WILL WE EVER DETECT HIGH FREQUENCY GRAVITATIONAL WAVES?

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[Based on RTD, S. Ellis, arXiv:2412.17897, *JHEP* 04 (2025) 164]

$$\omega_g$$


ω_g
nHz
μHz
10 Hz
kHz
PULSAR TIMING
LIGO-VIRGO-KAGRA

$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)}$$ Causality

$$\omega_g$$

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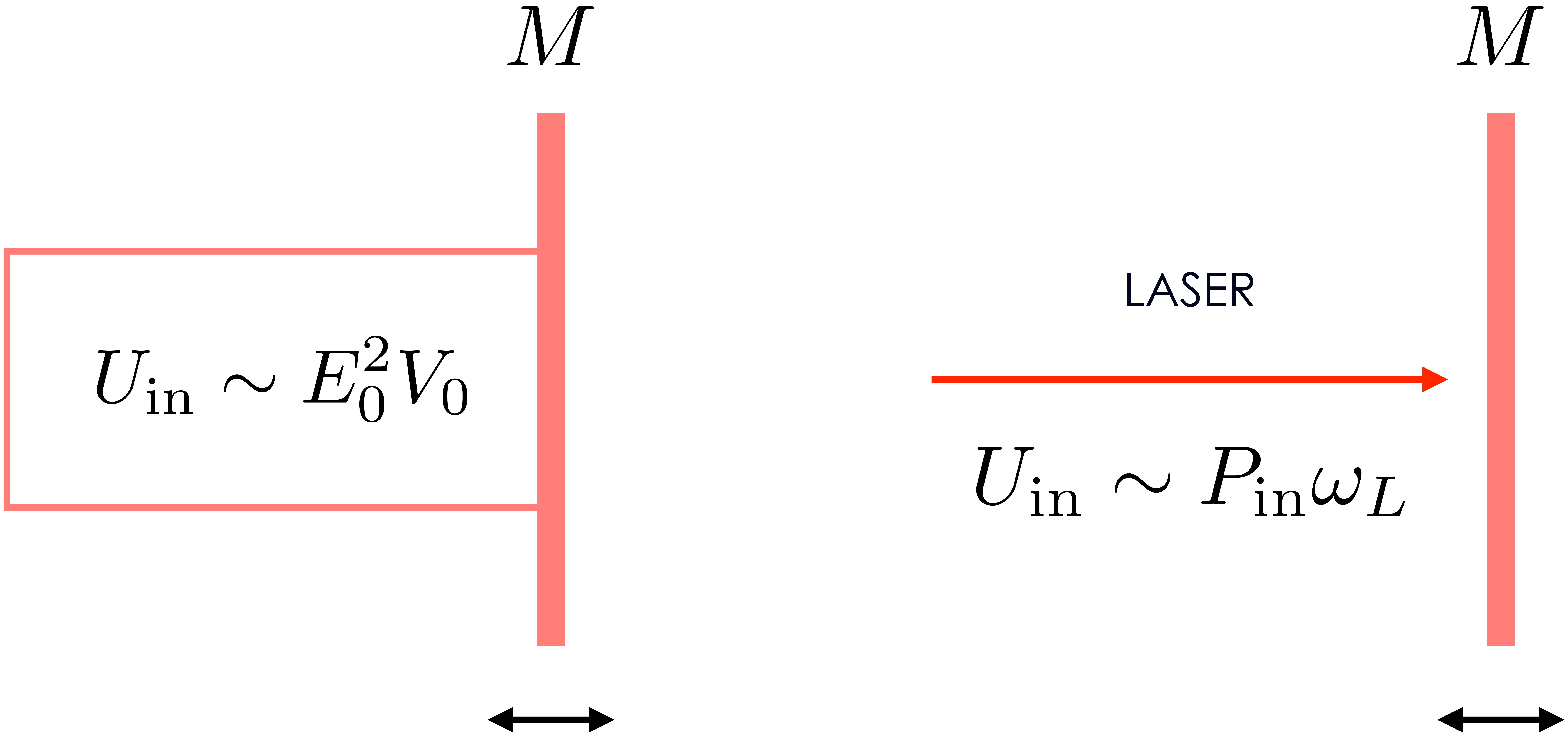
$$\omega_0(T_*) = \omega(T_*)\frac{a(T_*)}{a_0} \gtrsim \boxed{100\ \mathrm{MHz}}\left(\frac{T_*}{10^{15}\mathrm{GeV}}\right)\left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

DETECTORS

DETECTOR I

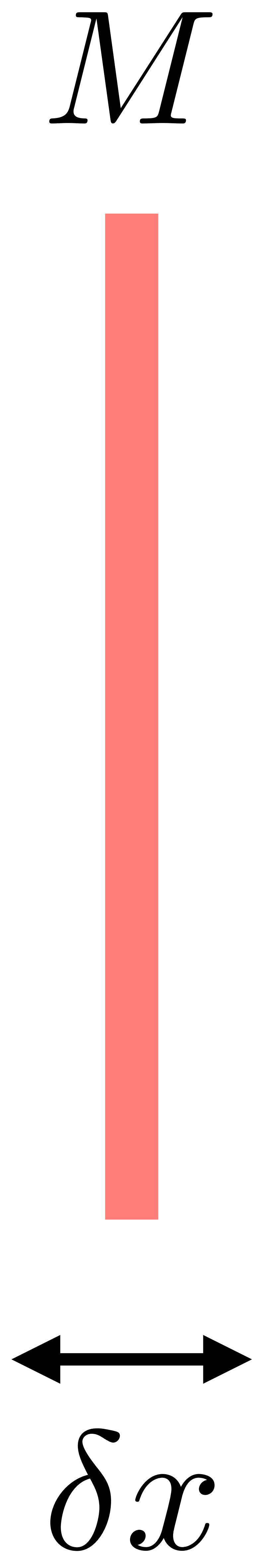
$$U_{\text{in}} \sim E_0^2 V_0$$

DETECTOR II



DETECTOR II

$$M\ddot{\delta x} = M\ddot{h}x + F_{\text{ext}}$$



DETECTOR II

$$M \to \infty$$

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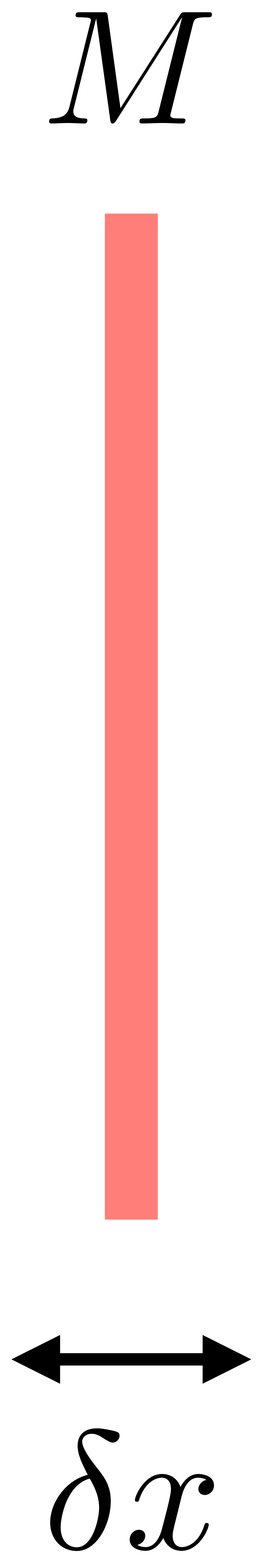

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$$M \to \infty$$

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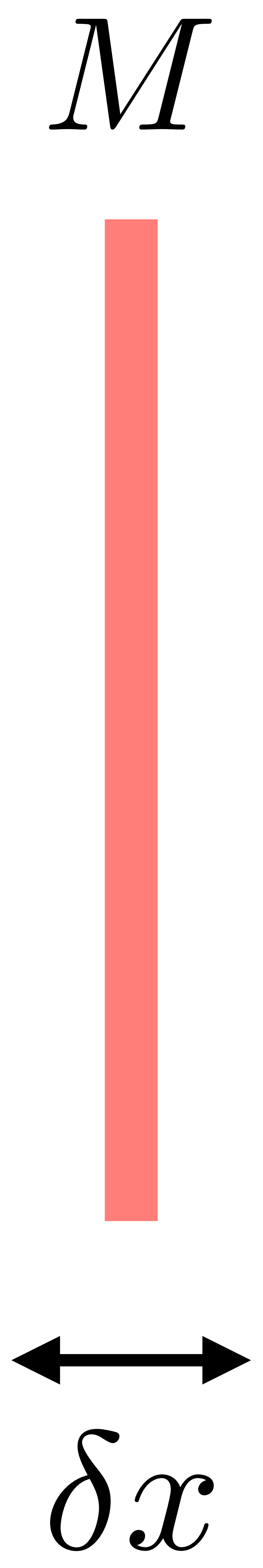
$$\ddot{\delta x}_{\text{sig}} \sim \ddot{h}x \sim \text{const}$$



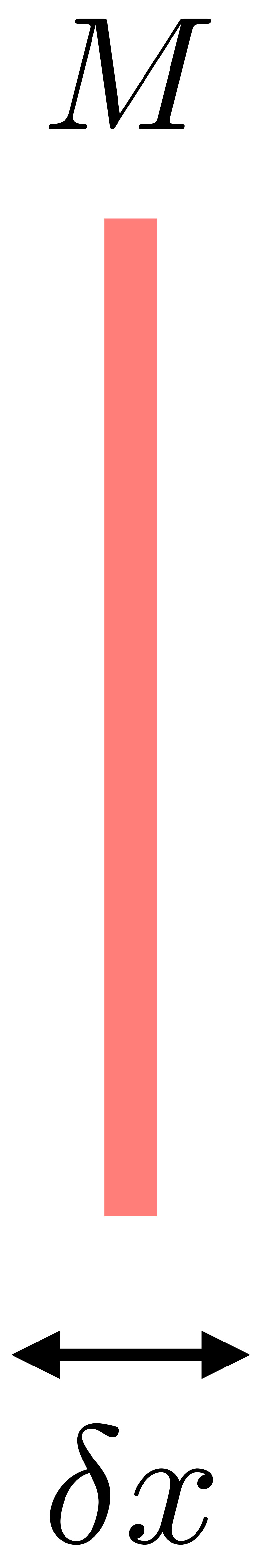
DETECTOR II

$$M \to \infty$$

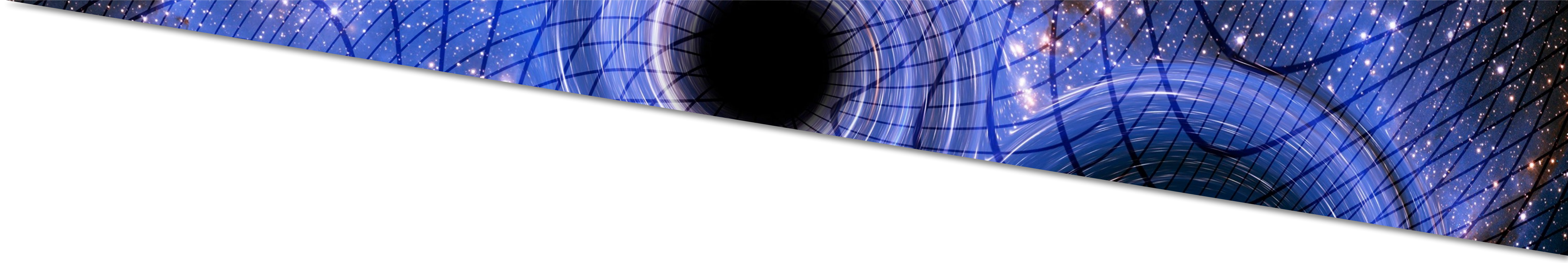
$$M\ddot{\delta x} = M\ddot{h}x + F_{\text{ext}}$$

$$\ddot{\delta x}_{\text{sig}} \sim \ddot{h}x \sim \text{const}$$

$$\ddot{\delta x}_{\text{noise}} \sim \frac{F_{\text{ext}}}{M} \to 0$$



THE BEST POSSIBLE SENSITIVITY



IN THE FOLLOWING I WILL ALWAYS CONSIDER

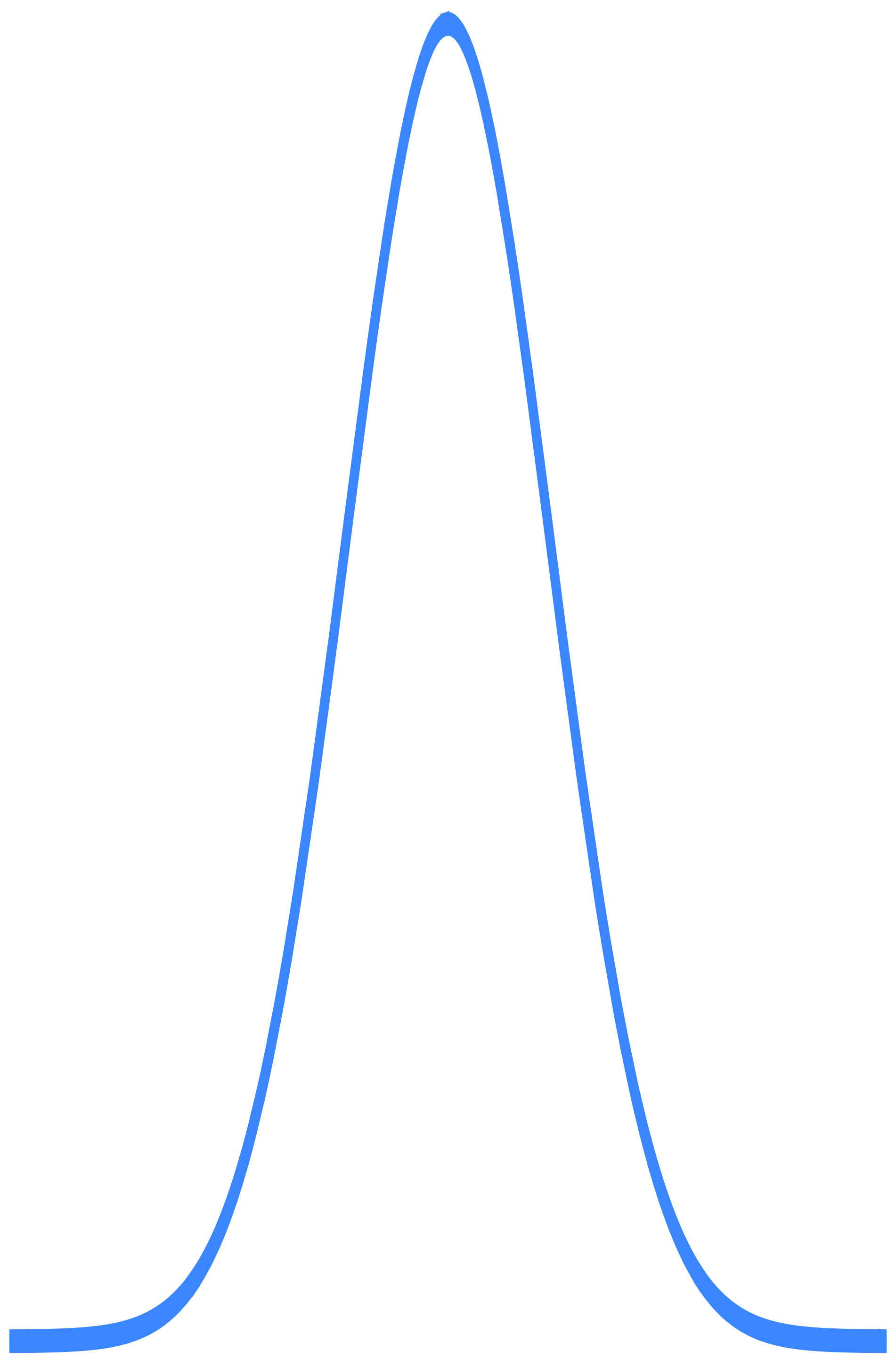
$$M \to \infty$$

$$\ddot{\delta} x_{\mathrm{noise}} \sim \frac{F_{\mathrm{ext}}}{M} \to 0$$

THE BEST POSSIBLE SENSITIVITY

In this limit I can ignore noise from the test mass and focus on the signal (and noise) photons that I can detect

DETECTOR I → $$U_{\text{in}} \sim E_0^2 V_0$$ ← DETECTOR II

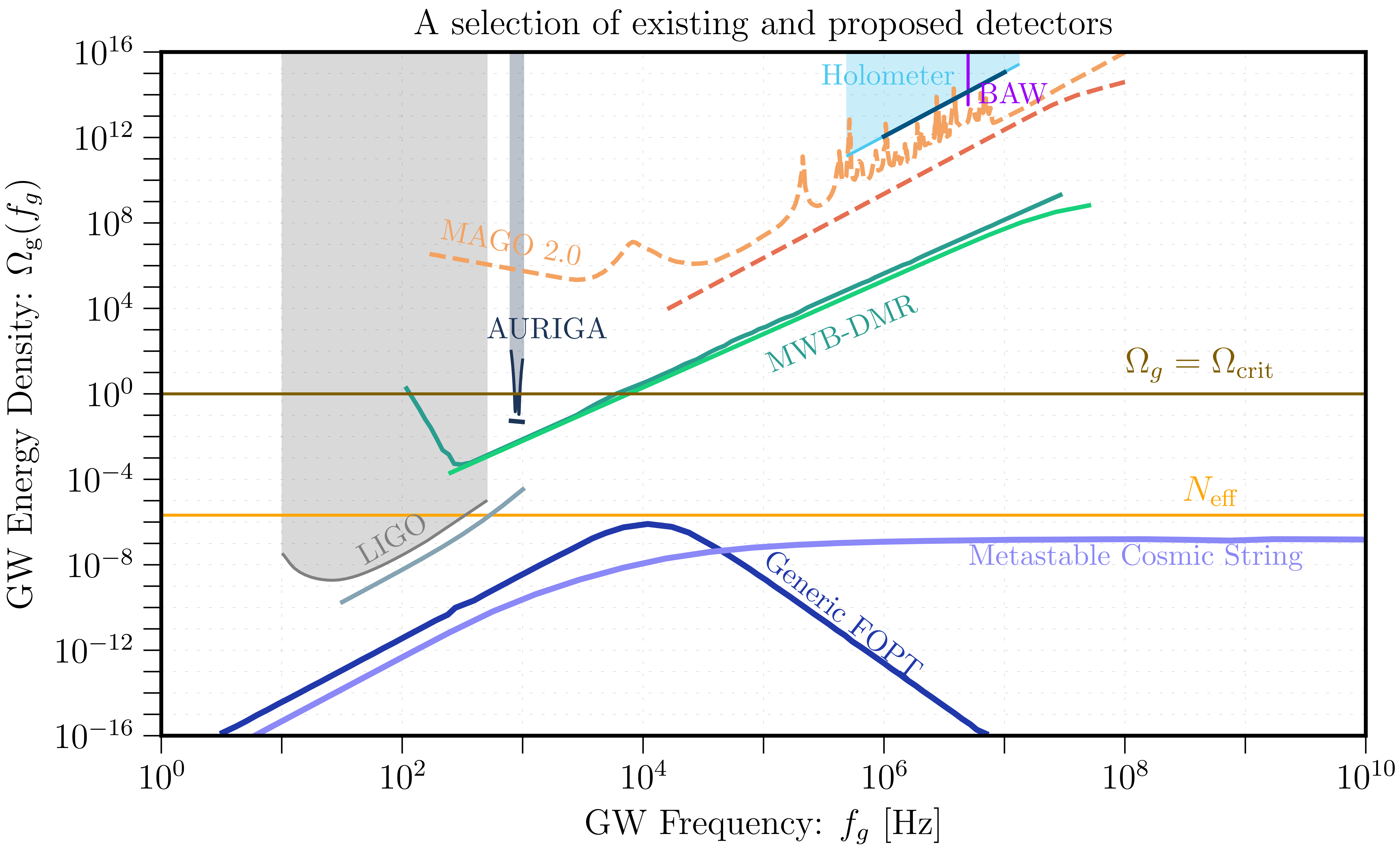


$$E_0(\omega_0)$$


$\omega_0 - \omega_g$
$\omega_0 + \omega_g$
E_h
$E_0(\omega_0)$
E_h

$$U_{\text{in}} \sim E_0^2 V_0$$

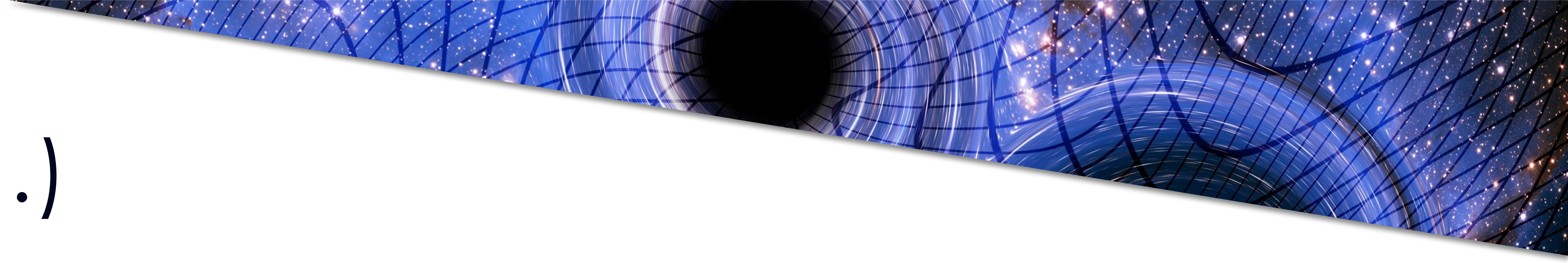
$$\text{———} \mathcal{T}(\omega) \text{———}$$



A selection of existing and proposed detectors
GW Energy Density: $\Omega_g(f_g)$
GW Frequency: f_g [Hz]
10^{16}
10^{12}
10^{8}
10^{4}
10^{0}
10^{-4}
10^{-8}
10^{-12}
10^{-16}
10^{0}
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
Holometer
BAW
MAGO 2.0
AURIGA
MWB-DMR
$\Omega_g = \Omega_{crit}$
N_{eff}
LIGO
Metastable Cosmic String
Generic FOPT

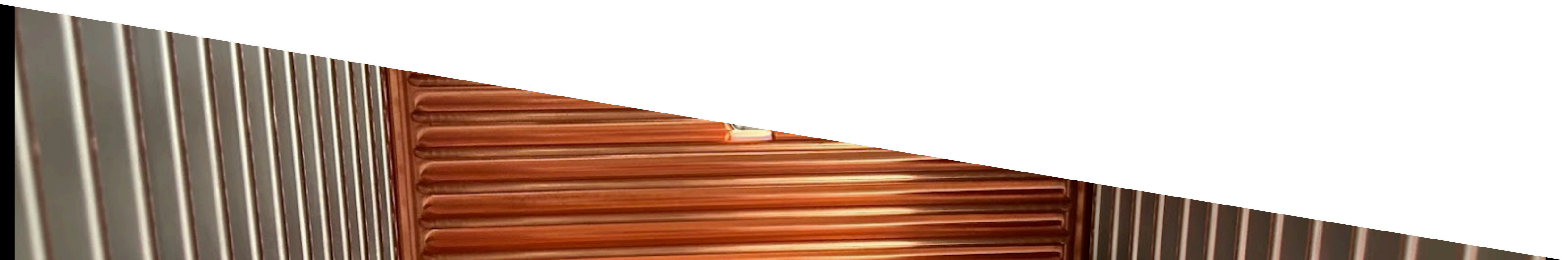

TRANSFER FUNCTIONS

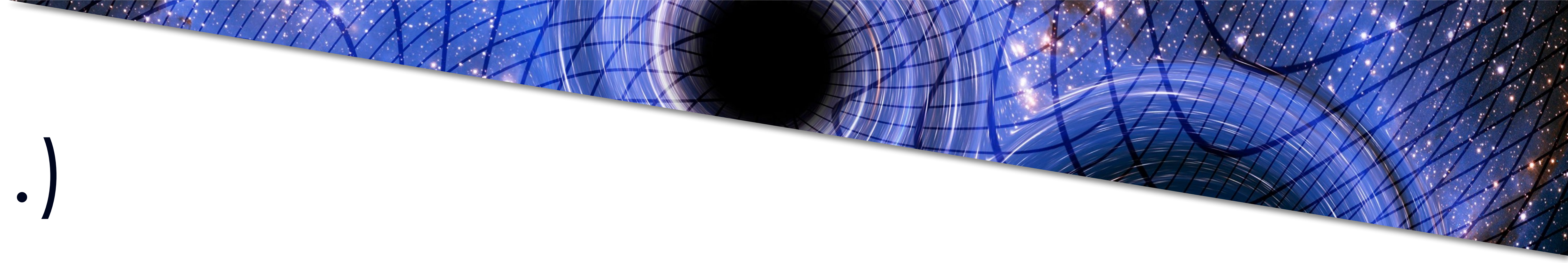
MOST DETECTORS (LIGO, OPTOMECHANICAL, WEBER BARS…)



$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{1}{2}\omega_g^2 V^{1/3}\tilde{h}(\omega,\omega_g)$$

The wave excites a mechanical mode



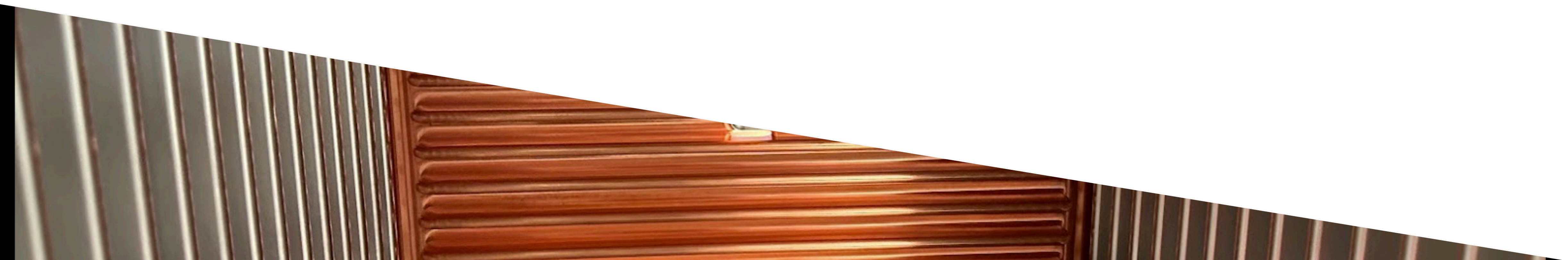


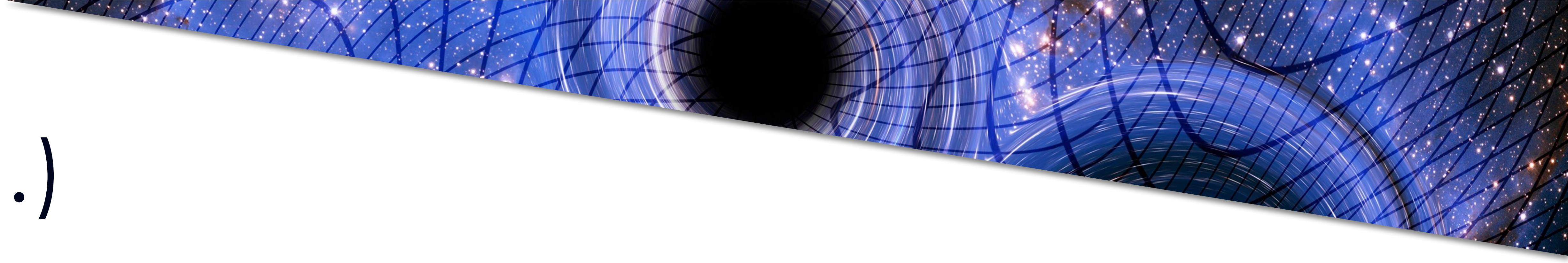
MOST DETECTORS (LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\left(\omega_m^2 - \omega^2 + i\frac{\omega\omega_m}{Q_m}\right)\tilde{u}_m(\omega) \simeq -\frac{1}{2}\omega_g^2 V^{1/3}\tilde{h}(\omega,\omega_g)$$

$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_h(\omega) \simeq -2\omega_1^2 V^{-1/3}\int d\omega'\,\tilde{u}_m(\omega'-\omega)\tilde{E}_0(\omega')$$

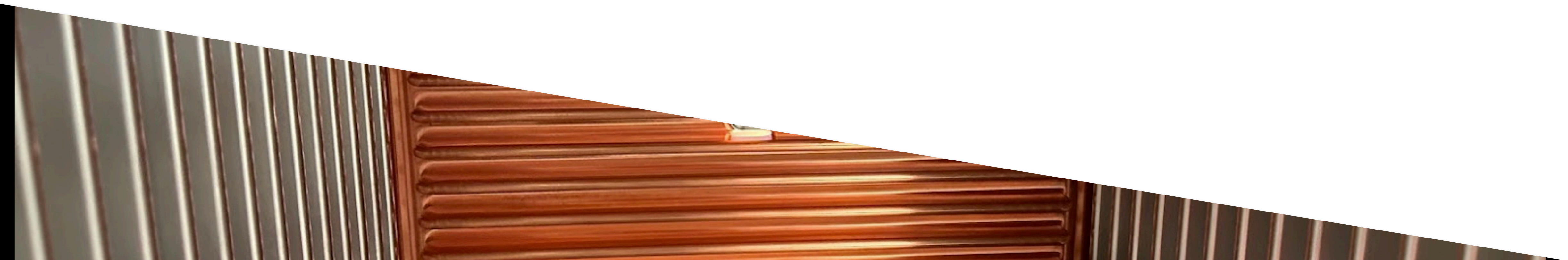
An EM resonator does the readout



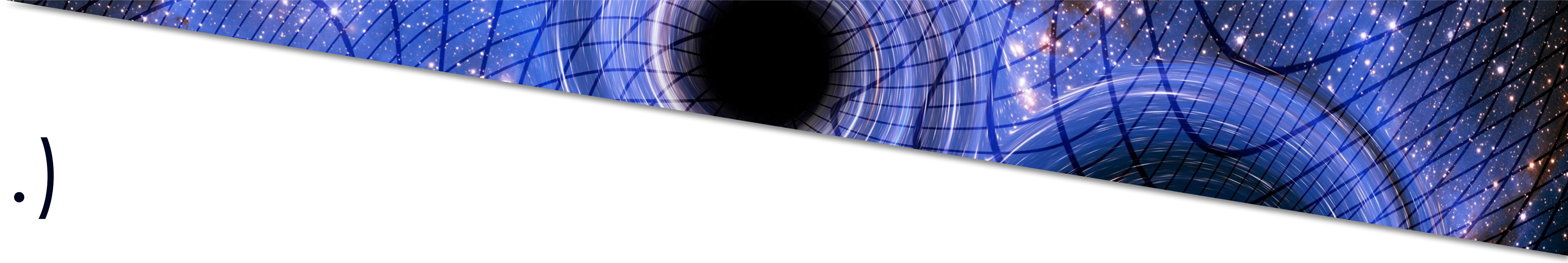


MOST DETECTORS (LIGO, OPTOMECHANICAL, WEBER BARS…)

$$\mathcal{T}^2(\omega) = \frac{\omega_g^4 \omega_1^4}{\left((\omega_1^2 - \omega^2)^2 + \frac{\omega^2 \omega_1^2}{Q^2}\right)\left((\omega_m^2 - \omega_g^2)^2 + \frac{\omega_g^2 \omega_m^2}{Q_m^2}\right)}$$

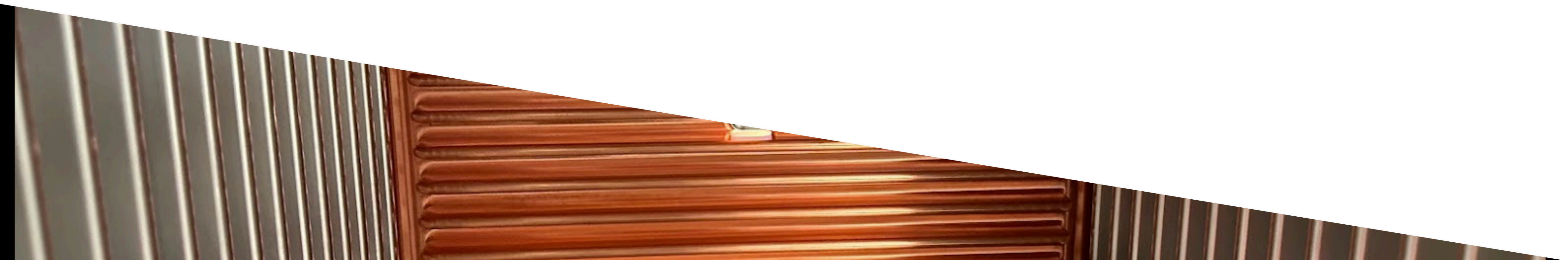


MOST DETECTORS (LIGO, OPTOMECHANICAL, WEBER BARS...)

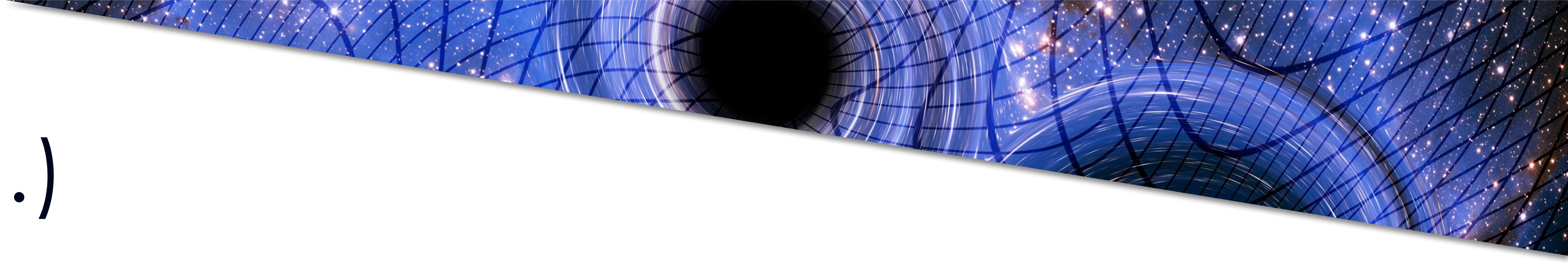


$$\left(\mathcal{T}^{\mathrm{LVK}}\right)^2(\omega_g) \simeq \frac{\omega_L^2}{\left(4\omega_g^2 + \frac{\omega_L^2}{Q^2}\right)} \simeq \frac{\omega_L^2 L_{\mathrm{eff}}^2}{(4\omega_g^2 L_{\mathrm{eff}}^2 + 1)}$$

$$\omega_0 \simeq \omega_1 \simeq \omega_L \gg \omega_g \gg \omega_m$$

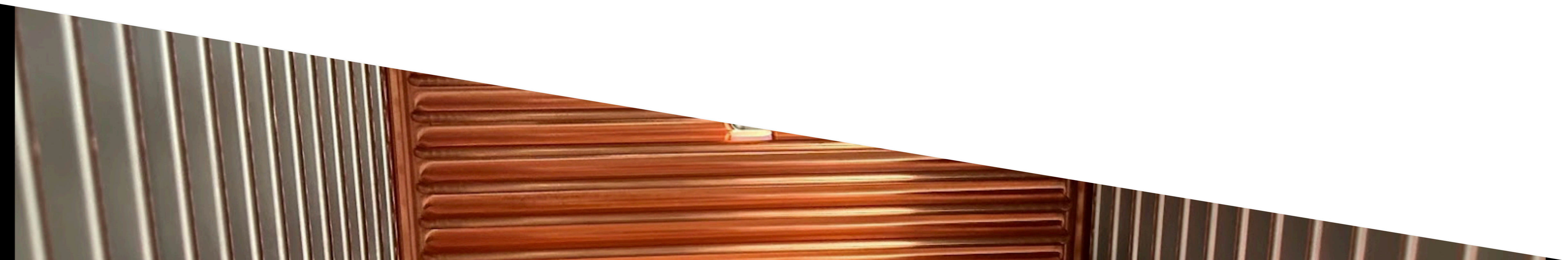


MOST DETECTORS (LIGO, OPTOMECHANICAL, WEBER BARS…)



$$(\mathcal{T}^{\text{Weber}})^2 \lesssim Q^2 Q_m^2$$

$$\omega_m \simeq \omega_{1,0} \simeq \omega_g$$

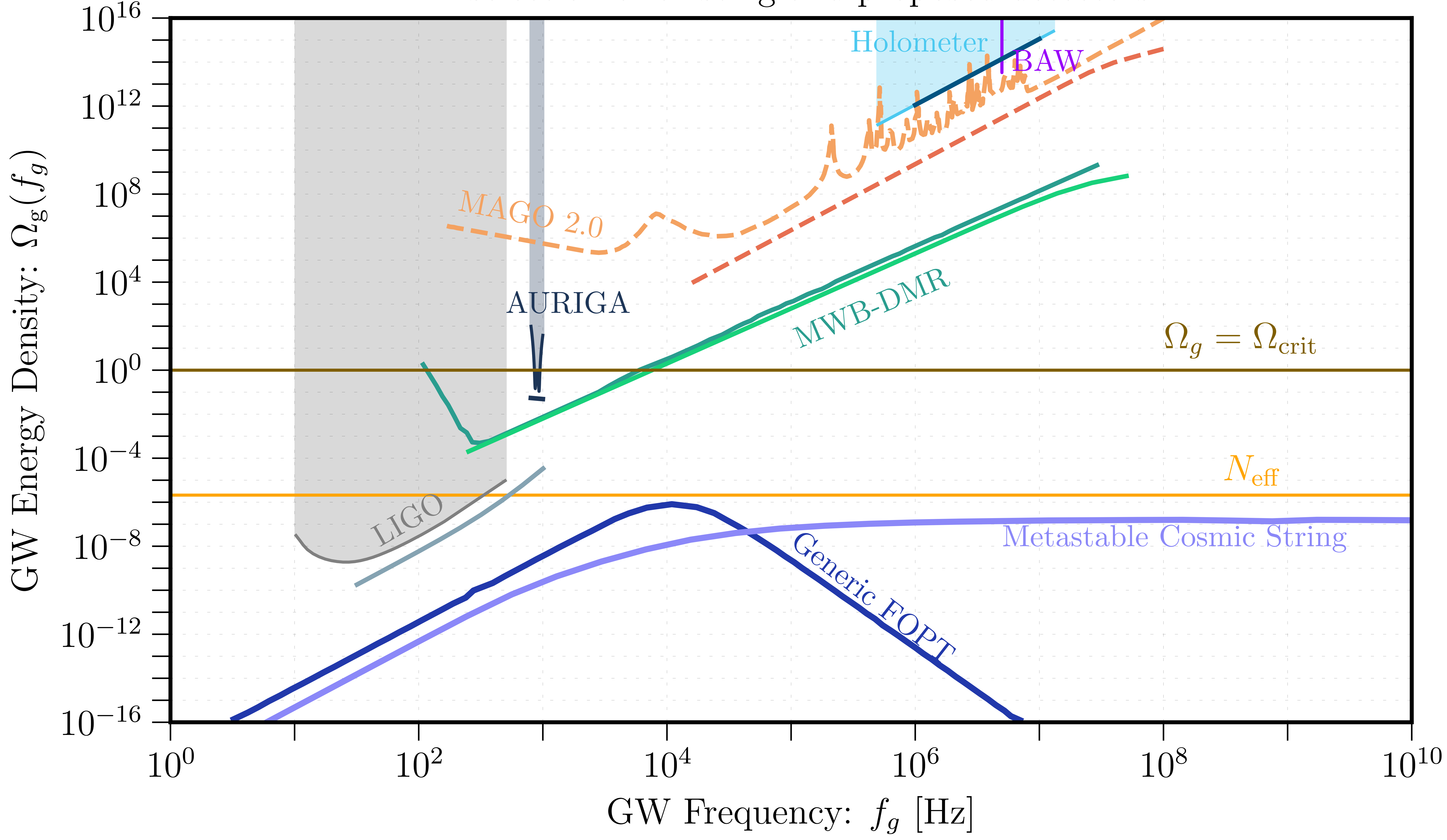


EM DETECTORS

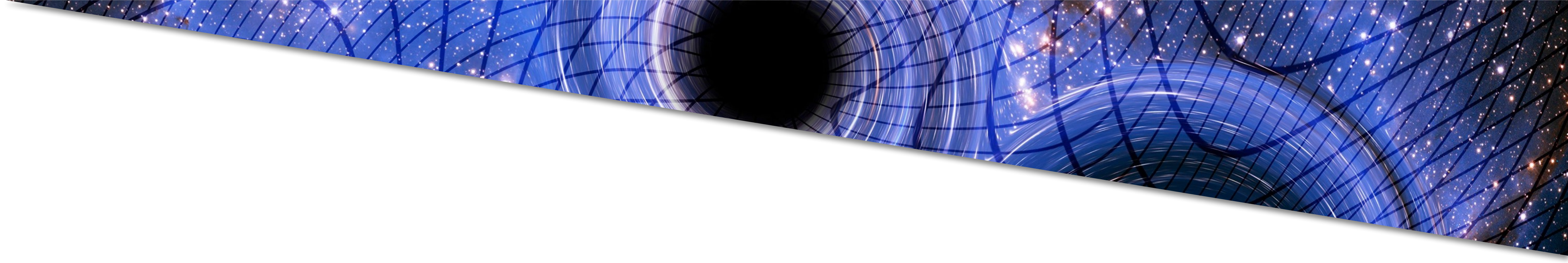
$$\left(\omega_1^2 - \omega^2 + i\frac{\omega\omega_1}{Q}\right)\tilde{E}_1(\omega) \simeq g_e \int d\omega'\,\tilde{E}_0(\omega - \omega')\omega\,\tilde{h}^{\mathrm{TT}}(\omega')$$

$$\mathcal{T}_{\mathrm{EM}}^2(\omega) = \frac{\omega_g^2\omega^2(\omega_g L + \omega_0 L + 1)^2}{\left((\omega_1^2 - \omega^2)^2 + \frac{\omega^2\omega_1^2}{Q^2}\right)}\,\mathrm{min}[1, \omega_g^2 L^2]$$

A selection of existing and proposed detectors

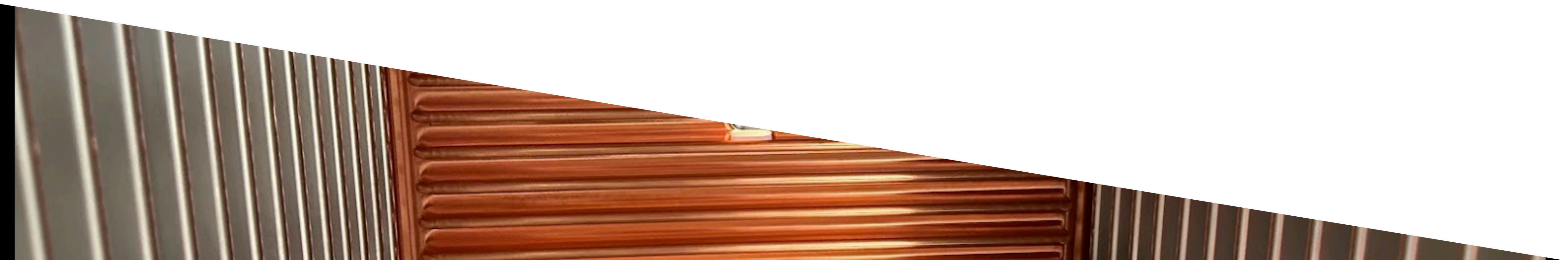


NAIVE COMPARISON

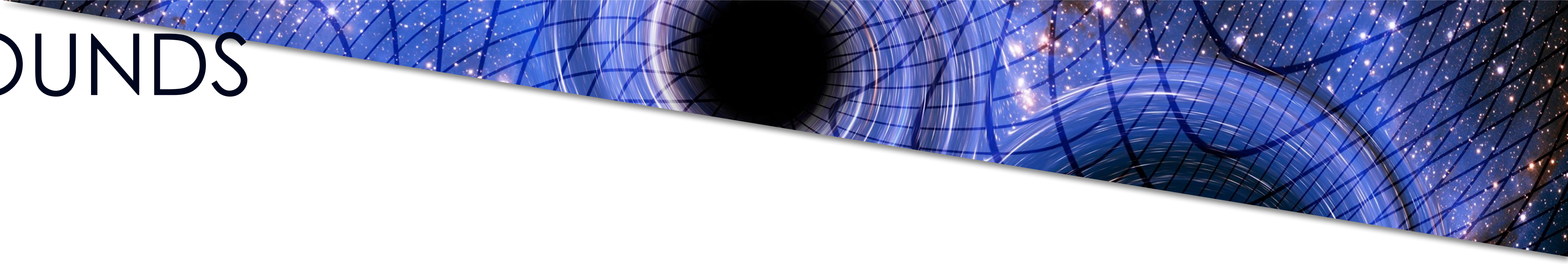


$$\mathcal{T}^{\mathrm{LVK}} \lesssim 10^{10}$$

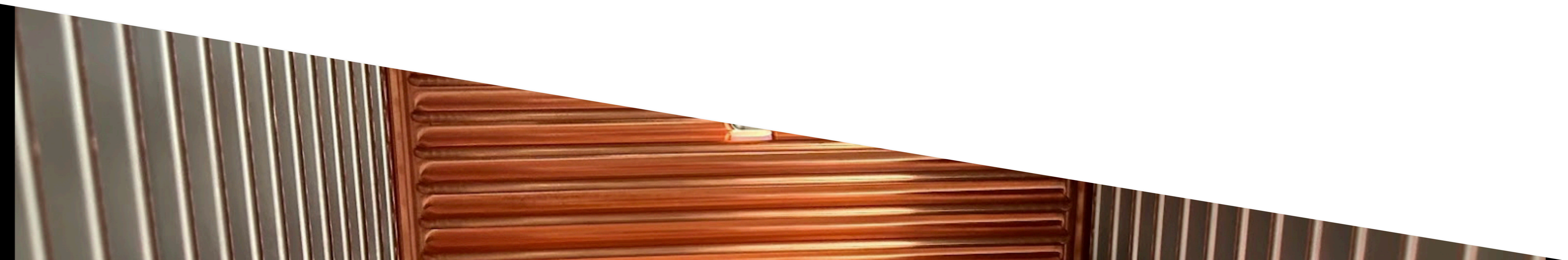
$$\mathcal{T}^{\mathrm{res}} \simeq Q \lesssim 10^{12}$$



COMPARISON ON STOCHASTIC BACKGROUNDS



$$\Omega_g \sim \frac{h^2}{\Delta\omega}$$



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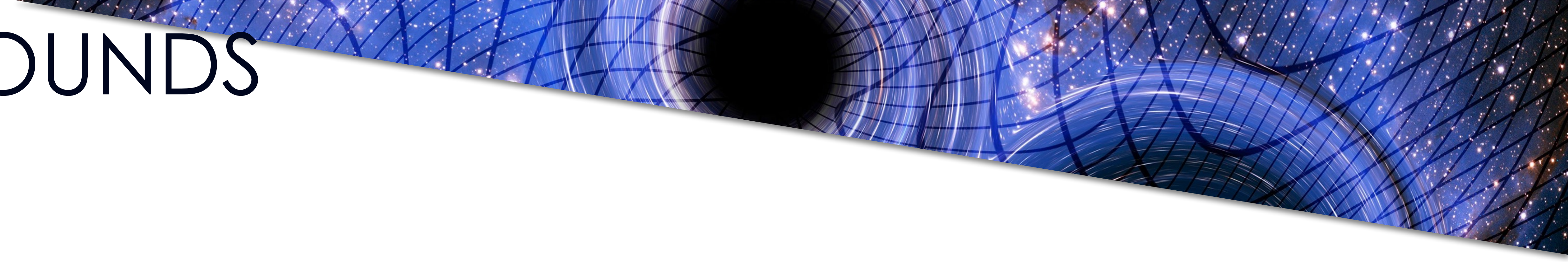
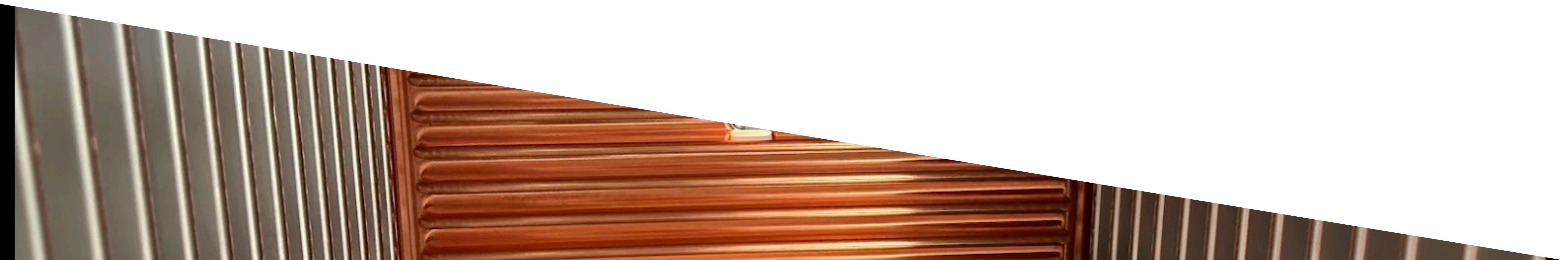


$$\Omega_g \sim \frac{h^2}{\Delta\omega}$$

$$\Omega_g \sim \left(\mathcal{T}^{\mathrm{res}} \sqrt{\frac{\omega}{\Delta\omega}} \simeq \sqrt{Q} \right)^{-2}$$



COMPARISON ON STOCHASTIC BACKGROUNDS

$$\Omega_g \sim \left(\mathcal{T}^{\mathrm{LVK}}\right)^{-2}$$

$$\Omega_g \sim \left(\mathcal{T}^{\mathrm{res}}\sqrt{\frac{\omega}{\Delta\omega} \simeq \sqrt{Q}}\right)^{-2}$$


SIGNALS

$$U_{\text{sig}} \sim U_{\text{in}} \times \begin{cases} (h\mathcal{T})^2 \\ h\mathcal{T} \end{cases}$$

$$U_{\text{in}} \sim E_0^2 V_0$$

CASE I: QUADRATIC SIGNALS

$$\langle E_h(t) E_0(t) \rangle \propto \langle \tilde{E}_h(\omega) \tilde{E}_0(\omega) \rangle = 0$$

$$\langle E_h(t)E_0(t)\rangle \propto \langle \tilde{E}_h(\omega)\tilde{E}_0(\omega)\rangle = 0$$

$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

$\Delta\omega_d$

Detector Bandwidth

$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

In absence of a signal,
the detector is empty (classically)

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the detector is empty (classically)

$$P_{\mathrm{min}} \simeq \frac{2\pi\omega}{t_{\mathrm{int}}}$$

But we have one quantum of noise
from vacuum fluctuations

QUADRATIC SIGNAL

$$P_{\mathrm{sig}} \lesssim \underline{h^2 U_{\mathrm{in}}} \omega_s \mathcal{T}^2(\omega_s)$$

Signal Energy

QUADRATIC SIGNAL

$$P_{\mathrm{sig}} \lesssim h^2 U_{\mathrm{in}} \underline{\omega_s} \mathcal{T}^2(\omega_s)$$

Maximum power
from Poynting's theorem

QUADRATIC SIGNAL

$$\frac{P_{\rm sig}}{P_{\rm noise}} \simeq 1 \quad \longrightarrow \quad h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}}}\,\frac{1}{\tau}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\text{int}}}{3H_0^2}$$

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$$U_{\text{in}}^{\text{BBN}} = 10^{12}\ J \left(\frac{\omega}{2\pi \times 80\text{kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

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$$U_{\text{in}}^{\text{BBN}} \simeq U_{\text{ITER}}$$

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CASE II: LINEAR SIGNALS

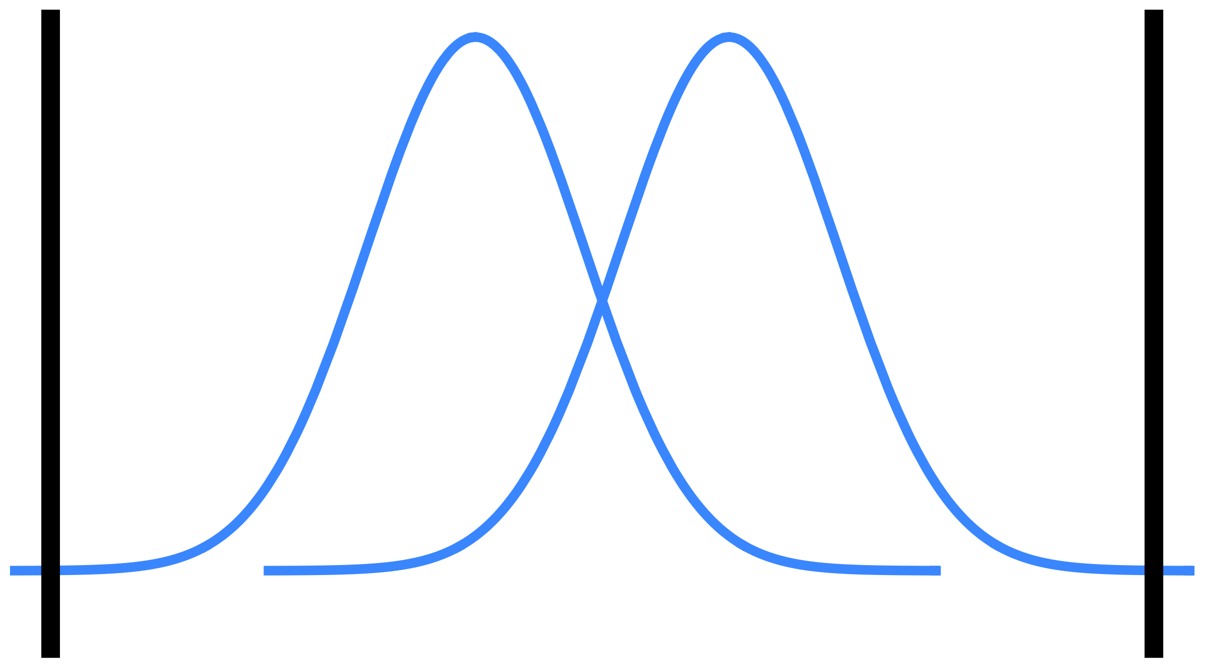
$$\langle E_h(t)E_0(t)\rangle \propto \langle \tilde{E}_h(\omega)\tilde{E}_0\omega)\rangle \neq 0$$

$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

CASE II: LINEAR SIGNALS

$\Delta\omega_d$

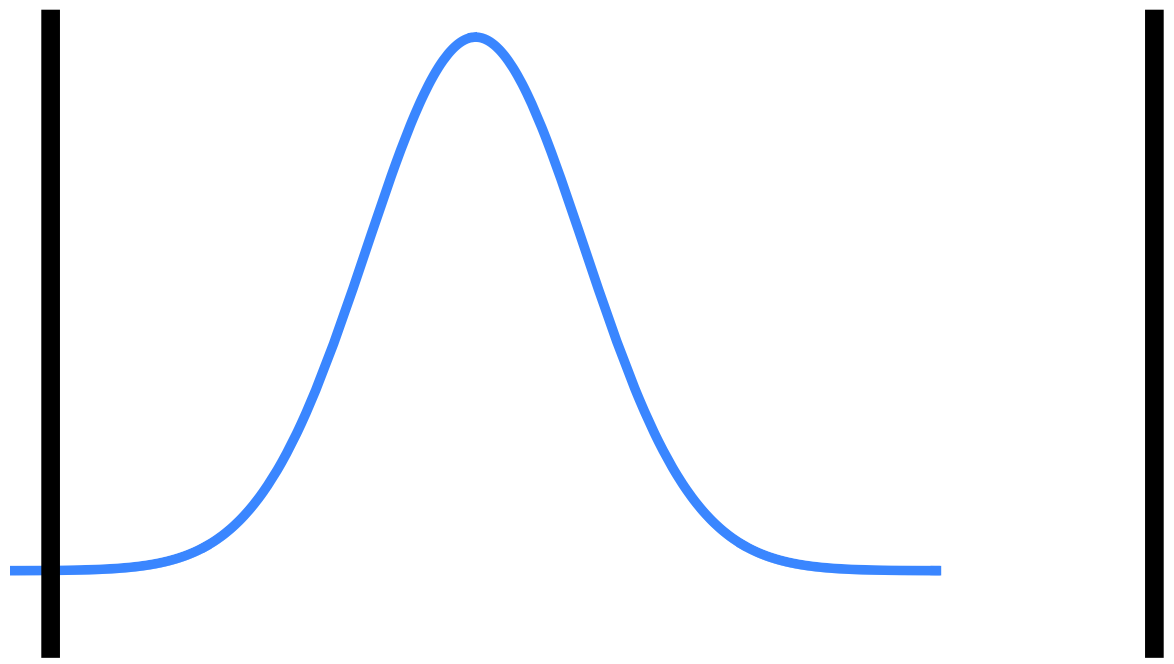
Detector Bandwidth



$\tilde{E}_0(\omega)$ $\tilde{E}_h(\omega)$

CASE II: LINEAR SIGNALS

In absence of a signal,
the detector is not empty



$$P^{\min}_{\text{noise}} \simeq \frac{2\pi\omega}{t_{\text{int}}}\left(1+\sqrt{\frac{P_{\text{in}}t_{\text{int}}}{2\pi\omega}}\right)$$

NARROW LINEAR SIGNAL

$$\frac{P_{\rm sig}}{P_{\rm noise}} \simeq 1 \quad \longrightarrow \quad h_{\rm min} \gtrsim \sqrt{\frac{2\pi}{U_{\rm in} t_{\rm int}}}\,\frac{1}{\mathcal{T}}$$

Same as quadratic!

STATISTICS INTERLUDE

Stationary Gaussian Process with Zero Mean

$$\langle h(t) \rangle = 0$$

$$\langle h(t) h(t') \rangle = H(t' - t)$$

POWER SPECTRAL DENSITY

$$P_{\text{sig,noise}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, S_{\text{sig,noise}}(\omega)$$

SNR

In practice we are comparing two Gaussians with zero mean and different variance

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int \frac{d\omega}{2\pi} \frac{S_{\mathrm{sig}}^2(\omega)}{S_{\mathrm{noise}}^2(\omega)} \right)^{1/2} \simeq 1$$

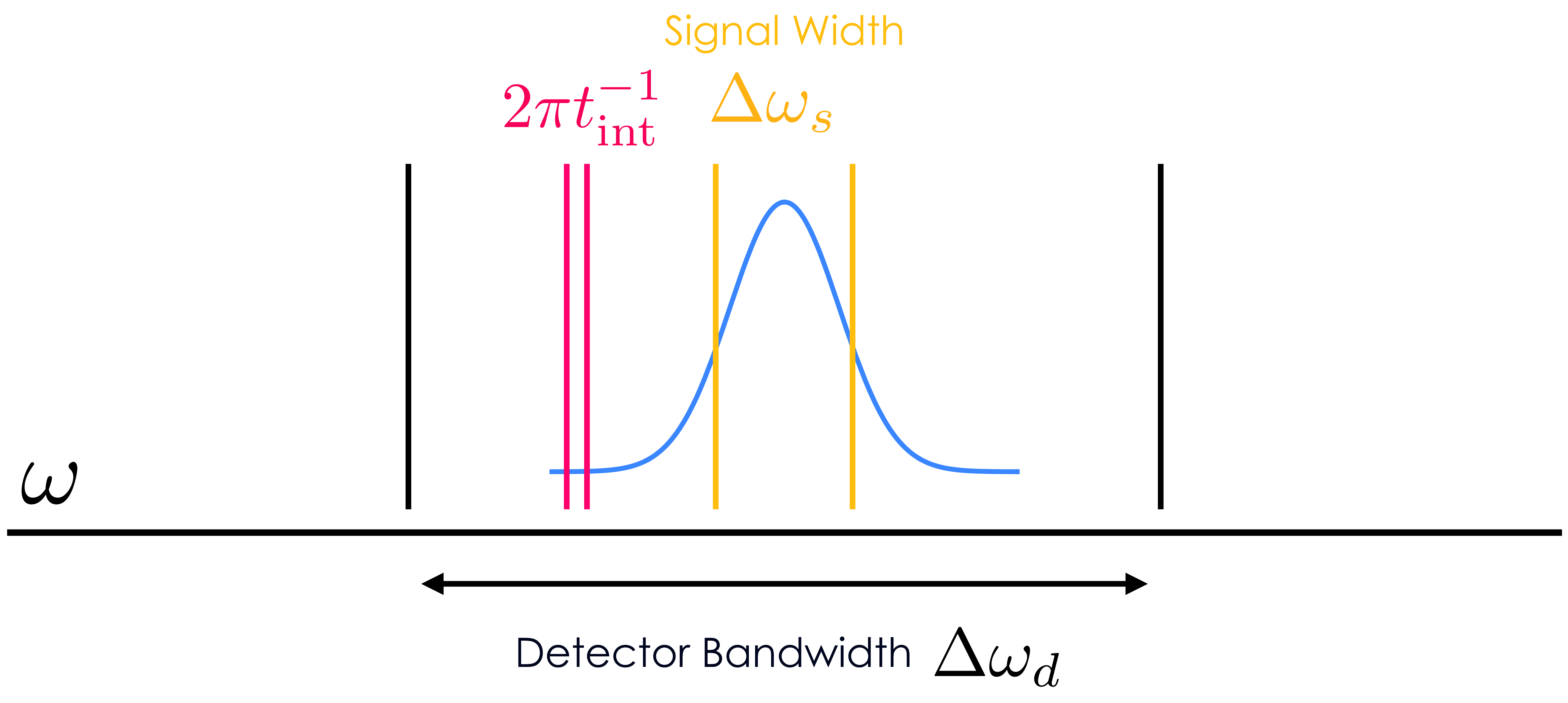
From maximum likelihood

SNR

$$h_{\min} \gtrsim \sqrt{\frac{1}{U_{\text{in}}}} \left(\frac{2\pi\Delta\omega}{t_{\text{int}}}\right)^{1/4} \frac{1}{\mathcal{T}(\omega)}$$

$$\Delta\omega \equiv \max[\min[\Delta\omega_d, \Delta\omega_s], (2\pi)\, t_{\text{int}}^{-1}]$$

SNR



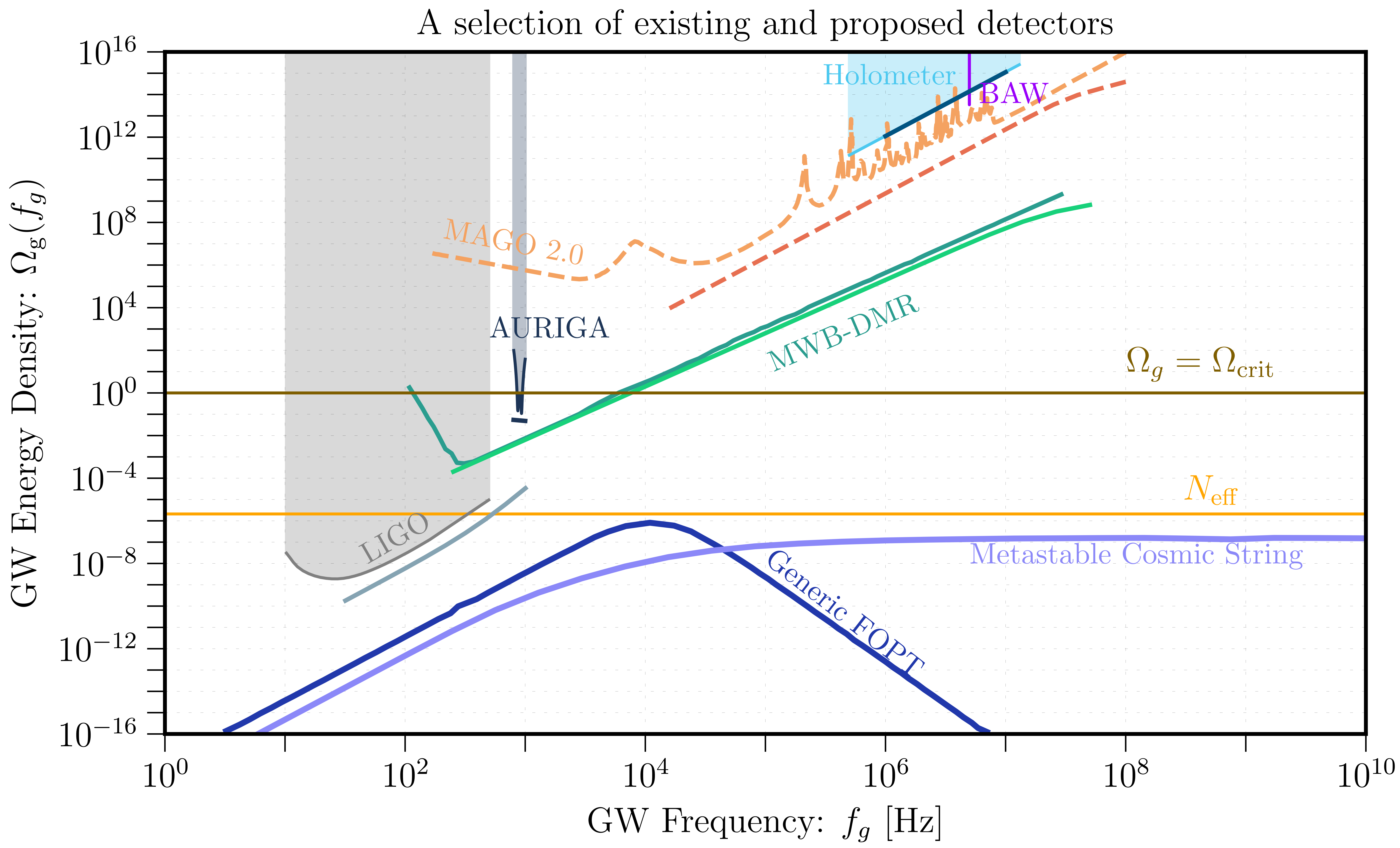
$$\Delta\omega \equiv \max[\min[\Delta\omega_d, \Delta\omega_s], (2\pi)\, t_{\text{int}}^{-1}]$$

SNR

Exception

$$\mathrm{SNR} = \left(t_{\mathrm{int}} \int \frac{d\omega}{2\pi} \frac{S_{\mathrm{sig}}(\omega)}{S_{\mathrm{noise}}(\omega)} \right)^{1/2} \simeq 1$$

Photon counting [2211.04016, 2404.07524]



A selection of existing and proposed detectors
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GW Frequency: f_g [Hz]
Holometer
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MAGO 2.0
AURIGA
MWB-DMR
$\Omega_g = \Omega_{crit}$
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Metastable Cosmic String
Generic FOPT

WHAT DID WE LEARN?

QUADRATIC vs LINEAR

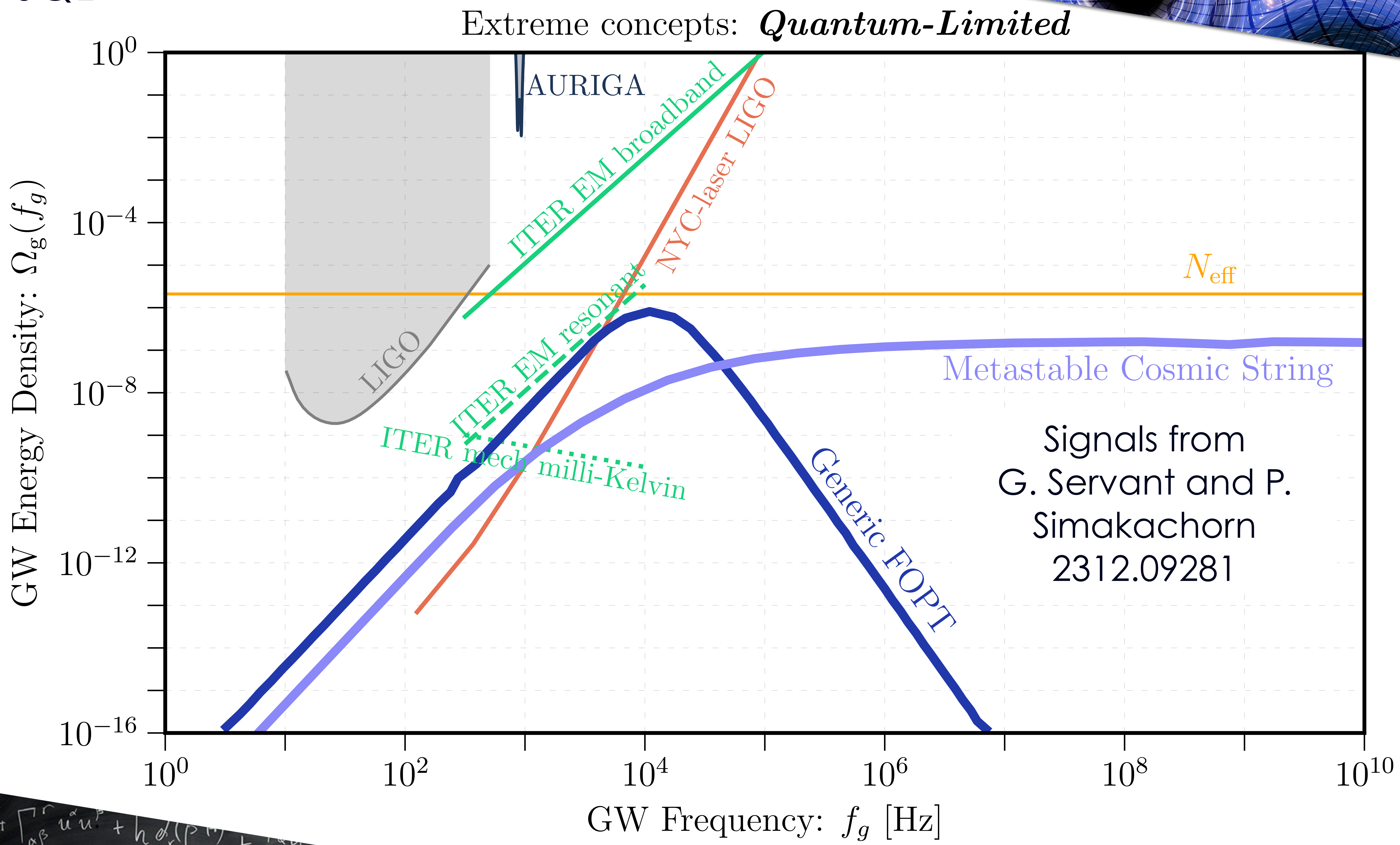
$$h_{\mathrm{min}} \gtrsim \sqrt{\frac{2\pi}{U_{\mathrm{in}} t_{\mathrm{int}}}}\,\frac{1}{\mathcal{T}}$$

ENERGY DENSITY

$$\Omega_g^{\min}(\omega) \simeq \frac{\omega^3 h_{\min}^2 t_{\rm int}}{3H_0^2}$$

$$U_{\rm in}^{\rm BBN} = 10^{12}\ J \left(\frac{\omega}{2\pi \times 80{\rm kHz}}\right)\left(\frac{10^{-6}}{\Omega_g}\right)\left(\frac{10^{12}}{\mathcal{T}^2 \frac{\omega}{\Delta\omega}}\right)$$

CRAZY SQL



Signals from G. Servant and P. Simakachorn 2312.09281

QUADRATIC vs LINEAR

QUADRATIC

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$

LINEAR

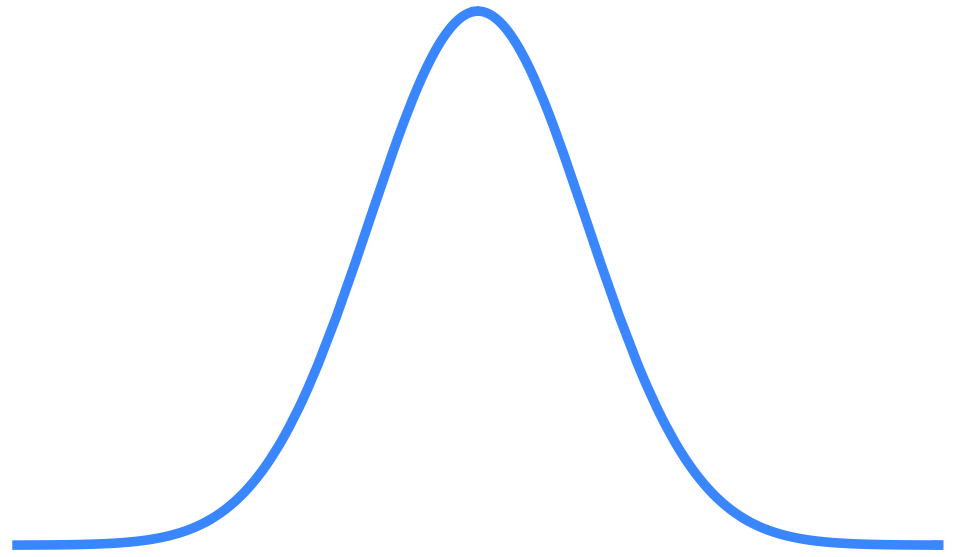
$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{\frac{P_{\text{in}} t_{\text{int}}}{2\pi\omega}}\right)$$

We can gain a lot
from
quantum techniques

BEYOND THE QUANTUM LIMIT

NO QUALITATIVE CHANGE FOR QUADRATIC SIGNALS

$$P_{\text{SQL}} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$



$$P_{\min} \simeq \frac{2\pi\omega}{t_{\text{int}}}$$

LINEAR

$$P_{\text{noise}}^{\text{min}} \simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{\frac{P_{\text{in}} t_{\text{int}}}{2\pi\omega}} \right)$$

$$\simeq \frac{2\pi\omega}{t_{\text{int}}} \left(1 + \sqrt{N_\gamma} \right)$$

LINEAR

In principle

$$P_{\mathrm{noise}}^{\mathrm{min}} \to \frac{2\pi\omega}{t_{\mathrm{int}}}$$

Heisenberg Limit

In practice

$$P_{\text{noise}}^{\text{min}} \to \frac{2\pi\omega}{t_{\text{int}}} \quad \longrightarrow \quad (\sqrt{N_\gamma} + 1) \to 0$$

You need to control an Avogadro's number of photons
at the single photon level

SCANNING

Heisenberg limited sensors
+
Classical computer

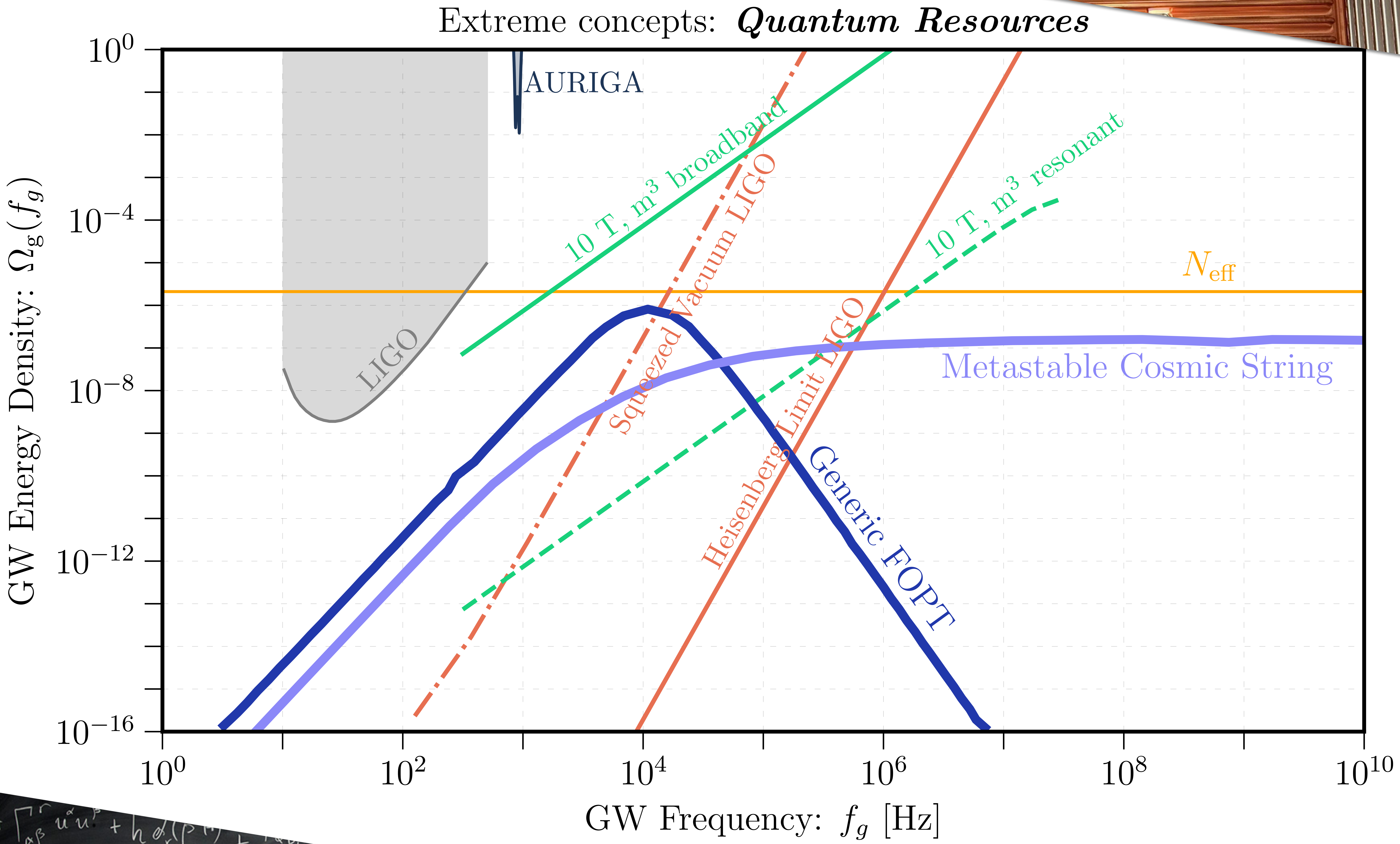
$$t_{\text{int}} = \mathcal{O}\left(\frac{1}{U_{\text{in}} h_{\text{min}}} \left\lceil \frac{|\Delta\omega|}{U_{\text{in}} h_{\text{min}}} \right\rceil\right)$$

Heisenberg limited sensors
+
Grover algorithm

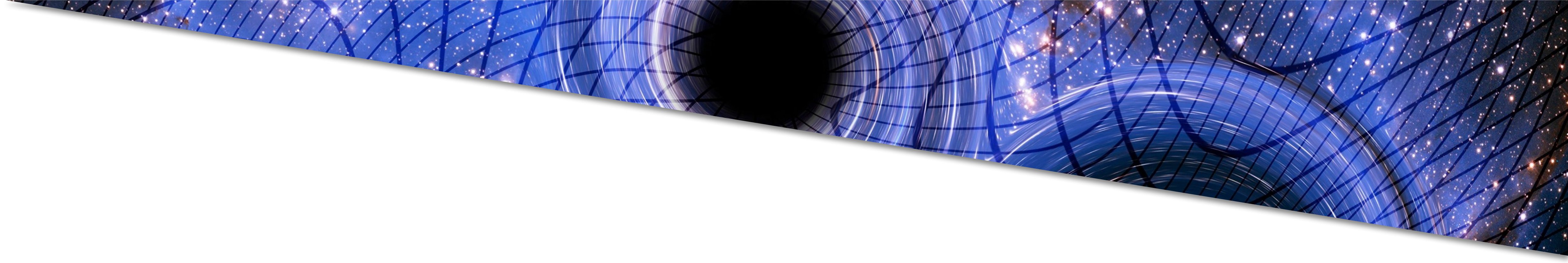
$$t_{\text{int}} = \mathcal{O}\left(\frac{1}{U_{\text{in}} h_{\text{min}}} \sqrt{\left\lceil \frac{|\Delta\omega|}{U_{\text{in}} h_{\text{min}}} \right\rceil}\right)$$

[2501.07625]

CRAZY BEYOND SQL



CONCLUSION



$$\Omega_g(\omega_g) \sim \omega_g^3 h^2$$