho = (\delta_\mu{}^\lambda + c_\mu^\lambda)\partial_\lambda(\delta_\nu{}^\rho + c_\nu^\rho) - (\mu \leftrightarrow \nu)$$
$$= \partial_\mu c_\nu^\rho - \partial_\nu c_\mu^\rho + O(c^2)$$
$$\left[A_\nu,[A_\mu,A_\nu]\right]^\lambda = \partial_\nu(\partial_\mu c_\nu^\lambda - \partial_\nu c_\mu^\lambda) + \mathrm{O}(\mathrm{c}^2) = 0$$

The gauge transformation becomes

$$\delta_v \, c_\mu^\lambda = [v,\; A_\mu]^\lambda = v^\nu\partial_\nu(\delta_\mu{}^\lambda + c_\mu^\lambda) - (\delta_\mu{}^\nu + c_\mu^\nu)\partial_\nu v^\lambda$$
$$= -\partial_\mu v^\lambda + O(c)$$

EOM $\partial_\nu(\partial_\mu c_\nu^{\lambda} - \partial_\nu c_\mu^{\lambda}) = 0\,,\ \partial_\nu c_\mu^{\nu} = 0\,.$

Gauge transformation $\delta_v\, c_\mu^{\lambda} = -\partial_\mu v^\lambda\,, \partial_\mu v^\mu = 0.$

Using the gauge symmetry, we can fix the gauge to Lorenz like gauge: $\partial_\nu c_\nu^{\lambda} = 0.$

Then we have

$$\Box c_\mu^{\nu} = 0\,,\ \partial_\nu c_\mu^{\nu} = 0,\ \partial_\mu c_\mu^{\nu} = 0\,.$$

Thus we have massless states corresponding to a rank 2 tensor in $D - 2$ directions, that is, graviton, K-R, and dilaton, which is the same as closed string.

The interactions do not break these constraints due to the closure of Lie algebra.

Generalized Low energy effective theory

9. Generalized Minimal Model

In principle we can consider any Lie algebra $\mathcal{L}$ instead of the matrix algebra because any Lie algebra is a sub Lie algebra of the matrix algebra:

$$\mathrm{End}(V) \implies \mathcal{L} \subset \mathrm{End}(V)$$

A simple choice of such $\mathcal{L}$ is the set of differential operators of the form:

$$\mathcal{L} = \left\{ v^{\mu}(x)\partial_{\mu} + t^{a} b^{a}(x), \partial_{\mu} v^{\mu} = 0 \right\}.$$

Here t^{a} are the generators of a finite dimensional Lie algebra.

In other words, $\mathcal{L} = \left\{ v^{\mu}(x) D_{\mu} \,,\ \partial_{\mu} v^{\mu} = 0 \right\}$.

Here, $D_{\mu} = \partial_{\mu} + t^{a} a_{\mu}^{a}(x)$ is the covariant derivative.

Therefore a simple possible low energy effective theory around flat $\mathbb{R}^{10}$ with gauge group G is given by

$$A_\mu = e_\mu{}^\nu(x)(\partial_\nu + t^a a^a_\nu(x)) \, , \partial_\nu e^\nu_\mu = 0 \, ,$$

$$\Psi_s = t^a \, \psi_s{}^a(x) \, .$$

EOM: $\left[A_\nu, [A_\mu, A_\nu]\right] + \bar{\Psi}\gamma^\mu\Psi = 0 \, ,$

$$\bar{\Psi}\gamma^\mu[A_\mu, \Psi] = 0 \, .$$

Gauge symmetry: $\delta A_\mu = \left[v, A_\mu\right] , \quad \delta\Psi = [v, \Psi] \, ,$

where $v = v^\mu(x)\partial_\mu + t^a b^a(x), \; \partial_\mu v^\mu = 0.$

This model consists of

YM + Maj-Weyl fermion with adj. rep. + gravity .

SM from generalized Minimal Model

10. SM as a low energy effective theory

We can consider the following back ground for the generalized minimal model:

$$A_\mu^{(0)} = \begin{cases} \dfrac{\partial}{\partial x^m} & (m = 1\sim4), \\ \dfrac{\partial}{\partial y^i} + t^a a_i^{(0)\,a}(y) & (i = 1\sim6)\,. \end{cases}$$

$$\Psi_s^{(0)} = 0.$$

Here $x \in \mathbb{R}^4$ and $y \in M_6$, where M_6 is a 6D compact mfd.

Question

Can we realize the SM, especially chiral fermions?

Major difficulty

All fields are adjoint representation. How can we get SM chiral fermions?

Comment: Abe, Kobayashi, Ohki, Oikawa1Sumita

Abe et.al. have considered various models of this type with low energy SUSY.

Here we consider models without SUSY.

11. E8 minimal model

A simple model can be made by considering $G = E8$.

According to $E8 \supset SO(16) \supset SO(10) \otimes SO(6)$,

$$\mathrm{adj(E8)} = \mathrm{adj}\big(\mathrm{SO(16)}\big) \oplus \mathrm{s(SO(16))} ,$$

$$\mathrm{s}\big(\mathrm{SO(16)}\big) = \mathrm{s}\big(\mathrm{SO(10)}\big) \otimes \mathrm{s}\big(\mathrm{SO(6)}\big) \oplus c\big(SO(10)\big) \otimes c(SO(6)) .$$

s: spinor with positive chirality

c: negative

If the b.g. gauge field on M_6 , $t^a a_i^{(0)\,a}(y)$, consists of the above SO(6) and has index= 3, we have 3 generations of 4D chiral spinor with $\mathrm{s(SO(10))}$, which represents quarks and leptons in SM.

Decomposition of Ψ_s^a

space time index s

According to $SO(9,1) \supset SO(3,1) \times SO(6)$, we have

$$s(SO(9,1)) \\ = s(SO(3,1)) \otimes s(SO(6)) \oplus c(SO(3,1) \otimes c(SO(6)).$$

Therefore,

$$\mathrm{MajWeyl}(SO(9,1)) \cong s(SO(3,1)) \otimes s(SO(6)).$$

In other words, 16 real compts = 2× 4 complex compts.

$\Psi_s^a \cong \Psi_{\alpha,j}^a$ $\quad s = 1\sim16 : 10D$ Majorana Weyl

$\alpha = 1\sim2 : 4D$ Weyl $\in s(SO(3,1))$

$j = 1\sim 4 : 6D$ spinor $\in s(SO(6))$

internal index a

According to E8⊃ $SO(16)$,

$$\mathrm{adj(E8) = adj(SO(16)) \oplus s(SO(16)).}$$

According to $SO(16) \supset SO(10) \times SO(6)$,

$$\mathrm{s(SO(16)) = s\big(SO(6)\big) \otimes s\big(SO(10)\big)}$$
$$\mathrm{\oplus\, c\big(SO(6)\big) \otimes c\big(SO(10)\big)}$$

Therfore, each $\Psi^{a}_{\alpha,j}$ is decomposed as

$$\Psi^{a}_{\alpha,j} \Leftrightarrow \Psi^{B}_{\alpha,j}, \Psi^{k,l}_{\alpha,j}, \Psi^{\bar{k},\bar{l}}_{\alpha,j}$$

$$B = 1 \sim 120 \in \mathrm{adj(SO(16))}$$

$$k = 1 \sim 4 \in s\big(SO(6)\big), l = 1 \sim 16 \in s\big(SO(10)\big).$$

Thus we have

$$\Psi^{k,l}_{\alpha\ j}(x,y) = \sum_{g=1}^{3} \psi^{(g)l}_{\alpha}(x)\, \xi^{(g)k}_{j}(y)\,,$$

where $\xi^{(g)}_{j}(y)$ $(g = 1\sim3)$ are the zero modes of $D_6^{(0)}$:

$$D_6^{(0)}\,\xi^{(g)} = 0,\ \ D_6^{(0)} = \gamma^{i}\left(\frac{\partial}{\partial y^{i}} + t^{a} a_i^{(0)\,a}(y)\right).$$

Note that t^{a} are generators of $SO(6)$ so that they act on the $s(SO(6))$ index k.

Therefore, we have 3 generations of $s(SO(10))$.

An example

$M_6 = T_2 \times T_4$

$SO(6) \cong SU(4) \supset SU(3) \times U(1)$

$$t^a a_i^{(0)\,a}(y) = \begin{cases} \text{U(1) const. mag. field on } T_2 \\ \text{SU(3) instanton on } T_4 \end{cases}$$

$$\text{index} = \frac{1}{3!} \int \mathrm{tr}_{s(SO(6))} F_6^3 = \text{index}_{\mathrm{T}_2} \times \text{index}_{\mathrm{T}_4} = 3$$

Note:

This back ground satisfies EOM of the MM without mass term.

SM Higgs

We can show that SM Higgs doublet can be obtained as a massless mode of the fluctuations of A_i^a around the back ground:

$$A_i^a = A_i^{(0)a} + \phi_i^a \,, (i = 1 \sim 6).$$

In fact, the SM Higgs comes from a vector rep. of $SO(10)$.

According to $\mathrm{SO}(16) \supset \mathrm{SO}(6) \times \mathrm{SO}(10)$,

$$\mathrm{adj}(\mathrm{SO}(16)) = \mathrm{adj}\big(\mathrm{SO}(6)\big) \oplus \mathrm{adj}(\mathrm{SO}(10)) \oplus \mathrm{vec}(\mathrm{SO}(6)) \otimes \mathrm{vec}(\mathrm{SO}(10)) \,.$$

Therefore, $\phi_i^a \supset \phi_i^{bc}, b = 1 \sim 6 \in \mathrm{vec}\big(\mathrm{SO}(6)\big)$

$$, \mathrm{c} = 1 \sim 10 \in \mathrm{vec}\big(\mathrm{SO}(10)\big) \,.$$

From the EOM for A_μ we find that the mass of ϕ in 4D is determined by the 6D Laplacian:

$$D_j^2\phi_i - D_i\left(D_j\phi_j\right) + 2iF_{ij}\phi_j = -M^2\phi_i\,,$$

where $D_i = \frac{\partial}{\partial y^i} + t^a a_i^{(0)\,a}(y)$ and $F_{ij} = -i\left[D_i, D_j\right]$.

Note that $t^a \in SO(6)$ so that it acts on the index b of ϕ^{bc} as a vector representation.

Unlike fermions, there is no index theorem, so eigenvalues are generally nonzero. However, by adjusting the M_6 metric and the Wilson lines, it is possible to make only one mode have zero mass, while the m² of the other modes remains positive.

In this way, we have one Higgs doublet.

Summary

If the emergent spacetime can be described by matrices that fluctuate around diagonal matrices, it is useful to represent the matrices as bi-local fields.

The action of the IIB matrix model in terms of bilocal field can be regarded as a kind of pre-geometric theory.

GR arises as a low energy effective theory consisting of NG bosons around the emergent geometry.

Furthermore, the Standard Model may naturally emerge as the low-energy effective theory of the IIB matrix model. However, it is not yet clear whether the E8 structure is truly realized from the IIB matrix model.

E8 minimal model looks nice, but it is not clear whether it is

IIB or not IIB, that is the question.

Thank you very much.