

K-theoretic Matrix Theory and D-branes (review)

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based on works with

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ref) hep-th/**0108085**, 0202165, **0212188**, 0305006

“Matrix model for superstring/M-theory” @ YITP, December 5, 2025

1. Introduction

What is “K-theoretic matrix theory” ?

Matrix theory based on non-BPS D-particles or D-instantons

- e.g. Type IIA: non-BPS $D(-1)$, $D0-\overline{D0}$
Type IIB: $D(-1)-\overline{D(-1)}$, non-BPS $D0$
Type I: non-BPS $D(-1)$, non-BPS $D0$

As I will explain later, it has a structure of K-theory.

Let me call these matrix theories as “**K-matrix theories**” in this talk.

(Naïve) Motivation

BFSS \rightarrow D0-branes $\times N$

IKKT \rightarrow D(-1)-branes $\times N$

\Rightarrow The D0/D(-1) charge is fixed to be N .

e.g. fuzzy sphere config. In polarized IKKT

\rightarrow spherical D1 with N unit of flux $\frac{1}{2\pi} \int_{S^2} F = N$ (= D(-1) charge)

Can one find matrix theories that include configurations with arbitrary number (including negative values) of D0/D(-1) charge?

D0- $\overline{\text{D0}}$ pairs $\times \infty$ (IIA)

D(-1)- $\overline{\text{D(-1)}}$ pairs $\times \infty$ (IIB)

Or, we could consider non-BPS D(-1) $\times \infty$ (IIA)

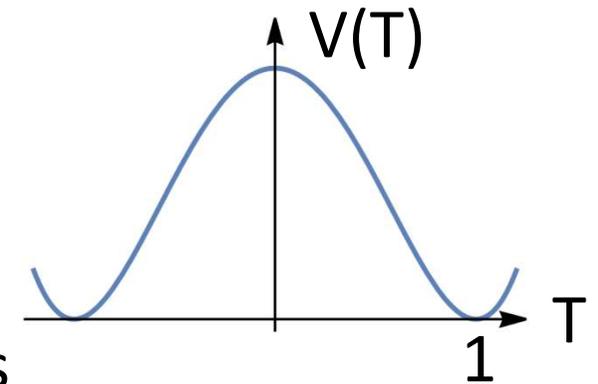
non-BPS D0 $\times \infty$ (IIB)

Tachyon matrix T

Consider, for example, $D(-1)-\overline{D(-1)}$ pairs $\times \infty$

There is a complex $\infty \times \infty$ matrix T with a Mexican-hat-type potential

Suppose T is normalized so that the minimum of the potential is $|T|=1$



$$T = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix} \Rightarrow \text{All the } D(-1)-\overline{D(-1)} \text{ pairs are annihilated}$$

$$T = \begin{pmatrix} \overleftarrow{n} \overrightarrow{1} & & & \\ & 1 & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix} \Rightarrow n \text{ } D(-1) \text{ remain}$$

In general,

$$\text{Index } T := \text{Ker } T - \text{Ker } T^\dagger = D(-1) \text{ charge}$$

Messages

- 1) Any D-brane configuration can, in principle, be realized in K-matrix theories.
- 2) D-brane configurations in K-matrix theories are classified by K-homology group
- 3) The effective actions of D-branes can be derived from K-matrix theories.
- 4) Atiyah–Singer index theorem is derived from physical consideration

Bigger picture

Geometry \Leftrightarrow **Algebra**

(Spacetime \Leftrightarrow **Matrix)**

cf) **Gel'fand-Naimark theorem**

compact Hausdorff space \Leftrightarrow commutative C^* -algebra

equivalent

\Downarrow

operator (matrix)

Geometric properties in physics can be described by Matrices

Matrix descriptions have advantages in describing non-commutative geometry etc.

\rightarrow Next talk by S. Terashima

Geometry \Leftrightarrow Algebra

Today:

D-brane with
smooth world-volume

\Leftrightarrow
equivalent

D-brane in
matrix theory

path integral
formulation of QM

\Leftrightarrow
equivalent

operator
formulation of QM

$$\frac{1}{8\pi^2} \int \text{tr}(F \wedge F)$$

\Leftrightarrow
equivalent

Index of \mathcal{D}

Plan

- ✓ **1. Introduction**
- 2. Non-BPS D-brane systems**
- 3. BSFT action**
- 4. D-brane solution**
- 5. D-brane action**
- 6. K-homology**
- 7. Application to index theorem**
- 8. Summary and discussion**

2. Non-BPS D-brane systems

The non-BPS D-brane systems we consider are as follows:

- $D_p\text{-}\overline{D}_p$ system $p=\text{even}$ for type IIA
 $p=\text{odd}$ for type IIB
 $G = U(N) \times U(N)$

	gauge field	adjoint scalar	bifund. scalar	+fermions
field	$A_\alpha, \overline{A}_\alpha$	$\Phi^i, \overline{\Phi}^i$	T	
rep. of G	(adj., 1), (1, adj.)	(adj., 1), (1, adj.)	(\square, \square^*)	
	$\alpha = 0 \sim p$	$i = p + 1 \sim 9$		

- non-BPS D_p -branes $p=\text{odd}$ for type IIA
 $p=\text{even}$ for type IIB
 $G = U(N)$

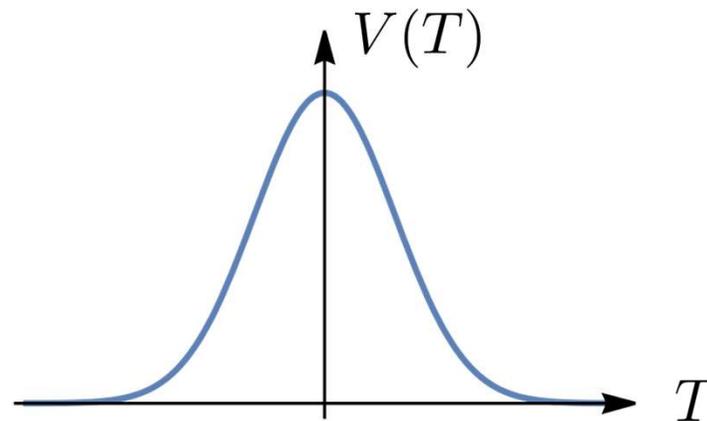
	gauge field	adjoint scalar	adjoint scalar	+fermions
field	A_α	Φ^i	T	
rep. of G	adj.	adj.	adj.	
	$\alpha = 0 \sim p$	$i = p + 1 \sim 9$		

 $T = T^\dagger$

- The scalar field T is called “tachyon field”
It has a non-trivial potential:

$$V(T) = \text{Tr} e^{-TT^\dagger} + \text{Tr} e^{-T^\dagger T} \quad \text{Dp-}\overline{\text{Dp}} \text{ system}$$

$$V(T) = \text{Tr} e^{-T^2} \quad \text{non-BPS Dp-branes}$$



- $T=0$ is unstable

The minimum of the potential is $|T| \rightarrow \infty$

It corresponds to annihilation of $\text{Dp-}\overline{\text{Dp}}$ or non-BPS Dp

Construction of lower dim D-branes

Sen, Witten, Horava, ... around 1999

- D-branes can be constructed as solitons in non-BPS D9 (for IIA) or D9- $\overline{\text{D9}}$ system (for IIB)

$$T \sim \sum_{i=p+1}^9 \gamma_i x^i \quad (\text{kink } (p=8), \text{ vortex } (p=7), \dots)$$

⇒ Dp-brane localized at $x^i=0$ ($i = p+1, \dots, 9$)

- D-brane charges are classified by K-theory

$$K^0(X) \text{ for type IIB, } K^1(X) \text{ for type IIA,}$$

3. BSFT action (= disk partition function for superstring)

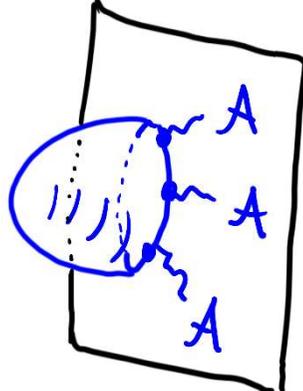
Boundary String Field Theory

[Witten 1992, Shatashvili 1993, Kutasov-Marino-Moore 2000]

$$S[\mathcal{A}] = \langle 0 | e^{-S_b(\mathcal{A})} | Bp \rangle$$

for Dp-brane

boundary interaction boundary state

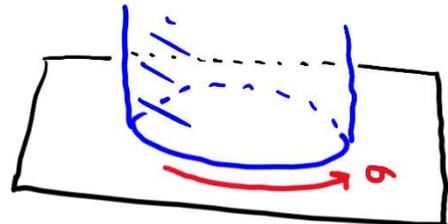


The diagram shows a rectangular sheet representing a disk. A blue circle is drawn on the left side, representing a boundary state. Three blue wavy lines labeled 'A' extend from the boundary into the interior of the disk, representing boundary interactions.

$$S_b(\mathcal{A}) = \oint d\hat{\sigma} \sum_I \mathcal{A}_I(\mathbf{X}(\hat{\sigma})) \mathcal{O}_I(\hat{\sigma})$$

open string field vertex operator

superfield



The diagram shows a rectangular sheet representing a disk. A blue circle is drawn on the left side, representing a boundary state. A red arrow labeled 'g' points to the right along the bottom edge of the disk, representing a boundary interaction.

$\hat{\sigma} = (\sigma, \theta)$
superspace

$$\mathbf{X}^\mu(\hat{\sigma}) = X^\mu(\sigma) + i\theta\Psi_+^\mu(\sigma)$$

$$X^\mu(\sigma) = \hat{x}_0 + i \sum_{m \neq 0} \left(\frac{\alpha_m^\mu}{m} e^{-im\sigma} + \frac{\tilde{\alpha}_m^\mu}{m} e^{im\sigma} \right)$$

$$\Psi_+^\mu(\sigma) = \sum_r \left(\Psi_r^\mu e^{-ir\sigma} + i\tilde{\Psi}_r^\mu e^{ir\sigma} \right)$$

Boundary interaction

- For BPS Dp-brane with $\mathcal{A}_I = (A_\alpha, \Phi^i)$

$$e^{-S_b(A_\alpha, \Phi^i)} = \text{Tr} \hat{\mathcal{P}} \exp \left(- \int d\hat{\sigma} A_\alpha(\mathbf{X}) D\mathbf{X}^\mu + i\Phi^i(\mathbf{X}) \mathbf{P}_i \right)$$

susy version of path ordering (pointing to $\hat{\mathcal{P}}$)
susy version of Wilson loop (pointing to the integral)

$$\mathbf{X}^\mu(\hat{\sigma}) = X^\mu(\sigma) + i\theta\Psi^\mu(\sigma) \quad D = \partial_\theta + \theta\partial_\sigma$$

$$\mathbf{P}_i(\hat{\sigma}) = \theta P_i(\sigma) + i\Pi_i(\sigma) \quad (P_i, \Pi_i : \text{momenta conj. to } X^i, \Psi^i)$$

- For Dp- $\overline{\text{Dp}}$ system with $\mathcal{A}_I = (A_\alpha, \overline{A}_\alpha, \Phi^i, \overline{\Phi}^i, T)$

$$e^{-S_b(A_\alpha, \Phi^i, T, \dots)} = \text{Tr} \hat{\mathcal{P}} \exp \left(\int d\hat{\sigma} \mathbf{M}(\hat{\sigma}) \right)$$

$$\mathbf{M} = \begin{pmatrix} -A_\alpha(\mathbf{X})D\mathbf{X}^\alpha - i\Phi^i(\mathbf{X})\mathbf{P}_i & T(\mathbf{X}) \\ T(\mathbf{X})^\dagger & -\overline{A}_\alpha(\mathbf{X})D\mathbf{X}^\alpha - i\overline{\Phi}^i(\mathbf{X})\mathbf{P}_i \end{pmatrix}$$

(for non-BPS Dp-brane, set $A_\mu = \overline{A}_\mu, \Phi^i = \overline{\Phi}^i, T = T^\dagger$) 13

Boundary states

$$|Bp\rangle = \int \mathcal{D}\mathbf{x}^\alpha \underbrace{|\mathbf{x}^\alpha, \mathbf{x}^i = 0\rangle}_{\text{Coherent state}}$$

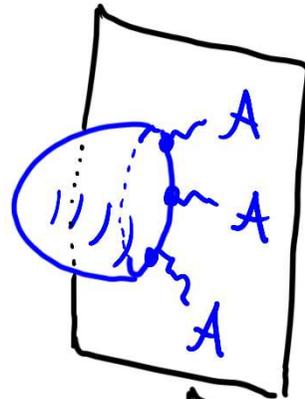
$\alpha = 0 \sim p$
 $i = p+1 \sim 9$

Coherent state

$$\mathbf{X}^\mu(\hat{\sigma})|\mathbf{x}^\mu\rangle = \mathbf{x}^\mu(\hat{\sigma})|\mathbf{x}^\mu\rangle$$

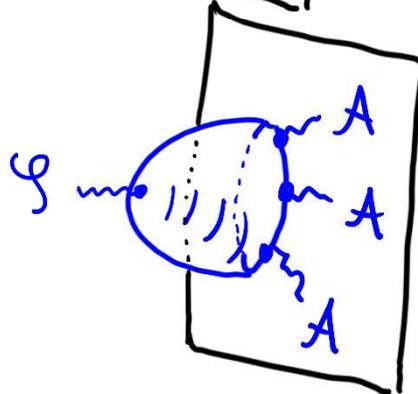
$$\mathbf{x}^\mu(\hat{\sigma}) = x^\mu(\sigma) + i\theta\psi^\mu(\sigma)$$

BSFT action



$$= \langle 0 | e^{-S_b(\mathcal{A})} |Bp\rangle$$

Coupling
to closed string



$$= \langle \varphi | e^{-S_b(\mathcal{A})} |Bp\rangle$$

For example, for a single non-BPS D9-brane, the BSFT action is calculated as

[Kutasov-Marino-Moore 2000,
Kraus-Larsen 2000,
Takayanagi-Terashima-Uesugi 2000,
Terashima-Uesugi 2001]

$$S[T, A_\mu] = -\sqrt{2}T_{\text{D9}} \int d^{10}x e^{-\frac{1}{4}T^2} \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})} \mathcal{F}(\mathcal{G}^{\mu\nu} \partial_\mu T \partial_\nu T) \\ + (\text{terms with } \partial_\mu F_{\nu\rho}, \partial_\mu \partial_\nu T)$$

where

$$\mathcal{G}^{\mu\nu} = \frac{1}{4\pi} \left(\frac{1}{1+F} \right)^{(\mu\nu)} \quad \mathcal{F}(x) = \frac{4^x x (\Gamma(x))^2}{2\Gamma(2x)} = 1 + 2(\log 2)x + \mathcal{O}(x^2)$$

When $T=0$, it reduces to DBI action.

The matrix theories we mainly consider in this talk are

Type IIA D(-1) theory

$$S \sim \text{Tr} \left(e^{-T^2} \left(1 + [\Phi^\mu, \Phi^\nu]^2 + [\Phi^\mu, T]^2 + \dots \right) \right)$$

Type IIB D(-1)- $\overline{\text{D}(-1)}$ theory

$$S \sim \text{Tr} \left(e^{-T^\dagger T} \left(1 + [\Phi^\mu, \Phi^\nu]^2 + (T\Phi^\mu - \overline{\Phi}^\mu T)(\Phi^\mu T^\dagger - T^\dagger \overline{\Phi}^\mu) + \dots \right) \right) \\ + \text{Tr} \left(e^{-TT^\dagger} \left(1 + [\overline{\Phi}^\mu, \overline{\Phi}^\nu]^2 + (T^\dagger \overline{\Phi}^\mu - \Phi^\mu T^\dagger)(\overline{\Phi}^\mu T - T\Phi^\mu) + \dots \right) \right)$$

4. D-brane solution

[Terashima 2001]

Chern-Simons term for non-BPS D(-1) : [Takayanagi-Terashima-Uesugi 2000]

$$S_{\text{CS}} \sim \text{Tr}([T, \Phi^\mu] e^{-T^2} C_\mu) + \dots$$

RR 1-form

$$[T, \Phi^0] \neq 0 \Rightarrow \text{D0-brane charge}$$

A Dp-brane is obtained by

$$T = u \sum_{\alpha=0}^p \gamma^\alpha \hat{p}_\alpha, \quad \Phi^\alpha = \hat{x}^\alpha, \quad \Phi^i = 0.$$

($\alpha = 0, \dots, p$), ($i = p+1, \dots, 9$)

$$[\hat{x}^\alpha, \hat{p}_\beta] = i\delta_\beta^\alpha \quad \gamma^\alpha : SO(p+1) \text{ } \Gamma\text{-matrix}$$

The BSFT action for this configuration can be calculated exactly.

The Dp-brane tension and RR-charge are reproduced in $u \rightarrow \infty$ limit

5. D-brane action

Consider the following ansatz

$$T = u \sum_{\alpha=0}^p \gamma^\alpha (\hat{p}_\alpha - i A_\alpha(\hat{x})) , \quad \Phi^\alpha = \hat{x}^\alpha , \quad \Phi^i = \phi^i(\hat{x}) .$$

One can show

$$e^{-S_b(\Phi^\mu, T)} |B(-1)\rangle \rightarrow e^{-S_b(A_\alpha, \phi^i)} |Bp\rangle$$

in the $u \rightarrow \infty$ limit

... (★)

This implies

$$S[\Phi^i, T] = \langle 0 | e^{-S_b(\Phi^\mu, T)} |B(-1)\rangle \xrightarrow{u \rightarrow \infty} \langle 0 | e^{-S_b(A_\alpha, \phi^i)} |Bp\rangle = S_{Dp}[A_\alpha, \phi^i] !$$

Matrix theory action

$u \rightarrow \infty$

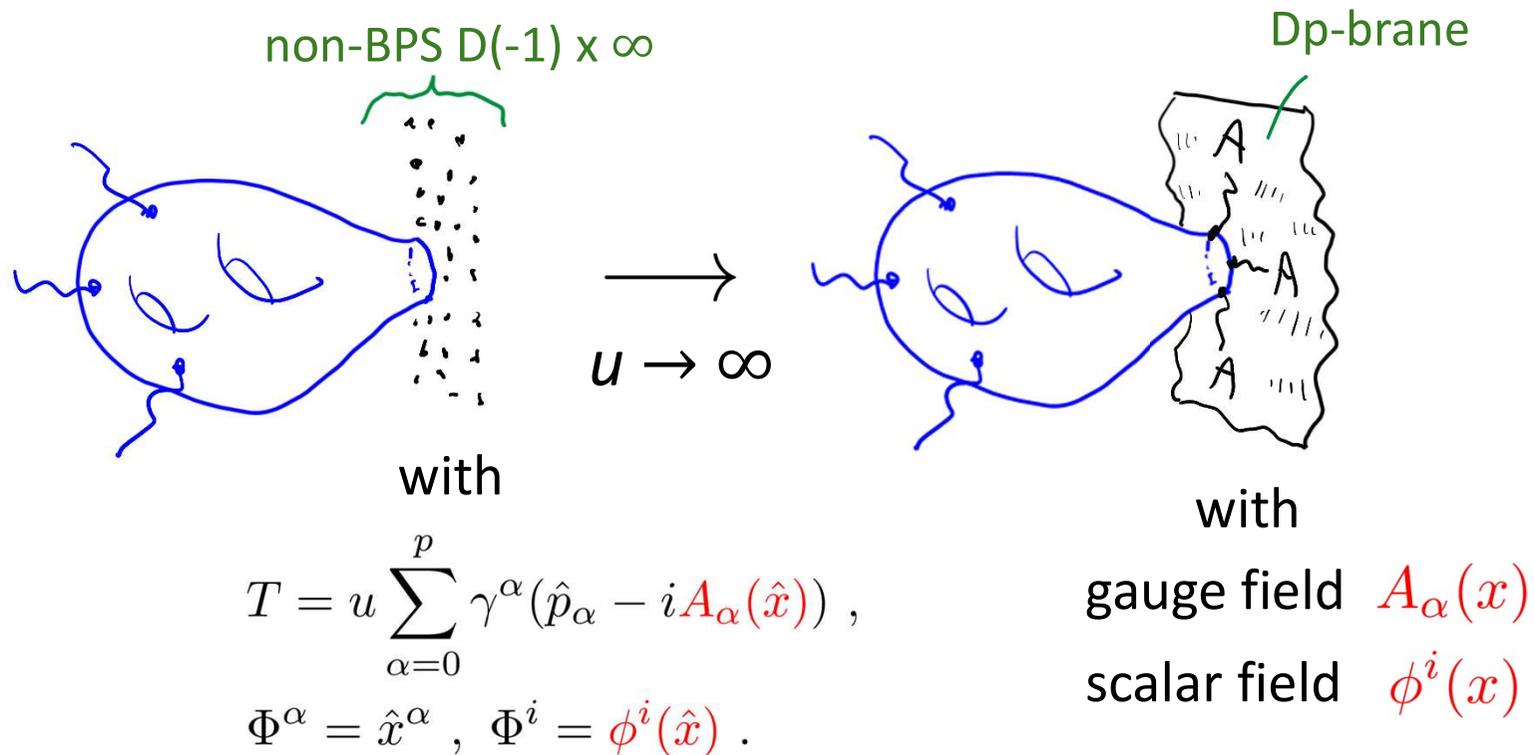
Dp-brane action

Moreover,

for any closed string state $|\varphi\rangle$,

$$\langle\varphi|e^{-S_b(\Phi^\mu, T)}|B(-1)\rangle \rightarrow \langle\varphi|e^{-S_b(A_\alpha, \phi^i)}|Bp\rangle$$

$$u \rightarrow \infty$$



A simple derivation of (★)

$$e^{-S_b(\Phi^\mu, T)} |B(-1)\rangle \rightarrow e^{-S_b(A_\alpha, \phi^i)} |Bp\rangle$$

in the $u \rightarrow \infty$ limit

...(★)

••

$$\begin{aligned}
 & e^{-S_b(\Phi^\mu, T)} |B(-1)\rangle \\
 = & \text{Tr} \hat{P} \exp \left[\int d\hat{\sigma} \begin{pmatrix} -i\Phi^\mu \mathbf{P}_\mu & T \\ T & -i\Phi^\mu \mathbf{P}_\mu \end{pmatrix} \right] |x^\mu = 0\rangle \\
 = & \text{Tr} \hat{P} \exp \left[\int d\hat{\sigma} (-i(\hat{x}^\alpha \mathbf{P}_\alpha + \phi^i(\hat{x}) \mathbf{P}_i) + u(\hat{p}_\alpha - iA_\alpha(\hat{x}))\Gamma^\alpha) \right] |x^\mu = 0\rangle \\
 = & \int D\Gamma^\alpha D\mathbf{x}^\alpha D\mathbf{p}_\alpha \exp \left[\int d\hat{\sigma} \left(\frac{1}{4}\Gamma^\alpha D\Gamma^\alpha + i\mathbf{p}_\alpha D\mathbf{x}^\alpha - i\mathbf{x}^\alpha \mathbf{P}_\alpha - i\phi^i(\mathbf{x})\mathbf{P}_i + u(\mathbf{p}_\alpha - iA_\alpha(\mathbf{x}))\Gamma^\alpha \right) \right] |x^\mu = 0\rangle \\
 = & \int D\Gamma^\alpha D\mathbf{x}^\alpha D\mathbf{p}_\alpha \exp \left[\int d\hat{\sigma} \left(\frac{1}{4}\Gamma^\alpha D\Gamma^\alpha + i\mathbf{p}_\alpha D\mathbf{x}^\alpha - i\phi^i(\mathbf{x})\mathbf{P}_i + u(\mathbf{p}_\alpha - iA_\alpha(\mathbf{x}))\Gamma^\alpha \right) \right] |x^\alpha, x^i = 0\rangle \\
 = & \int D\mathbf{x}^\alpha \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} D\mathbf{x}^\alpha D^2 \mathbf{x}^\alpha - i\phi^i(\mathbf{x})\mathbf{P}_i + iA_\alpha(\mathbf{x})D\mathbf{x}^\alpha \right) \right] |x^\alpha, x^i = 0\rangle \\
 \rightarrow & \underbrace{\exp \left[\int d\hat{\sigma} (-i\phi^i(\mathbf{X})\mathbf{P}_i + iA_\alpha(\mathbf{X})D\mathbf{X}^\alpha) \right]}_{e^{-S_b(A_\alpha, \phi^i)}} \underbrace{\int D\mathbf{x}^\alpha}_{|Bp\rangle} |x^\alpha, x^i = 0\rangle
 \end{aligned}$$

$T = u \sum_{\alpha=0}^p \gamma^\alpha (\hat{p}_\alpha - iA_\alpha(\hat{x}))$,
 $\Phi^\alpha = \hat{x}^\alpha$, $\Phi^i = \phi^i(\hat{x})$. $\Gamma^\alpha = \begin{pmatrix} & \gamma^\alpha \\ \gamma^\alpha & \end{pmatrix}$
 Path integral
 $iD\mathbf{x}^\alpha + u\Gamma^\alpha = 0$

6. K-homology

$$K_1(X) := \{(\mathcal{H}, \phi, T)\} / \sim$$

- \mathcal{H} is a separable Hilbert space
- $\phi : C_0(X) \rightarrow \mathbf{B}(\mathcal{H})$ homomorphism
- $T \in \mathbf{B}(\mathcal{H}), T = T^\dagger,$
 $T^2 - 1 \in \mathbf{K}(\mathcal{H}), [T, \phi(a)] \in \mathbf{K}(\mathcal{H})$ for $\forall a \in C_0(X)$

$C_0(X)$: continuous complex functions on X (that vanish at infinity)

$\mathbf{B}(\mathcal{H})$: bounded operators on \mathcal{H} ($T \in \mathbf{B}(\mathcal{H}) \Leftrightarrow \exists C > 0$ s.t. $\|Tv\| \leq C\|v\|$ for $\forall v \in \mathcal{H}$)

$\mathbf{K}(\mathcal{H})$: compact operator on \mathcal{H} (operators obtained as a limit of finite rank operators)

Interpretation

T is the tachyon operator on non-BPS D(-1) normalized so that the minimum of the potential is $|T|=1$.

ϕ gives a configuration of the scalar operators by

$$\phi : f(x^\mu) \mapsto f(\Phi^\mu)$$

$T^2 - 1 \in \mathbf{K}(\mathcal{H}) \Rightarrow$ non-BPS D(-1) are mostly annihilated.

$[T, \phi(a)] \in \mathbf{K}(\mathcal{H}) \Rightarrow [T, \Phi^\mu]$ term in the action is not too large

\sim : addition of annihilated non-BPS D(-1) etc.

$$K_0(X) := \{(\mathcal{H}, \overline{\mathcal{H}}, \phi, \overline{\phi}, T)\} / \sim$$

- $\mathcal{H}, \overline{\mathcal{H}}$ are separable Hilbert spaces \Rightarrow associated with $D(-1)-\overline{D(-1)}$
- $\phi : C_0(X) \rightarrow \mathbf{B}(\mathcal{H})$ homomorphism \Rightarrow configuration of Φ^μ
- $\overline{\phi} : C_0(X) \rightarrow \mathbf{B}(\overline{\mathcal{H}})$ homomorphism \Rightarrow configuration of $\overline{\Phi}^\mu$
- $T \in \mathbf{B}(\mathcal{H}, \overline{\mathcal{H}})$, \Rightarrow tachyon
 $T^\dagger T - 1 \in \mathbf{K}(\mathcal{H}), TT^\dagger - 1 \in \mathbf{K}(\overline{\mathcal{H}}),$

$\Rightarrow D(-1)-\overline{D(-1)}$ pairs are mostly annihilated.

$$T\phi(a) - \overline{\phi}(a)T \in \mathbf{K}(\mathcal{H}, \overline{\mathcal{H}}) \text{ for } \forall a \in C_0(X)$$

$\Rightarrow T\Phi^\mu - \overline{\Phi}^\mu T$ is not too large.

\sim : addition of annihilated $D(-1)-\overline{D(-1)}$ pairs etc.

There is a duality analogous to the Poincare duality:

$$K_i(X) \simeq K^{n-i}(X) \quad (i = 0, 1 \pmod{2})$$

K-homology K-theory

for a compact n-dim manifold X.

⇒ consistent with the K-theory classification of D-brane charges

7. Application to index theorem

Consider type IIB $D(-1)-\overline{D(-1)}$ matrix theory

$$\langle \varphi | e^{-S_b(\Phi^\mu, T)} | B(-1) \rangle \xrightarrow{u \rightarrow \infty} \langle \varphi | e^{-S_b(A_\alpha, \phi^i)} | Bp \rangle$$

$$T = u \sum_{\alpha=0}^p \gamma^\alpha (\hat{p}_\alpha - i A_\alpha(\hat{x})),$$

$$\Phi^\alpha = \bar{\Phi}^\alpha = \hat{x}^\alpha, \quad \Phi^i = \bar{\Phi}^i = \phi^i(\hat{x}).$$

Choose φ to be the RR 0-form field C_0 (with $k^\mu=0$) \Rightarrow D(-1) charge

One can show

$$\langle C_0 | e^{-S_b(\Phi^\mu, T)} | B(-1) \rangle_{\text{RR}} = C_0 \left(\text{Tr}(e^{-TT^\dagger}) - \text{Tr}(e^{-T^\dagger T}) \right) = C_0 \text{Index } \mathcal{D}$$

$$\langle \varphi | e^{-S_b(A_\alpha, \phi^i)} | Bp \rangle_{\text{RR}} = C_0 \int_{Dp} \text{Tr} e^{F/2\pi}$$



Index theorem !

8. Summary and discussion

- We considered matrix theories based on non-BPS D0 or D(-1) systems.
- Arbitrary D-brane configurations can be constructed.
- D-brane action is obtained
- We found a nice relation

$$T = u \sum_{\alpha=0}^p \gamma^\alpha (\hat{p}_\alpha - i A_\alpha(\hat{x})), \quad \Phi^\alpha = \hat{x}^\alpha, \quad \Phi^i = \phi^i(\hat{x}).$$

$$e^{-S_b(\Phi^\mu, T)} |B(-1)\rangle \rightarrow e^{-S_b(A_\alpha, \phi^i)} |Bp\rangle \quad (u \rightarrow \infty)$$

- D-brane configurations are classified by K-homology

- We have only considered classical configurations. It would be interesting to consider loop effect to see how gravity appears in the model.
- By looking at the coupling to RR 0-form, we find Atiyah-Singer index theorem.
But, the relation

$$\langle \varphi | e^{-S_b(\Phi^\mu, T)} | B(-1) \rangle \rightarrow \langle \varphi | e^{-S_b(A_\alpha, \phi^i)} | Bp \rangle \quad (u \rightarrow \infty)$$

holds for any closed string state $|\varphi\rangle$.

Can we find more interesting mathematical relations?

- In type IIA, we can have arbitrary number of D0-branes, which means arbitrary number of momentum along the 11th direction in M-theory. Is there any interesting application for the construction of M-theory?

- Speculative observation:

Type IIA non-BPS D(-1) contains

T, Φ^μ ($\mu = 0 \sim 9$) \Rightarrow 11 adjoint scalars

ψ_L, ψ_R \Rightarrow 32 component spinors

Any hints towards 11 dim theory?

Thank you!