Consequences of entanglement suppression on light decuplet baryons

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Disclaimer

I am NOT going beyond the standard model…

- **All discussions in this talk are within QCD.**
  ~ example of application of entanglemnt

- **Entanglement for Higgs, electroweak, neutrino, …**

  M. Carena, I. Low, C.E.M. Wagner, and M.-L. Xiao, PRD109, L051901 (2024);
  M. Carena, G. Coloretti, W. Liu, M. Littmann, I. Low, and C.E.M. Wagner, arXiv:2505.00873 [hep-ph];
  J. Thaler, S. Trifinopoulos, PRD111, 056021 (2025);
  J. Liu, M. Tanaka, X.-P. Wang, J.-J. Zhang, and Z. Zheng, arXiv:2505.06001 [hep-ph];
  I. Chernyshev, C.E.P. Robin, M.J. Savage, PRResearch 7, 023228 (2025);…

I am NOT an expert on quantum information science…

Contents

- **Overview**

  S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019);
  I. Low, T. Mehen, PRD104, 074014 (2021)

  - Entanglement power
  - Entanglement suppression for NN scattering

- **SU(3) baryon decuplet scattering**

  T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]

  - Spin 3/2 scattering
  - Emergent symmetries
  - Sub-unitary S-matrix

- **Summary**

Basic concept of entanglement suppression

Entanglement power (EP) of S-matrix $E(\hat{S})$

- ability of generating entanglement

$$|\psi_{\rm out}\rangle = \hat{S}\,|\psi_{\rm in}\rangle$$

entangle?

Entanglement suppression

S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019)

$E(\hat{S}) = 0$ **—> emergent symmetries of NN scattering**

Generalization to other hadrons and nuclei, …

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023);
T. Kirchner, W. Elkamhawy, H.-W. Hammer, Few-Body Syst. 65, 29 (2024);
T.R. Hu, S. Chen, F.K. Guo, PRD110, 014001 (2024); …

Symmetries of NN scattering

Fundamental symmetries $\mathrm{SU}(2)_{\mathrm{spin}} \times \mathrm{SU}(2)_{\mathrm{isospin}}$ **<— QCD**

- **spin** $\mathrm{SU}(2)_{\mathrm{spin}}$ $\{ |\uparrow\rangle, |\downarrow\rangle \}$
- **isospin** $\mathrm{SU}(2)_{\mathrm{isospin}}$ $\{ |p\rangle, |n\rangle \}$

$$\frac{1}{2} \otimes \frac{1}{2} = \underbrace{0}_{\mathrm{A}} \oplus \underbrace{1}_{\mathrm{S}}$$

s-wave NN **scattering: spin** $J \otimes$ **isospin** I **should be** A

$|J=0, I=1\rangle$ $({}^1\mathrm{S}_0)$ **and** $|J=1, I=0\rangle$ $({}^3\mathrm{S}_1)$, **independent**

Emergent symmetries <— entanglement suppression?

- **large scattering length** $|a_0| \gg 1\ \mathrm{fm}$: **NR conformal symmetry**

  T. Mehen, I.W. Stewart, and M.B. Wise, PLB474, 145 (2000)

- **both** $J=0$ **and** $J=1$ **has large** $|a_0|$: **spin-flavor** $\mathrm{SU}(4)$

  $\{ |p\uparrow\rangle, |p\downarrow\rangle, |n\uparrow\rangle, |n\downarrow\rangle \}$

  E. Wigner, PR51, 106 (1937), …, D.B. Kaplan and M.J. Savage, PLB365, 244 (1996), …

Entanglement measures

Entanglement measure for bipartite state $|\psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_B$

- von Neumann entropy

$$\mathscr{E}_{\rm vN}(|\psi\rangle) = -\,\mathrm{Tr}_A[\rho_A \ln \rho_A], \quad \rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$$

- linear entropy (expansion $\ln \rho_A \sim (\rho_A - 1)$ **)**

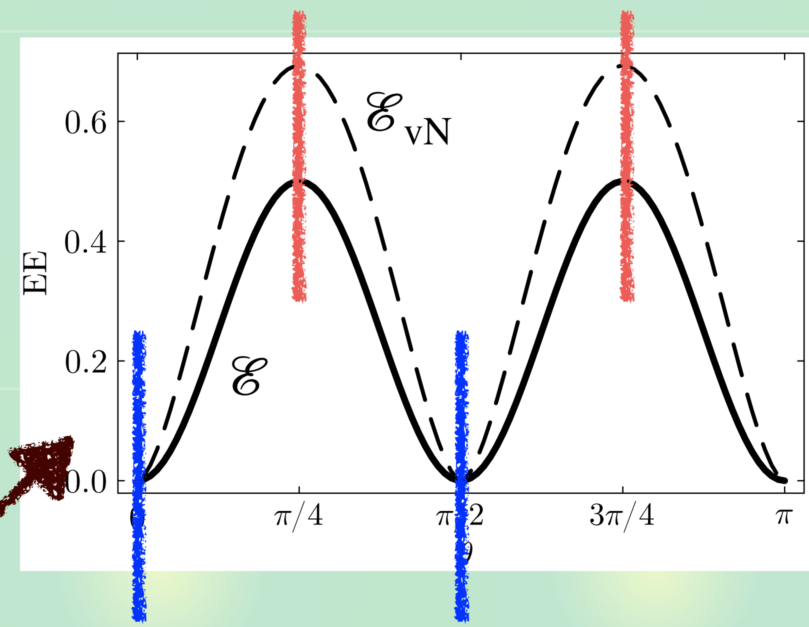
$$\mathscr{E}(|\psi\rangle) = 1 - \mathrm{Tr}_A[\rho_A^2]$$

- product state: minimum (=0)

$$|\uparrow\downarrow\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle$$

- Bell state: maximum (=1/2)

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$



Linear combination:

$$|\psi\rangle = \cos\theta|\uparrow\downarrow\rangle + \sin\theta|\downarrow\uparrow\rangle$$

Entanglement power

Entanglement power (EP) of operator $\hat{U}$ acting on $|\psi\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

- average over entanglement measure of $\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle$

$\mathcal{E}(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$

$$E(\hat{U}) = \overline{\mathcal{E}\left(\hat{U}|\psi_A\rangle \otimes |\psi_B\rangle\right)}$$

$$= \int d\omega_A d\omega_B \; \mathcal{E}(\hat{U}|\psi_A(\omega_A)\rangle \otimes |\psi_B(\omega_B)\rangle)$$

—> ability of $\hat{U}$ to generate entanglement

- **spin 1/2 (qubit):** rays in $\mathbb{C}^2 \simeq \mathbb{CP}^1$ **—> Fubini-Study measure**

$$|\psi(\omega_2)\rangle = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 e^{i\nu_1} \end{pmatrix}, \quad d\omega_{\mathbf{2}} = \frac{1}{\pi} d\theta_1 \cos\theta_1 \sin\theta_1 d\nu_1 \qquad \begin{matrix} \theta_1 \in [0,\pi/2) \\ \nu_1 \in [0,2\pi) \end{matrix}$$

(equivalent to Bloch sphere with $\theta_1 = \theta/2,\ \nu_1 = \phi,\ d\omega_{\mathbf{2}} = d\Omega/4\pi$)

S-matrix for *NN* scattering

S-matrix for *pn* scattering: 2 phase shifts δ_0, δ_1

$$\hat{S} = \underbrace{\exp\{2i\delta_0\}}_{J=0}\frac{1-\boldsymbol{\sigma}_A\cdot\boldsymbol{\sigma}_B}{4} + \underbrace{\exp\{2i\delta_1\}}_{J=1}\frac{3+\boldsymbol{\sigma}_A\cdot\boldsymbol{\sigma}_B}{4}$$

(isospin is automatically determined by Pauli principle)

Entanglement power of *NN* scattering

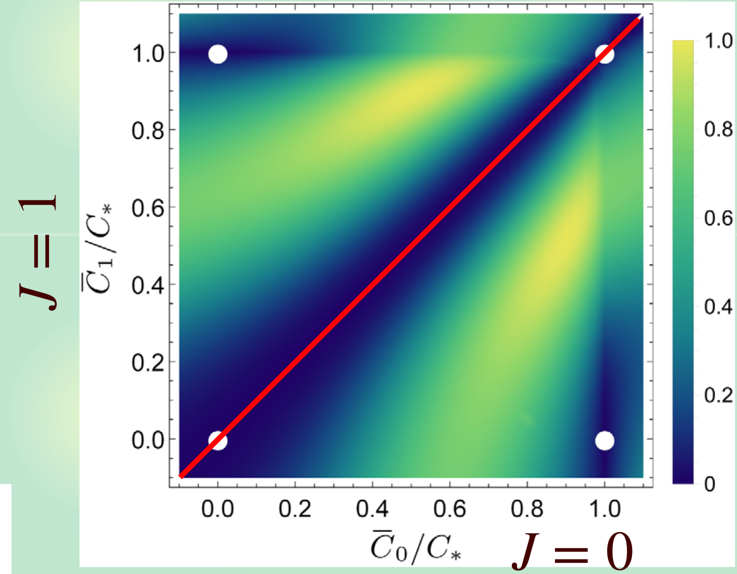
S.R. Beane, D.B. Kaplan, N. Klco, and M.J. Savage, PRL122, 102001 (2019)

$$E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0-\delta_1)] \geq 0$$

$E(\hat{S}) = 0$ **is achieved if**

$$\begin{cases} \delta_0 = \delta_1 & \textbf{SU(4) symmetry} \\ (\delta_0, \delta_1) = (0,0), (0,\pi/2), (\pi/2,0), (\pi/2,\pi/2) \end{cases}$$

NR conformal symmetry



Interpretation by quantum information

S-matrix with $E(\hat{S}) = 0$: product state —> product state

I. Low, T. Mehen, PRD104, 074014 (2021)

- **Identity gate:** $|\delta_0 - \delta_1| = 0$

$$\hat{S} \propto \frac{1 - \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} + \frac{3 + \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} = 1, \quad 1\,|\psi_A\rangle \otimes |\psi_B\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

- **SWAP gate:** $|\delta_0 - \delta_1| = \pi/2$ **(spin exchange operator P_s)**

$$\hat{S} \propto -\frac{1 - \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} + \frac{3 + \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{4} = \frac{1 + \boldsymbol{\sigma}_A \cdot \boldsymbol{\sigma}_B}{2} = \mathrm{SWAP}$$

$$\mathrm{SWAP}\,|\psi_A\rangle \otimes |\psi_B\rangle = |\psi_B\rangle \otimes |\psi_A\rangle$$

Identity and SWAP are the only minimal entanglers

<— parametrization of $\mathrm{SU}(4)/(\mathrm{SU}(2) \times \mathrm{SU}(2))$

Summary of overview part

- **Entanglement power** of S-matrix ~ ability of generating entanglement

$$E(\hat{S}) = \overline{\mathcal{E}(\hat{S}\,|\,\psi_A\rangle \otimes |\,\psi_B\rangle)}$$

- Entanglement suppression: **requireing** $E(\hat{S}) = 0$ for NN scattering, the following symmetries emerge

  - $\mathrm{SU}(4)$ spin-flavor symmetry ($\hat{S} \propto 1$)

  - NR conformal symmetry ($\hat{S} \propto \mathrm{SWAP}$)

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I. Low, T. Mehen, PRD104, 074014 (2021)

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SU(3) baryon decuplet scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]

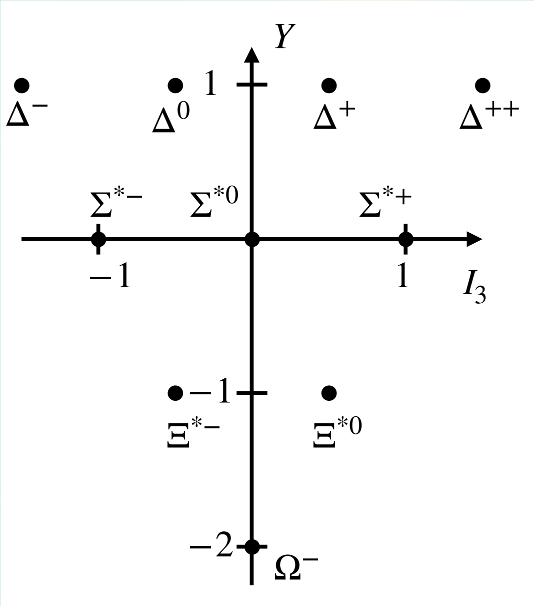
- Spin 3/2 scattering
- Emergent symmetries
- Sub-unitary S-matrix

Summary

Motivation

Decuplet (10d representation) baryons

- spin 3/2
- ground states with Octet in quark model



Possibly large scattering length a_0

- $\Delta\Delta$ dibaryon, $d^*(2380)$?

  F. Dyson, N.H. Xuong, PRL13, 815 (1964);
  P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for $\Omega\Omega$ scattering: $a_0(J=0) = 4.6\ \mathrm{fm} \gg 1\ \mathrm{fm}$

  S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Quantum information viewpoint

- spin 3/2 scattering = 4d-qudit quantum gates

Spin 3/2 state

Spin 3/2 (4d-qudit): rays in $\mathbb{C}^4 \simeq \mathbb{CP}^3$

$$|\psi_{\mathbf{4}}\rangle = \begin{pmatrix} \cos\theta_1 \sin\theta_2 \sin\theta_3 \\ \sin\theta_1 \sin\theta_2 \sin\theta_3 e^{i\nu_1} \\ \cos\theta_2 \sin\theta_3 e^{i\nu_2} \\ \cos\theta_3 e^{i\nu_3} \end{pmatrix}, \quad d\omega_{\mathbf{4}} = \frac{3!}{\pi^3} \prod_{i=1}^{3} d\theta_i d\nu_i \cos\theta_i \sin^{2i-1}\theta_i,$$

$$\theta_i \in [0,\pi/2), \quad \nu_i \in [0,2\pi)$$

Spin decomposition (2 symmetric, 2 anti-symmetric)

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\mathrm{A}} \oplus \underbrace{1 \oplus 3}_{\mathrm{S}}$$

S-matrix: 4 components, 4 phase shifts $\delta_0, \delta_1, \delta_2, \delta_3$

$$\hat{S} = \mathcal{J}_0 \exp\{2i\delta_0\} + \mathcal{J}_1 \exp\{2i\delta_1\} + \mathcal{J}_2 \exp\{2i\delta_2\} + \mathcal{J}_3 \exp\{2i\delta_3\}$$

$$\mathcal{J}_0 = \frac{33}{128} + \frac{31}{96}\left(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}\right) - \frac{5}{72}\left(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}\right)^2 - \frac{1}{18}\left(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}\right)^3, \cdots$$

Entanglement power

Entanglement power (~3 days on mathematica)

$$E(\hat{S}) = \frac{1}{200000}\Big\{77482 - 2100\cos\left[2\left(\delta_0+\delta_1-\delta_2-\delta_3\right)\right]$$
$$-2100\cos\left[2\left(\delta_0-\delta_1+\delta_2-\delta_3\right)\right] - 2100\cos\left[2\left(\delta_0-\delta_1-\delta_2+\delta_3\right)\right]$$
$$-1200\cos\left[2\left(\delta_0-2\delta_1+\delta_2\right)\right] - 4200\cos\left[2\left(\delta_0+\delta_2-2\delta_3\right)\right]$$
$$-8400\cos\left[2\left(\delta_1-2\delta_2+\delta_3\right)\right]$$
$$-375\cos\left[4\left(\delta_0-\delta_1\right)\right] - 10800\cos\left[2\left(\delta_0-\delta_2\right)\right] - 625\cos\left[4\left(\delta_0-\delta_2\right)\right]$$
$$-875\cos\left[4\left(\delta_0-\delta_3\right)\right] - 2175\cos\left[4\left(\delta_1-\delta_2\right)\right] - 26376\cos\left[2\left(\delta_1-\delta_3\right)\right]$$
$$-5481\cos\left[2\left(\delta_1-\delta_3\right)\right] - 10675\cos\left[2\left(\delta_2-\delta_3\right)\right]\Big\}$$

$E(\hat{S}) = 0$ **is achieved if (all cos = +1)**

$$\delta_0 = \delta_2 = \delta_{\text{even}}, \quad \delta_1 = \delta_3 = \delta_{\text{odd}}, \quad \begin{cases} |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 \\ |\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 \end{cases}$$

Minimal entanglers

S-matrix with $E(\hat{S}) = 0$

- **Identity gate:** $|\delta_{\rm even} - \delta_{\rm odd}| = 0$

$$\hat{S} \propto \mathcal{J}_0 + \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 = 1$$

- **SWAP gate:** $|\delta_{\rm even} - \delta_{\rm odd}| = \pi/2$

$$\hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_1 - \mathcal{J}_2 + \mathcal{J}_3 = \sum_i \mathcal{S}_i - \sum_i \mathcal{A}_i$$

—> Minimal entanglement is achived by Identity and SWAP, also for spin 3/2 scattering

Identity and SWAP are the only minimal entanglers for any bipartite system with same dimension

E. Alfsen, F. Shultz, JMP51, 052201 (2010);…

Decuplet-decuplet scattering

Spin-flavor decomposition: spin⊗flavor should be A

- **spin** $\mathrm{SU}(2)_{\mathrm{spin}}$ $\qquad \frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\mathrm{A}} \oplus \underbrace{1 \oplus 3}_{\mathrm{S}}$

- **flavor** $\mathrm{SU}(3)_{\mathrm{flavor}}$ $\qquad \mathbf{10} \otimes \mathbf{10} = \underbrace{\mathbf{27} \oplus \mathbf{28}}_{\mathrm{S}} \oplus \underbrace{\overline{\mathbf{10}} \oplus \mathbf{35}}_{\mathrm{A}}$

8 phase shifts $\delta_{0,\mathbf{27}},\ \delta_{0,\mathbf{28}},\ \delta_{1,\overline{\mathbf{10}}},\ \delta_{1,\mathbf{35}},\ \delta_{2,\mathbf{27}},\ \delta_{2,\mathbf{28}},\ \delta_{3,\overline{\mathbf{10}}},\ \delta_{3,\mathbf{35}}$

$$\hat{S} = \mathcal{J}_0 \otimes \left(\mathcal{F}_{\mathbf{27}}\, e^{2i\delta_{0,\mathbf{27}}} + \mathcal{F}_{\mathbf{28}}\, e^{2i\delta_{0,\mathbf{28}}}\right) + \mathcal{J}_1 \otimes \left(\mathcal{F}_{\overline{\mathbf{10}}}\, e^{2i\delta_{1,\overline{\mathbf{10}}}} + \mathcal{F}_{\mathbf{35}}\, e^{2i\delta_{1,\mathbf{35}}}\right)$$

$$+\mathcal{J}_2 \otimes \left(\mathcal{F}_{\mathbf{27}}\, e^{2i\delta_{2,\mathbf{27}}} + \mathcal{F}_{\mathbf{28}}\, e^{2i\delta_{2,\mathbf{28}}}\right) + \mathcal{J}_3 \otimes \left(\mathcal{F}_{\overline{\mathbf{10}}}\, e^{2i\delta_{3,\overline{\mathbf{10}}}} + \mathcal{F}_{\mathbf{35}}\, e^{2i\delta_{3,\mathbf{35}}}\right)$$

$E(\hat{S}) = 0$ is achieved if

$$\delta_{0,F} = \delta_{2,F'} = \delta_{\mathrm{even}}, \quad \delta_{1,F} = \delta_{3,F'} = \delta_{\mathrm{odd}}, \quad \begin{cases} |\,\delta_{\mathrm{even}} - \delta_{\mathrm{odd}}\,| = 0 \\ |\,\delta_{\mathrm{even}} - \delta_{\mathrm{odd}}\,| = \pi/2 \end{cases}$$

Emergent symmetries

Emergent symmetry <— degeneracy counting

	$\delta_{\rm even}(J=0,2,\ F=\mathbf{27},\mathbf{28})$		$\delta_{\rm odd}(J=1,3,\ F=\overline{\mathbf{10}},\mathbf{35})$	
spin	$1+5=6$		$3+7=10$	
flavor	$27+28=55$		$10+35=45$	
total	330	+	450	= 780

- **Identity gate:** $|\delta_{\rm even}-\delta_{\rm odd}|=0$ **—> spin-flavor** $\mathrm{SU}(40)$

  $$\mathbf{40}\otimes\mathbf{40}=\mathbf{820}\oplus\mathbf{780}$$

- **SWAP gate:** $|\delta_{\rm even}-\delta_{\rm odd}|=\pi/2$

  —> $\mathrm{SU}(4)_{\rm spin}\times\mathrm{SU}(10)_{\rm flavor}$ **+ NR conformal symmetry**

  $$\mathbf{4}\otimes\mathbf{4}=\mathbf{10}\oplus\mathbf{6},\quad \mathbf{10}\otimes\mathbf{10}=\mathbf{55}\oplus\mathbf{45}$$

Justified by effective Lagrangian approach

Idntical particles and sub-unitary S-matrix

Identical particles: symmetric spin states are forbidden

- nn **scattering/ΩΩ scattering: only antisymmetric component**

$$\hat{S}_{nn} = \mathscr{I}_0 \exp\{2i\delta_0\}$$ **spin 1 absent**

$$\hat{S}_{\Omega\Omega} = \mathscr{I}_0 \exp\{2i\delta_{0,\mathbf{28}}\} + \mathscr{I}_2 \exp\{2i\delta_{2,\mathbf{28}}\}$$ **spin 1 and 3 absent**

- **naive calculation of EP: larger than Bell state (E=1/2)?**

$$E(\hat{S}_{nn}) = \frac{23}{24}$$

S-matrix is not unitary in full space: $\hat{S}_{nn}\hat{S}^\dagger_{nn} = \mathscr{I}_0 \neq 1$

$$\hat{S} = \begin{pmatrix} (\text{Antisymmetric}) & 0 \\ 0 & (\text{Symmetric}) \end{pmatrix}, \quad \mathscr{I}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Density matrix of out state is not properly normalized

$$\rho = |\psi_{\text{out}}\rangle\langle\psi_{\text{out}}| = \hat{S}_{nn}|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|\hat{S}^\dagger_{nn}, \quad \text{Tr}[\rho] = \langle\psi_{\text{in}}|\mathscr{I}_0|\psi_{\text{in}}\rangle < 1$$

ΩΩ scattering

Properly normalized EP of ΩΩ S-matrix (maximum EE=3/4)

$$E_2(\hat{S}_{\Omega\Omega}) = \frac{1}{48}\left\{25 - \cos\left[4\left(\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}\right)\right]\right\}$$

$E_2(\hat{S}_{\Omega\Omega})$ **is minimized (not 0) if**

$$\begin{cases} |\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0 & \hat{S} \propto \mathcal{J}_0 + \mathcal{J}_2 \equiv \mathcal{P}_{\mathrm{A}} \\ |\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2 & \hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_2 \equiv \mathrm{SWAP}_{\mathrm{A}} \end{cases}$$

Lattice QCD shows $a_0(J=0) = 4.6\ \mathrm{fm} \gg 1/m_\pi$ **—>** $\delta_{0,\mathbf{28}} \sim \pi/2$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Entanglement suppression suggests either

- **spin 2 channel is near the unitary limit (**$|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0$**), or**
- **spin 2 channel is almost noninteracting (**$|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2$**)**

Summary

- Entanglement suppression for baryon decuplet
- $E(\hat{S}) = 0$ is achieved only by $\hat{S} \propto 1$ **or** $\hat{S} \propto \mathrm{SWAP}$ for spin 3/2 scattering
- Largest emergent symmetries
  - $\mathrm{SU}(40)$ spin-flavor symmetry ($\hat{S} \propto 1$)
  - $\mathrm{SU}(4)_{\mathrm{spin}} \times \mathrm{SU}(10)_{\mathrm{flavor}}$ + NR conformal ($\hat{S} \propto \mathrm{SWAP}$)
- Sub-unitary S-matrix
  - Final state should be properly normalized
  - $\Omega\Omega$ scattering: implication to spin 2 channel

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]