International College
OSAKA UNIVERSITY

Top Yukawa at Muon Collider

The Frontier of Particle Physics: Exploring Muon, Quantum Science and the Cosmos
Yukawa Institute for Theoretical Physics

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June 17, 2025

Based on

[1] Vernon Barger, Kaoru Hagiwara and YJZ, Phys.Lett.B 850 (2024) 138547.
[2] Morgan Cassidy, Zhongtian Dong, Kyoungchul Kong, Ian Lewis, Yanzhe Zhang and YJZ, JHEP05(2024) 176.
[3] Vernon Barger, Kaoru Hagiwara and YJZ, in preparation.

LHC searches and constraints

$$\mathcal{L}_{ttH} = -gH\bar{t}(\cos\xi + i\gamma_5 \sin\xi)t$$

g is real and positive, $-\pi < \xi < \pi$

When $g=g_{SM}=m_t/v$, $\xi=0$ ⟶ SM

- X. Zhang, S. K. Lee, K. Whisnant, and B. L. Young, "Phenomenology of a nonstandard top quark Yukawa coupling," Phys. Rev. D 50 (1994) 7042–7047, arXiv:hep-ph/9407259.
- H. Bahl, E. Fuchs, S. Heinemeyer, J. Katzy, M. Menen, K. Peters, M. Saimpert, and G. Weiglein, "Constraining the CP structure of Higgs-fermion couplings with a global LHC fit, the electron EDM and baryogenesis," Eur. Phys. J. C 82 (2022) no. 7, 604, arXiv:2202.11753 [hep-ph].
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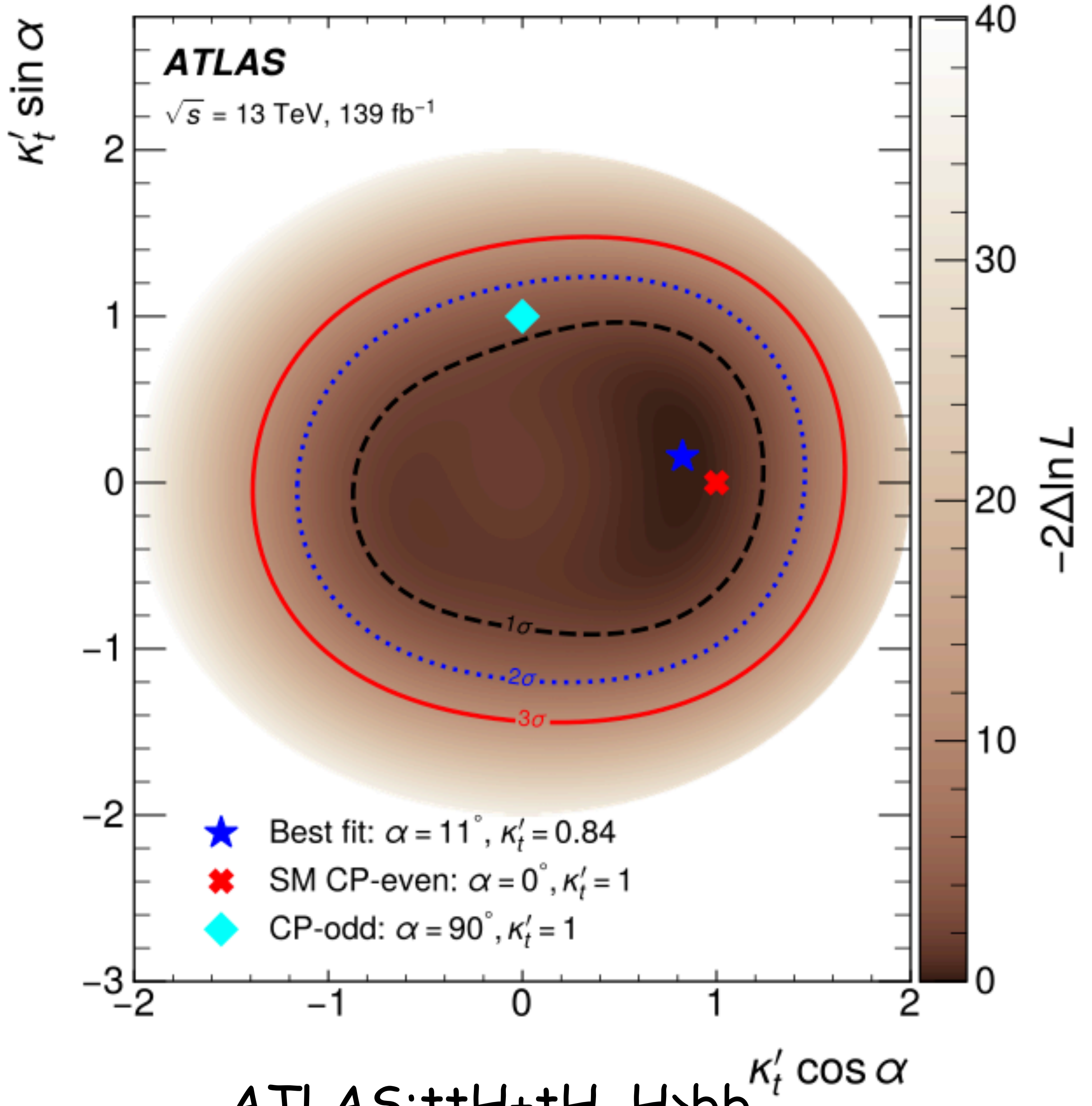
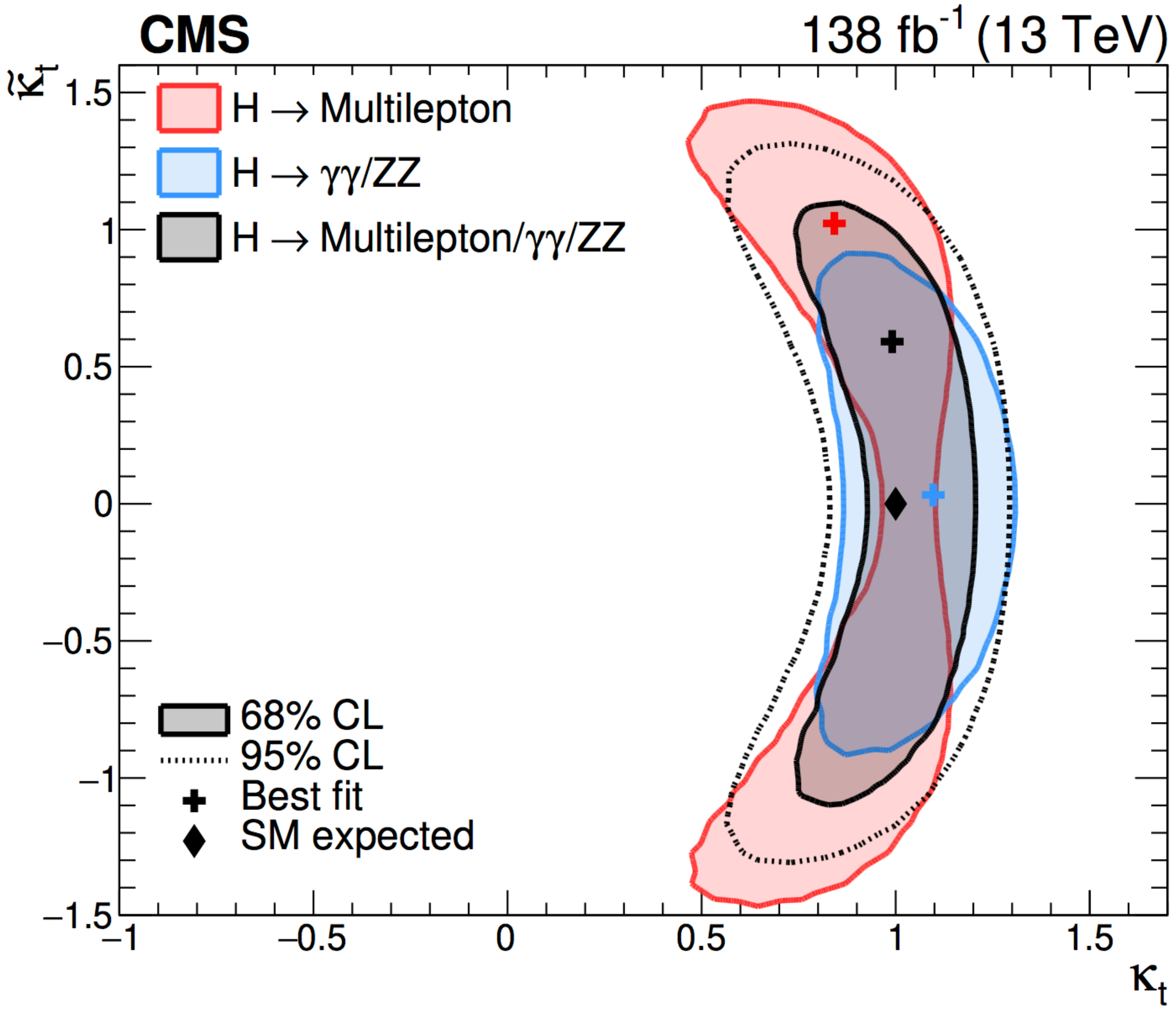


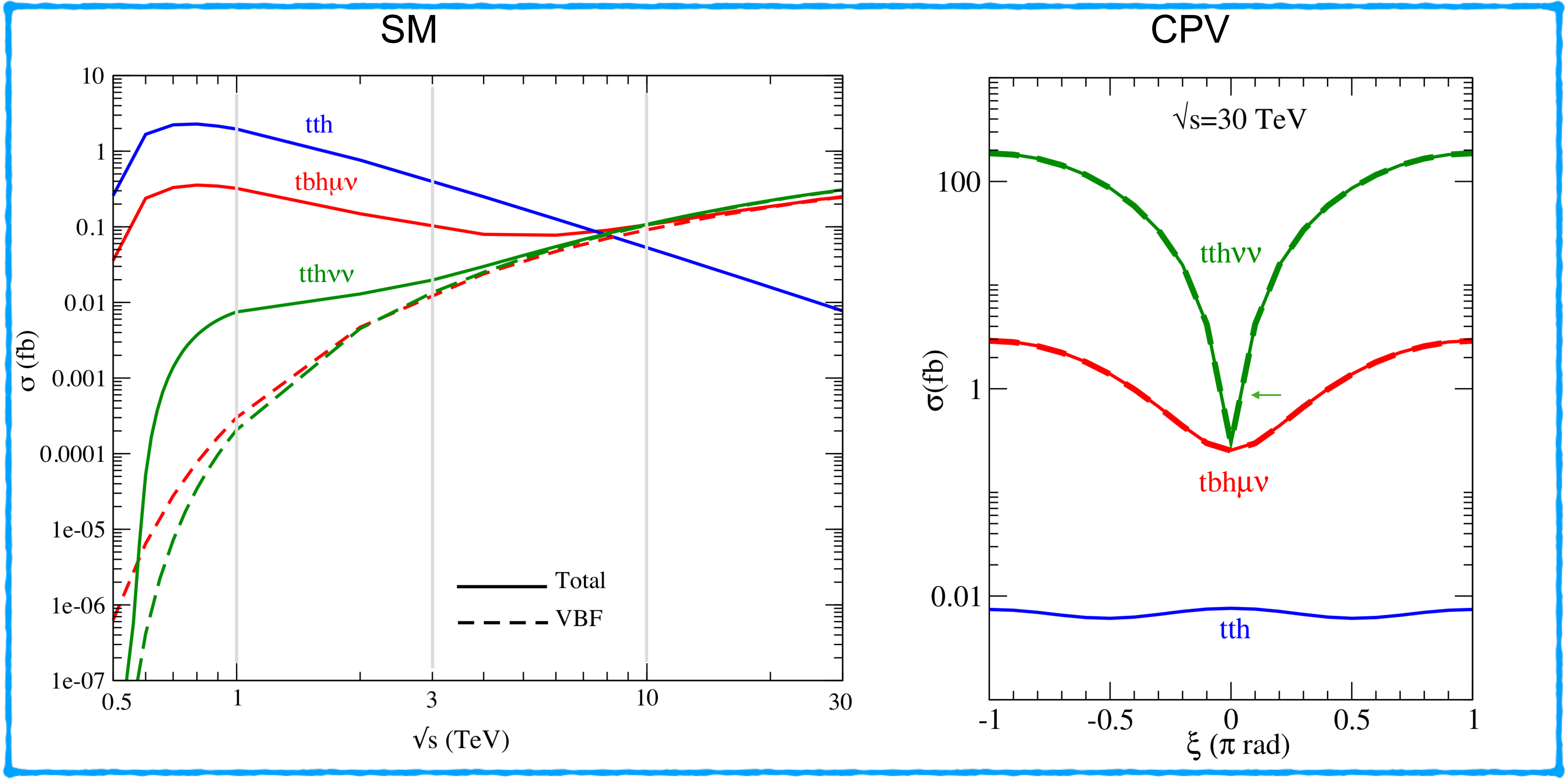
ATLAS:ttH+tH, H>bb
arXiv:2303.05974



CMS:ttH+tH, H>ls/rr/ZZ
JHEP07(2023)092

LHC direct searches:
ATLAS best fit:11+52-73 degree

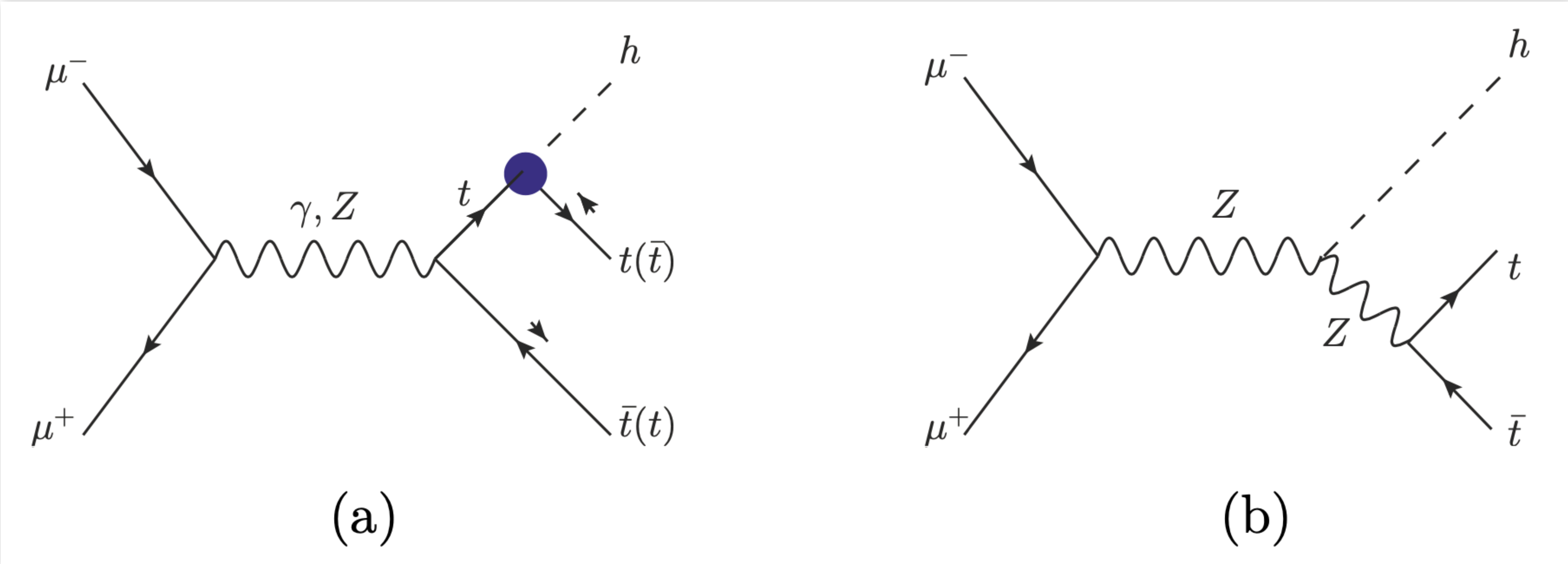
Top Yukawa processes at muon collider



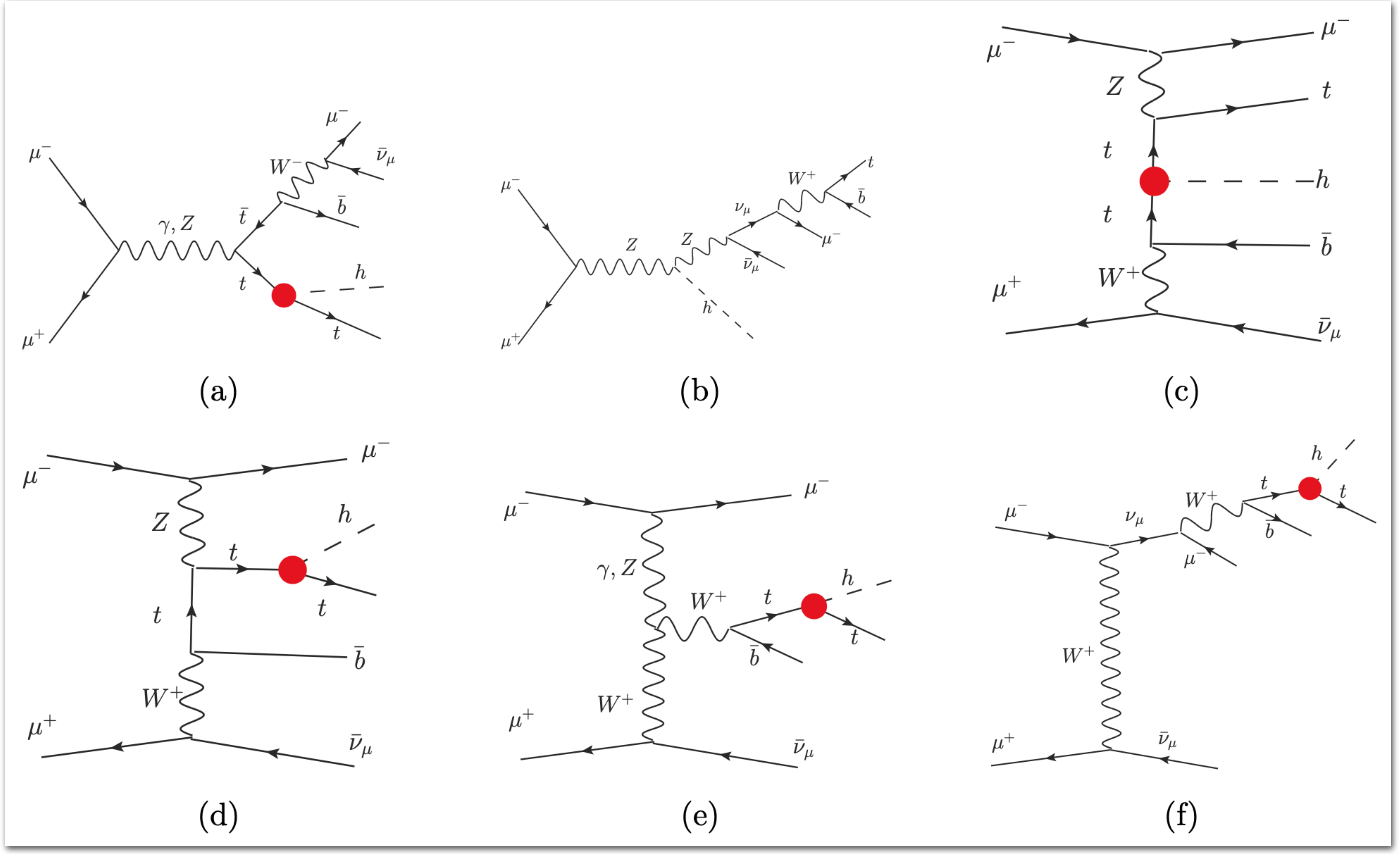
M.Cassidy, Z.Dong, KC.Kong, I. Lewis, Y.Zhang and YJZ, JHEP 05 (2024) 176.

Representative Feynman diagrams

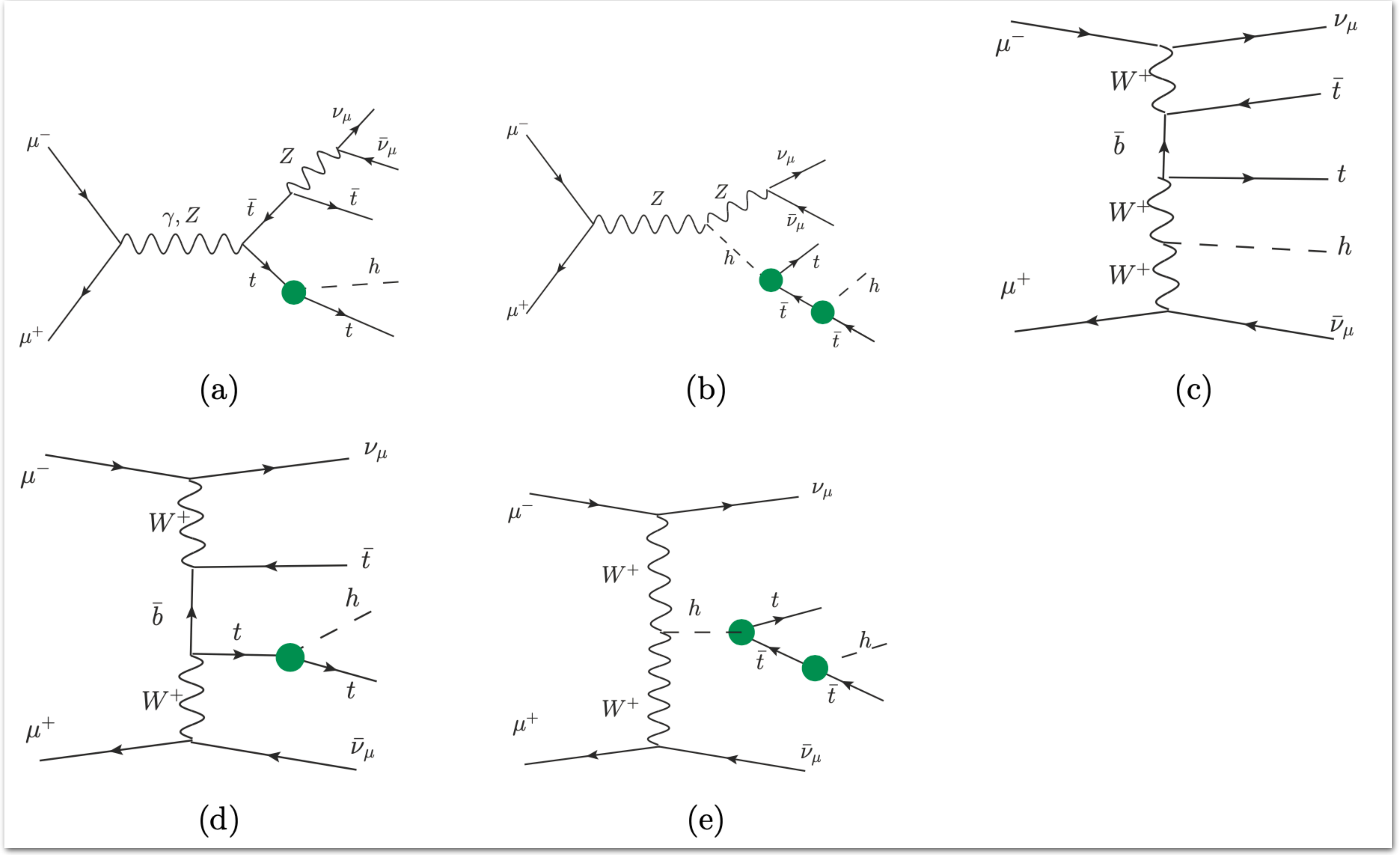
tth



tbhμν

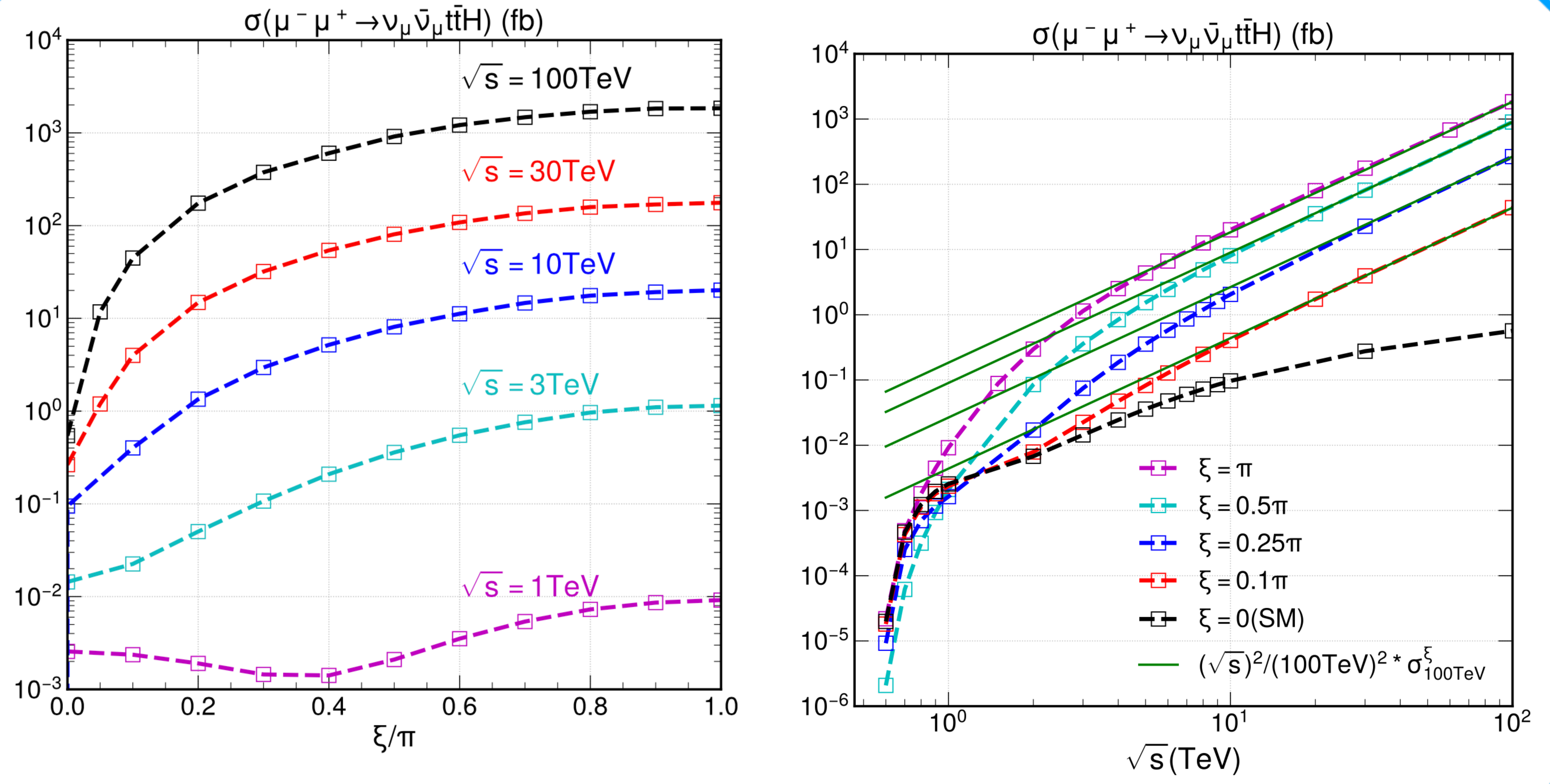


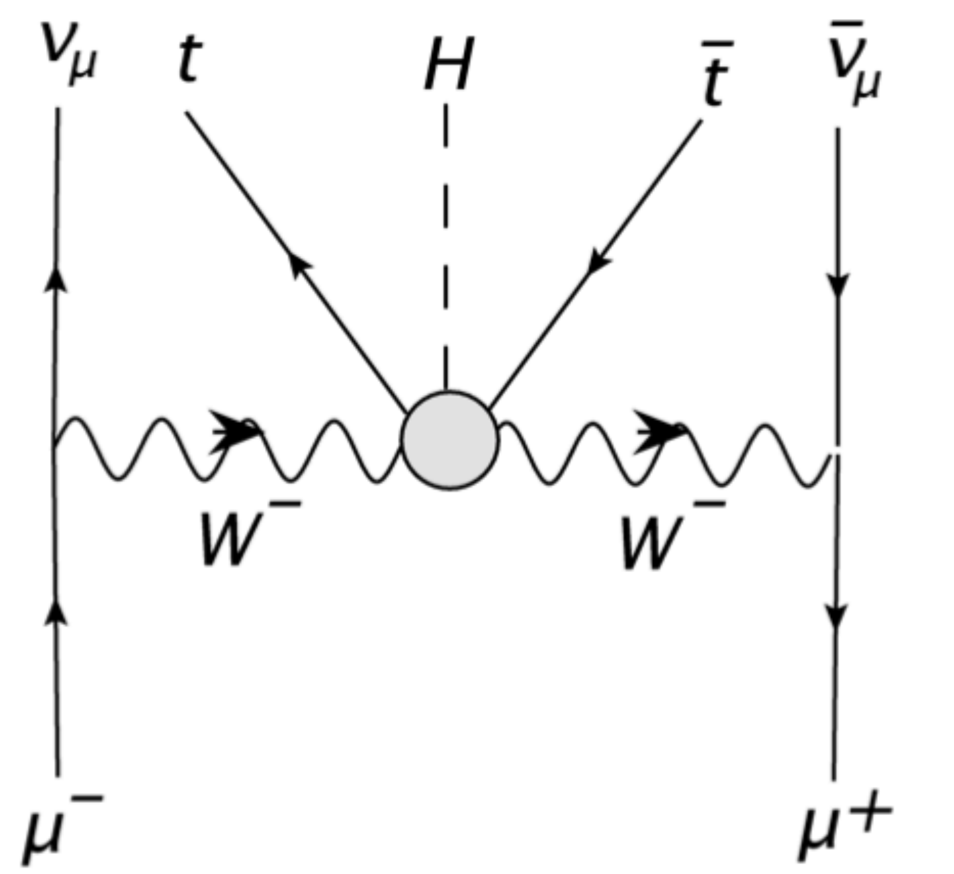
tthνν



M.Cassidy, Z.Dong, KC.Kong, I. Lewis, Y.Zhang and YJZ, JHEP 05 (2024) 176.

$\nu_\mu \bar{\nu}_\mu t\bar{t}H$ at future muon collider





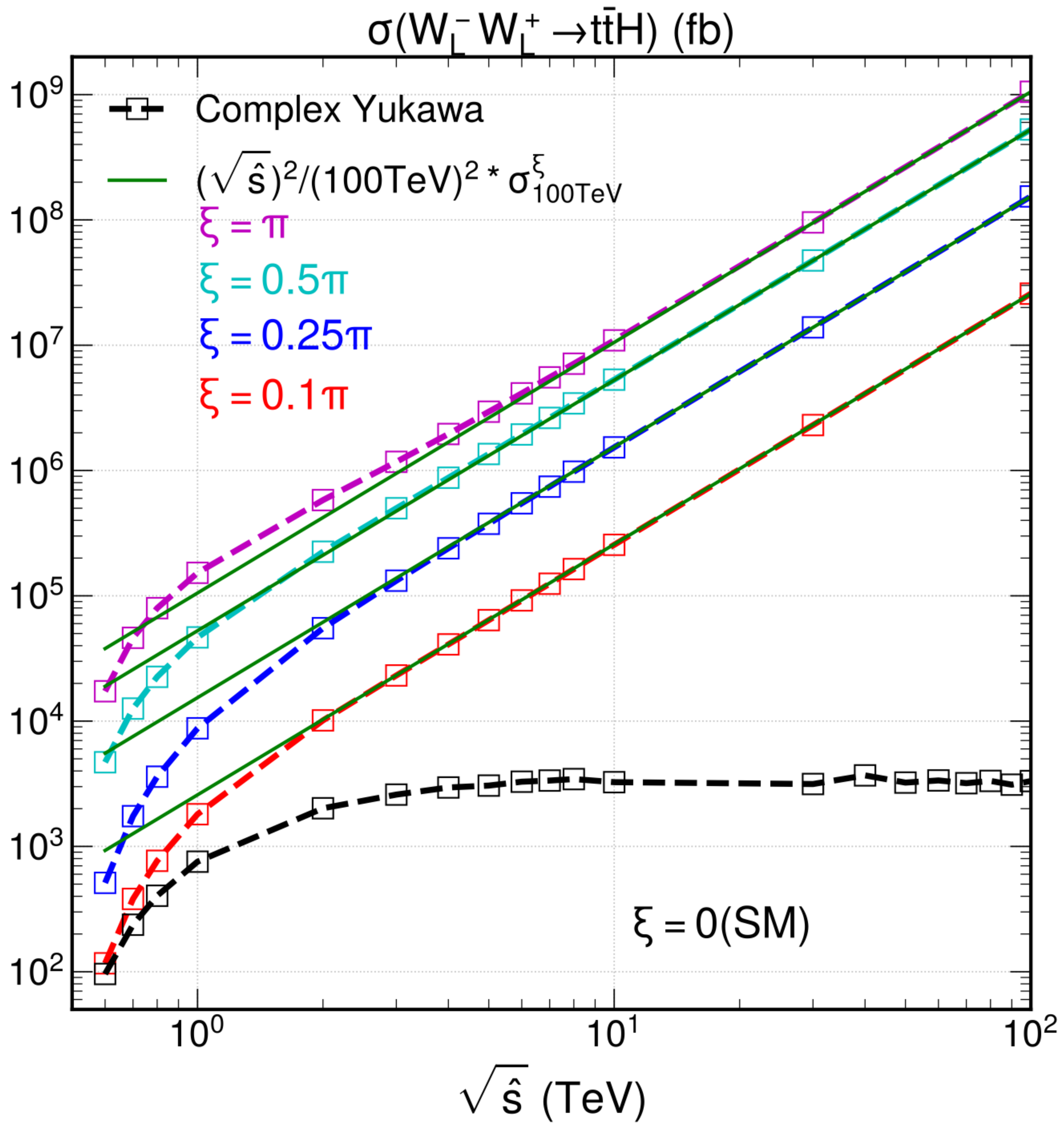
88 diagrams generated from Madgraph5, 20 diagrams are Vector Boson Fusions

ξ dependence:
at low energy: s-channel diagrams contribute.
at high energy: t-channel (VBF) dominates.

Energy dependence at high energy:
BSM: quadratic growth
SM: logarithmically growth

→ Vector Boson Fusion

Dominate sub-diagrams $W_L^- W_L^+ \to t\bar{t}H$



$$W^-(q, h=0)\, W^+(\bar{q}, \bar{h}=0) \to t\bar{t}H$$

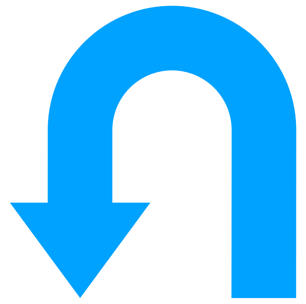
$\xi \neq 0$ **quadratic energy growth** from the longitudinally polarized weak boson wave functions **E/mW**.

$\xi = 0$ In the SM, **E/mW** from individual diagram **cancels** after summing up, leading to the **Goldstone boson equivalence theorem** (GBET) as a manifestation of gauge invariance.

$$\mathcal{L}_{ttH} = -gH\bar{t}(\cos\xi + i\gamma_5 \sin\xi)t$$

Gauge invariant formulation: Models like two Higgs doublet models,etc

Energy dependence at high energy:
BSM: quadratic growth from (**E/mW**)², with E=$\sqrt{\hat{s}}/2$
SM: constant



SMEFT

A gauge invariant top Yukawa sector

Dimension-6 operator

$$\mathcal{L} = -y_{\rm SM} Q^\dagger \phi t_R + \frac{\lambda}{\Lambda^2} Q^\dagger \phi t_R \left(\phi^\dagger \phi - \frac{v^2}{2}\right) + \text{h.c.}$$

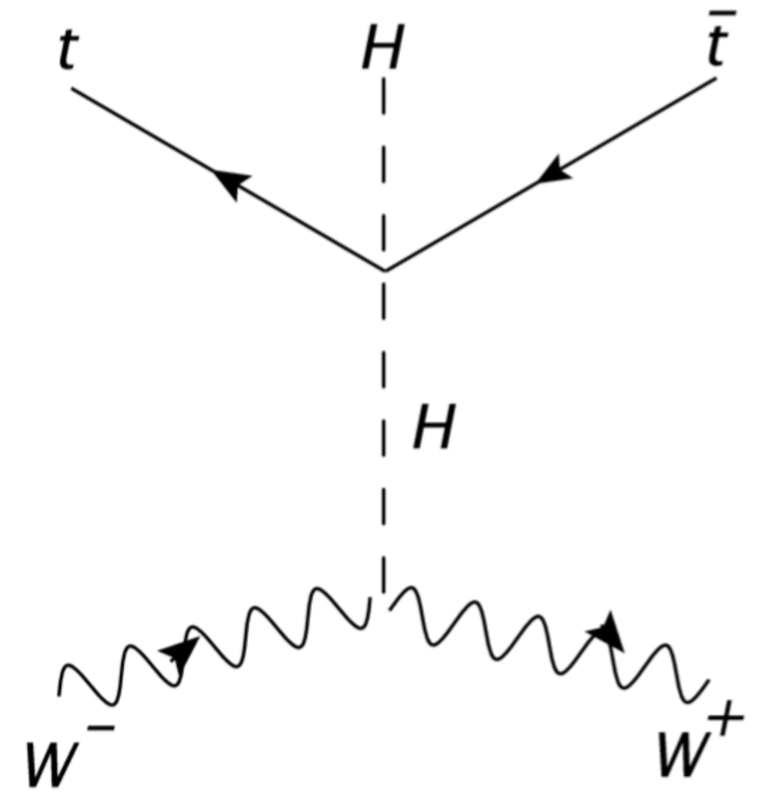
$$Q = (t_L, b_L)^T$$

$$\phi = ((v + H + i\pi^0)/\sqrt{2}, i\pi^-)^T$$

$$\begin{aligned}\mathcal{L}_{ttH}^{\rm SMEFT} = & -m_t t_L^\dagger t_R - g_{\rm SM}\left[(H + i\pi^0)t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger\right] t_R \\ & -(g e^{i\xi} - g_{\rm SM})\left\{H t_L^\dagger t_R + \frac{H}{v}\left[(H + i\pi^0)t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger\right] t_R\right\} \\ & -(g e^{i\xi} - g_{\rm SM})\left\{\left[\frac{H^2 + (\pi^0)^2}{2v} + \frac{\pi^+\pi^-}{v}\right] t_L^\dagger t_R \right. \\ & \left. + \frac{H^2 + (\pi^0)^2 + 2\pi^+\pi^-}{2v^2}\left[(H + i\pi^0)t_L^\dagger + i\sqrt{2}\pi^- b_L^\dagger\right] t_R\right\} + h.c.,\end{aligned}$$

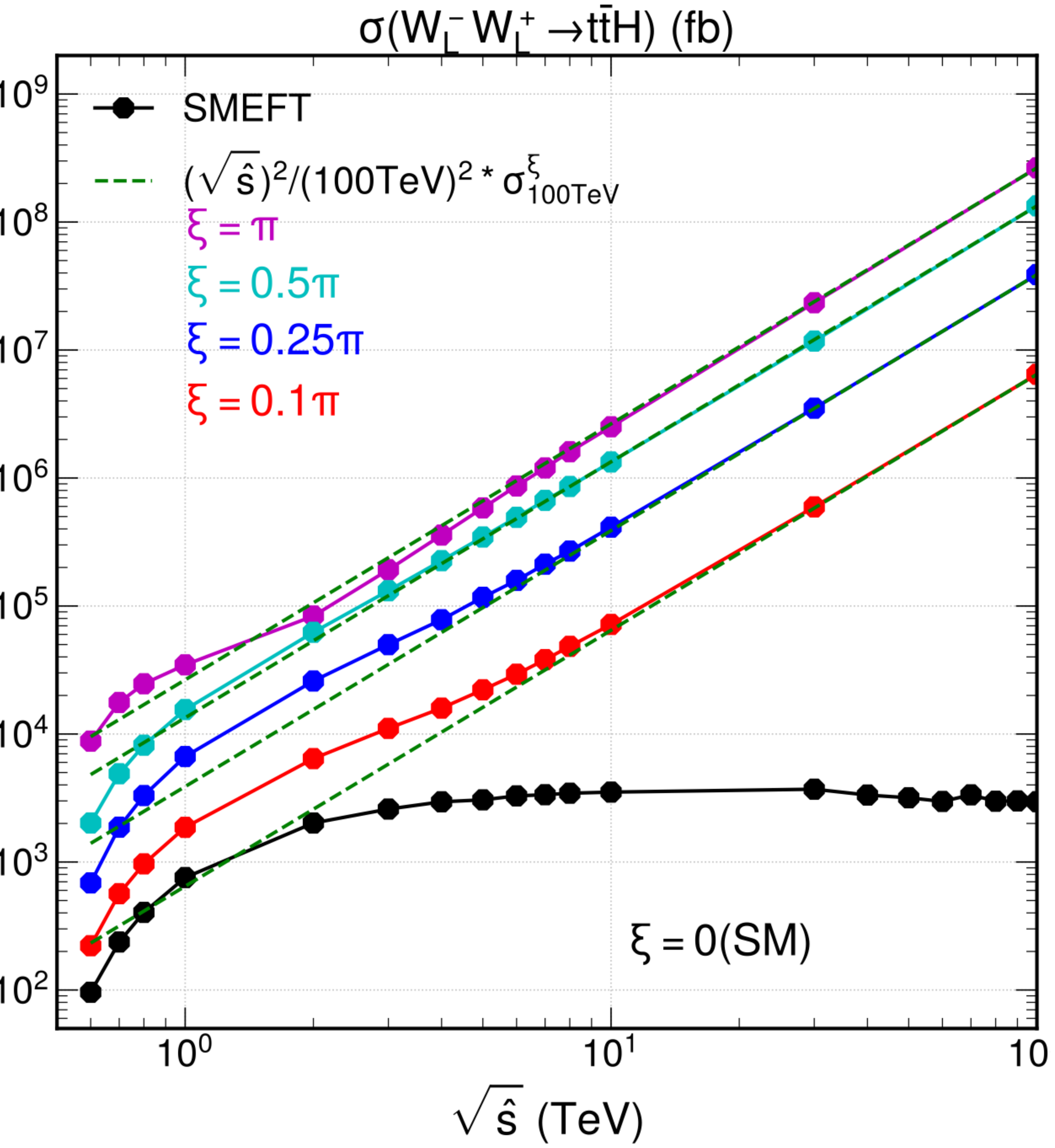
$$g_{\rm SM} = \frac{y_{\rm SM}}{\sqrt{2}} = \frac{m_t}{v} \qquad \frac{g_{\rm SM} - g e^{i\xi}}{v^2} = \frac{\lambda}{\Lambda^2}$$

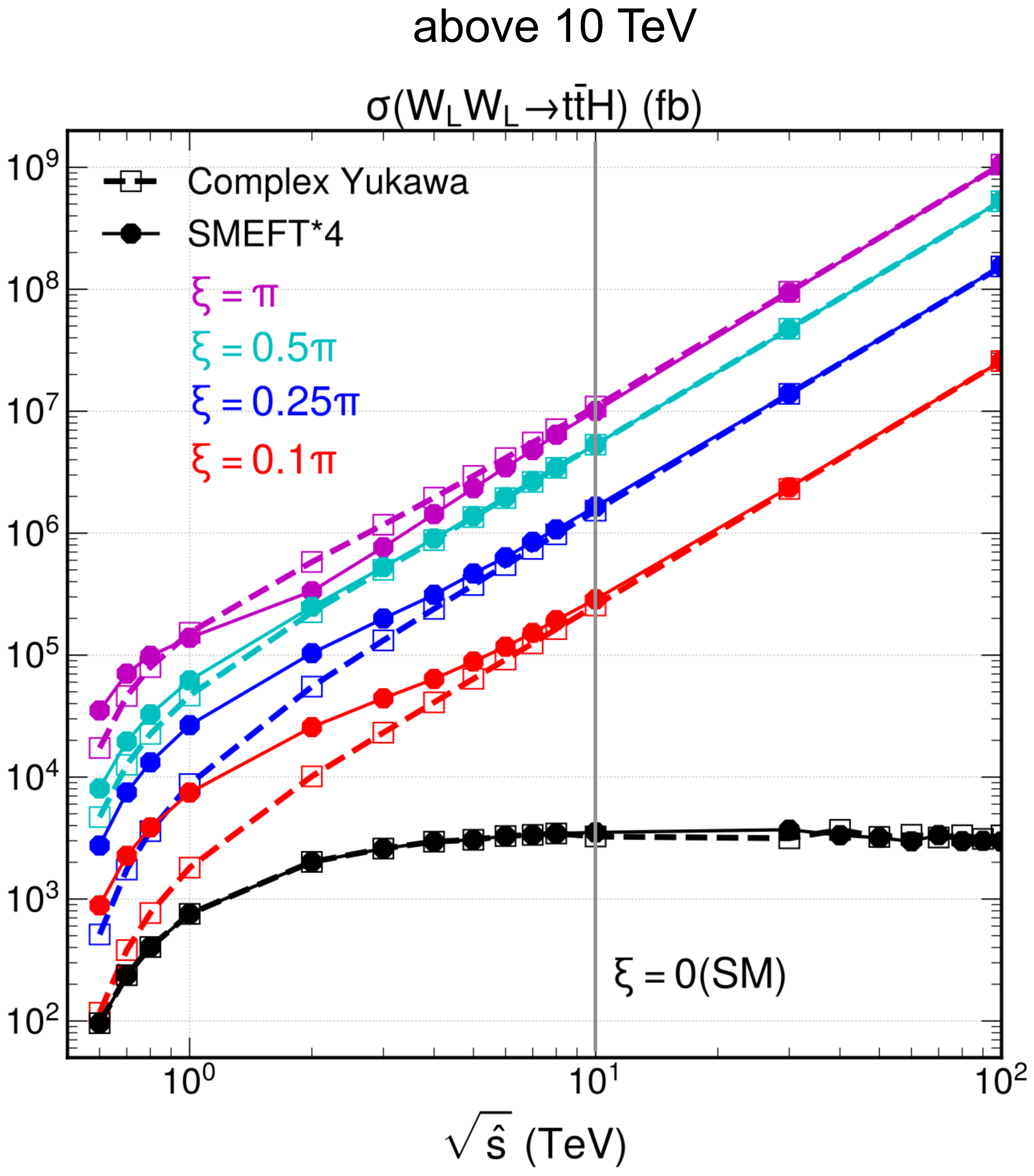
Additional ttHH and ttHHH coupling

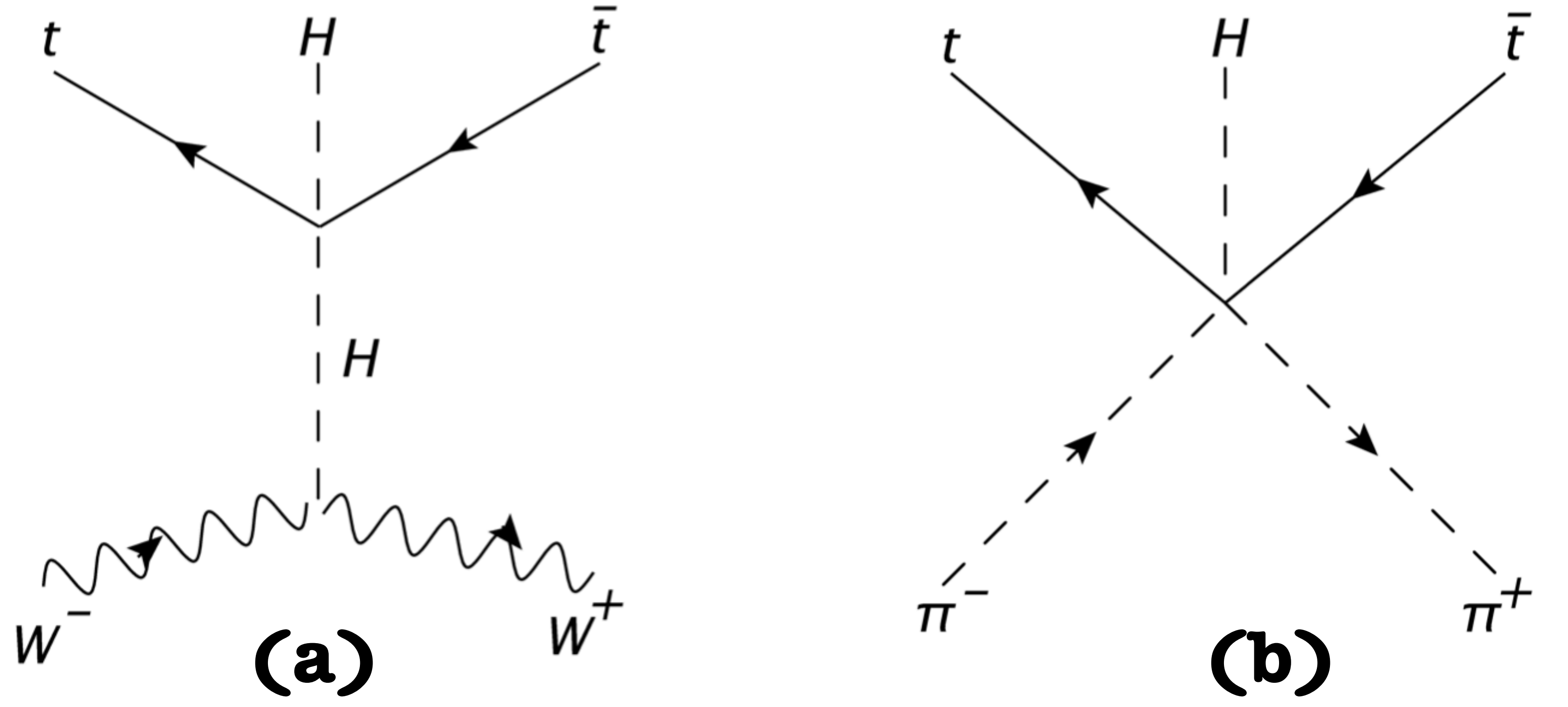


$$\mathcal{L}_{ttHH}^{\rm SMEFT} = \frac{3(g_{\rm SM} - g e^{i\xi})}{v}\frac{H^2}{2} t_L^\dagger t_R + \text{h.c.}$$

$$W_L^- W_L^+ \to t\bar{t}H$$







Complete amplitude in SMEFT:

$$\mathcal{M}(W_L^- W_L^+ \to t\bar{t}H)_{\rm SMEFT} = \sum_{k=1}^{20} \mathcal{M}_k + \mathcal{M}_{\rm Fig.\,\mathbf{a}}$$

$$\mathcal{M}^{\pm\pm}_{\rm Fig\,\mathbf{b}} = \frac{1}{v^2}\left[\mp 2p_t\,(g_{\rm SM} - g\cos\xi) - i m_{tt}\,(g\sin\xi)\right]$$

$$\mathcal{M}^{\pm\pm}_{\rm Fig.\,\mathbf{a}} = \frac{3}{v^2}\left[\mp 2p_t(g_{\rm SM} - g\cos\xi) - i m_{tt}(g\sin\xi)\right]\frac{(\hat{s} - 2m_W^2)}{(\hat{s} - m_H^2)}$$

$$\mathcal{M}^{\pm\pm}_{\rm Fig.\,\mathbf{a}} = 3\mathcal{M}^{\pm\pm}_{\rm Fig.\,\mathbf{b}} \cdot \left\{1 + \mathcal{O}(\frac{1}{\hat{s}})\right\}$$

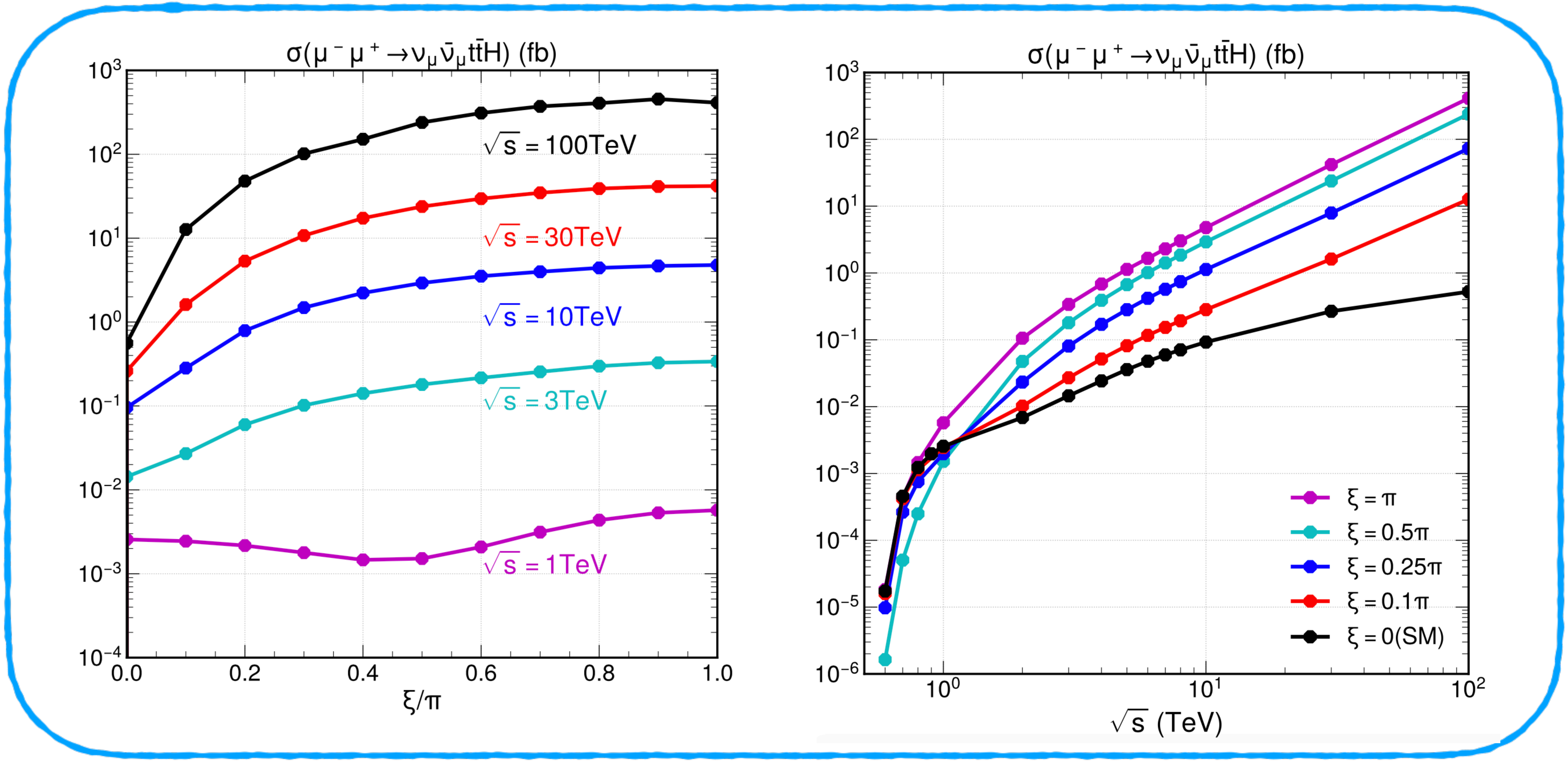
GBET tells:

$$\sum_{k=1}^{20} \mathcal{M}_k + \mathcal{M}_{\rm Fig.\,\mathbf{a}} = \mathcal{M}_{\rm Fig.\,\mathbf{b}} \cdot \left\{1 + \mathcal{O}(\frac{1}{\hat{s}})\right\}$$

$$\sum_{k=1}^{20} \mathcal{M}_k \approx -2\mathcal{M}_{\rm Fig.\,\mathbf{b}}$$

$$\sigma_{\rm tot}(W_L^- W_L^+ \to ttH)_{\rm SMEFT} \approx \frac{1}{4}\sigma_{\rm tot}(W_L^- W_L^+ \to ttH)_{\rm complex\ Yukawa}$$

$\mu^-\mu^+ \to \nu_\mu \bar{\nu}_\mu t\bar{t}H$ in SMEFT



Perturbative Unitarity

In the SM, $g_{\rm SM} = m_t/v$, unitarity should be valid at all scales

When $\xi \neq 0$, perturbation unitarity can be violated at $\sqrt{s} \geq o\left(\frac{\Lambda}{\sqrt{\lambda}}\right)$

$$g_{\rm SM} - g e^{i\xi} = \frac{\lambda v^2}{\sqrt{2}\Lambda^2}$$

$$SS^\dagger = 1$$

$$(1 + iT)(1 - iT^\dagger) = 1$$

$$-i(T - T^\dagger) = TT^\dagger$$

$$-i < i|(T - T^\dagger)|i > = < i|TT^\dagger|i >$$

$$-i(T_{ii} - T_{ii}^*) = < i|T|f > \Phi_f < f|T^\dagger|i >$$

All final states with phase space Φ_f

Optical Theorem: $2Im(T_{ii}) = \sum_f |T_{fi}|^2 \Phi_f = 2\hat{s} \sum_f \sigma(i \to f)$

Unitarity bound:
for $J=0$
i=j

$$2Im(T_{ii}) > |T_{ii}|^2 \frac{1}{8\pi}$$

2-body phase space

$$|T_{ii}|^2 < 16\pi Im(T_{ii}) < 16\pi |T_{ii}|$$

$$|T_{ii}| < 16\pi$$

$$2\hat{s} \sum_f \sigma(i \to f)_{J=0} = 2Im(T_{ii})_{J=0} < 32\pi$$

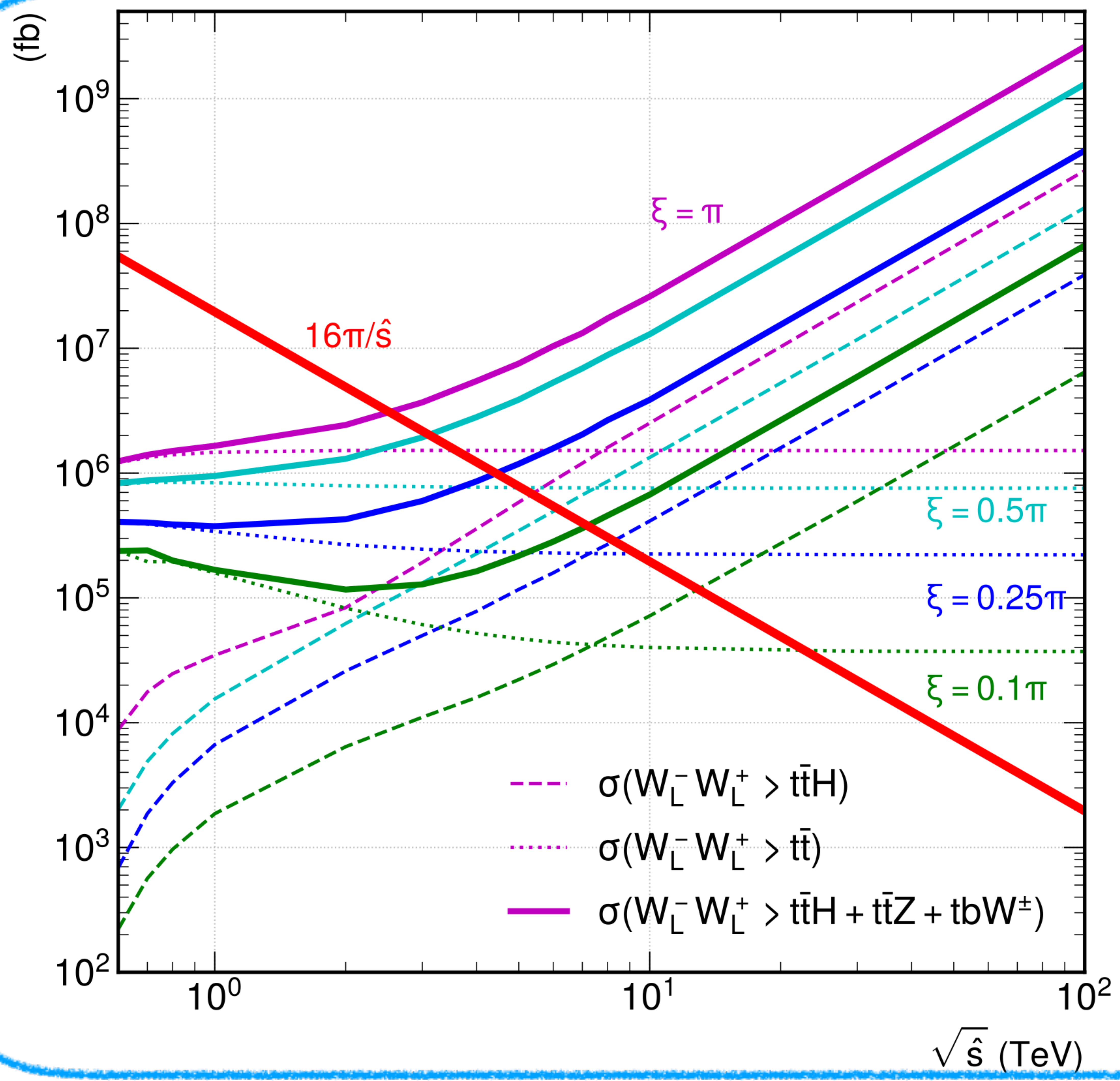
$$\sum_f \sigma(i \to f)_{J=0} < \frac{16\pi}{\hat{s}}$$

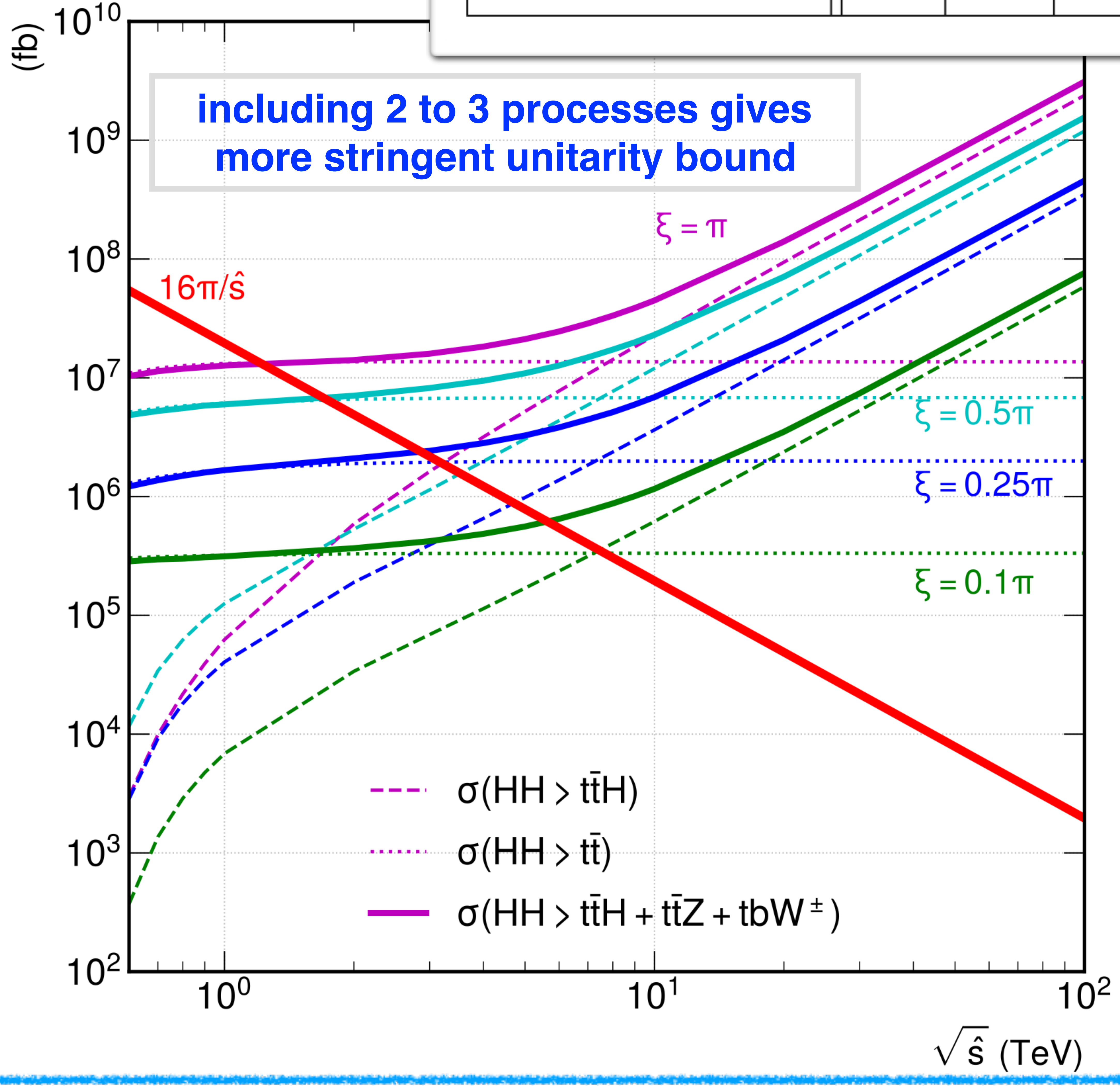
Unitarity bound

$$\sum_f \sigma_{\rm tot}\,(W_L^- W_L^+ \to f; J=0) < \frac{16\pi}{\hat{s}}$$

$$\sum_f \sigma_{tot}\,(HH \to f; J=0) < \frac{16\pi}{\hat{s}}$$

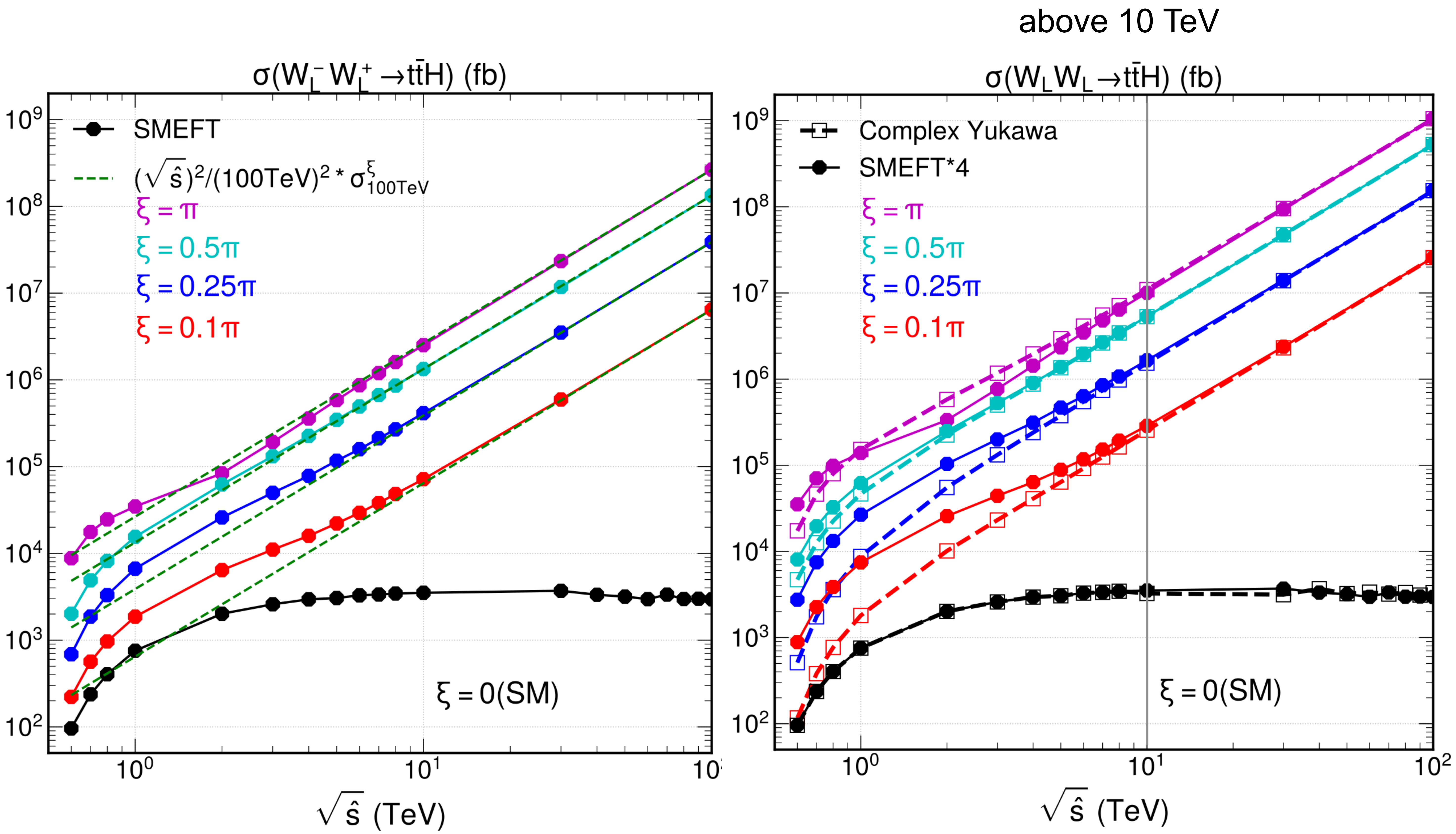
$\lvert\xi\rvert$	π	0.5π	0.25π	0.1π
$\lvert\lambda\rvert \cdot \Lambda^{-2}(\mathrm{TeV}^{-2})$	32.9	23.2	12.6	5.14
$\sqrt{\hat{s}}_{W_L W_L}(\mathrm{TeV})$	2.5	3.1	4.4	7.2
$\sqrt{\hat{s}}_{HH}$ (TeV)	1.2	1.7	2.9	5.6





Questions Unanswered

Why the high energy behavior of the WW→Htt cross section in the original complex Yukawa model is 4 times that of the SMEFT?



There may be a gauge invariant representation of the complex Yukawa coupling with 2 times the $\pi^+\pi^-$Htt GB coupling of SMEFT.

Going to dimension-8?

$$\mathcal{L}_{eff}^{ttH} = Q^\dagger \phi t_R \left\{ -y_{\rm SM} + \frac{\lambda_6}{\Lambda^2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right) + \frac{\lambda_8}{\Lambda^4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \ldots \right\} + h.c.$$

$$= -\frac{\mathbf{v} y_{\rm SM}}{\sqrt{2}} \boxed{t_L^\dagger t_R} - \left(\frac{y_{\rm SM}}{\sqrt{2}} - \frac{\lambda_6 v^2}{\sqrt{2}\Lambda^2}\right) \boxed{H t_L^\dagger t_R}$$

$$+ \left(\frac{3\lambda_6 v}{2\sqrt{2}\Lambda^2} + \frac{\lambda_8 v^3}{\sqrt{2}\Lambda^4}\right) \boxed{H^2 t_L^\dagger t_R} + \cdots$$ **=0 →Scenario 2**

$$+ \frac{\lambda_6 v}{\sqrt{2}\Lambda^2} \boxed{\left(\pi^+\pi^- + \frac{(\pi^0)^2}{2}\right) t_L^\dagger t_R} + \cdots$$

$$+ \left(\frac{\lambda_6}{\sqrt{2}\Lambda^2} + \frac{2\lambda_8 v^2}{\sqrt{2}\Lambda^4}\right) \boxed{\pi^+\pi^- H t_L^\dagger t_R} + \cdots + h.c.$$ **=0 →Scenario 3**

dim. of SMEFT operator	Dim-4	Dim-6	Dim-8	
Scenario 1	$y_{\rm SM}$	$y_{\rm SM} - y e^{i\xi}$	Negelible	SMEFT
Scenario 2	$y_{\rm SM}$	$y_{\rm SM} - y e^{i\xi}$	$-3/2(y_{\rm SM} - y e^{i\xi})$	HEFT
Scenario 3	$y_{\rm SM}$	$y_{\rm SM} - y e^{i\xi}$	$-1/2(y_{\rm SM} - y e^{i\xi})$	No E^2 energy growth

SMEFT expects the couplings from each operator follows |dim4|>>|dim6|>>|dim8|.
Violation of the above condition tells that the new physics scale is not much above the SM scale.

Vernon Barger, Kaoru Hagiwara and YJZ, in preparation.

Summary

- We identify the cause of a power law increase of the $\mu^-\mu^+ \to \nu\nu ttH$ cross section when the top Yukawa coupling is complex as due to the power law increase of the weak boson fusion subprocess ($WW \to ttH$)cross section.

- We identify the dimension-six SMEFT operator which gives a gauge invariant description for complex Yukawa coupling and confirm that the total cross section for $W\ W \to ttH$ satisfies the Goldstone Boson Equivalence Theorem.

- We obtain a novel perturbative unitarity bound on the SMEFT operator by summing over all $2 \to 2$ and $2 \to 3$ processes which contribute to the J=0 $HH \to HH$ amplitude.

- We comment on the case with dimension-8 operator.