Overview of Matrix Models

1. Motivation

"textbook" str. theory ... perturbative in g_s

amplitude $\mathcal{A}^{\text{str}} = \sum_{\chi = 2, 0, -2, \cdots} g_s^{2-\chi} \mathcal{A}_\chi^{\text{str}}$

... no non-pert. effect

(branes, transitions b/w states, etc.)
"theories"

⇝ Matrix model as a non-perturbative formulation

2. Matrix models

$$S = \begin{cases} \mathrm{tr}\, \mathcal{L}[X] & \cdots \text{0D MM.} \\ \int dt\, \mathrm{tr}\, \mathcal{L}[X(t)] & \cdots \text{1D MM.} \end{cases}$$

	0D MM	1D MM.
bos.	One-Mat. model Two-Mat. model ⋮	c = 1 Mat. model
super	IKKT / Polarised IKKT K-Mat. theory	BFSS / BMN BD

Conjectures:

- The one-mat. model (DT) ⟷ $(2, q)$ minimal str. theory
  $q = 3$: 2D pure grav. (1D crit. bos. str.)
- IKKT ⟷ (10D) Type IIB string theory
- BFSS ⟷ M-theory

3. Ideas of the Mat. Models

- Quantum (Loop) corrections

$$\mathcal{A}^{str} = \sum_{h=0}^{\infty} g_s^{2h} \mathcal{A}^{str}_{2-2h}$$

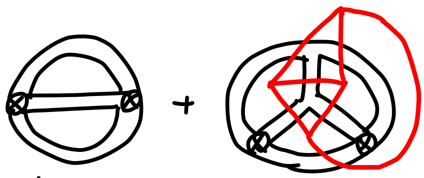
$$\updownarrow$$

$$\mathcal{A}^{MM}_{(N,\lambda)} = \sum_{h=0}^{\infty} N^{-2h} \mathcal{A}^{MM}_{2-2h}(\lambda)$$

E.g. One-mat. model $S = N\,\mathrm{tr}\left[\frac{1}{2}M^2 + \frac{\lambda}{3}M^3\right]$

propagator $\,{}^{i}_{j}\!=\!{}^{l}_{k} = \frac{1}{N}\delta^i_l\,\delta^k_j$ ... ribbon

vertex  $= N\lambda$

$\langle \mathrm{tr}M^3\,\mathrm{tr}M^3\rangle =$  $+ \cdots$

$\frac{1}{N^3}\cdot N^3 = 1$, $\frac{1}{N^6}\cdot(N\lambda)^2\cdot N^4 = \lambda^2$

$$\rightsquigarrow \mathcal{A}^{MM} = \frac{1}{N^2}\sum N^{-I+V+L}\lambda^{V-n}$$

I : # of propagators $\longleftrightarrow$ $\tilde{E}$ edges

V : # of vertices $\longleftrightarrow$ $\tilde{F}$ faces

L : # of index loops $\longleftrightarrow$ $\tilde{V}$ vertices

n : # of insertions

Dual diagrams

 . . .

$$\therefore A^{MM} = \frac{1}{N^2}\sum N^{-\tilde{E}+\tilde{F}+\tilde{V}}\lambda^{\tilde{F}-n} = \sum N^{-2h}\lambda^{\tilde{F}-n}$$

$$= \sum_{h=0}^{\infty} N^{-2h}\mathcal{A}^{MM}_{2-2h}(\lambda)$$

$\downarrow$ ∞ as $\lambda \to \lambda_c$

Singularity at $\lambda = \lambda_c$.

str. susceptibility

$$\Gamma_s = \frac{-2}{p+q-1} = -\frac{1}{2}$$

μ : cosmo. const.

$\rightsquigarrow$ Double Scaling lim. $N \to \infty$

$$N^{-2}(\lambda-\lambda_c)^{-(2-\Gamma_s)} = g_s^2\,\mu^{-(2-\Gamma_s)}$$

- Many-body system of str. theory objects

  Matrices can contain geometric info.

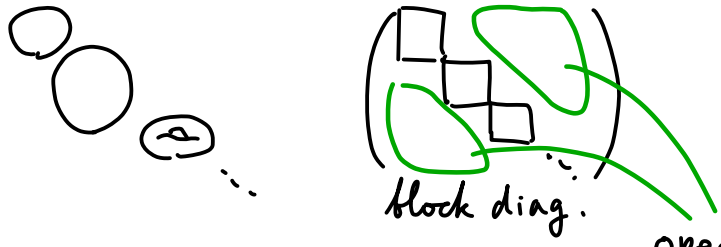
  eg. fuzzy sphere

  $[L_a, L_b] = i\varepsilon_{abc}L_c$ ⋯ SO(3)

  spin-ℓ irrep. $(N=2\ell+1)$

  $X_a = \frac{2r}{\sqrt{N^2-1}} L_a \rightsquigarrow X_1^2+X_2^2+X_3^2 = r^2\mathbb{1}$ ⋯ S^2

  branes $\longleftrightarrow$ classical solution



open str. b/w branes

- Low dimensionality

  Matrices can contain more info. than just geometric one.

- Computability

  "Textbook" str. theory is defined by asymptotic exp.

  String theory should be formulated by finite-valued quantities.

  $Z = \int_{N\times N \text{ mat.}} [dX]\, e^{-S[X]}$ ⋯ finite $\rightsquigarrow$ large-N lim.

4. M.M. for superstr. / M theory

A mat. model for a $D>2$ critical str. theory cannot be obtained by a simple extension of the bos. mat. models.

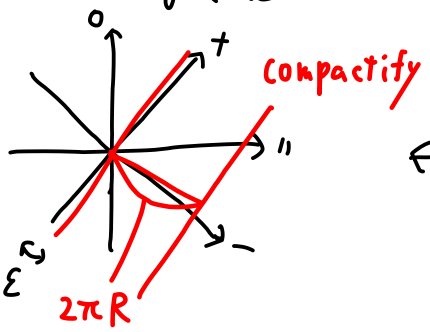
$\rightsquigarrow$ Matrix model from branes (new idea)

BFSS

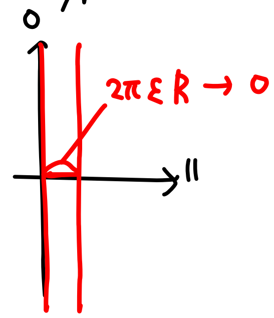
Let's formulate M-theory by using duality to IIA str.

DLCQ M theory (11D SUGRA) $\qquad$ 10D type IIA str.

$$\begin{pmatrix} x^- \cong x^- - 2\pi R + \varepsilon^2 \\ x^+ \cong x^+ + \varepsilon^2 \end{pmatrix}$$



$\longleftrightarrow$ Lorentz trf.



l_p : 11D Planck length

$l_p' = g_s^{1/3} l_s$, $\varepsilon R = g_s l_s$

Let $\dfrac{\varepsilon R^2}{l_p'^2} = \dfrac{R^2}{l_p^2}$: fixed

$\rightsquigarrow g_s \sim \varepsilon^{\frac{3}{4}} \to 0$

Weak coupl.

Finite theory only on D0s.

$$\begin{cases} g_{Dp}^2 = \dfrac{(2\pi l_s)^p l_s g_s}{(2\pi l_s^2)^2} \sim \varepsilon^{\frac{p}{4}} R^{p-3} \\ T_{F1}^{-1} \sim \varepsilon^{\frac{1}{2}} R^2 \to 0 \\ T_{NS5}^{-1} \sim \varepsilon^3 R^6 \to 0 \end{cases}$$

$p^+ = \dfrac{N}{R}$ (KK mom. of gravitons) $\longleftrightarrow$ D0 Energy

$p^- = \dfrac{R}{2N} p_i^2$

$$E \simeq p^{10} + \frac{p_i^2}{2p^{10}} \quad (i = 1, \cdots, 9)$$

$$= \frac{N}{\varepsilon R} + \frac{\varepsilon R}{2N} p_i^2$$

$\frac{N}{\varepsilon R} \to \infty$; $\frac{\varepsilon R}{2N} p_i^2$: lowest level open str. on D0s.

$\rightsquigarrow$ Conjecture

$S_{\text{non-rel D0}}$ (BFSS) describes (DLCQ) M-theory.

IKKT

- 0D M.M. could contain BFSS.
- Super membrane theory (M2) $\xrightarrow{\text{Mat. reg.}}$ BFSS

$\rightsquigarrow$ 10D IIB Superstr. $\xrightarrow{\text{Mat. reg.}}$ IKKT

What's special in 0D M.M.

- Difficulty in holography (state? energy? entropy?)
  ⇝ mass Ω

- Definition of path int. $Z = \int (dx) \exp[i S[X; \eta_{\mu\nu}]]$

$$S[X; G_{\mu\nu}] = N \mathrm{tr}\left[\frac{1}{4} G_{\mu\rho} G_{\nu\sigma} [X^\mu, X^\nu][X^\rho, X^\sigma] + \text{fermions}\right]$$

... There have been recent developments in both problems.